

Title: PSI 2019/2020 - Relativity (Kubiznak)

Speakers:

Collection: PSI 2019/2020 - Relativity (Kubiznak)

Date: September 03, 2019 - 9:00 AM

URL: <http://pirsa.org/19090043>

RELATIVITY CORE

1) OVERVIEW OF SPECIAL RELATIVITY

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a) CAUSAL STRUCTURE

SPACETIME = CONTINUUM OF SPACETIME EVENTS

3 MUTUALLY DISTINCT
POSSIBILITIES

+ p (OBSERVER)

i) $\nearrow q$ $q \dots$ TO THE FUTURE OF p

+ p (OBSERVER)

ii) $\nwarrow p$ $q \dots$ TO THE PAST OF p
 $\downarrow q$

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+ p (OBSERVER)

ii) $\nwarrow p$ $q \dots$ TO THE PAST OF p

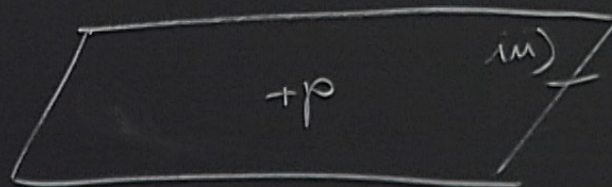
$\nearrow q$

iii) IMPOSSIBLE FOR THE OBSERVER
TO BE AT BOTH p & q

NEWTONIAN PHYSICS

t
↑
SPACE
→

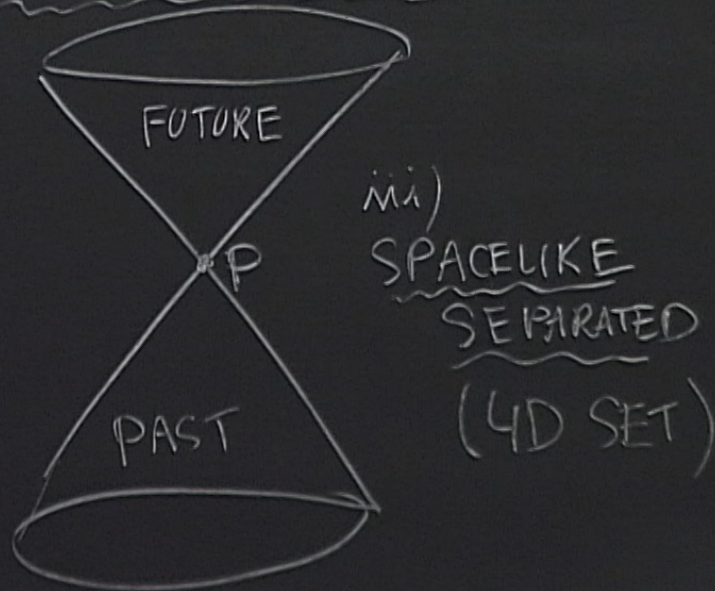
FUTURE



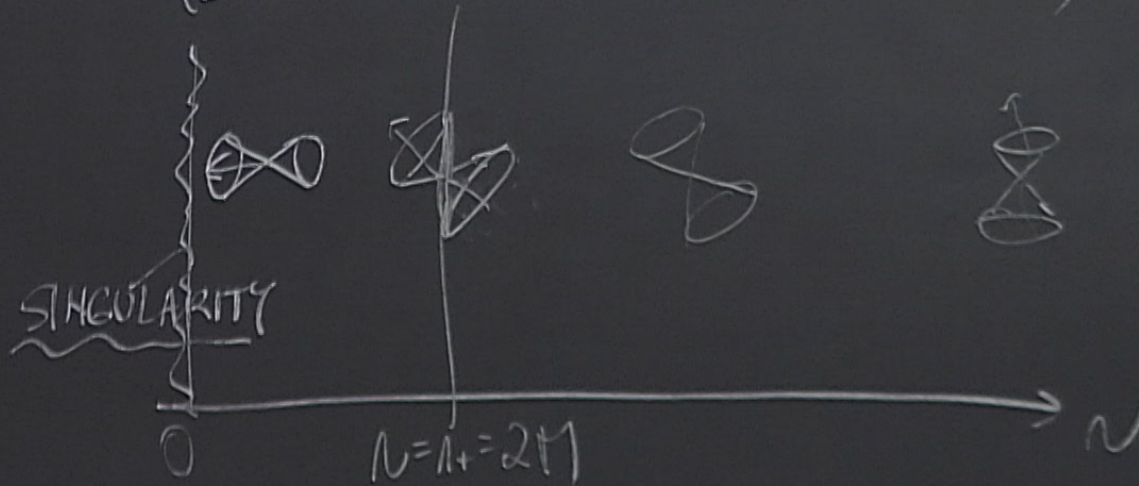
PAST

3D SIMULTANEITY
SURFACE

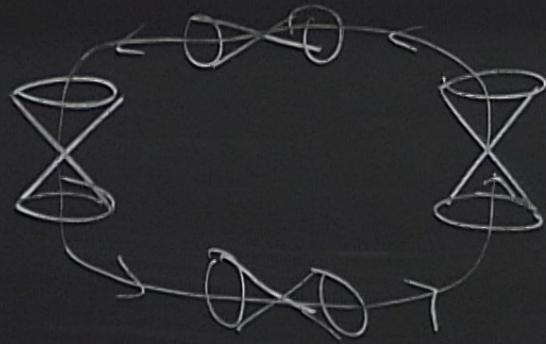
SPECIAL RELATIVITY



REMARK: LIGHTCONE STRUCTURE IS ALSO IMPORTANT
 IN GENERAL RELATIVITY .. BUT ONLY LOCALLY
 (LIGHT CONES CAN "TWIST AROUND")

