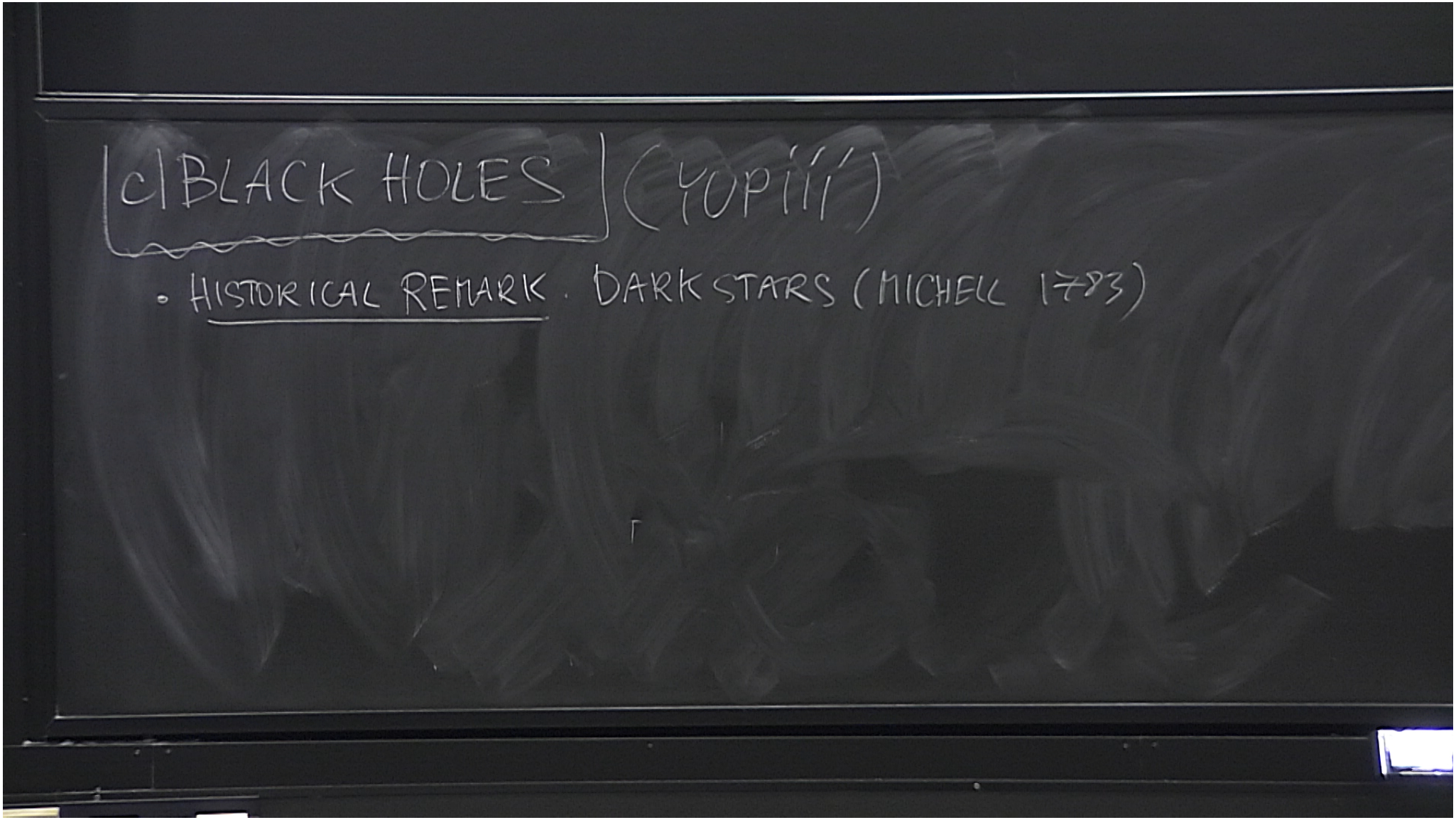
Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 14

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

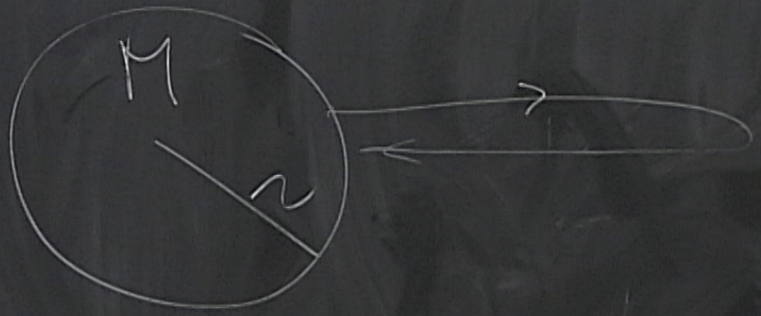
Date: September 19, 2019 - 10:45 AM

URL: http://pirsa.org/19090041


c) BLACK HOLES (YOPiii)
• HISTORICAL REMARK. DARK STARS (MICHELL 1783)

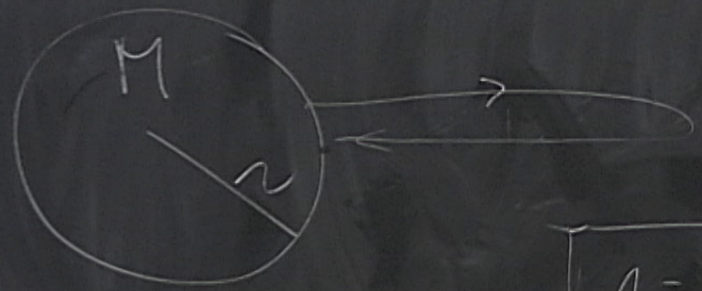
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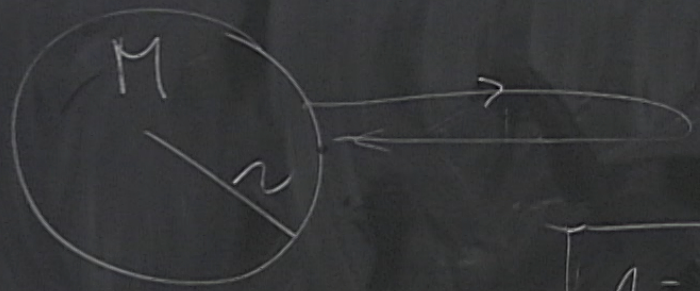
$$\frac{1}{2}mc^2 - \frac{GMm}{r} = 0$$

ESCAPE VELOCITY IS $v = c$

$$\boxed{r = r_+ = \frac{2GM}{c^2}}$$

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- HISTORICAL REMARK. DARK STARS (MICHELL 1783)



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ESCAPE VELOCITY IS $v = c$

$$r = r_+ = \frac{2GM}{c^2}$$

COINCIDES WITH RADIUS OF SCHW. BH.

- TRUE BHs : DESCRIBED BY GR. FOR EXAMPLE SCHWARZSCHILD BH

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2, \quad d\Omega^2 = \sin^2\theta d\varphi^2 + d\theta^2$$

$$f = 1 - \frac{2M}{r}$$

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WE TAKE THIS ALL THE WAY TO $r = 0$

- TRUE BHS . DESCRIBED BY GR. FOR EXAMPLE

SCHWARZSCHILD BH

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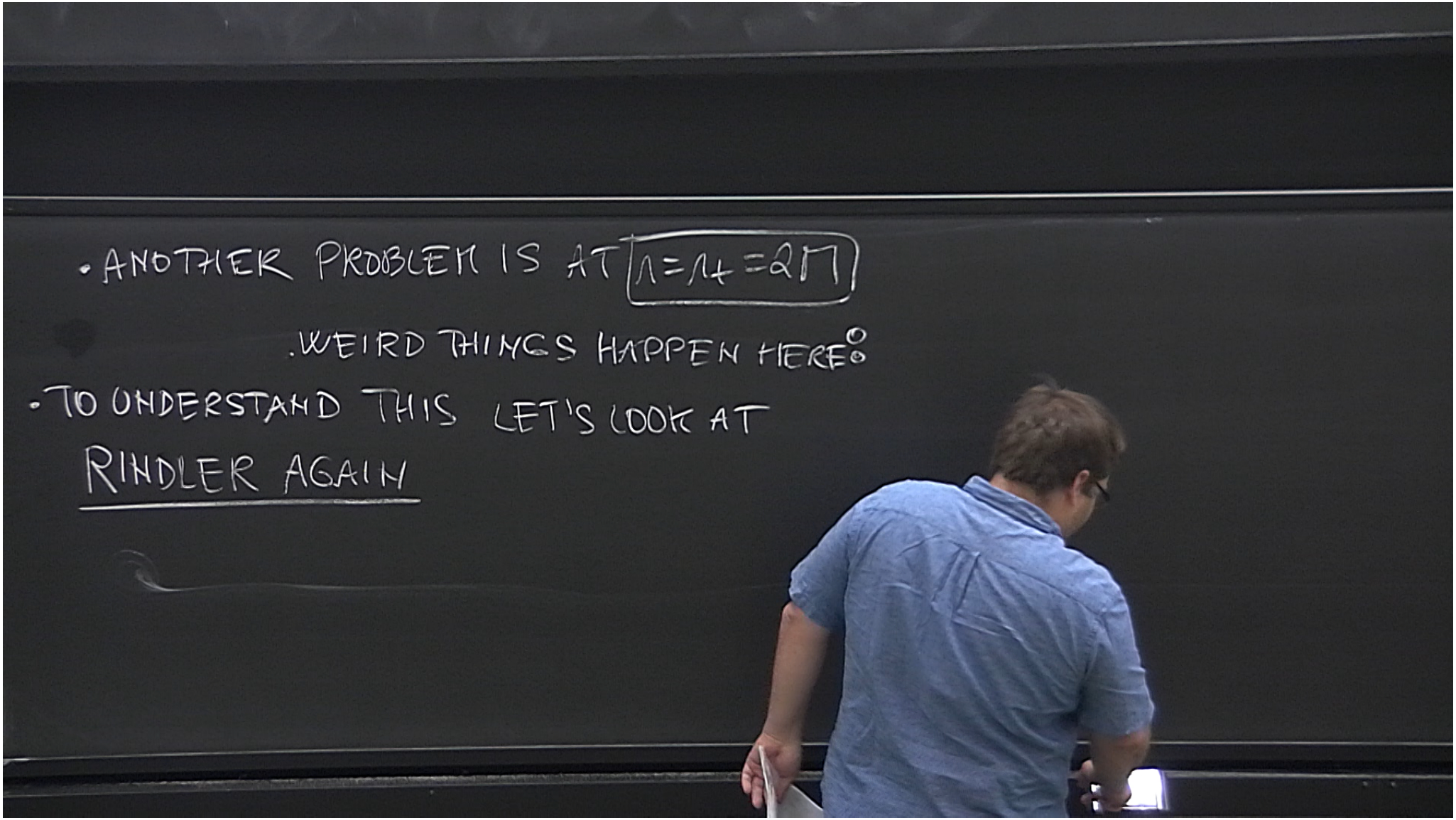
.. TRUE SINGULARITY AT $\boxed{r = 0}$

