Title: PSI 2019/2020 - Quantum Theory (Branczyk) - Lecture 1

Speakers: Agata Branczyk

Collection: PSI 2019/2020 - Quantum Theory (Branczyk/Dupuis)

Date: September 04, 2019 - 9:00 AM

URL: http://pirsa.org/19090044

Quick review

QT can be divided:

(i) Identifying the level structure of some systems.

$$(\mathcal{H}, |\psi\rangle, H, A)$$

(ii) Predicting the statistics associated with an agent's interaction w/ system.

Two approaches to fix this:

1) by Dirac: "rigged" Hilbert Space
→ extends Hilbert space.

2) by von Neumann: introduces
a generalized spectral decomposition.

Lectures by J. Emerson PIRSA: 15090045 & 15090046

Preparations

Postulate 1: An ideal preparation is described by a Hilbert space vector called a pure state $|\psi\rangle$.

Some properties of Hilbert spaces:

- There is a dual space $\mathcal{H}^\dagger$ to a Hilbert space $\mathcal{H}$ that of all continuous linear functionals mapping elements of the Hilbert space to comlex scalars.

- Riesz' representation theorem, in Dirac notation $\langle\psi| = |\psi\rangle^{\dagger}$ refers to a unique element of the dual space.

Also:

- $e^{i\phi}|\psi\rangle \overset{\text{physically indistinguishable}}{\Longleftrightarrow} |\psi\rangle$
- If you don't like this, $\hat{\rho} = e^{i\phi}|\psi\rangle\langle\psi|e^{-i\phi} = |\psi\rangle\langle\psi|$

Can express any pure state.

$$|\psi\rangle = \sum_j c_j |\phi\rangle$$

where $\{\phi_j\}$ is an orthonormal basis of $\mathcal{H}$

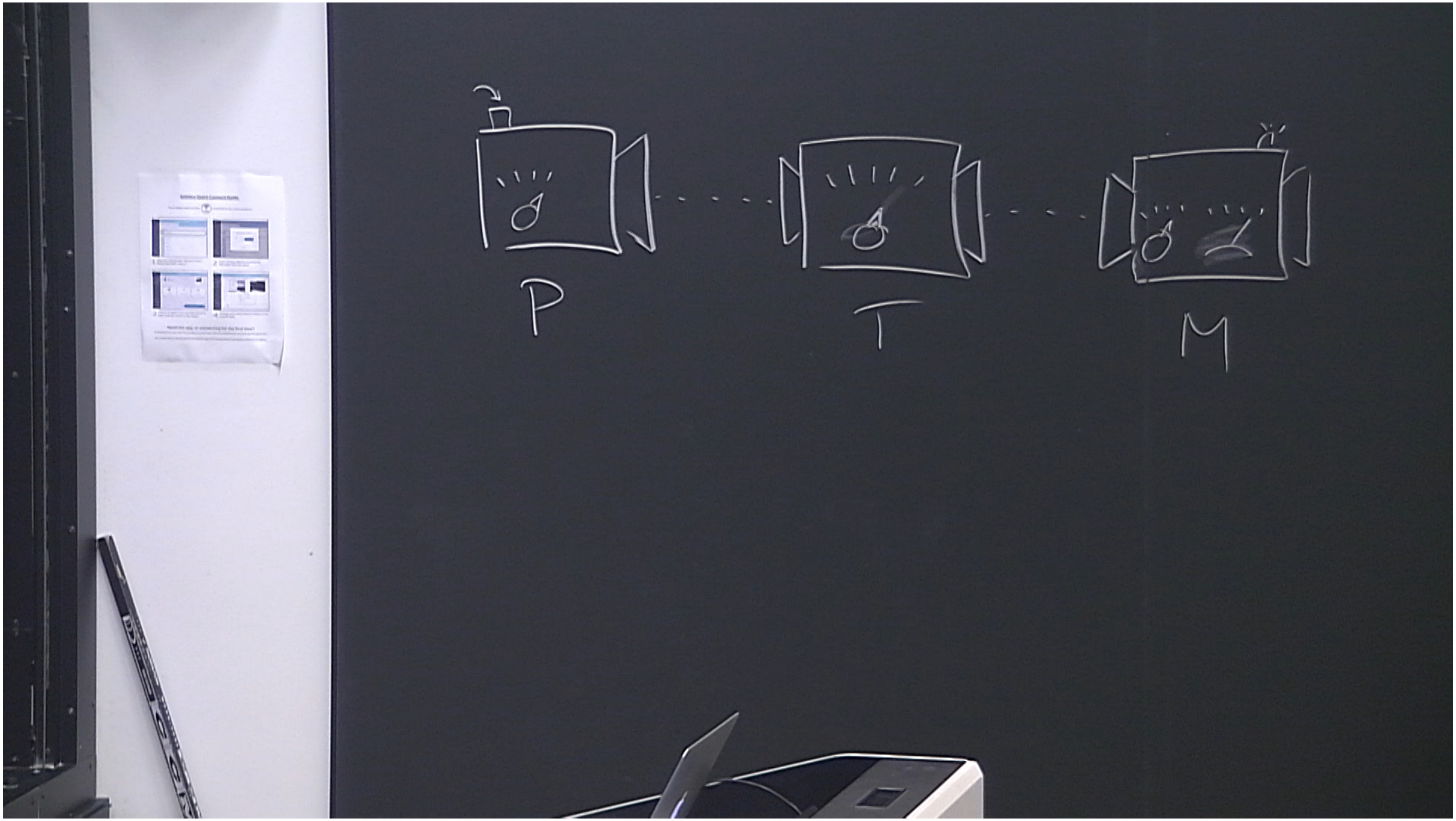
satisfying $\langle\phi_j|\phi_i\rangle = \delta_{ij}$

