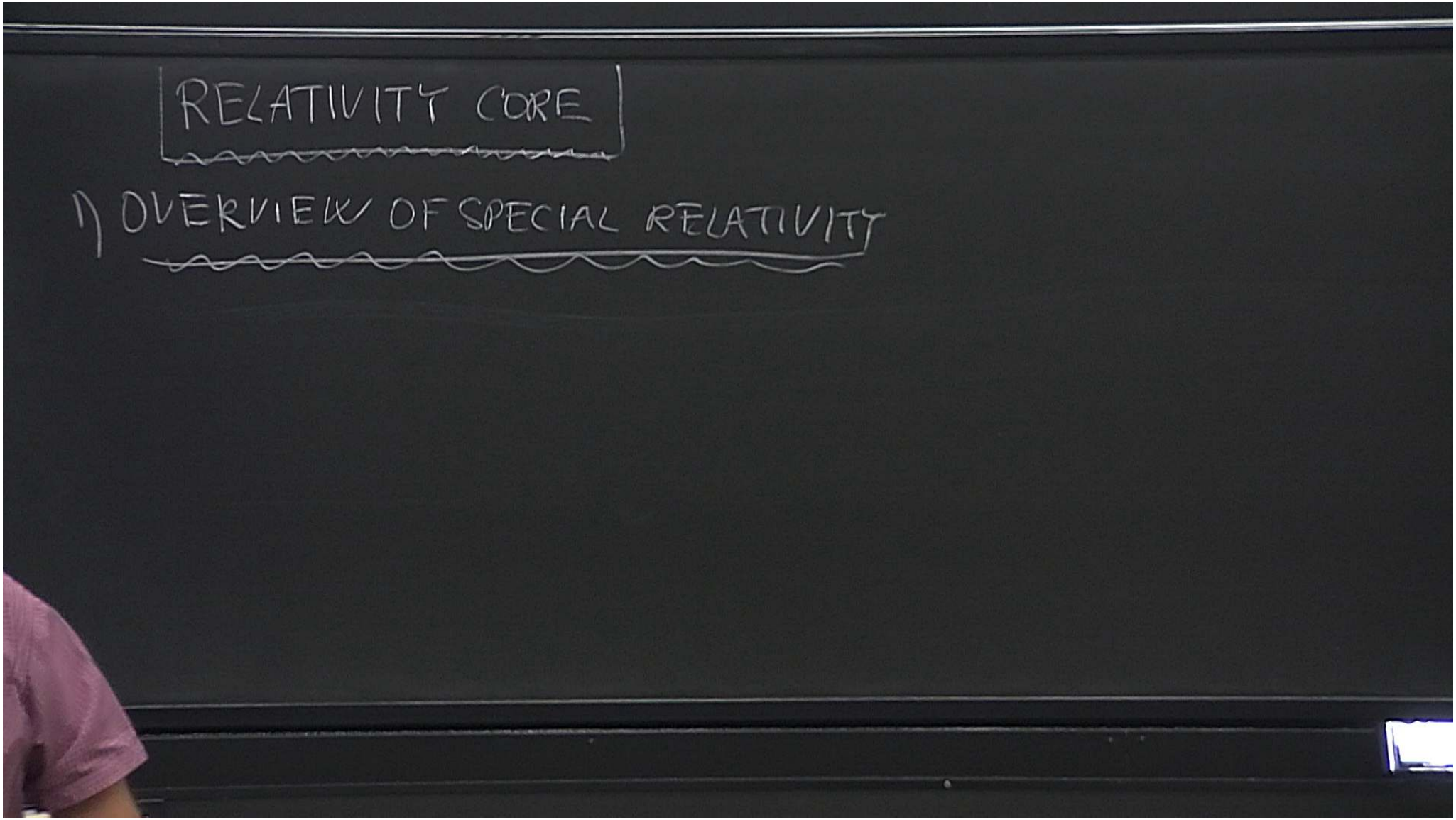
Title: PSI 2019/2020 - Relativity (Kubiznak)

Speakers:

Collection: PSI 2019/2020 - Relativity (Kubiznak)

Date: September 03, 2019 - 9:00 AM

URL: http://pirsa.org/19090043


RELATIVITY CORE
1) OVERVIEW OF SPECIAL RELATIVITY

RELATIVITY CORE

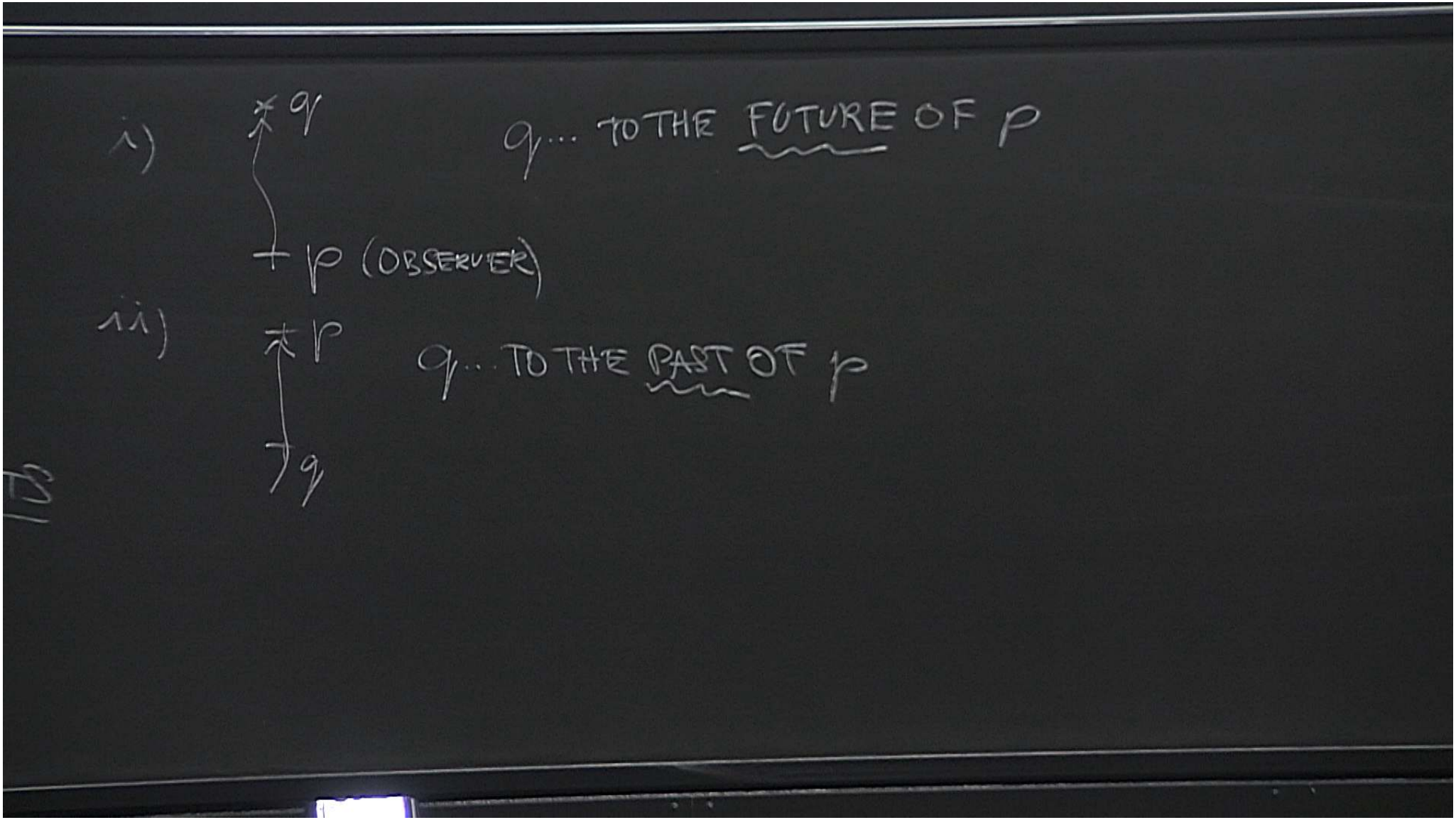
1) OVERVIEW OF SPECIAL RELATIVITY

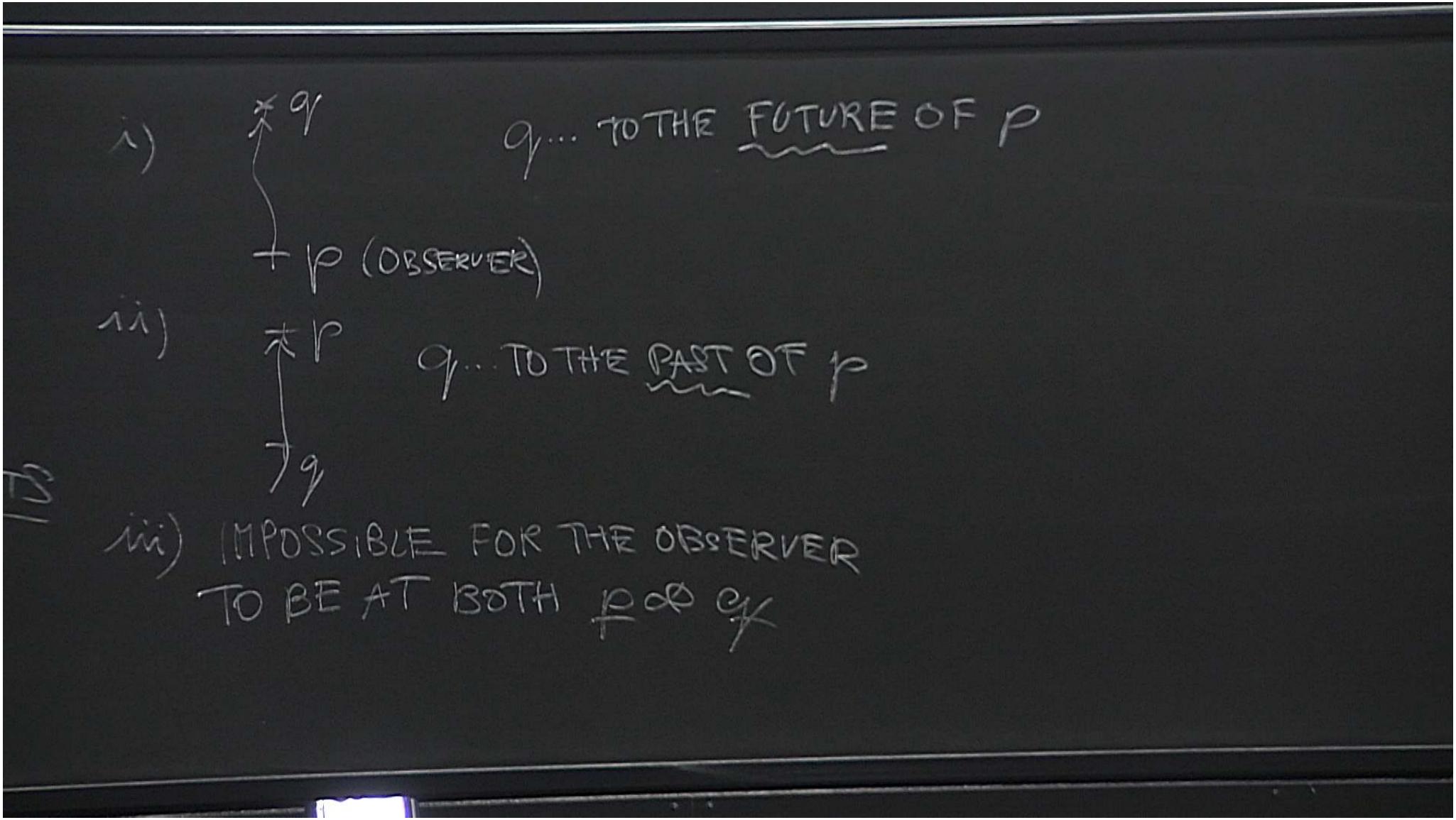
+ P (OBSERVER)

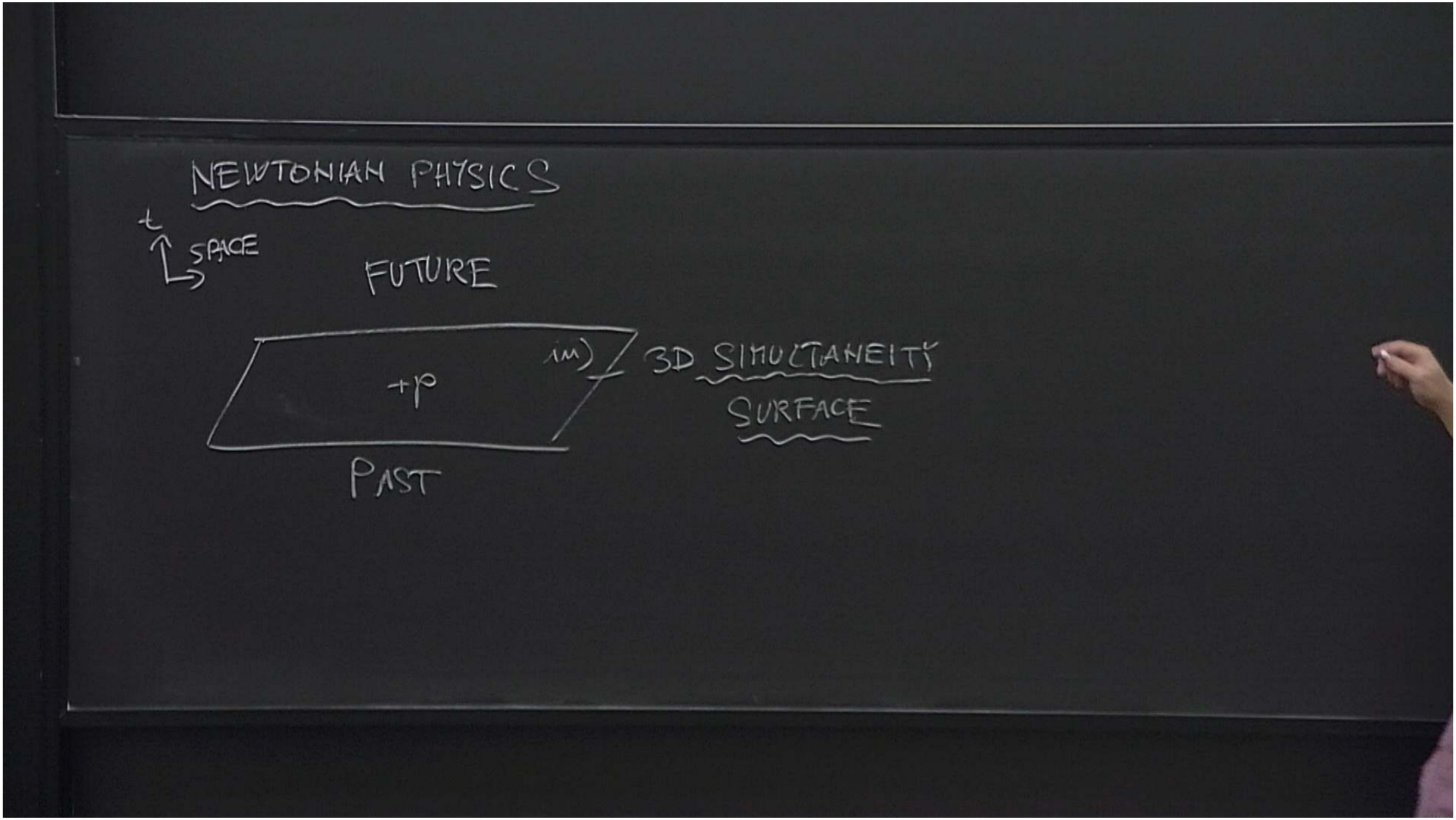
a) CAUSAL STRUCTURE

SPACETIME = CONTINUUM OF SPACETIME EVENTS

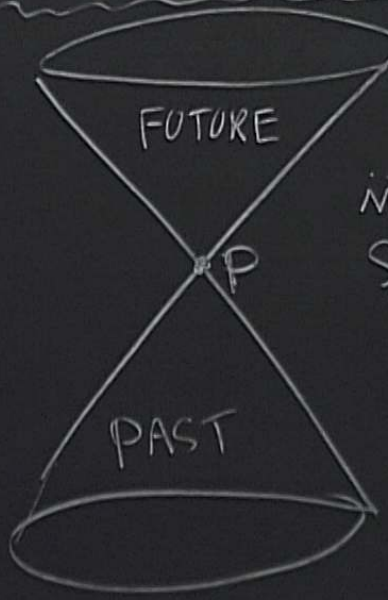
3 MUTUALLY DISTINCT POSSIBILITIES


i)
q
q... TO THE FUTURE OF p
p (OBSERVER)
ii)
p
q... TO THE PAST OF p
q


i)
q
q... TO THE FUTURE OF p
p (OBSERVER)
ii)
p
q... TO THE PAST OF p
q
iii) IMPOSSIBLE FOR THE OBSERVER
TO BE AT BOTH p & q