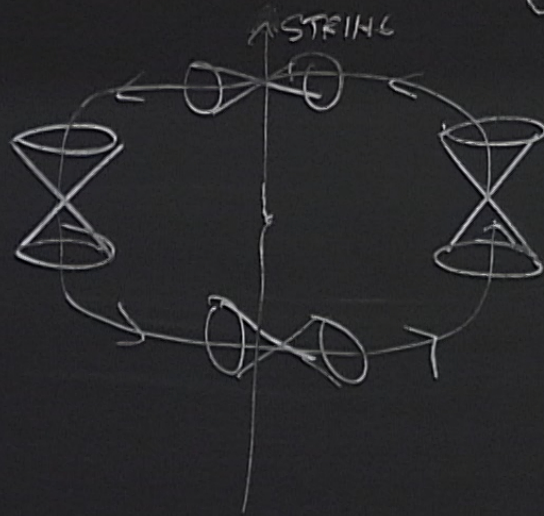


BEWARE: SOLUTIONS WITH CLOSED TIMELIKE CURVES (CTCs)



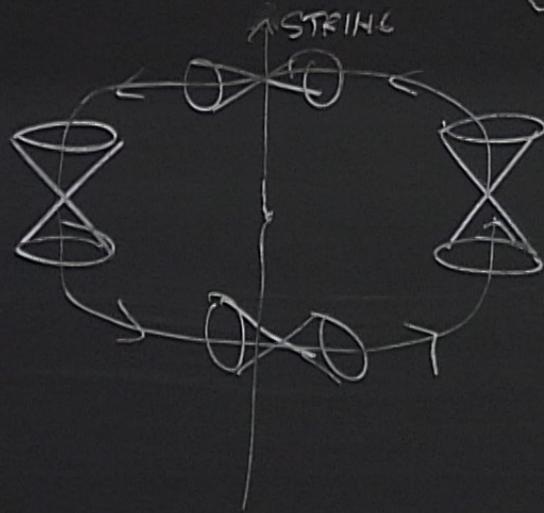
FOR EXAMPLE: KERR SPACETIME
(ONLY ROTATING BH SOLUTION)
... HIDING BEHIND HORIZON

BEWARE: SOLUTIONS WITH CLOSED TIMELIKE CURVES (CTCs)



FOR EXAMPLE: KERR SPACETIME
(ONLY ROTATING BH SOLUTION)
... HIDING BEHIND HORIZON
TAUB-NOT...

BEWARE: SOLUTIONS WITH CLOSED TIMELIKE CURVES (CTCs)



FOR EXAMPLE: KERR SPACETIME
(ONLY ROTATING BH SOLUTION)
... HIDING BEHIND HORIZON

TAUB-NOT ... BH SOLUTION WITH
ROTATING STRINGS

(?) PATHOLOGICAL (?)

• INERTIAL OBSERVERS PLAY IMPORTANT ROLE IN BOTH
NEWTONIAN & SR PHYSICS

.. WE HAVE ∞ -MANY .. CONNECTED BY

(GALILEAN TRANSF & POINCARÉ TRANSF)

BOOST, SPACE ROTATIONS, SPACETIME TRANSL.

• INERTIAL OBSERVERS PLAY IMPORTANT ROLE IN BOTH
NEWTONIAN & SR PHYSICS

.. WE HAVE ∞ -MANY .. CONNECTED B.Y

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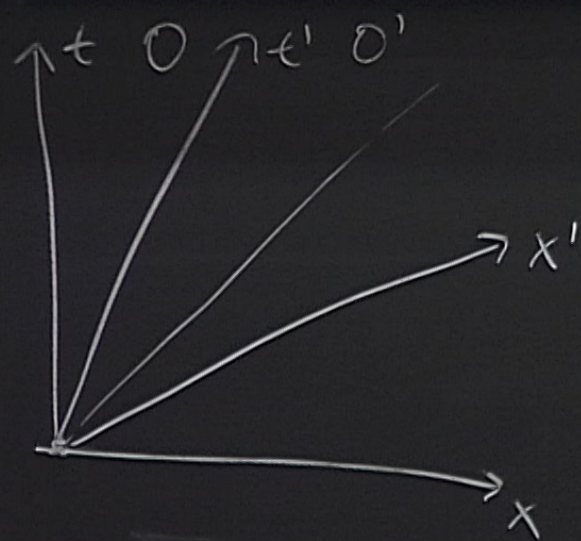
EX:

