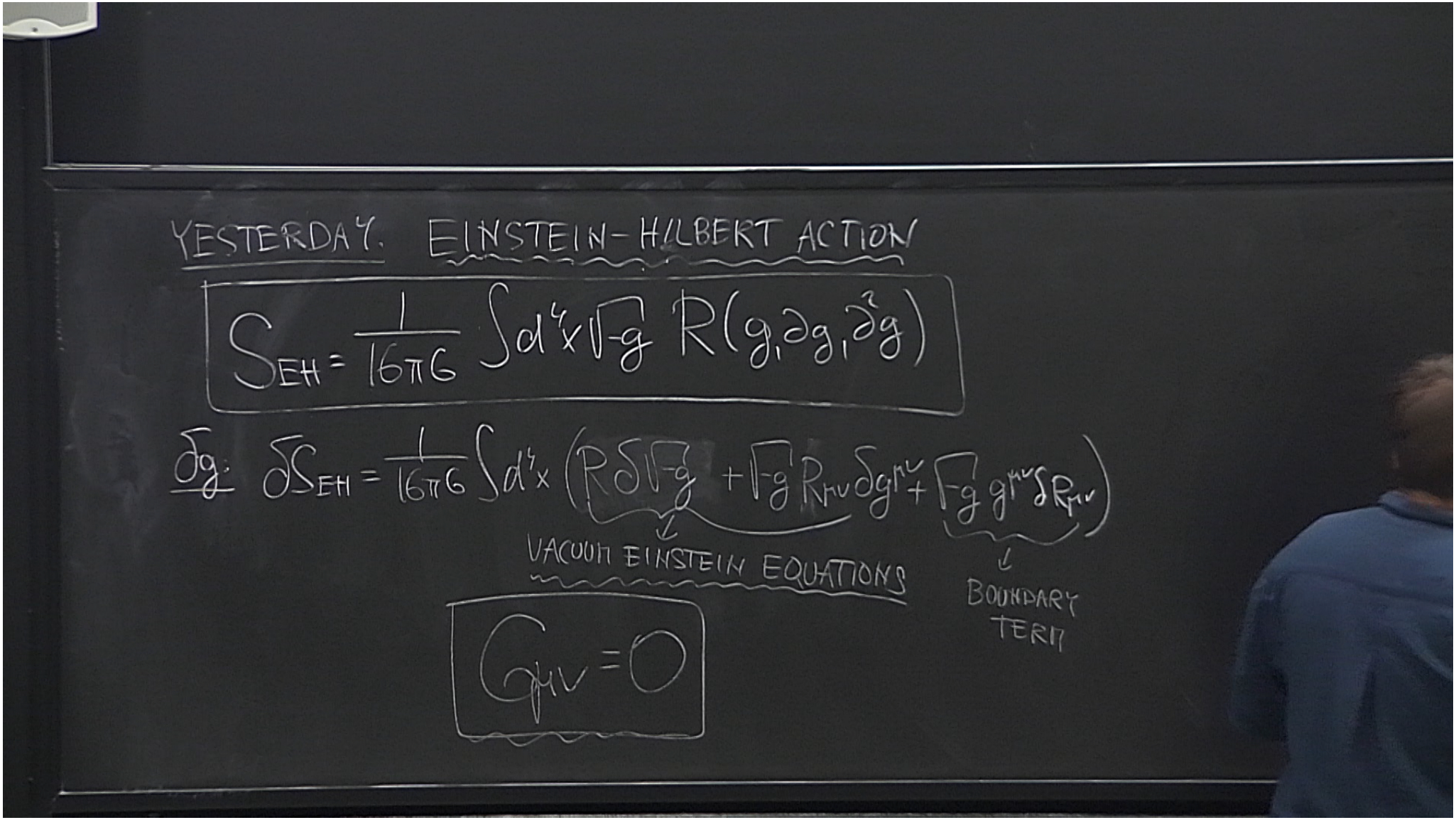
Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 10

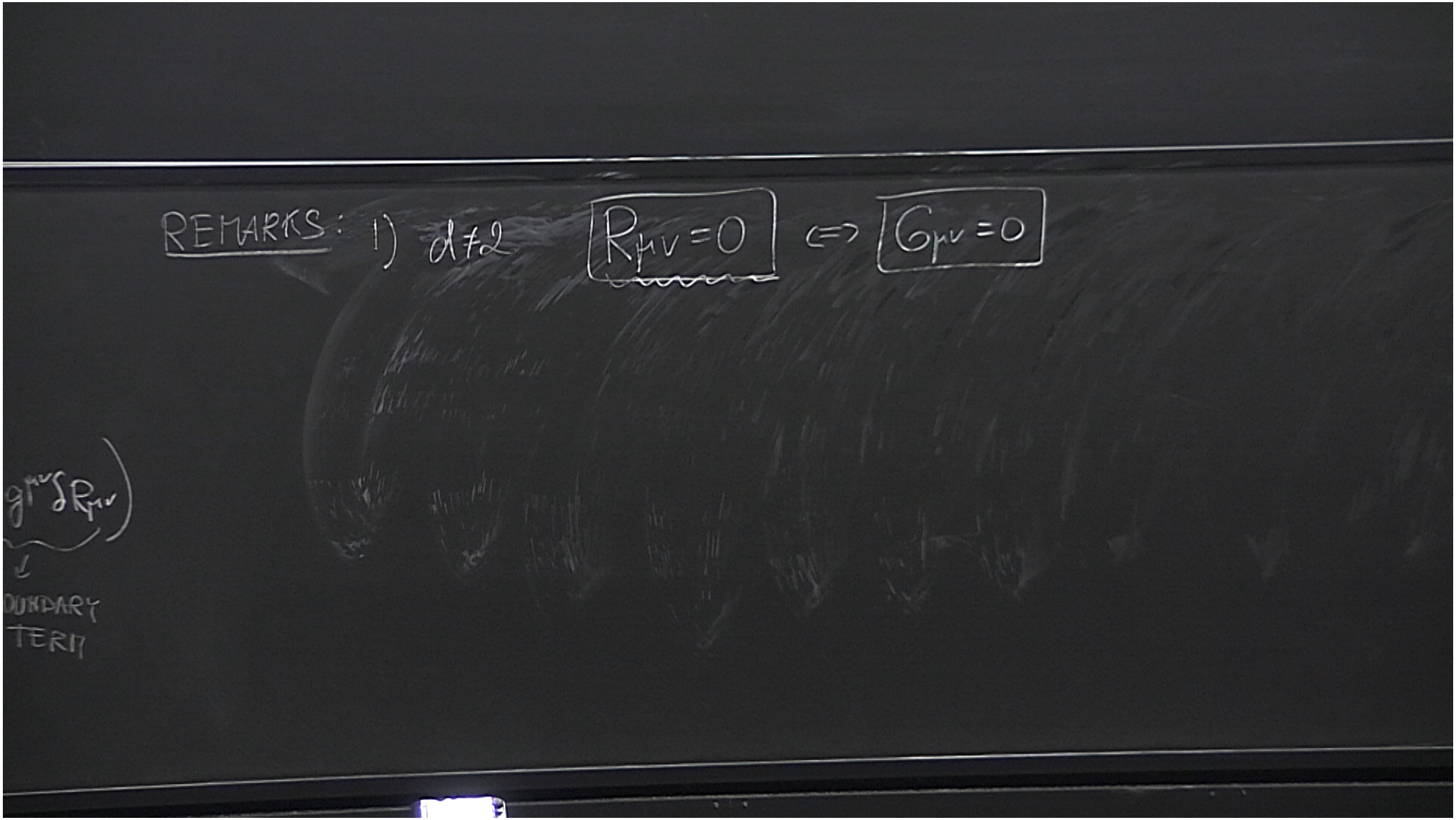
Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