NEWTONIAN PHYSICS
t
SPACE
FUTURE
+p
3D SIMULTANEITY
SURFACE
PAST

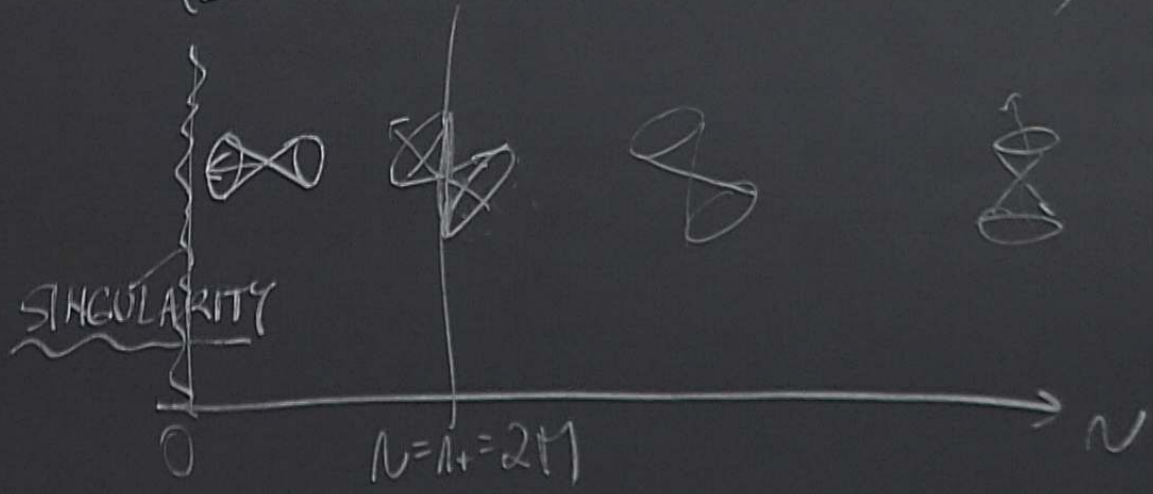
SPECIAL RELATIVITY



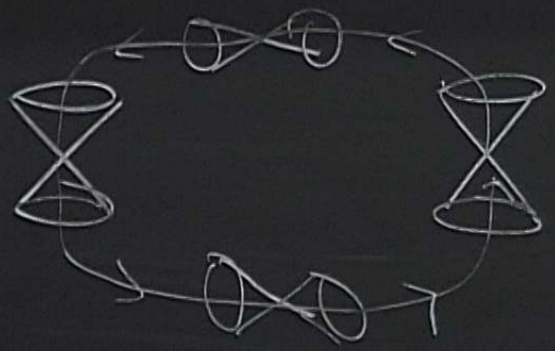
iii)
SPACELIKE
SEPARATED
(4D SET)

REMARK: LIGHT CONE STRUCTURE IS ALSO IMPORTANT IN GENERAL RELATIVITY .. BUT ONLY LOCALLY

(LIGHT CONES CAN "TWIST AROUND")



BEWARE: SOLUTIONS WITH CLOSED TIMELIKE CURVES (CTCs)

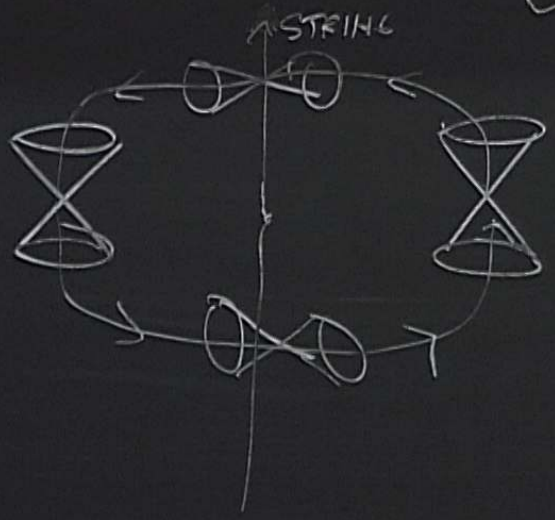


FOR EXAMPLE: KERR SPACETIME

(ONLY ROTATING BH SOLUTION)

... HIDING BEHIND HORIZON

BEWARE. SOLUTIONS WITH CLOSED TIMELIKE CURVES (CTCs)



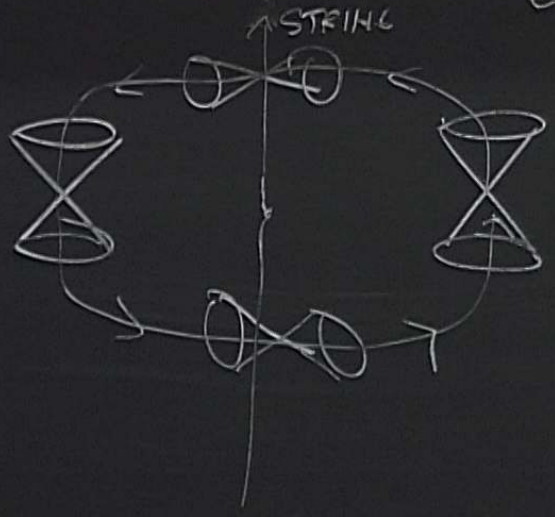
FOR EXAMPLE: KERR SPACETIME

(ONLY ROTATING BH SOLUTION)

... HIDING BEHIND HORIZON

TAUB-NOT ..

BEWARE. SOLUTIONS WITH CLOSED TIMELIKE CURVES (CTCs)



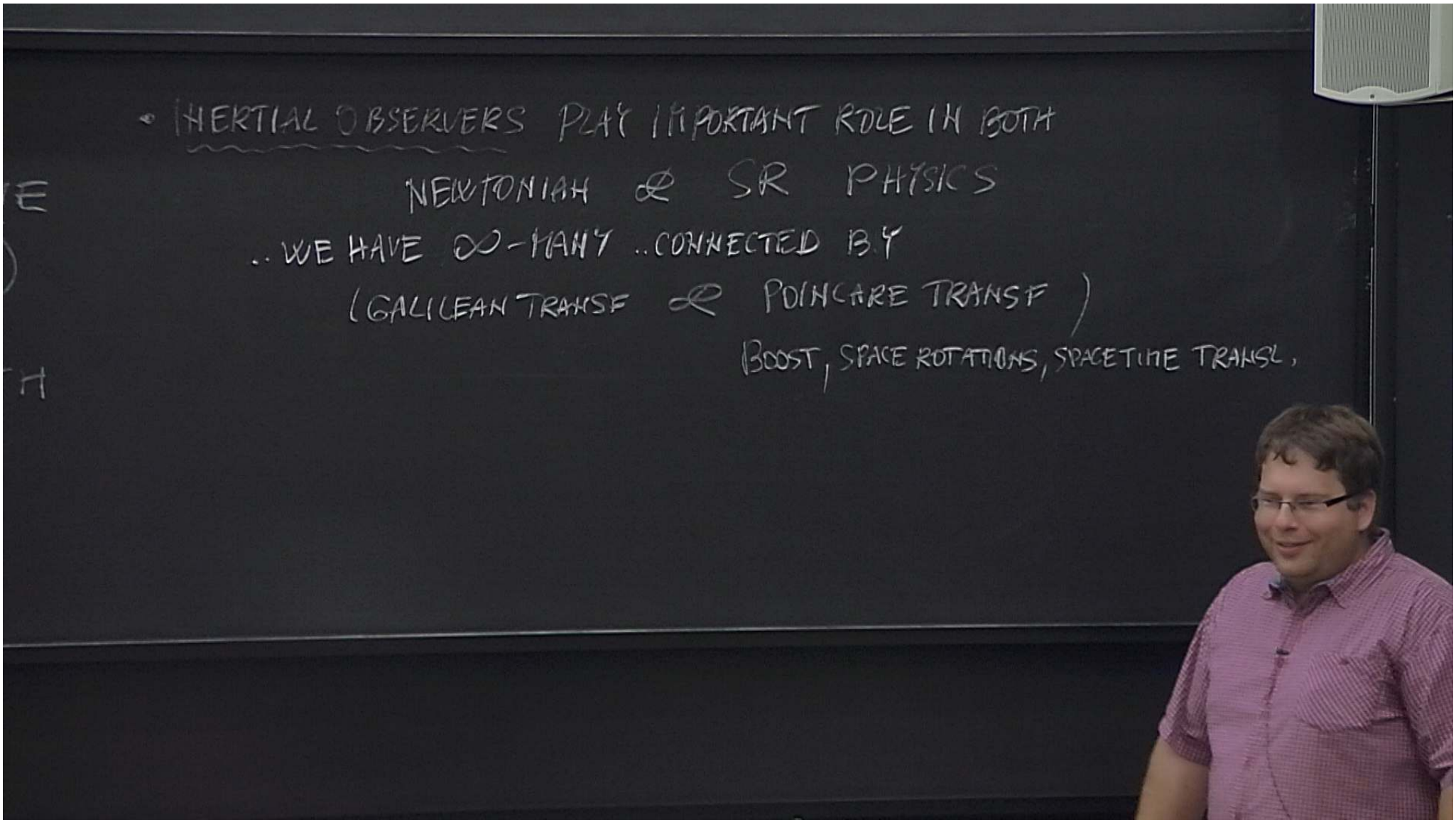
FOR EXAMPLE: KERR SPACETIME

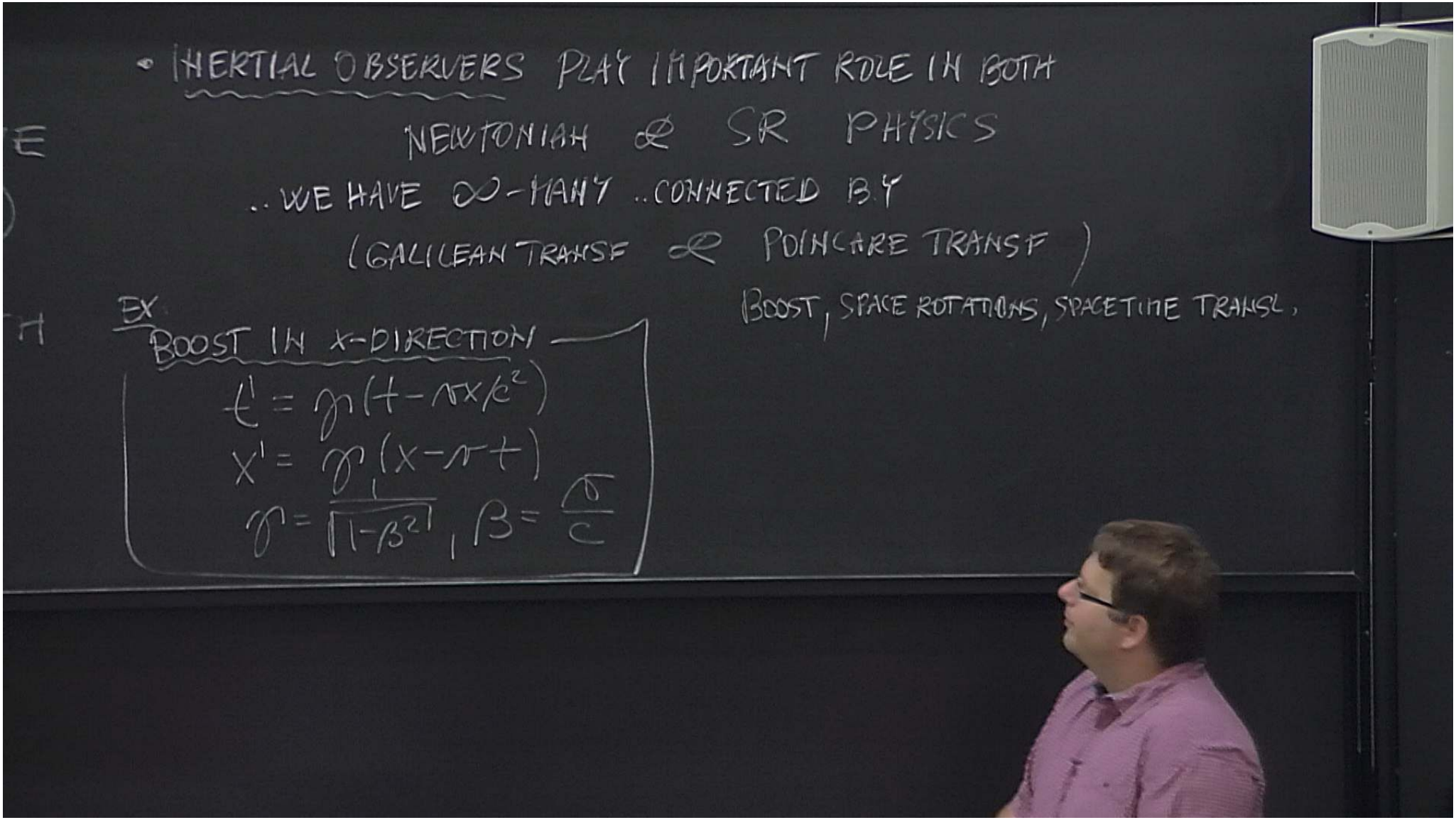
(ONLY ROTATING BH SOLUTION)