0

T IS $\Lambda = 0$

ES WITH

OF SCHW. BH.


• ANOTHER PROBLEM IS AT $r = r_+ = 2M$
. WEIRD THINGS HAPPEN HERE!
• TO UNDERSTAND THIS LET'S LOOK AT
RINDLER AGAIN

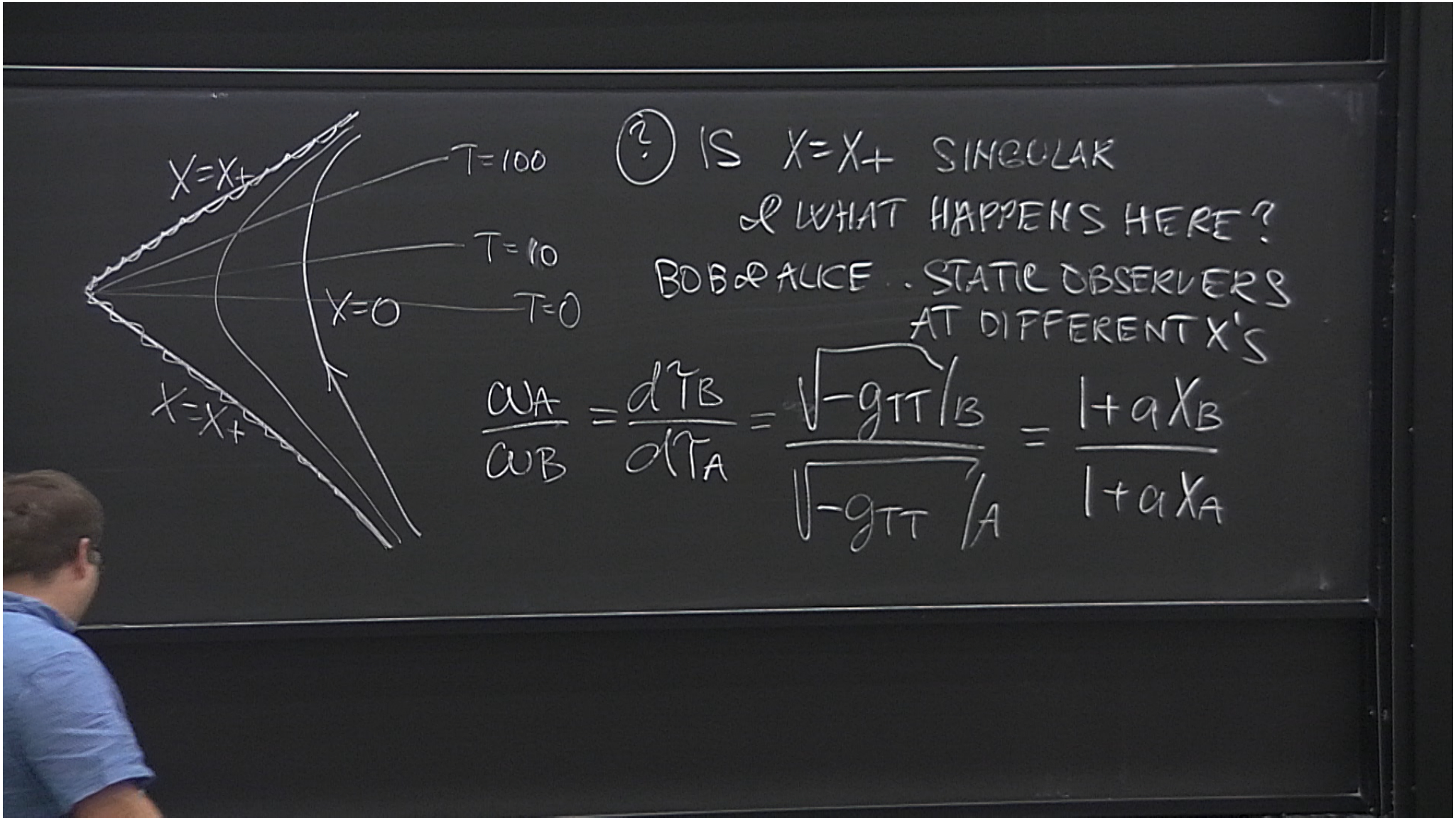
- ANOTHER PROBLEM IS AT $\Lambda = \Lambda_+ = 2\Pi$

  . WEIRD THINGS HAPPEN HERE.

- TO UNDERSTAND THIS LET'S LOOK AT RINDLER AGAIN

$$ds^2 = -(1+aX)^2 dT^2 + dX^2$$

$T \in (-\infty, \infty)$, $X > X_+ = -\frac{1}{a}$


X=X+
T=100
T=10
X=0
T=0
X=X+
? IS X=X+ SINGULAR
& WHAT HAPPENS HERE?
BOB & ALICE . STATIC OBSERVERS
AT DIFFERENT X'S
$\frac{\omega_A}{\omega_B} = \frac{d\tau_B}{d\tau_A} = \frac{\sqrt{-g_{TT}}|_B}{\sqrt{-g_{TT}}|_A} = \frac{1+aX_B}{1+aX_A}$

$ds = -(1+aX)dT + dX$

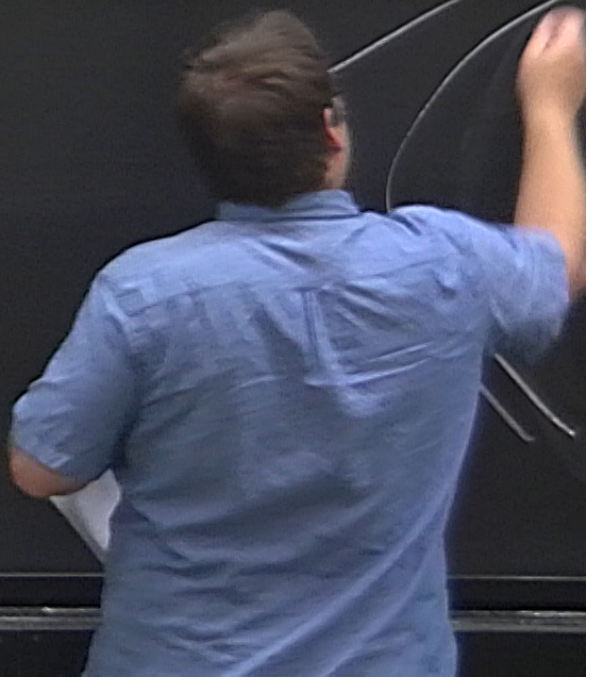
$T \in (-\infty, \infty)$, $X > \boxed{X_+ = -\frac{1}{a}}$

- LET'S SEND BOB TO RINDLER HORIZON

$X_B \Rightarrow X_+$ $\quad \boxed{\frac{\omega_A}{\omega_B} = 0}$

RINDLER HORIZON $\equiv$ SURFACE OF INFINITE REDSHIFT

- FROZEN FLASH



$ds = -(1+aX)dT + dX$

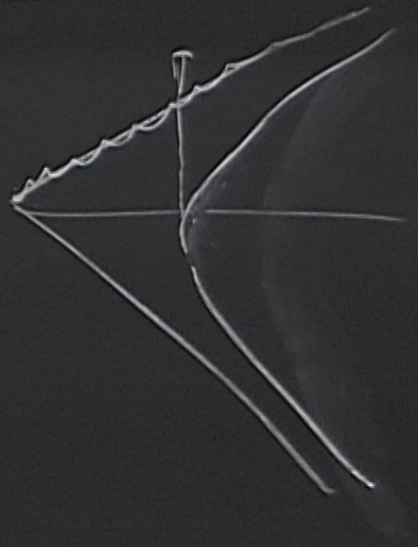
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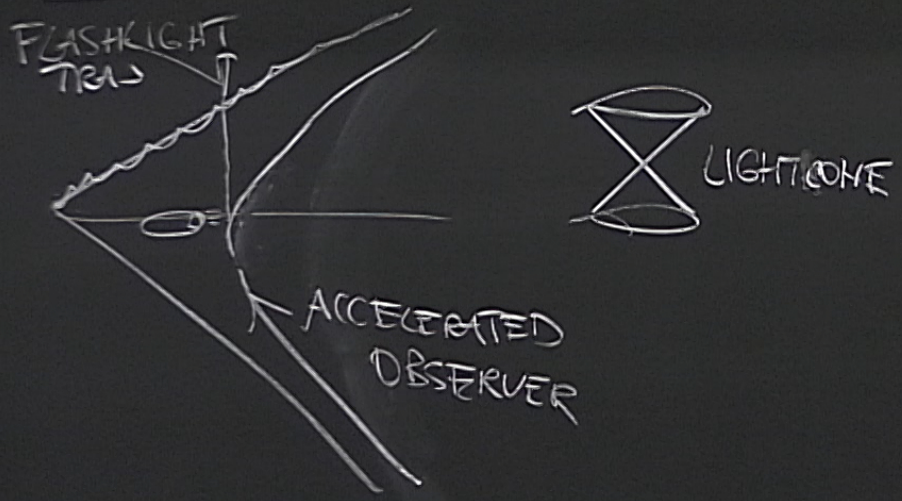
RINDLER HORIZON ≡ SURFACE OF INFINITE REDSHIFT