The condition $\langle\psi|\psi\rangle = 1$ $\Rightarrow$ normalization condition for the ket.

e.g. $\sum_j |c_j|^2 = 1$.

e.g. Preparations.

1. Two-level atom : $|\psi\rangle = \frac{1}{\sqrt{2}}|g\rangle + \frac{1}{\sqrt{2}}|e\rangle$
2. Photon polarization: $|R\rangle = \frac{1}{\sqrt{2}}|H\rangle + \frac{i}{\sqrt{2}}|V\rangle$
3. Photon # superposition : $|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$
4. Coherent state: $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$
5. Spin 1 particle $|\psi\rangle = \frac{1}{\sqrt{3}}|+1\rangle + \frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|-1\rangle$
6. Cat $|cat\rangle = \frac{1}{\sqrt{2}}|alive\rangle + \frac{1}{\sqrt{2}}|dead\rangle$


P
T
M

Transformations

Postulate 2: Ideal transformations are generated by a linear operator U,

$$|\psi(t)\rangle = U(t_2,t_1)\,|\psi(t_1)\rangle$$

satisfying

$$i\hbar\,\frac{\partial U(t_2,t_1)}{\partial t_2} = H(t_2)\,U(t_2,t_1)$$

where $H(t)$ is a self-Adjoint operator reprenting the system energy function and t is time, subject to the initial condition $U(t_i,t_i)=\mathbb{1}$.

Some comments.

- (1) implies that U is unitary
- for bounded operators self-adjoint & Hermitian are same thing

Q: Why should H be self adjoint a priori?

Q: precise definition of "bounded"

Examples on Bloch spere

$$|\psi\rangle = \cos\frac{\chi}{2}|0\rangle + e^{i\phi}\sin\frac{\chi}{2}|1\rangle$$

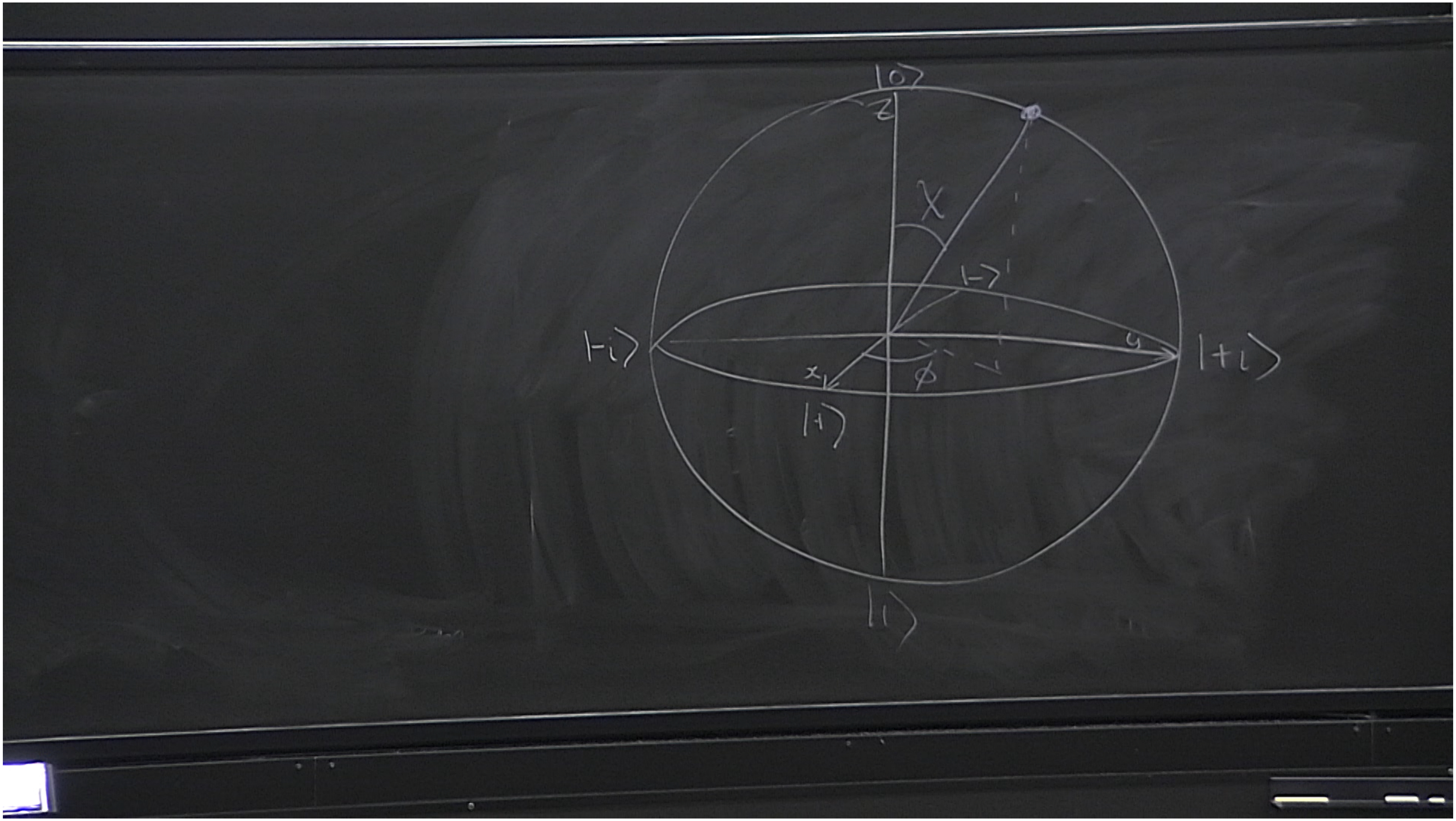
Unitaries = Rotations.

