Title: Discussion:  Is there novel physics at the black hole horizon?

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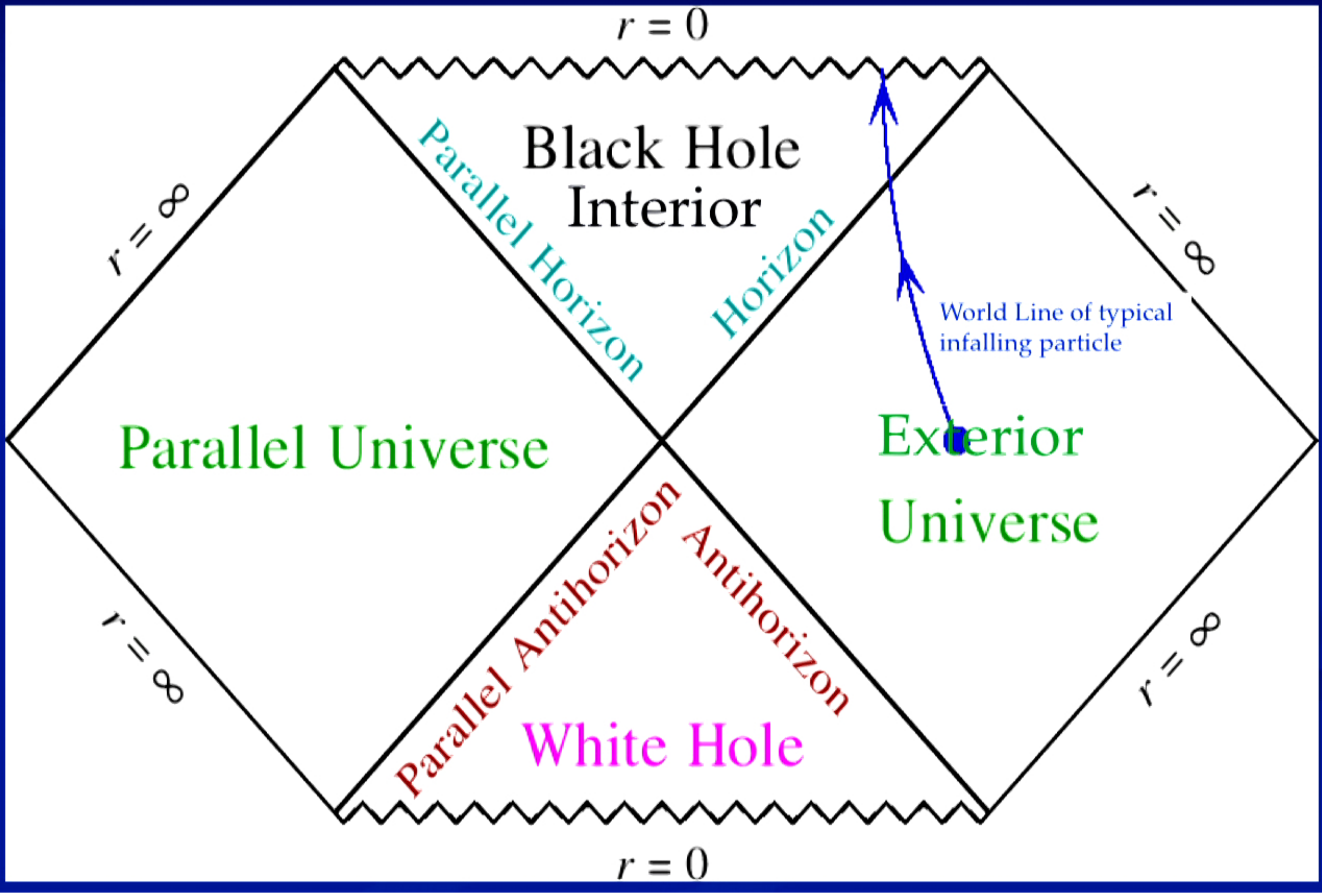
# Mathematical Black Holes

