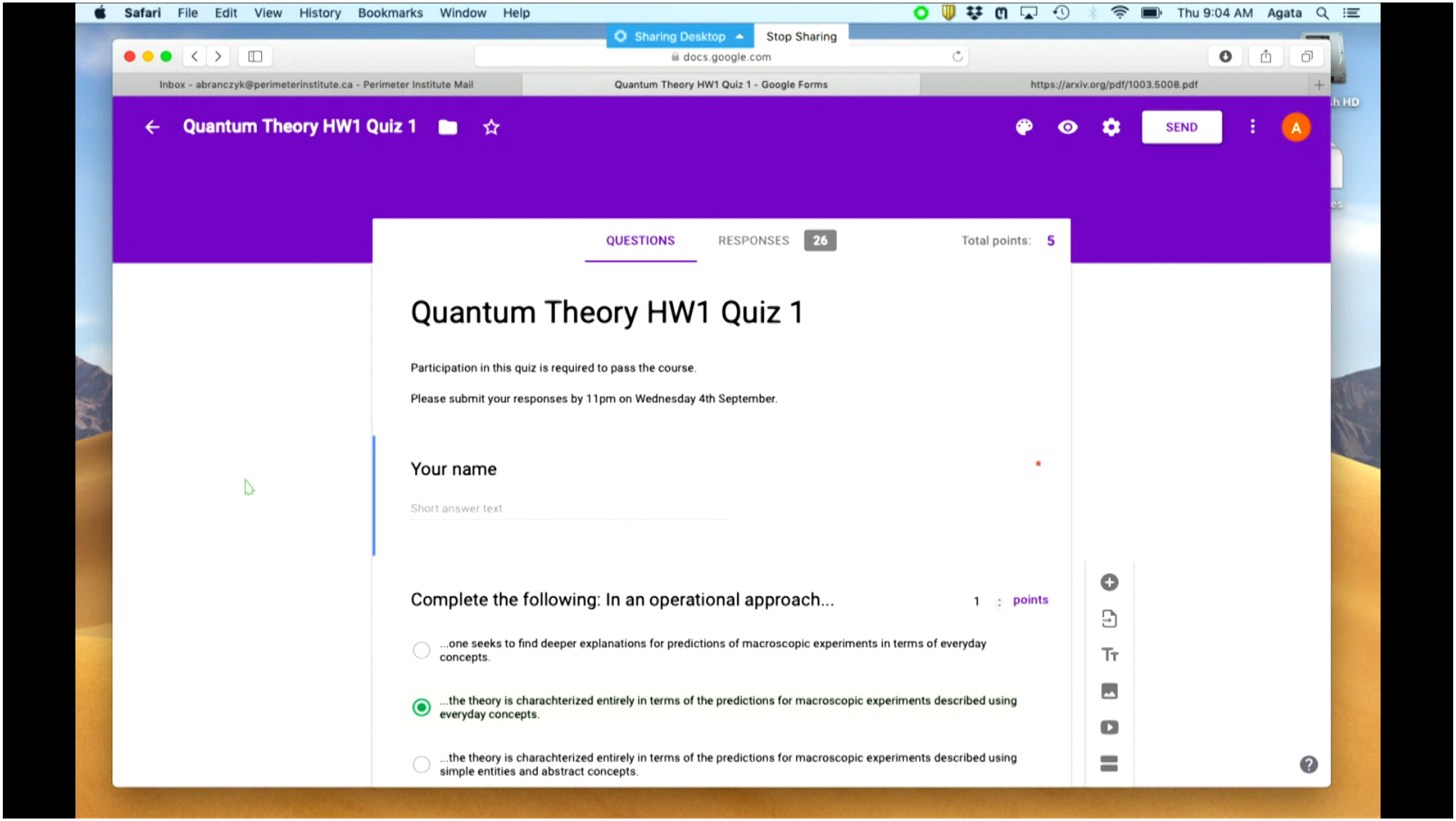
Title: PSI 2019/2020 - Quantum Theory (Branczyk) - Lecture 3

Speakers: Agata Branczyk

Collection: PSI 2019/2020 - Quantum Theory (Branczyk/Dupuis)

Date: September 05, 2019 - 9:00 AM

URL: http://pirsa.org/19090045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Quantum Theory HW1 Quiz 1 - Google Forms
https://arxiv.org/pdf/1003.5008.pdf
Quantum Theory HW1 Quiz 1
SEND
QUESTIONS
RESPONSES 26
Total points: 5
Quantum Theory HW1 Quiz 1
Participation in this quiz is required to pass the course.
Please submit your responses by 11pm on Wednesday 4th September.
Your name
*
Short answer text
Complete the following: In an operational approach...
1 points
...one seeks to find deeper explanations for predictions of macroscopic experiments in terms of everyday concepts.
...the theory is charachterized entirely in terms of the predictions for macroscopic experiments described using everyday concepts.
...the theory is charachterized entirely in terms of the predictions for macroscopic experiments described using simple entities and abstract concepts.

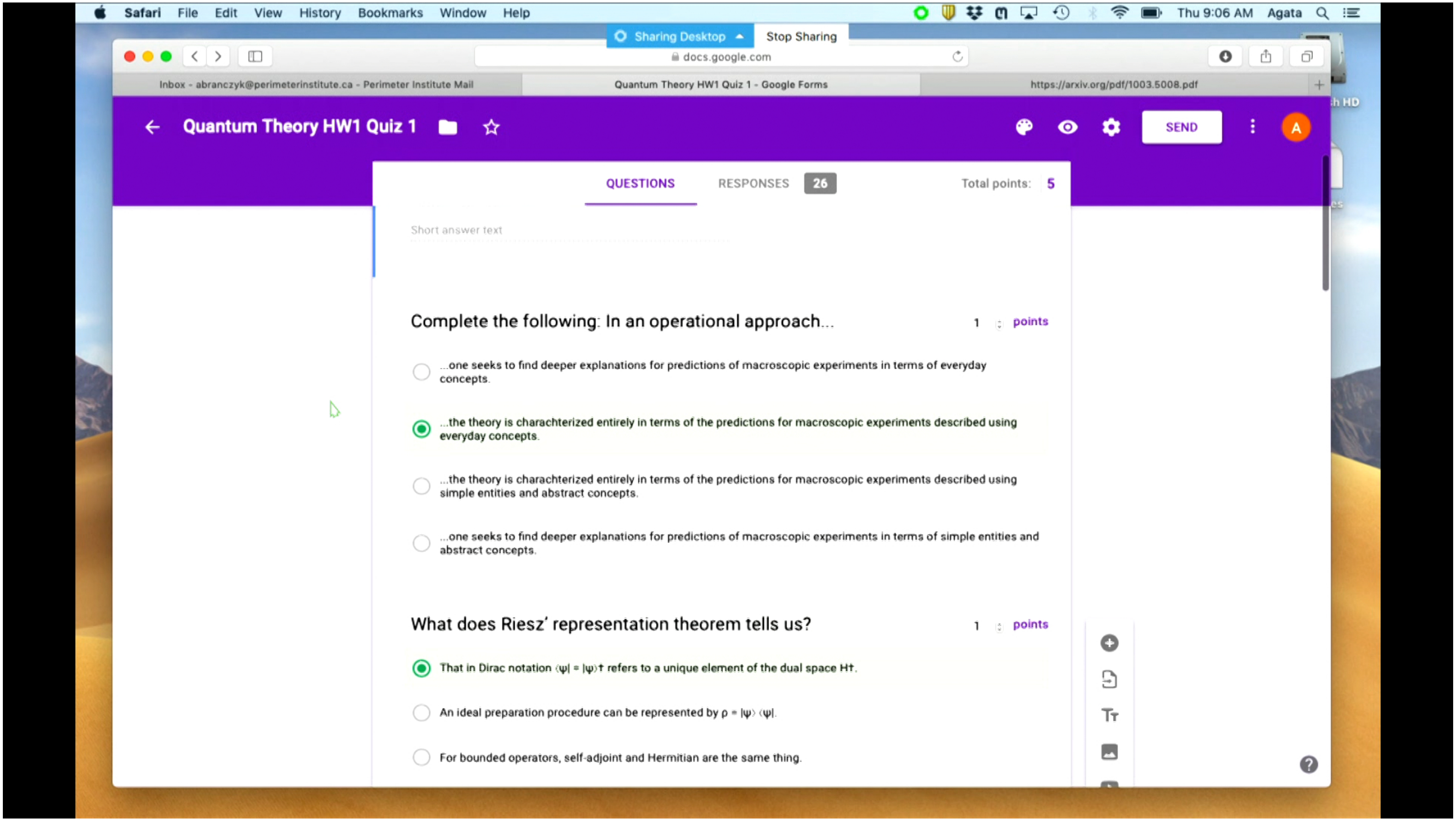
Why Physics Needs Quantum Foundations

by Lucien Hardy and Robert Spekkens

Perimeter Institute for Theoretical Physics, 31 Caroline St. N, Waterloo, Ontario, Canada N2L 2Y5

Quantum theory is a peculiar creature. It was born as a theory of atomic physics early in the twentieth century, but over time its scope has broadened, to the point where it now underpins all of modern physics with the exception of gravity. It has been verified to extremely high accuracy and has never been contradicted experimentally. Yet despite its enormous success, there is still no consensus among physicists about what this theory is saying about the nature of reality. There is no question that quantum theory works well as a tool for predicting what will occur in experiments. But just as understanding how to drive an automobile is different from understanding how it works or how to fix it should it break down, so too procedures, specified as lists of instructions of what to do in the lab. They are recipes with macroscopic activities as ingredients. The theory merely specifies what probabilities of outcomes will be observed when a given measurement follows a given preparation. For the realist, there is some deeper reality underlying the equations of quantum theory that ultimately accounts for why we see the relative frequencies we do. For the realist, quantum theory needs an interpretation. Does the wave function describe a real entity? Are there extra hidden variables in addition to the wave function needed to fully describe a quantum system?