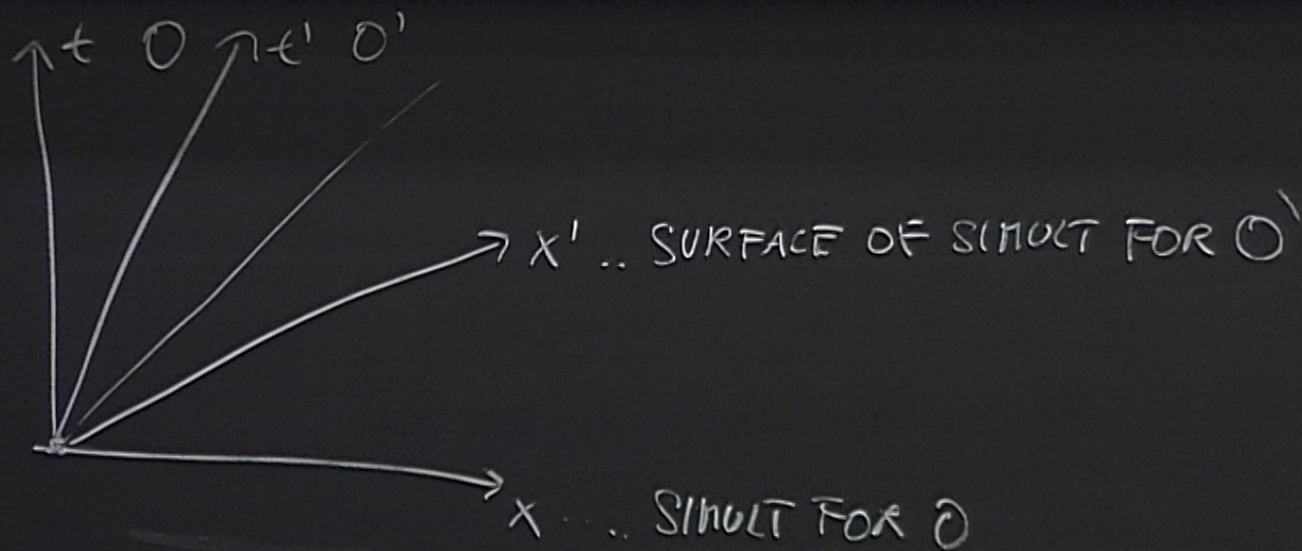
BOOST IN x-DIRECTION

$$t' = \gamma(t - vx/c^2)$$

$$x' = \gamma(x - vt)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \beta = \frac{v}{c}$$





- IN GENERAL COORDINATES HAVE NO INTRINSIC MEANING... SEARCH FOR QUANTITIES WITH ABSOLUTE MEANING

NEXTON: Δt (ABSOLUTE TIME)

- IN GENERAL COORDINATES HAVE NO INTRINSIC MEANING... SEARCH FOR QUANTITIES WITH ABSOLUTE MEANING

NEWTON: Δt (ABSOLUTE TIME)

$|\Delta \vec{x}|$ FOR SIMULTANEOUS EVENTS

SR: ONLY ONE... SPACETIME INTERVAL

$$\Delta S^2 = \eta_{\mu\nu} \Delta x^\mu \Delta x^\nu = -c^2 (\Delta t)^2 + \delta_{ij} \Delta x^i \Delta x^j$$

$\uparrow$ MINK METRIC $\uparrow$ (ct, x^i)

DIAG $(-1, 1, 1, 1)$

b) LORENZ TRANSFORMATIONS

LEAVE THE FORM OF MINK METRIC INVARIANT.

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}, \quad \eta_{\alpha\beta} = \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} \eta_{\mu\nu}$$

b) LORENTEZ TRANSFORMATIONS

LEAVE THE FORM OF MINK METRIC INVARIANT

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}, \quad \eta_{\alpha\beta} = \Lambda^{\mu}_{\alpha} \Lambda^{\nu}_{\beta} \eta_{\mu\nu}$$

($\Lambda^T = \Lambda^{-1}$) "ROTATIONS IN SPACETIME"

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$$x^{\mu} = \tilde{\Lambda}^{\mu}_{\nu} x'^{\nu}, \quad \Lambda^{\mu}_{\nu} \tilde{\Lambda}^{\nu}_{\alpha} = \delta^{\mu}_{\alpha}$$

$$(\mathbb{I} = \Lambda^T \Lambda)$$

"ROTATIONS IN
SPACETIME"

VECTOR: $V'^{\mu} = \Lambda^{\mu}_{\nu} V^{\nu}$

CO-VECTOR: $\omega_{\mu}' = \tilde{\Lambda}^{\nu}_{\mu} \omega_{\nu}$

SCALAR: $\phi'(x') = \phi(x)$

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EX: BOOST IN X-DIRECTION

$$\Lambda^{\alpha}_{\beta} = \begin{matrix} & \begin{matrix} t & x & y & z \end{matrix} \\ \begin{matrix} t \\ x \\ y \\ z \end{matrix} & \begin{pmatrix} \cosh \theta & -\sinh \theta & 0 & 0 \\ -\sinh \theta & \cosh \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