## Beyond the Horizon: Analytic Extension

## Carter-Penrose Conformal Diagram of Schwarzschild Soln.



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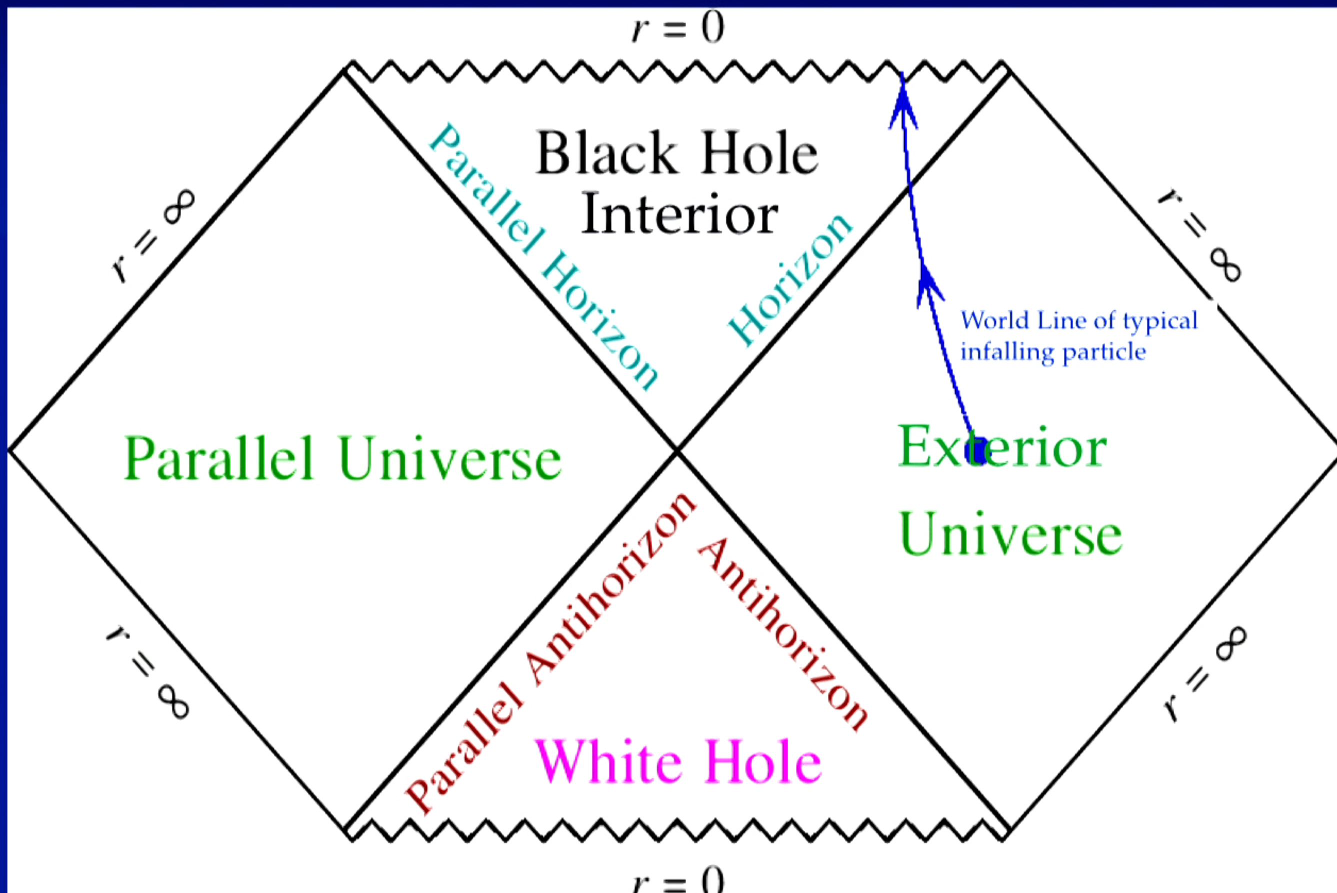
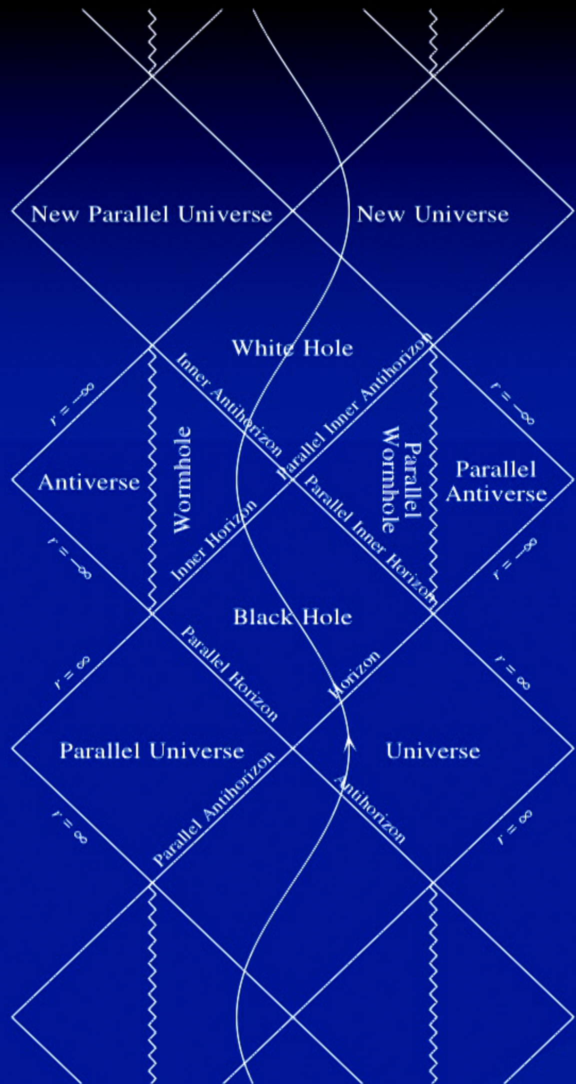


