Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 15

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

Date: September 20, 2019 - 10:45 AM

URL: http://pirsa.org/19090042

d) BLACK HOLE TDS

MOTIVATION: SCHWARZSCHILD BH

$$\boxed{ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 \qquad f = 1 - \frac{2M}{r}\,,\quad d\Omega^2 = \sin^2\theta\, d\varphi^2 + d\theta^2}$$

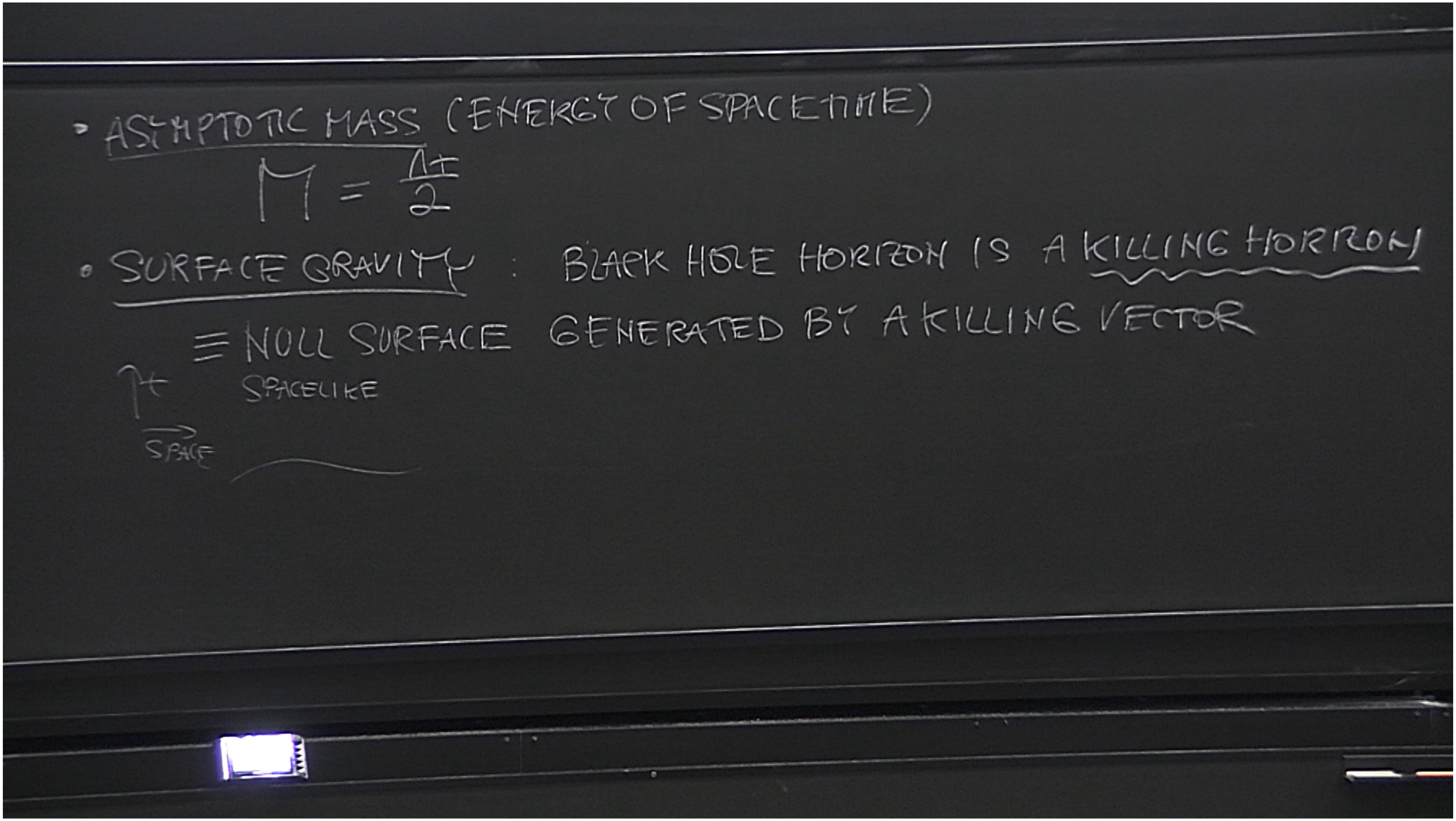
• AS

- ASYMPTOTIC MASS (ENERGY OF SPACETIME)

$$M = \frac{\Lambda_I}{2}$$

- SURFACE GRAVITY : BLACK HOLE HORIZON IS A KILLING HORIZON

$\equiv$ NULL SURFACE GENERATED BY A KILLING VECTOR



• ASYMPTOTIC MASS (ENERGY OF SPACETIME)
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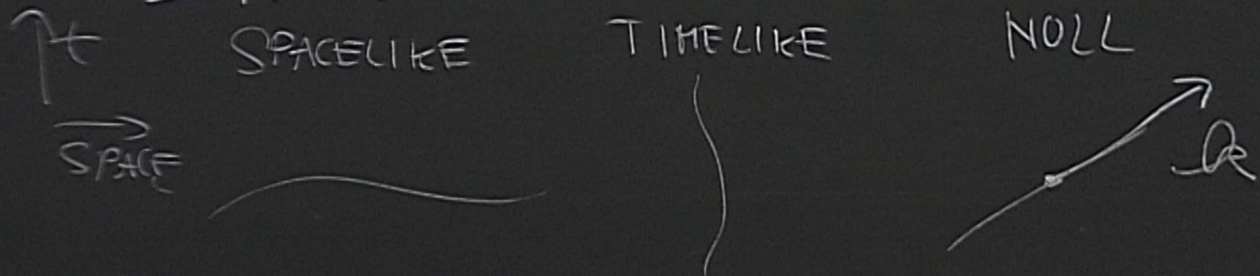
t
SPACELIKE
SPACE

- ASYMPTOTIC MASS (ENERGY OF SPACETIME)

$$M = \frac{r_+}{2}$$

- SURFACE GRAVITY : BLACK HOLE HORIZON IS A KIL

$\equiv$ NULL SURFACE GENERATED BY A KILLING VE



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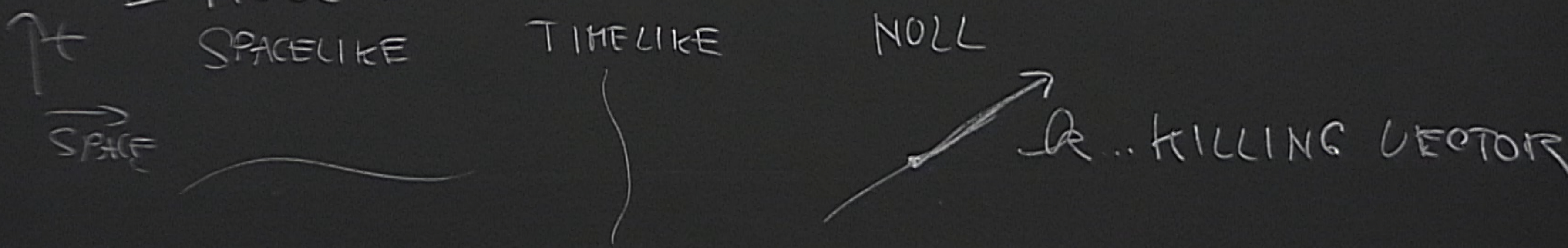