VECTOR: $V'^{\mu} = \Lambda^{\mu}_{\nu} V^{\nu}$

CO-VECTOR: $\omega_{\mu}' = \tilde{\Lambda}^{\nu}_{\mu} \omega_{\nu}$

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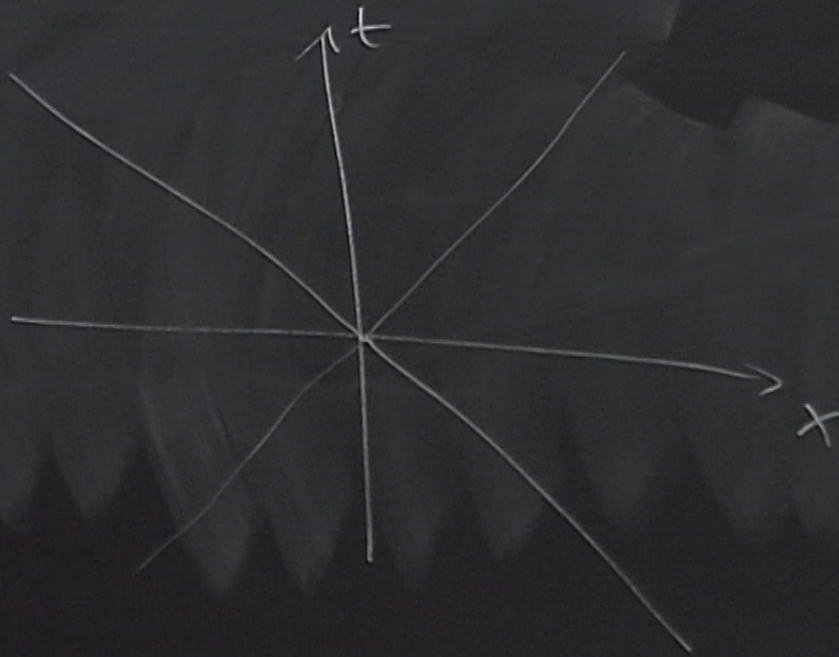
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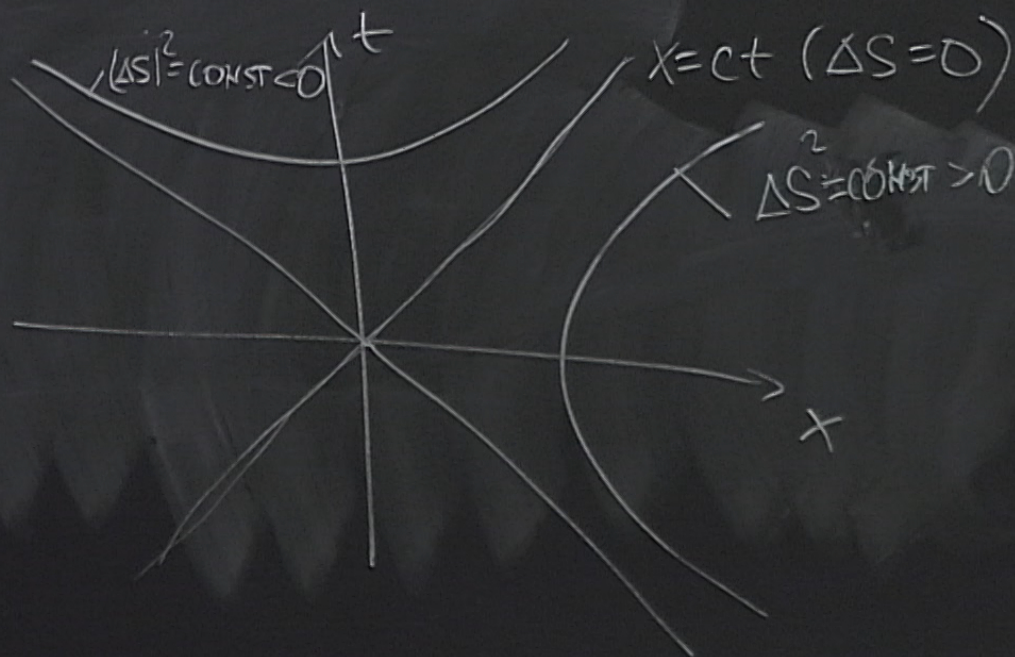
$$\beta = \frac{v}{c} = \tanh \theta$$