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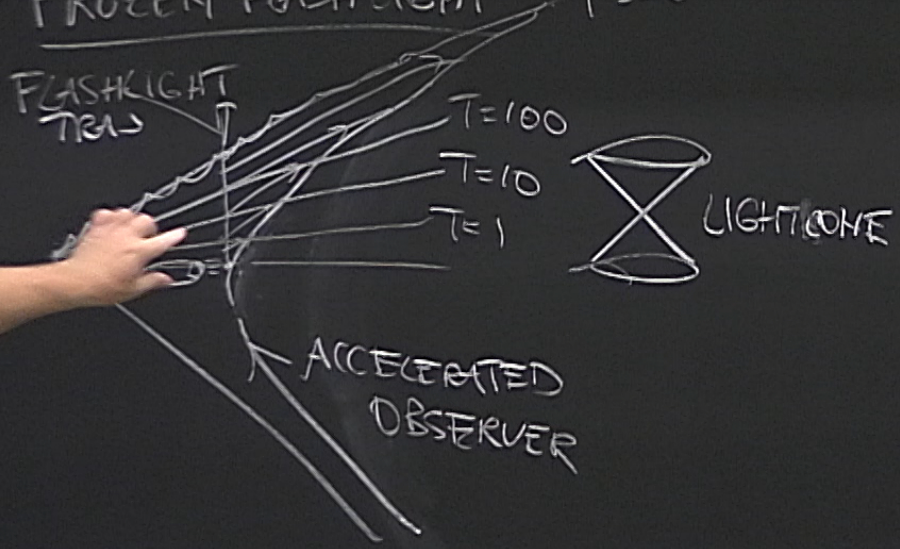
$\alpha_B \; d\tau_A = \frac{\sqrt{-g_{\tau\tau}}\,/_A}{1+a x_A}$

- FROZEN FLASHLIGHT



$\omega_B \, d\tau_A = \frac{\sqrt{-g_{\tau\tau}}\big|_B}{\sqrt{-g_{\tau\tau}}\big|_A} = \frac{1+a x_B}{1+a x_A}$

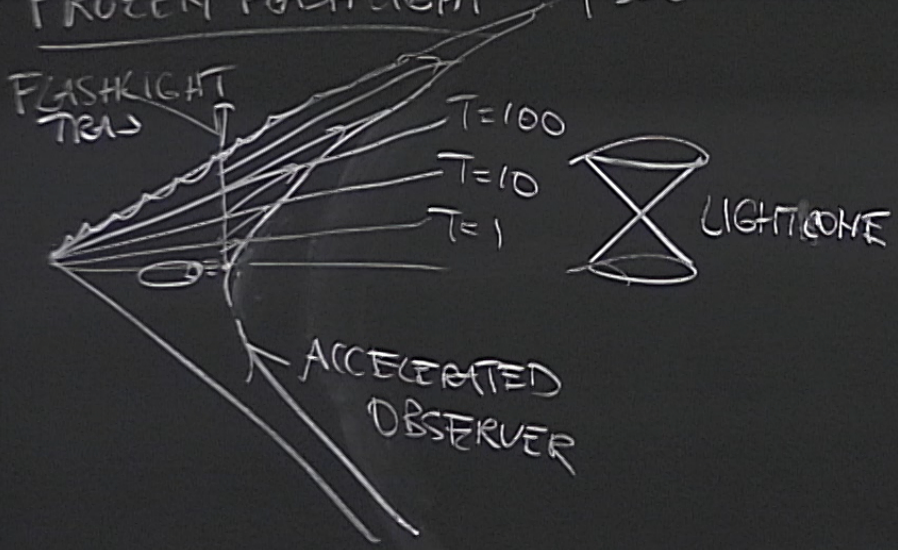
- FROZEN FLASHLIGHT $T=\infty$



OBSERVER SEES THE FLASHLIGHT 'SLOWING DOWN' AS IT GOES CLOSER TO HORIZON.
(THE CROSSING HAPPENS AT $T=\infty$)

$\omega_B \, d\tau_A = \frac{\sqrt{-g_{TT}}\,dT_A}{\sqrt{-g_{TT}}\big|_A} = \frac{dT_A}{1+aX_A}$

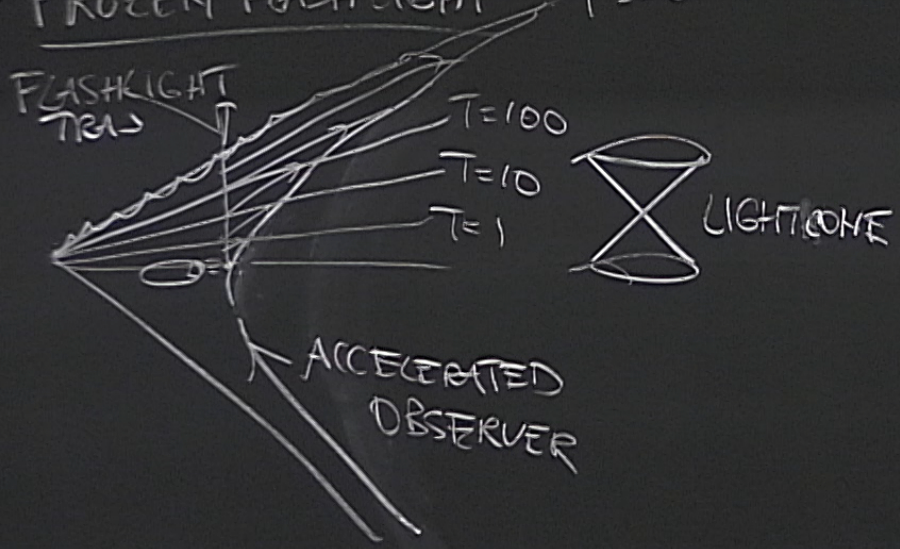
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$\omega_B \quad d\tau_A = \frac{\sqrt{-g_{\tau\tau}}\,dt}{\sqrt{-g_{\tau\tau}}\,/_A} = \frac{dt_{AB}}{1+a x_A}$

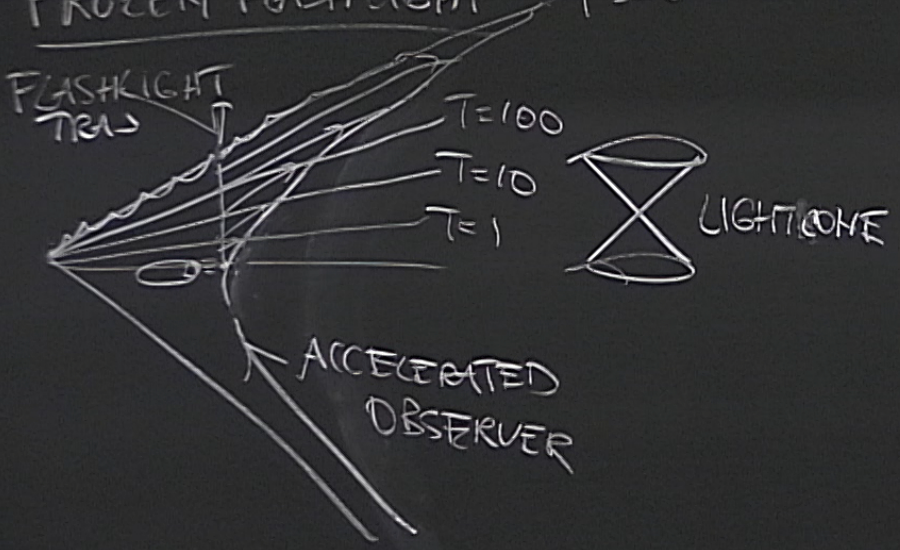
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- FROZEN FLASHLIGHT $T=\infty$



OBSERVER SEES THE FLASHLIGHT 'SLOWING DOWN' AS IT GOES CLOSER TO HORIZON.

(THE CROSSING HAPPENS AT $T=\infty$)

FLASHLIGHT FREEZES THERE

- WHAT REALLY HAPPENS WITH THE FLASHLIGHT?

CAN WE CROSS THE HORIZON (IF SITTING ON A FLASHLIGHT?

IS THERE AN EXTENSION OF RINDLER SPACE

IDEA: SHOOT GEODESICS AT HORIZON

USE NULL GEODESICS FOR SIMPLICITY.

$$0 = (1+aX)^2 \dot{T}^2 + \dot{X}^2$$

REALLY HAPPENS WITH THE FLASHLIGHT?