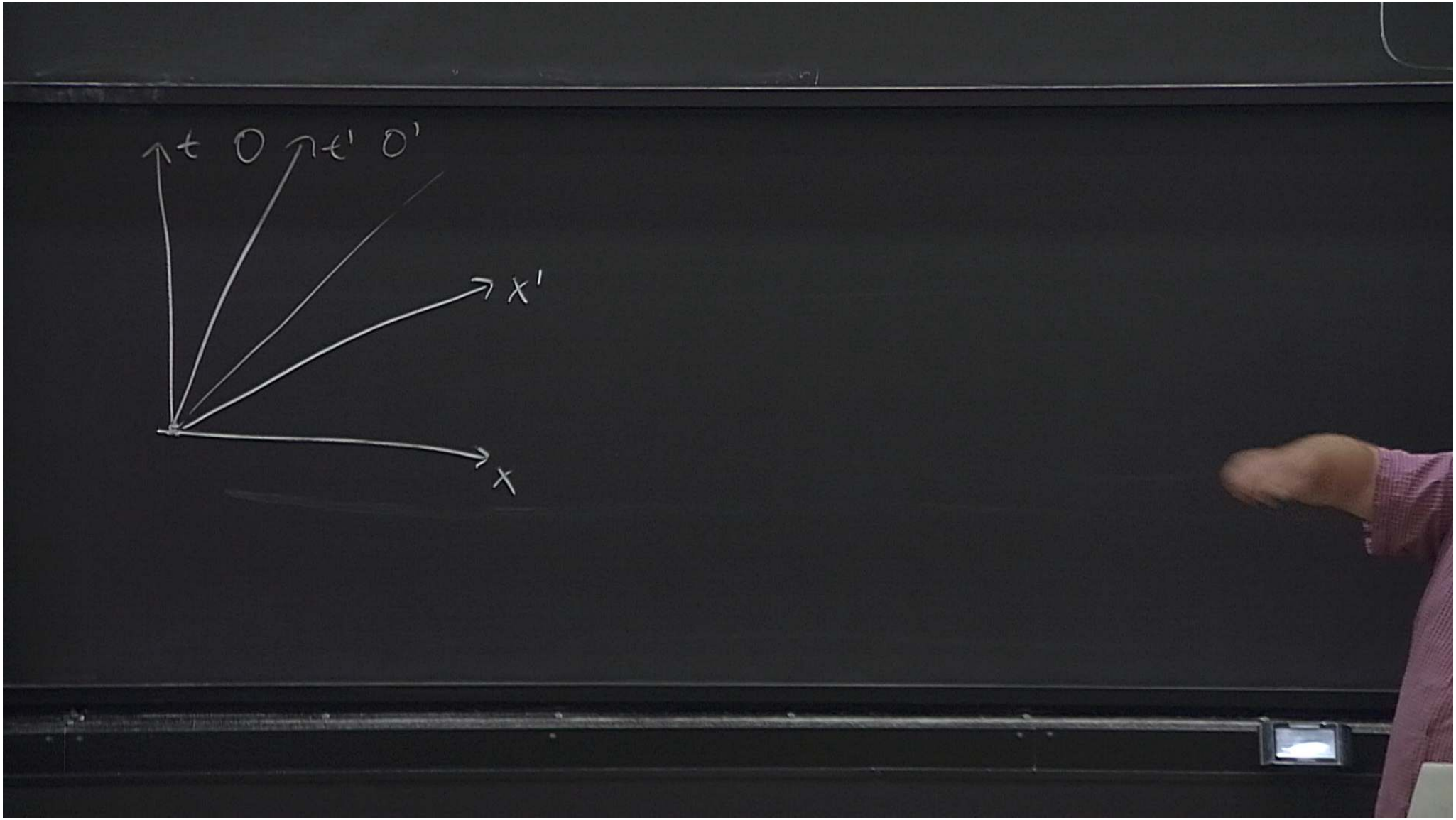
... HIDING BEHIND HORIZON

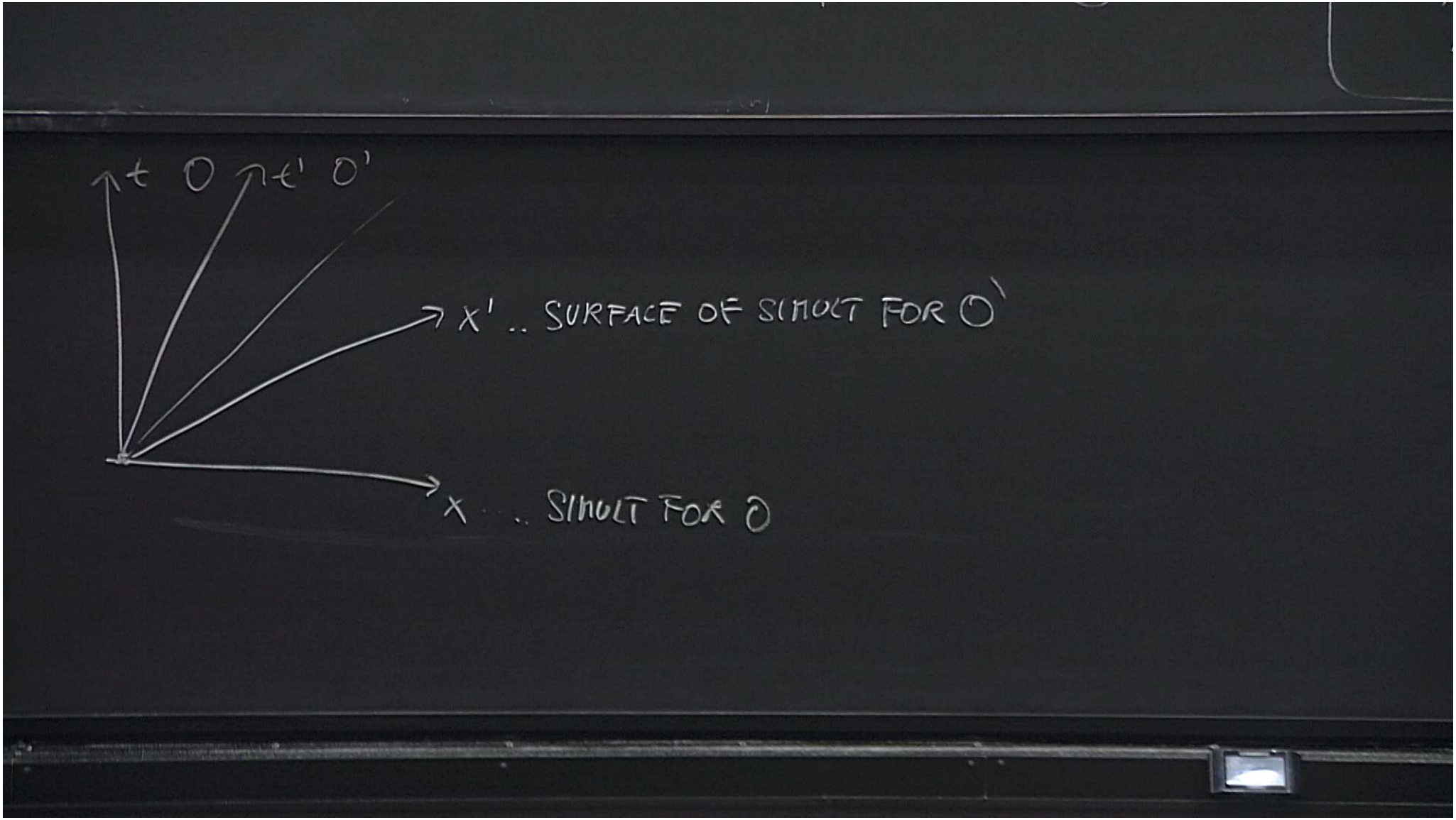
TAUB-NOT ... BH SOLUTION WITH ROTATING STRINGS

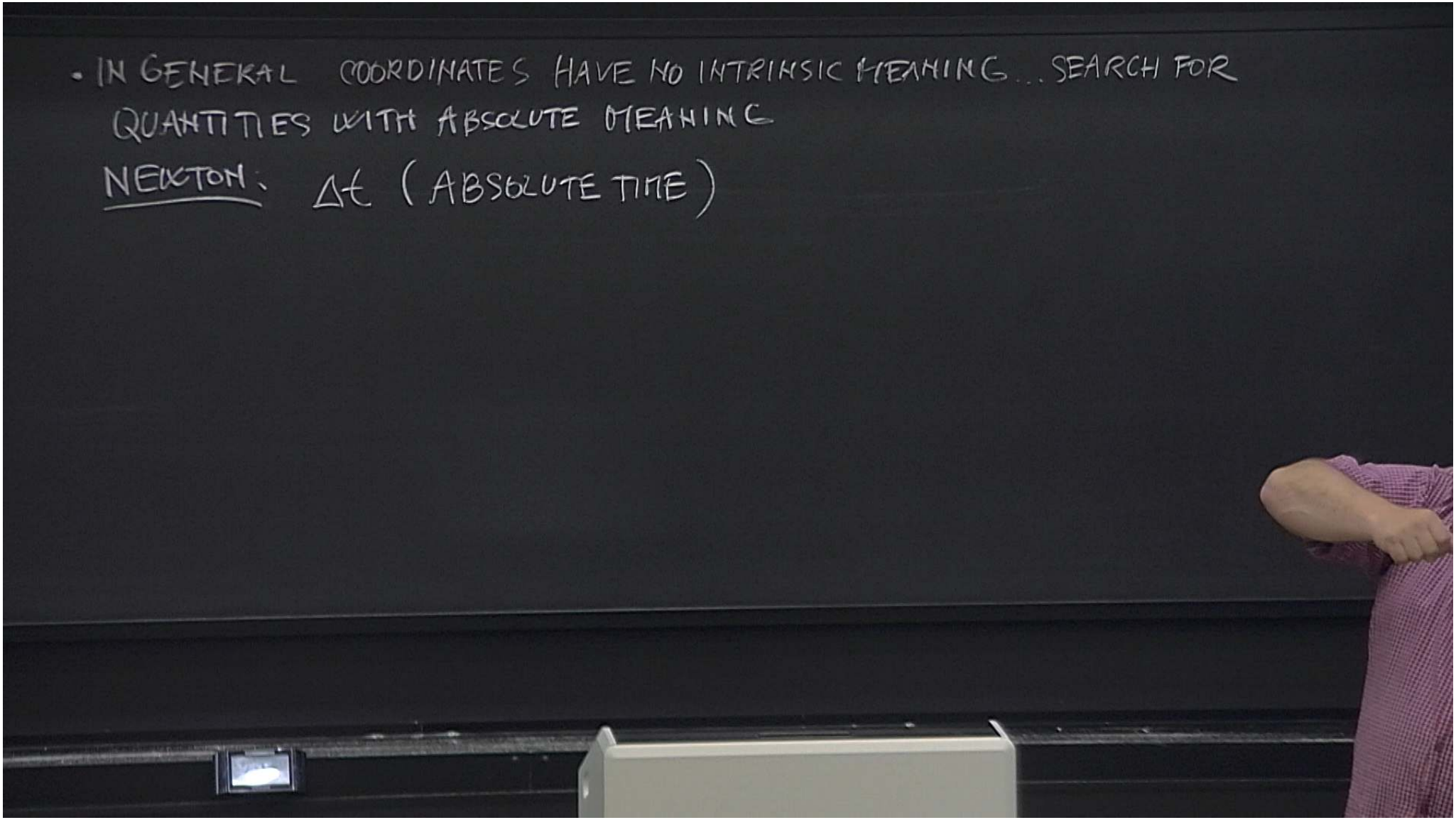
(?) PATHOLOGICAL (?)

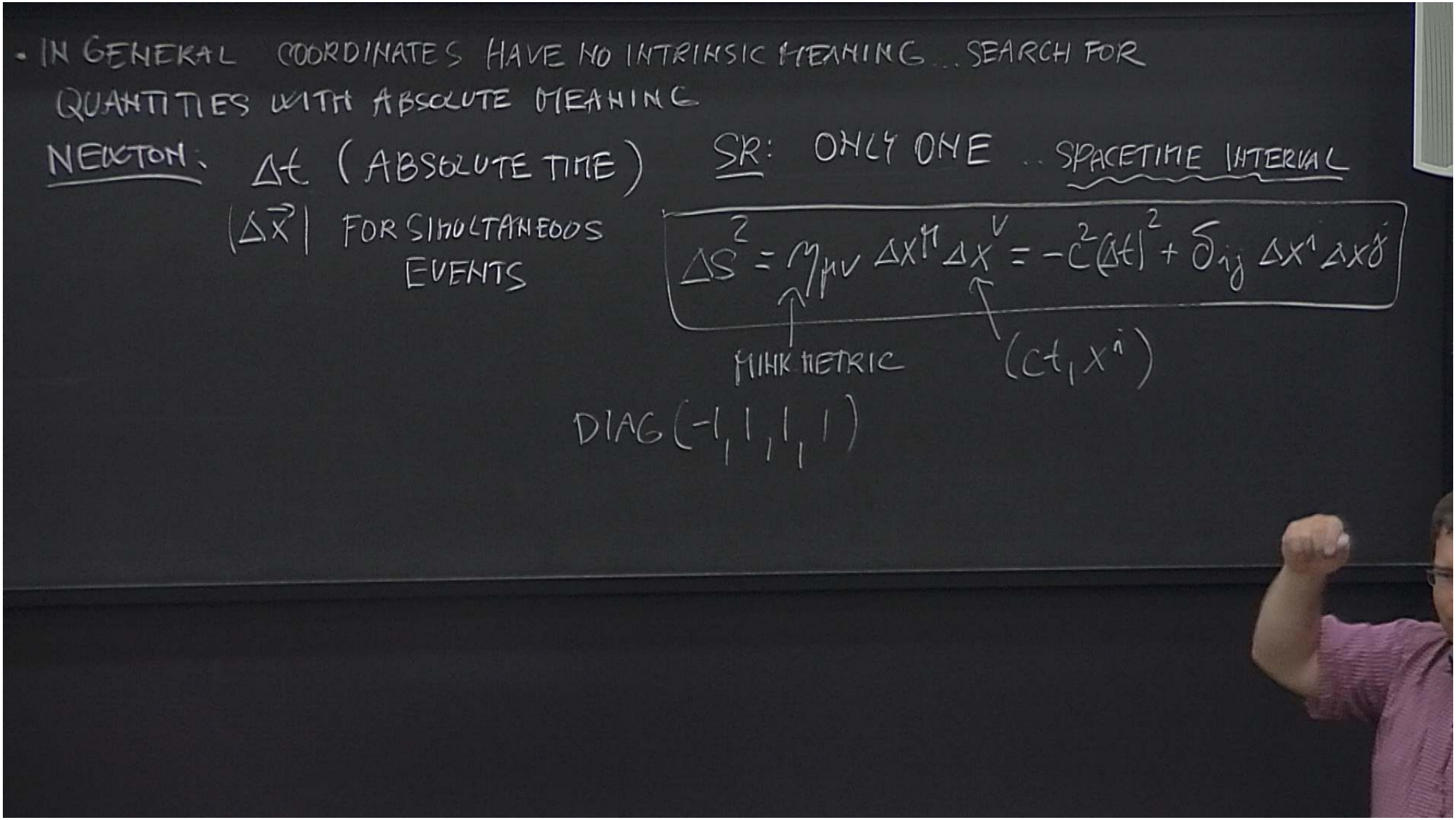

• INERTIAL OBSERVERS PLAY IMPORTANT ROLE IN BOTH
NEWTONIAN & SR PHYSICS
.. WE HAVE ∞-MANY .. CONNECTED BY
(GALILEAN TRANSF & POINCARE TRANSF)
BOOST, SPACE ROTATIONS, SPACETIME TRANSL,