$$\Rightarrow \cosh \theta = \gamma, \sinh \theta = \beta \gamma$$

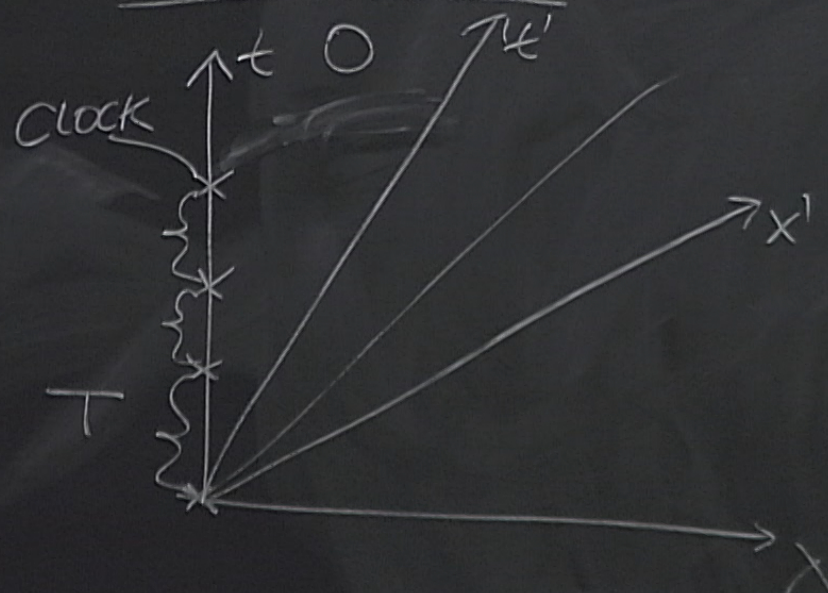
STRESS . LORENTZ INVARIANT INTERVAL

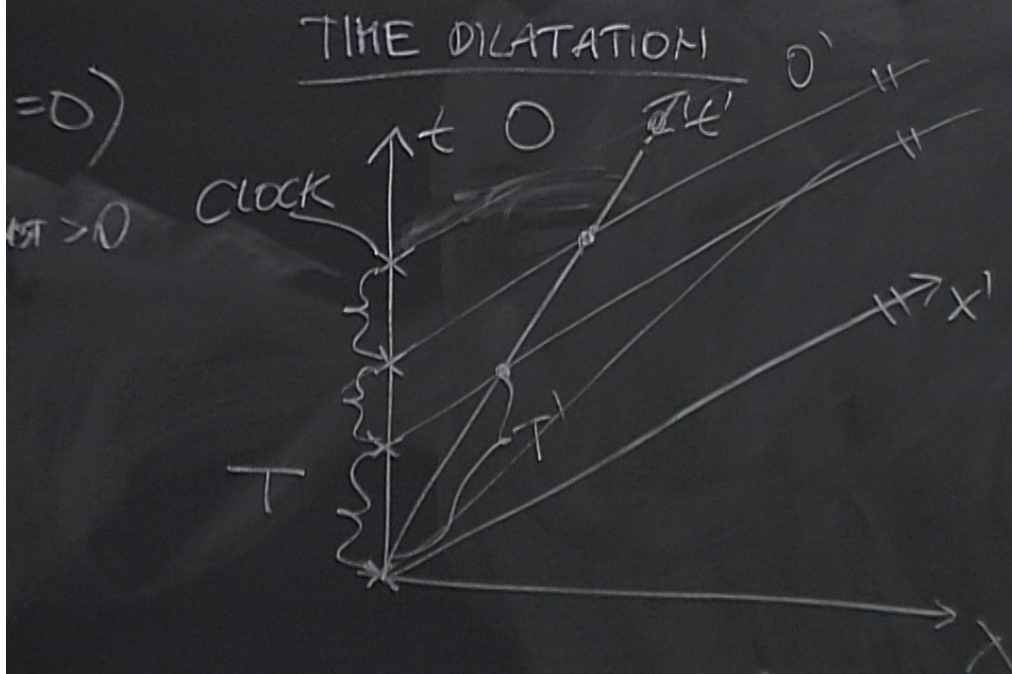


STRESS . LORENTZ INVARIANT INTERVAL



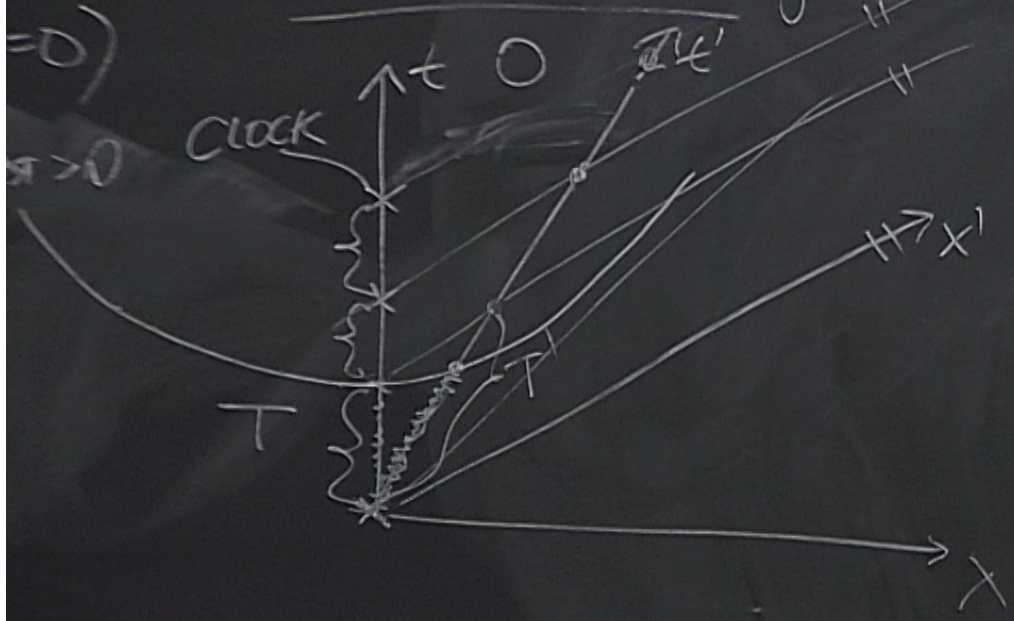
TIME DILATION





$$T' = \gamma T$$

TIME DILATION

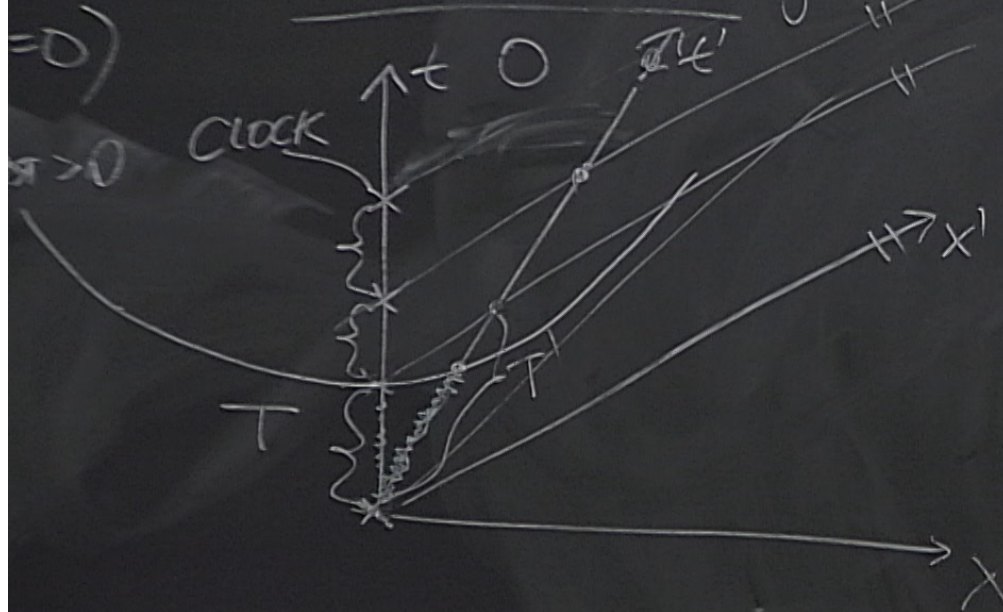


$$T' ? T$$

$$T' > T$$

FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY

TIME DILATION



$$T' ? T$$

$$T' > T$$

FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY

$$T' = \gamma T$$

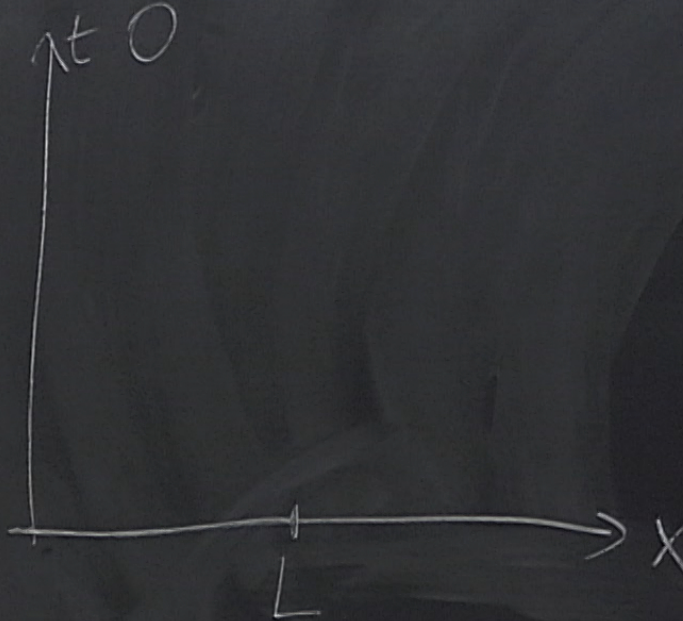
$$T' ? T$$

$$T' > T$$

FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY

$$T' = \gamma T$$

LORENTZ CONTRACTION



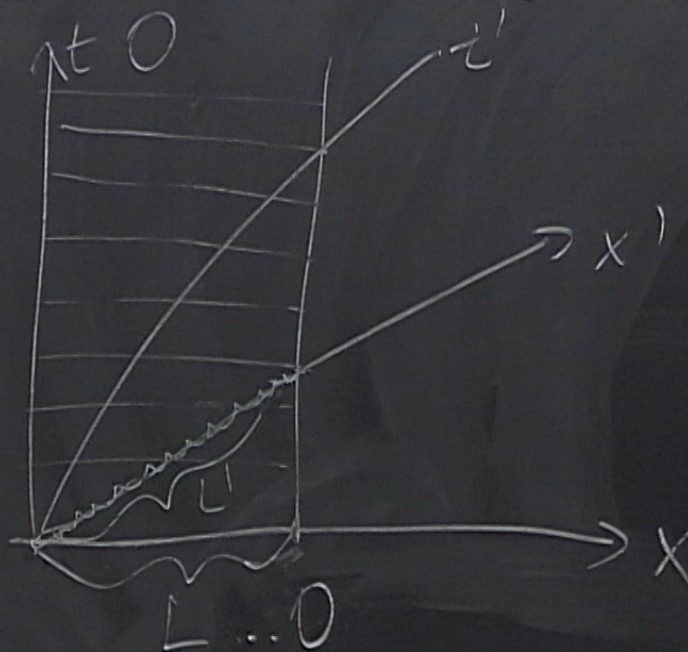
$$T' ? T$$

$$T' > T$$

FOR MOVING OBSERVER