E CROSS THE HORIZON (IF SITTING ON A FLASHLIGHT?

ERE AN EXTENSION OF RINDLER SPACE

SHOOT GEODESICS AT HORIZON

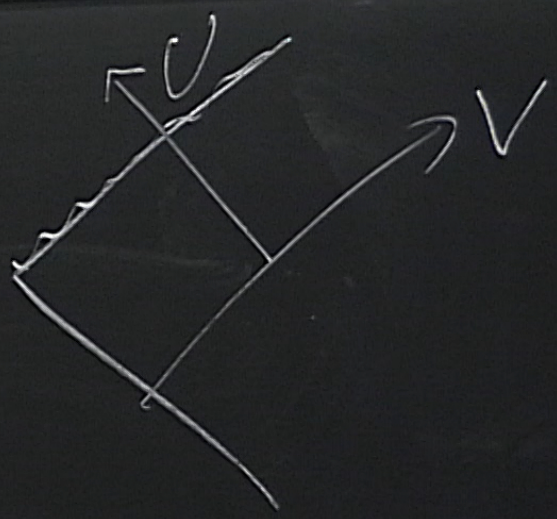
USE NULL GEODESICS FOR SIMPLICITY.

$$0 = (1+aX)^2 \dot{T}^2 + \dot{X}^2 \quad \Leftrightarrow \quad T = \pm \frac{1}{a} \log(1+aX) + \text{CONST}$$

REALLY HAPPENS WITH THE FLASHLIGHT?

E CROSS THE HORIZON (IF SITTING ON A FLASHLIGHT?

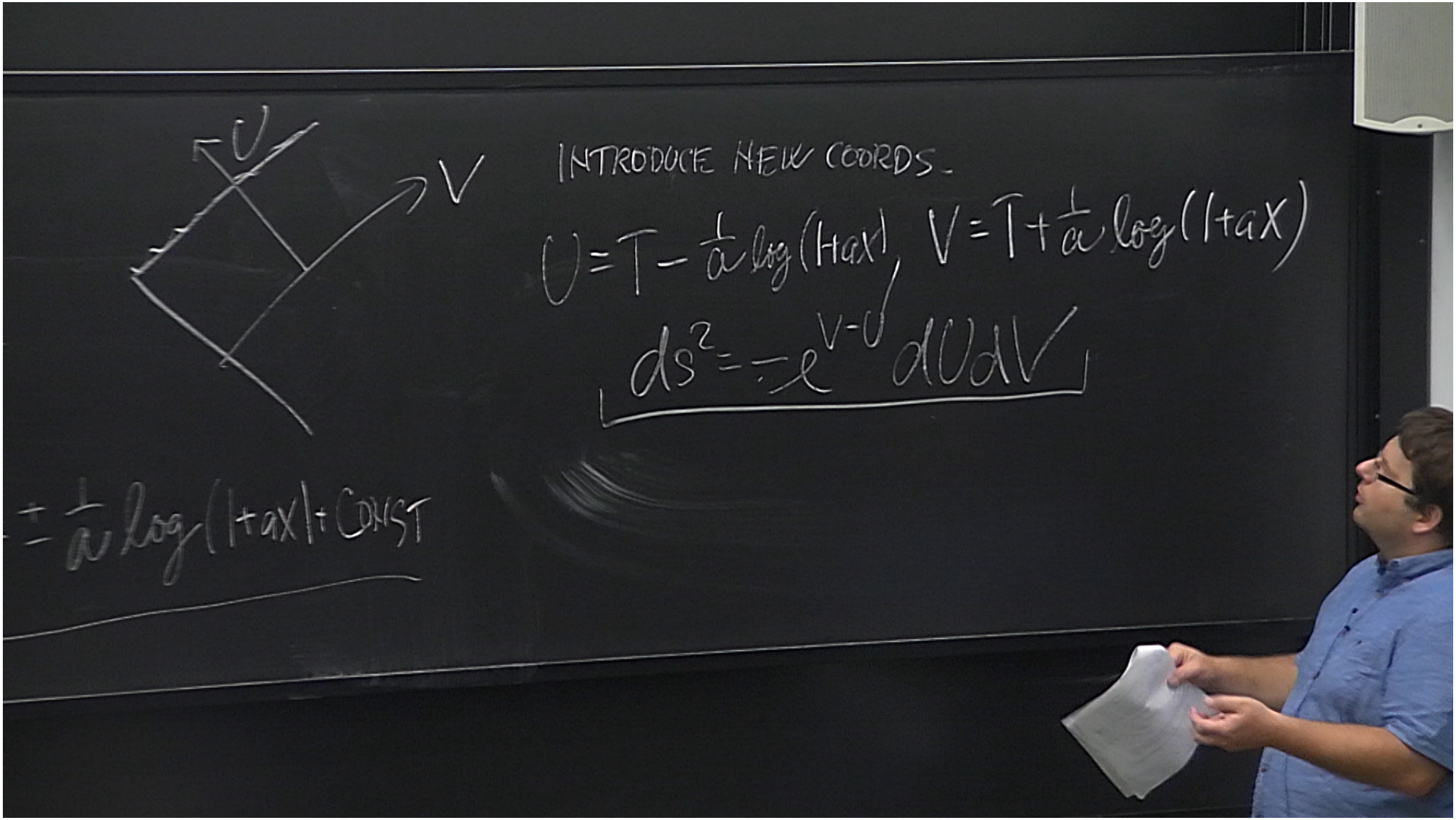
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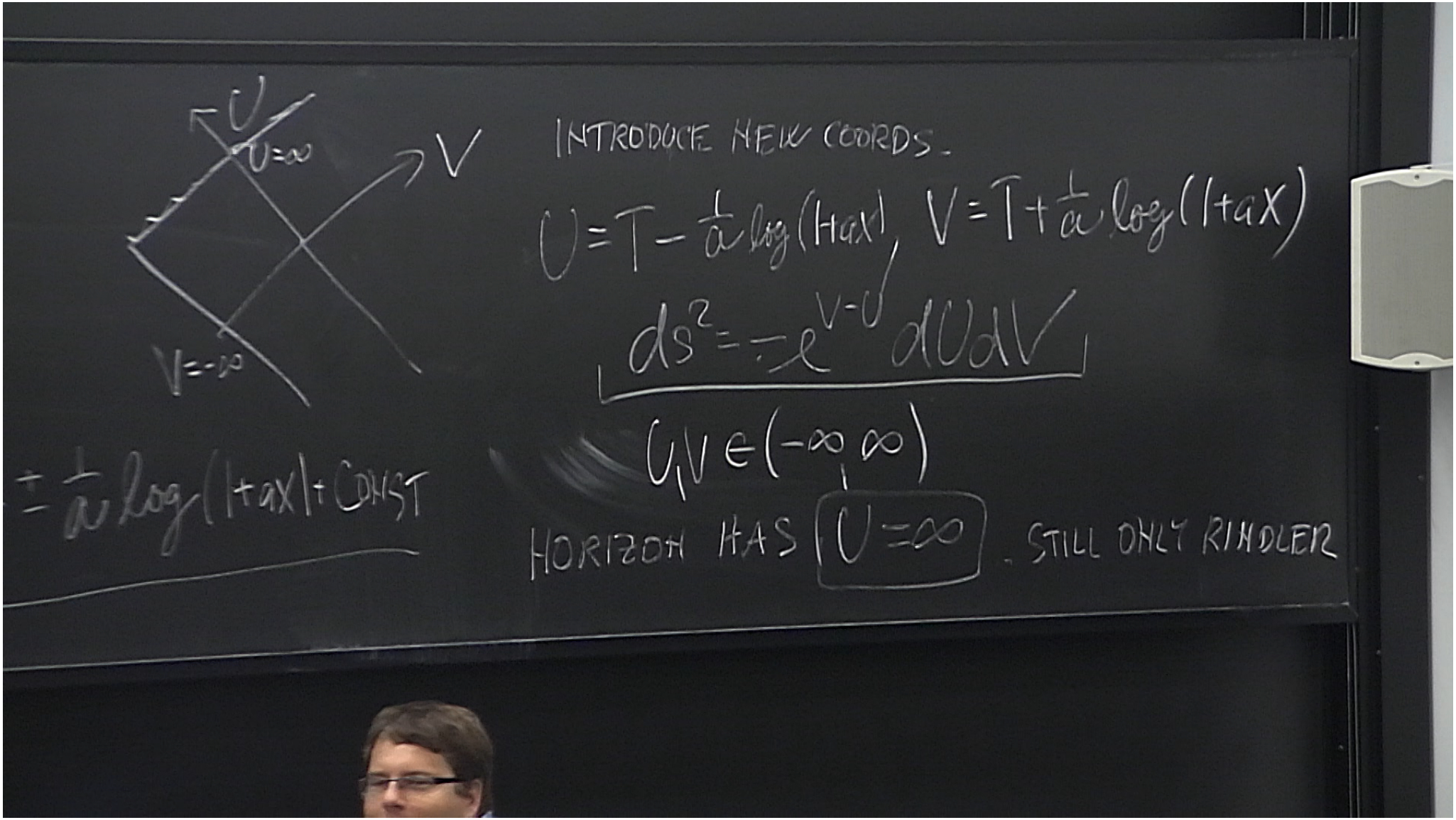