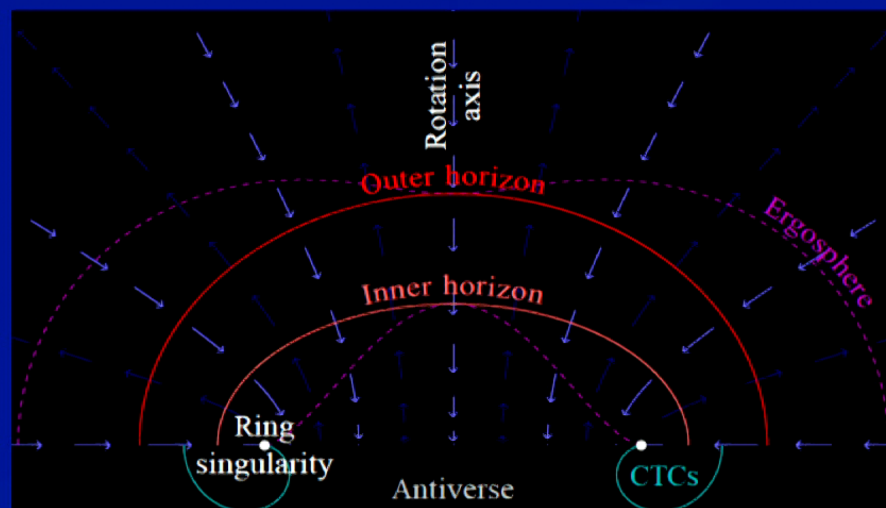



# Mathematical Kerr Black Hole Interiors

- More singularities
- More parallel universes
- Closed Timelike Curves:

(say hello to your greatgrandparents)

More unphysical



# Black 'Holes'… or Not
## Singularity Theorems

Black Holes 'inevitable' in Gen. Rel. if

- Trapped Surface forms
- Energy Conditions hold

- **Weak** Energy Condition (Penrose 1965)

$$\rho + p_i \geq 0 \qquad i = 1, 2, 3$$

Violated by Quantum Fields, e.g. by Casimir Effect

- **Strong** Energy Condition (Hawking-Penrose 1970)

$$\rho + \sum_{i=1}^{3} p_i \geq 0$$

Violated by Hadronic 'Bag', Cosmological Dark Energy,

Inflation V(φ): $p_i = -\rho < 0$

Negative Pressure → Defocusing → Effective **Repulsion**

# Black Holes and Entropy

- A fixed classical solution usually has **no entropy** :
  (What is the "entropy" of the Coulomb potential $\Phi = Q/r$ ?)
  … But if matter/radiation disappear into a black hole,
  what happens to its entropy? (Only M, J, Q remain)
- Maybe **Area** (which always increases **classically**) is a kind of "entropy"? To get entropy units need to divide Area, A by $(\text{length})^2$
  … But there is no fixed length scale in classical Gen. Rel.
- Planck length $\ell_{Pl}^2 = \hbar G/c^3$ involves $\hbar$
- Bekenstein proposed: $S_{BH} = \gamma\, k_B\, A/\ell_{Pl}^2$ with $\gamma \sim O(1)$
- Hawking (1974) argued black holes emit thermal radiation at

$$T_H = \frac{\hbar c}{8\pi G k_B M}$$

**Form of Surface Tension**

The **classical** relation $dE = \kappa\, dA/8\pi G$

then interpreted as Thermodynamics $dE = T_H\, dS_{BH}$ fixes $\gamma = 1/4$
***by a Quantum Effect but …***

# A few problems remained …

- **Multiply & Divide by $\hbar$ ?**
- $S_{BH} \propto A$ is *non-extensive* and ***HUGE*** (Factor of ~$10^{19}$)
- In the classical limit $T_H \to 0$ (cold) but $S_{BH} \to \infty$ ?
- $E \propto T^{-1}$ implies negative heat capacity

$$\frac{dE}{dT} << 0 \qquad \Rightarrow \text{highly unstable}$$

Equilibrium Thermodynamics applicable at all ?

- Calculations are done in **Fixed Classical** Background
- **Information Paradox**: Where does the information go?

  (Pure states → Mixed States? Unitarity ?)
- What is the statistical interpretation of $S_{BH}$ ?

  Boltzmann asks: $\mathbf{S = k_B \ln W}$ ??