TIME GOES MORE
SLOWLY

$$T' = \gamma T$$

LORENTZ CONTRACTION

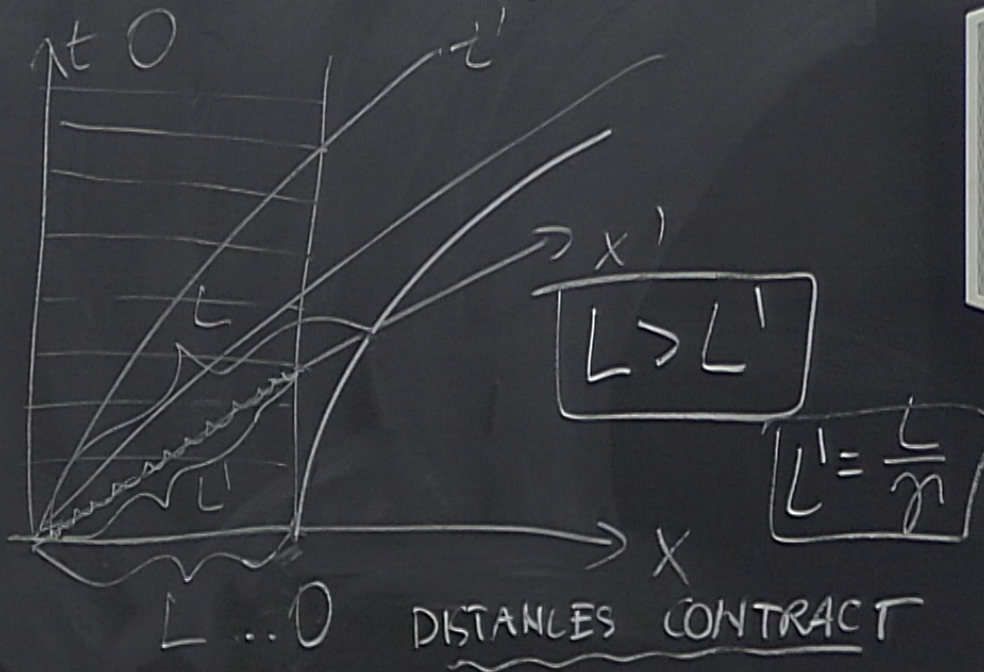


$$T' > T$$

FOR MOVING OBSERVER
TIME GOES MORE SLOWLY

$$T' = \gamma T$$

LORENTZ CONTRACTION



c) RELATIVISTIC OBJECTS

PARTICLE

$\uparrow ct$

$$x^\mu = x^\mu(\lambda)$$

WORLD LINE

TRAJECTORY

$$\frac{dx^\mu}{d\lambda} \dots 4\text{-VELOCITY}$$

$\rightarrow x$

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$$

$$\eta_{\mu\nu} \frac{dx^\mu}{dx^0} \frac{dx^\nu}{dx^0} \begin{cases} > 0 & \text{(NON-PHYSICAL)} \\ = 0 & \text{NULL (LIGHT)} \\ < 0 & \end{cases}$$

MINK METRIC

DIAG $(-1 \ 1 \ 1 \ 1)$

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} \begin{cases} > 0 & \text{(NON-PHYSICAL)} \\ = 0 & \text{NULL (LIGHT)} \\ < 0 & \text{MASSIVE PARTICLES} \end{cases}$$

FREE PARTICLE ... NO

PROPER TIME τ

$$\boxed{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1}$$

NEWTON. AT (ABSOLUTE TIME)

SR. ONLY

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} \begin{cases} > 0 & \text{(NON-PHYSICAL)} \\ = 0 & \text{NULL (LIGHT)} \\ < 0 & \text{MASSIVE PARTICLES} \end{cases}$$

PROPER TIME τ

$$\boxed{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1}$$

$$d\tau = \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} d\lambda$$

FREE PARTICLE

$$\boxed{S = -mc^2 \int d\tau = -mc^2 \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}}$$

ABSOLUTE TIME) SR: ONLY ONE SPACETIME INTERVAL

PHYSICAL)

FREE PARTICLE .. MOVES ON GEODESICS

(LIGHT)

VE PARTICLES

$$\begin{aligned} S &= -mc^2 \int d\tau \\ &= -mc^2 \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} d\lambda \end{aligned}$$