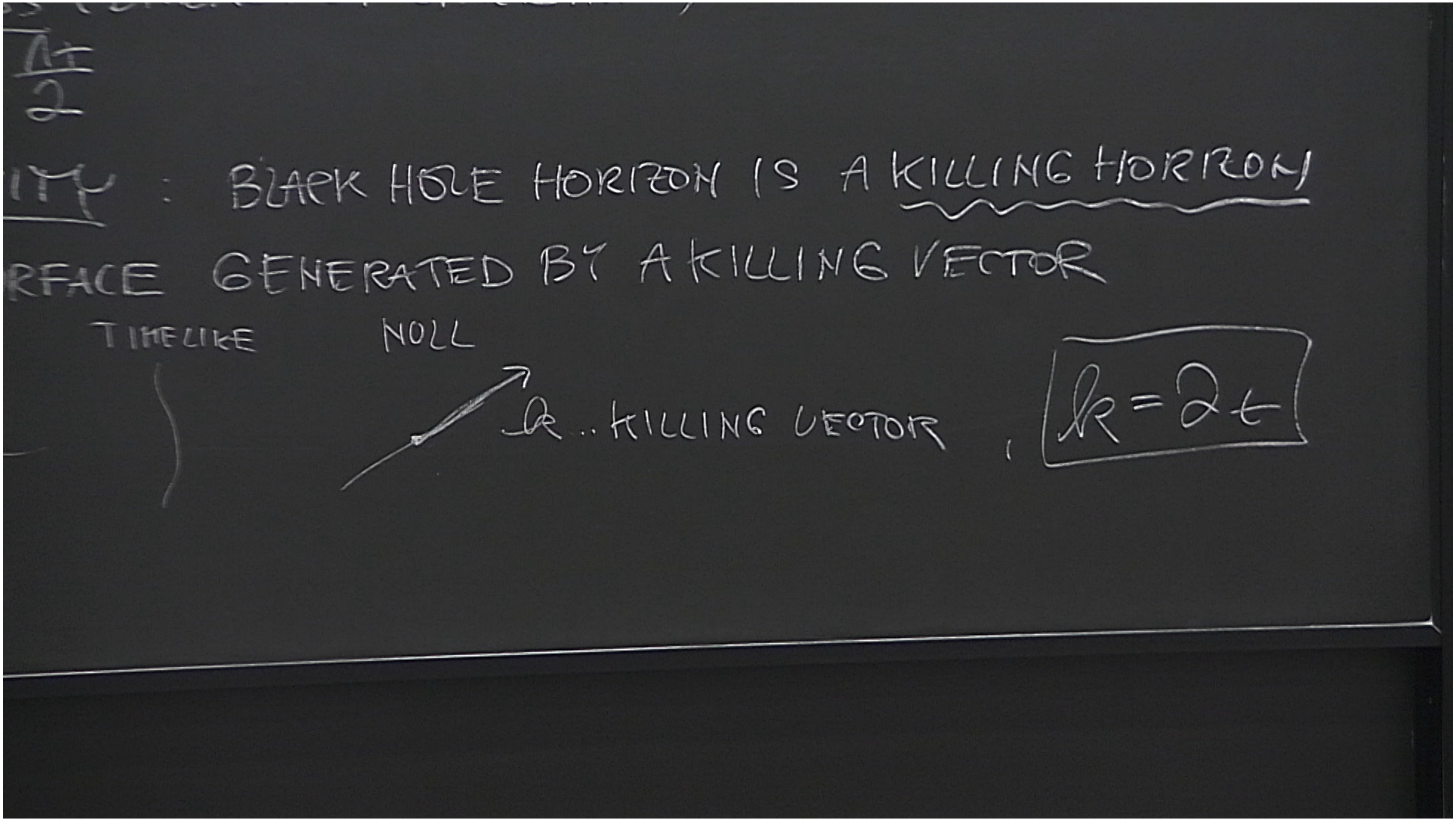

ITY : BLACK HOLE HORIZON IS A KILLING HORIZON
RFACE GENERATED BY A KILLING VECTOR
TIMELIKE
NULL
k .. KILLING VECTOR
k=2t

• SURFACE GRAVITY : BLACK HOLE HORIZON IS A KILLING

≡ NULL SURFACE GENERATED BY A KILLING VECTOR

t

SPACE

SPACELIKE TIMELIKE NULL

k .. KILLING VECTOR

k ... NULL GEODESIC

...RAVITY : BLACK HOLE HORIZON IS A KILLING HORIZON

... SURFACE GENERATED BY A KILLING VECTOR

...IKE   TIMELIKE   NULL

k ... KILLING VECTOR , $\boxed{k = \partial_t}$

...L GEODESIC

$$k^\mu \nabla_\mu k^\nu = \kappa k^\nu$$

RAVITY : BLACK HOLE HORIZON IS A KILLING HORIZON

SURFACE GENERATED BY A KILLING VECTOR

LIKE   TIMELIKE   NULL

k .. KILLING VECTOR , $\boxed{k = \partial_t}$

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$$k^\mu \nabla_\mu k^\nu = \kappa k^\nu$$

NON-AFFINE PARAMETRIZED

κ .. SURFACE GRAVITY

$$\kappa = \frac{f'(r_+)}{2} = \frac{2M}{r_+^2\, 2} = \frac{1}{4M} = \frac{1}{2r_+}$$

$\varkappa$ ... SURFACE GRAVITY

$$\varkappa = \frac{f'(r_+)}{2} = \frac{2M}{r_+^2\, 2} = \frac{1}{4M} = \frac{1}{2r_+}$$

- CONSIDER NEWTONIAN GRAV. ACC. ON HORIZON

$$a = \frac{M}{r_+^2} = \frac{1}{2r_+}$$

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COINCIDENCE.

• HORIZON AREA ($t = \text{CONST}$, $r = r_+$)

$$ds^2 \to r_+^2 \left(\sin^2\theta \, d\varphi + d\theta^2\right)$$

RIZON

CE.

- HORIZON AREA ($t = \text{const}$, $r = r_+$)

$$ds^2 \to r_+^2 \left(\sin^2\theta \, d\varphi + d\theta^2 \right) = \gamma$$

$$A = \int \sqrt{\det \gamma} \, d\theta \, d\varphi$$

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ON

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• OBSERVATION: $dM = \frac{1}{2} dr_+$

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- OBSERVATION: $dM = \frac{1}{2} dr_+$, $dA = 8\pi r_+ dr_+$

$$= \frac{1}{2r_+}$$

...C. ON HORIZON

COINCIDENCE.

$$ds^2 \to r_+^2 (\sin^2\theta \, d\varphi + d\theta^2) = \gamma$$

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(PHYSICAL PROCESS
→ THROW IN A PARTICLE)

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IZON

$$A = \int \sqrt{\det \gamma}\, d\theta\, d\varphi = \int r_+^2 \sin\theta\, d\theta\, d\varphi =$$

• OBSERVATION:

(PHYSICAL PROCESS
→ THROW IN A PARTICLE)

E.

$$dM = \frac{1}{2} dr_+ \,, \quad dA = 8\pi r_+ dr_+$$

$$dM = \frac{\kappa}{2\pi} \frac{dA}{4}$$

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1ST LAW OF BH MECHANICS

- LAWS OF BLACK HOLE MECHANICS (1973)

GENERAL BH: M, J, Q

0th : SURFACE GRAVITY $\varkappa = \text{CONST}$ (STATIONARY BH)

1st:

$$\boxed{dM = \frac{\varkappa}{2\pi}\frac{dA}{4} + \Omega\, dJ + \phi\, dQ}$$

ANG. VEL. EL. STAT. POTENTIAL

2ND LAW: CLASSICALLY, THE AREA OF BH NEVER DECREASES

$$\boxed{dA \geq 0}$$

3RD: IT IS IMPOSSIBLE TO REDUCE $\varkappa \to 0$ IN A FINITE # OF STEPS

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3RD: IT IS IMPOSSIBLE TO REDUCE $\kappa \to 0$ IN A FINITE # OF STEPS

PROOF: COMPLETELY GEOMETRICAL ☺

• LAWS OF BLACK HOLE MECHANICS (1973)

GENERAL BH: M, J, Q

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1st:

$$\boxed{dM = \frac{\varkappa}{2\pi}\frac{dA}{4} + \overbrace{\Omega\, dJ + \Phi\, dQ}^{\text{WORK TERMS}}}$$

ANG. VEL.