$$R_x(\theta) = e^{-i\frac{\theta\sigma_x}{2}}\ ;\ R_y(\theta) = e^{-i\frac{\theta\sigma_y}{2}}\ ;\ R_z(\theta) = e^{-i\frac{\theta\sigma_z}{2}}$$

$$\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

$$\sigma_x = |+\rangle\langle +| - |-\rangle\langle -| \quad \text{where } |\pm\rangle = \frac{|0\rangle \pm |1\rangle}{2}$$

$$\sigma_y = |+i\rangle\langle +i| - |-i\rangle\langle -i|$$



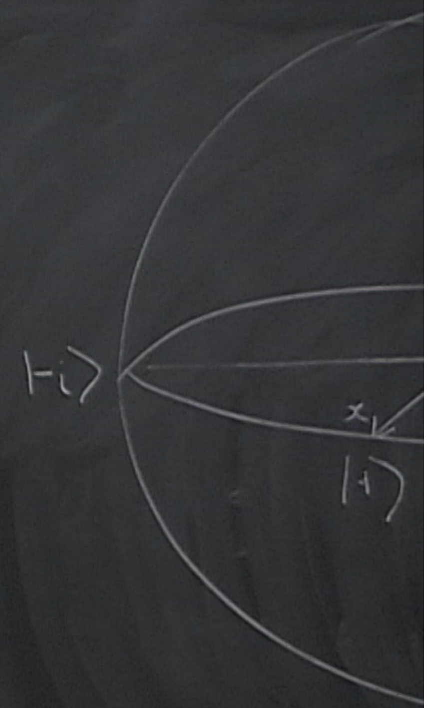
|0⟩
z
χ
|−⟩
|−i⟩
y
|+i⟩
x
ϕ
|+⟩
|1⟩

More generally

$$R_{\hat{n}}(\theta) = e^{-i\frac{\theta}{2}\hat{n}\cdot\vec{\sigma}}$$

where $\hat{n}$ is a unit vector

$$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$$



Examples on Bloch spere

$$|\psi\rangle = \cos\tfrac{\chi}{2}|0\rangle + e^{i\phi}\sin\tfrac{\chi}{2}|1\rangle$$

Unitaries = Rotations.

$$R_x(\theta) = e^{-i\frac{\theta\sigma_x}{2}}\ ;\ R_y(\theta) = e^{-i\frac{\theta\sigma_y}{2}}\ ;\ R_z(\theta) = e^{-i\frac{\theta\sigma_z}{2}}$$

$\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$

$\sigma_x = |+\rangle\langle +| - |-\rangle\langle -|$ where $|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{2}$