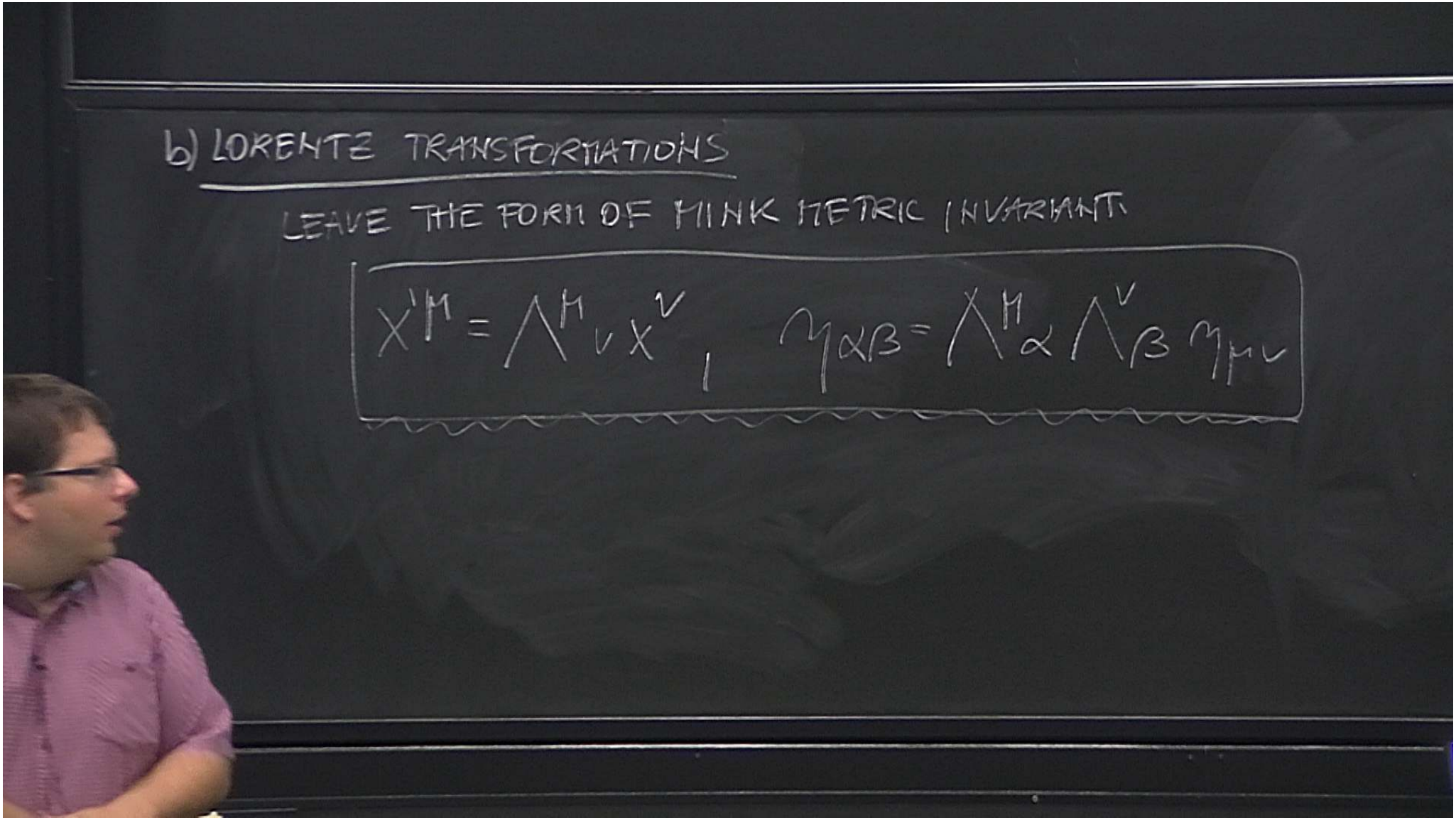

• INERTIAL OBSERVERS PLAY IMPORTANT ROLE IN BOTH
NEWTONIAN & SR PHYSICS
.. WE HAVE ∞-MANY .. CONNECTED BY
(GALILEAN TRANSF & POINCARE TRANSF)
BOOST, SPACE ROTATIONS, SPACETIME TRANSL.
EX
BOOST IN x-DIRECTION
$t' = \gamma(t - vx/c^2)$
$x' = \gamma(x - vt)$
$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \beta = \frac{v}{c}$

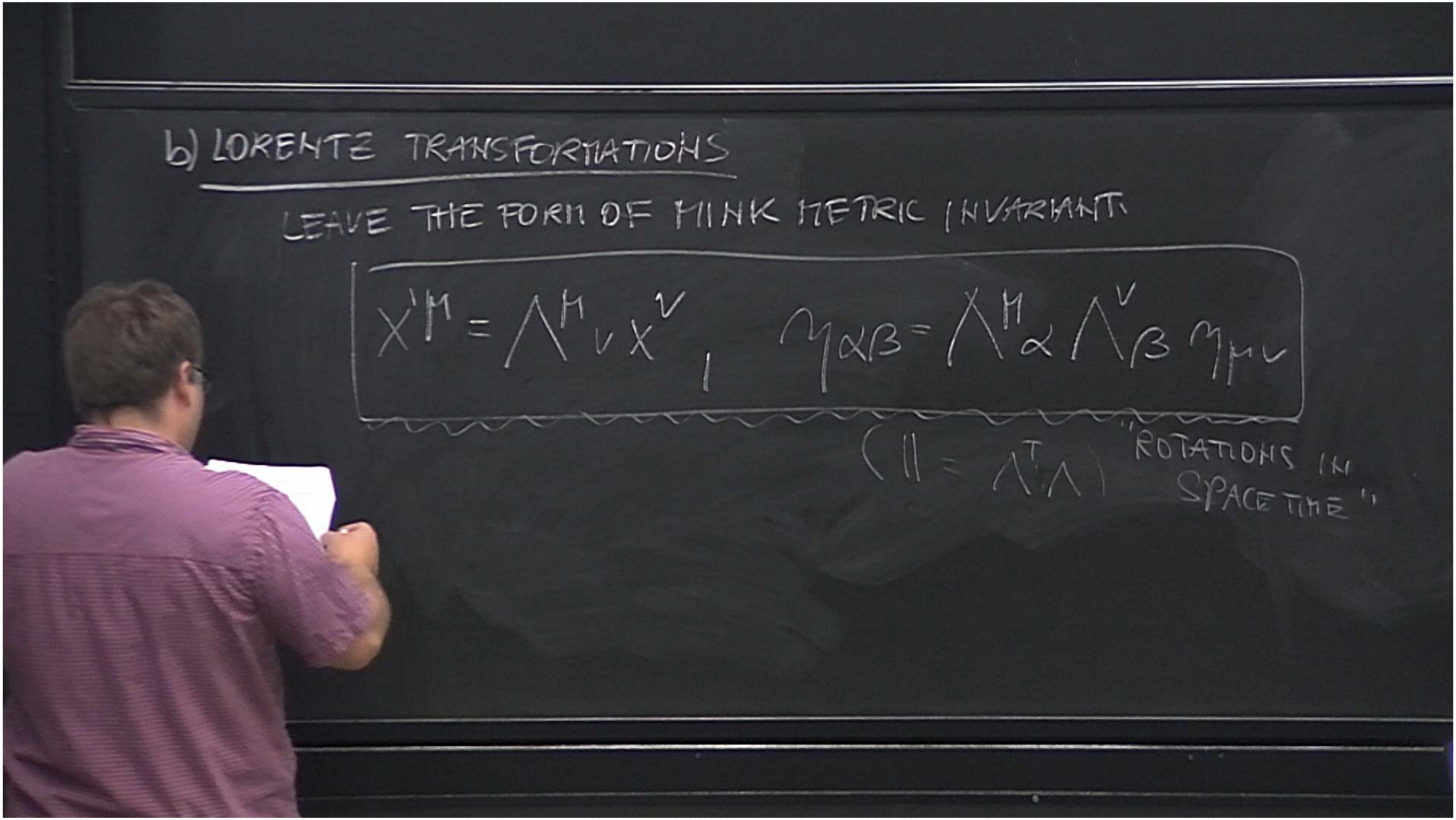

t
O
t'
O'
x'
x


t O
t' O'
x' .. SURFACE OF SIMULT FOR O'
x .. SIMULT FOR O


• IN GENERAL COORDINATES HAVE NO INTRINSIC MEANING... SEARCH FOR QUANTITIES WITH ABSOLUTE MEANING
NEWTON: Δt (ABSOLUTE TIME)


• IN GENERAL COORDINATES HAVE NO INTRINSIC MEANING... SEARCH FOR QUANTITIES WITH ABSOLUTE MEANING
NEWTON: Δt (ABSOLUTE TIME)
|Δx⃗| FOR SIMULTANEOUS EVENTS
SR: ONLY ONE .. SPACETIME INTERVAL
$\Delta s^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu = -c^2(\Delta t)^2 + \delta_{ij}\Delta x^i \Delta x^j$
MINK METRIC
(ct, x^i)
DIAG(-1, 1, 1, 1)


b) LORENTZ TRANSFORMATIONS
LEAVE THE FORM OF MINK METRIC INVARIANT
$X'^{\mu} = \Lambda^{\mu}_{\ \nu} X^{\nu}$, $\eta_{\alpha\beta} = \Lambda^{\mu}_{\ \alpha} \Lambda^{\nu}_{\ \beta} \eta_{\mu\nu}$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LEAVE THE FORM OF MINK METRIC INVARIANT
$X'^{\mu} = \Lambda^{\mu}_{\ \nu} X^{\nu}$, $\eta_{\alpha\beta} = \Lambda^{\mu}_{\ \alpha} \Lambda^{\nu}_{\ \beta} \eta_{\mu\nu}$
$(\mathbb{1} = \Lambda^{T} \Lambda)$ "ROTATIONS IN SPACETIME"

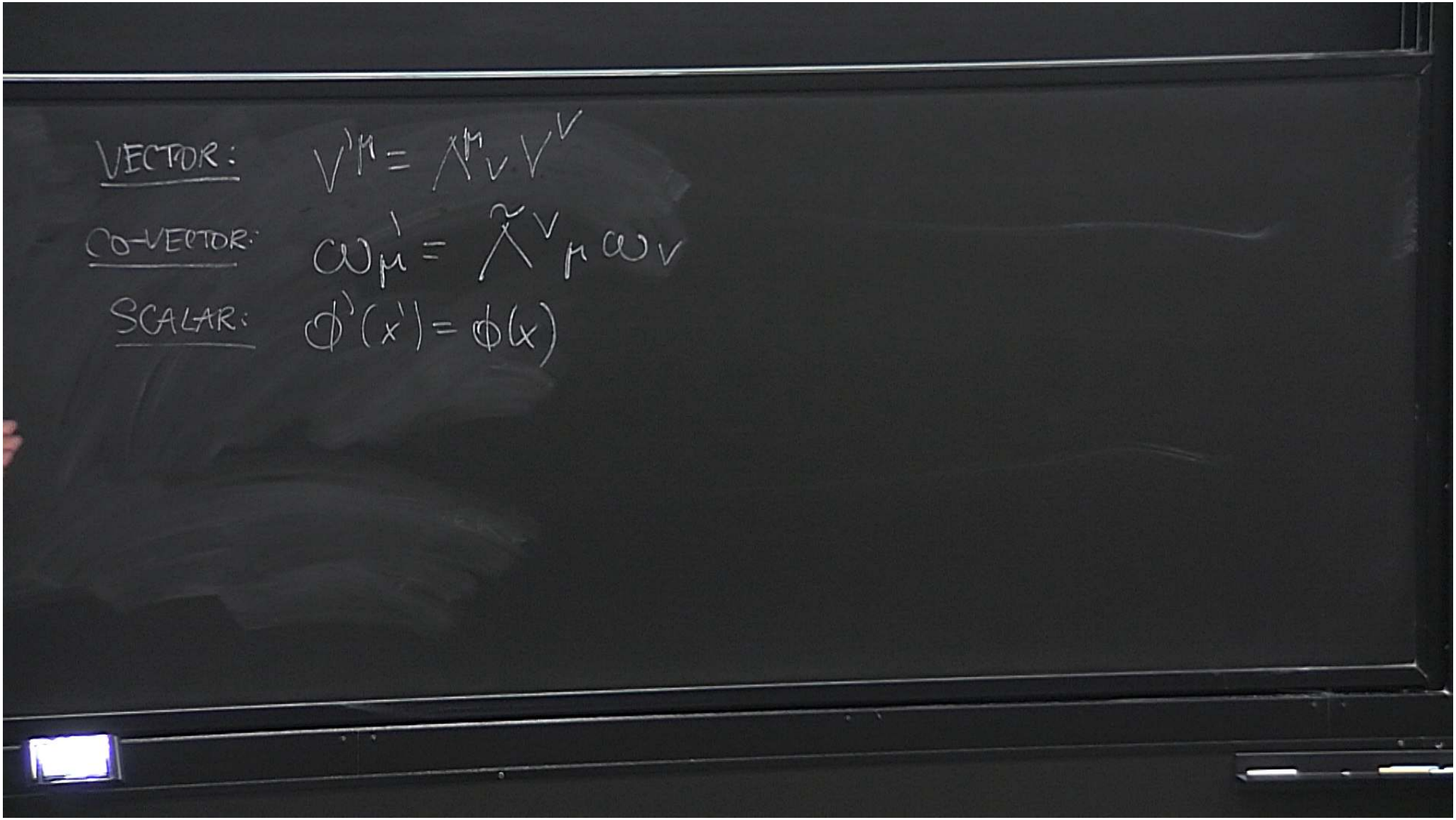
b) LORENTZ TRANSFORMATIONS

LEAVE THE FORM OF MINK METRIC INVARIANT

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu}, \quad \eta_{\alpha\beta} = \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} \eta_{\mu\nu}$$

$(\mathbb{1} = \Lambda^{T} \Lambda)$ "ROTATIONS IN SPACETIME"

$$x^{\mu} = \tilde{\Lambda}^{\mu}{}_{\nu} x'^{\nu}, \quad \Lambda^{\mu}{}_{\nu} \tilde{\Lambda}^{\nu}{}_{\alpha} = \delta^{\mu}{}_{\alpha}$$


VECTOR: $V'^{\mu} = \Lambda^{\mu}{}_{\nu} V^{\nu}$
CO-VECTOR: $\omega'_{\mu} = \tilde{\Lambda}^{\nu}{}_{\mu} \omega_{\nu}$
SCALAR: $\phi'(x') = \phi(x)$

VECTOR: $V'^{\mu} = \Lambda^{\mu}{}_{\nu} V^{\nu}$

CO-VECTOR: $\omega'_{\mu} = \tilde{\Lambda}^{\nu}{}_{\mu} \omega_{\nu}$

SCALAR: $\phi'(x') = \phi(x)$

EX: BOOST IN X-DIRECTION

$$\Lambda^{\alpha}{}_{\beta} = \begin{array}{c} \\ t \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \begin{pmatrix} \cosh\theta & -\sinh\theta & 0 & 0 \\ -\sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

VECTOR: $V'^{\mu} = \Lambda^{\mu}{}_{\nu} V^{\nu}$

CO-VECTOR: $\omega'_{\mu} = \tilde{\Lambda}^{\nu}{}_{\mu} \omega_{\nu}$

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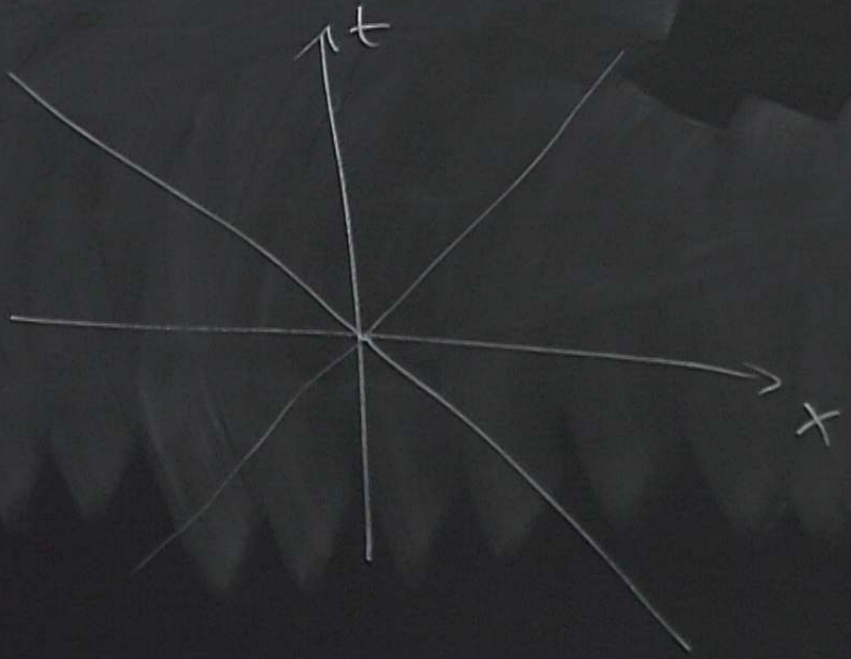
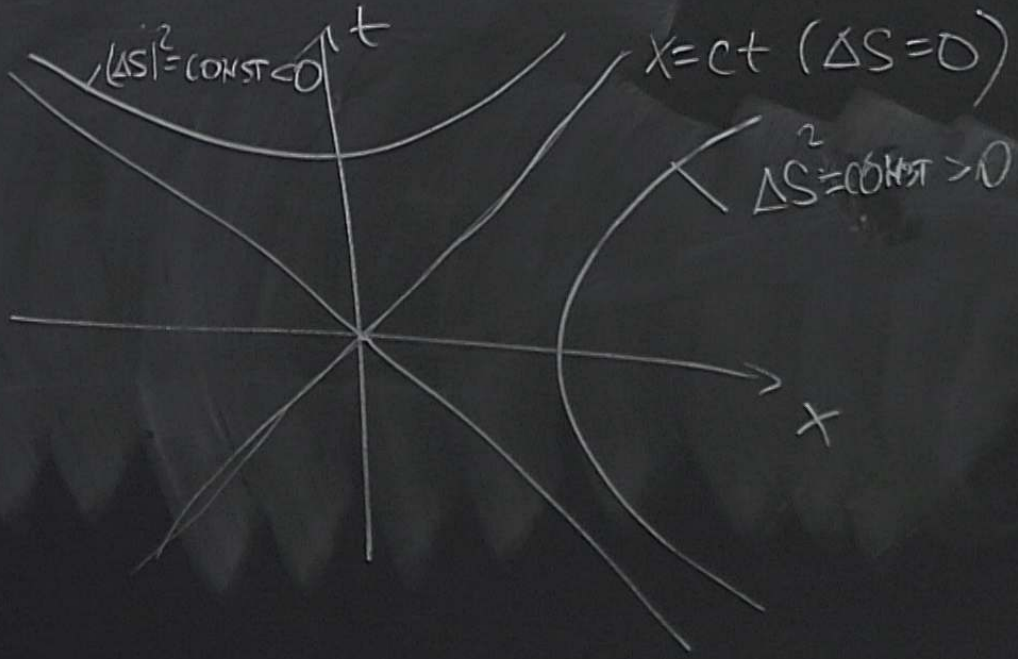
$$\beta = \frac{\Delta S}{c} = \tanh\theta$$