EL. STAT. POTENTIAL

INTRIQUING: $\mathcal{H} \sim T$?

$A \sim S$

BUT CLASSICALLY THIS IS
NOT THE CASE

BH.. PERFECT ABSORBER

INTRIGUING: $\kappa \sim T$?

$A \sim S$

BUT <u>CLASSICALLY</u> THIS IS NOT THE CASE

BH.. PERFECT ABSORBER

WHICH DOES NOT EMIT

$T = 0$

ORIGIN OF BH THERMODYNAMICS.

WHEELER'S CUP OF TEA

WHERE DOES THE ENTROPY GO?

ER

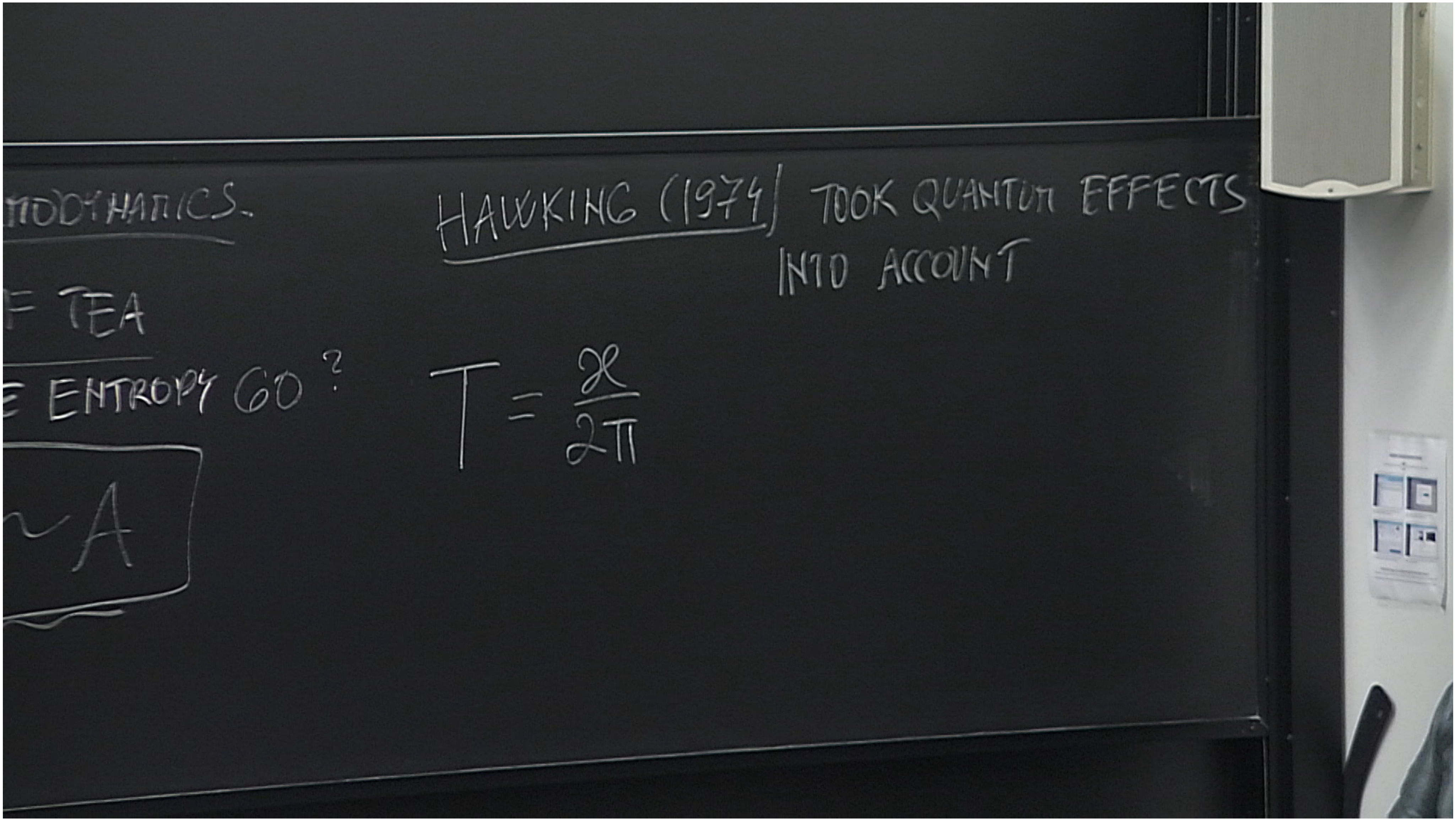
ORIGIN OF BH THERMODYNAMICS.