# Horizon in Quantum Theory

- <u>Infinite</u> Blueshift Surface

$$\omega_{local} = \omega_\infty (1 - 2GM/r)^{-1/2}$$

No problem classically, but in quantum theory

$$E = \hbar\omega = \hbar\omega_\infty (1 - 2GM/r)^{-1/2} \to \infty$$

$\hbar \to 0$ and $r \to 2GM$ limits do not commute

Quantum Vacuum is very different at horizon from $r \to \infty$

- Vacuum Polarization can produce large stress-energy

$$\langle T_a^{\ b} \rangle \sim \hbar\omega_{local}^{\ 4} \sim \hbar M^{-4} (1 - 2GM/r)^{-2}$$

The geometry need not remain unchanged down to $r = 2GM$

Scale at which Quantum Vacuum Polarization is O(1) is

$$\Delta r = r - 2GM \sim \ell_{Pl} \qquad \Delta L_{phys} \sim \sqrt{GM\,\ell_{Pl}} \sim 0.25\,\text{fm}\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}$$

## Quantum Effects Are Non-Local

- Isn't the horizon 'just a coordinate artifact,' that can be transformed away ( 'symmetry transformation'?
- But the coordinate transformation is itself **singular**
- ***Singular*** coordinate/gauge transformations need not be harmless and can contain new physics

E.g. from U(1) QED: Vortices in Superfluids/BECs

$$\oint_{\mathbb{S}^1} A_\lambda dx^\lambda = \oint A \overset{A=d\theta}{=} \int d\theta = 2\pi \quad \text{is gauge invariant}$$

Aharonov-Bohm Effect also "Pure Gauge" = no Local $F_{\mu\nu}$ Field

Quantum Effects are Non-Local

(Not Acausal e.g. Entanglement)

***DO NOT REQUIRE LARGE CURVATURES***

## Resolution Already Inherent in Purely Classical Gen. Relativity

Assuming classical Einstein eqs. &

- Static Killing time: $K^\mu \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial t}$
- Spherical Symmetry:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{h(r)} + r^2\,d\Omega^2$$

- Isotropic Pressure: $p_i = p(r)$
- Positive Monotonically Decreasing Density: $\frac{d\rho}{dr} \leq 0$
- Metric Continuity at Surface of Star r=R
- Then $R > \frac{9}{8} R_s = \frac{9}{4} GM$ **Buchdahl Bound**

or the pressure must diverge in the Interior

Note this ***R*** is outside horizon

# Buchdahl Bound & Schwarzschild Interior

- Importance of Buchdahl Bound is:

  Under **Adiabatic Compression**

  **Pressure grows unbounded Inside**

  -- Before the Event Horizon is Reached
- Holds for **Any** isotropic equation of state
- Bound is Saturated by Schwarzschild Interior Soln.

  **(Second paper 1916**

  **Back to the Beginning of GR)**
- Constant Density

  $p_\perp = p$ $\quad \frac{d\rho}{dr} = 0 \quad$ $\rho(r) = \bar{\rho} \equiv \frac{3M}{4\pi R^3}$
- Solve for Pressure *p(r)* , Metric Functions $f(r), h(r)$

# Schwarzschild Interior Solution

- Constant Density

$$\rho' = 0$$

Saturates

Buchdahl Bound

$$h(r) = 1 - H^2 r^2$$

$$H^2 = \frac{8\pi G}{3}\bar{\rho} = \frac{2GM}{R^3}$$

- Pressure $\quad p(r) = \bar{\rho}\left[\frac{\sqrt{1 - H^2 r^2} - \sqrt{1 - H^2 R^2}}{3\sqrt{1 - H^2 R^2} - \sqrt{1 - H^2 r^2}}\right]$

- Diverges at $\boxed{R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}}$ iff $\boxed{R < \frac{9}{8}R_s = \frac{9}{4}GM}$

- Pressure becomes negative for $0 < r < R_0$

- $f(r) = \frac{1}{4}\left[3\sqrt{1 - H^2 R^2} - \sqrt{1 - H^2 r^2}\right]^2$ **Vanishes at same $R_0$** **Never Negative**

- If $\boxed{R = R_S = R_0\,, \quad H^2 R^2 = 1 \Longrightarrow p = -\bar{\rho}}$ **= constant (de Sitter)**

# Komar Mass-Energy

$$\frac{1}{G}\frac{d}{dr}\left(r^2\kappa\right) = 4\pi\sqrt{\frac{f}{h}}\; r^2\left(\rho + p + 2p_\perp\right)$$

