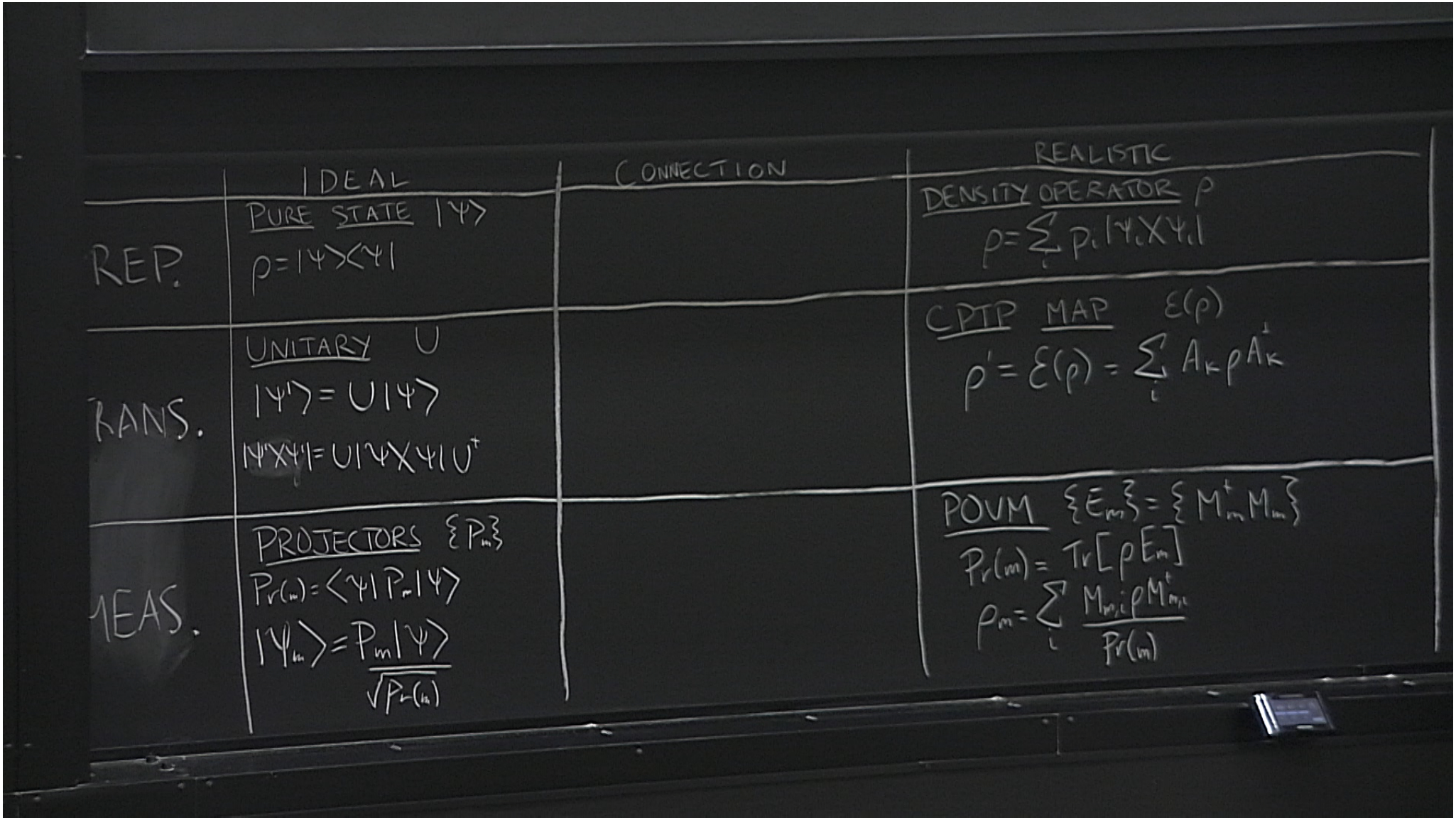
Title: PSI 2019/2020 - Quantum Theory (Branczyk) - Lecture 7