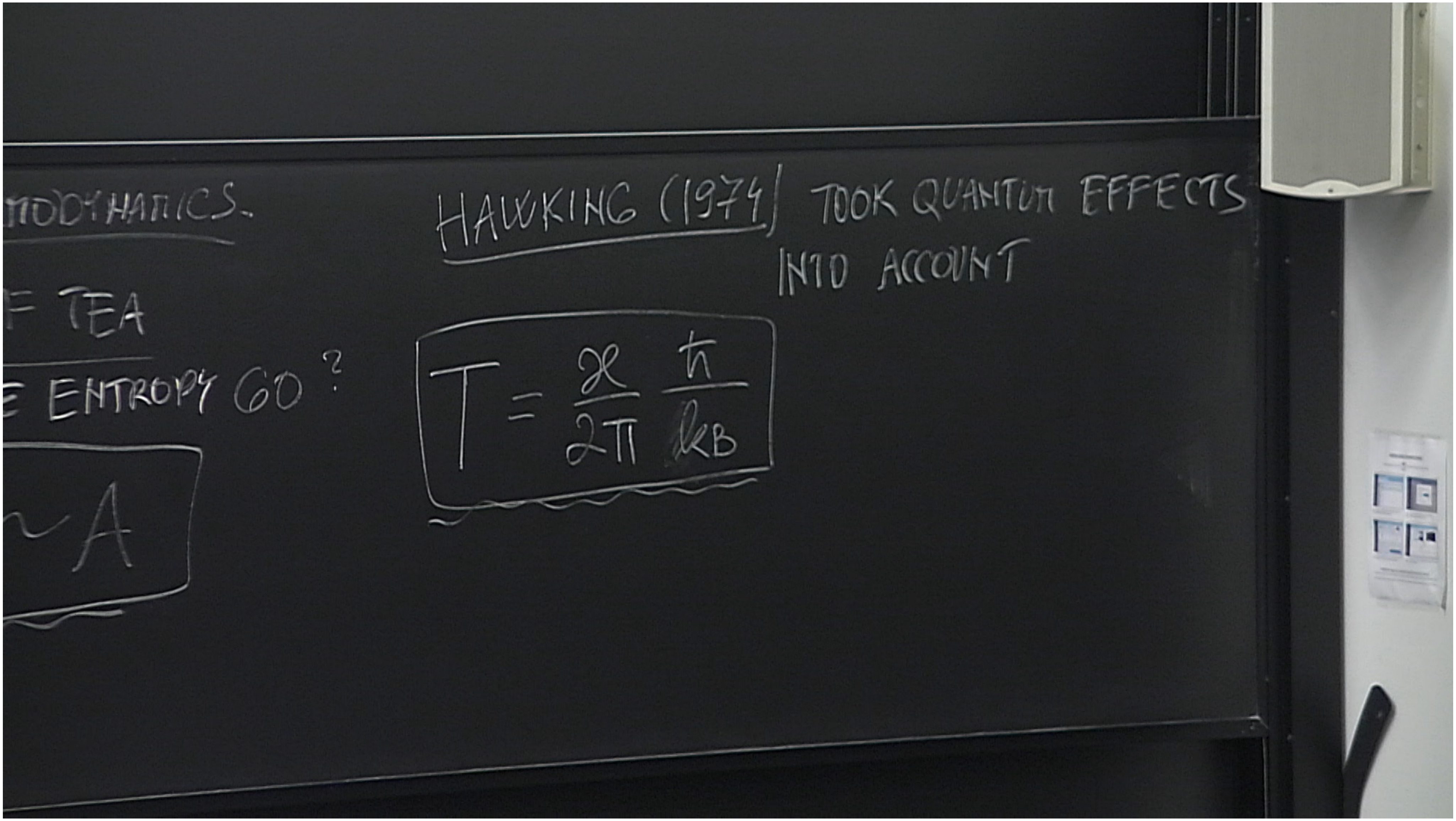
- WHEELER'S CUP OF TEA

WHERE DOES THE ENTROPY GO ?