Date: September 13, 2019 - 10:45 AM

URL: http://pirsa.org/19090037


YESTERDAY. EINSTEIN-HILBERT ACTION
$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R(g,\partial g,\partial^2 g)$
δg: $\delta S_{EH} = \frac{1}{16\pi G}\int d^4x\,(R\delta\sqrt{-g} + \sqrt{-g}R_{\mu\nu}\delta g^{\mu\nu} + \sqrt{-g}g^{\mu\nu}\delta R_{\mu\nu})$
VACUUM EINSTEIN EQUATIONS
BOUNDARY TERM
$G_{\mu\nu} = 0$


REMARKS: 1) d ≠ 2
$R_{\mu\nu} = 0$
$\Leftrightarrow$
$G_{\mu\nu} = 0$
$g^{\mu\nu}\delta R_{\mu\nu}$
OUNDARY
TERM

REMARKS: 1) $d \neq 2$ $\boxed{R_{\mu\nu} = 0} \iff \boxed{G_{\mu\nu} = 0}$

2) 2ND-ORDER PDE'S FOR THE METRIC $g_{\mu\nu}$.

REMARKS: 1) $d \neq 2$ $\boxed{R_{\mu\nu} = 0} \Leftrightarrow \boxed{G_{\mu\nu} = 0}$

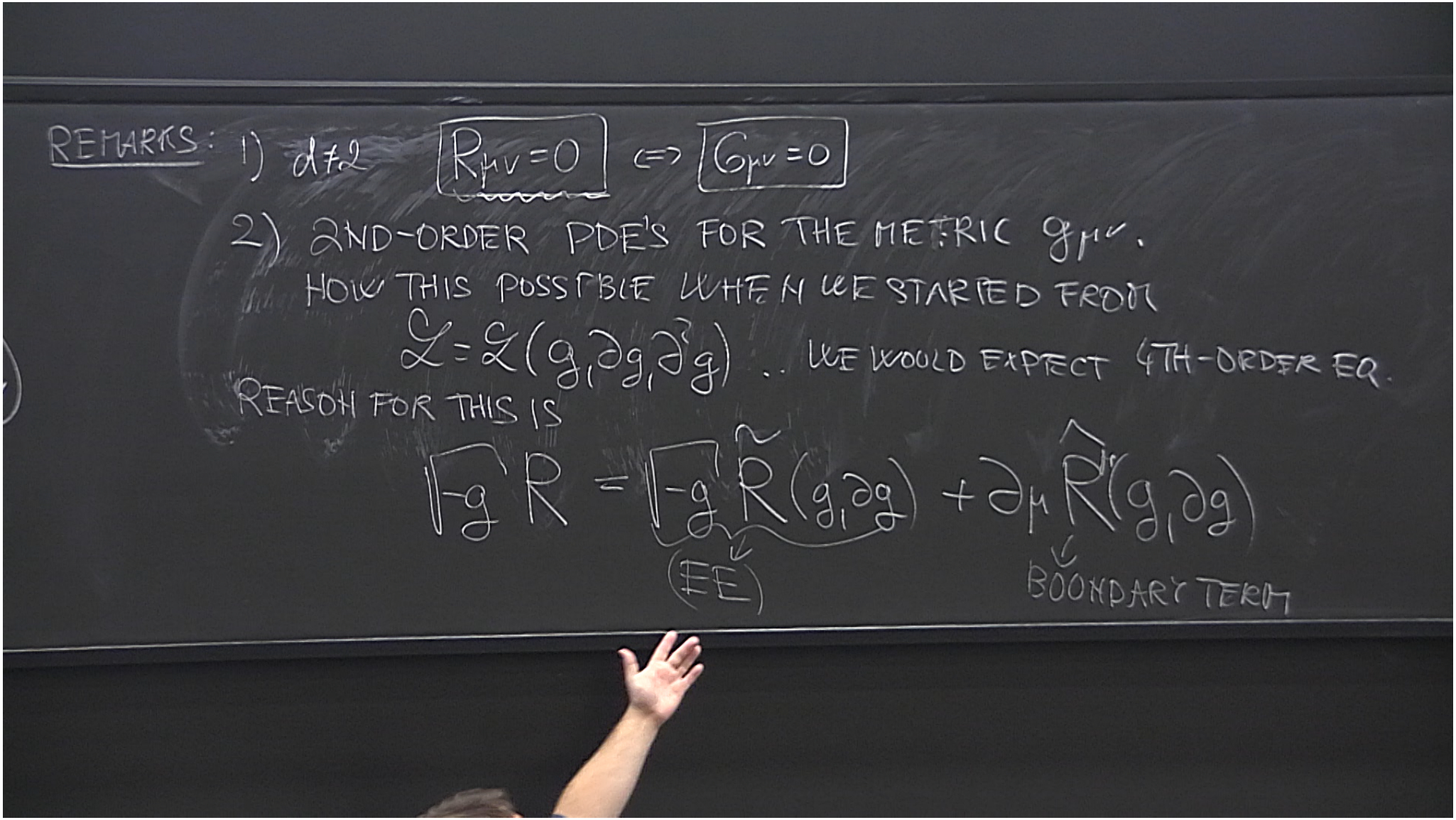
2) 2ND-ORDER PDE'S FOR THE METRIC $g_{\mu\nu}$.