$L(x^\mu, \dot{x}^\mu, \lambda)$

$$\frac{dx^\mu}{d\tau} = 0$$

$$u^\mu = \frac{dx^\mu}{d\tau}$$

NOTE . $\frac{d}{d\tau} = \frac{dx^\nu}{d\tau} \frac{\partial}{\partial x^\nu} = u^\nu \frac{\partial}{\partial x^\nu}$

$$\frac{du^\mu}{d\tau} = u^\nu \frac{\partial u^\mu}{\partial x^\nu} = u^\nu \partial_\nu u^\mu$$
$$= \boxed{\partial_\mu u^\mu = 0}$$

NOTE : $\frac{d}{d\tau} = \frac{dx^\nu}{d\tau} \frac{\partial}{\partial x^\nu} = u^\nu \frac{\partial}{\partial x^\nu}$

$$\frac{du^\mu}{d\tau} = u^\nu \frac{\partial u^\mu}{\partial x^\nu} = u^\nu \partial_\nu u^\mu$$

GR: $\boxed{\nabla_\mu u^\mu = 0} = \boxed{\partial_\mu u^\mu = 0}$



$$p^\mu = m_0 u^\mu$$

$$\frac{dp^\mu}{d\tau} = F^\mu$$

4-FORCE

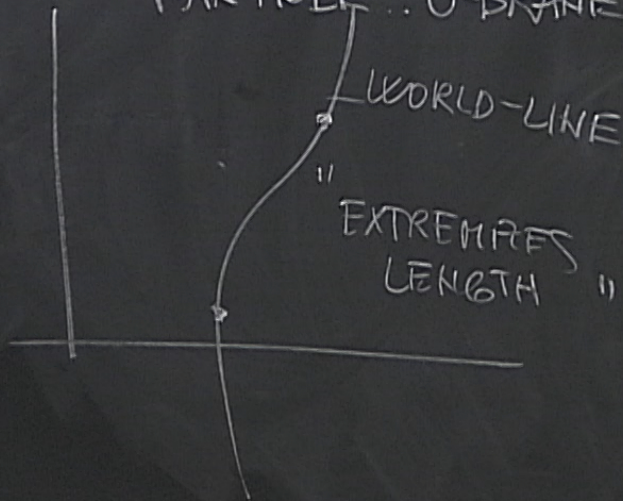
2ND NEWTON



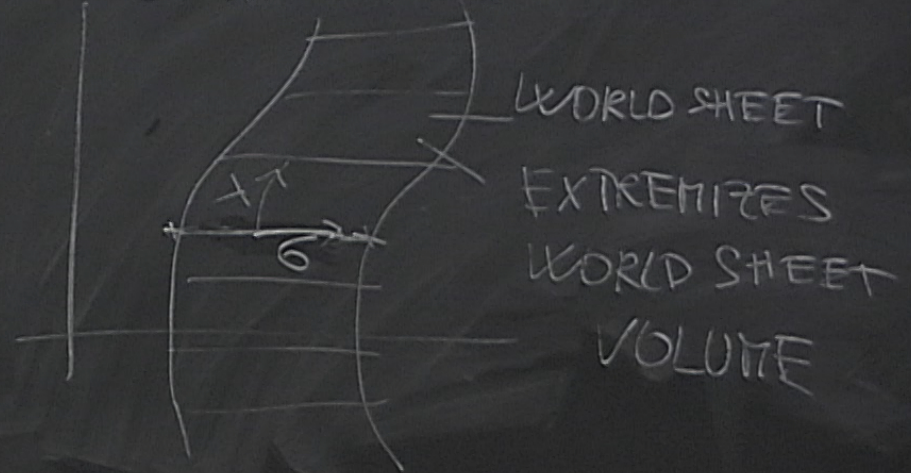
MOTION OF p-BRANES

p-BRANE . HIGHER-DIM. OBJECT
THAT SWEEPS (p+1)-DIM TRAJECTORY

PARTICLE .. 0-BRANE



STRING = 1-BRANE





WORLD SHEET

$$X^M = X^M(\xi^A)$$

$$A = 0 \dots 1, p$$

PARTICLE

$$\xi^A = \xi^\lambda$$

STRING

$$\xi^A = \xi^{\lambda, \sigma}$$

WORLD SHEET

$$X^M = X^M(\xi^A)$$

$$A = 0 \dots 1/p$$

PARTICLE

$$\xi^A = \{\tau\}$$

STRING

$$\xi^A = \{\tau, \sigma\}$$

MOTION OF FREE p-BRANE

EXTREMIZES

WORLD VOLUME

WORLD SHEET

$$X^M = X^M(\xi^A)$$

$$A = 0 \dots 1, p$$

PARTICLE $\xi^A = \{\tau\}$

STRING $\xi^A = \{\tau, \sigma\}$

MOTION OF FREE p -BRANE EXTREMIZES WORLD VOLUME

$$S_{NG} = T \int \sqrt{-\det(g_{AB})} d^{p+1} \xi$$

γ_{AB} ... INDUCED METRIC

$$\gamma_{AB} = \frac{\partial x^\mu}{\partial \xi^A} \frac{\partial x^\nu}{\partial \xi^B} \eta_{\mu\nu}$$