$\sigma_y = |+i\rangle\langle +i| - |-i\rangle\langle -i|$

More gener

$R_{\hat{n}}(\theta) =$

where $\hat{n}$ is

$\vec{\sigma} = (\sigma_x, \sigma_y$

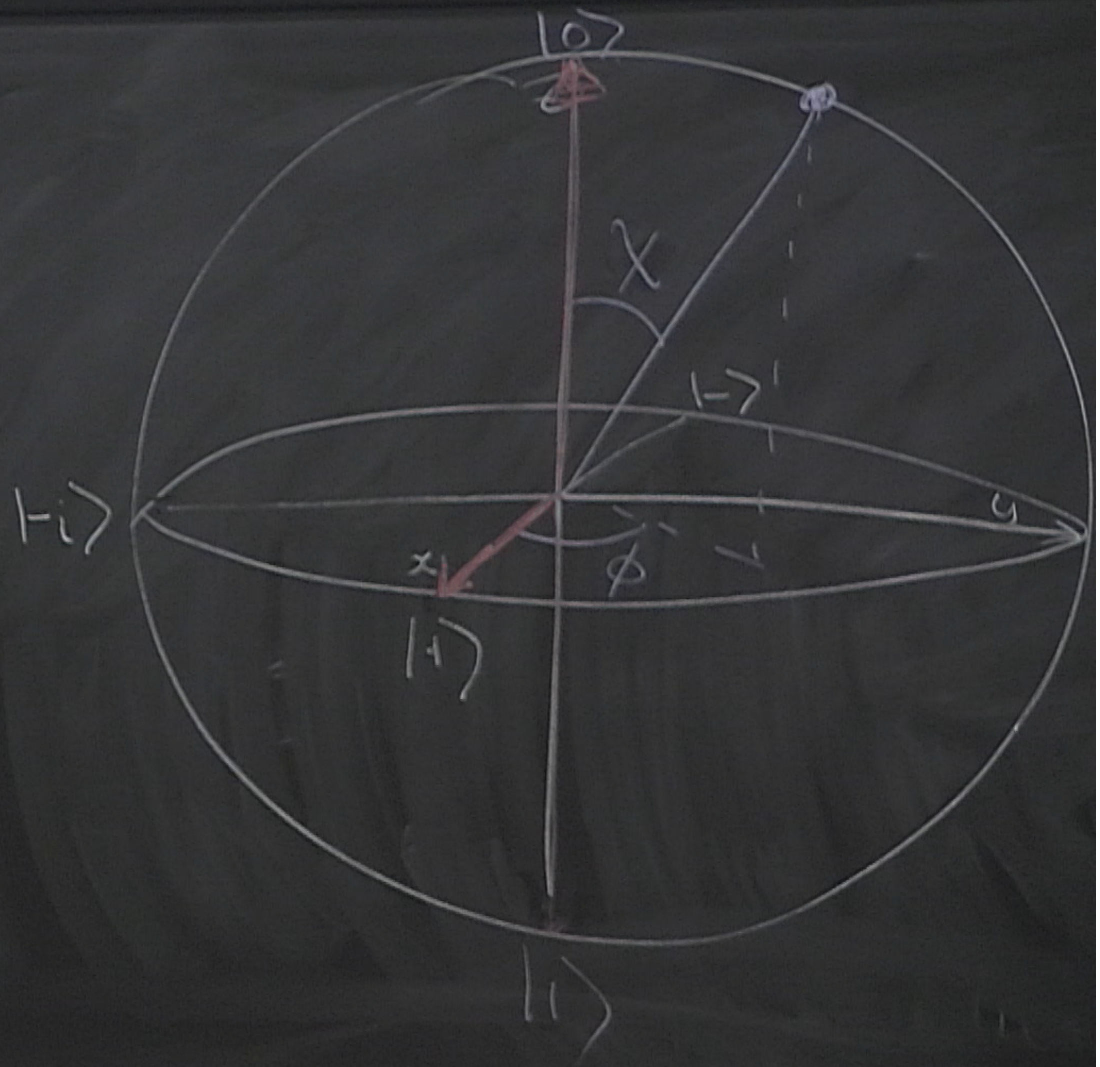
tor

initial $|+\rangle$

apply $R_y(\frac{\pi}{2})$

initial $|0\rangle$

apply $R_z(\frac{\pi}{3})$



Measurements

Postulate 3: An ideal measurement is represented by a collection $\{P_m\}$ of projection operators. The index m refers to the measurement outcomes. The probability of finding outcome m, given preparation $|\psi\rangle$, is $Pr(m) = \langle\psi|P_m|\psi\rangle$.