HOW THIS POSSIBLE WHEN WE STARTED FROM

$\mathcal{L} = \mathcal{L}(g, \partial g, \partial^2 g)$ .. WE WOULD EXPECT 4TH-ORDER EQ.

REASON FOR THIS IS

$$\sqrt{-g}\, R = \sqrt{-g}\, \tilde{R}(g, \partial g) + \partial_\mu \hat{R}^\mu(g, \partial g)$$


REMARKS: 1) d ≥ 2 $R_{\mu\nu}=0$ ⇔ $G_{\mu\nu}=0$
2) 2ND-ORDER PDE'S FOR THE METRIC $g_{\mu\nu}$.
HOW THIS POSSIBLE WHEN WE STARTED FROM
$\mathcal{L}=\mathcal{L}(g,\partial g,\partial^2 g)$ .. WE WOULD EXPECT 4TH-ORDER EQ.
REASON FOR THIS IS
$\sqrt{-g}R=\sqrt{-g}\tilde{R}(g,\partial g)+\partial_\mu \hat{R}^\mu(g,\partial g)$
(EE)
BOUNDARY TERM

SEE LaPL. $\tilde{R} = g^{\mu\nu}\left(\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}\right)$

THIS GIVES EINSTEIN EQS.

3) E-H ACTION IS COMPLICATED BECAUSE WE UNDERSTAND IT AS

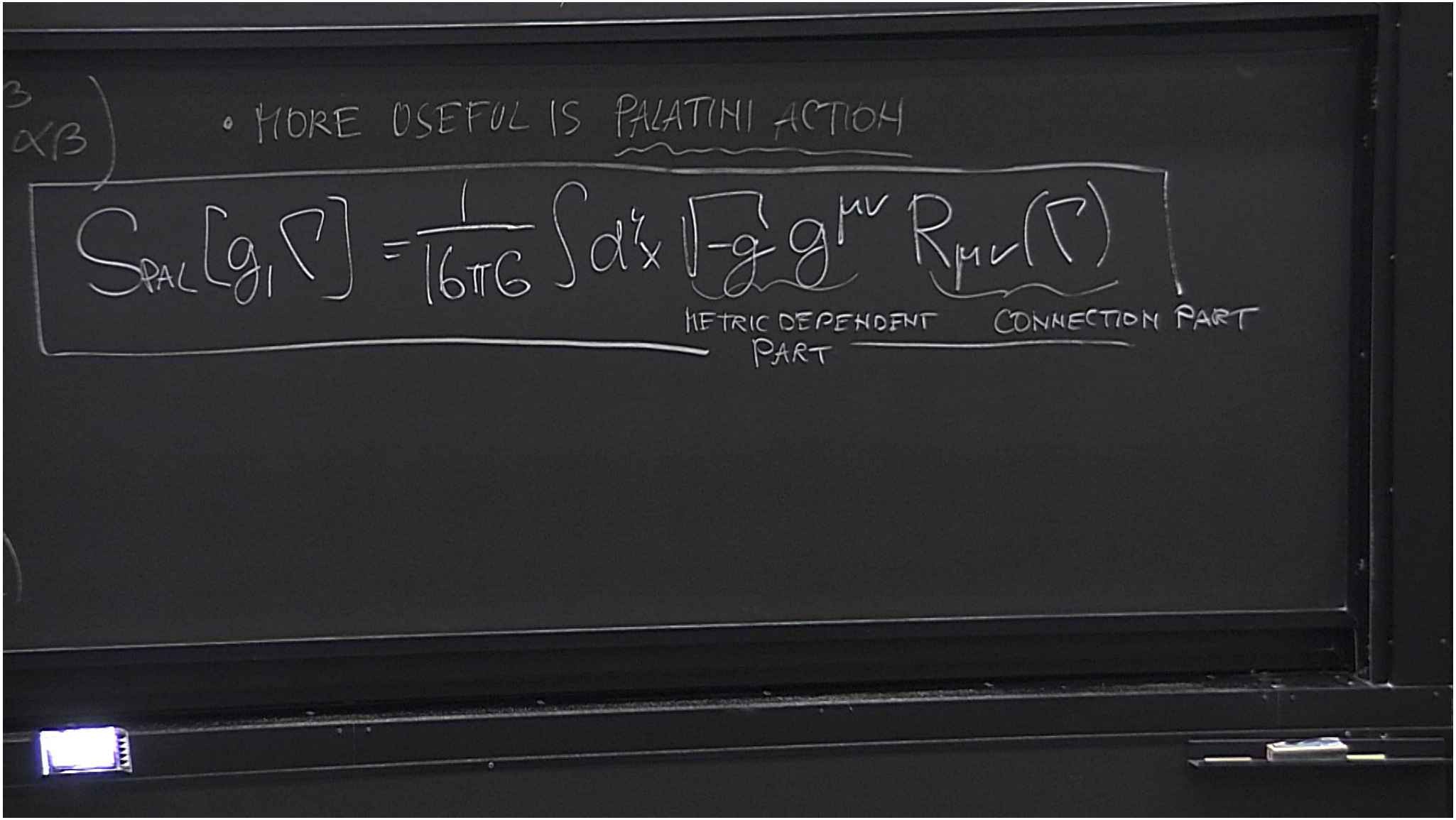
$$S_{EH} = S_{EH}[g]$$

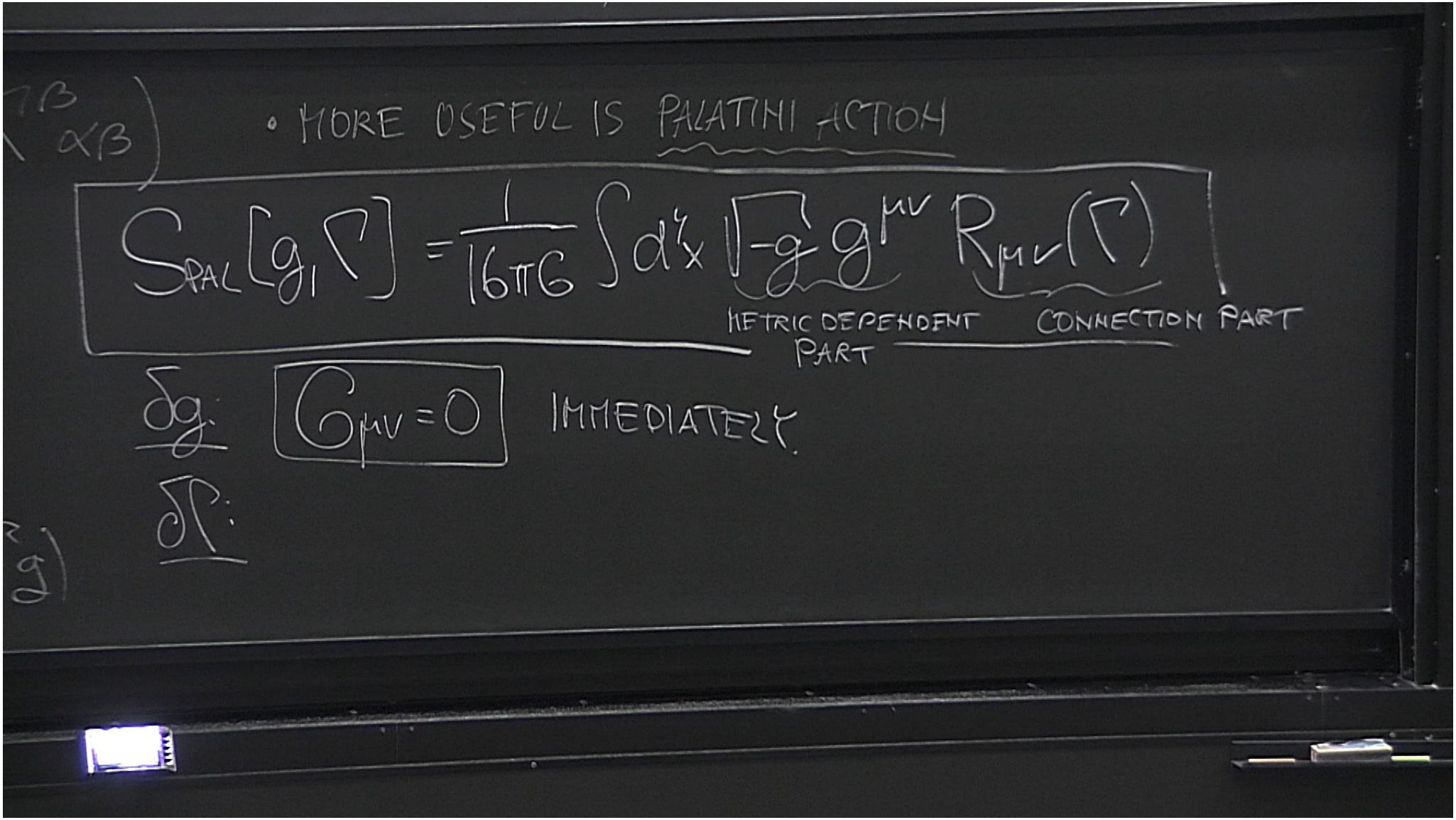
SEE LaPL. $\tilde{R} = g^{\mu\nu}\left(\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}\right)$

THIS GIVES EINSTEIN EQS.

