Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 12

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

Date: September 17, 2019 - 10:45 AM

URL: http://pirsa.org/19090039

TODAY

P.I.
* semi-classical expansion
* perturbative theory.

YESTERDAY: GRAVITATIONAL WAVES

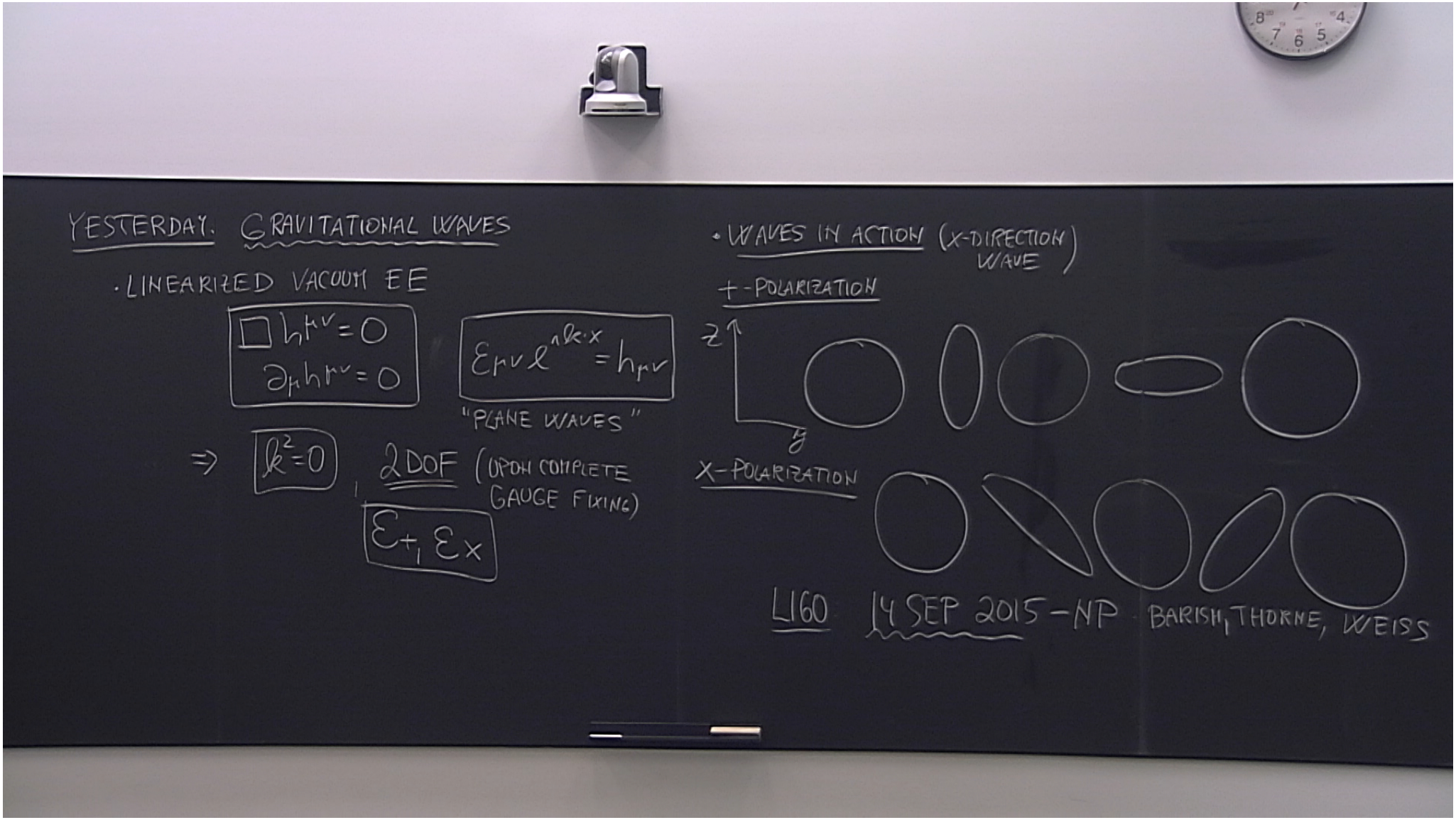
- LINEARIZED VACUUM EE

$$\Box h^{\mu\nu} = 0$$
$$\partial_\mu h^{\mu\nu} = 0$$

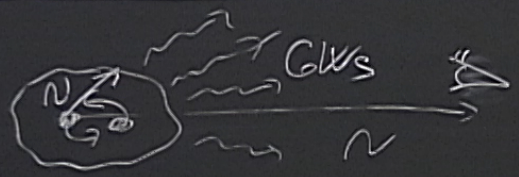
$$\varepsilon_{\mu\nu} e^{\pm i k\cdot x} = h_{\mu\nu}$$

"PLANE WAVES"

$\Rightarrow$ $k^2 = 0$, 2 DOF (UPON COMPLETE GAUGE FIXING)


YESTERDAY. GRAVITATIONAL WAVES
· LINEARIZED VACUUM EE
$\Box h^{\mu\nu} = 0$
$\partial_\mu h^{\mu\nu} = 0$
$\varepsilon_{\mu\nu} e^{\pm ik\cdot x} = h_{\mu\nu}$
"PLANE WAVES"
$\Rightarrow k^2 = 0$
2 DOF (UPON COMPLETE GAUGE FIXING)
$\varepsilon_+, \varepsilon_\times$
· WAVES IN ACTION (X-DIRECTION WAVE)
+-POLARIZATION
z
y
X-POLARIZATION
LIGO 14 SEP 2015 - NP
BARISH, THORNE, WEISS

GRAVITATIONAL RADIATION FROM ISOLATED SYSTEM



$$\bar{h}^{\mu\nu} = 4G \int d^3x' \, \frac{T^{\mu\nu}(t_R, x')}{|x - x'|}, \quad t_R = t - |x - x'|$$

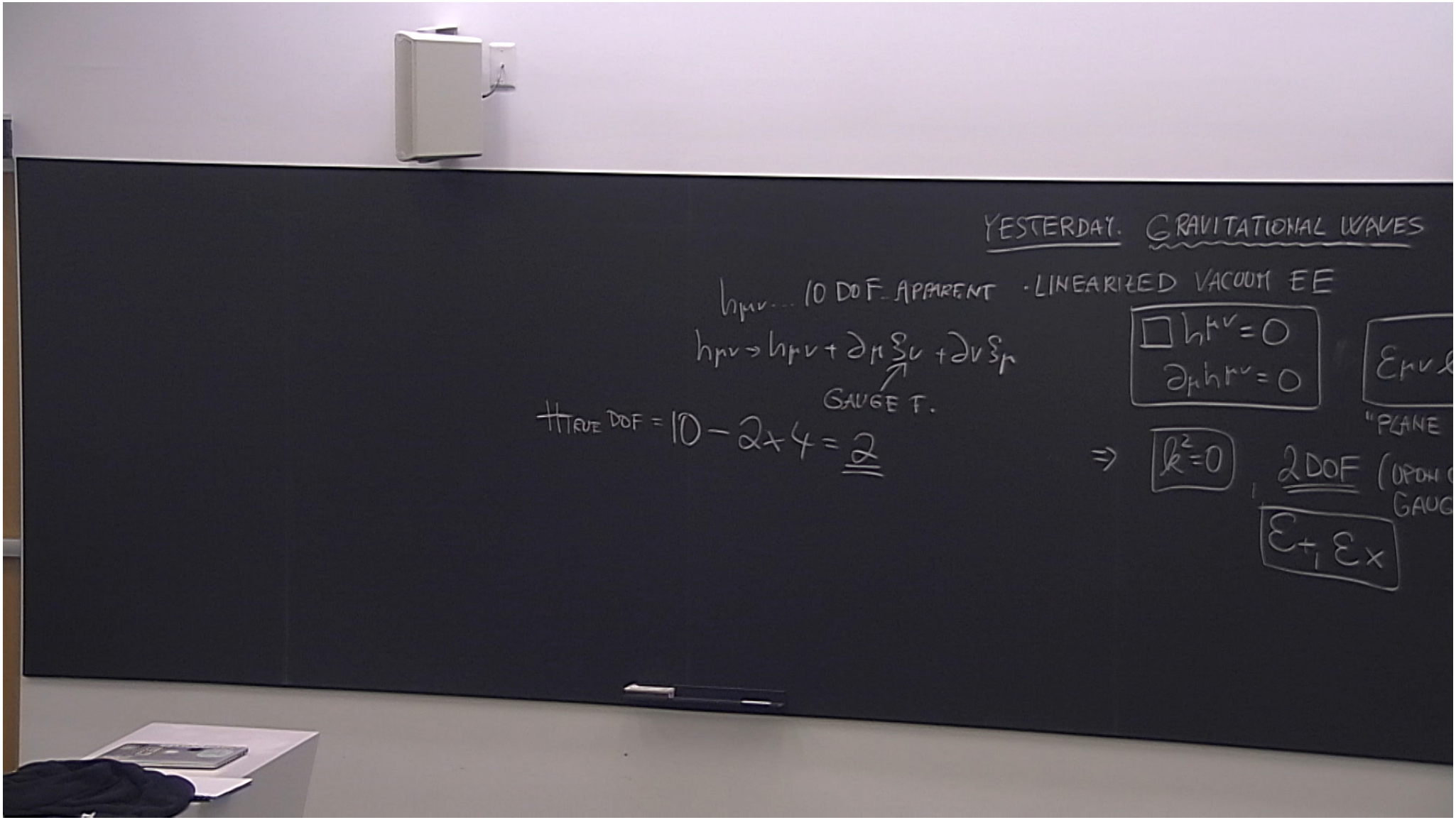
$r' \ll r, \quad \omega \ll \frac{1}{r'}$