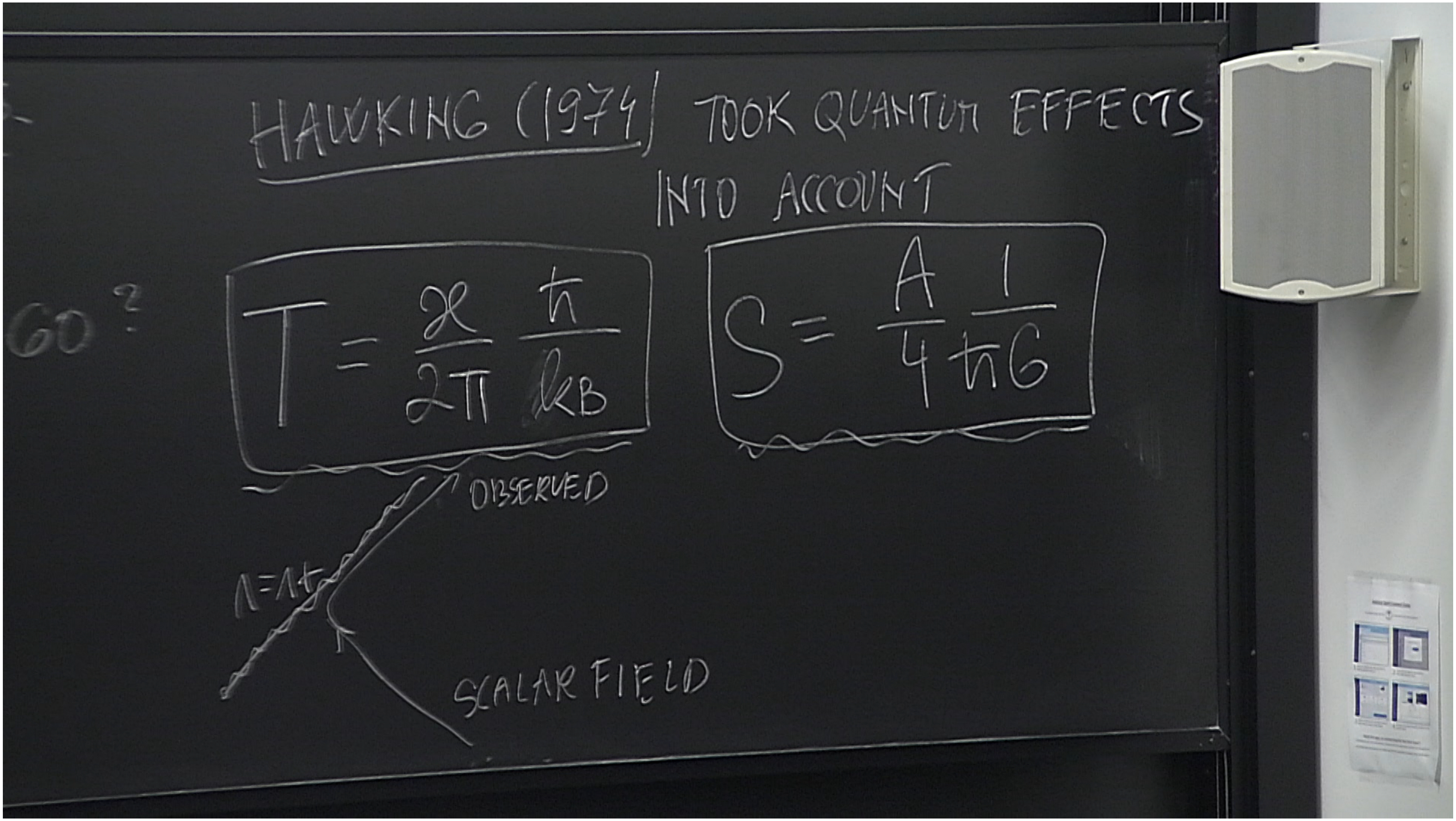
BEKENSTEIN : $\boxed{S \sim A}$

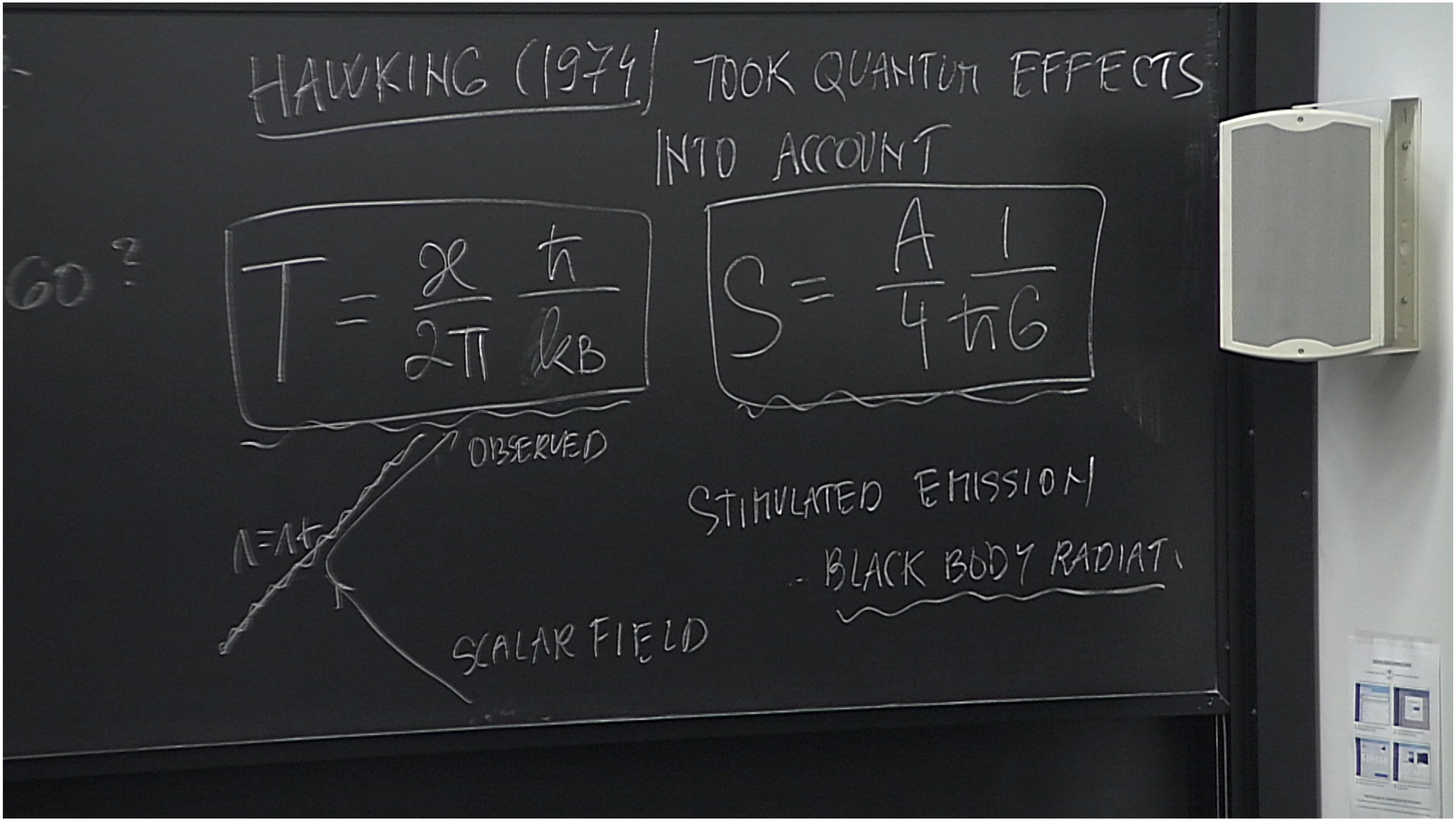
ER


HAWKING (1974) TOOK QUANTUM EFFECTS
INTO ACCOUNT
F TEA
E ENTROPY GO ?
$T = \frac{\varkappa}{2\pi}$
~A



MODYNAMICS.
F TEA
E ENTROPY GO ?
$\sim A$
HAWKING (1974) TOOK QUANTUM EFFECTS INTO ACCOUNT
$T = \frac{\kappa}{2\pi} \frac{\hbar}{k_B}$


HAWKING (1974) TOOK QUANTUM EFFECTS
INTO ACCOUNT
$T = \frac{\varkappa}{2\pi}\frac{\hbar}{k_B}$
$S = \frac{A}{4}\frac{1}{\hbar G}$
OBSERVED
SCALAR FIELD


HAWKING (1974)
TOOK QUANTUM EFFECTS INTO ACCOUNT
$T = \frac{\kappa}{2\pi} \frac{\hbar}{k_B}$
$S = \frac{A}{4} \frac{1}{\hbar G}$
OBSERVED
$\Lambda = \Lambda +$
STIMULATED EMISSION
BLACK BODY RADIAT
SCALAR FIELD

EUCLIDEAN TRICK VERIFICATION

- THERMAL GREEN FUNCTIONS HAVE PERIODICITY IN IMAGINARY (EUCLIDEAN) TIME $\boxed{\tau = it}$

$$G(\tau) = G(\tau + \beta), \quad \beta = \frac{1}{T}$$

EUCLIDEAN TRICK VERIFICATION

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$$\boxed{ds^2 = -(1+aX)^2 dT^2 + dX^2 + dy^2 + dz^2}$$

- WICK ROTAT. $\tau = i T$

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- WICK ROTAT. $\tau = i T$

$$ds^2 = (1+aX)^2 d\tau^2 + dX^2 + \dots$$

- CHANGE OF COORDS

$$\rho = \frac{1+aX}{a}$$

ET'S TRY THIS FOR RIHDLER

• RIH

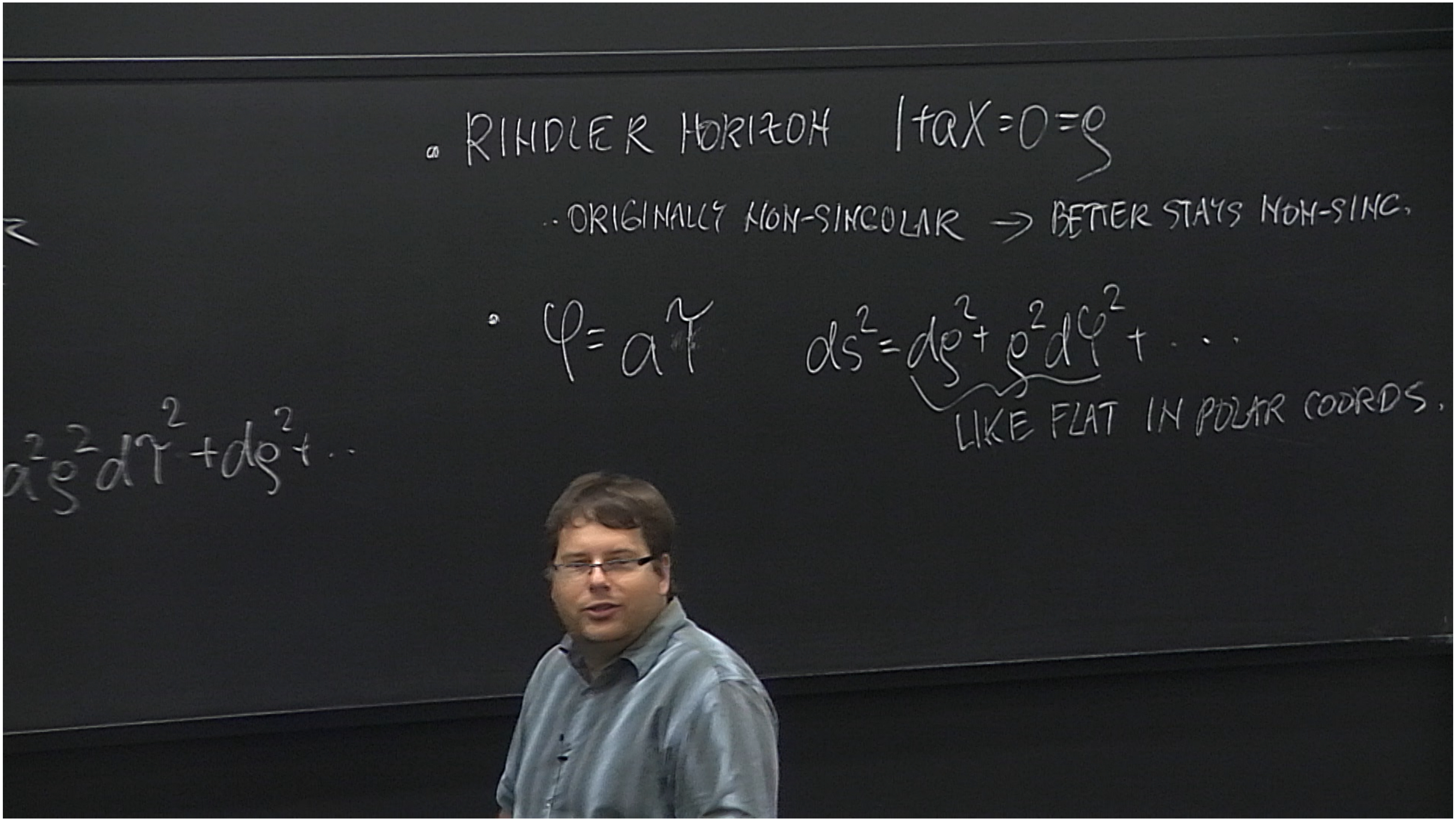
$$\boxed{ds^2 = -(1+aX)^2 dT^2 + dX^2 + dy^2 + dz^2}$$