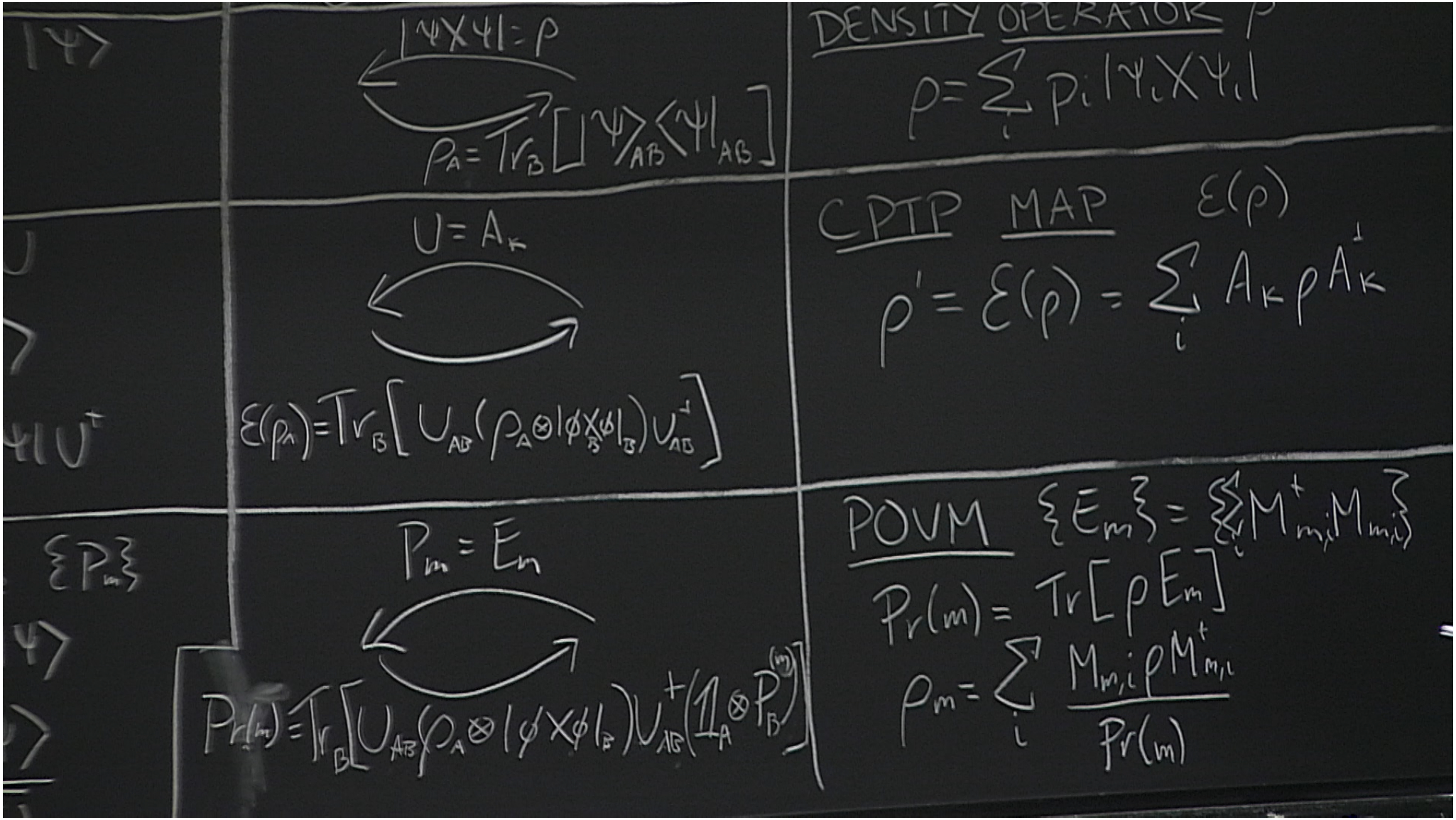
Speakers: Agata Branczyk

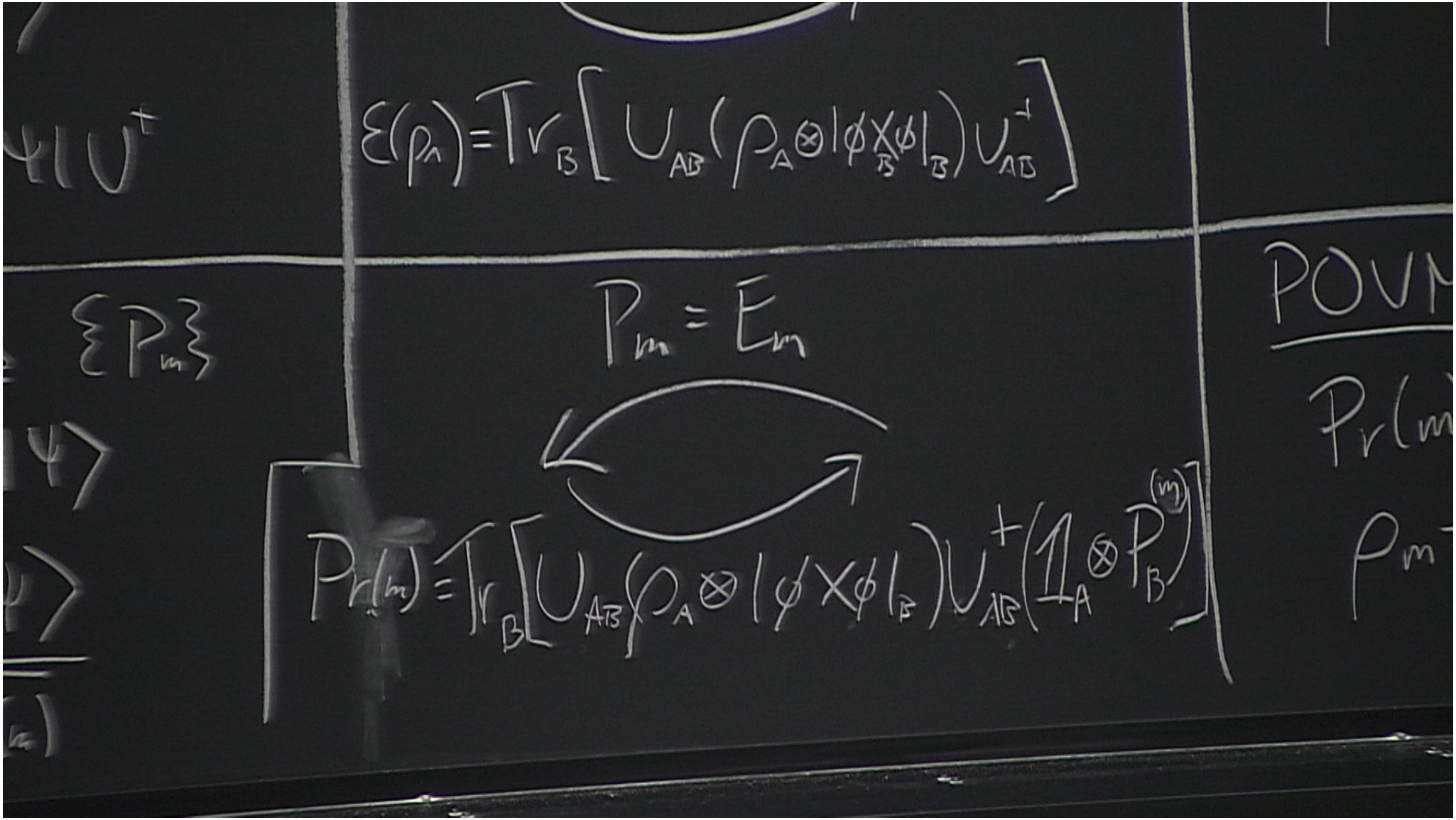
Collection: PSI 2019/2020 - Quantum Theory (Branczyk/Dupuis)