A classic example of the power of applying operational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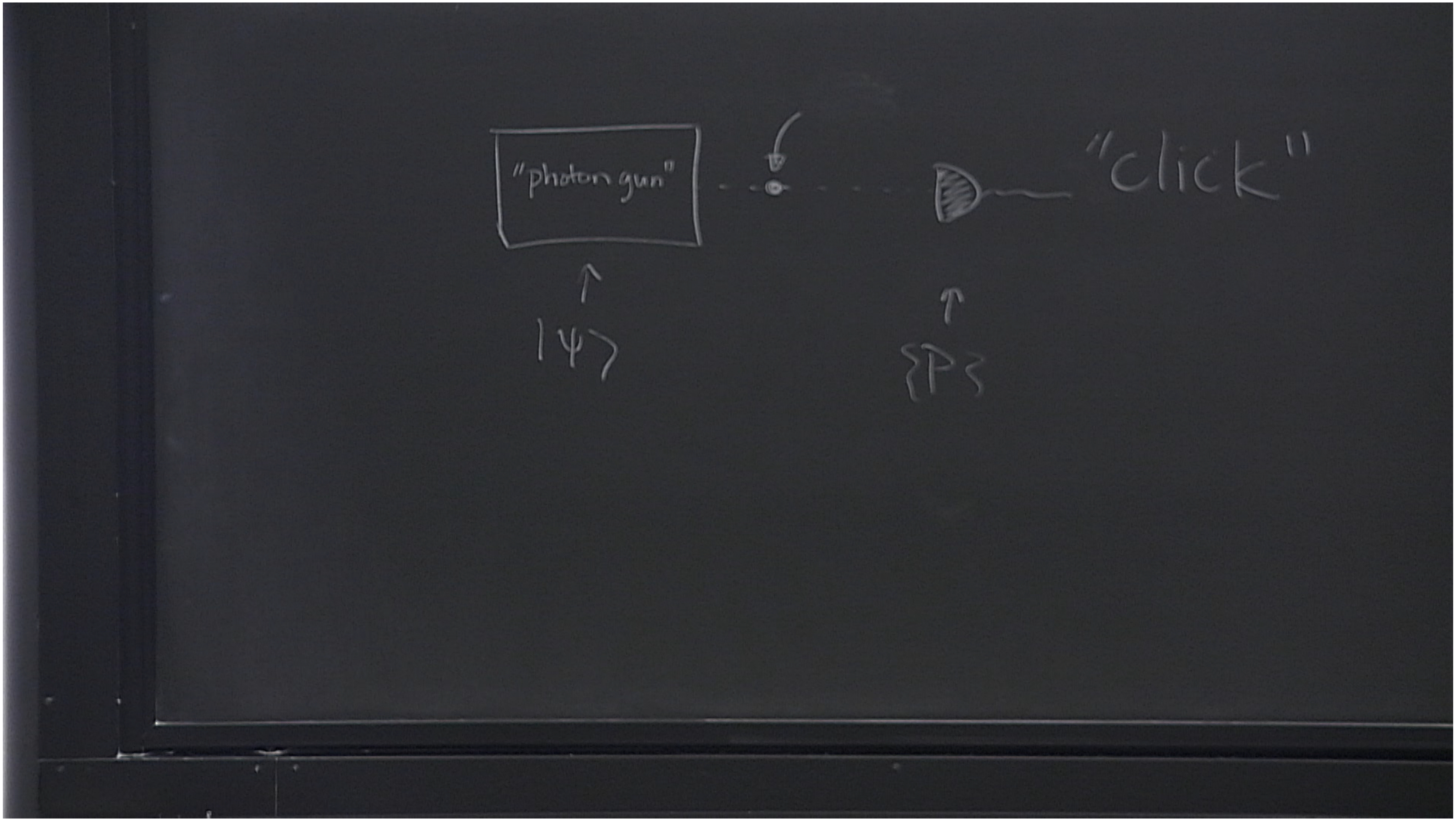
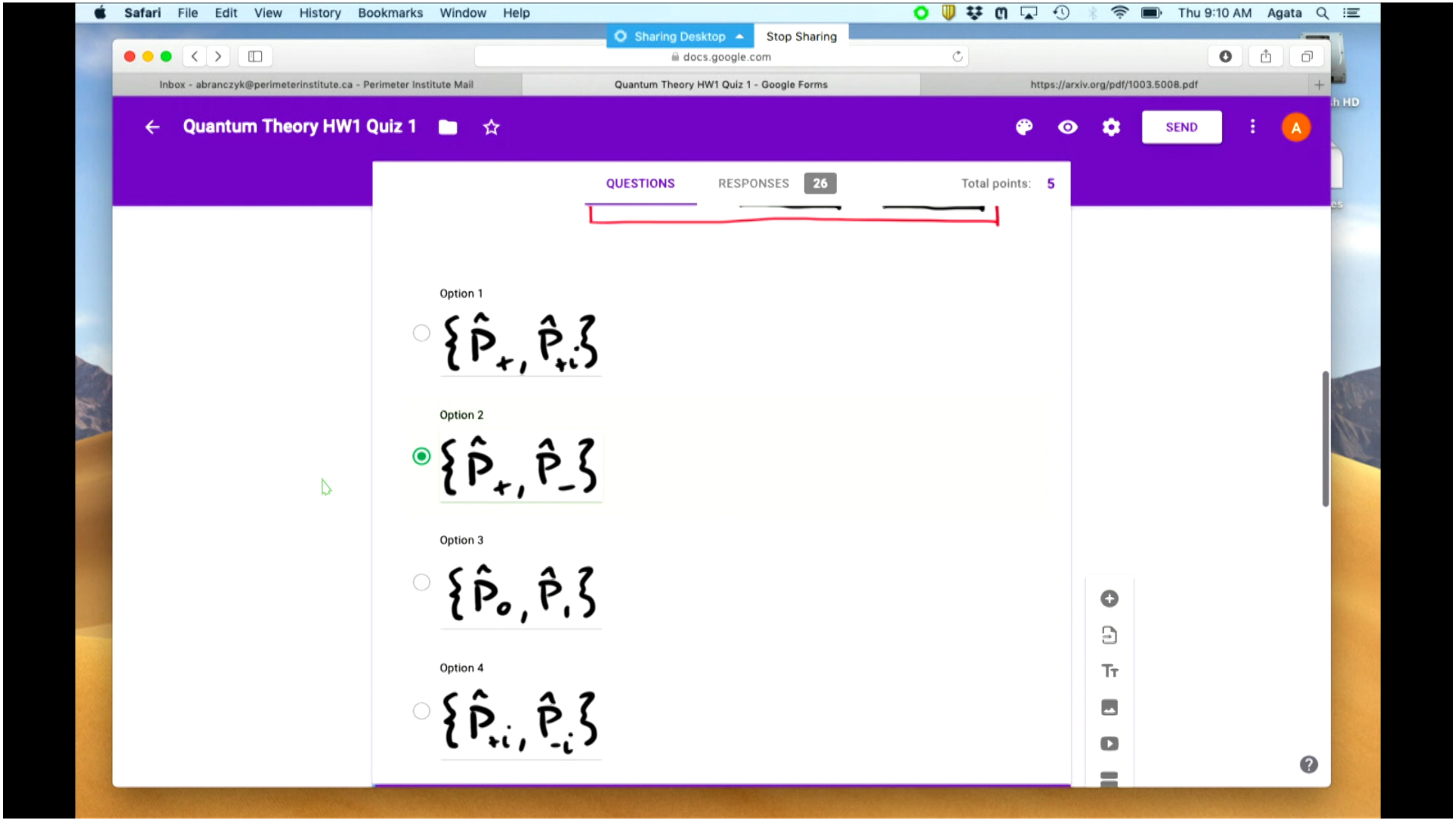
...the theory is charachterized entirely in terms of the predictions for macroscopic experiments described using simple entities and abstract concepts.
...one seeks to find deeper explanations for predictions of macroscopic experiments in terms of simple entities and abstract concepts.
What does Riesz' representation theorem tells us?
1 points
That in Dirac notation $\langle\psi| = |\psi\rangle^\dagger$ refers to a unique element of the dual space $H^\dagger$.
An ideal preparation procedure can be represented by $\rho = |\psi\rangle\langle\psi|$.
For bounded operators, self-adjoint and Hermitian are the same thing.