• WICK ROTAT. $\tau = i T$

$$ds^2 = (1+aX)^2 d\tau^2 + dX^2 + \dots = a^2 \rho^2 d\tau^2 + d\rho^2 + \dots$$

• CHANGE OF COORDS

$$\rho = \frac{1+aX}{a}, \quad d\rho = dX$$

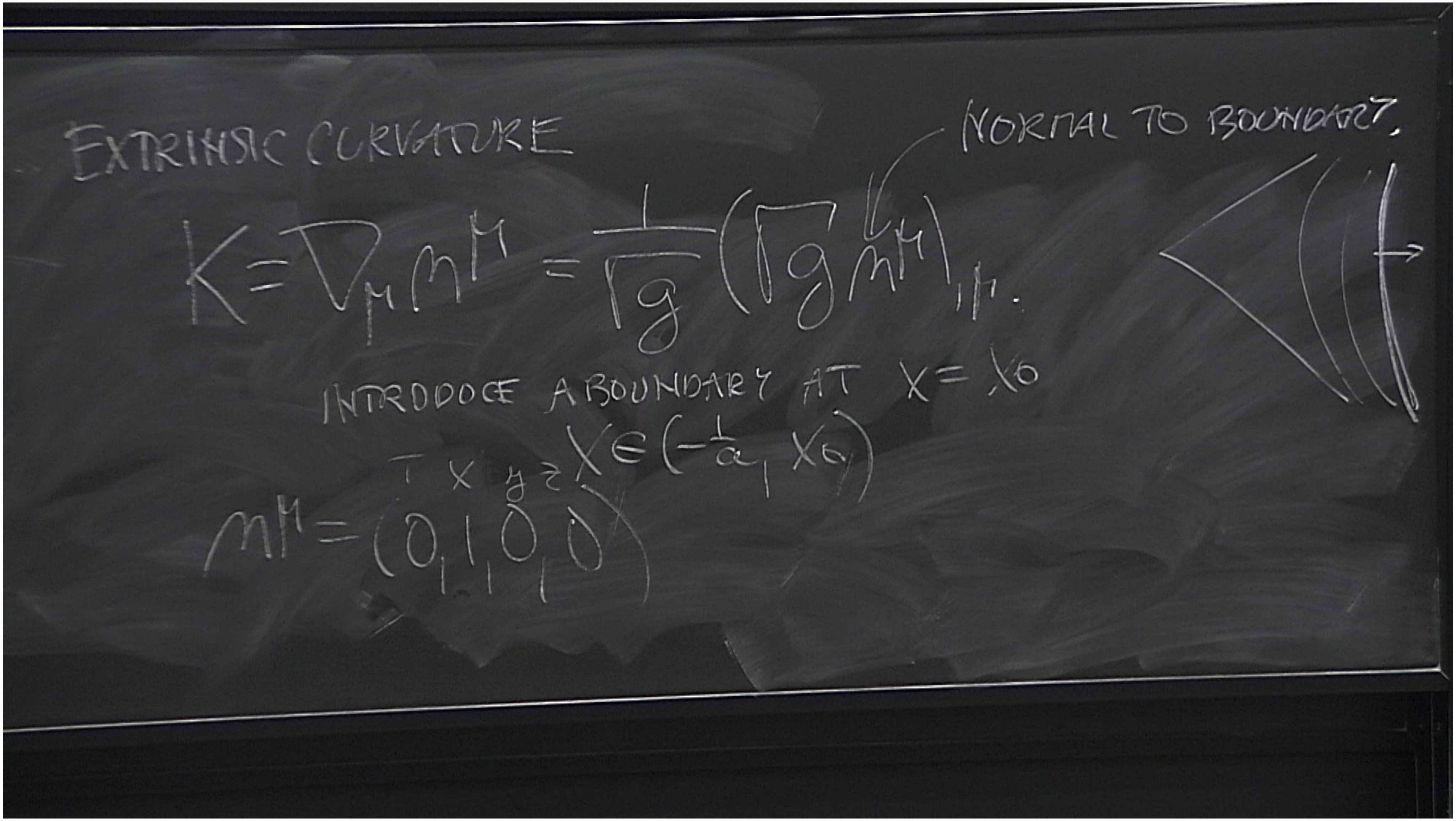

RINDLER HORIZON 1+aX = 0 = ρ
ORIGINALLY NON-SINGULAR → BETTER STAYS NON-SING.
φ = aτ
ds² = dρ² + ρ²dφ² + . . .
LIKE FLAT IN POLAR COORDS.
a²ρ²dτ² + dρ² + . .

- LAWS OF BLACK HOLE MECHANICS (1973)

ACTION CALCULATION:

$$S = \underbrace{\int_{\Omega} \frac{d^4x \sqrt{g}\, R}{16\pi G}}_{\text{E-H}} = \underbrace{\int_{\partial\Omega} \frac{d^3x \sqrt{h}\, K}{8\pi G}}_{\text{GIBBONS-HAWKING}}$$