ISOL. SYSTEM $\Rightarrow$

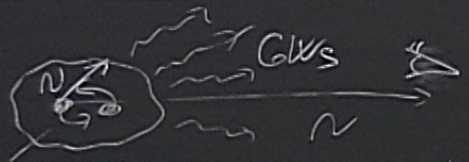
$$\tilde{h}_{\mu\nu} = \frac{1}{\sqrt{2\pi}} \int dt \, e^{i\omega t} \bar{h}_{\mu\nu} = \ldots$$

$$= 4G \frac{e^{i\omega r}}{r} \int d^3x' \, \tilde{T}_{\mu\nu}(\omega, x')$$

RADIATIVE FIELD


YESTERDAY. GRAVITATIONAL WAVES
$h_{\mu\nu}$... 10 DOF. APPARENT
· LINEARIZED VACUUM EE
$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
GAUGE F.
#TRUE DOF = 10 − 2 + 4 = 2
$\Box h^{\mu\nu} = 0$
$\partial_\mu h^{\mu\nu} = 0$
$\Rightarrow$ $k^2 = 0$
2 DOF
$\varepsilon_+, \varepsilon_\times$
"PLANE

GRAVITATIONAL RADIATION FROM ISOLATED SYSTEM



$T_{\mu\nu} \neq 0 \quad r' \ll r, \quad \omega \ll \frac{1}{r'}$

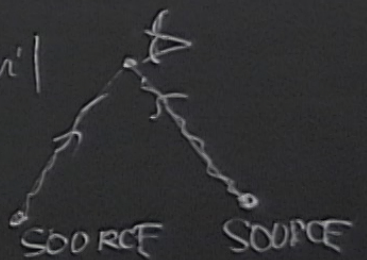
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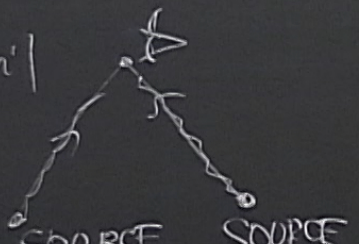
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$$= 4G \frac{e^{i\omega r}}{r} \int d^3x' \, \tilde{T}_{\mu\nu}(\omega, x')$$

RADIATIVE FIELD



$t_R = t - |n - x'|$



$= \ldots$

$\ldots(\omega, n')$

ELD)

NOTE: WE DON'T NEED ALL $\tilde{h}_{\mu\nu}$
ONLY NEED $\tilde{h}_{ij}$

- WE HAVE $\partial_\mu \tilde{h}^{\mu\nu} = 0$

$$\Leftrightarrow \tilde{h}^{t\mu} = \frac{i}{\omega} \partial_i \tilde{h}^{i\mu}$$

$\mu = j$: $\tilde{h}^{tj} = \frac{i}{\omega} \partial_i \tilde{h}^{ij}$

$\mu = t$: $\tilde{h}^{tt} = \frac{i}{\omega} \partial_i \tilde{h}^{it} = -\frac{1}{\omega^2} \partial_i \partial_j \tilde{h}^{ij}$

- BLOODY TRICK.

ALSO HAVE $\partial_\mu T^{\mu\nu} = 0$

$$-\partial_k \tilde{T}^{k\mu} = i\omega \tilde{T}^{t\mu}$$

$$\int d^3x\, \tilde{T}^{ij}(\omega, r) = \int d^3x\, \partial_k \left(x^i \tilde{T}^{kj} \right) - \int x^i \left(\partial_k \tilde{T}^{kj} \right) d^3x$$

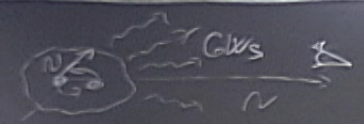
BOUNDARY TERM

BUT SYSTEM IS ISOLATED

$\downarrow$

0

GRAVITATIONAL RADIATION FROM ISOLATED SYSTEM



$T^{\mu\nu} \neq 0$ $\quad r' \ll r$, $\omega \ll \frac{1}{r'}$

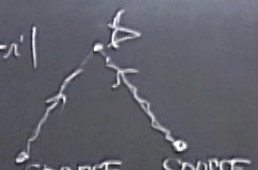
$$\bar{h}^{\mu\nu} = 4G \int d^3x' \frac{T^{\mu\nu}(t_R, x')}{|x - x'|}, \quad t_R = t - |x - x'|$$

ISOL SYSTEM $\Rightarrow$

$$\tilde{h}_{\mu\nu} = \frac{1}{\sqrt{2\pi}} \int dt\, e^{i\omega t} \bar{h}_{\mu\nu} = \ldots$$

$$= 4G \frac{e^{i\omega r}}{r} \int d^3x' \tilde{T}_{\mu\nu}(\omega, x')$$

RADIATIVE FIELD



NOTE: WE DON'T NEED ALL $\tilde{h}_{\mu\nu}$ ONLY NEED $\tilde{h}_{ij}$

- WE HAVE $\partial_\mu \bar{h}^{\mu\nu} = 0$

$$\Leftrightarrow \tilde{h}^{tt} = \frac{i}{\omega} \partial_i \tilde{h}^{it}$$

$\mu = j$: $\tilde{h}^{tj} = \frac{i}{\omega} \partial_i \tilde{h}^{ij}$

$\mu = t$: $\tilde{h}^{tt} = \frac{i}{\omega} \partial_i \tilde{h}^{it} = -\frac{1}{\omega^2} \partial_i \partial_j \tilde{h}^{ij}$

- **BLOODY TRICK.**

ALSO HAVE $\partial_\mu \bar{T}^{\mu\nu} = 0$

$$-\partial_k \tilde{T}^{kn} = i\omega \tilde{T}^{tn}$$

$$\int d^3x\, \tilde{T}^{ij}(\omega, x) = \int d^3x\, \partial_k \left(x^i \tilde{T}^{kj} \right) - \int x^i \left(\partial_k \tilde{T}^{kj} \right) d^3x = i\omega \int d^3x\, x^i \tilde{T}^{tj} \overset{\text{TRICK AGAIN}}{=} -\frac{\omega^2}{2} \int d^3x\, x^i x^j \tilde{T}^{tt}(\omega, x)$$

BOUNDARY TERM

BUT SYSTEM IS ISOLATED $\Rightarrow 0$

$\tilde{I}^{ij}(\omega)$

QUADRUPOLE

- **BLOODY TRICK.**

ALSO HAVE $\partial_\mu T^{\mu\nu} = 0$

$$-\partial_k \tilde{T}^{k\mu} = i\omega \tilde{T}^{t\mu}$$

$$\int d^3r\, \tilde{T}^{ij}(\omega, r) = \int d^3r\, \partial_k \left(x^i \tilde{T}^{kj} \right) - \int x^i \left(\partial_k \tilde{T}^{kj} \right) d^3r = \dots$$

BOUNDARY TERM

BUT SYSTEM IS ISOLATED

$\downarrow$

0

TRICK AGAIN

$$\int (\partial_t \tilde{T}^{0ij})\, d^3x = i\omega \int d^3x\, x^i \tilde{T}^{tj} = -\frac{\omega^2}{2} \int \underbrace{d^3x\, x^i x^j \tilde{T}^{tt}(\omega, x)}_{\tilde{I}^{ij}(\omega)}$$