$$\kappa(r) = \frac{1}{2}\sqrt{\frac{h}{f}}\;\frac{df}{dr}$$

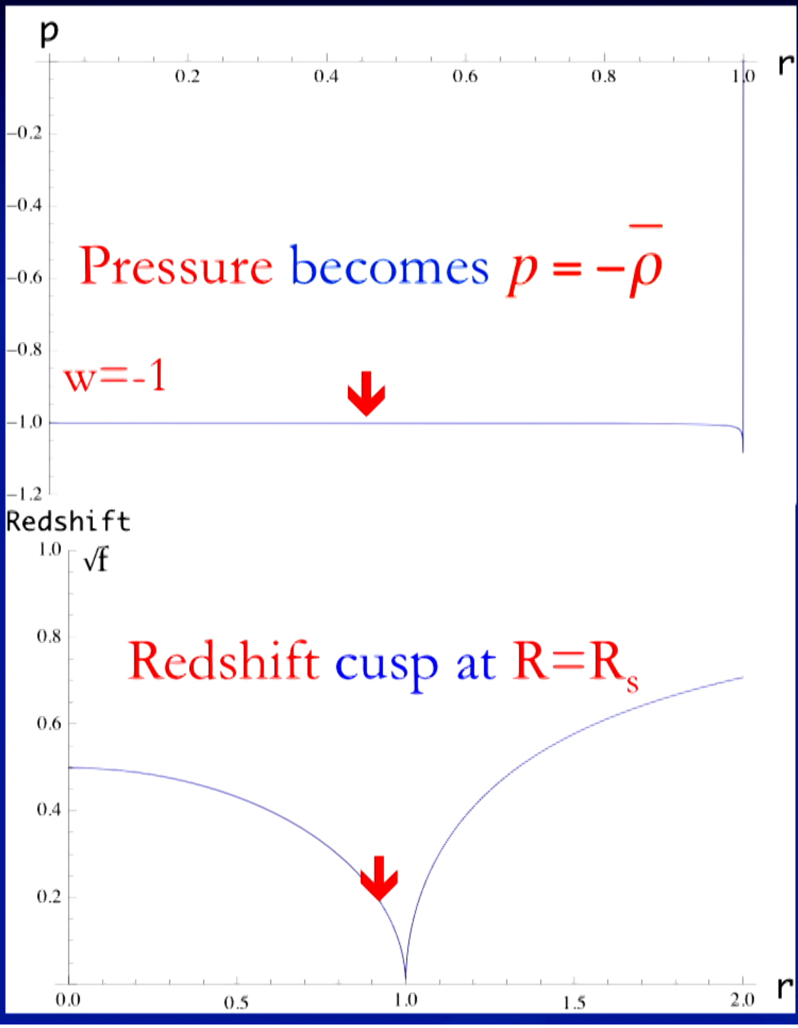
**Surface Gravity**

**Total Mass: 'Gauss' Law' for Static Solns.**

$$M = 4\pi\int_0^{R_s} dr\,\sqrt{\frac{f}{h}}\; r^2\left(\rho + p + 2p_\perp\right)$$

is finite integrable across discontinuity

# $R=R_s$ Limit is Grav. Condensate Star (2001)




No divergence in $p$

$$p = -\bar{\rho}$$

$$h(r) = 1 - H^2 r^2$$

$$f(r) = \tfrac{1}{4}\, h(r)$$

$$H = 1/R_s$$

but non-analytic cusp
Discontinuity (classically)
Interior is de Sitter space in (modified) static coordinates
(Time runs slower inside)

# Surface Tension

[Class.Quant.Grav. 32 (2015) 215024]

## Discontinuity in Surface Gravities

$$\kappa_\pm \equiv \lim_{r \to R_0^\pm} \kappa(r) = \pm \frac{4\pi G}{3} \bar{\rho}\, R_0$$

$$\Delta\kappa \equiv \kappa_+ - \kappa_- = \frac{R_s R_0}{R^3} \to \frac{1}{R_s}$$

**is surface tension**

$$\boxed{\tau_s = \frac{E_s}{2A} = \frac{\Delta\kappa}{8\pi G}} \to \frac{1}{8\pi G R_s} > 0$$

Interior is not analytic continuation of exterior

# First Law

**Classical Mechanical Conservation of Energy**

$$dM = dE_v + \tau_s \, dA$$

Schw. Interior Soln. in $R \rightarrow R_s$ Limit describes a **Zero Entropy/Zero Temperature Condensate**

(not a 'firewall')

Gibbs Relation $p + \rho = s\,T + \mu\,N = 0$

Discontinuity in κ implies non-analytic behavior

No Trapped Surface, Truly Static, **t** is a Global Time

**Surface Area is Surface Area not Entropy**

**Surface Gravity is Surface Tension not Temperature**

# Surface Oscillations

$$dM = dE_v + \tau_s \, dA$$

- Energy Minimized by minimizing A for fixed Volume
- Surface Tension acts as a restoring force
- Surface Oscillations are **Stable** $\tau_s > 0$
- Surface Normal Modes are **Discrete**
- Characteristic Frequency

$$\omega \sim \frac{c}{4\pi R_s} = 8.1 \left(\frac{M_\odot}{M}\right) \mathrm{kHz}$$