"photon gun"
"click"
|ψ⟩
{P}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Option 1
$\{\hat{P}_+, \hat{P}_{+i}\}$
Option 2
$\{\hat{P}_+, \hat{P}_-\}$
Option 3
$\{\hat{P}_0, \hat{P}_1\}$
Option 4
$\{\hat{P}_{+i}, \hat{P}_{-i}\}$

$\xrightarrow{t}$

$$\boxed{|\psi\rangle\langle\psi|} - \overbrace{\boxed{R_z(\tfrac{\pi}{2})} - \boxed{R_x(\tfrac{\pi}{2})} - \boxed{\{\hat{P}_0, \hat{P}_1\}}}^{P'_m}$$

$|\psi'\rangle$

$$|\psi'\rangle = R_x(\tfrac{\pi}{2}) R_z(\tfrac{\pi}{2}) |\psi\rangle$$

$$\Pr(m) = \langle \psi' | P_m | \psi' \rangle$$

$$= \langle \psi | \underbrace{R_z^{\dagger}(\tfrac{\pi}{2}) R_x^{\dagger}(\tfrac{\pi}{2}) P_m R_x(\tfrac{\pi}{2}) R_z(\tfrac{\pi}{2})}_{P'_m} | \psi \rangle$$

Quantum Theory HW1 Quiz 1

QUESTIONS RESPONSES 26 Total points: 5

Choose correct answers:

Consider a preparation $(|a\rangle+|b\rangle+|c\rangle)/\sqrt{3}$, where $|a\rangle$, $|b\rangle$ & $|c\rangle$ are orthogonal to each other, followed by a measurement in the $\{|a\rangle\langle a|, |b\rangle\langle b|+|c\rangle\langle c|\}$ basis. Given the outcome corresponding to $|b\rangle\langle b|+|c\rangle\langle c|$, what is the post-measurement state?

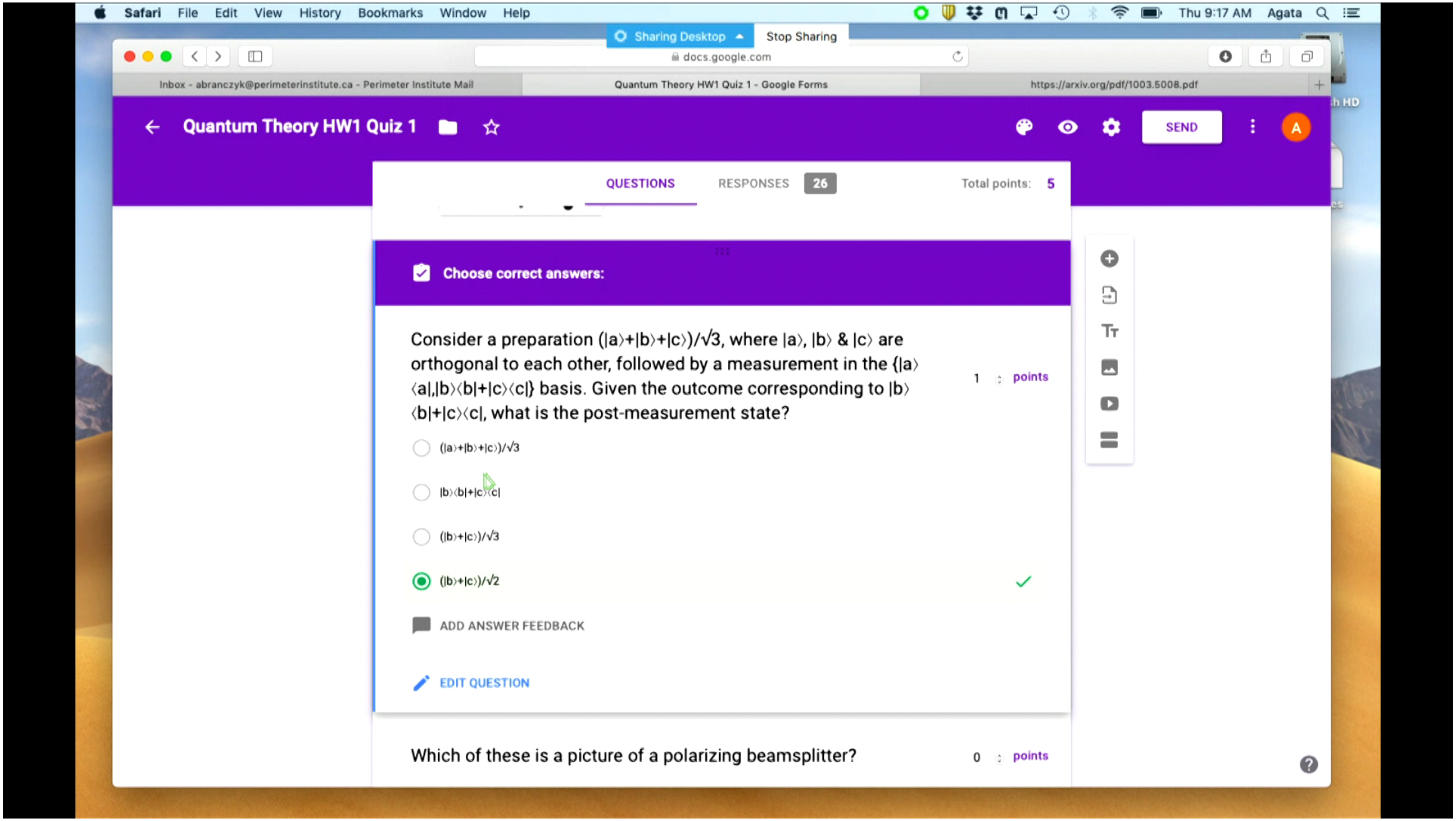
1 points

- $(|a\rangle+|b\rangle+|c\rangle)/\sqrt{3}$
- $|b\rangle\langle b|+|c\rangle\langle c|$
- $(|b\rangle+|c\rangle)/\sqrt{3}$
- $(|b\rangle+|c\rangle)/\sqrt{2}$ ✓

ADD ANSWER FEEDBACK

EDIT QUESTION

Which of these is a picture of a polarizing beamsplitter? 0 points


Quantum Theory HW1 Quiz 1
QUESTIONS
RESPONSES 26
Total points: 5
Choose correct answers:
Consider a preparation (|a⟩+|b⟩+|c⟩)/√3, where |a⟩, |b⟩ & |c⟩ are orthogonal to each other, followed by a measurement in the {|a⟩⟨a|,|b⟩⟨b|+|c⟩⟨c|} basis. Given the outcome corresponding to |b⟩⟨b|+|c⟩⟨c|, what is the post-measurement state?
1 points
(|a⟩+|b⟩+|c⟩)/√3
|b⟩⟨b|+|c⟩⟨c|
(|b⟩+|c⟩)/√3
(|b⟩+|c⟩)/√2
ADD ANSWER FEEDBACK
EDIT QUESTION
Which of these is a picture of a polarizing beamsplitter?
0 points

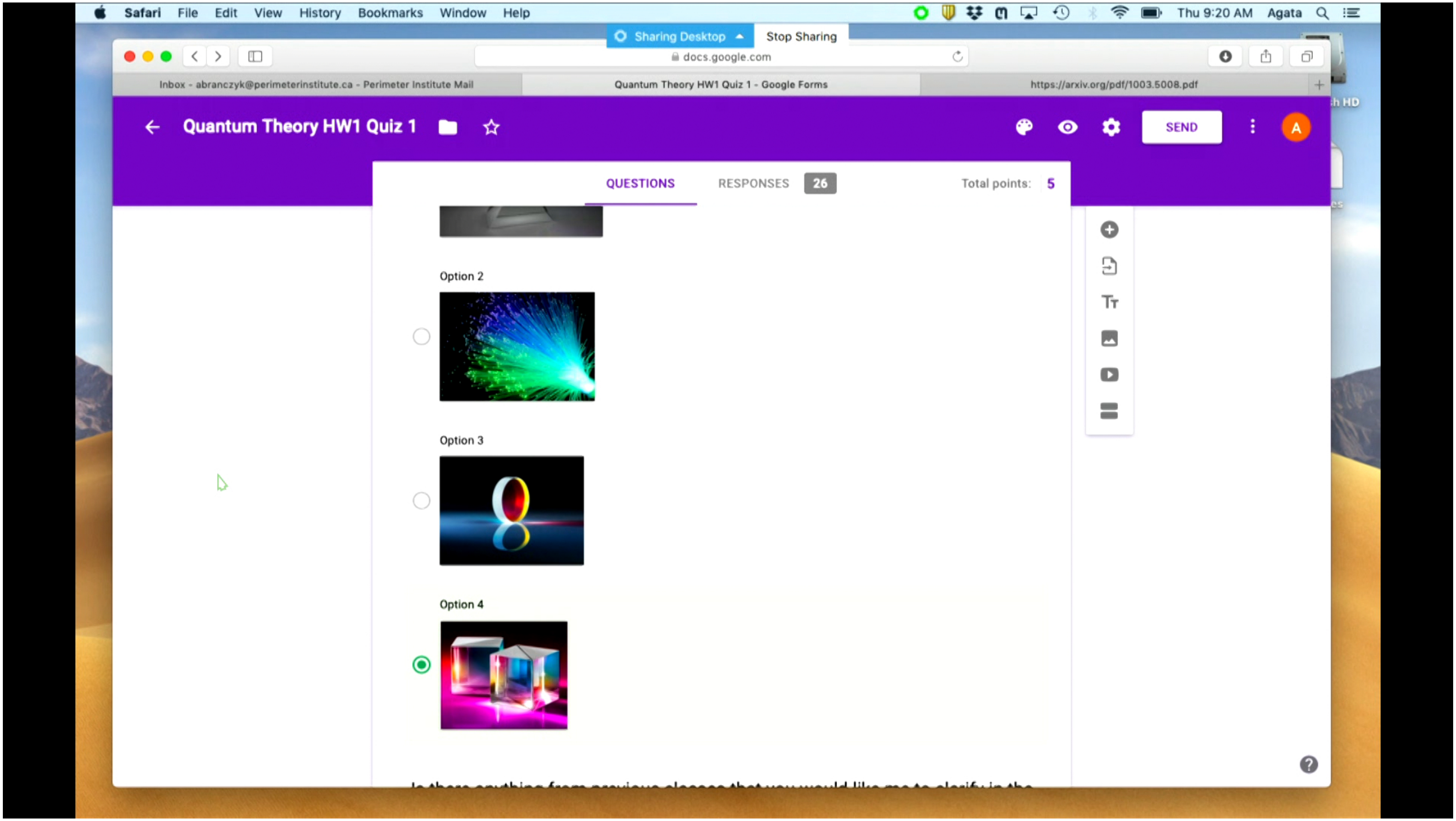
$$|\psi\rangle = \frac{|a\rangle + |b\rangle + |c\rangle}{\sqrt{3}}$$