QUADRUPOLE

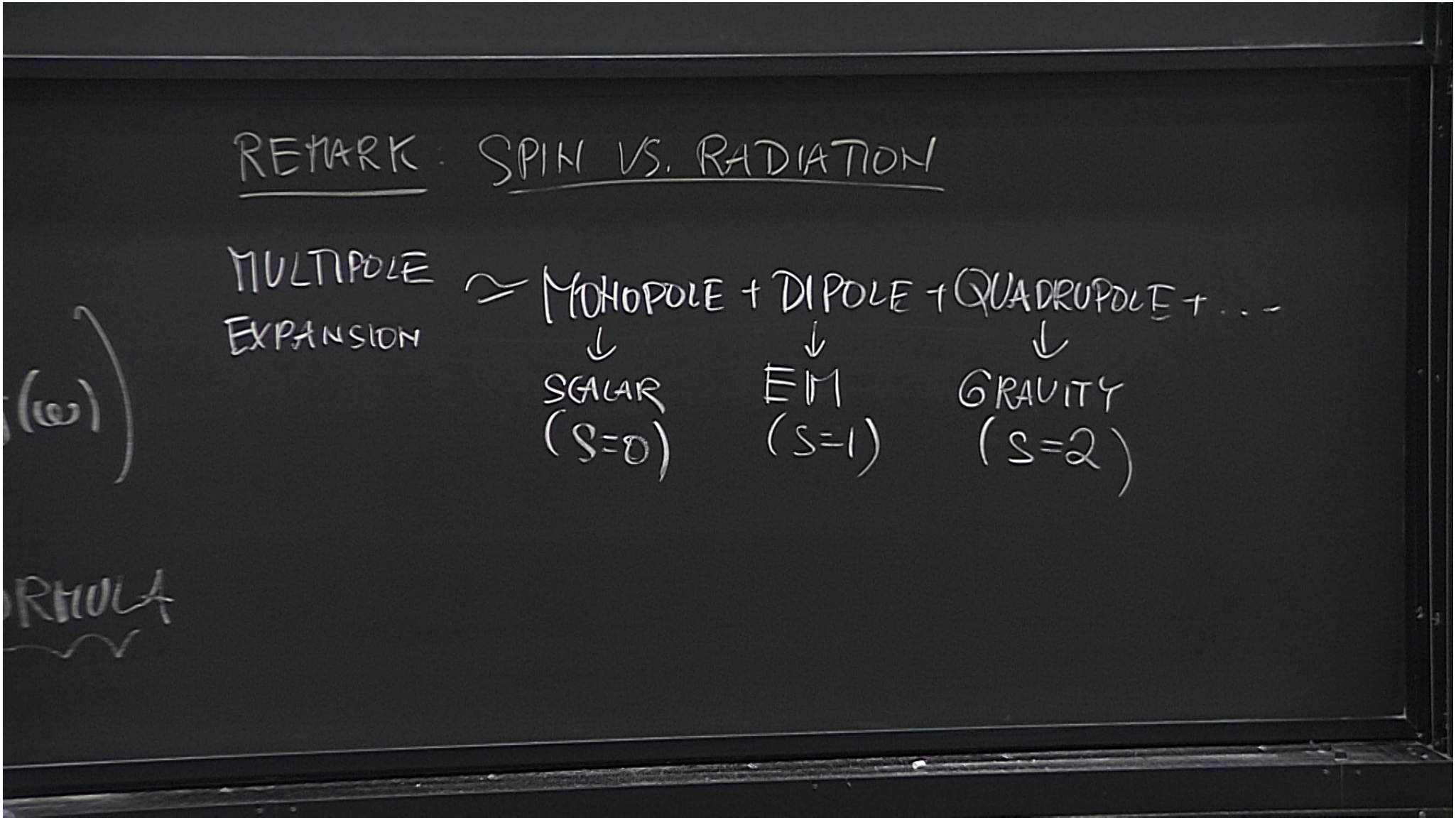
$$\tilde{h}_{ij}(\omega, r) = 4G \frac{e^{i\omega r}}{r}\left(-\frac{\omega^2}{2}\tilde{I}_{ij}(\omega)\right)$$

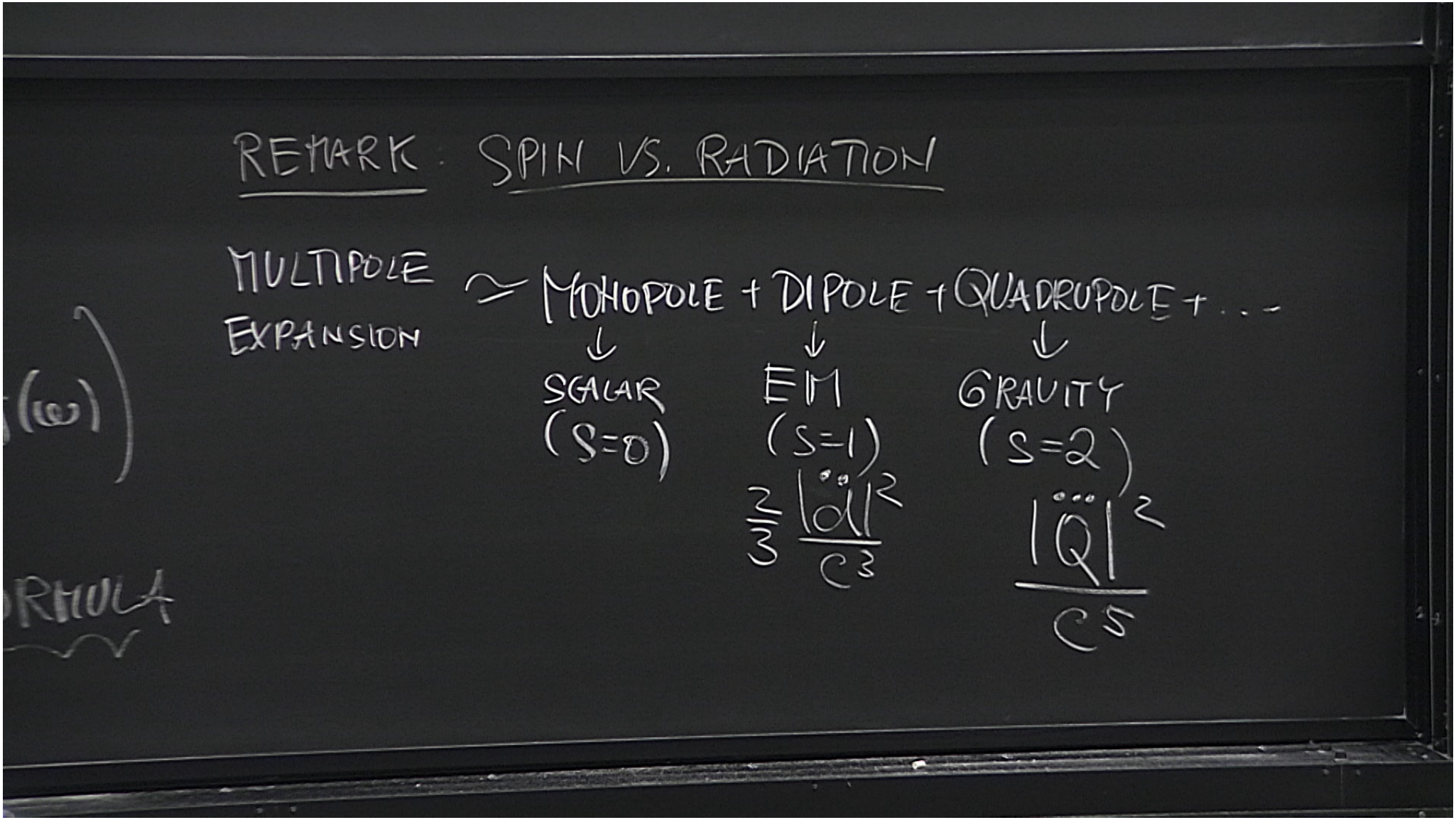
$$h_{ij}(t, r) = \frac{1}{\sqrt{2\pi}}\int e^{-i\omega t_R} d\omega \left(-2G\omega^2 \frac{1}{r}\tilde{I}_{ij}(\omega)\right)$$