3) E-H ACTION IS COMPLICATED BECAUSE WE UNDERSTAND IT AS

$$\boxed{S_{EH} = S_{EH}[g]}$$ .. 2ND ORDER


• MORE USEFUL IS PALATINI ACTION
$S_{PAL}[g,\Gamma] = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)$
METRIC DEPENDENT PART
CONNECTION PART


• MORE USEFUL IS PALATINI ACTION
$S_{PAL}[g,\Gamma] = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)$
METRIC DEPENDENT PART
CONNECTION PART
δg: $G_{\mu\nu} = 0$ IMMEDIATELY
$\delta\Gamma$:

• MORE USEFUL IS PALATINI ACTION

$$S_{PAL}[g,\Gamma] = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)$$

METRIC DEPENDENT PART — CONNECTION PART

δg: $G_{\mu\nu} = 0$ IMMEDIATELY

$\delta\Gamma$: EQ OF MOTION FOR Γ: $\Gamma \ldots$ CHRISTOFFEL

• MORE USEFUL IS PALATINI ACTION

$$S_{PAL}[g,\Gamma] = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)$$

METRIC DEPENDENT PART — CONNECTION PART

δg: $G_{\mu\nu} = 0$ IMMEDIATELY

1ST-ORDER FORMALISM

$\delta\Gamma$: EQ OF MOTION FOR Γ: $\Gamma \ldots$ CHRISTOFFEL

4) OTHER CURVATURE INVARIANTS

EX: R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, ...