$$P_1 = |a\rangle\langle a|$$

$$P_2 = |b\rangle\langle b| + |c\rangle\langle c|$$

$$|\psi_2\rangle = \frac{P_2|\psi\rangle}{\sqrt{Pr(2)}} = \left(\frac{|b\rangle\langle b| + |c\rangle\langle c|}{\sqrt{2/3}}\right)\left(\frac{|a\rangle + |b\rangle + |c\rangle}{\sqrt{3}}\right)$$

$$= \frac{|b\rangle + |c\rangle}{\sqrt{2}}$$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Option 2
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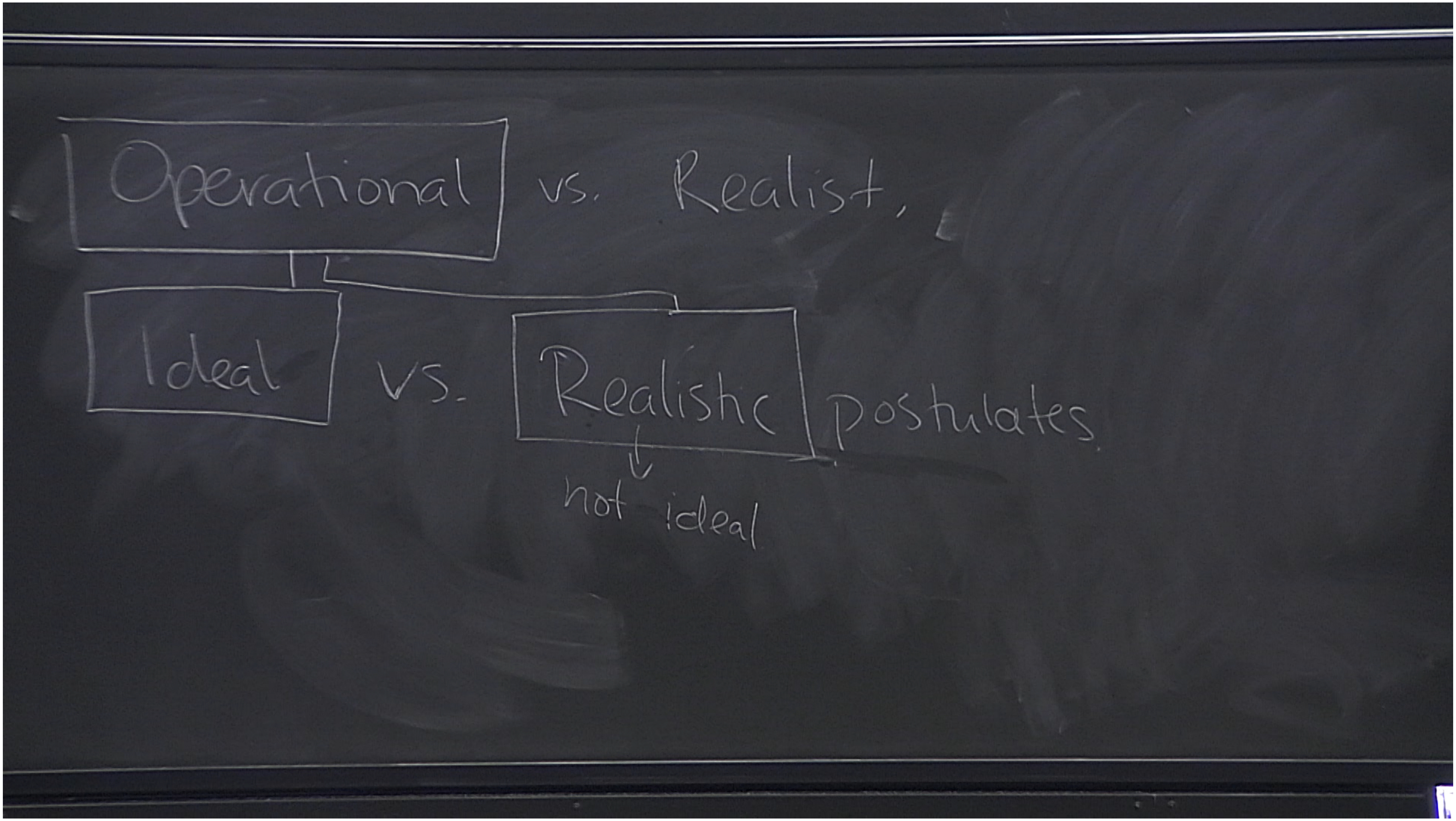

Operational vs. Realist,
Ideal vs. Realistic postulates
not ideal

onal vs. Realist,

$$P_1 = |a\rangle\langle a|$$

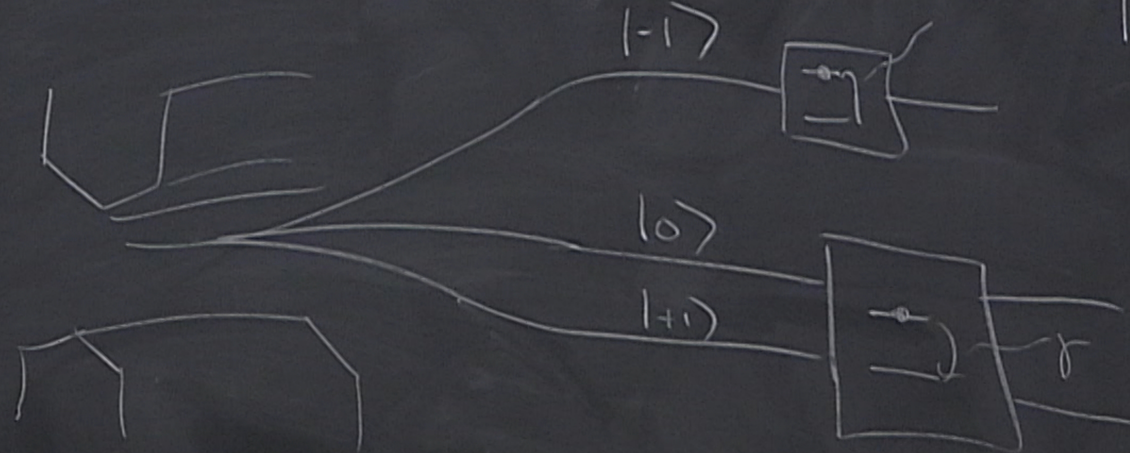
$$P_2 = |b\rangle\langle b| + |c\rangle\langle c|$$

vs. $\boxed{\text{Realistic}}$ postulates

not ideal

$|0\rangle, |1\rangle, |2\rangle$

$|-1\rangle, |0\rangle, |+\rangle$



$P_1 =$

$P_2 =$

Review of Bloch sphere

$$\rho = \frac{1}{2}\left(\mathbb{1} + \vec{a}\cdot\vec{\sigma}\right) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{a_x}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{a_y}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} +$$

$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$

$\vec{a} \in \mathbb{R}^3$ Bloch vector

$\hat{\rho}$ is positive-semi definite.
(non-negative e-values).

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + \frac{a_z}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

vector

$\hat{\rho}$ is positive-semi definite.
(non-negative e-values).

Can show that e-values of $\hat{\rho}$ are

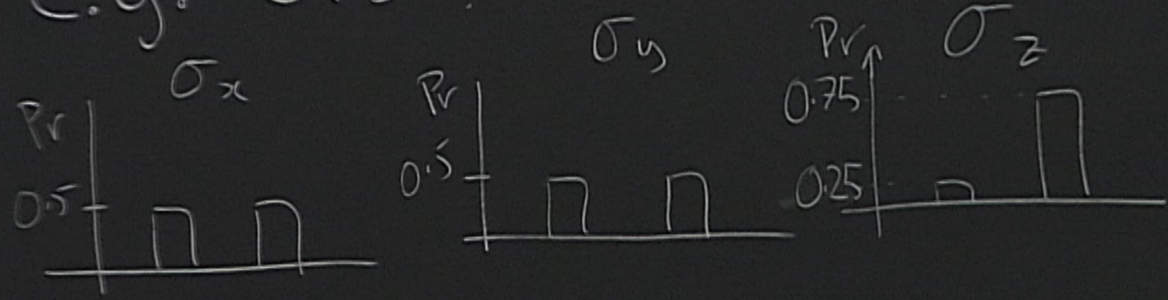
$$\frac{1}{2}\left(1 \pm |\vec{a}|\right) \Rightarrow |\vec{a}| \leq 1$$

For pure states $\rho = |\psi\rangle\langle\psi|$

$$\mathrm{Tr}(\hat{\rho}) = 1 = \mathrm{Tr}(\rho^2) = \frac{1}{2}\left(1 + |\vec{a}|^2\right) \rightarrow |\vec{a}| = 1$$