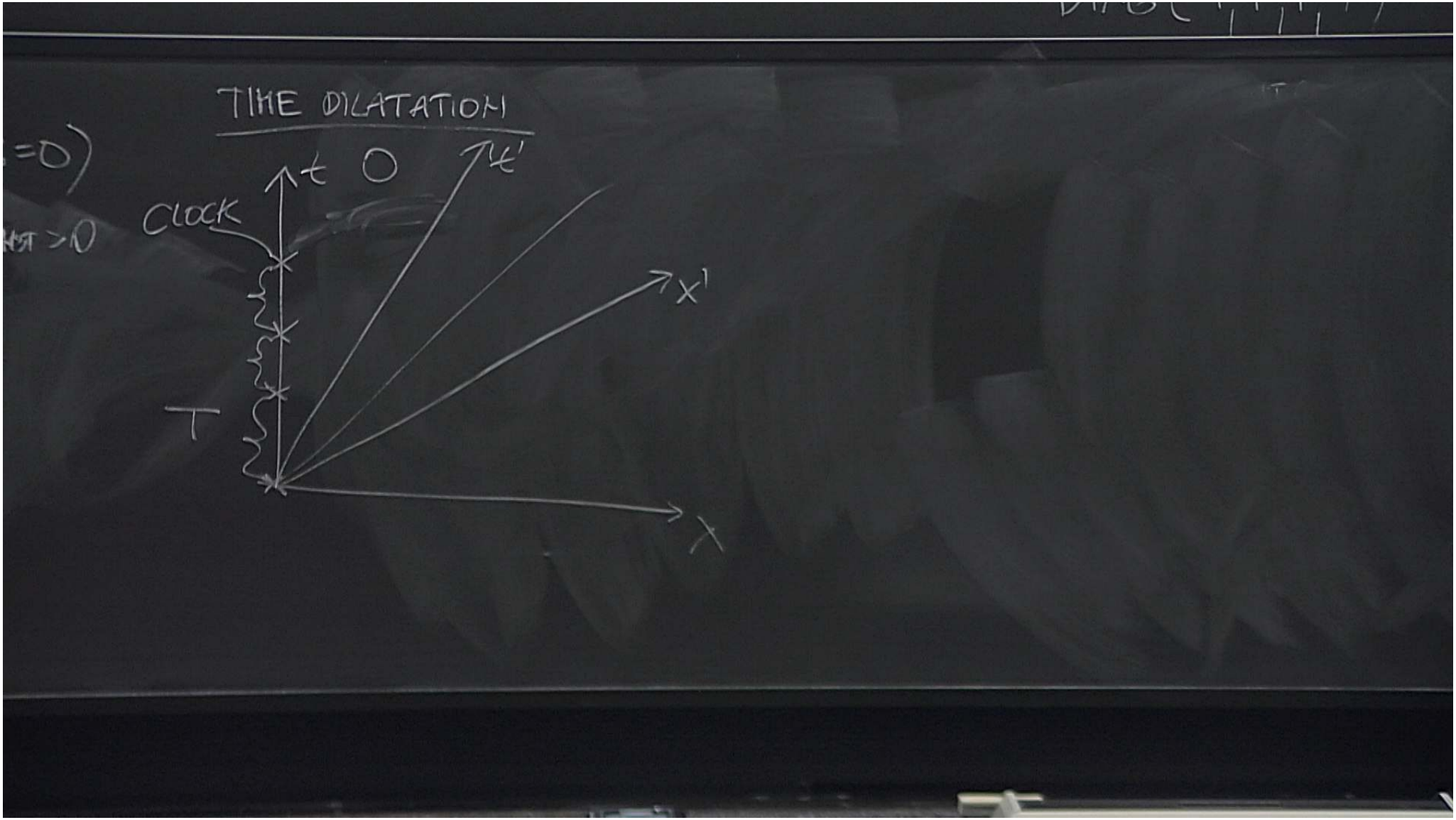
$$\Rightarrow \cosh\theta = \gamma, \quad \sinh\theta = \beta\gamma$$