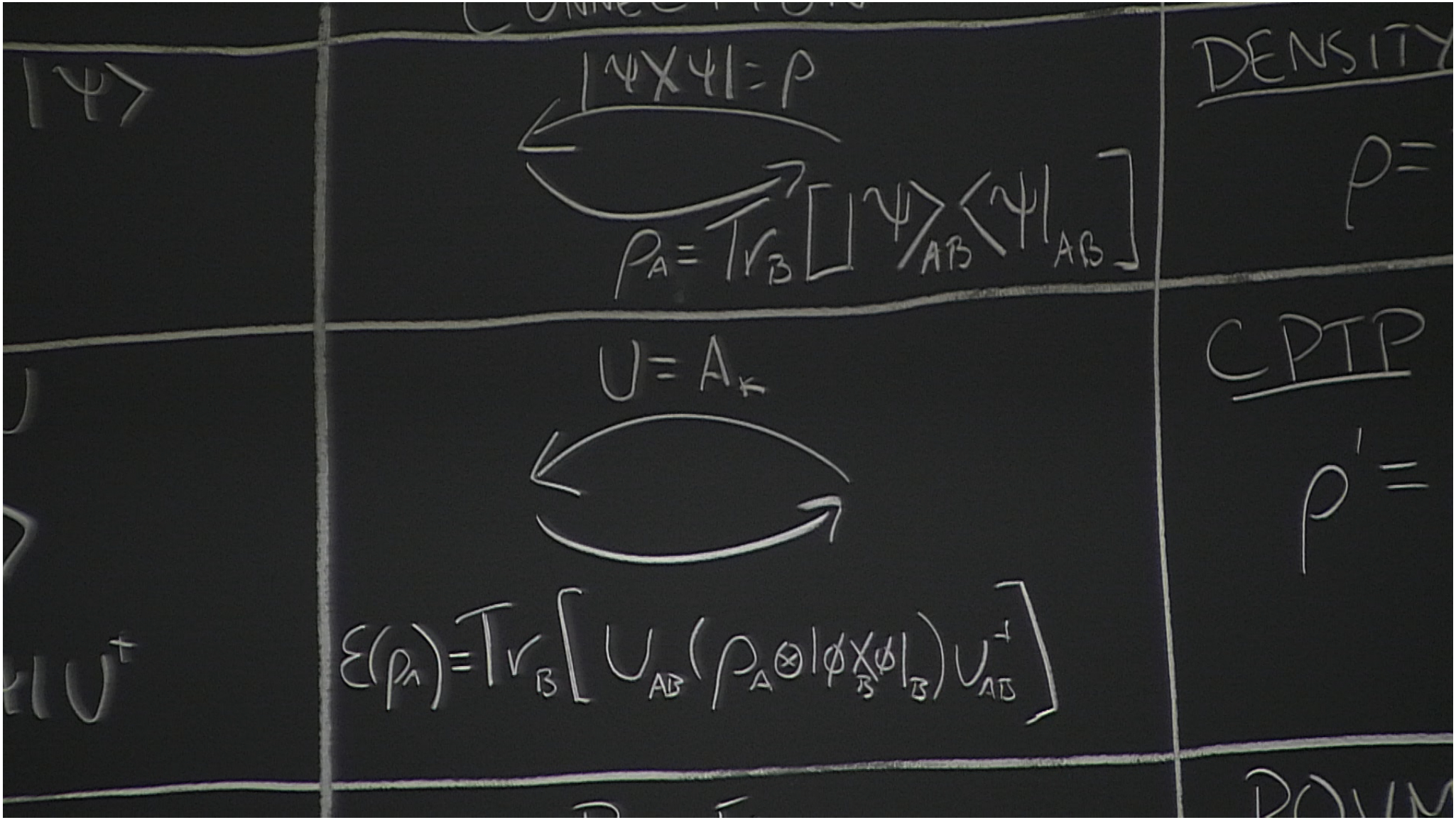
Date: September 11, 2019 - 9:00 AM

URL: http://pirsa.org/19090049


IDEAL
CONNECTION
REALISTIC
REP.
PURE STATE $|\psi\rangle$
$\rho = |\psi\rangle\langle\psi|$
DENSITY OPERATOR ρ
$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
TRANS.
UNITARY U
$|\psi'\rangle = U|\psi\rangle$
$|\psi\rangle\langle\psi| = U|\psi\rangle\langle\psi|U^\dagger$
CPTP MAP $\mathcal{E}(\rho)$
$\rho' = \mathcal{E}(\rho) = \sum_i A_k \rho A_k^\dagger$
MEAS.
PROJECTORS $\{P_m\}$
$Pr(m) = \langle\psi|P_m|\psi\rangle$
$|\psi_m\rangle = \frac{P_m|\psi\rangle}{\sqrt{Pr(m)}}$
POVM $\{E_m\} = \{M_m^\dagger M_m\}$
$Pr(m) = Tr[\rho E_m]$
$\rho_m = \sum_i \frac{M_{m,i}\rho M_{m,i}^\dagger}{Pr(m)}$


$|\psi\rangle$
$|\psi\rangle\langle\psi| = \rho$
$\rho_A = Tr_B[|\psi\rangle_{AB}\langle\psi|_{AB}]$
DENSITY OPERATOR ρ
$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$
U
$U = A_k$
$\mathcal{E}(\rho_A) = Tr_B[U_{AB}(\rho_A \otimes |\phi\rangle\langle\phi|_B) U_{AB}^\dagger]$
CPTP MAP $\mathcal{E}(\rho)$
$\rho' = \mathcal{E}(\rho) = \sum_i A_k \rho A_k^\dagger$
$\{P_m\}$
$P_m = E_m$
$Pr(m) = Tr_B[U_{AB}(\rho_A \otimes |\phi\rangle\langle\phi|_B) U_{AB}^\dagger (\mathbb{1}_A \otimes P_B^{(m)})]$
POVM $\{E_m\} = \{\sum_i M_{m,i}^\dagger M_{m,i}\}$
$Pr(m) = Tr[\rho E_m]$
$\rho_m = \sum_i \frac{M_{m,i} \rho M_{m,i}^\dagger}{Pr(m)}$


$\mathcal{E}(\rho_A) = \mathrm{Tr}_B\left[U_{AB}\left(\rho_A \otimes |\phi\rangle_B\langle\phi|_B\right)U_{AB}^{\dagger}\right]$
$\{P_m\}$
$P_m = E_m$
POVM
$|\psi\rangle$
$P_r(m) = \mathrm{Tr}_B\left[U_{AB}\left(\rho_A \otimes |\phi\rangle\langle\phi|_B\right)U_{AB}^{\dagger}\left(\mathbb{1}_A \otimes P_B^{(m)}\right)\right]$
ρ_m


$|\psi\rangle$
$|\psi\rangle\langle\psi| = \rho$
$\rho_A = Tr_B[|\psi\rangle_{AB}\langle\psi|_{AB}]$
DENSITY
$\rho =$
$U = A_k$
CPTP
$\rho' =$
$\mathcal{E}(\rho_A) = Tr_B[U_{AB}(\rho_A \otimes |\phi\rangle_B\langle\phi|_B)U_{AB}^{\dagger}]$
$U^{\dagger}$
POVM

Quantum Control

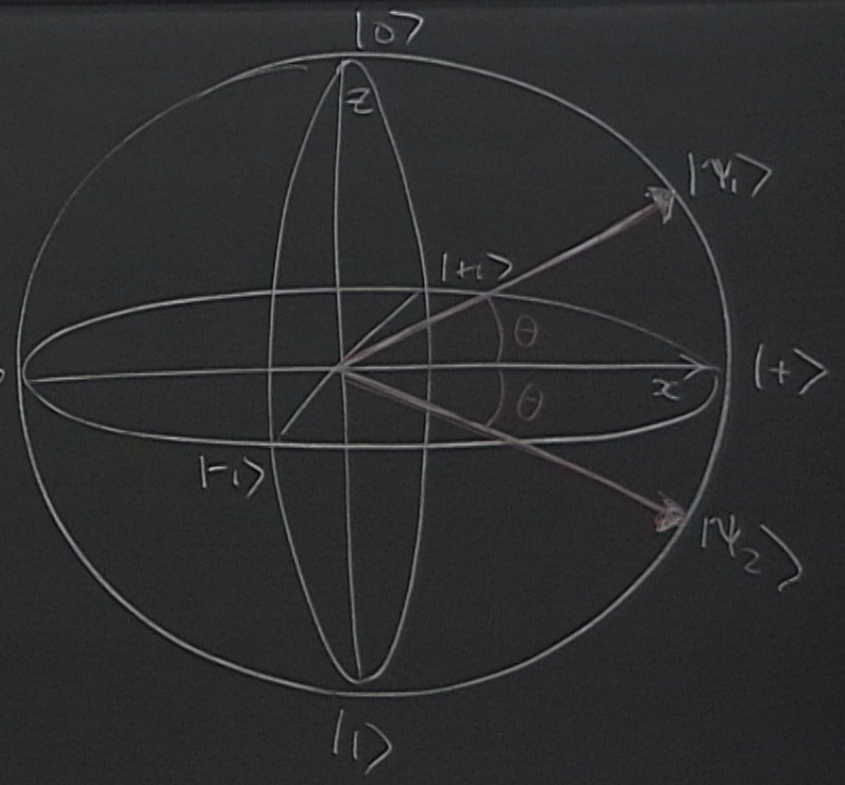
The task.