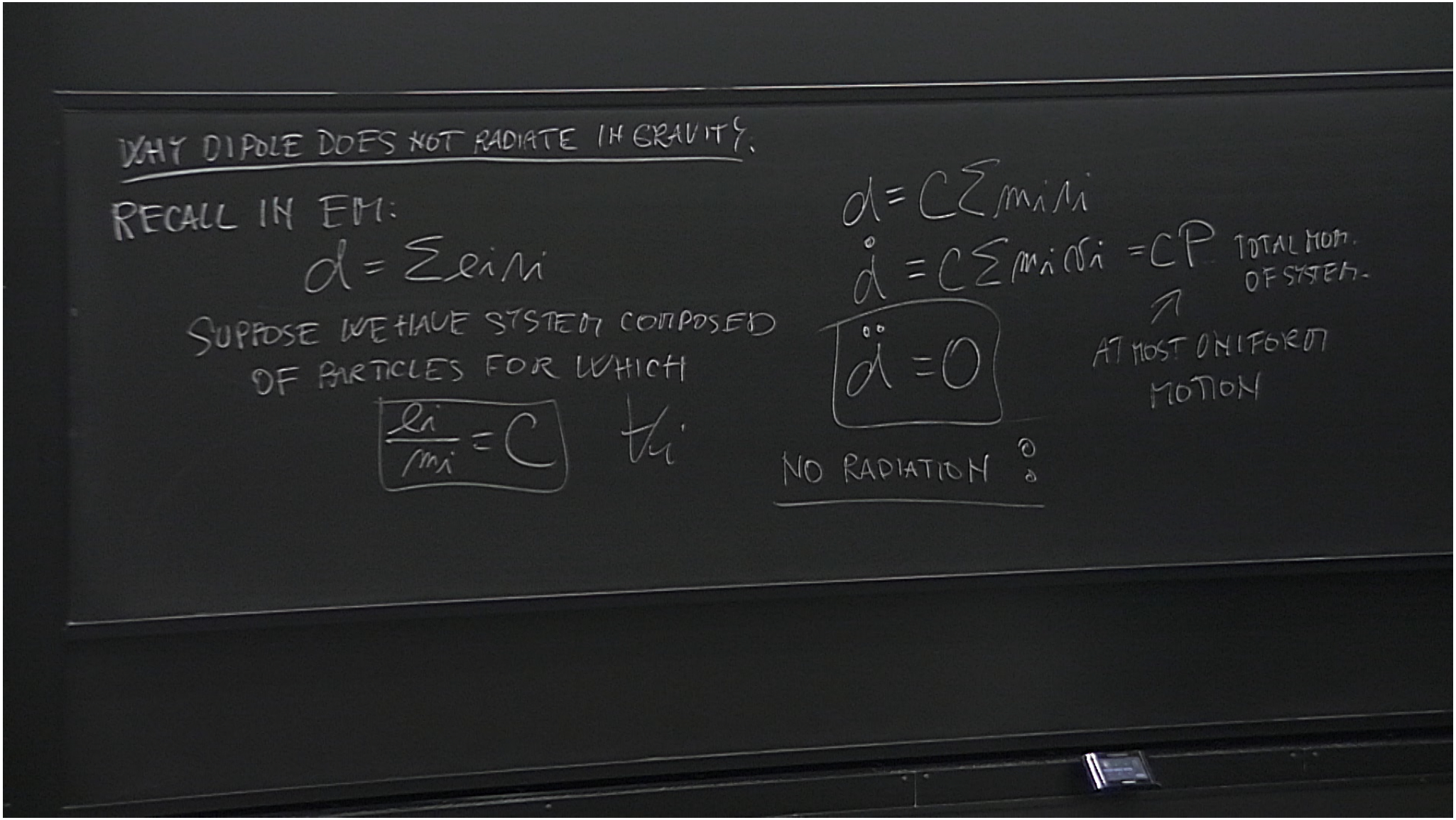
$\frac{1}{\sqrt{2\pi}}\frac{d^2}{dt^2}$

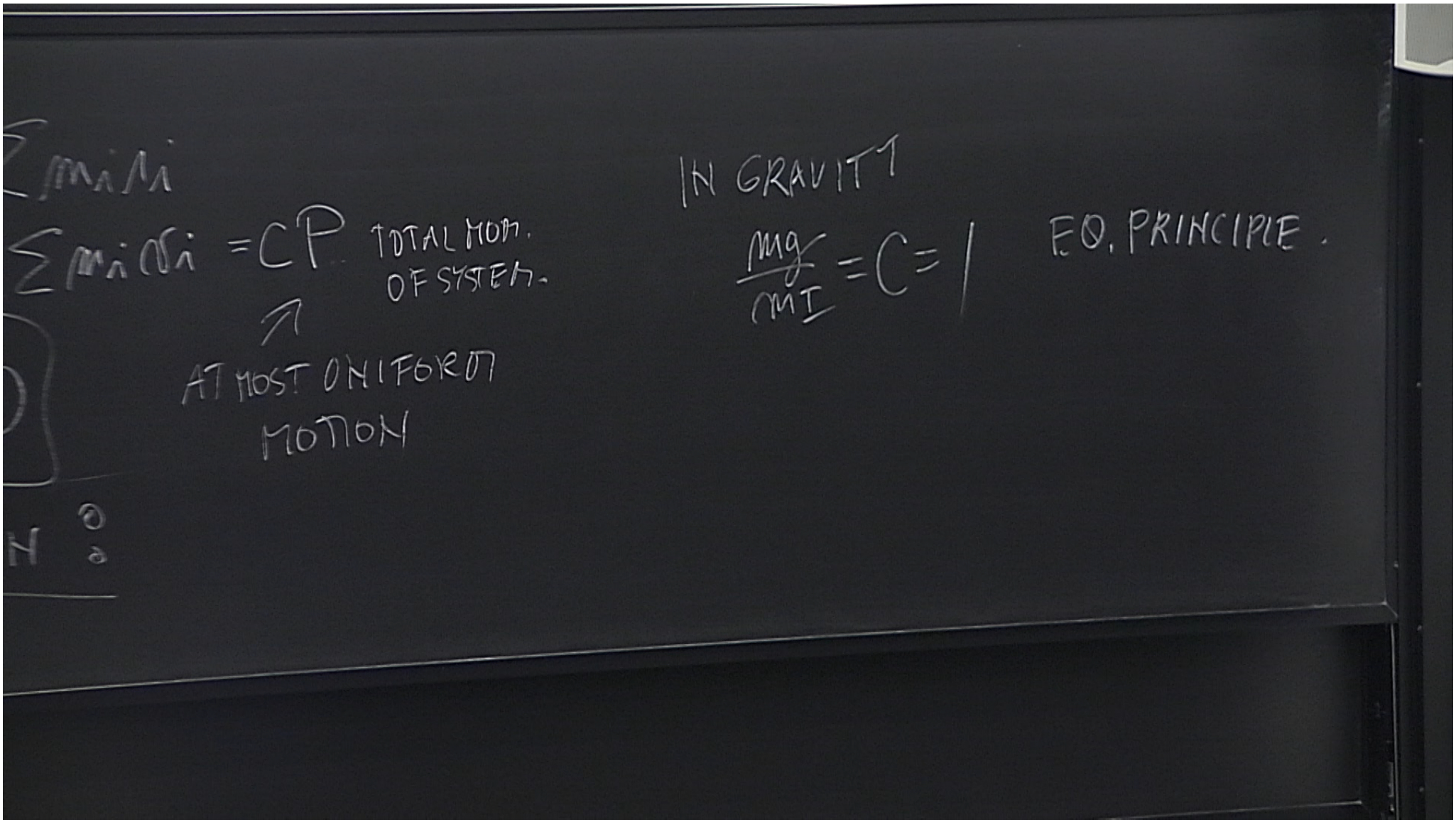
$$h_{ij} = \frac{2G}{r}\frac{d^2 I_{ij}}{dt^2}\Big|_{t=t_R}$$