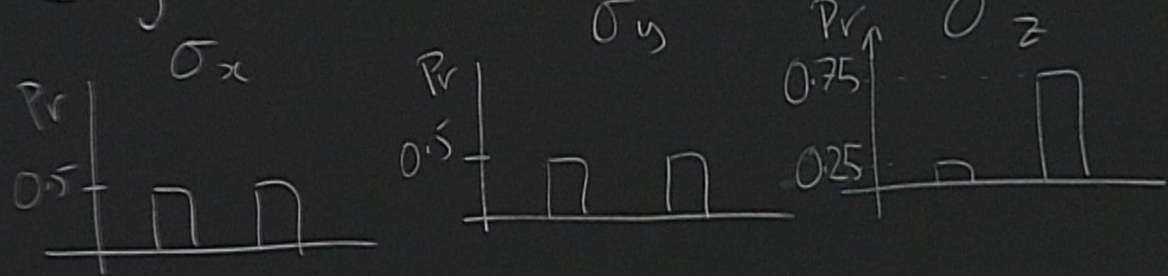
Motivation for Realistic (non-ideal) postulates

e.g. QST



Motivation for Realistic (non-ideal) postulates

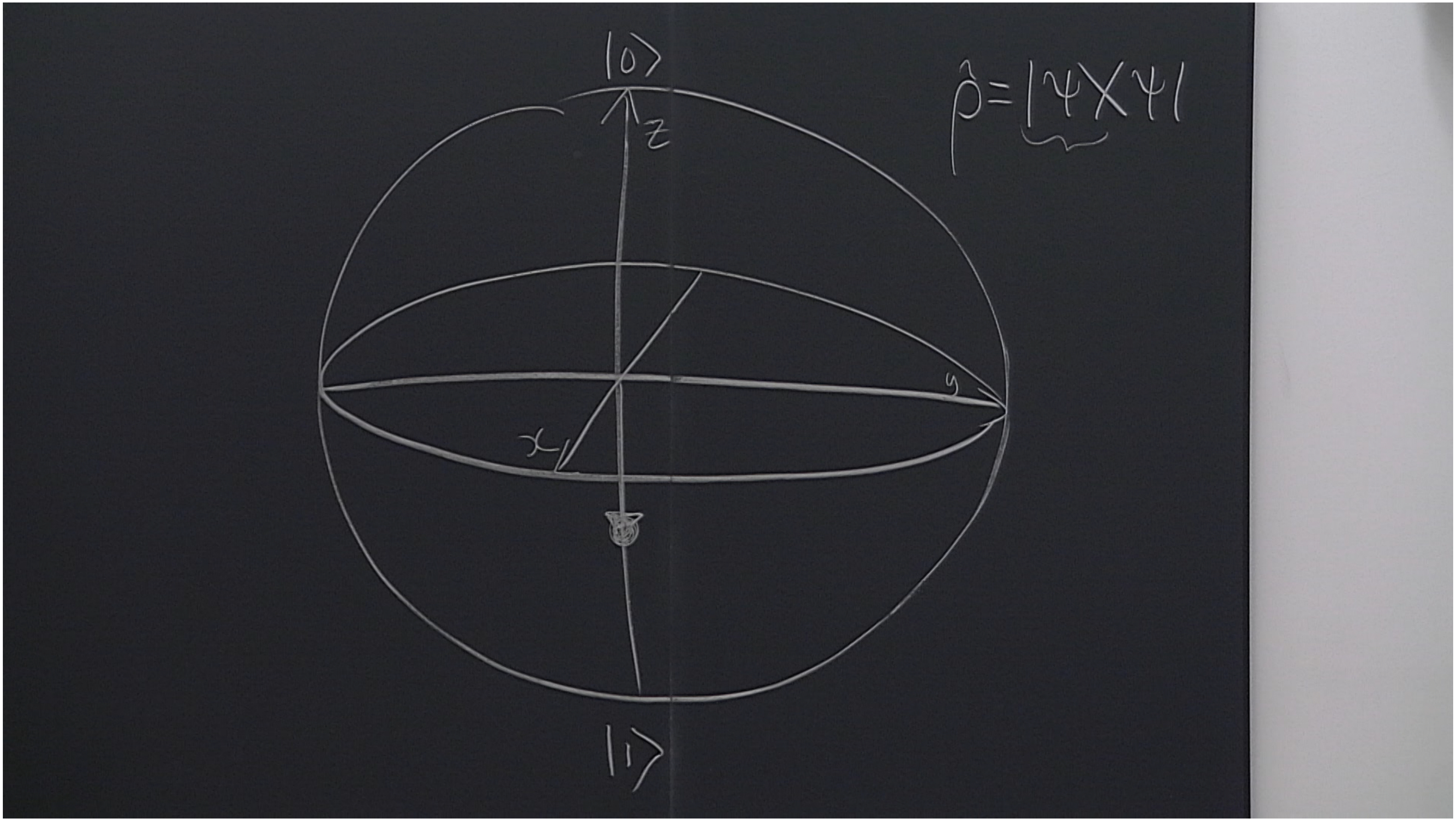
e.g. QST



$\langle \sigma_x \rangle = \frac{1}{2} - \frac{1}{2} = 0$

$\langle \sigma_y \rangle = 0$

$\langle \sigma_z \rangle = 0.25 - 0.75$
$= -0.5$


$|0\rangle$
z
x
y
$|1\rangle$
$\hat{\rho} = |\psi\rangle\langle\psi|$

es

Need more general description of $\hat{\rho}$ than $|\psi\rangle\langle\psi|$

Preparations

Postulate 1. A realistic preparation procedure is described by a density operator, which is a positive semi-definite operator ρ with unit trace, acting on the Hilbert space $\mathcal{H}$.

$\hat{\rho}$ is positive-semi definite $\Rightarrow \langle\psi|A|\psi\rangle \geq 0$
(non-negative e-values) for all $|\psi\rangle$

$\begin{matrix}1 & 0\\ 0 & -1\end{matrix}\Big)$

Can show that e-values of $\hat{\rho}$ are

$$\frac{1}{2}\left(1 \pm |\vec{a}|\right) \Rightarrow |\vec{a}| \leq 1$$

For pure states $\rho = |\psi\rangle\langle\psi|$

$$\mathrm{Tr}(\hat{\rho}) = 1 = \mathrm{Tr}(\rho^2) = \frac{1}{2}\left(1 + |\vec{a}|^2\right) \rightarrow |\vec{a}| = 1$$

Normalization condition.

$$\mathrm{Tr}[\rho] = \sum_i \langle \phi_i | \rho | \phi_i \rangle = 1$$

where $\{|\phi_i\rangle\}$ is an O.N. basis for $\mathcal{H}$ that $\hat{\rho}$ acts on.

Any $\hat{\rho}$ can be decomposed:

$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|$$

where p_j are non-negative real no.s

$$\sum_j p_j = 1.$$

n be decomposed:

$$\sum p_j |\psi_j\rangle\langle\psi_j|$$

"probabilistic mixture" or "statistical mixture" of pure states $|\psi_j\rangle$

re non-negative real no.s

mposed:

$\psi_i|$

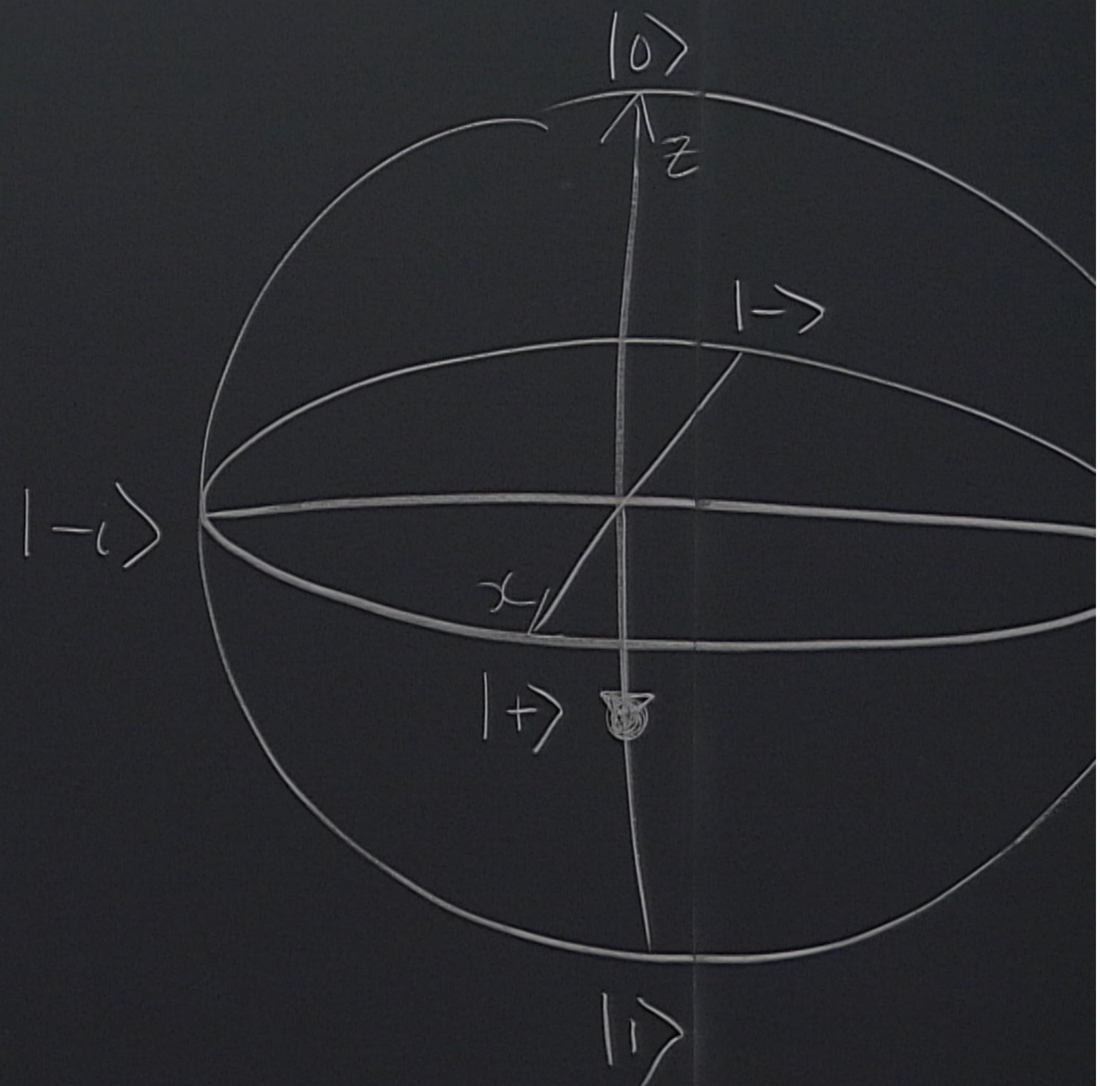
gative real nos.