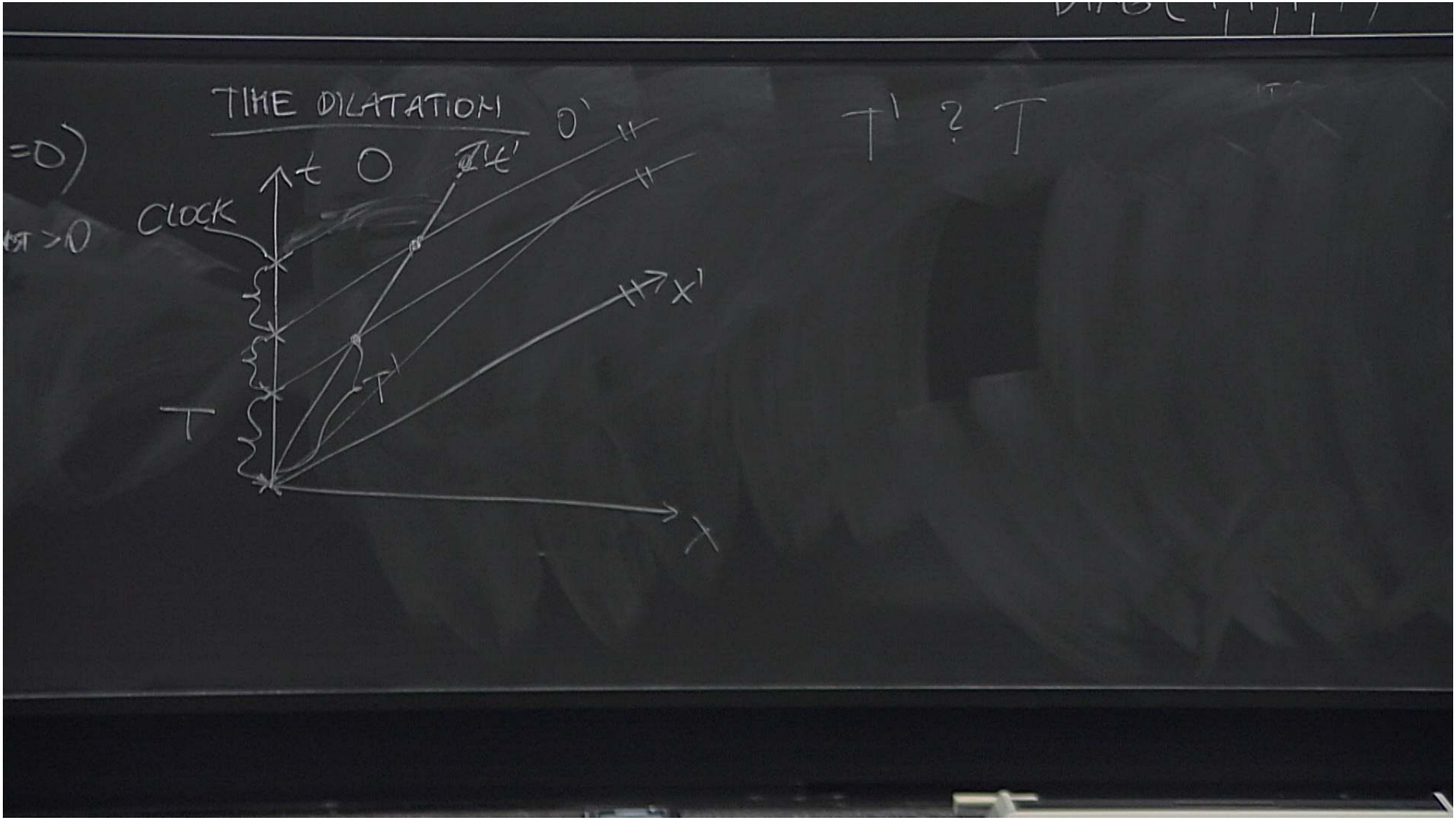
STRESS. LORENTZ INVARIANT INTERVAL

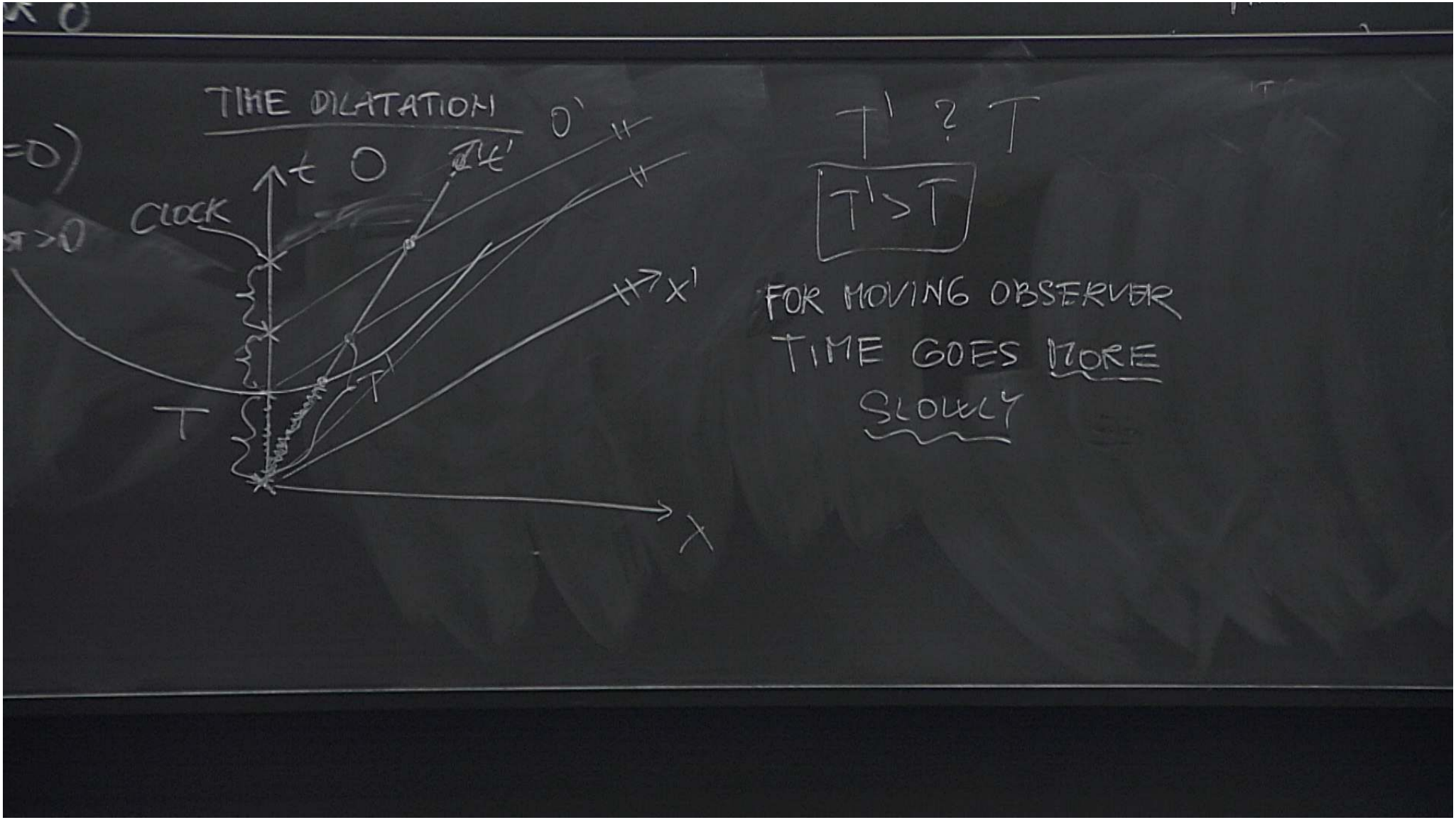


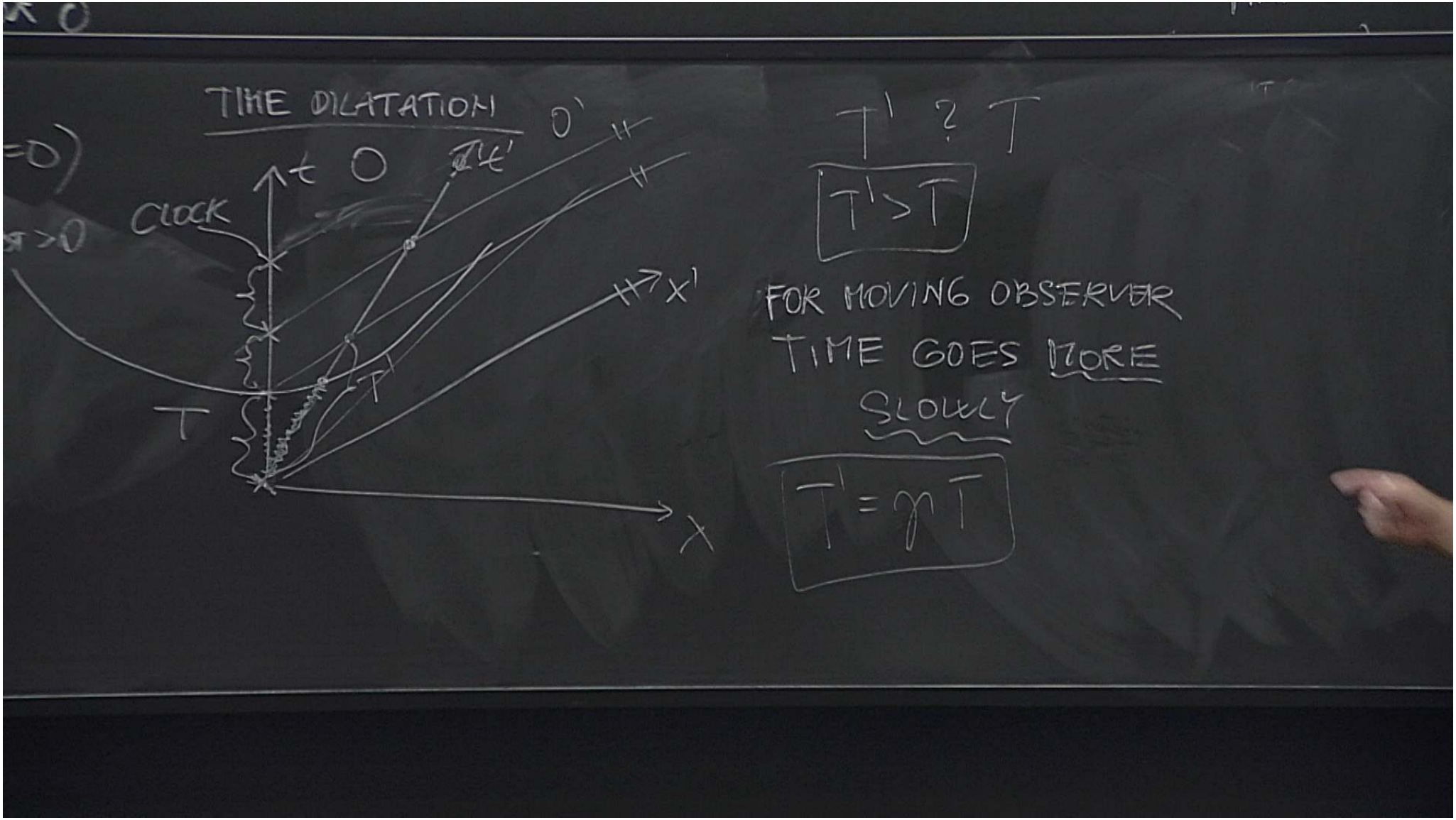
STRESS. LORENTZ INVARIANT INTERVAL

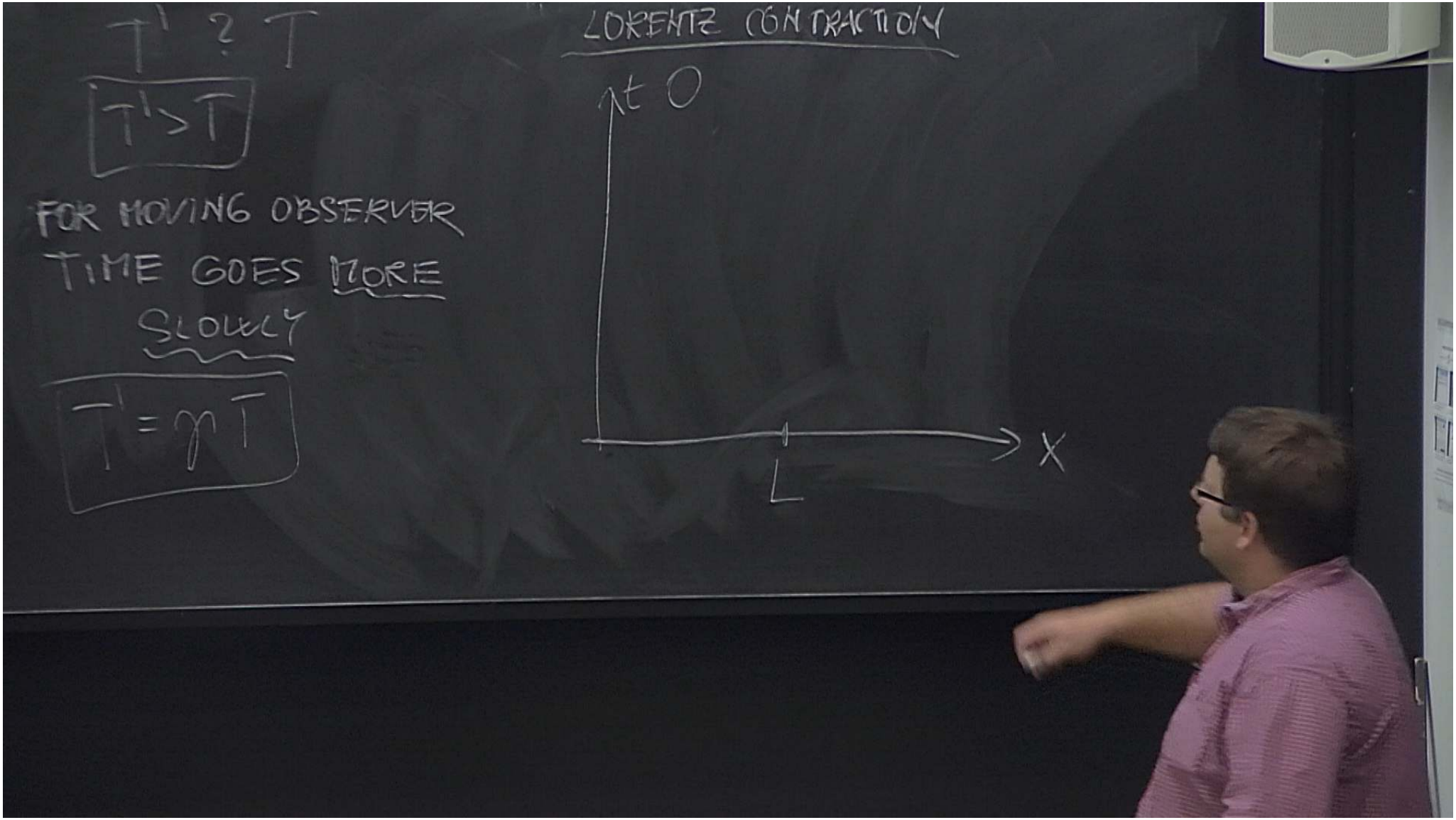


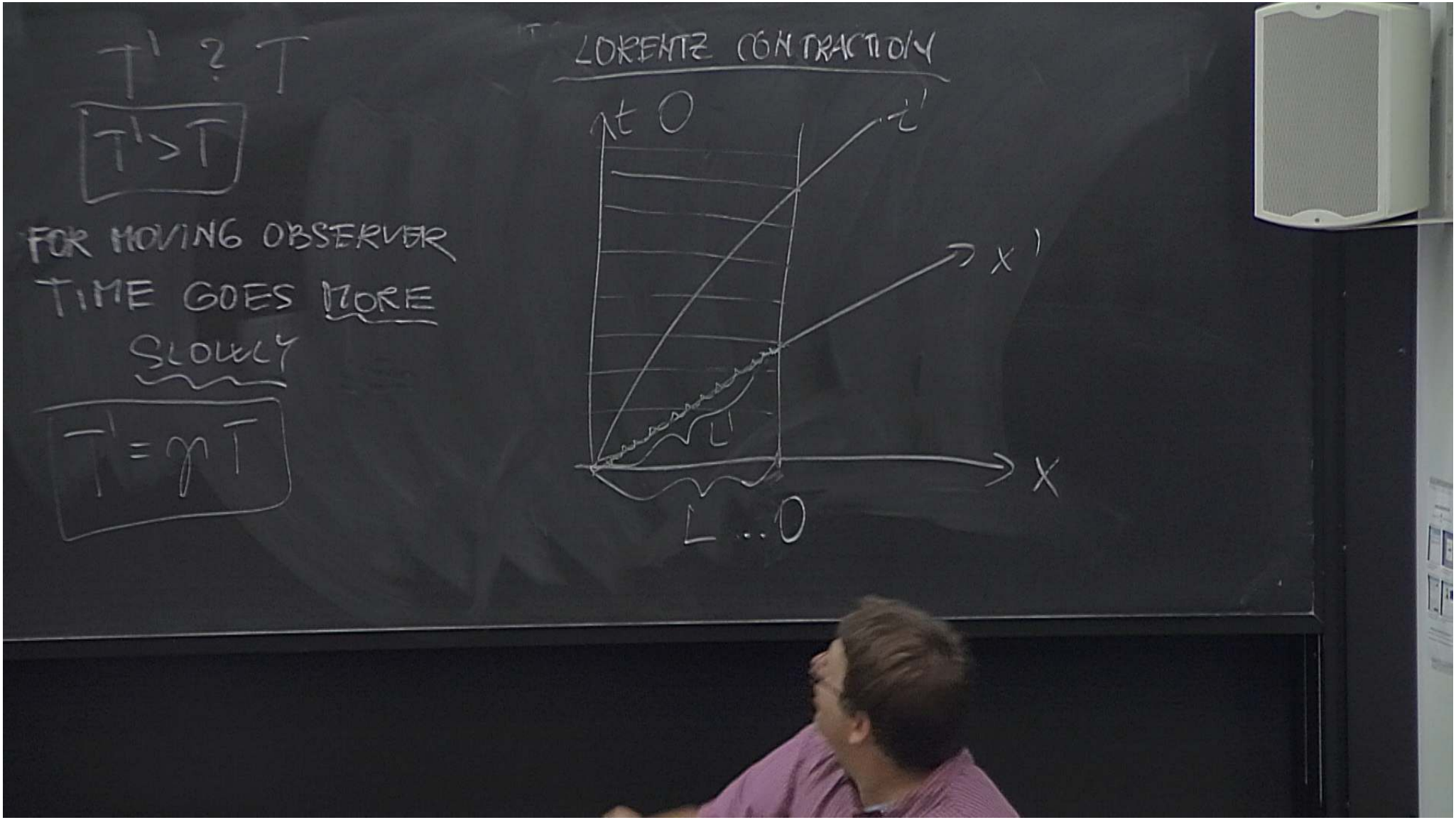

TIME DILATATION
t
0
t'
CLOCK
x'
T
x

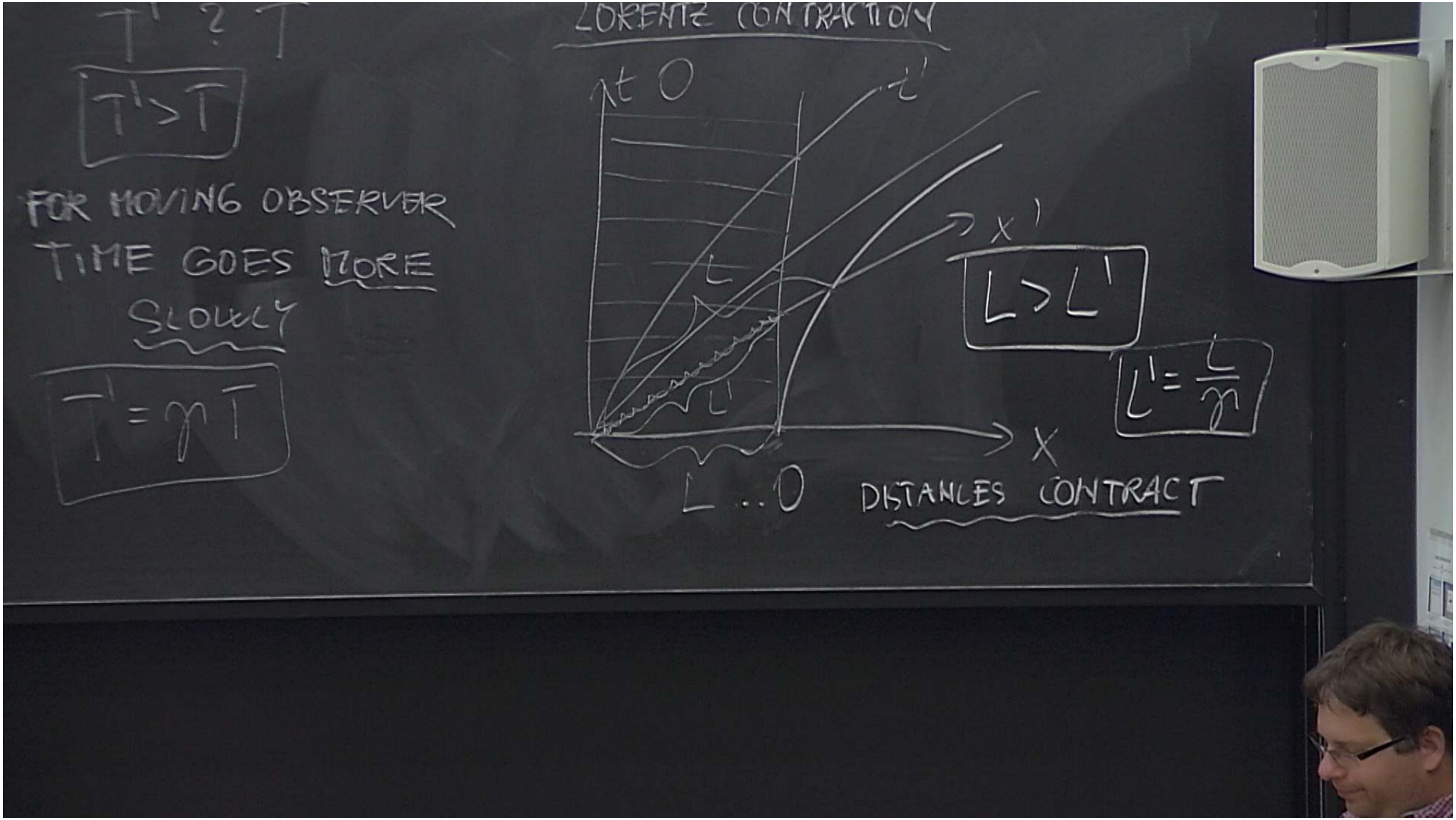

TIME DILATATION
CLOCK
t
O
O'
t'
x'
x
T
T'
T' ? T

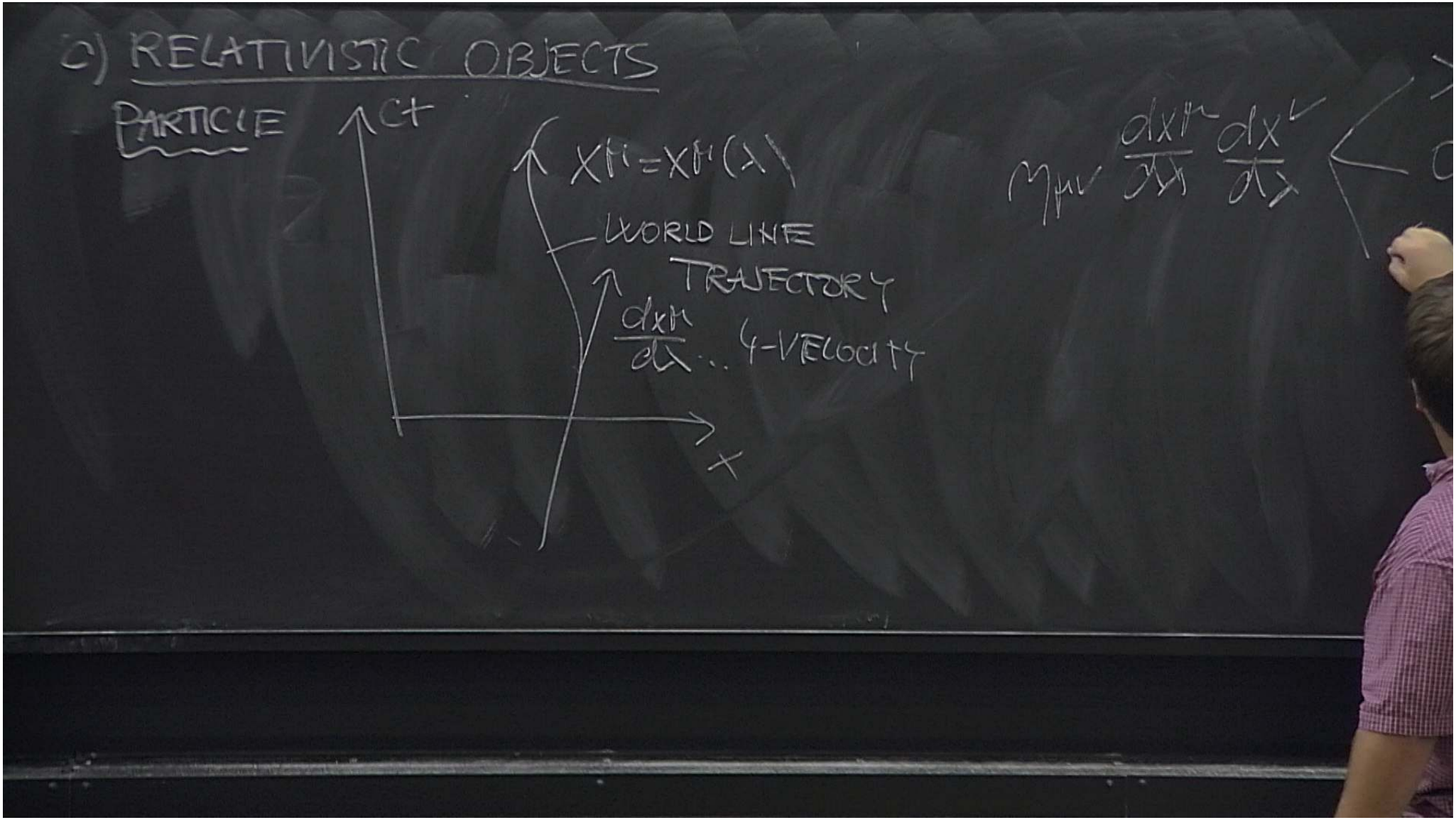

TIME DILATATION
t
O
O'
t'
CLOCK
x'
T'
T
x
T' ? T
T' > T
FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY

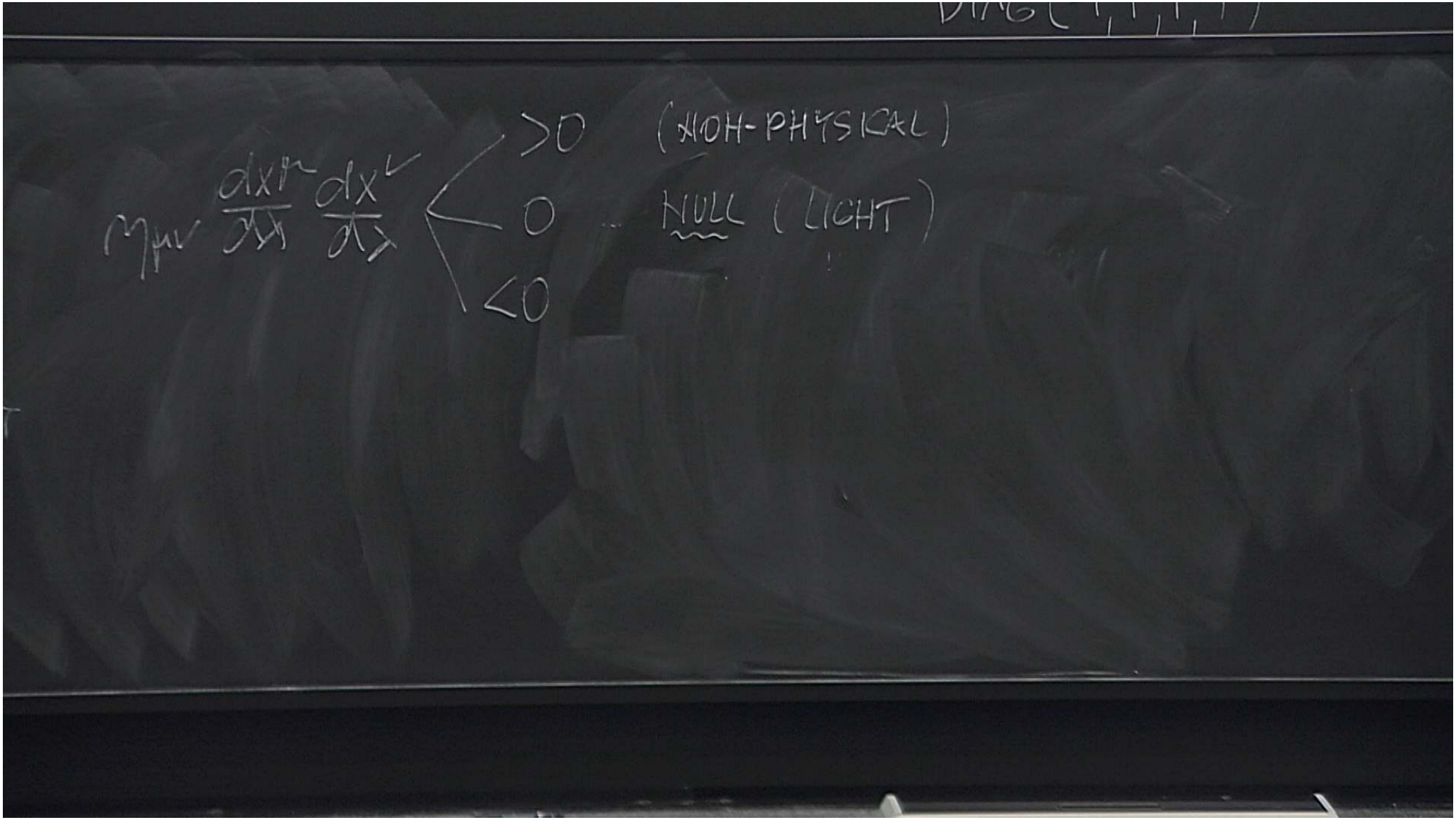

TIME DILATATION
CLOCK
t
O
O'
x'
x
T
T'
T' ? T
T' > T
FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY
T' = γT


T' ? T
T'>T
FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY
T'=γT
LORENTZ CONTRACTION
t
0
X
L


T' ? T
T'>T
FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY
T'=γT
LORENTZ CONTRACTION
t 0
t'
x'
L'
X
L ..0


T' ? T
T'>T
FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY
T'=γT
LORENTZ CONTRACTION
t 0
t'
x'
L
L'
L>L'
L'=L/γ
x
L ..0
DISTANCES CONTRACT


c) RELATIVISTIC OBJECTS
PARTICLE
ct
x
$x^\mu = x^\mu(\lambda)$
WORLD LINE
TRAJECTORY
$\frac{dx^\mu}{d\lambda}$ .. 4-VELOCITY
$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$


$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$
> 0 (NON-PHYSICAL)
0 NULL (LIGHT)
< 0

MINK METRIC

DIAG (-1 1 1 1)

$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$ >0 (NON-PHYSICAL) FREE PARTICLE ..MO

<0 NULL (LIGHT)

<0 MASSIVE PARTICLES

PROPER TIME τ

$$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$

NEWTON. Δt (ABSOLUTE TIME) SR: ONLY $\Delta\tau$

$\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$ $\begin{cases} >0 & \text{(NON-PHYSICAL)} \\ 0 & \text{NULL (LIGHT)} \\ <0 & \text{MASSIVE PARTICLES} \end{cases}$

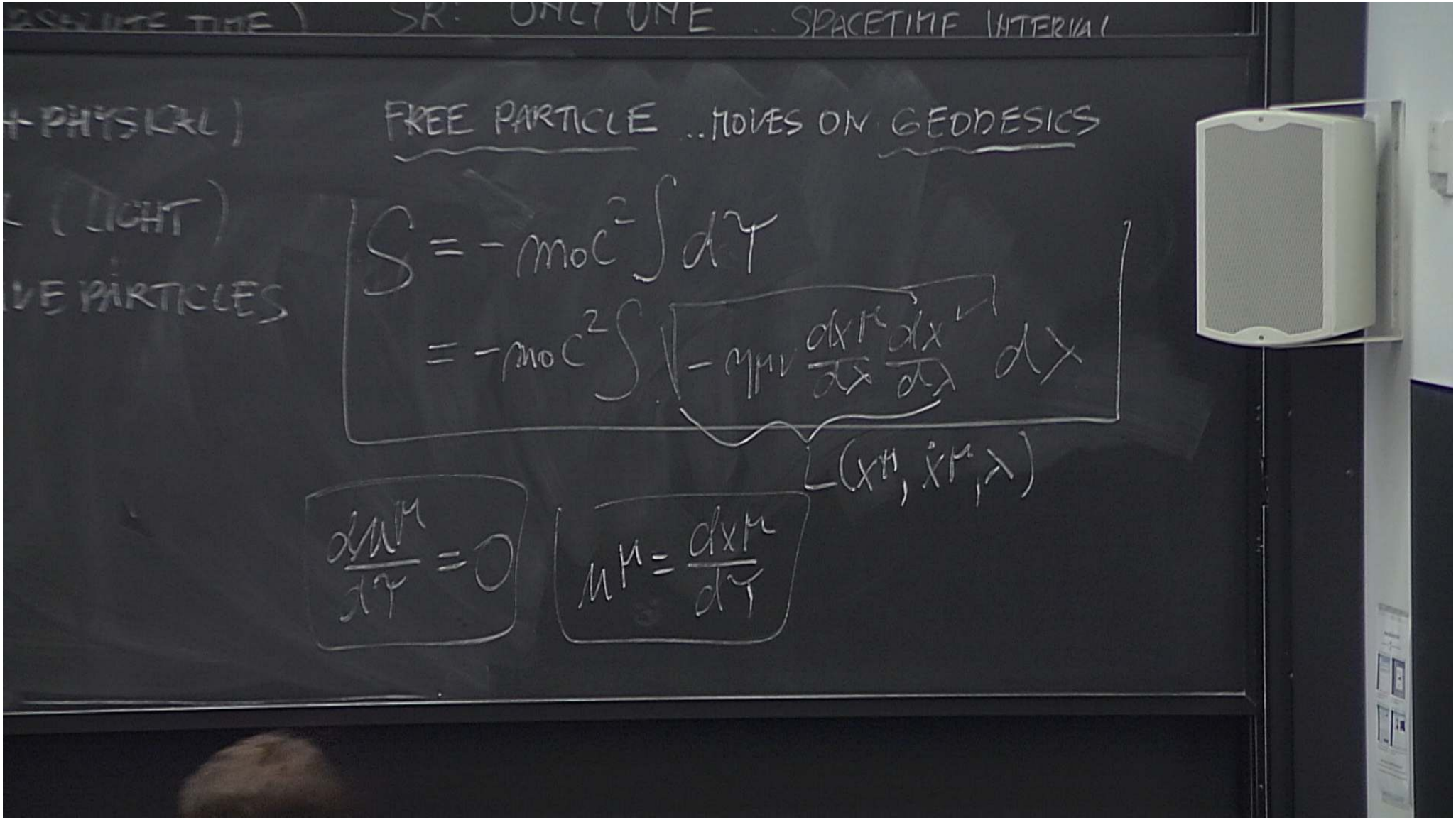
FREE PARTICLE

$$S = -m_0 c^2 \int$$
$$= -m_0 c^2 \int$$

PROPER TIME τ

$$\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = -1$$

$$d\tau = \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\, d\lambda$$


SR: ONLY ONE ... SPACETIME INTERVAL
PHYSICAL)
(LIGHT)
VE PARTICLES
FREE PARTICLE ..MOVES ON GEODESICS
$S = -m_0 c^2 \int d\tau$
$= -m_0 c^2 \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, d\lambda$
$\mathcal{L}(x^\mu, \dot{x}^\mu, \lambda)$
$\frac{dp^\mu}{d\tau} = 0$
$p^\mu = \frac{dx^\mu}{d\tau}$

NOTE.

$$\frac{d}{d\tau} = \frac{dx^\nu}{d\tau}\frac{\partial}{\partial x^\nu} = u^\nu \frac{\partial}{\partial x^\nu}$$

$$\frac{du^\mu}{d\tau} = u^\nu \frac{\partial u^\mu}{\partial x^\nu} = u^\nu \partial_\nu u^\mu$$

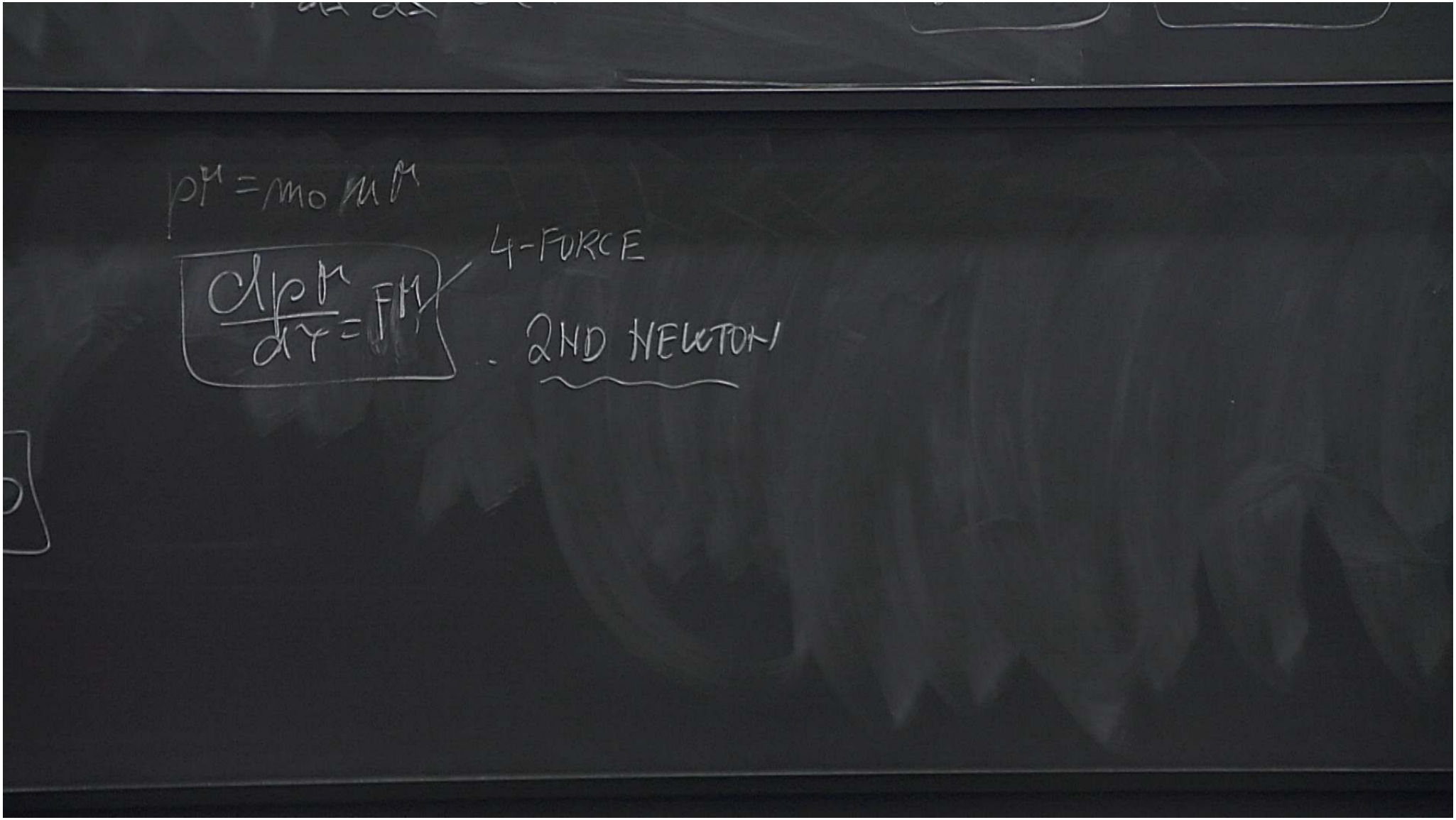
$$= \boxed{\partial_\nu u^\mu = 0}$$

NOTE: $\frac{d}{d\tau} = \frac{dx^\nu}{d\tau}\frac{\partial}{\partial x^\nu} = u^\nu \frac{\partial}{\partial x^\nu}$

$$\frac{du^\mu}{d\tau} = u^\nu \frac{\partial u^\mu}{\partial x^\nu} = u^\nu \partial_\nu u^\mu$$