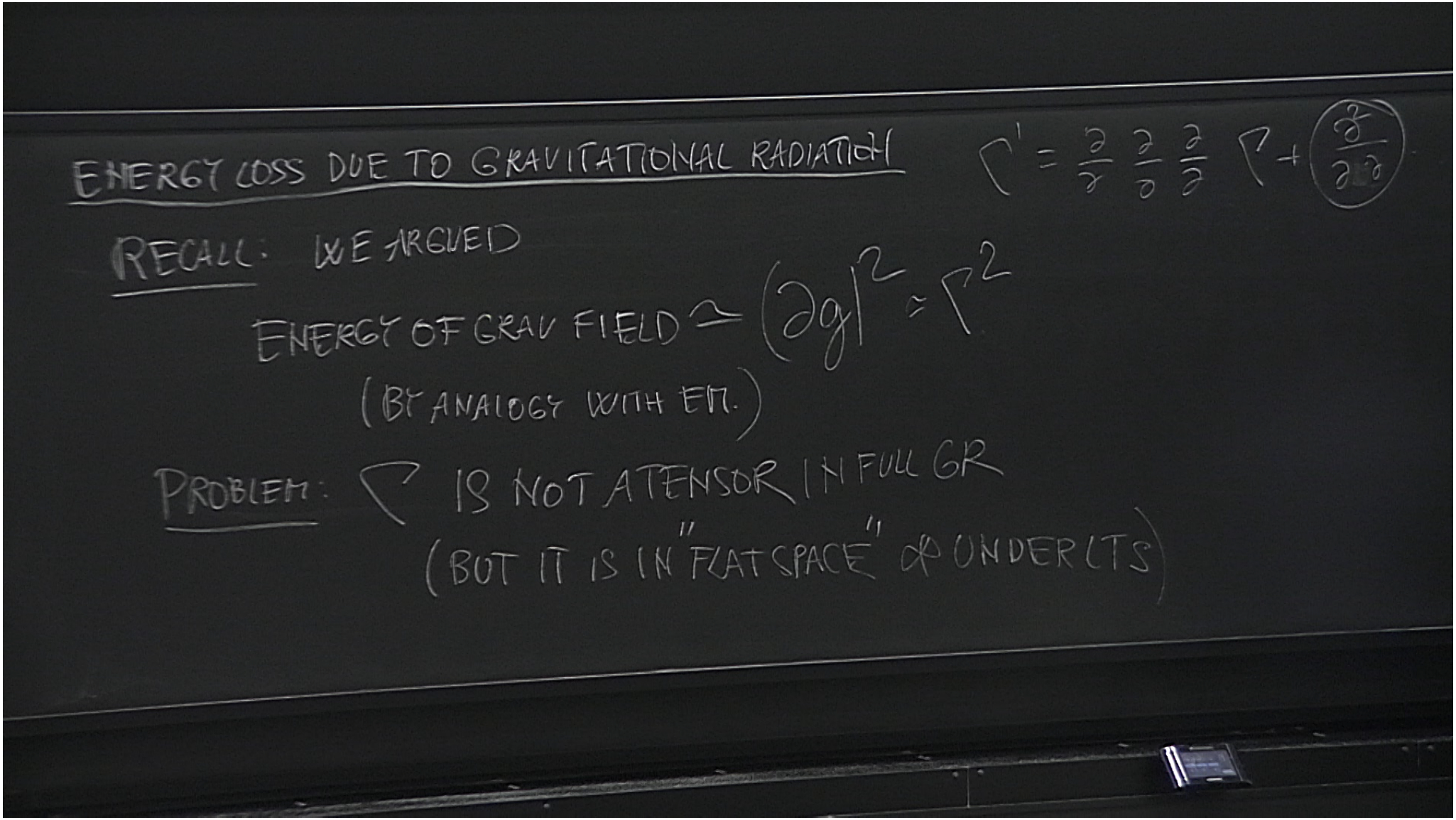
QUADRUPOLE FORMULA

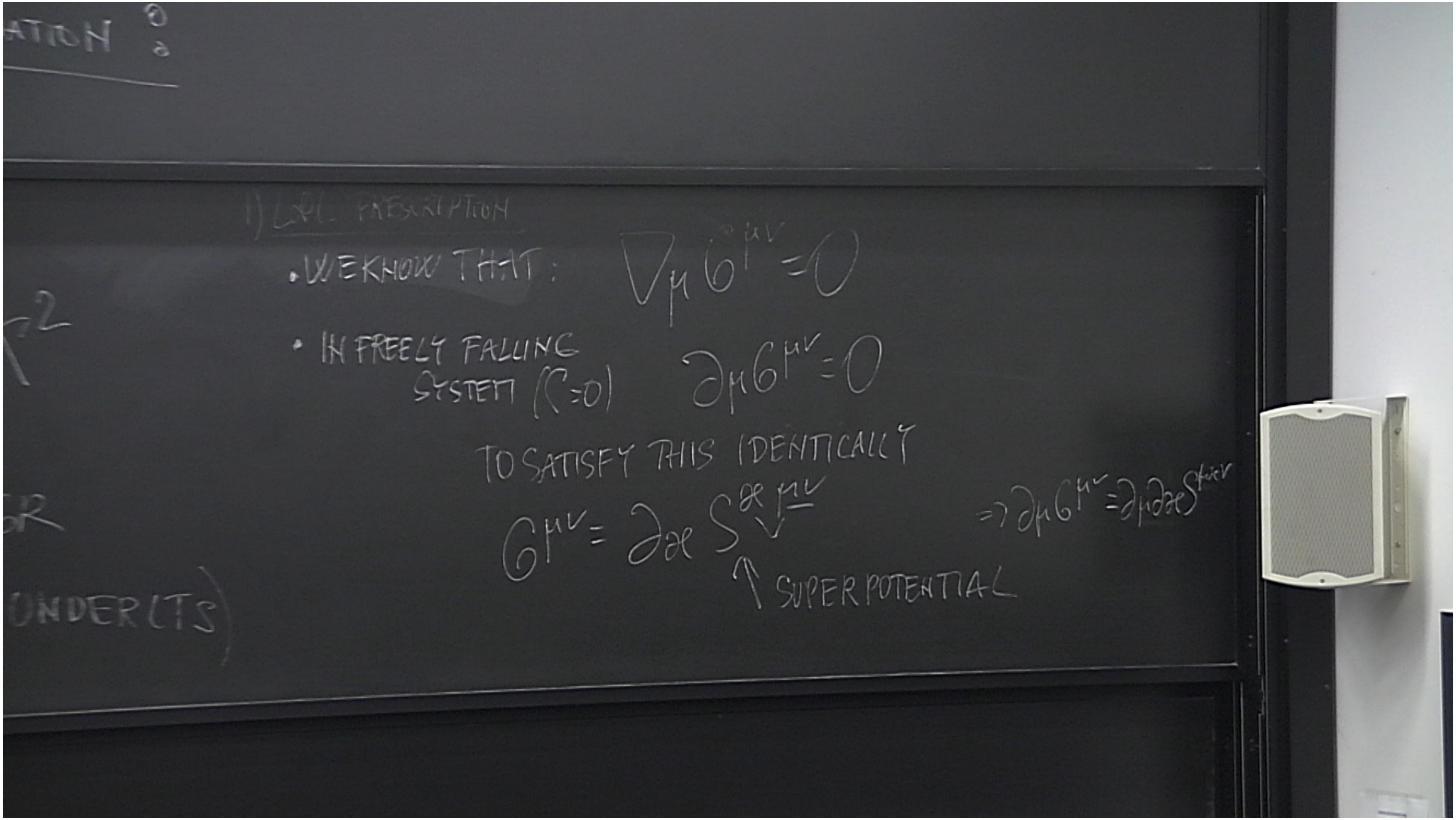

REMARK: SPIN VS. RADIATION
MULTIPOLE EXPANSION
≃ MONOPOLE + DIPOLE + QUADRUPOLE + ...
SCALAR (S=0)
EM (S=1)
GRAVITY (S=2)


REMARK: SPIN VS. RADIATION
MULTIPOLE EXPANSION
≃ MONOPOLE + DIPOLE + QUADRUPOLE + ...
SCALAR (S=0)
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GRAVITY (S=2)


WHY DIPOLE DOES NOT RADIATE IN GRAVITY.
RECALL IN EM:
$d = \sum e_i r_i$
SUPPOSE WE HAVE SYSTEM COMPOSED OF PARTICLES FOR WHICH
$\frac{e_i}{m_i} = C$ $\forall i$
$d = C \sum m_i r_i$
$\dot{d} = C \sum m_i v_i = CP$ TOTAL MOM. OF SYSTEM.
AT MOST UNIFORM MOTION
$\ddot{d} = 0$
NO RADIATION !!


$\sum m_i M_i$
$\sum m_i \delta_i = CP$
TOTAL MOM. OF SYSTEM.
AT MOST ONIFORM MOTION
IN GRAVITY
$\frac{mg}{mI} = C = 1$
EQ. PRINCIPLE.


ENERGY LOSS DUE TO GRAVITATIONAL RADIATION
$\Gamma' = \frac{\partial}{\partial} \frac{\partial}{\partial} \frac{\partial}{\partial} \Gamma + \frac{\partial^2}{\partial \partial}$
RECALL: WE ARGUED
ENERGY OF GRAV FIELD $\simeq (\partial g)^2 \simeq \Gamma^2$
(BY ANALOGY WITH EM.)
PROBLEM: Γ IS NOT A TENSOR IN FULL GR
(BUT IT IS IN "FLAT SPACE" & UNDERLTS)


ATION
1) LOCAL PRESCRIPTION
• WE KNOW THAT: $\nabla_\mu G^{\mu\nu} = 0$
• IN FREELY FALLING SYSTEM ($\Gamma = 0$) $\partial_\mu G^{\mu\nu} = 0$
TO SATISFY THIS IDENTICALLY
$G^{\mu\nu} = \partial_\alpha S^{\alpha\mu\nu}$
$\Rightarrow \partial_\mu G^{\mu\nu} = \partial_\mu \partial_\alpha S^{\alpha\mu\nu}$
↑ SUPERPOTENTIAL
UNDERLTS)

CAN FIND $S^{\alpha\beta\mu\nu}$ EXPLICITLY.

- IN GENERAL FRAME

$$\partial_\alpha S^{\alpha\beta\mu\nu} - G^{\mu\nu} \equiv 8\pi G\, \tau^{\mu\nu} \approx T^{\mu\nu}$$

CAL GRAV. PSEUDO TENSOR

- SYMMETRIC IN $\mu\nu$
- ALSO USING (FE)

$$\partial_\alpha S^{\alpha\beta\mu\nu} = 8\pi G\,(T^{\mu\nu} + \tau^{\mu\nu})$$

$8\pi G\,\tau^{\mu\nu} \approx \rho\rho$

GRAV. PSEUDO TENSOR

$\partial_\mu \tau^{\mu\nu} = 8\pi G\,(T^{\mu\nu} + \tau^{\mu\nu})$

• APPLYING ∂_μ:

$$0 = \partial_\mu (T^{\mu\nu} + \tau^{\mu\nu})$$

TOTAL ENERGY MOM. TENSOR OF MATTER PLUS GRAVITY IS CONSERVED.

- APPLYING ∂_μ:

$$0 = \partial_\mu (T^{\mu\nu} + \tau^{\mu\nu})$$

$$\nabla_\mu T^{\mu\nu} = 0$$

TOTAL ENERGY MOM. TENSOR OF MATTER PLUS GRAVITY IS CONSERVED.