$\hat{A}$ admits a spectral decomposition.

$$\hat{A} = \sum_{l} a_l P_l$$

Can derive:

$$\langle A \rangle = \sum_{l} Pr(l)\, a_l$$

$$= \sum_{l} a_l \langle \psi | P_l | \psi \rangle$$

$$= \langle \psi | \hat{A} | \psi \rangle$$

After measurement, given an outcome m, the state is

$$|\psi_m\rangle = \frac{P_m|\psi\rangle}{\sqrt{Pr(m)}}$$

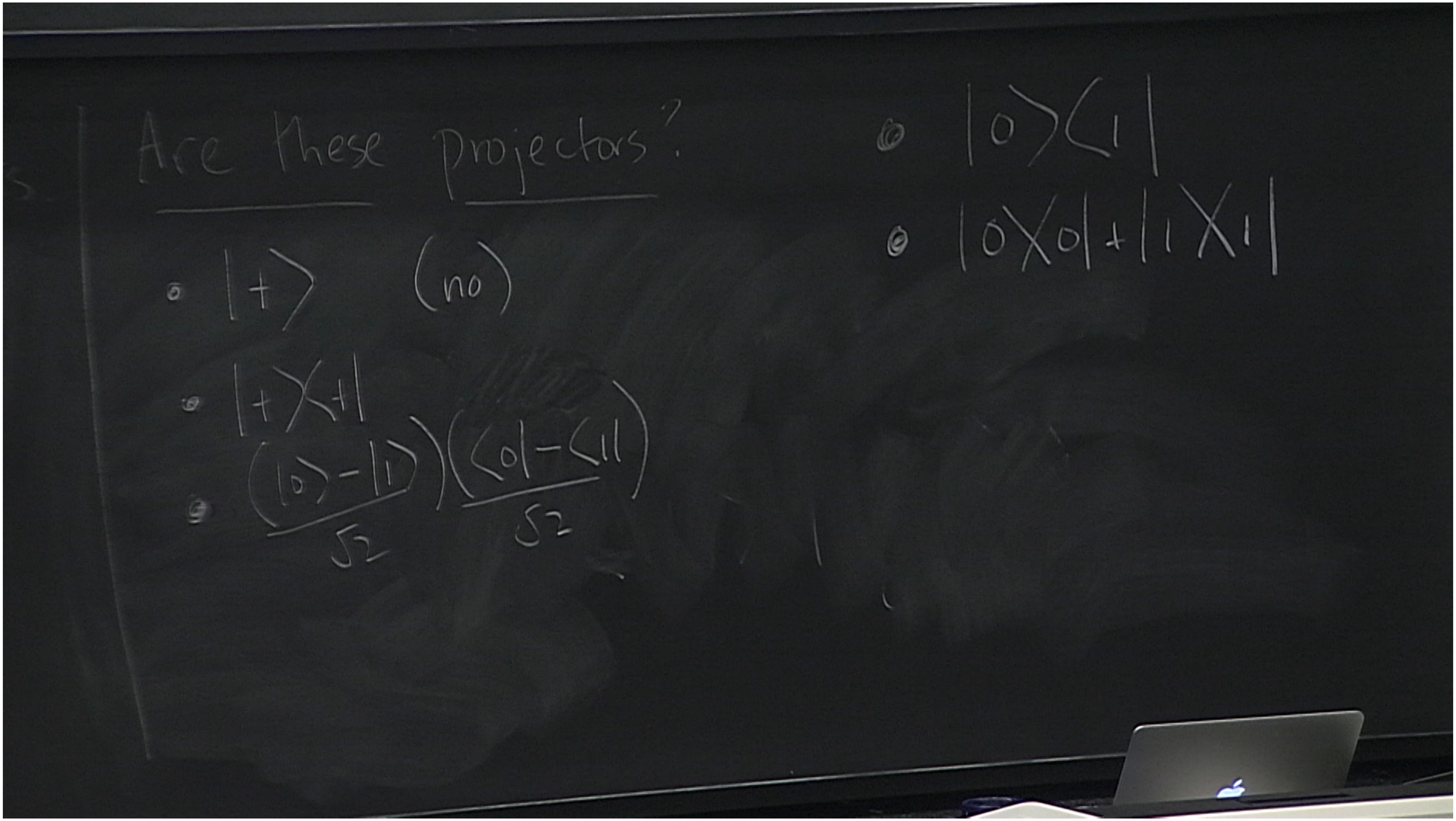
The projection operators salisfy:

$$P_\ell P_k = \delta_{\ell k} P_\ell \quad \text{and} \quad P_\ell = P_\ell^\dagger.$$

Q: Why should H be self adjoint a priori?

Q: precise definition of "bounded"

Q: are projectors implicitly orthogonal?


Are these projectors?
$|+\rangle$ (no)
$|+\rangle\langle+|$
$\frac{(|0\rangle-|1\rangle)}{\sqrt{2}}\frac{(\langle0|-\langle1|)}{\sqrt{2}}$
$|0\rangle\langle1|$
$|0\rangle\langle0|+|1\rangle\langle1|$

e.g. Preparations.