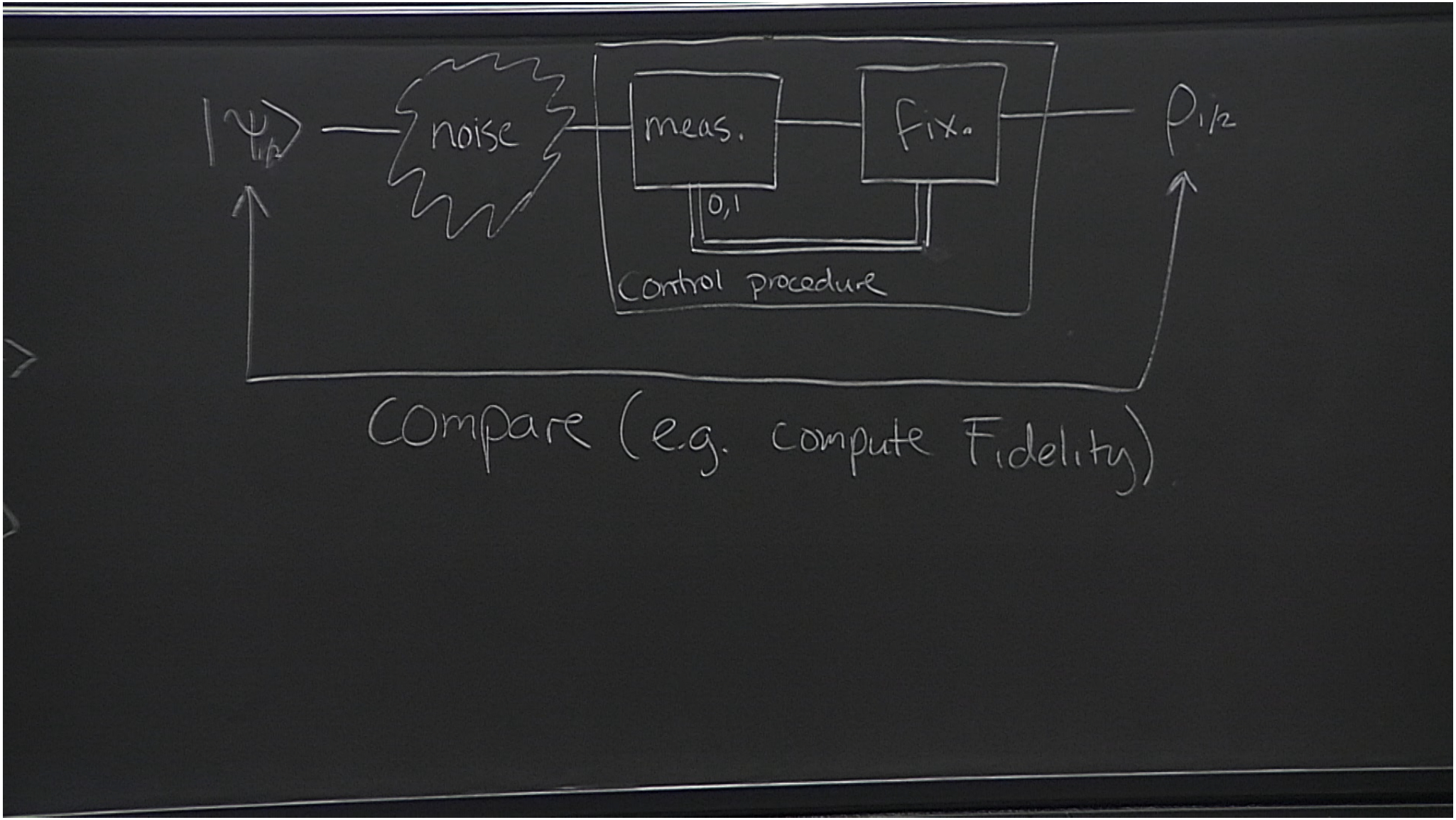
Consider one of two qubits:

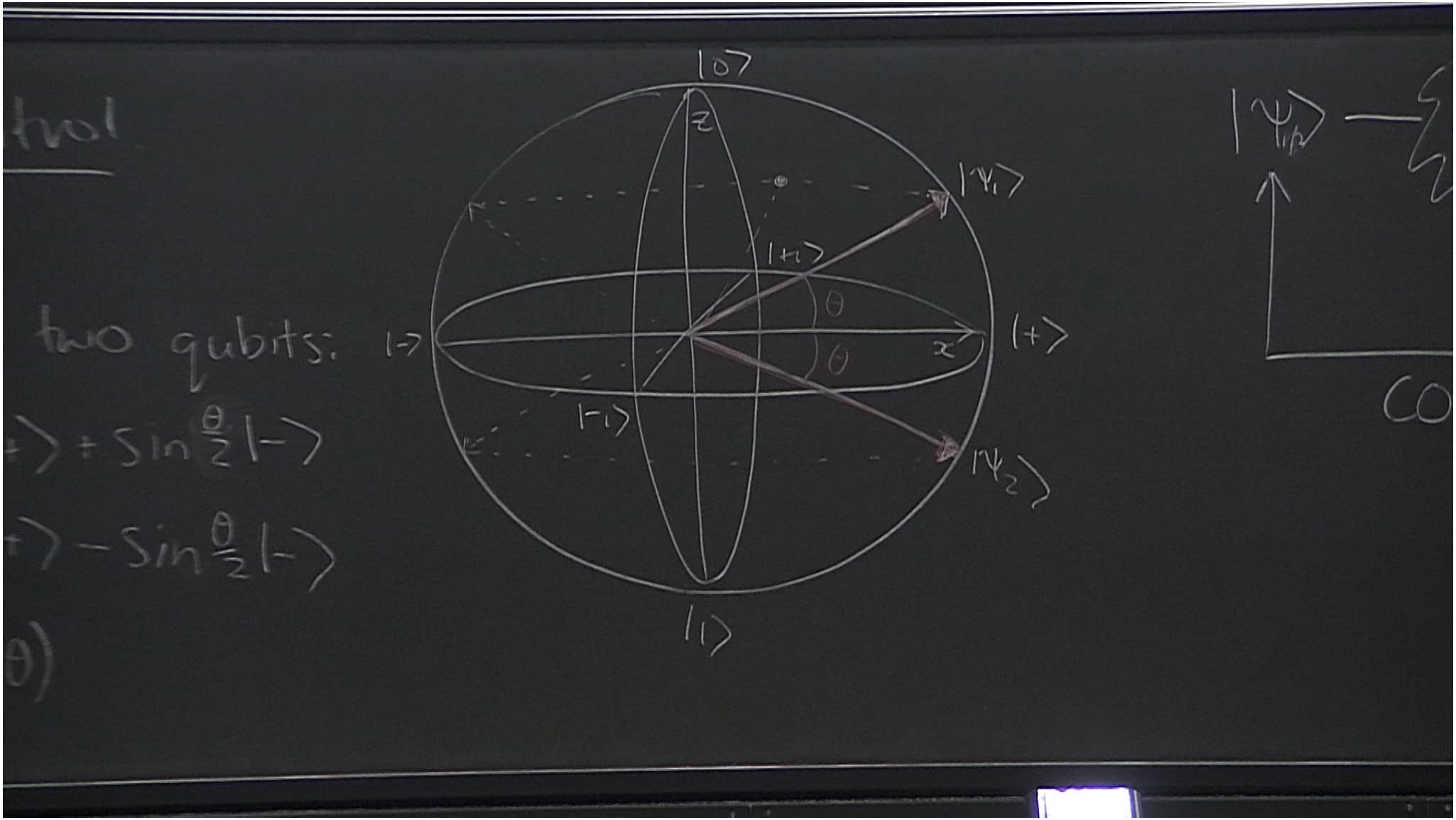
$$|\psi_1\rangle = \cos\tfrac{\theta}{2}|+\rangle + \sin\tfrac{\theta}{2}|-\rangle$$

$$|\psi_2\rangle = \cos\tfrac{\theta}{2}|+\rangle - \sin\tfrac{\theta}{2}|-\rangle$$

$$\langle\psi_1|\psi_2\rangle = \cos(\theta)$$




$|\psi_{1/2}\rangle$
noise
meas.
fix.
$\rho_{1/2}$
0,1
Control procedure
Compare (e.g. compute Fidelity)


|0⟩
z
|ψ1⟩
|+⟩
θ
θ
|-⟩
x
|+⟩
|-⟩
|ψ2⟩
|1⟩
|ψ1⟩
CO

Dephasing noise (phase-flip channel)

$$\mathcal{E}(\rho) = p(\sigma_z \rho \sigma_z) + (1-p)\rho$$

or: $A_1 = \sqrt{p}\,\sigma_z \qquad A_2 = \sqrt{1-p}\,\mathbb{1}$

States after noise.

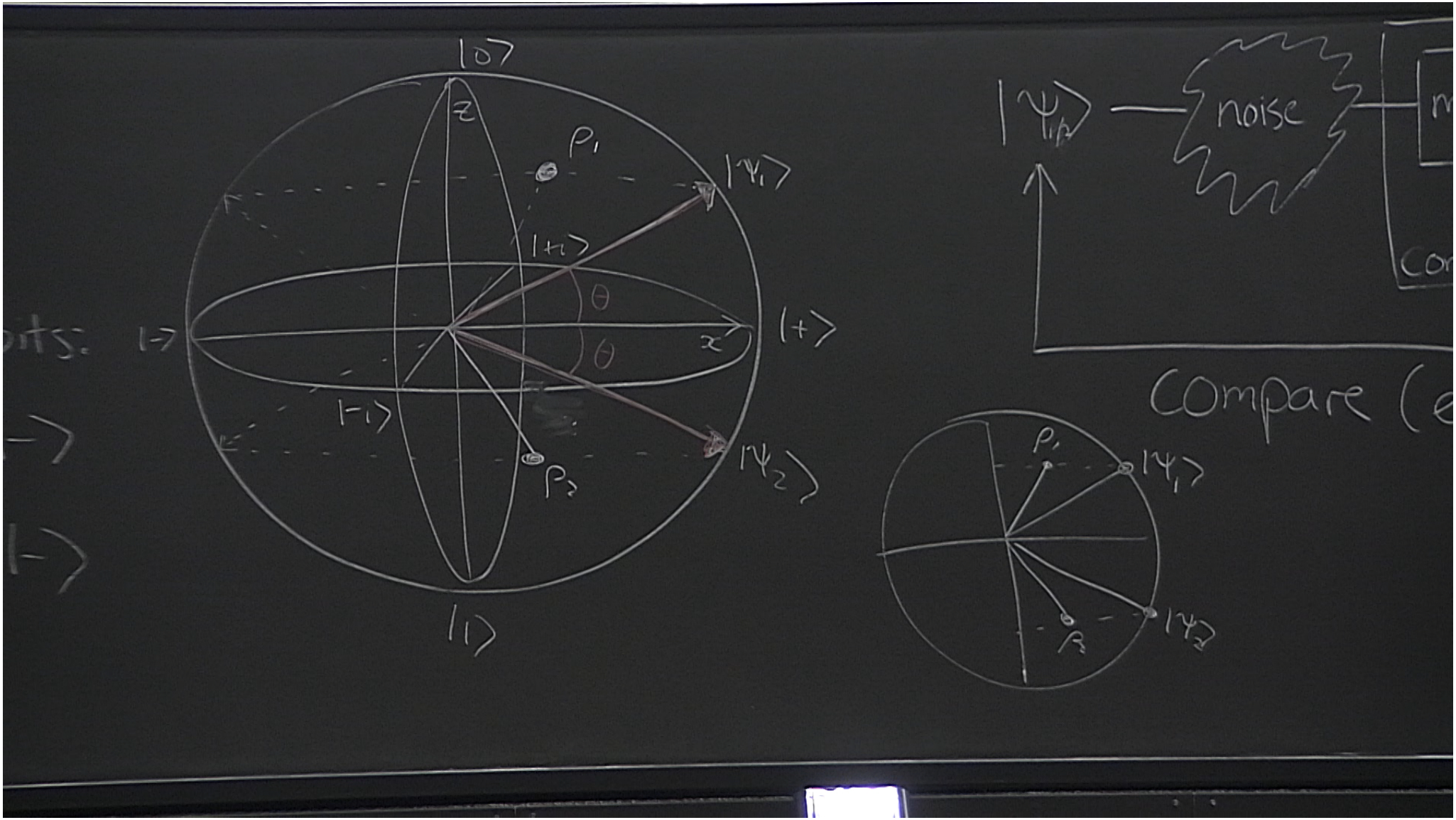
$$\rho_{1/2} = p\,\sigma_z |\psi_{1/2}\rangle\langle\psi_{1/2}| \sigma_z + (1-p)\,|\psi_{1/2}\rangle\langle\psi_{1/2}|$$