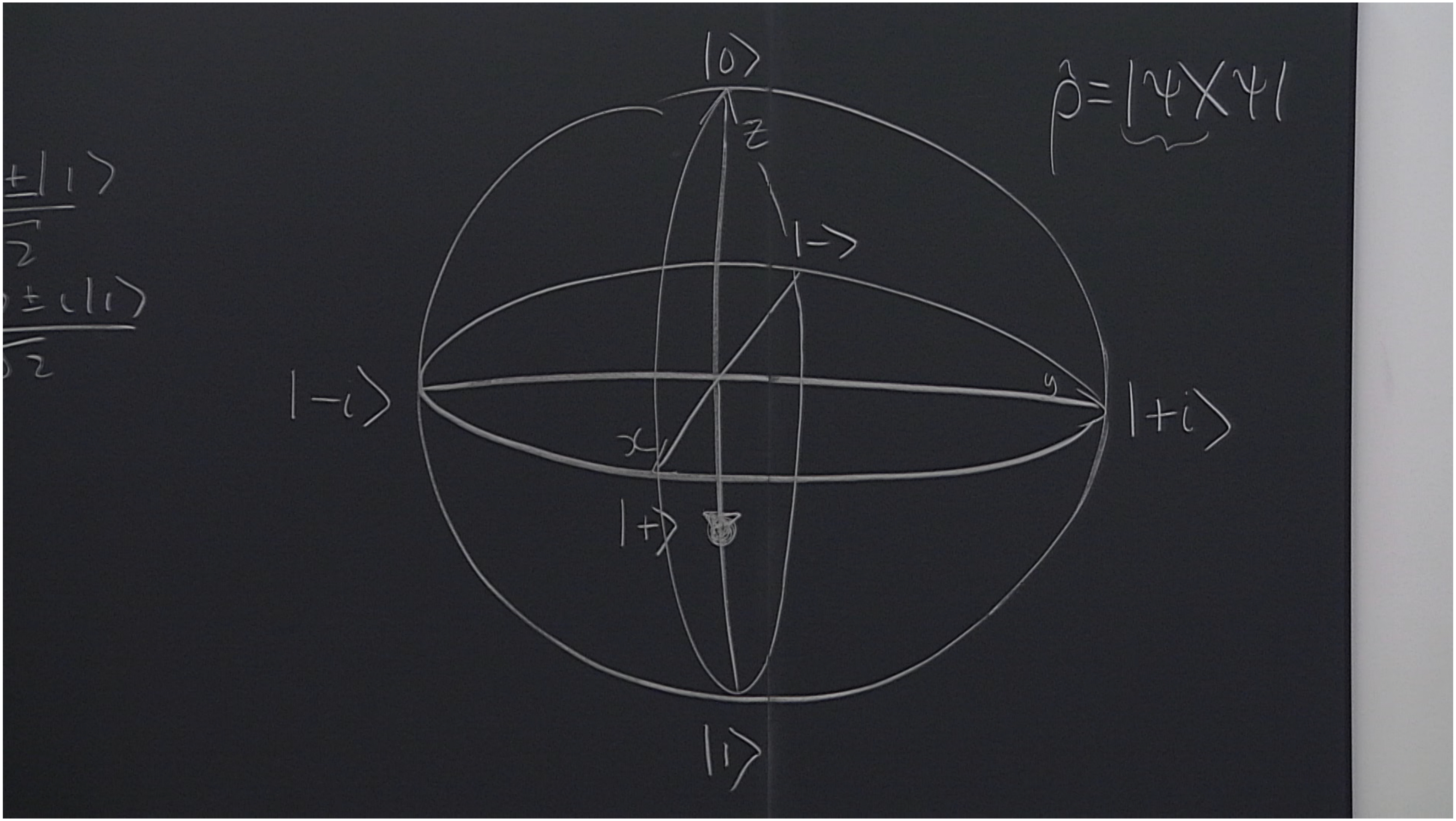
"probabilistic mixture" or "statistical mixture" of pure states $|\psi_i\rangle$

Note: $|\psi_i\rangle$ do not have to be orthogonal.

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

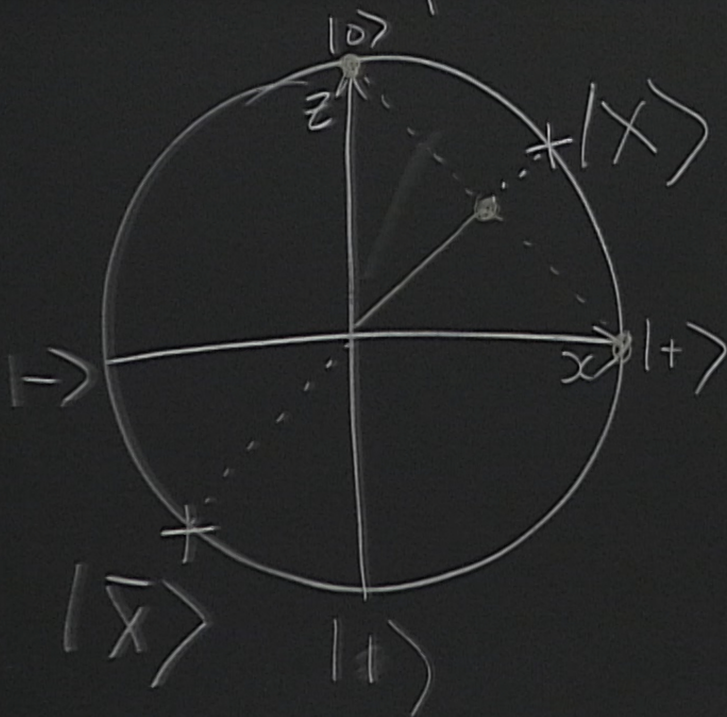
$$|\pm i\rangle = \frac{|0\rangle \pm i|1\rangle}{\sqrt{2}}$$