GR: $\boxed{\nabla_u u^\mu = 0} = \boxed{\partial_u u^\mu = 0}$

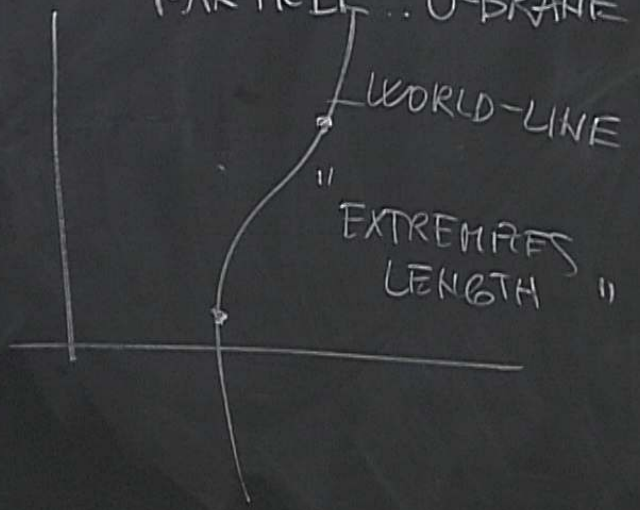
$\partial_V = \frac{\partial}{\partial \vec{v}} = V^\mu \frac{\partial}{\partial x^\mu}$


$p^\mu = m_0 u^\mu$
$\frac{dp^\mu}{d\tau} = F^\mu$
4-FORCE
2ND NEWTON

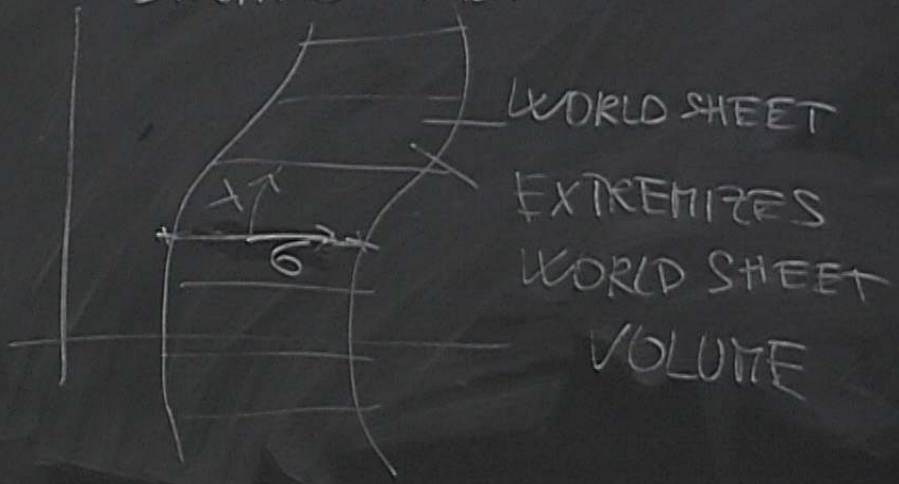
MOTION OF p-BRANES

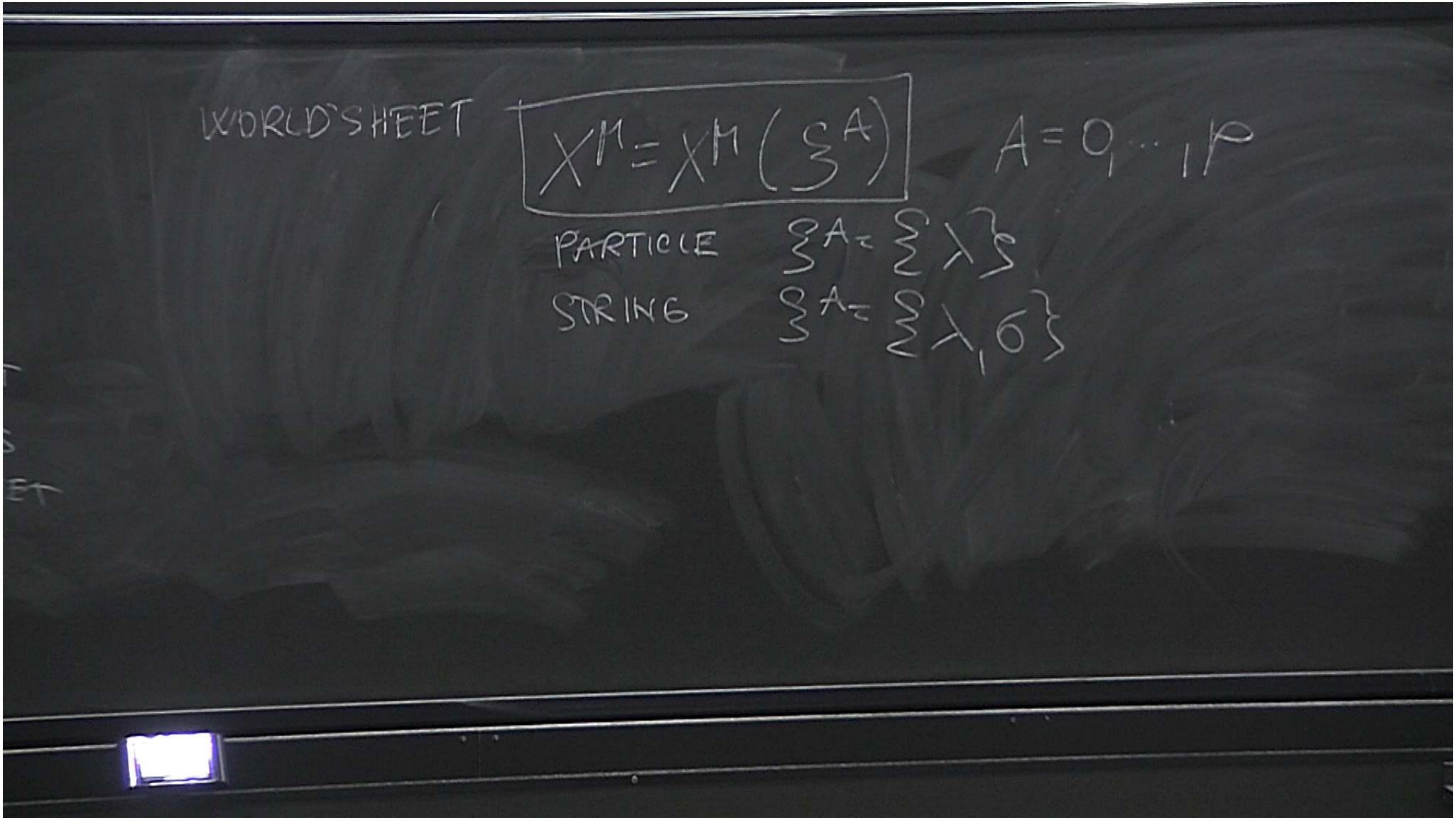
p-BRANE : HIGHER-DIM. OBJECT
THAT SWEEPS (p+1)-DIM TRAJECTORY

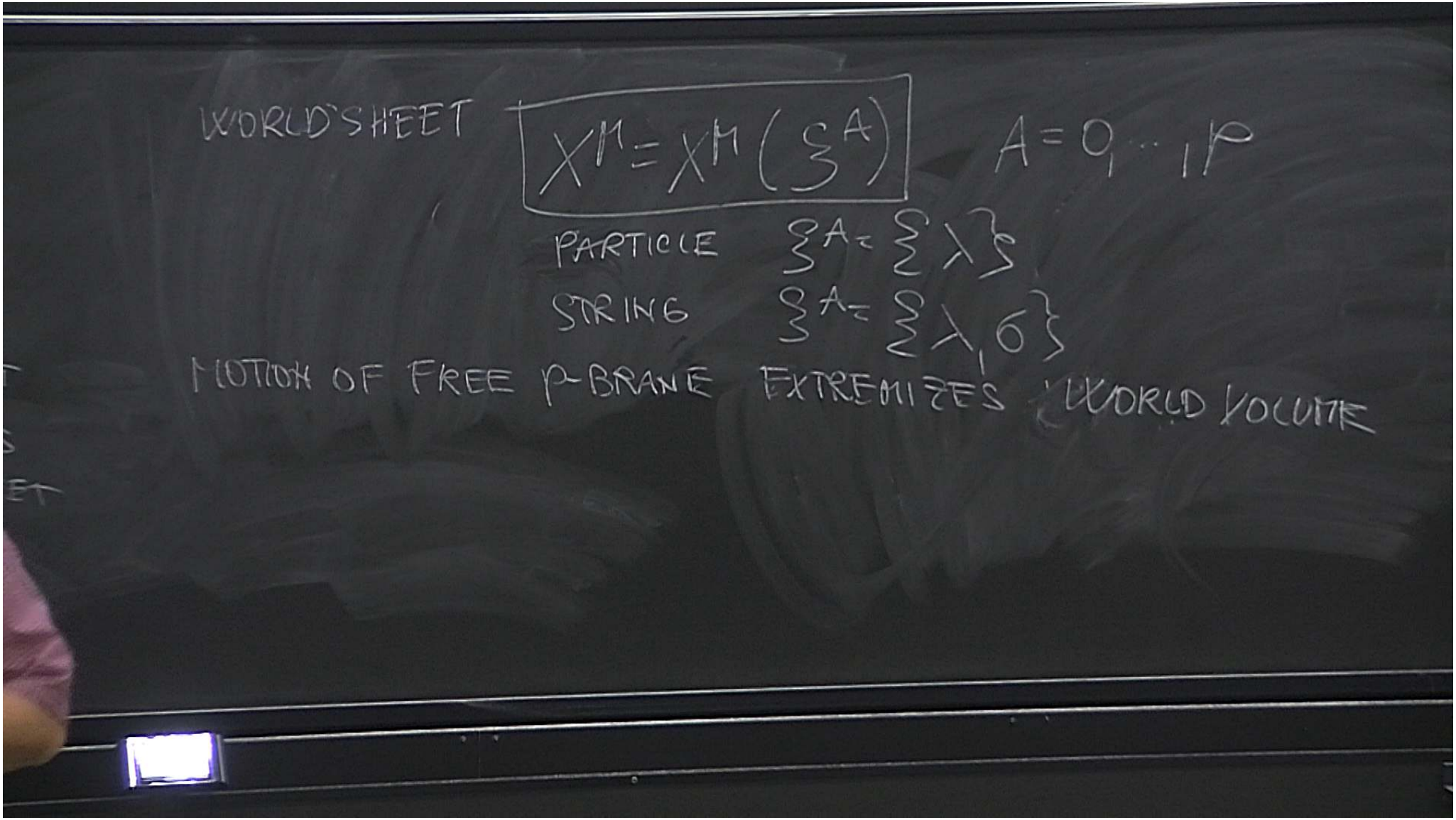
PARTICLE : 0-BRANE

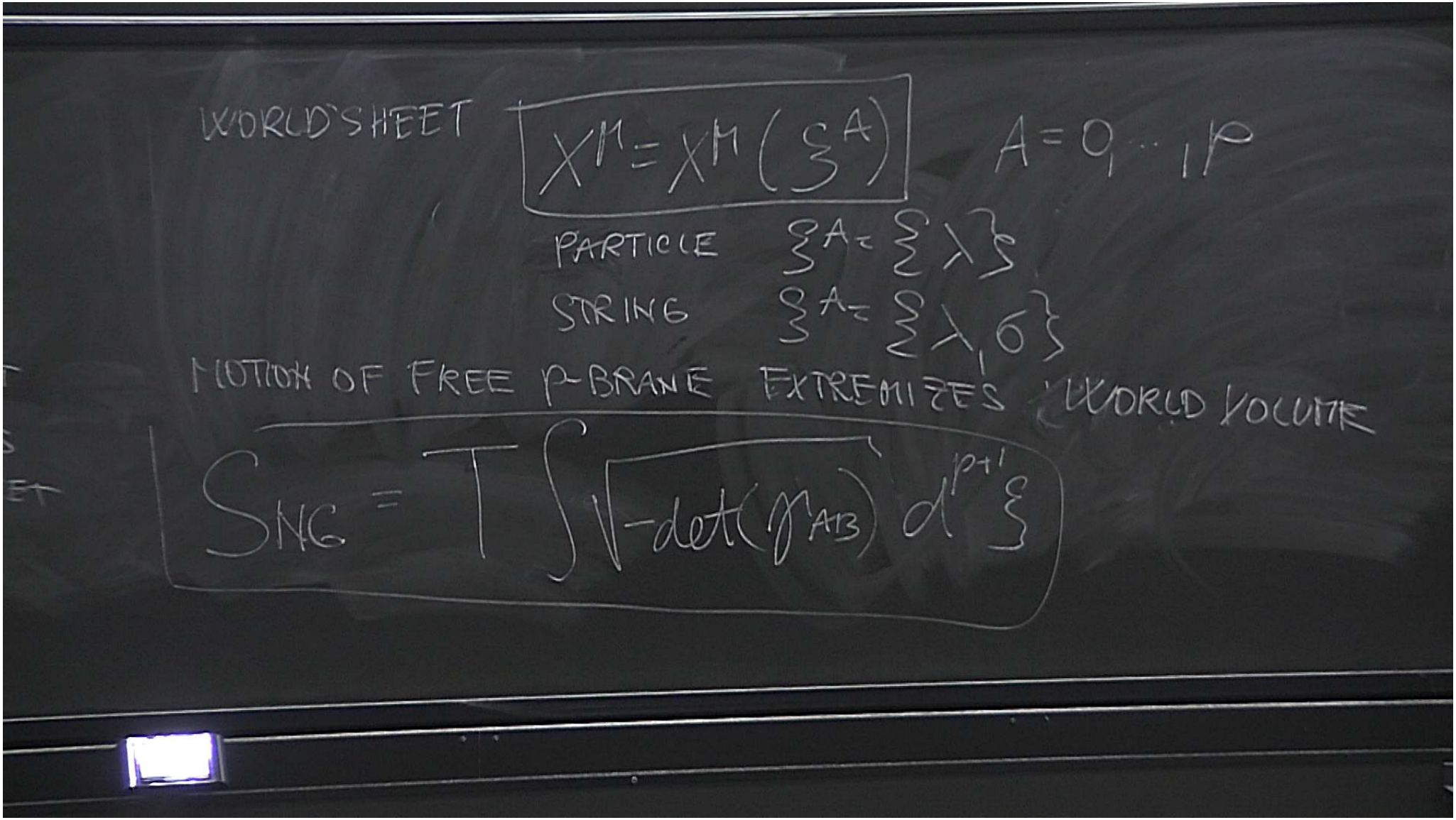


STRING = 1-BRANE




WORLD SHEET
$X^\mu = X^\mu(\xi^A)$
$A = 0 \cdots 1p$
PARTICLE $\xi^A = \xi\{\lambda\}$
STRING $\xi^A = \xi\{\lambda, \sigma\}$


WORLD SHEET
$X^\mu = X^\mu(\xi^A)$
$A = 0 \cdots p$
PARTICLE $\xi^A = \{\lambda\}$
STRING $\xi^A = \{\lambda, \sigma\}$
MOTION OF FREE p-BRANE EXTREMIZES WORLD VOLUME


WORLD SHEET
$X^\mu = X^\mu(\xi^A)$
$A = 0 \dots p$
PARTICLE $\xi^A = \{\lambda\}$
STRING $\xi^A = \{\lambda, \sigma\}$
MOTION OF FREE p-BRANE EXTREMIZES WORLD VOLUME
$S_{NG} = T \int \sqrt{-\det(\gamma_{AB})}\, d^{p+1}\xi$

γ_{AB} .. INDUCED METRIC

$$\gamma_{AB} = \frac{\partial X^\mu}{\partial \xi^A} \frac{\partial X^\nu}{\partial \xi^B} \eta_{\mu\nu}$$