$$= p\left(\cos\tfrac{\theta}{2}|-\rangle \pm \sin\tfrac{\theta}{2}|+\rangle\right)\left(\cos\tfrac{\theta}{2}\langle -| \pm \sin\tfrac{\theta}{2}\langle +|\right)$$

$$+ (1-p)\left(\cos\tfrac{\theta}{2}|+\rangle \pm \sin\tfrac{\theta}{2}|-\rangle\right)\left(\cos\tfrac{\theta}{2}\langle +| \pm \sin\tfrac{\theta}{2}\langle -|\right)$$

$$= \left(\cos^2\tfrac{\theta}{2} - p\cos\theta\right)|+\rangle\langle +| + \left(\sin^2\left(\tfrac{\theta}{2}\right) + p\cos\theta\right)|-\rangle\langle -| \pm \frac{\sin\theta}{2}\left(|+\rangle\langle -| + |-\rangle\langle +|\right)$$

$$D(\rho_1,\rho_2) = \frac{1}{2}\mathrm{Tr}\left[\sqrt{(\rho_1-\rho_2)^2}\right] = \sin\theta$$


|0>
z
ρ1
|ψ1>
|+i>
θ
θ
|->
|+>
x
|-i>
ρ2
|ψ2>
|1>
|ψin>
noise
Compare
ρ1
|ψ1>
ρ2
|ψ2>

States after noise.

$$\rho_{1/2} = p\,\sigma_z |\psi_{1/2}\rangle\langle\psi_{1/2}| \sigma_z + (1-p)\,|\psi_{1/2}\rangle\langle\psi_{1/2}|$$

$$= p\left(\cos\tfrac{\theta}{2}|-\rangle \pm \sin\tfrac{\theta}{2}|+\rangle\right)\left(\cos\tfrac{\theta}{2}\langle -| \pm \sin\tfrac{\theta}{2}\langle +|\right)$$

$$+ (1-p)\left(\cos\tfrac{\theta}{2}|+\rangle \pm \sin\tfrac{\theta}{2}|-\rangle\right)\left(\cos\tfrac{\theta}{2}\langle +| \pm \sin\tfrac{\theta}{2}\langle -|\right)$$

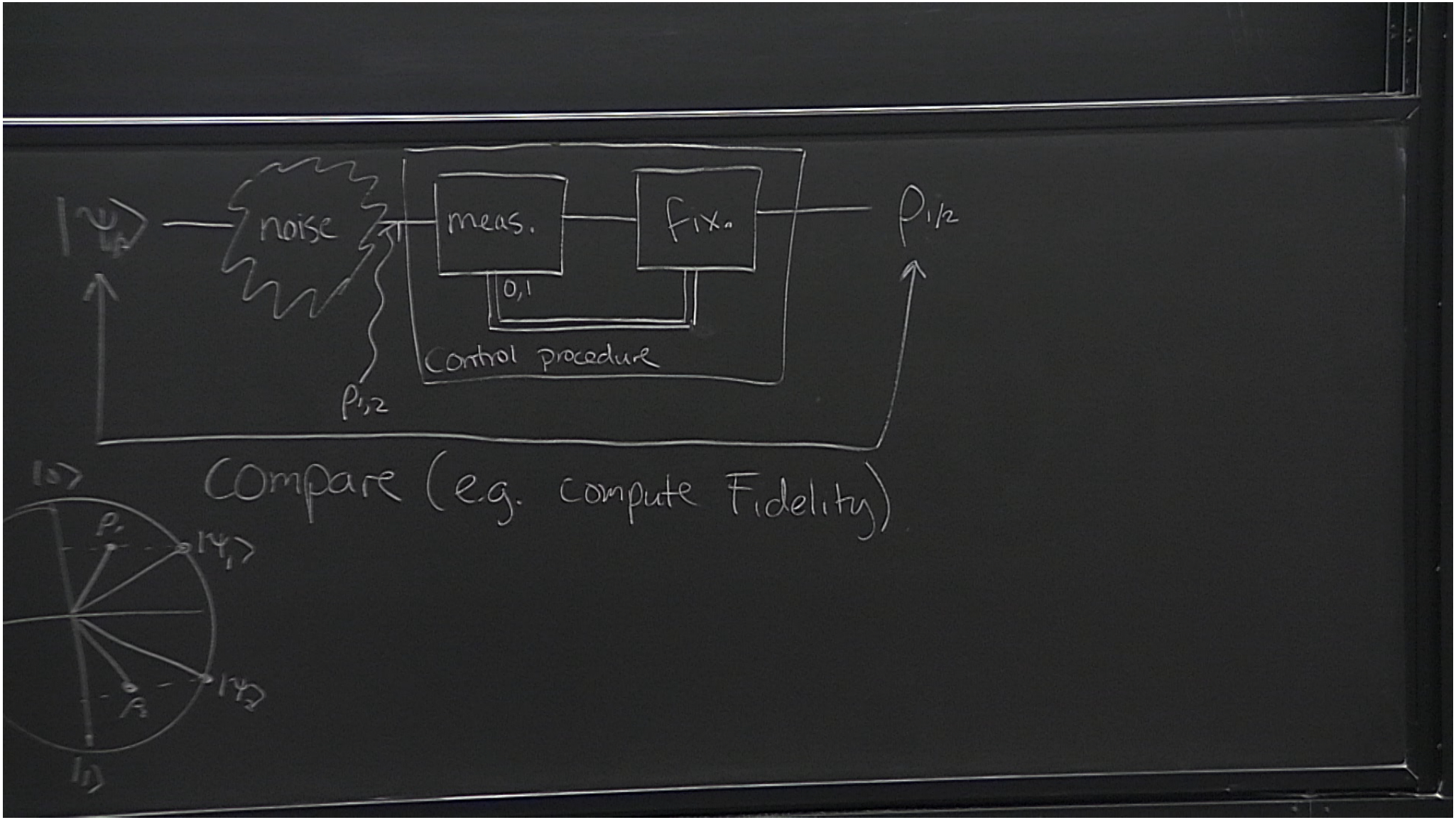
$$= \left(\cos^2\tfrac{\theta}{2} - p\cos\theta\right)|+\rangle\langle +| + \left(\sin^2\left(\tfrac{\theta}{2}\right) + p\cos\theta\right)|-\rangle\langle -| \pm \frac{\sin\theta}{2}\left(|+\rangle\langle -| + |-\rangle\langle +|\right)$$

$$D(\rho_1,\rho_2) = \frac{1}{2}\,\mathrm{Tr}\left[\sqrt{(\rho_1-\rho_2)^2}\right] = \sin\theta$$

"Classical" Strategy (measure → reprepare).