1. Two-level atom : $|\psi\rangle = \frac{1}{\sqrt{2}}|g\rangle + \frac{1}{\sqrt{2}}|e\rangle$
2. Photon polarization: $|R\rangle = \frac{1}{\sqrt{2}}|H\rangle + \frac{i}{\sqrt{2}}|V\rangle$
3. Photon # superposition : $|\psi\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{\sqrt{2}}|2\rangle$
4. Coherent state: $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle$
5. Spin 1 particle $|\psi\rangle = \frac{1}{\sqrt{3}}|+1\rangle + \frac{1}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|-1\rangle$
6. Cat $|cat\rangle = \frac{1}{\sqrt{2}}|alive\rangle + \frac{1}{\sqrt{2}}|dead\rangle$

en an outcome m, the state is.

satisfy:

nd $P_\ell = P_\ell^\dagger$.

Don't say: "I make measurement P_m," because P_m is an outcome of the meas.

Are these projectors

- $|+\rangle$ (no)
- $|+\rangle\langle+|$
- $\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right)\left(\frac{\langle 0|-\langle 1|}{\sqrt{2}}\right)$

Changing meas. basis

in σ_x basis.

Say I want to meas in basis $\{P_+, P_-\}$

but I can only measure in basis $\{P_0, P_1\}$.

in σ_z basis

Changing meas. basis

Say I want to meas in basis $\{P_+, P_-\}$ in σ_x basis.

but I can only measure in basis $\{P_0, P_1\}$. in σ_z basis

$$
\begin{aligned}
Pr(m) &= \langle \psi' | P_m | \psi' \rangle \\
&= \langle \psi | \underbrace{U^\dagger P_m U} | \psi \rangle \\
&= \langle \psi | P'_m | \psi \rangle
\end{aligned}
$$

where $P'_m = U^\dagger P_m U$

basis $\quad$ in σ_x basis.

meas In basis $\{P_+, P_-\}$

measure in basis $\{P_0, P_1\}$.

$\rightarrow$ in σ_z basis

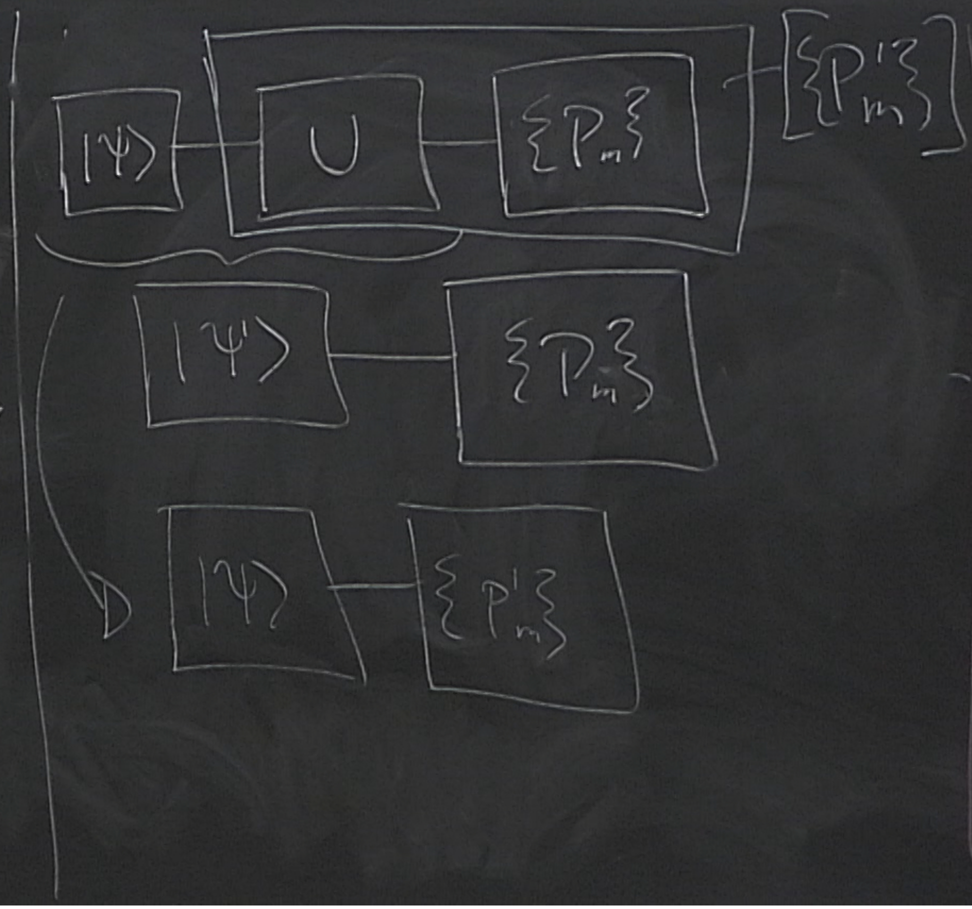
$|\psi'\rangle$

where

$P'_m = U^\dagger P_m U$

$U|\psi\rangle$

$|\psi\rangle$



Quantum State Tomography.