ONLY $g, \partial g, \partial^2 g$

4) OTHER CURVATURE INVARIANTS

EX: R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, ...

ONLY $g, \partial g, \partial^2 g$

THEOREM (LOVELOCK): IN 4d, $\mathcal{L}=R$, IS THE ONLY POSSIBILITY (UP TO TOP. TERMS) WHICH YIELDS 2ND-ORDER EQS FOR g

IN HIGHER DIMS.. WE HAVE ALSO LOVELOCK GRAVITIES

(FOR EXAMPLE .. GAUSS-BONNET GRAVITY)

a) EINSTEIN EQUATIONS (ADDING MATTER)

Λ: $R \to \boxed{R - 2\Lambda}$ $\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = 0}$

2g) 01: EQ OF MOTION FOR Γ: Γ ... CHRISTOFFEL

e) EINSTEIN EQUATIONS (ADDING MATTER)

Λ: $R \to \boxed{R - 2\Lambda}$ $\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = 0}$

$$\boxed{S_{EDDINGTON}[\Gamma] = \frac{1}{8\pi G \Lambda} \int d^4x \sqrt{-\det R_{\mu\nu}(\Gamma)}}$$

• OTHER MATTER:

$$S = S_{EH} + \int d^4x \sqrt{-g}\, \mathcal{L}_m$$

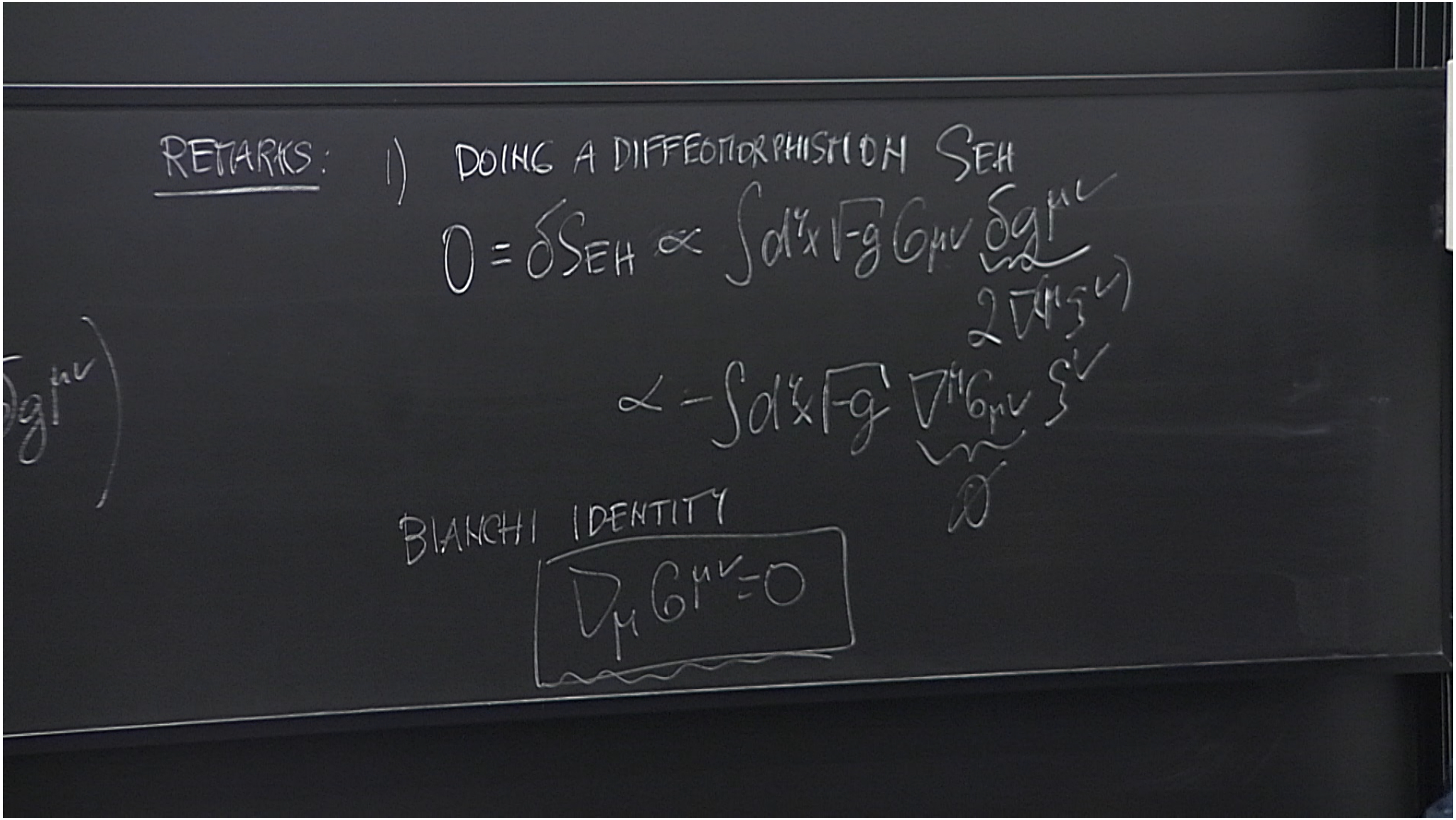
$$\delta_g S = \int d^4x \sqrt{-g} \left(\frac{G_{\mu\nu}\, \delta g^{\mu\nu}}{16\pi G} \right.$$