1) Measure $\{|0\rangle, |1\rangle\}$

2) If $m=0$, prepare $|\psi_1\rangle$
   If $m=1$, prepare $|\psi_2\rangle$


noise
meas.
Fix.
0,1
Control procedure
Compare (e.g. compute Fidelity)

$$Pr_0(\rho_{1/2}) = Tr[|0\rangle\langle 0|\,\rho_{1/2}]$$

$$= \frac{1}{2}\left(\left(\cos^2\frac{\theta}{2} - p\cos\theta\right) + \left(\sin^2\frac{\theta}{2} + p\cos\theta\right) \pm 2\sin\frac{\theta}{2}\right)$$

$$= \frac{1}{2}(1 \pm \sin\theta)$$

$$Pr_1(\rho_{1/2}) = \frac{1}{2}(1 \mp \sin\theta)$$

Given input $|\psi_1\rangle$, what is output after control strategy?

$$\rho_1^{(cc)} = Pr_0(\rho_1)\,|\psi_1\rangle\langle\psi_1| + Pr_1(\rho_1)\,|\psi_2\rangle\langle\psi_2|$$

$$\rho_2^{(cc)} = Pr_0(\rho_2)\,|\psi_1\rangle\langle\psi_1| + P_1(\rho_2)\,|\psi_2\rangle\langle\psi_2|$$

Fidelity measures overlap between state.

$F(|\psi\rangle, \rho) = \langle\psi|\rho|\psi\rangle$

$0 \leq F \leq 1$

If $F = 0$ Orthogonal

$F = 1$ identical



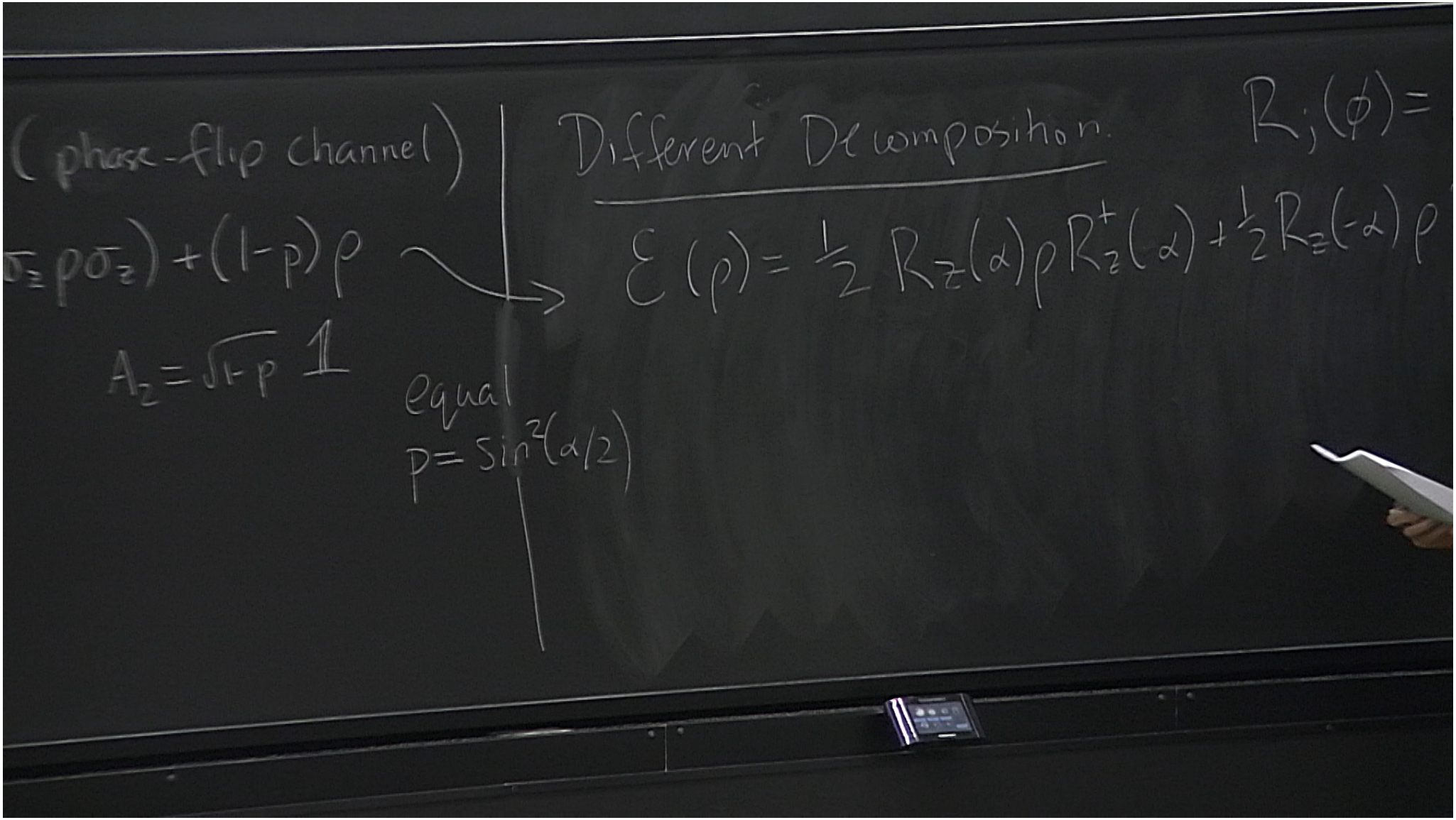
Given input $|\psi_i\rangle$, what is output after control strategy?

$$\rho_1^{(cc)} = Pr_0(\rho_1)\,|\psi_1\rangle\langle\psi_1| + Pr_1(\rho_1)\,|\psi_2\rangle\langle\psi_2|$$

$$\rho_2^{(cc)} = Pr_0(\rho_2)\,|\psi_1\rangle\langle\psi_1| + P_1(\rho_2)\,|\psi_2\rangle\langle\psi_2|$$

How well does it work? Compute average Fidelity.

$$F_{cc} = \frac{1}{2}\langle\psi_1|\rho_1^{(cc)}|\psi_1\rangle + \frac{1}{2}\langle\psi_2|\rho_2^{(cc)}|\psi_2\rangle = 1 - \frac{1}{2}(\sin^2\theta - \sin^3\theta)$$


(phase-flip channel)
$\sigma_z \rho \sigma_z) + (1-p)\rho$
$A_2 = \sqrt{1-p}\, 1$
equal
$p = \sin^2(\alpha/2)$
Different Decomposition
$\mathcal{E}(\rho) = \frac{1}{2} R_z(\alpha) \rho R_z^+(\alpha) + \frac{1}{2} R_z(-\alpha) \rho$
$R_j(\phi) =$

ifferent Decomposition. $R_j(\phi) = \cos\frac{\phi}{2}\mathbb{1} - i\sin\frac{\phi}{2}\sigma_j$

$$\mathcal{E}(\rho) = \tfrac{1}{2} R_z(\alpha)\rho R_z^\dagger(\alpha) + \tfrac{1}{2} R_z(-\alpha)\rho R_z(-\alpha)$$

Possible control strategy: find which direction the noise rotated the state.