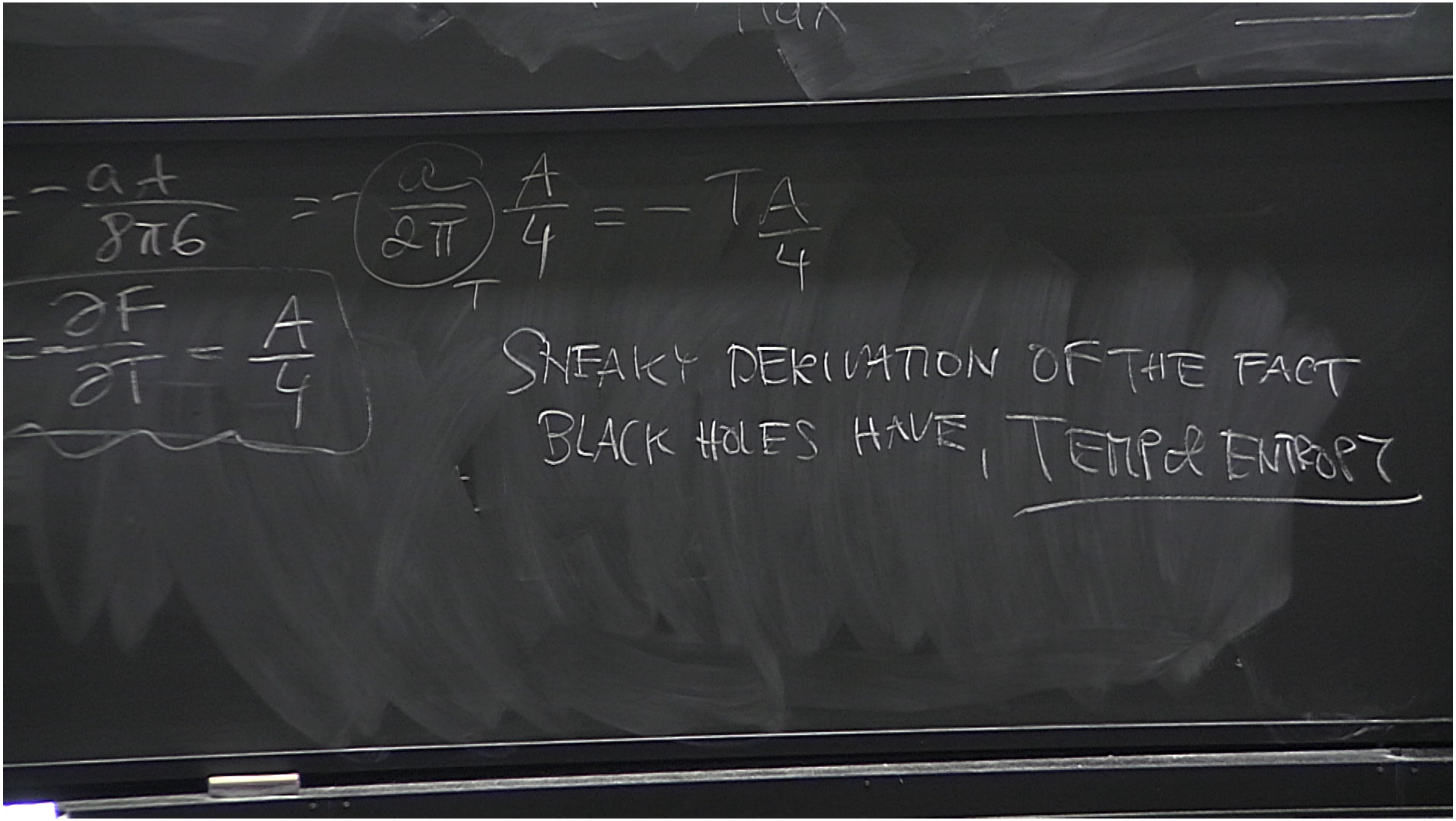
K


EXTRINSIC CURVATURE
NORMAL TO BOUNDARY.
$K = \nabla_\mu n^\mu = \frac{1}{\sqrt{g}} \left(\sqrt{g}\, n^\mu \right)_{,\mu}$.
INTRODUCE A BOUNDARY AT $X = X_0$
$X \in \left(-\frac{1}{a}, X_0\right)$
T X y z
$n^\mu = (0, 1, 0, 0)$

$$S_E = -\frac{1}{8\pi G}\int d\tau\, dy\, dz\, \cancel{(1+a x_0)}\, \frac{a}{\cancel{1+a x_0}}$$

$$= -\frac{a}{8\pi G}\,\beta \underbrace{\int dy\, dz}_{A} = -\frac{a\beta A}{8\pi G}$$

$F =$


$= -\frac{aA}{8\pi G} = -\frac{a}{2\pi}\frac{A}{4} = -T\frac{A}{4}$
$-\frac{\partial F}{\partial T} = \frac{A}{4}$
SNEAKY DERIVATION OF THE FACT
BLACK HOLES HAVE, TEMP & ENTROPY

RECALL

- $T \sim \varkappa \sim \frac{1}{M}$
- SPECIFIC HEAT OF BH IS NEGATIVE
- BH EVENTUALLY EVAPORATES

$$\frac{dM}{dt} = -\sigma T^4 A \sim -\frac{1}{M^2}$$

CHANGE OF COORDS

RECALL

- $T \sim \kappa \sim \frac{1}{M}$

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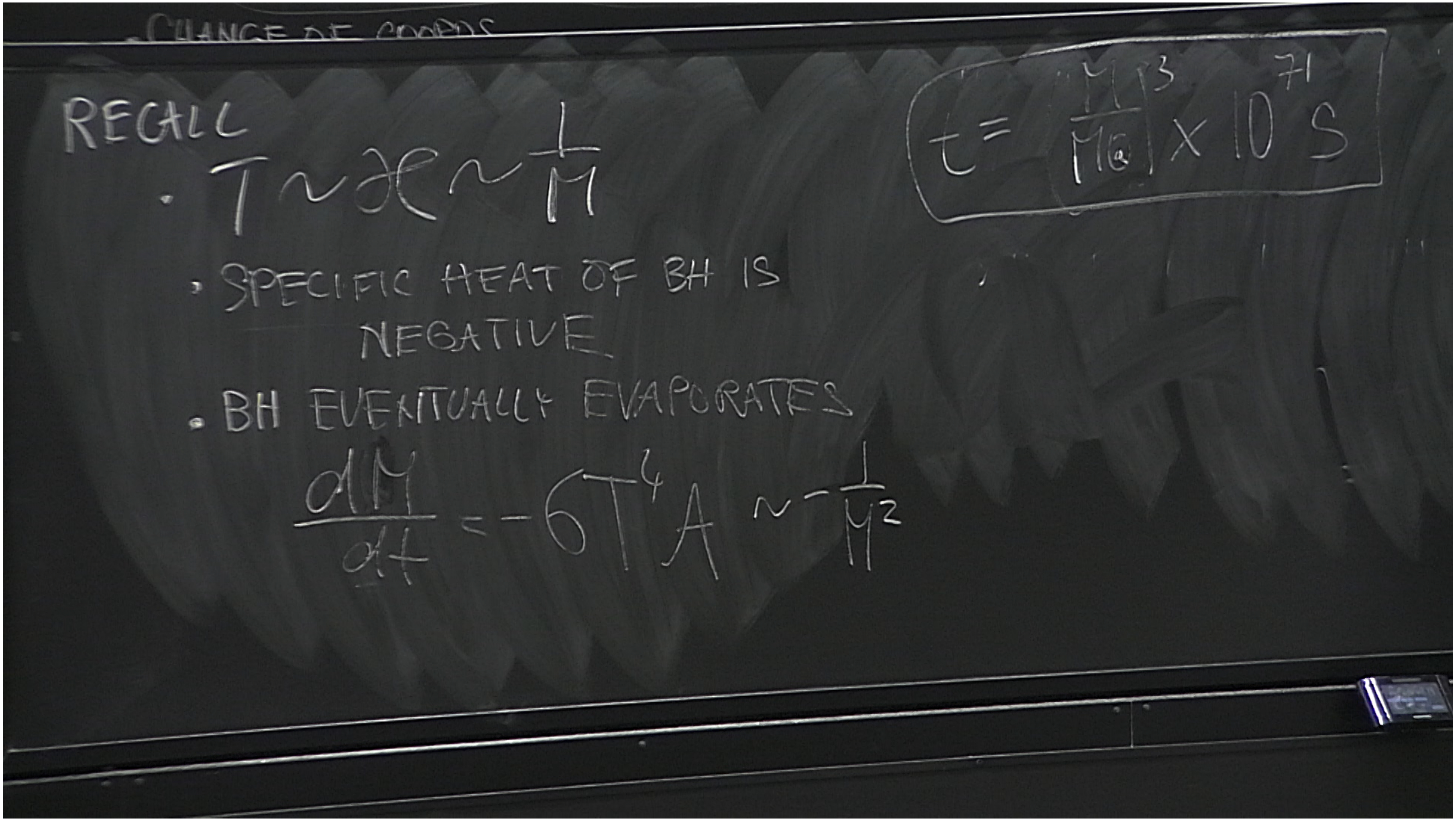
- BH EVENTUALLY EVAPORATES