A DIFFERENT CONSTRUCTION

$G_{\mu\nu} = 0$ .. V. EE .. SOLVE PERTURBATIVELY
ORDER BY ORDER

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + h^{(2)}_{\mu\nu} + O(h^3)$$

0th: $G_{\mu\nu}(\eta) = 0$ AUTOMATICALLY. $O(h^3)$

1st: $G_{\mu\nu}(\eta + h) = G^{LIN}_{\mu\nu}(h) = 0$ LINEARIZED GRAVITY

WE ALSO HAVE $\boxed{\partial_\mu G^{\mu\nu}_{LIN} = 0}$ FOR ANY METRIC h

2ND-ORDER:

$$G^{\mu\nu}(\eta + h + h^{(2)}) = G^{LIN}_{\mu\nu}(h) + G^{LIN}_{\mu\nu}(h^{(2)}) + G^{QUADR}_{\mu\nu}(h)$$

0 (1ST ORDER) $= 0$

$$\boxed{G^{\mu\nu}_{LIN}(h^{(2)}) = 8\pi G \tau^{\mu\nu}}$$

$$\tau^{\mu\nu} = -\frac{1}{8\pi G} G^{\mu\nu}_{QUADR}(h)$$

EINSTEIN TENSOR EXPANDED TO 2ND-ORDER

GRAVITY

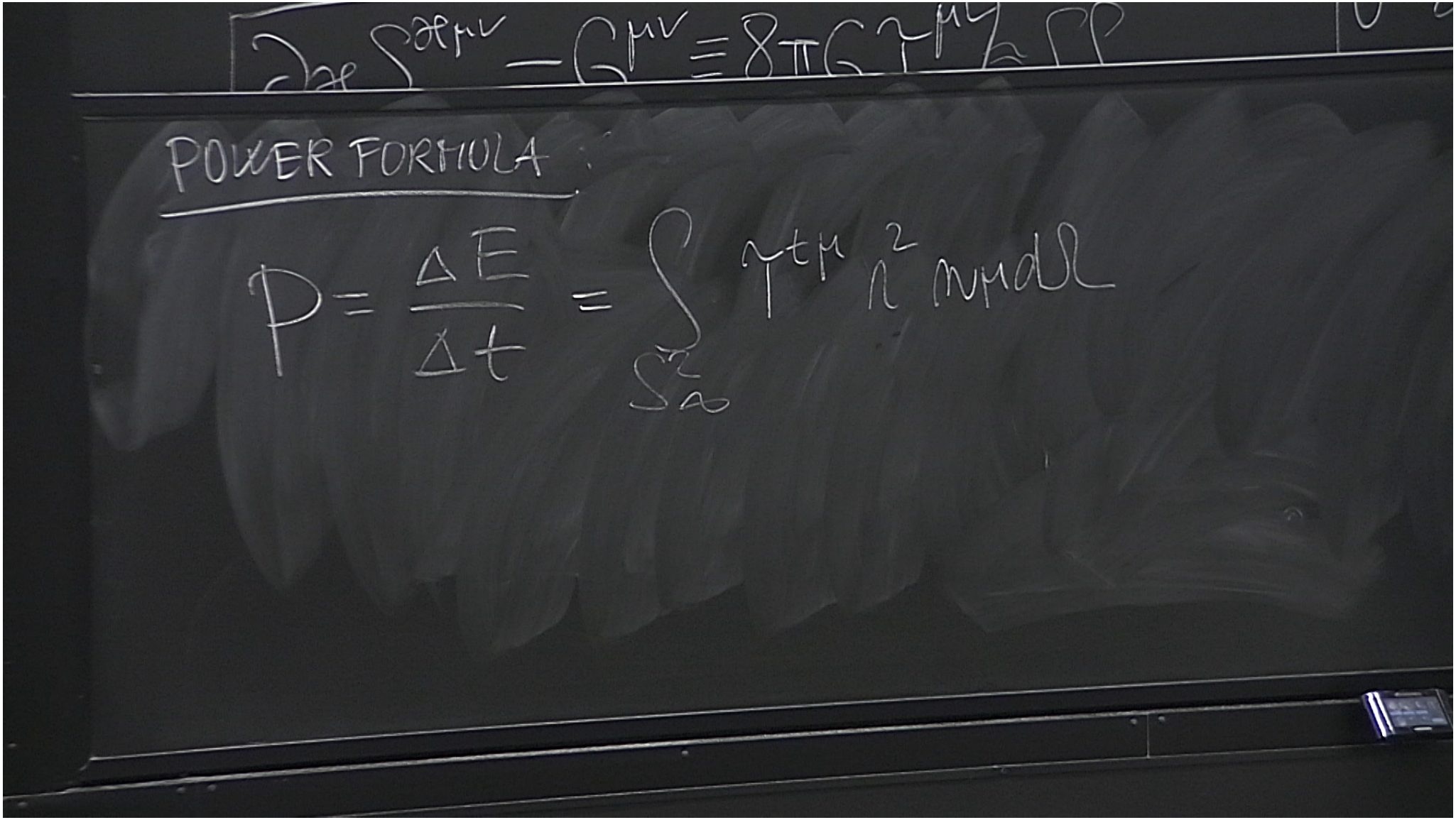
$\tilde{T}^{\mu\nu}$ ... ENERGY MOM. OF GRAV. FIELD. (PSEUDOTENSOR)

$\boxed{\partial_\mu \tilde{T}^{\mu\nu} = 0}$ BECAUSE $\partial_\mu G^{\mu\nu}_{LIN} = 0$ ON ANY METRIC

BY INTEGRATING BY PARTS (COARSE GRAINING OF SPACETIME)

$$\tilde{T}_{\mu\nu} = \frac{1}{32\pi G}\left\langle \partial_\mu h_{\rho\sigma}\,\partial_\nu h^{\rho\sigma} + \ldots \right\rangle$$

↑ VANISH USING LORENZ GAUGE.


POWER FORMULA:
$P = \frac{\Delta E}{\Delta t} = \int_{S_A^2} T^{t\mu} r^2 n_\mu d\Omega$

$\partial_\mu \tilde{T}^{\mu\nu} = 0$ BECAUSE $\partial_\mu G^{\mu\nu}_{LIN} = 0$ ON ANY METRIC

BY INTEGRATING BY PARTS (COARSE GRAINING OF SPA

$$\tilde{T}_{\mu\nu} = \frac{1}{32\pi G}\left\langle \partial_\mu h_{\rho\sigma}\, \partial_\nu h^{\rho\sigma} + \dots \right\rangle$$

↑ QUADRUPOLE $\bar{h}$.