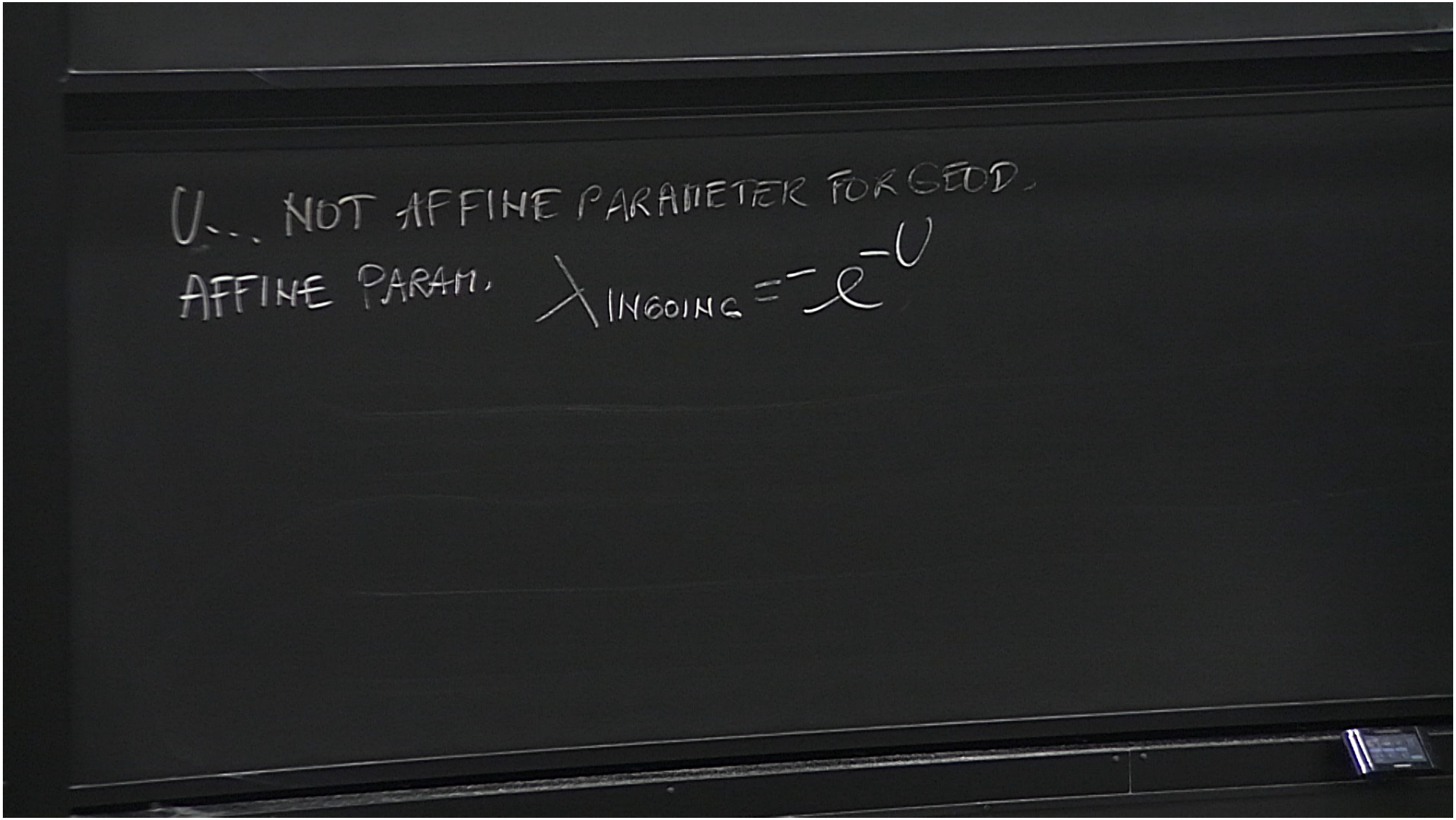
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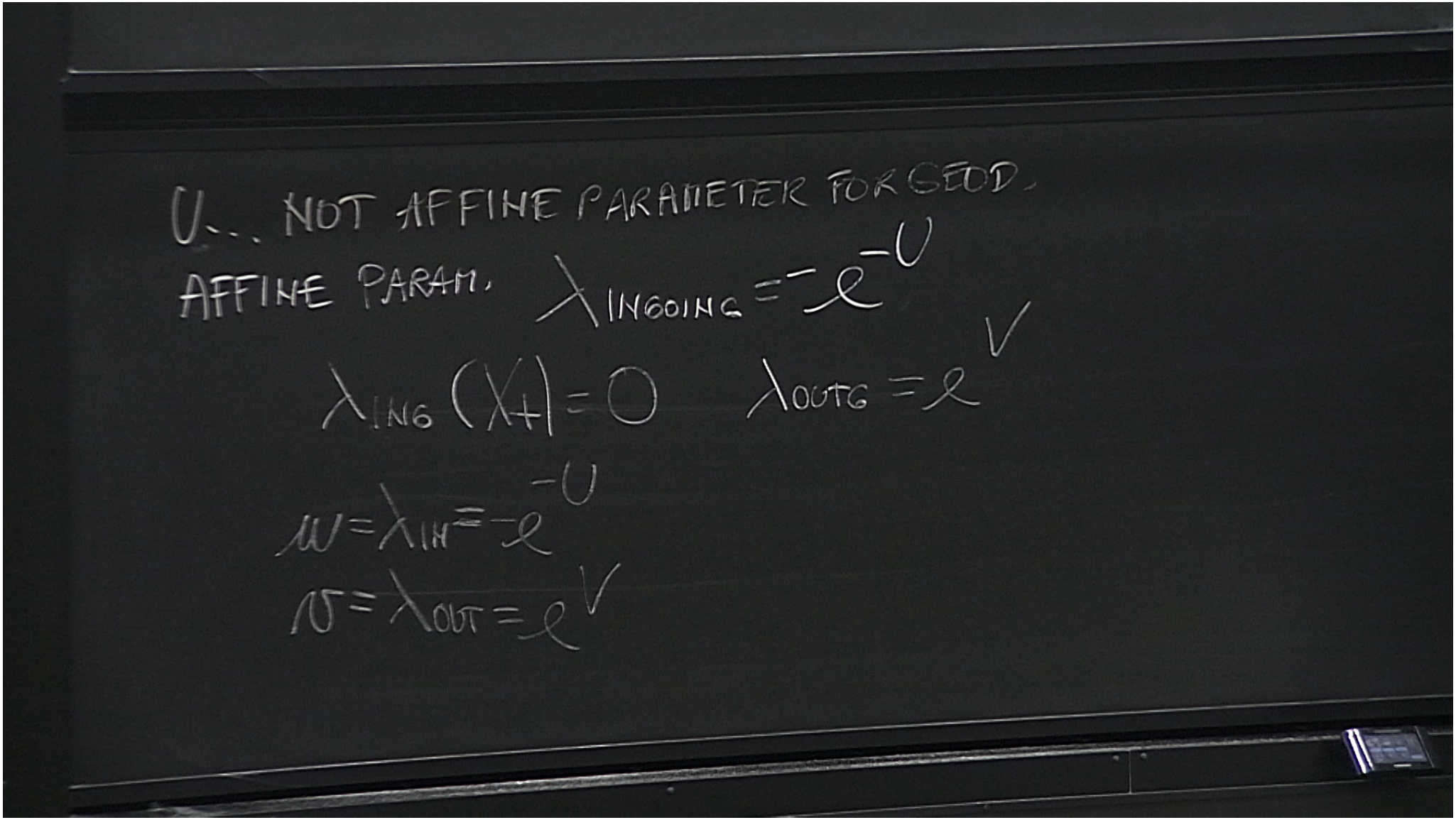
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$$0 = (1+aX)^2 \dot{T}^2 + \dot{X}^2 \quad \Leftrightarrow \quad T = \pm \frac{1}{a} \log(1+aX) + \text{CONST}$$


U
V
INTRODUCE NEW COORDS.
$U = T - \frac{1}{a}\log(1+aX)$, $V = T + \frac{1}{a}\log(1+aX)$
$ds^2 = -e^{V-U}\,dU\,dV$
$= \frac{1}{a}\log|1+aX| + \text{CONST}$


U
V
U=∞
V=-∞
INTRODUCE NEW COORDS.
$U = T - \frac{1}{a}\log(1+aX)$, $V = T + \frac{1}{a}\log(1+aX)$
$ds^2 = -e^{V-U}\,dU\,dV$
$U, V \in (-\infty, \infty)$
HORIZON HAS $U = \infty$. STILL ONLY RINDLER
$= \frac{1}{a}\log(|1+aX|) + \text{CONST}$


U... NOT AFFINE PARAMETER FOR GEOD.
AFFINE PARAM.
$\lambda_{\text{INGOING}} = -e^{-U}$


U... NOT AFFINE PARAMETER FOR GEOD.
AFFINE PARAM. $\lambda_{INGOING} = -e^{-U}$
$\lambda_{ING}(X_+) = 0$ $\lambda_{OUTG} = e^{V}$
$U = \lambda_{IN} = -e^{-U}$
$V = \lambda_{OUT} = e^{V}$

U... NOT AFFINE PARAMETER FOR GEOD.