|0⟩
$\hat{\rho} = |\psi\rangle\langle\psi|$
z
|−⟩
|−i⟩
y
|+i⟩
x
|+⟩
|1⟩

e.g. $\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|+\rangle\langle +|$



This decomposition is not unique

$$= a|x\rangle\langle x| + (1-a)|\bar{x}\rangle\langle\bar{x}|$$

where $a = (2+\sqrt{2})/4$

$$|x\rangle = \cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle$$

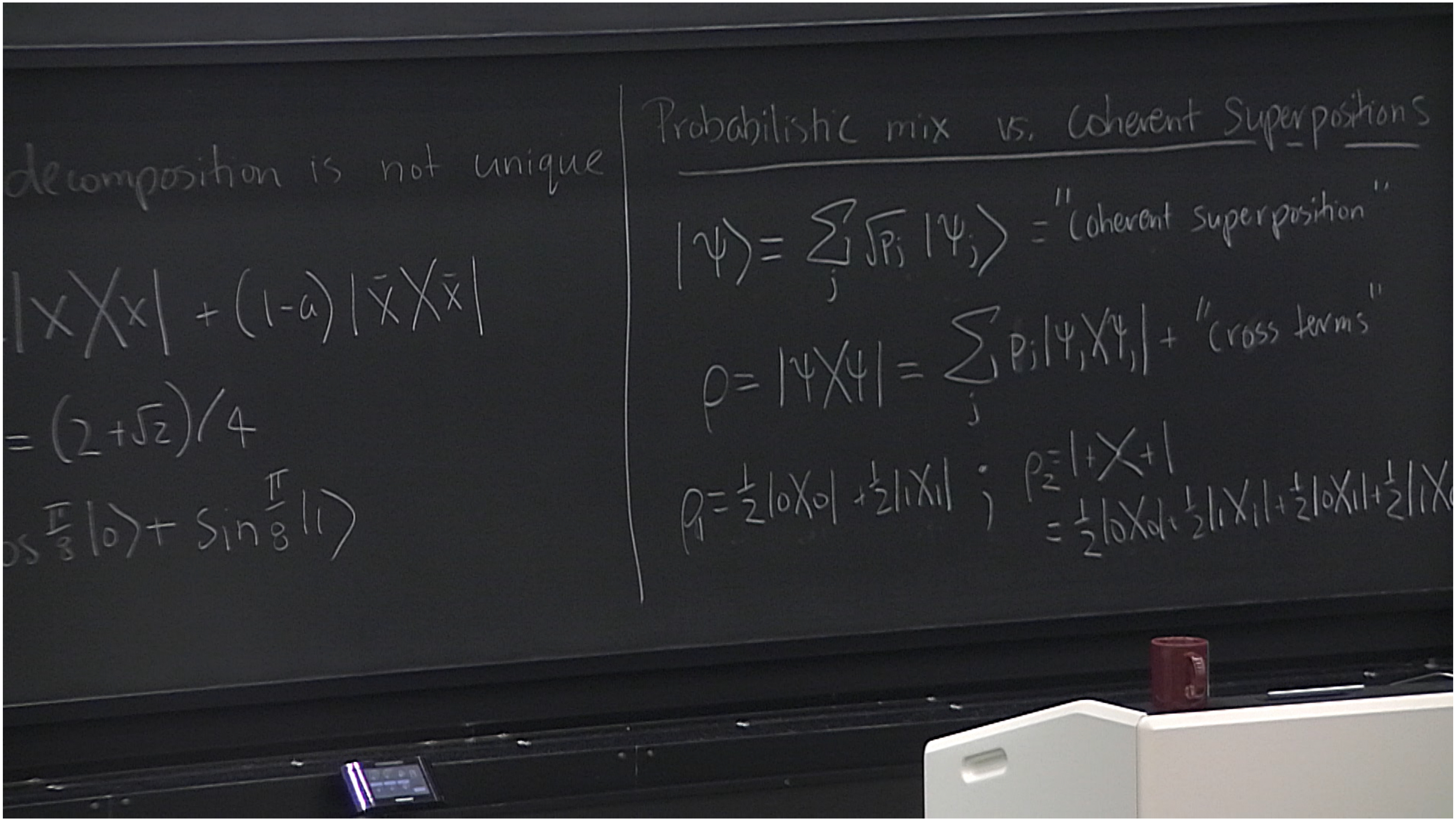
unique

Probabilistic mix vs. Coherent Superpositions

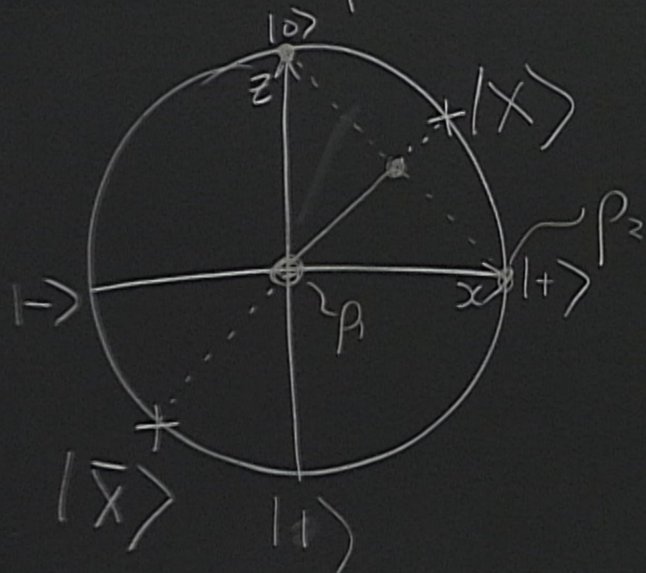
$$|\psi\rangle = \sum_j \sqrt{p_j}\, |\psi_j\rangle = \text{"coherent superposition"}$$

$$\rho = |\psi\rangle\langle\psi| = \sum_j p_j |\psi_j\rangle\langle\psi_j| + \text{"cross terms"}$$

$$\rho = \tfrac{1}{2}|0\rangle\langle 0| + \tfrac{1}{2}|1\rangle\langle 1| \; ; \quad \rho = |+\rangle\langle +|$$
$$= \tfrac{1}{2}|0\rangle\langle 0| + \tfrac{1}{2}|1\rangle\langle 1| + \tfrac{1}{2}|0\rangle\langle 1| + \tfrac{1}{2}|1\rangle\langle 0|$$


decomposition is not unique
$|x\rangle\langle x| + (1-a)|\bar{x}\rangle\langle\bar{x}|$
$= (2+\sqrt{2})/4$
$\cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle$
Probabilistic mix vs. Coherent Superpositions
$|\psi\rangle = \sum_j \sqrt{p_j}\,|\psi_j\rangle$ = "coherent superposition"
$\rho = |\psi\rangle\langle\psi| = \sum_j p_j |\psi_j\rangle\langle\psi_j|$ + "cross terms"
$\rho_1 = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$; $\rho_2 = |+\rangle\langle +|$
$= \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| + \frac{1}{2}|0\rangle\langle 1| + \frac{1}{2}|1\rangle\langle$

e.g. $\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|+\rangle\langle +|$



This decomposition is

$= a|\chi\rangle\langle\chi| + (1-a)$

where $a = (2+\sqrt{2})/4$

$|\chi\rangle = \cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle$

Purity

$\hat{\rho} = |\psi\rangle\langle\psi| = \hat{\rho}^2$

$[\quad]$ $[\quad]$

Can compare elements of ρ & ρ^2
but takes a long time for big systems

Or, can check

$\underbrace{Tr[\rho^2]}_{\text{"purity"}} = 1$

r, can check.

$$\underbrace{Tr[\rho^2]}_{\text{"purity"}} = 1$$

$$\underset{\text{max. mixed}}{\underset{\uparrow}{\frac{1}{d}}} \leq Tr[\rho^2] \leq \underset{\text{pure.}}{\underset{\uparrow}{1}}$$

$$\rho_{max,mix} = \frac{1}{d}\mathbb{1}$$

2D $\quad \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$

^{2}

systems

$\leq \mathrm{Tr}[\rho^2] \leq 1$

$$\frac{1}{3}|a\rangle\langle a| + \frac{1}{3}|b\rangle\langle b| + \frac{1}{3}|c\rangle\langle c|$$

mixed

pure

$\frac{1}{d}\mathbb{1}$

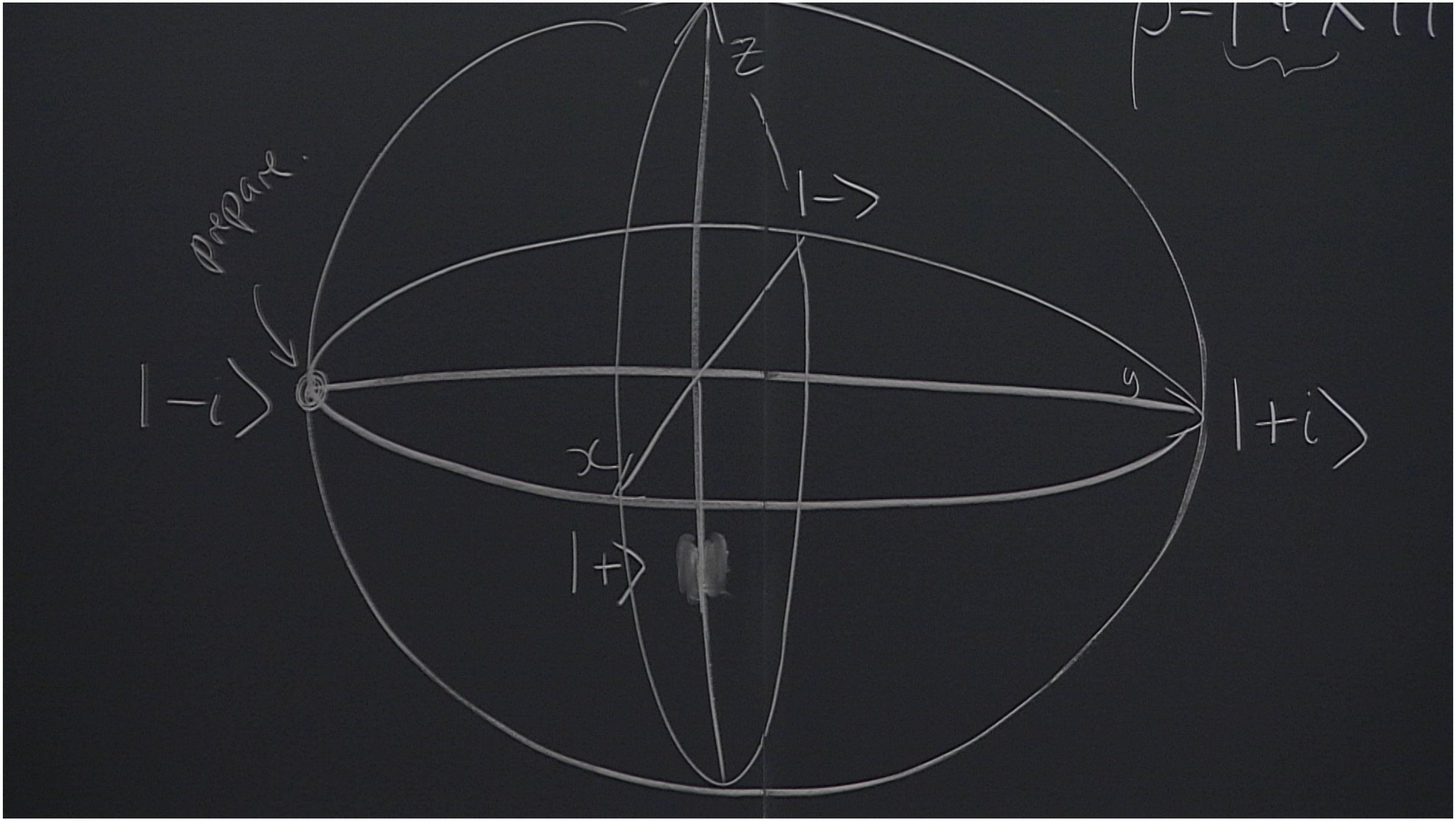
$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$