$$\frac{dM}{dt} = -\sigma T^4 A \sim -\frac{1}{M^2}$$

$$\boxed{t = \left(\frac{M}{M_\odot}\right)^3 \times 10^{71}\, s}$$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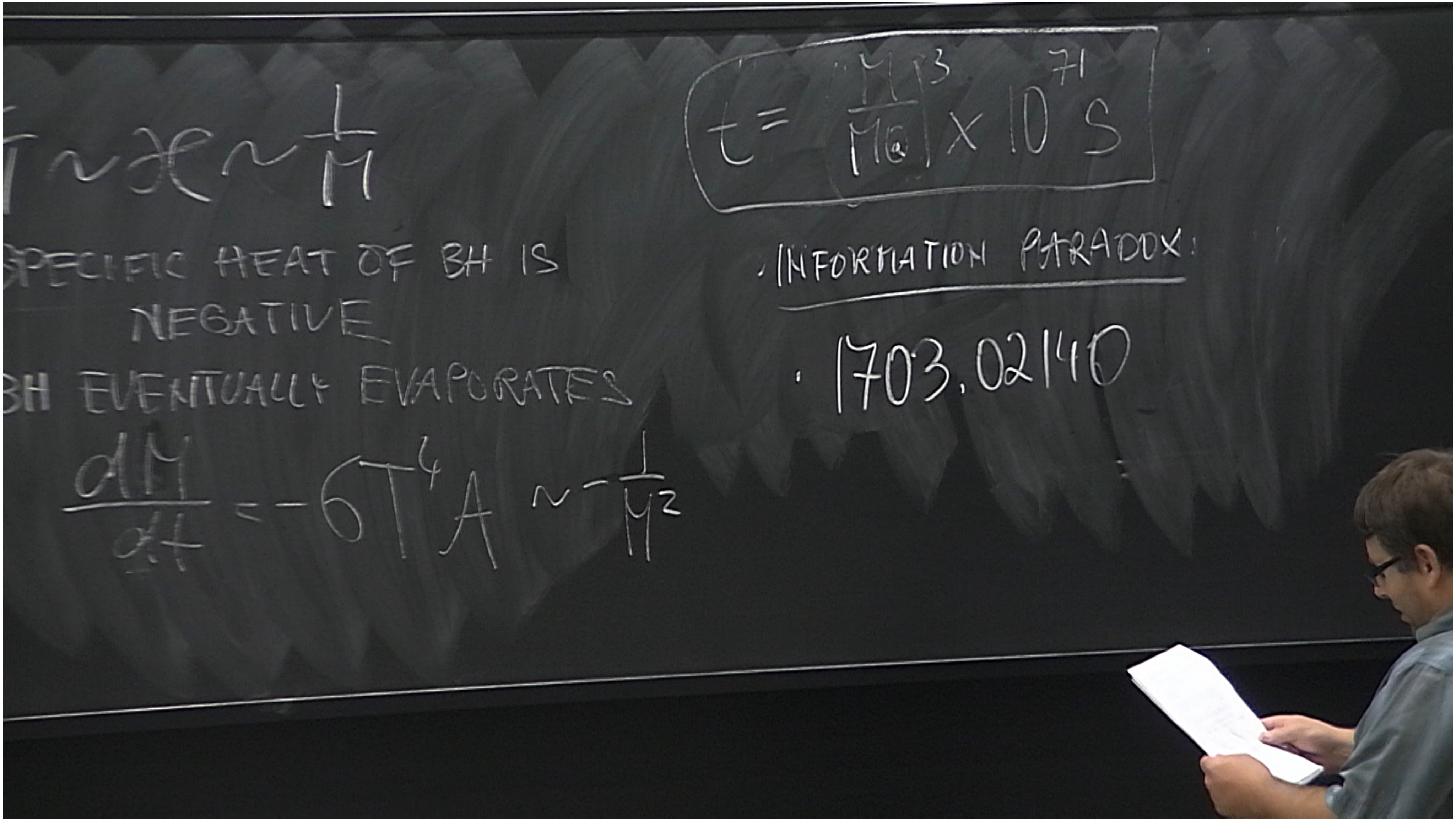
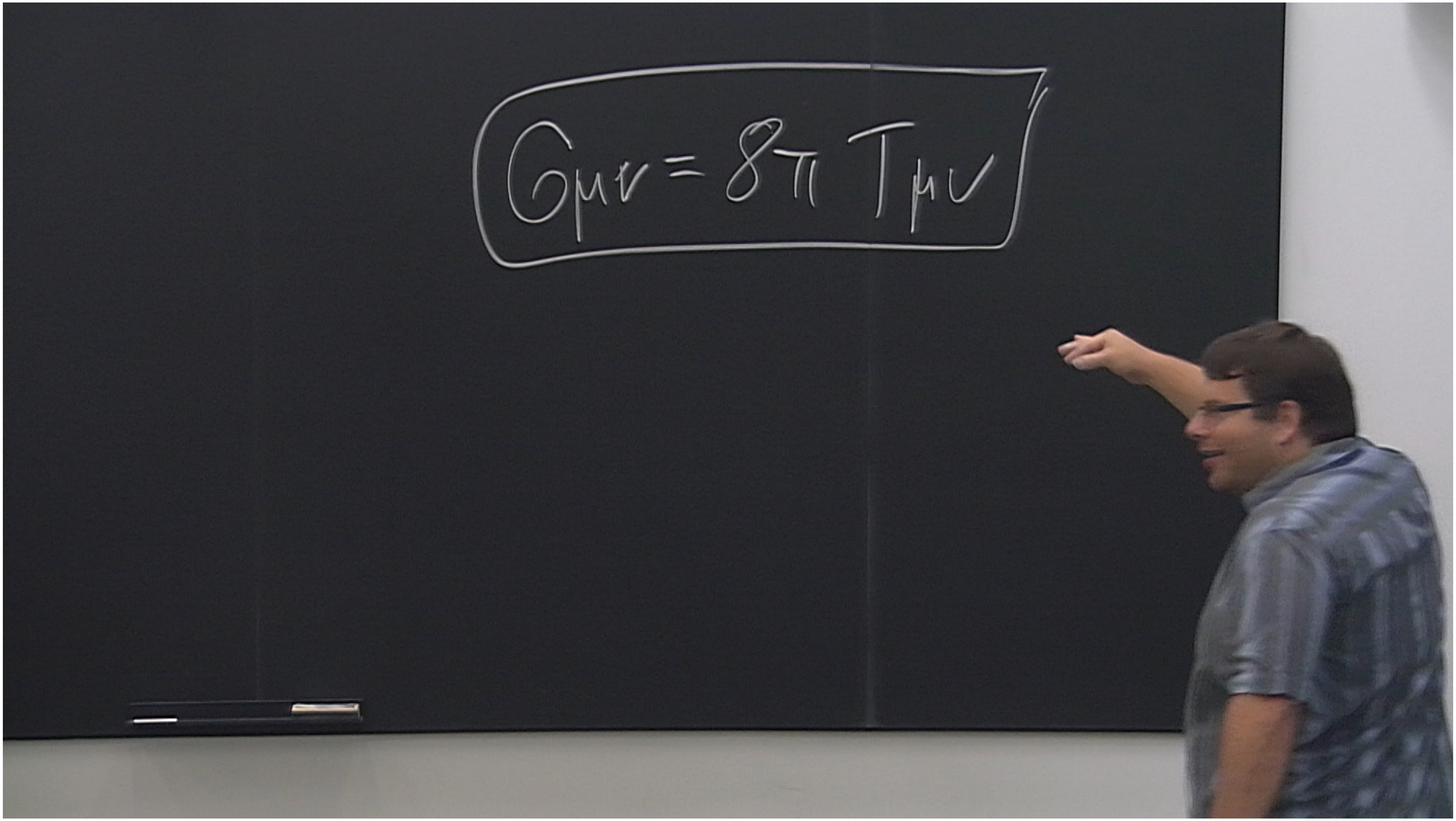


$T \sim \varkappa \sim \frac{1}{M}$
SPECIFIC HEAT OF BH IS NEGATIVE
BH EVENTUALLY EVAPORATES
$\frac{dM}{dt} = -\sigma T^4 A \sim -\frac{1}{M^2}$
$t = \left(\frac{M}{M_\odot}\right)^3 \times 10^{71}\,s$
INFORMATION PARADOX:
1703.02140


$G_{\mu\nu} = 8\pi T_{\mu\nu}$