↑ VANISH USING L.O

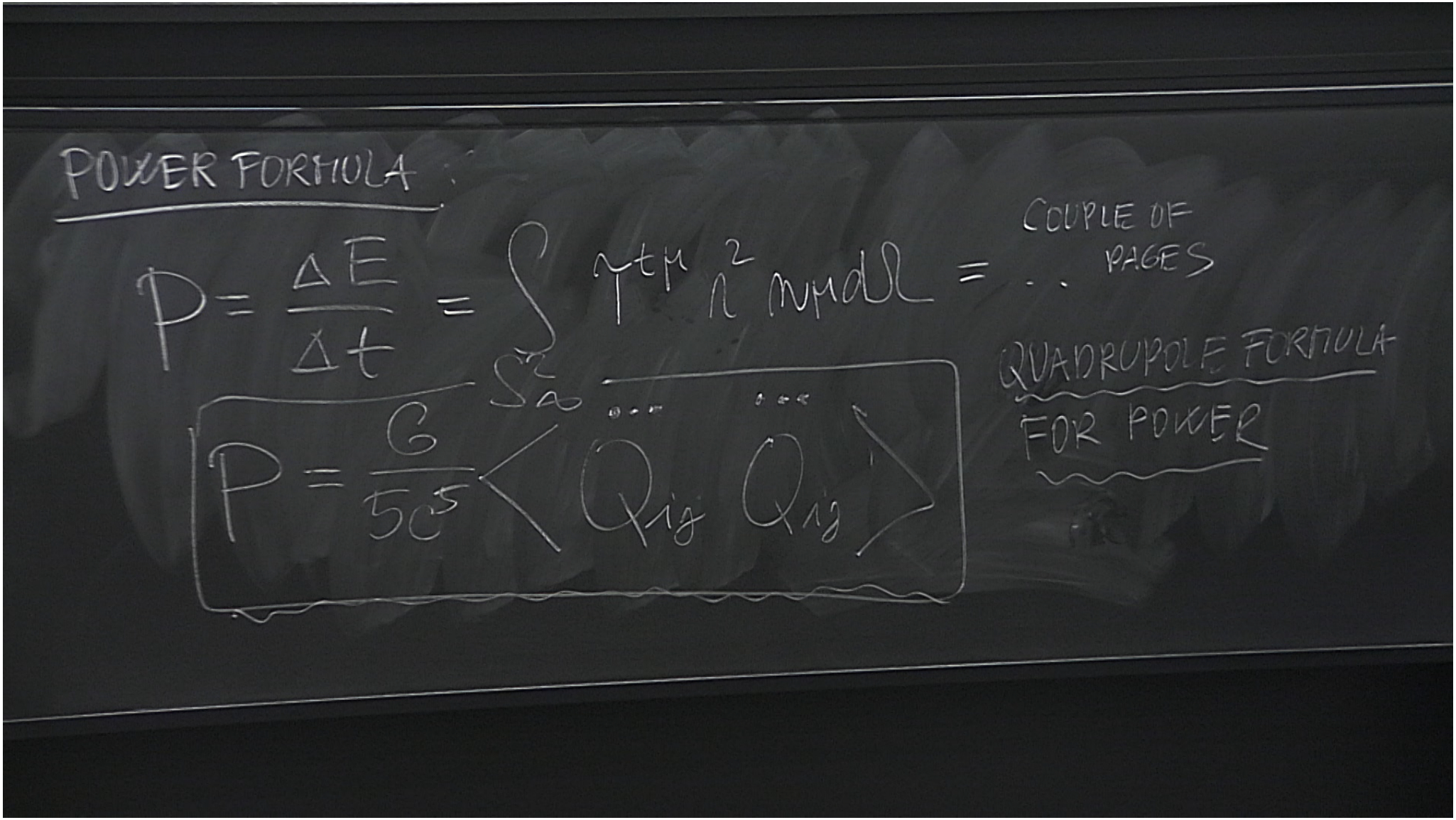
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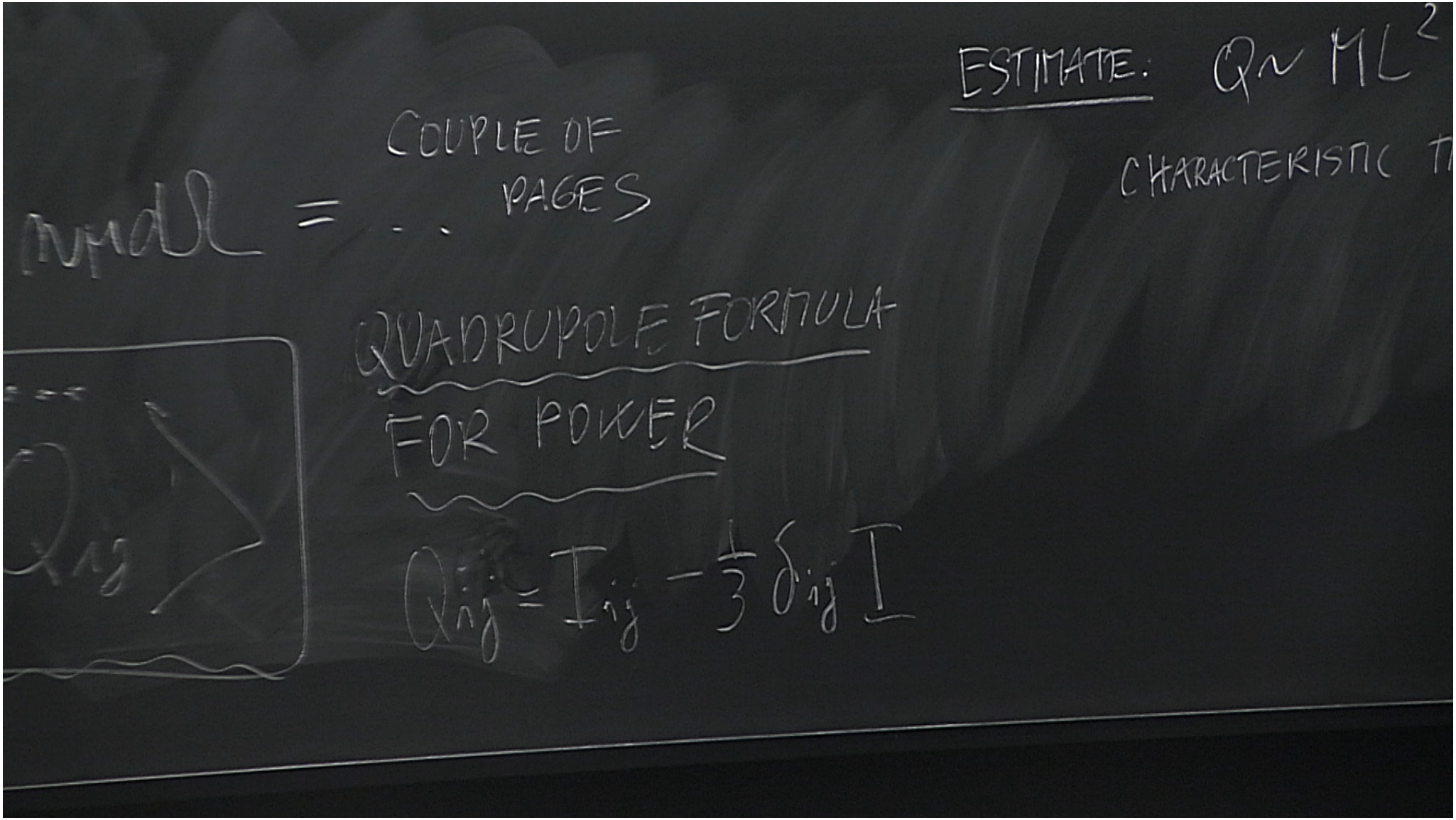
BY INTEGRATING BY PARTS (COARSE GRAINING OF SPA