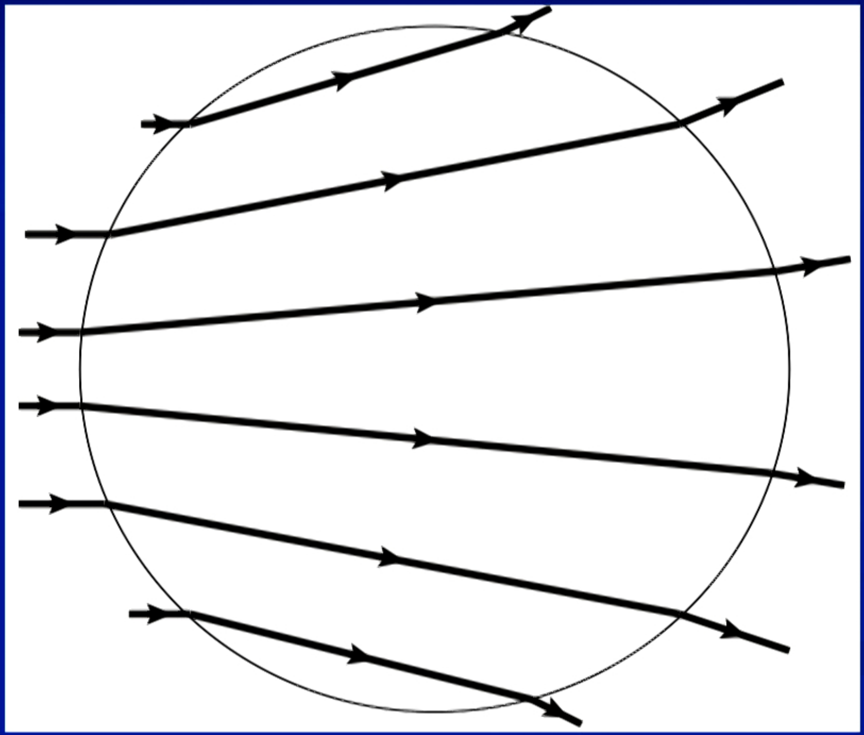
- **Discrete GW Spectrum--Different Ringdown**
- Signatures for LIGO/VIRGO for $M \simeq 10 \leftrightarrow 100 \; M_\odot$

  (but needs detailed modeling)

# Defocusing of Null Rays

No Horizon ➔ Light Rays Penetrate Interior

Geometric Optics Limit Assuming Transparency



Surface **Very Dark** due to Very Deep **Redshift** (< <10%)
Do not expect optical/IR emission

**Different Imaging Possible from a Black Hole**

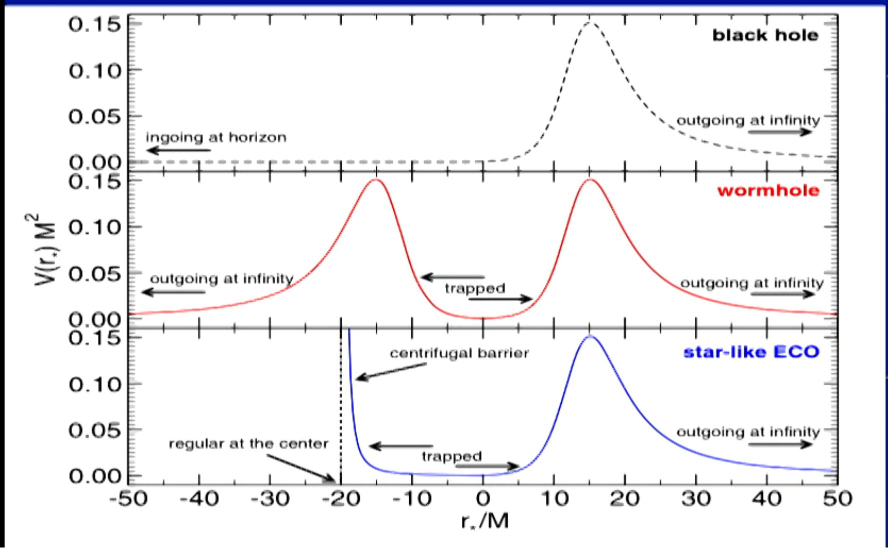
# Gravastar Interior Echoes

- Transmission (with log time delay) through surface
- Regge-Wheeler radial coordinate
- Scalar Wave or Grav. Wave Eq.

$$dr^* = \frac{dr}{\sqrt{fh}}$$

$$\left(-\frac{d^2}{dr^{*\,2}} + V_\ell\right)\phi_{\omega\ell} = \omega^2\phi_{\omega\ell}$$