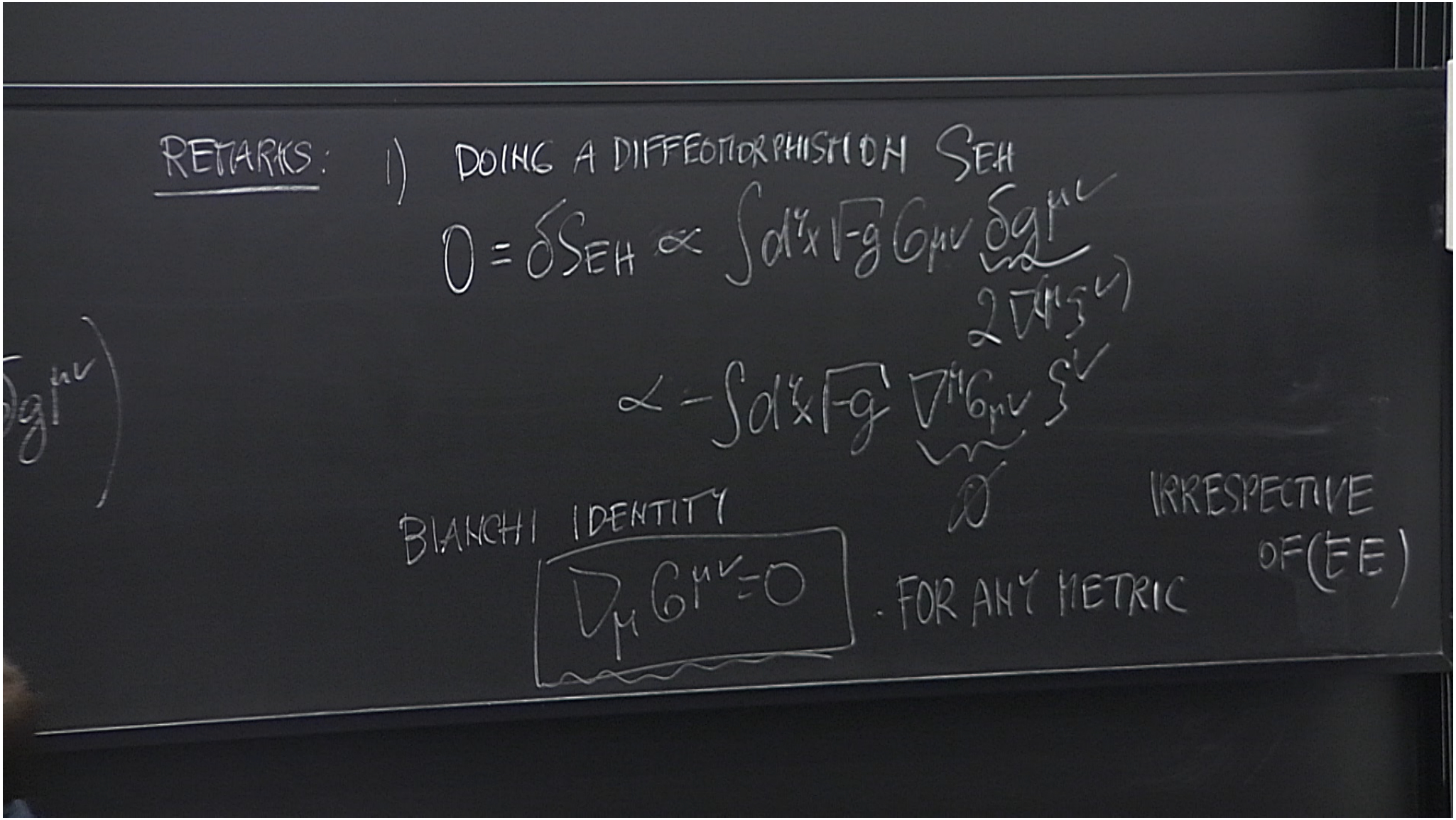
• OTHER MATTER:

$$S = S_{EH} + \int d^4x \sqrt{-g}\, \mathcal{L}_m$$

$$\delta_g S = \int d^4x \sqrt{-g} \left(\frac{G_{\mu\nu}\delta g^{\mu\nu}}{16\pi G} - \frac{1}{2} T_{\mu\nu}\delta g^{\mu\nu} \right)$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$


RETARKS: 1) DOING A DIFFEOTIORPHISTION SEH
$0 = \delta S_{EH} \propto \int d^4x \sqrt{-g}\, G_{\mu\nu} \frac{\delta g^{\mu\nu}}{2\nabla^{(\mu}\xi^{\nu)}}$
$\propto -\int d^4x \sqrt{-g}\, \nabla^\mu G_{\mu\nu}\, \xi^\nu$
$\nabla^\mu G_{\mu\nu} = 0$
BIANCHI IDENTITY
$D_\mu G^{\mu\nu} = 0$


REMARKS: 1) DOING A DIFFEOMORPHISM S_{EH}
$0 = \delta S_{EH} \propto \int d^4x \sqrt{-g}\, G_{\mu\nu}\, \delta g^{\mu\nu}$
$\delta g^{\mu\nu} = 2\nabla^{(\mu}\xi^{\nu)}$
$\propto -\int d^4x \sqrt{-g}\, \nabla^\mu G_{\mu\nu}\, \xi^\nu$
$\nabla^\mu G_{\mu\nu} = 0$
BIANCHI IDENTITY
$\nabla_\mu G^{\mu\nu} = 0$
. FOR ANY METRIC
IRRESPECTIVE OF (EE)

• OTHER MATTER:

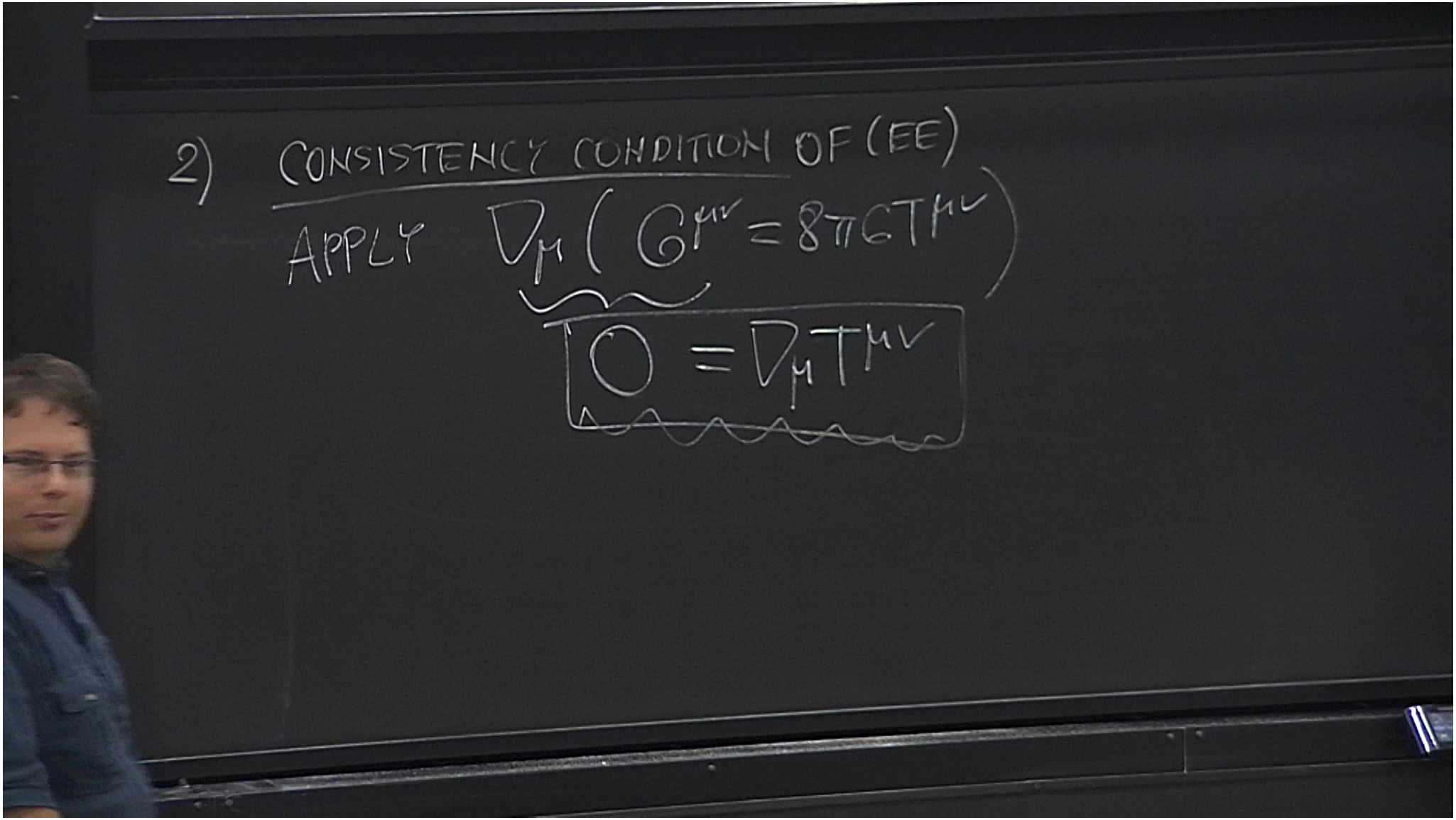
$$S = S_{EH} + \int d^4x \sqrt{-g}\, \mathcal{L}_m$$

$$\delta S = \int d^4x \sqrt{-g} \left(\frac{G_{\mu\nu}\delta g^{\mu\nu}}{16\pi G} - \frac{1}{2} T_{\mu\nu}\delta g^{\mu\nu} \right)$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (EE)$$

REMARKS: 1)

BI


2) CONSISTENCY CONDITION OF (EE)
APPLY $D_\mu (G^{\mu\nu} = 8\pi G T^{\mu\nu})$
$0 = D_\mu T^{\mu\nu}$

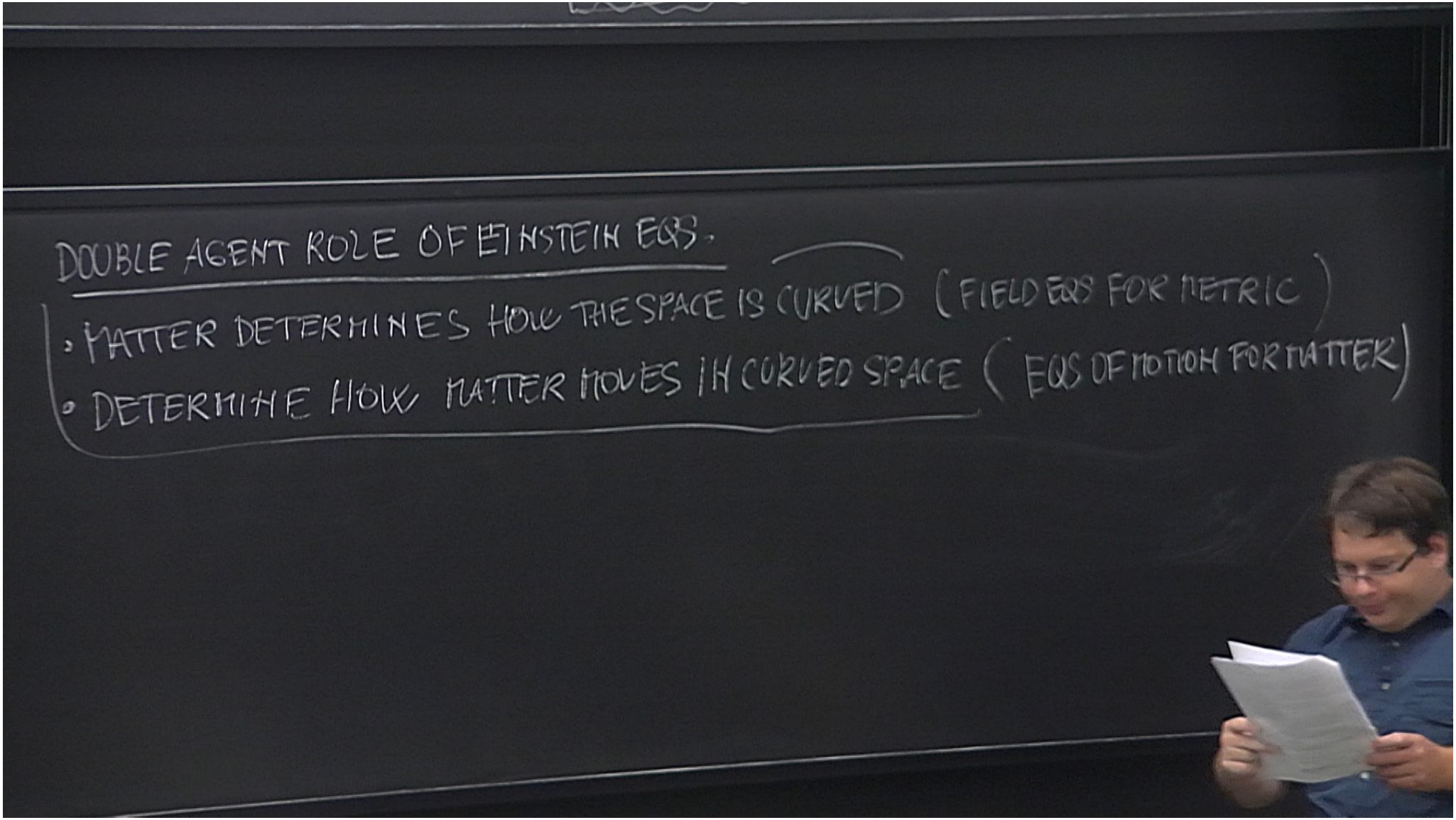
2) CONSISTENCY CONDITION OF (EE)