Weak measurement.

$$M_0 = \cos\frac{\chi}{2}\,|+iX+i| + \sin\frac{\chi}{2}\,|-iX-i|$$

$$M_1 = \sin\frac{\chi}{2}\,|+iX+i| + \cos\frac{\chi}{2}\,|-iX-i|$$

$0 < \chi < \frac{\pi}{2}$

$\chi = 0$ $\quad M_0 = |+iX+i|$ $\quad M_1 = |-iX-i|$

$\chi = \pi/2$ $\quad M_0 = M_1 = \frac{1}{\sqrt{2}}$

$M_0 = M_1 = \frac{}{\sqrt{2}}$

Given outcome 0, rotate state by $R_z(\eta)$

" " 1, rotate " " $R_z(-\eta)$

$$\mathcal{E}(\rho_S) = Pr(0) R_z(\eta) \frac{M_0 \rho_S M_0^\dagger}{Pr(0)} R_z^\dagger(\eta) + Pr(1) R_z(-\eta) \frac{M_1 \rho_S M_1^\dagger}{Pr(1)} R_z^\dagger(-\eta)$$

$$= R_z(\eta) M_0 \rho_S M_0^\dagger R_z^\dagger(\eta) + R_z(-\eta) M_1 \rho_S M_1^\dagger R_z^\dagger(-\eta)$$

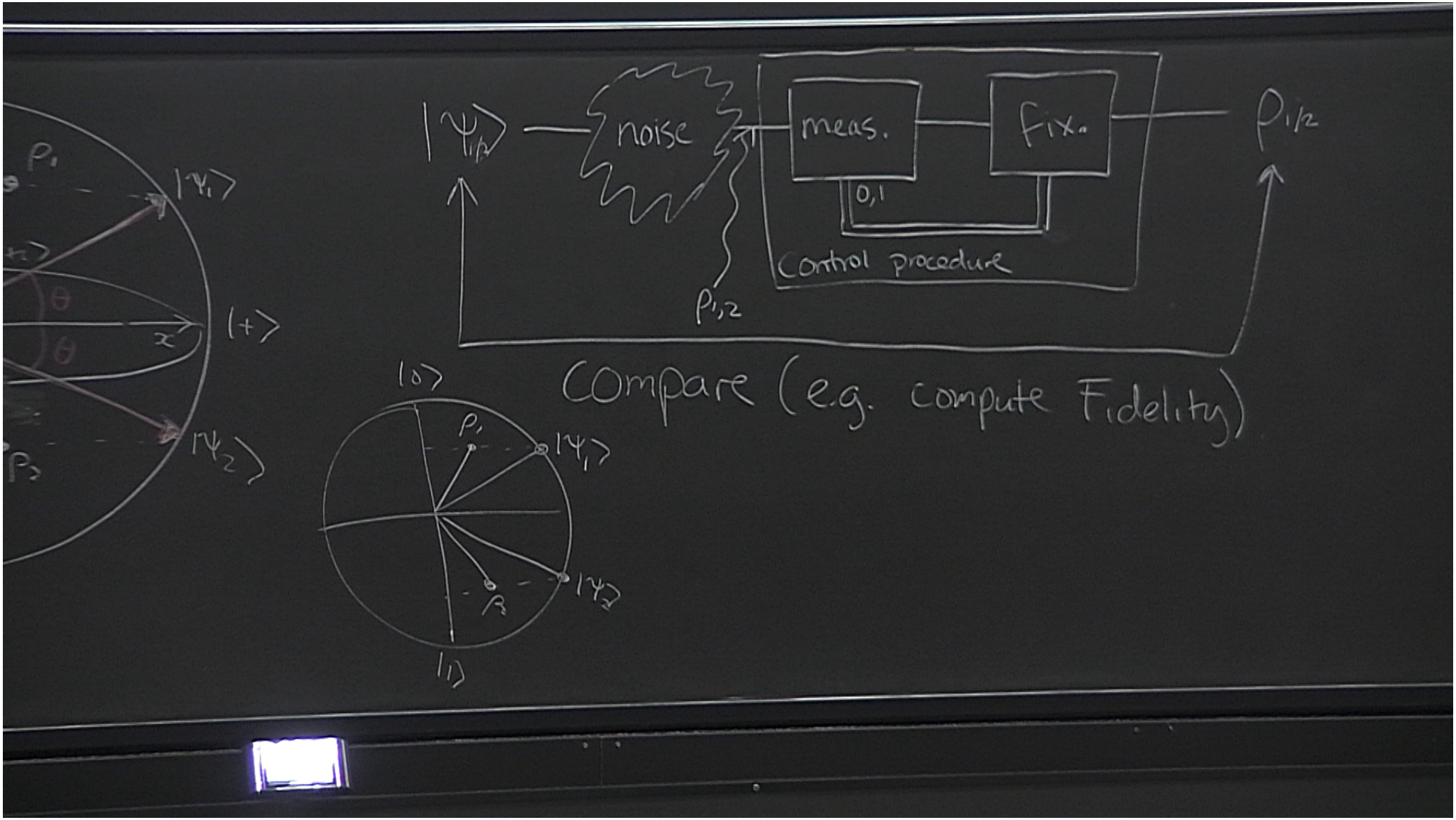
Compute average fidelity

$$F_{qc} = \frac{1}{2}\langle\psi_1|\mathcal{E}_{qc}(\rho_1)|\psi_1\rangle + \frac{1}{2}\langle\psi_2|\mathcal{E}_{qc}(\rho_2)|\psi_2\rangle$$

$$= f(\theta, p, \chi, \eta)$$

optimize

$$F_{optimal\text{-}qc} = \frac{1}{2} + \frac{1}{2}\sqrt{\cos^2\theta + \frac{\sin^4\theta}{1-(1-2p)^2\cos\theta}} > F_{optimal\text{-}cc}$$


noise
meas.
fix.
0,1
Control procedure
Compare (e.g. compute Fidelity)