Q. Does positive s.d.

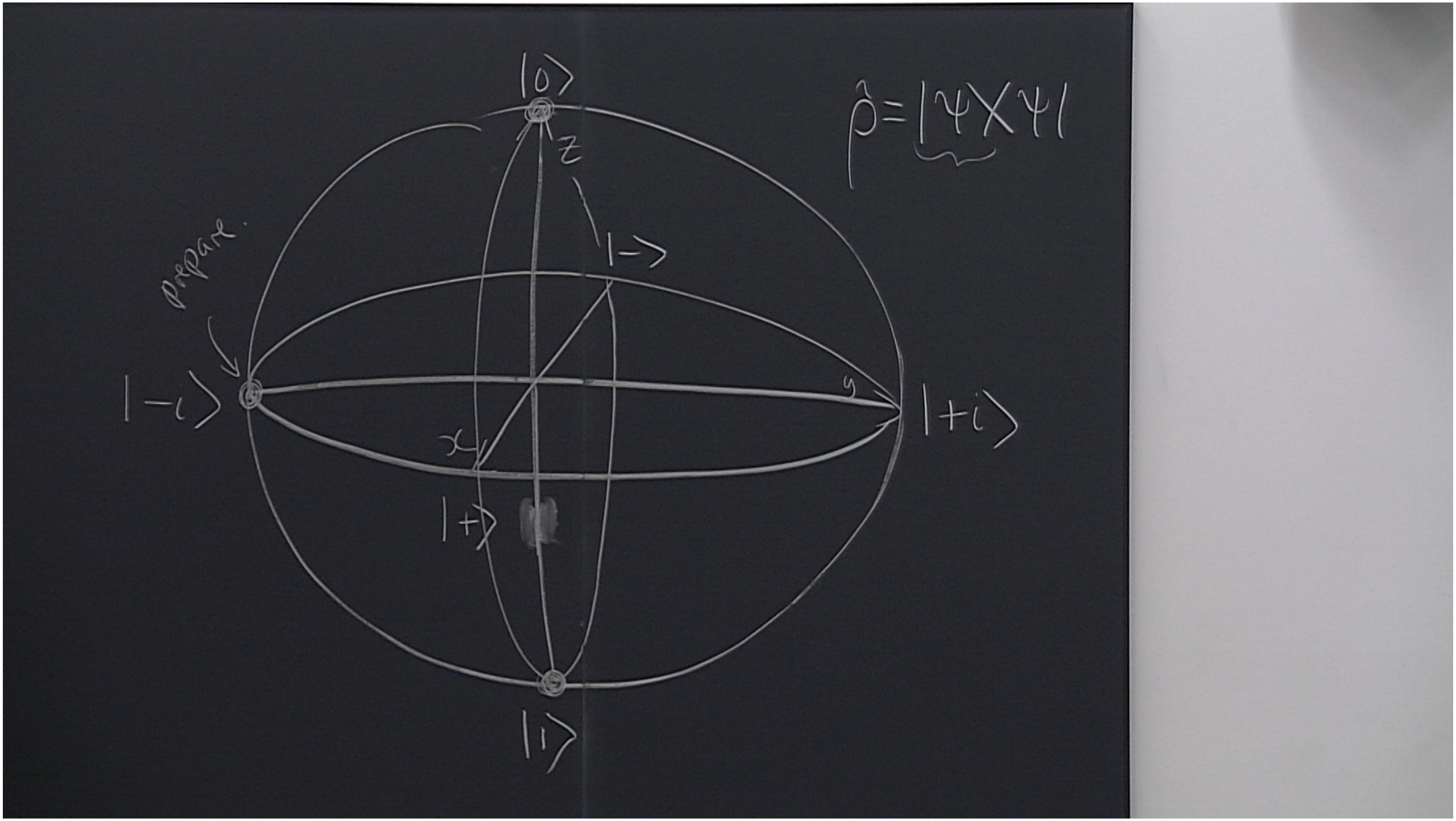
imply self adjoint?

$$|\pm\rangle = \frac{|0\rangle \pm |}{\sqrt{2}}$$

$$|\pm i\rangle = \frac{|0\rangle \pm}{\sqrt{2}}$$



z
prepare.
|−⟩
y
|−i⟩
|+i⟩
x
|+⟩


|0⟩
z
|−⟩
prepare.
|−i⟩
y
|+i⟩
x
|+⟩
|1⟩
$\hat{\rho} = |\psi\rangle\langle\psi|$

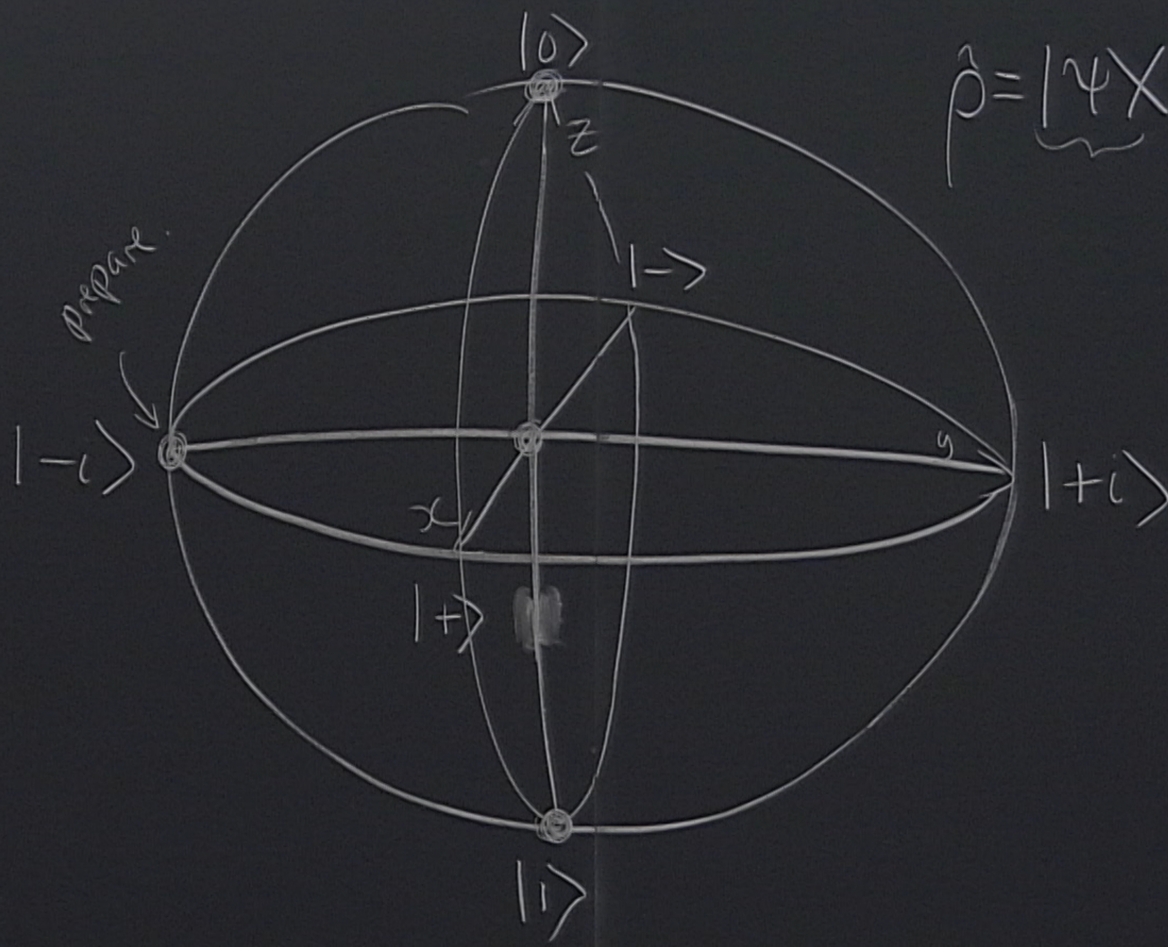
$$\hat{\rho} = \underbrace{|\psi\rangle}_{}\langle\psi|$$

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

$$|\pm i\rangle = \frac{|0\rangle \pm i|1\rangle}{\sqrt{2}}$$

$$|\psi_{in}\rangle = |-i\rangle\langle -i|$$

$$\rho_{out} = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$



Postulate 2. (Transformations)

Realistic dynamical transformations are prescribed by completely positive trace-preserving (CPTP) maps $\mathcal{E}(\rho)$.

e

A positive map takes a positive operator to a positive operator.

Complete positivity:

$$\rho_{AB}(t) = \underbrace{\mathcal{E}_A \otimes \mathbb{1}_B}(\rho_{AB}(0))$$ is positive if $\rho_{AB}(0)$ is positive

$$\frac{1}{d} \leq Tr[\rho^2] \leq 1$$

$\uparrow$ ax. mixed $\qquad \uparrow$ pure.

$$_{max} = \frac{1}{d}\mathbb{1}$$

$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

$$\frac{1}{3}|a\rangle\langle a| + \frac{1}{3}|b\rangle\langle b| + \frac{1}{3}|c\rangle\langle c|$$

$$\mathcal{H}_A \otimes \mathcal{H}_B$$