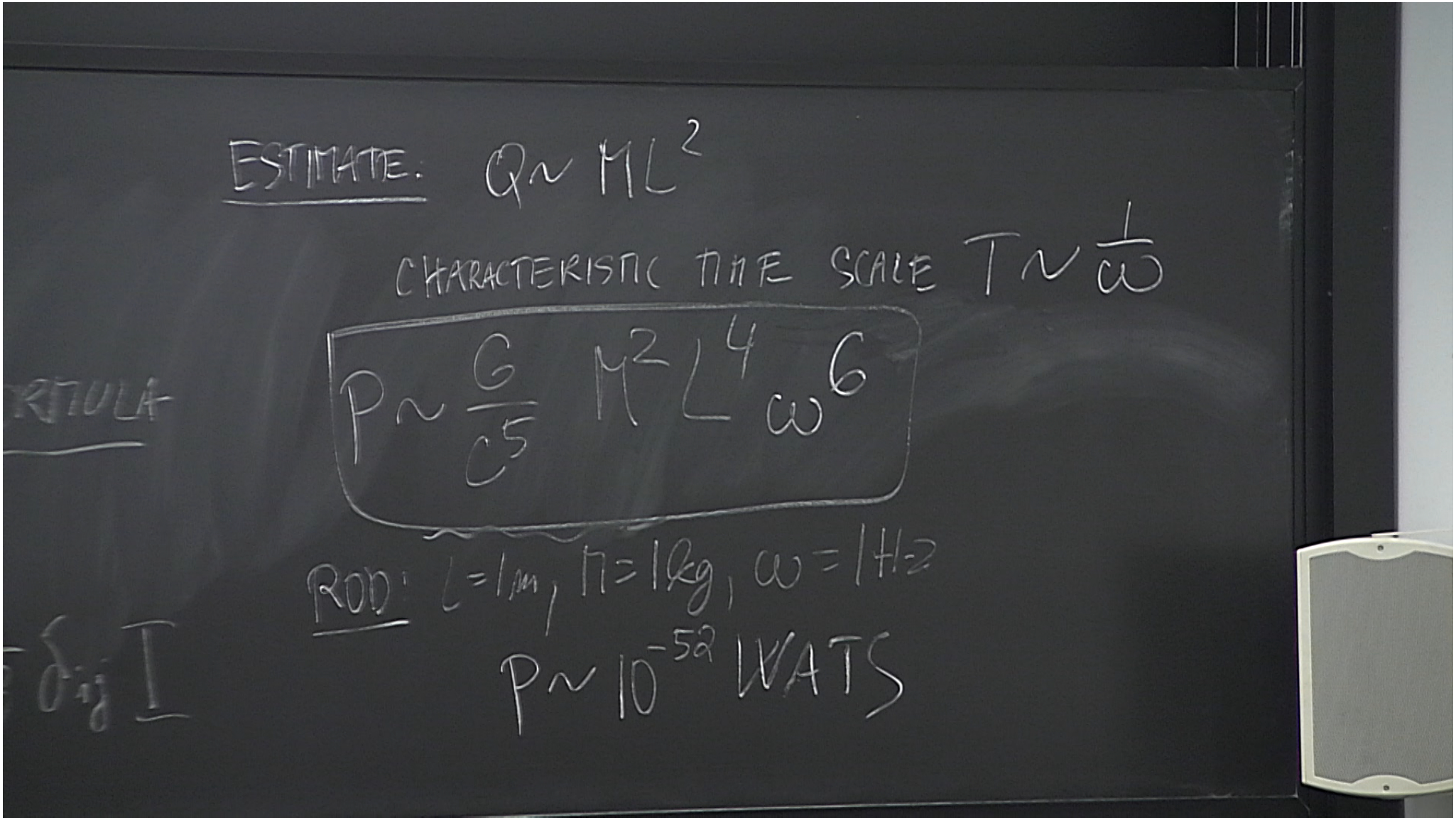
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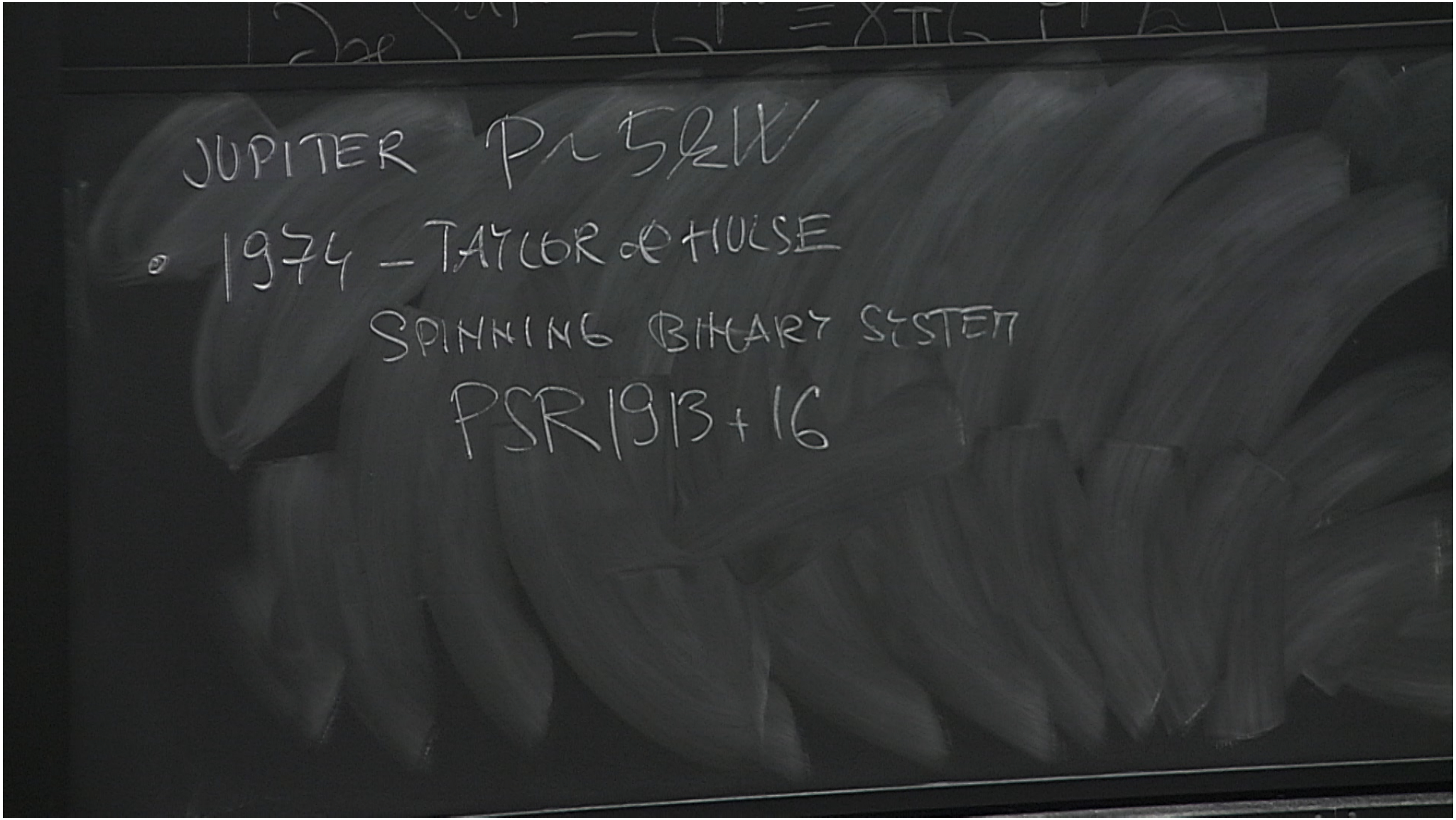
↑ QUADRUPOLE $\bar{E}$.

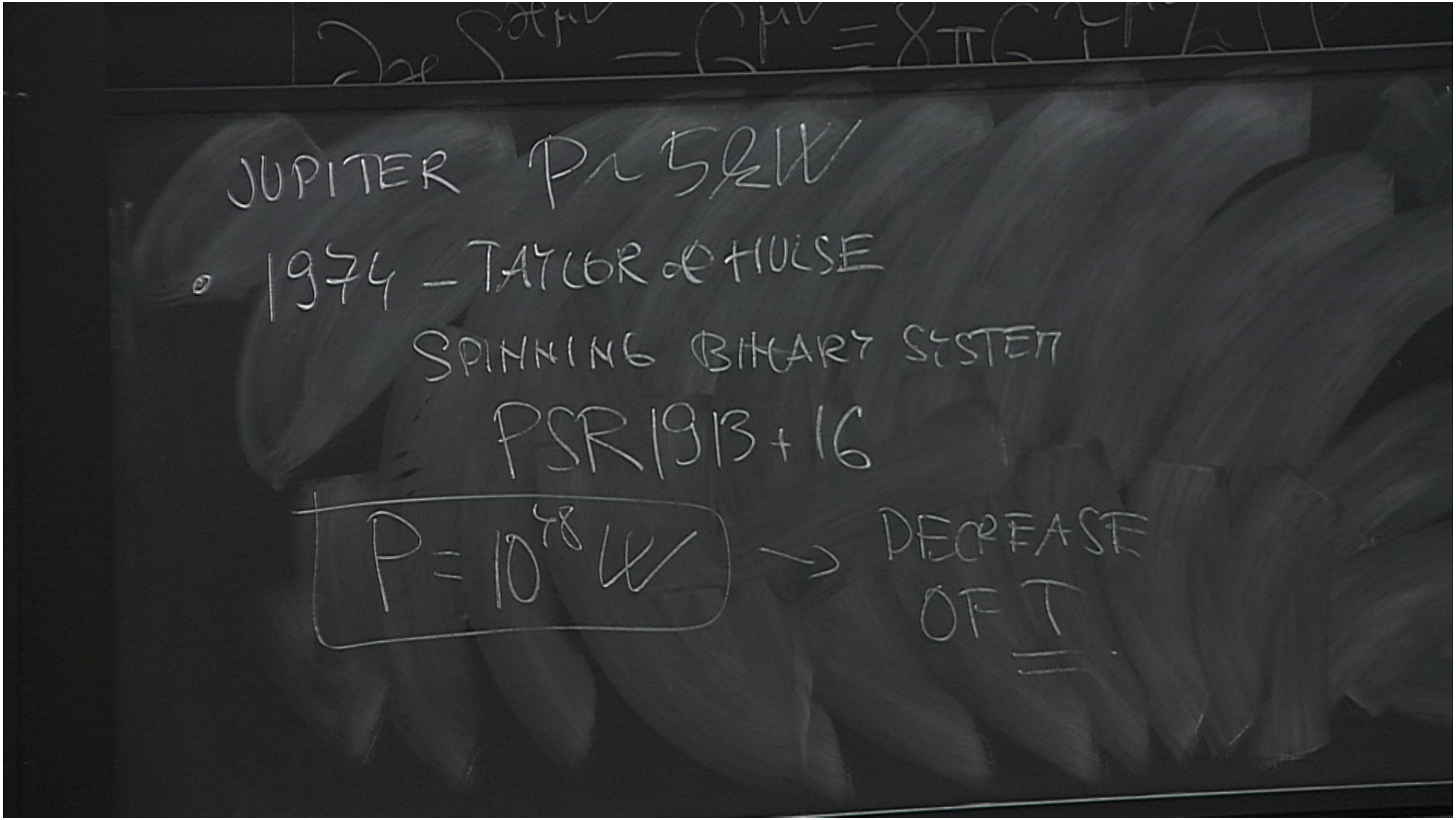
↑ VANISH USING L.O


POWER FORMULA :
$P = \frac{\Delta E}{\Delta t} = \int_{S^2} \eta^{t\mu} \lambda^2 N_\mu N_\nu dS^2 =$ .. COUPLE OF PAGES
$P = \frac{G}{5c^5} \langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle$
QUADRUPOLE FORMULA FOR POWER


ESTIMATE: $Q \sim ML^2$
COUPLE OF PAGES
QUADRUPOLE FORMULA
FOR POWER
$Q_{ij} = I_{ij} - \frac{1}{3}\delta_{ij} I$


ESTIMATE: $Q \sim ML^2$
CHARACTERISTIC TIME SCALE $T \sim \frac{1}{\omega}$
$P \sim \frac{G}{c^5} M^2 L^4 \omega^6$
ROD: $L = 1m$, $M = 1kg$, $\omega = 1Hz$
$P \sim 10^{-52}$ WATS


JUPITER PULSAR
• 1974 – TAYLOR & HULSE
SPINNING BINARY SYSTEM
PSR 1913+16


JUPITER P ~ 5kW
1974 – TAYLOR & HULSE
SPINNING BINARY SYSTEM
PSR 1913+16
P = 10^28 W
DECREASE OF T