$$|\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Can de

$$\hat{\rho} = \frac{1}{2} \sum_{j=}^{3}$$

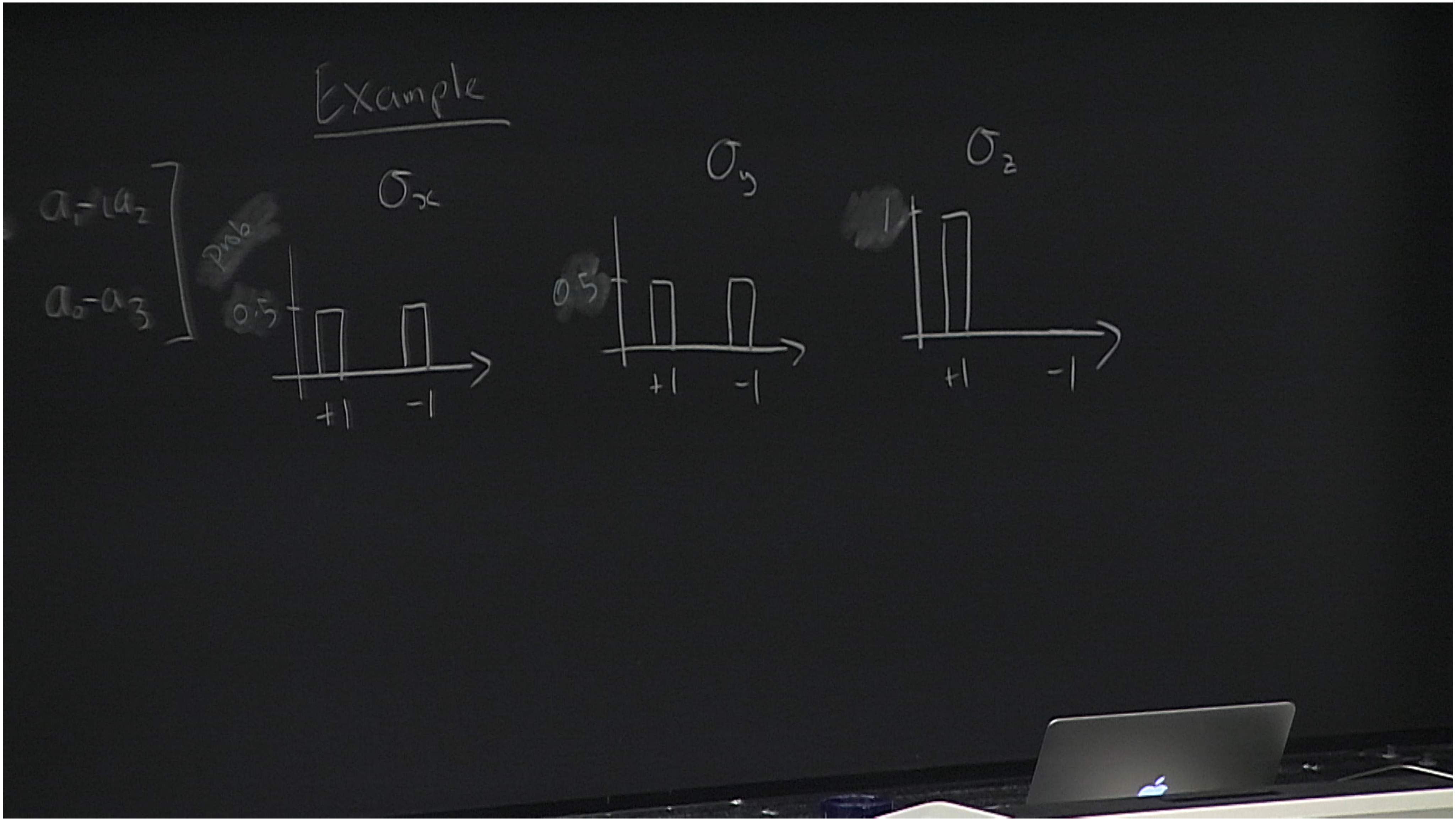
Can decompose:

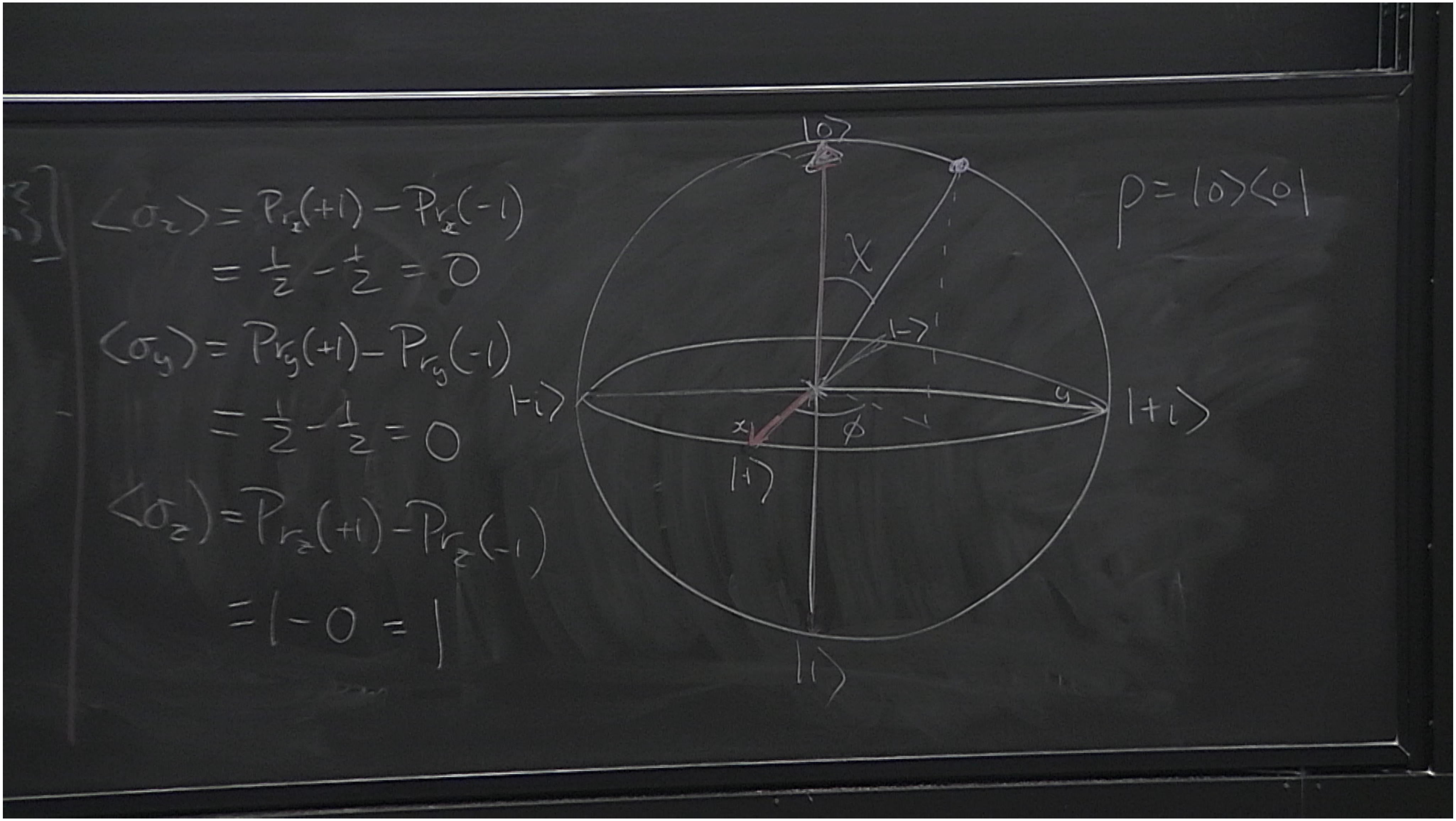
$$\hat{\rho} = \frac{1}{2}\sum_{j=0}^{3} a_j \sigma_j = \frac{1}{2}\begin{bmatrix} a_0 + a_3 & a_1 + i a_2 \\ a_1 - i a_2 & a_0 - a_3 \end{bmatrix}$$

where

$$a_j = \mathrm{Tr}[\sigma_j \hat{\rho}] = \langle \sigma_j \rangle$$

$$= \langle \psi | \sigma_j | \psi \rangle$$


Example
$a_1 - i a_2$
$a_0 - a_3$
prob
0.5
σ_x
+1
-1
σ_y
0.5
+1
-1
σ_z
1
+1
-1


$\langle \sigma_z \rangle = Pr_z(+1) - Pr_z(-1)$
$= \frac{1}{2} - \frac{1}{2} = 0$
$\langle \sigma_y \rangle = Pr_y(+1) - Pr_y(-1)$
$= \frac{1}{2} - \frac{1}{2} = 0$
$\langle \sigma_z \rangle = Pr_z(+1) - Pr_z(-1)$
$= 1 - 0 = 1$
$|0\rangle$
χ
$|-\rangle$
$|-i\rangle$
$|+i\rangle$
y
x
ϕ
$|+\rangle$
$|1\rangle$
$\rho = |0\rangle\langle 0|$