AFFINE PARAM. $\lambda_{INGOING} = -e^{-U}$

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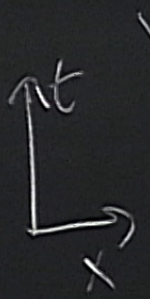
$w = \lambda_{IN} = -e^{-U}$

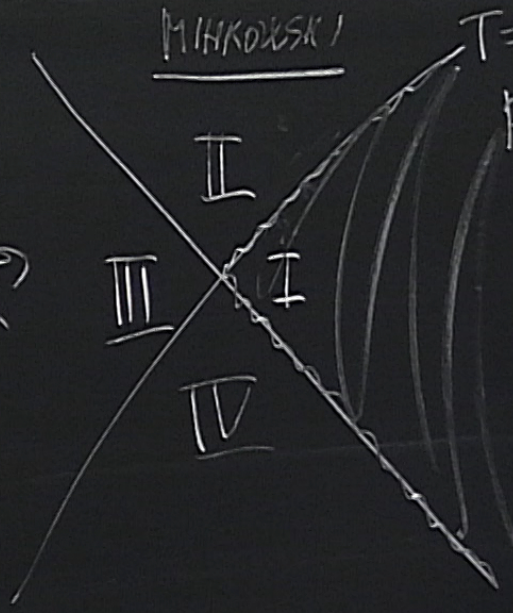
$v = \lambda_{OUT} = e^{V}$

$$ds^2 = -dw\,dv$$

$t = \frac{u+v}{2}$, $X = \frac{v-u}{2}$

$ds^2 = -dt^2 + dx^2$





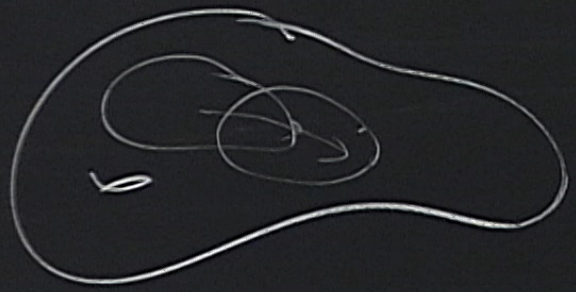
$T = \infty, X = -\frac{1}{a}$

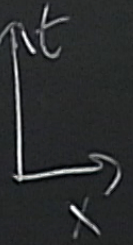
RINDLER

MAXIMAL EXTENSION OF RINDLER IS MINK.

$t = \frac{u+v}{2}$, $x = \frac{v-u}{2}$

$ds^2 = -dt^2 + dx^2$





MINKOWSKI

II

III

I

IV

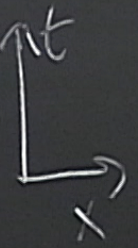
$T = \infty$, $X = -\frac{1}{a}$

RINDLER

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MINKOWSKI

II

III

I

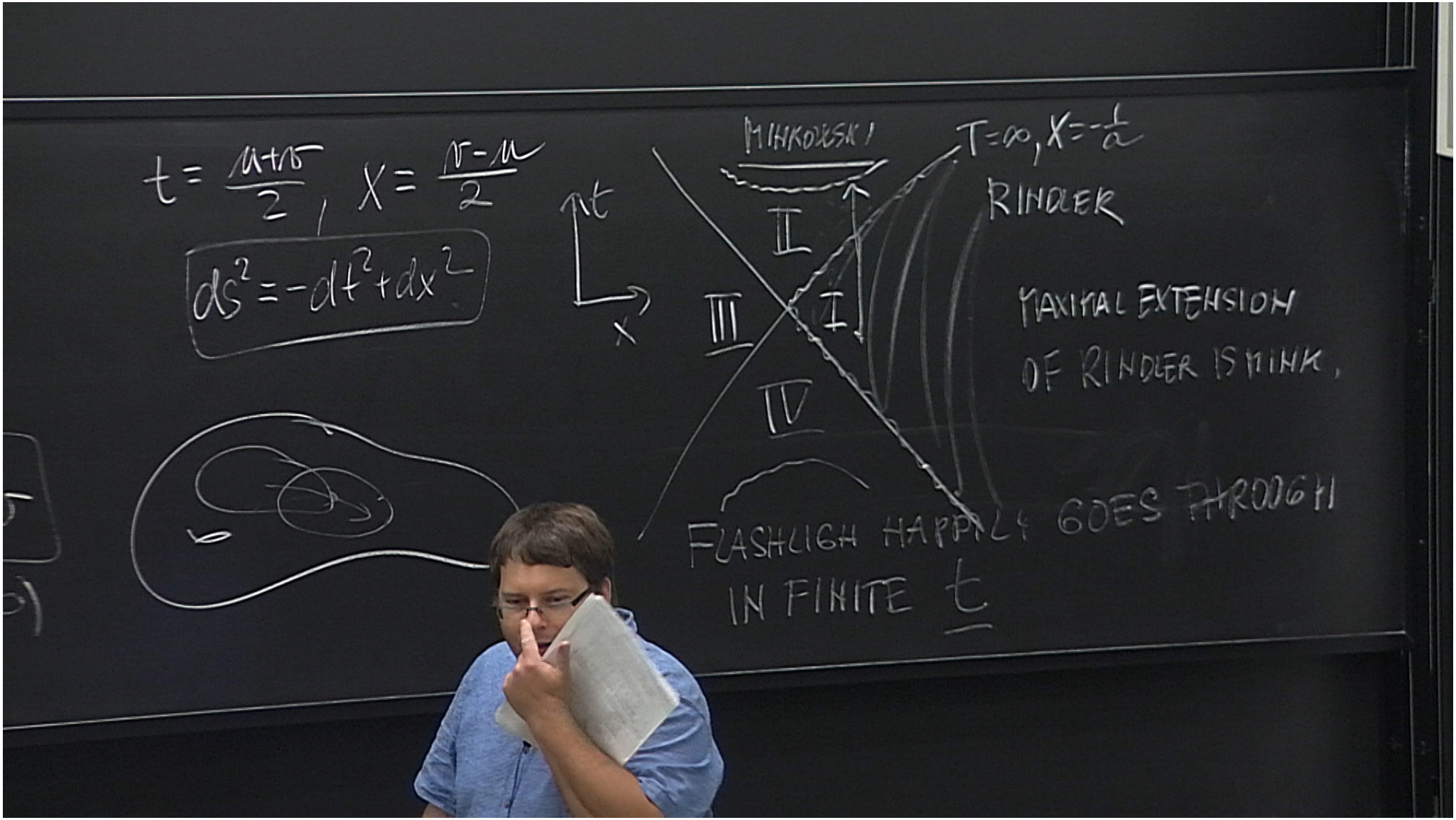
IV

$T = \infty, X = -\frac{1}{a}$

RINDLER

MAXIMAL EXTENSION OF RINDLER IS MINK.

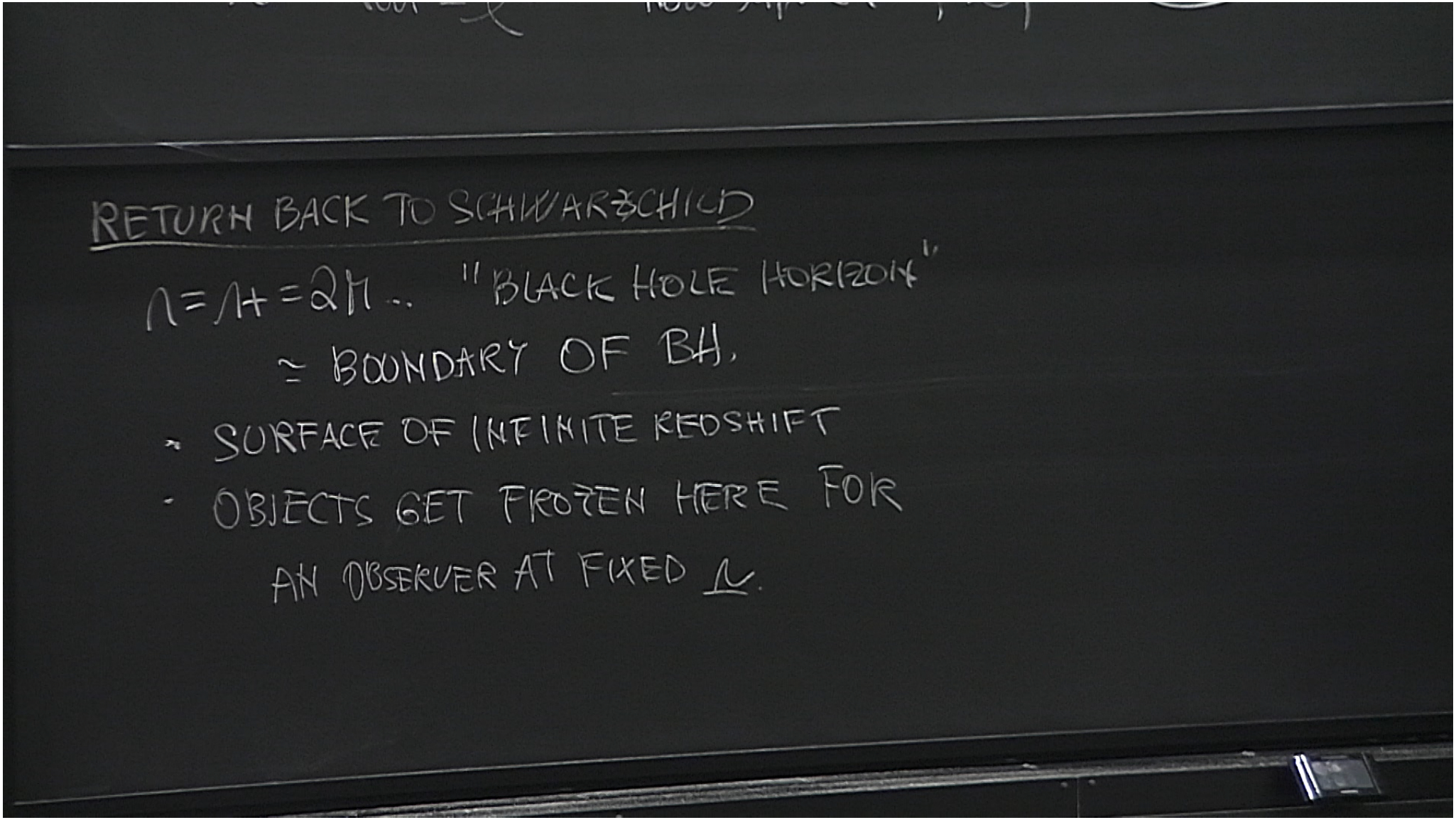
FLASHLIGH HAPPILY GOES THROUGH IN FINITE t


$t = \frac{u+v}{2}$, $x = \frac{v-u}{2}$
$ds^2 = -dt^2 + dx^2$
t
x
MINKOWSKI
II
III
I
IV
$T = \infty$, $X = -\frac{1}{a}$
RINDLER
MAXIMAL EXTENSION
OF RINDLER IS MINK.
FLASHLIGH
IN FINITE t
GOES THROUGH

RETURN BACK TO SCHWARZSCHILD

$\Lambda = \Lambda_+ = 2M$ .. "BLACK HOLE HORIZON"

$\simeq$ BOUNDARY OF BH.