- Reflection from de Sitter interior barrier: 'Echo'




$$V_\ell = \frac{1}{4}(1 - H^2r^2)\left(\frac{\ell(\ell+1)}{r^2} - 2H^2\right)$$

$$r < 2M = H^{-1}$$

$$V_\ell = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right)$$

$$r > 2M$$

# Summary

- Buchdahl Bound → Interior Pressure Divergence Develops (inside out) **before** Event Horizon forms $R > \frac{9}{8} R_s = \frac{9}{4} GM$
- Pressure is **Integrable-Sensible Boundary Layer (Shock)**
- Implies Formation of a Surface, $p_\perp$ **Surface Tension** & a Non-Singular (de Sitter $p = -\rho$) Interior
- Vacuum Energy must change (Quantum Phase Transition)
  - Note: QCD already provides $p = -\rho$ Vacuum Energy in Bag Constant (Gluon Condensate, possibly density dependent in NSs)
- **Gravitational Condensate Star negative pressure** already realized/inherent in Classical Gen. Rel. (1916)
- **Cold** Quantum **Final State** of Gravitational Collapse
- Very Thin Surface Layer very close to would-be horizon is consistent with EHT observation of M87

# Summary

- Surface Tension is Surface Tension not Temperature
- QM, Unitarity ✓ **No 'Information Paradox'**
- No large number of microstates: One Vacuum State
- Both Echoes and Discrete Surface Modes
- Has been extended to slow rotation C. Posada-Aguirre, MNRAS 468 (2017) 2128
- Full Soln. & Dynamical Evolution Requires Quantum Effective Theory of the Conformal Anomaly
- Dynamical Vacuum Condensate Energy
- Regulated Finite Thickness Boundary layer

$$\Delta L_{phys} \sim \sqrt{GM\,\ell_{Pl}} \sim 0.25\,\mathrm{fm}\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}$$