RETURN BACK TO SCHWARZSCHILD
$r = r_+ = 2M$ .. "BLACK HOLE HORIZON"
≃ BOUNDARY OF BH.
- SURFACE OF INFINITE REDSHIFT
- OBJECTS GET FROZEN HERE FOR
AN OBSERVER AT FIXED r.

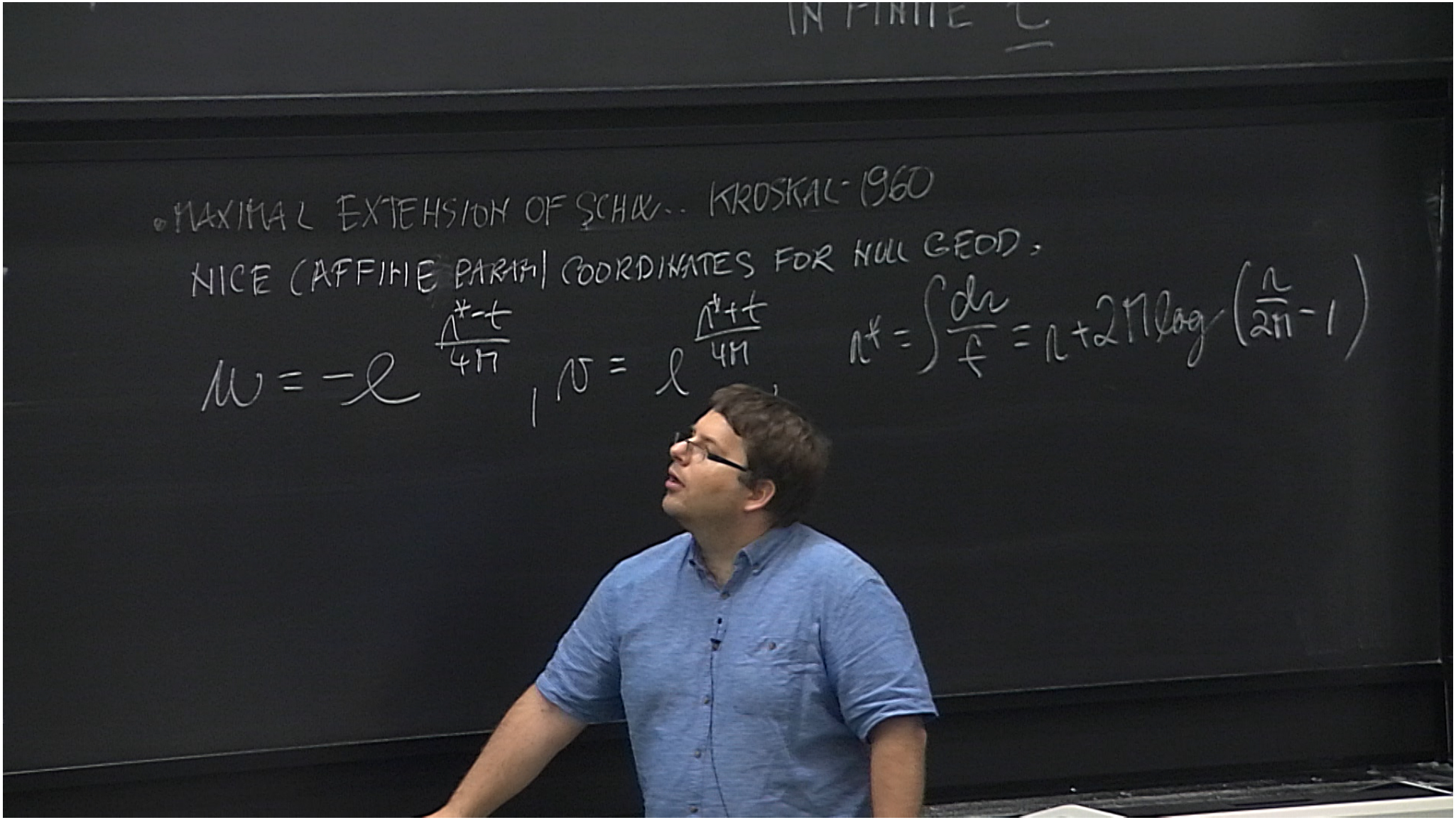
RETURN BACK TO SCHWARZSCHILD

$\Lambda = \Lambda_+ = 2M$ .. "BLACK HOLE HORIZON"

$\simeq$ BOUNDARY OF BH.

- SURFACE OF INFINITE REDSHIFT
- OBJECTS GET FROZEN HERE FOR AN OBSERVER AT FIXED $\underline{\Lambda}$.

FROZEN STARS → BLACK HOLES.

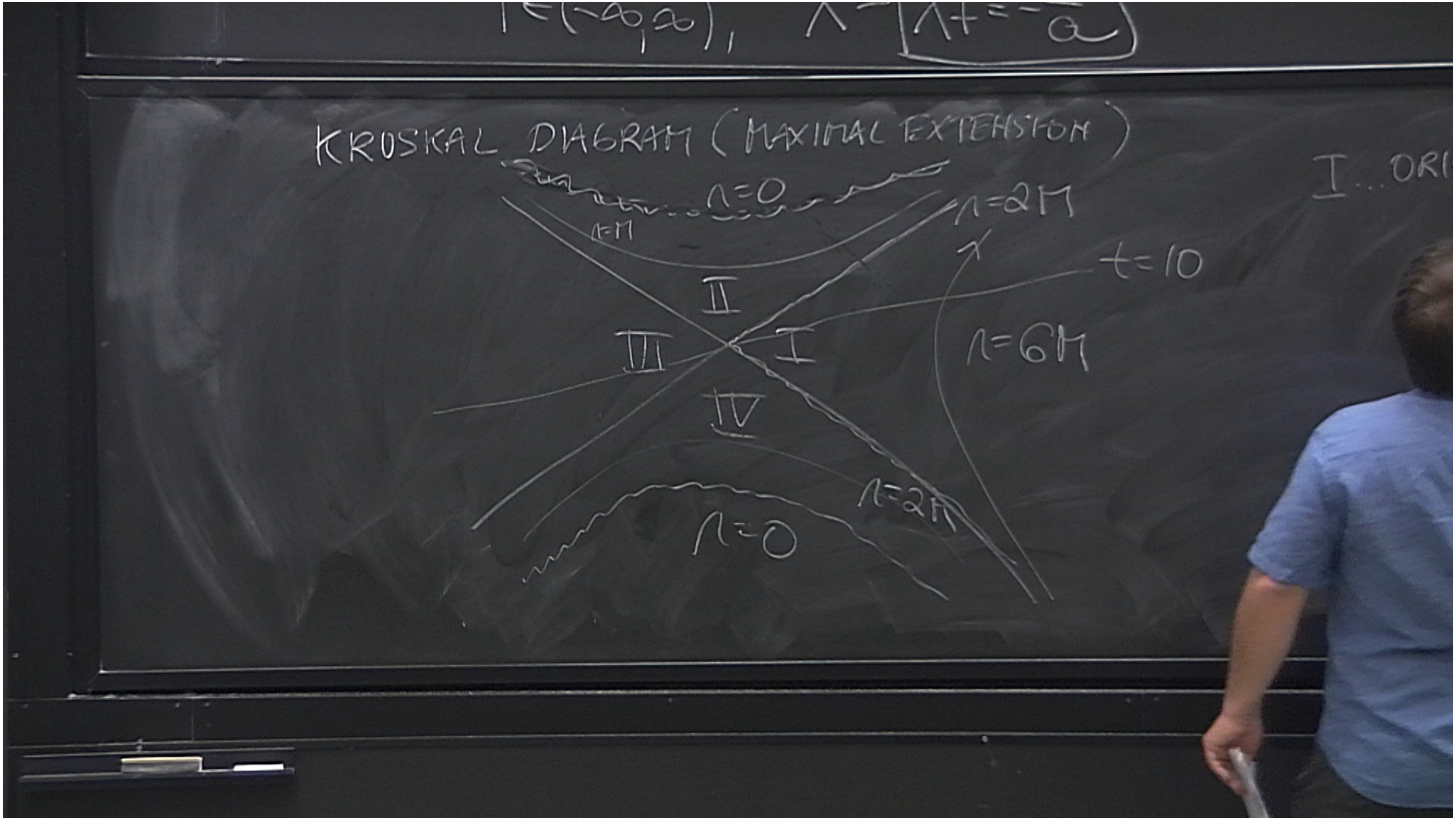

INFINITE C
• MAXIMAL EXTENSION OF SCHW.. KROSKAL-1960
NICE (AFFINE PARAM) COORDINATES FOR NULL GEOD.
$u = -e^{\frac{r^*-t}{4m}}$, $v = e^{\frac{r^*+t}{4m}}$
$r^* = \int \frac{dr}{f} = r + 2m \log\left(\frac{r}{2m} - 1\right)$

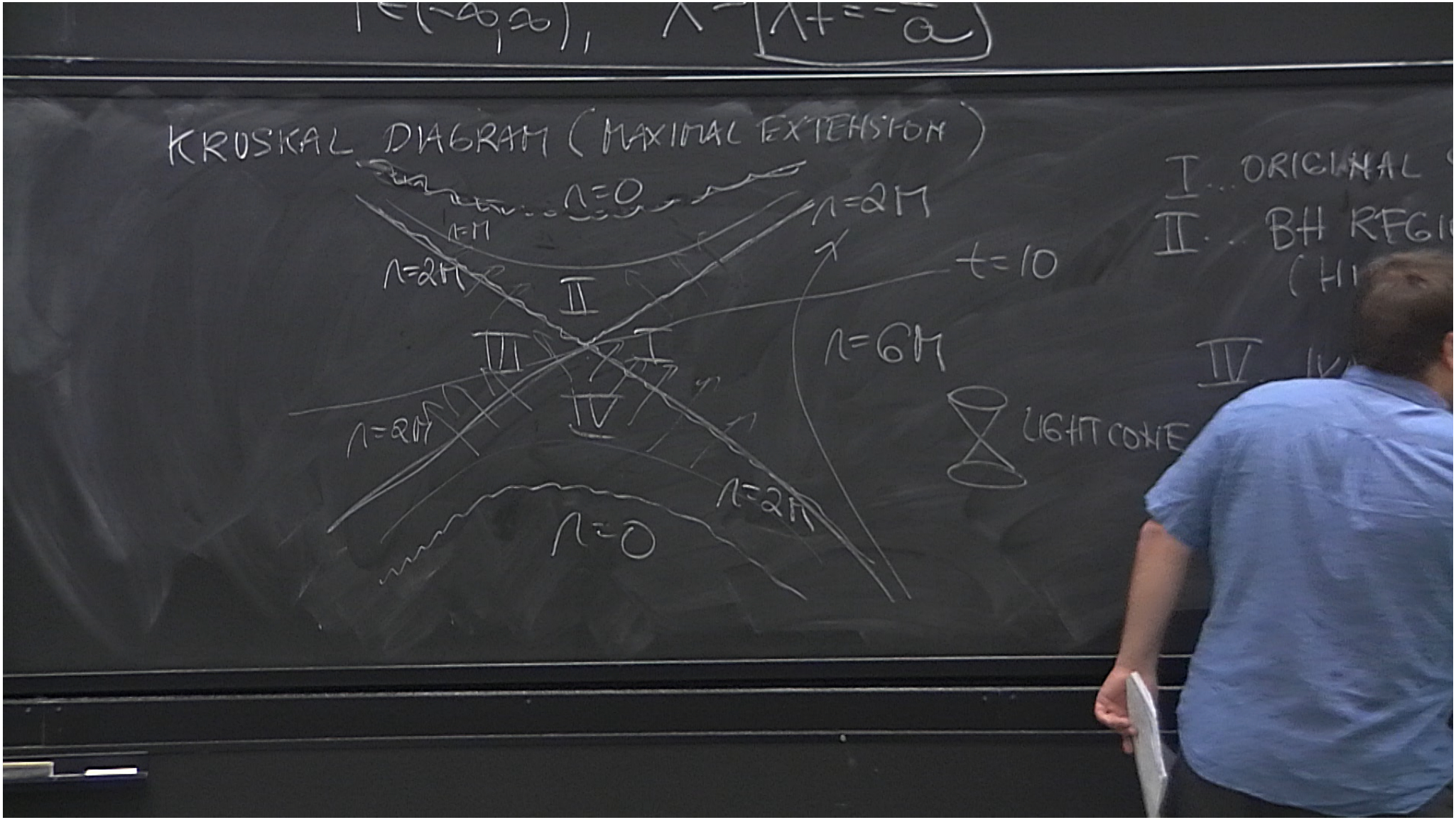
• MAXIMAL EXTENSION OF SCHW.. KRUSKAL 1960

NICE (AFFINE PARAM) COORDINATES FOR NULL GEOD.

$$u = -e^{\frac{r^*-t}{4M}}, \quad v = e^{\frac{r^*+t}{4M}}, \quad r^* = \int \frac{dr}{f} = r + 2M \log\left(\frac{r}{2M} - 1\right)$$

$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} du\,dv + r^2 d\Omega^2$$

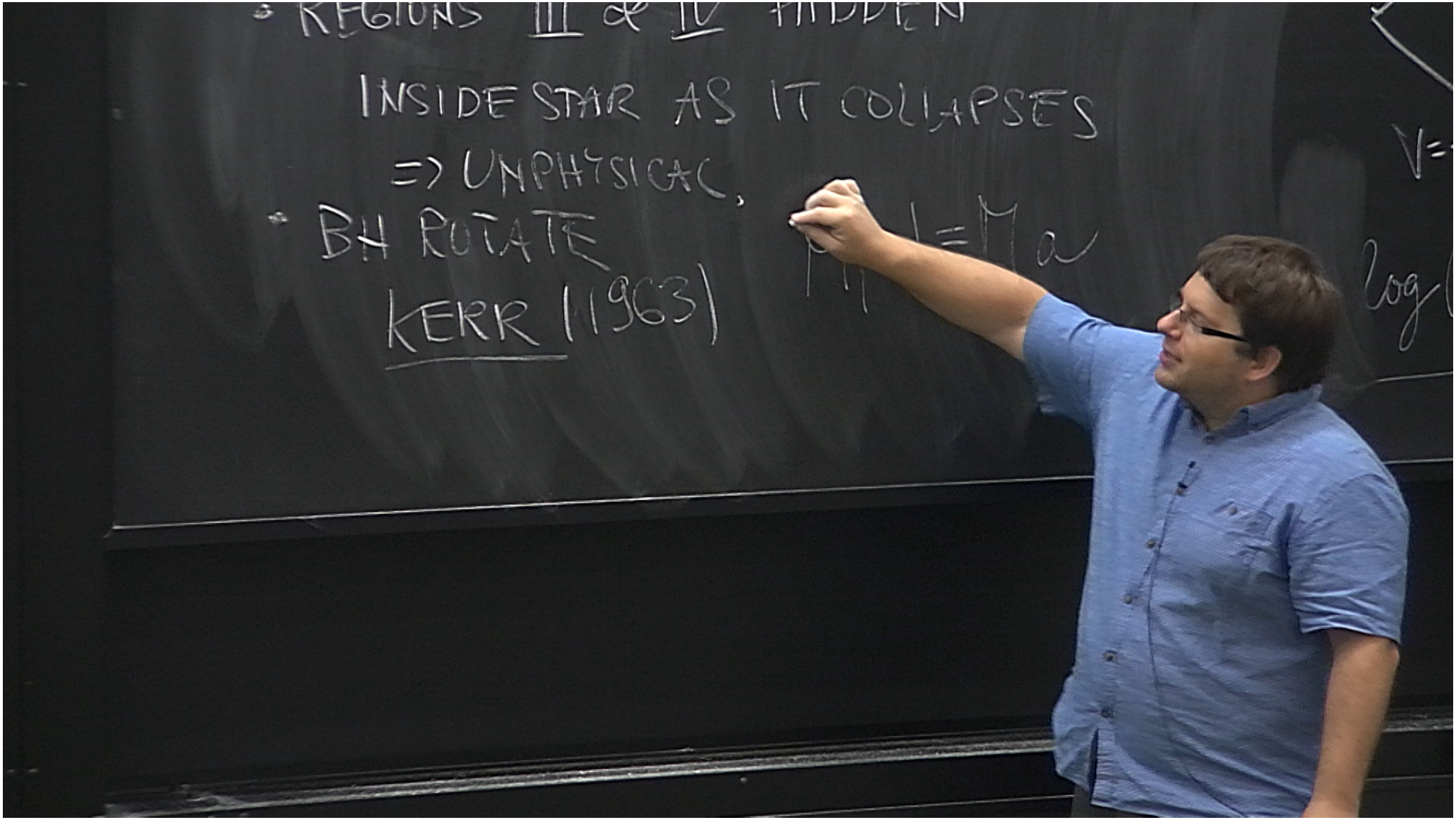

KRUSKAL DIAGRAM (MAXIMAL EXTENSION)
r=0
r=2M
t=10
II
III
I
IV
r=6M
r=2M
r=0


KRUSKAL DIAGRAM (MAXIMAL EXTENSION)
r=0
r=2M
r=2M
II
t=10
r=6M
III
I
IV
r=2M
r=2M
r=0
I... ORIGINAL
II... BH REGI
(HI
IV
LIGHT CONE

- LET'S LOOK AT t = CONST SLICE $\left(\theta = \frac{\pi}{2}\right)$

$$ds^2 = \frac{dr^2}{f} + r^2 d\varphi^2$$

TO VISUALIZE HOW THIS LOOKS LIKE WE EMBED IN $\mathbb{E}^3$


REGIONS III & IV HIDDEN
INSIDE STAR AS IT COLLAPSES
=> UNPHYSICAL.
BH ROTATE
KERR (1963)