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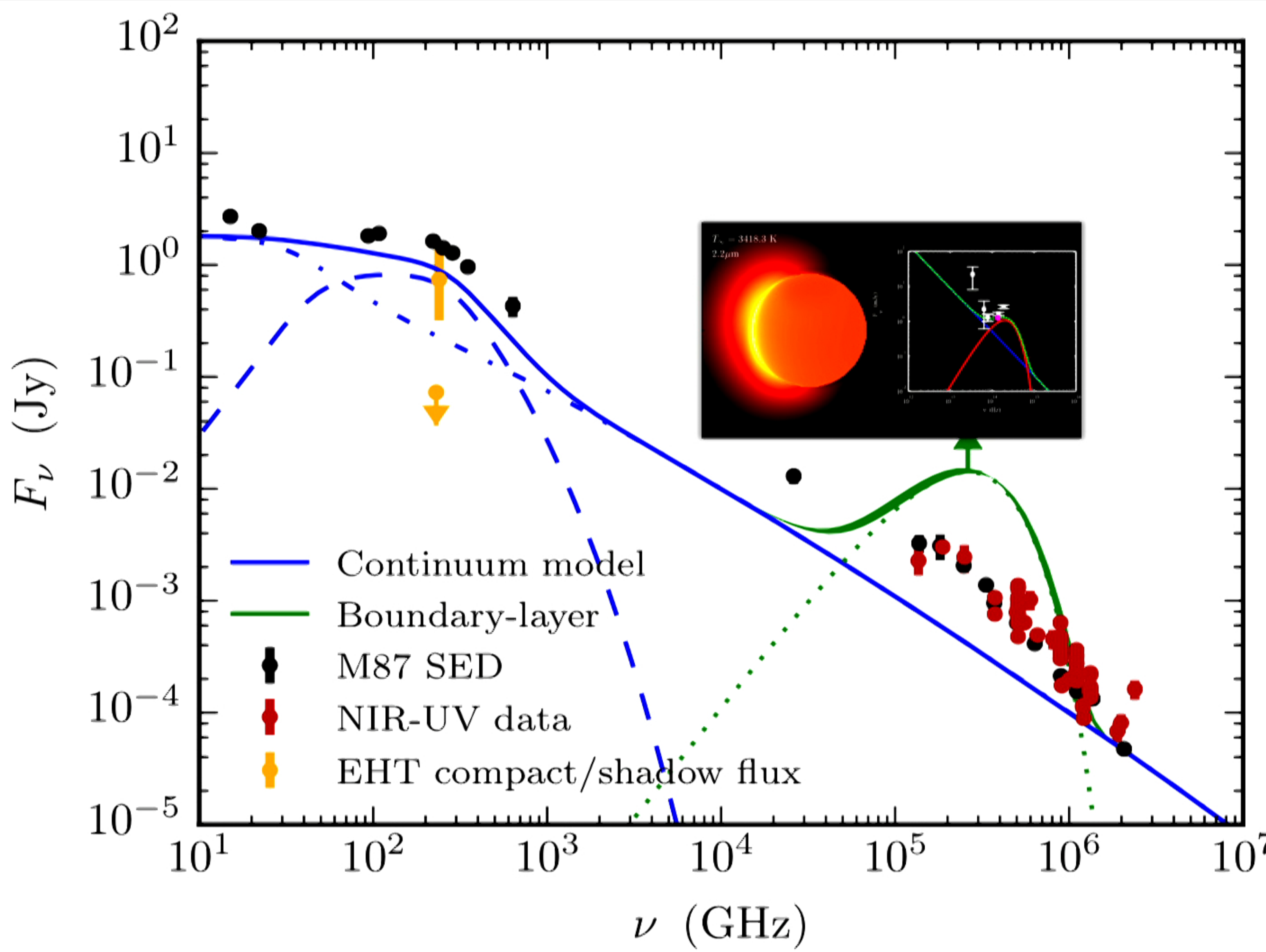
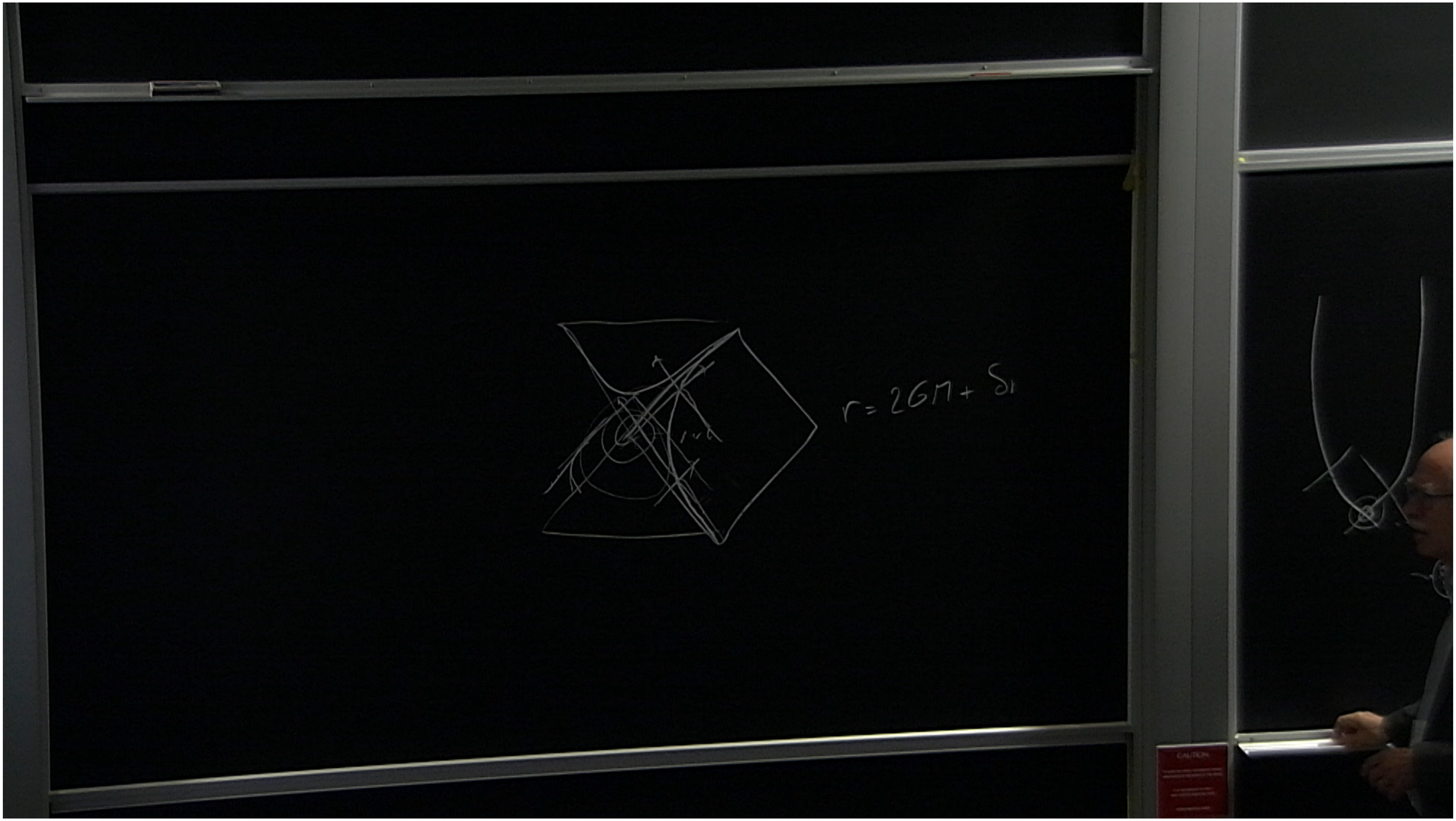


$10^2$
$10^1$
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$F_\nu$ (Jy)
$10^1$
$10^2$
$10^3$
$10^4$
$10^5$
$10^6$
$10^7$
$\nu$ (GHz)
Continuum model
Boundary-layer
M87 SED
NIR-UV data
EHT compact/shadow flux


r = 2GM + δr