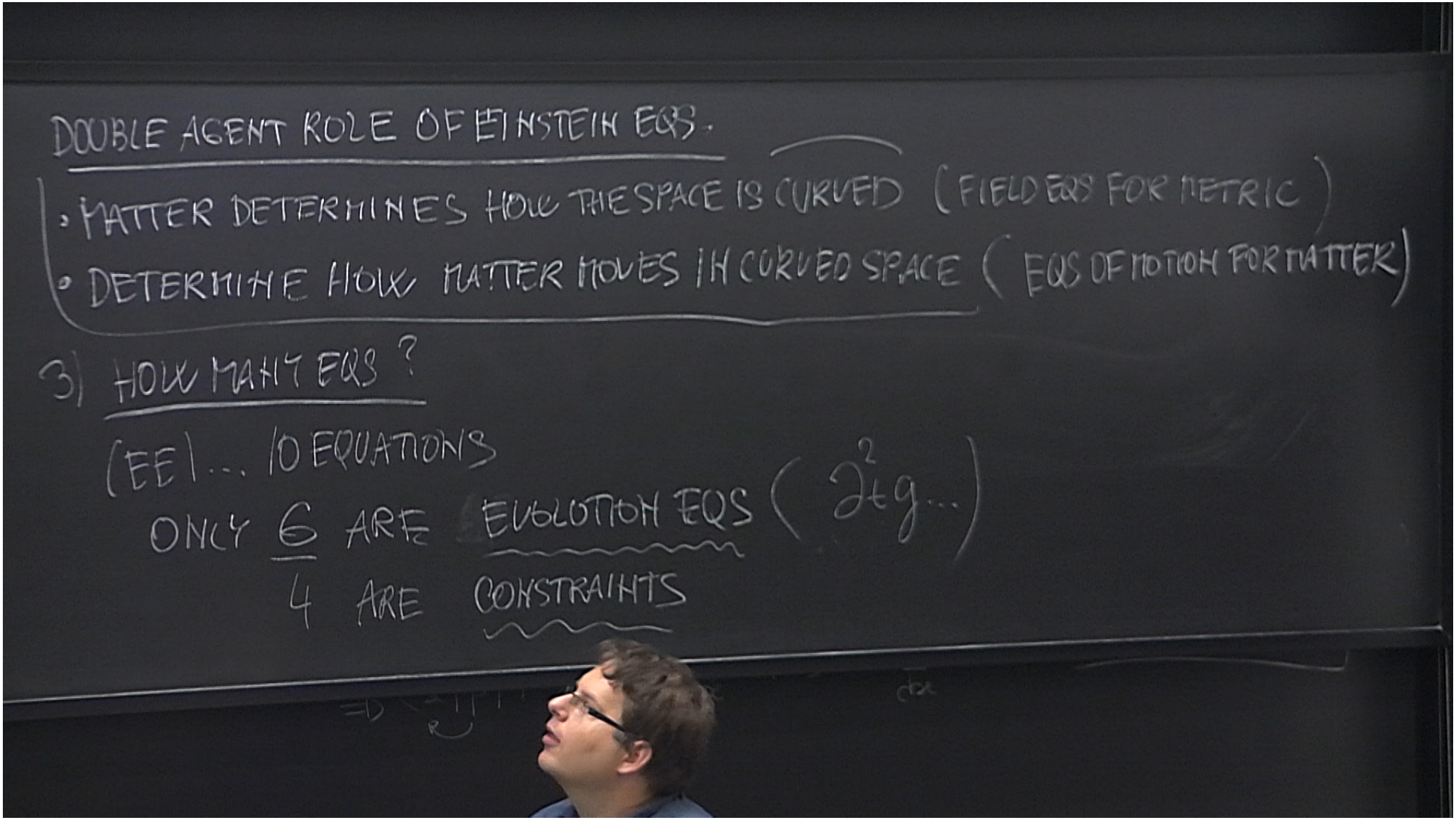
APPLY $D_\mu \left(G^{\mu\nu} = 8\pi G T^{\mu\nu} \right)$

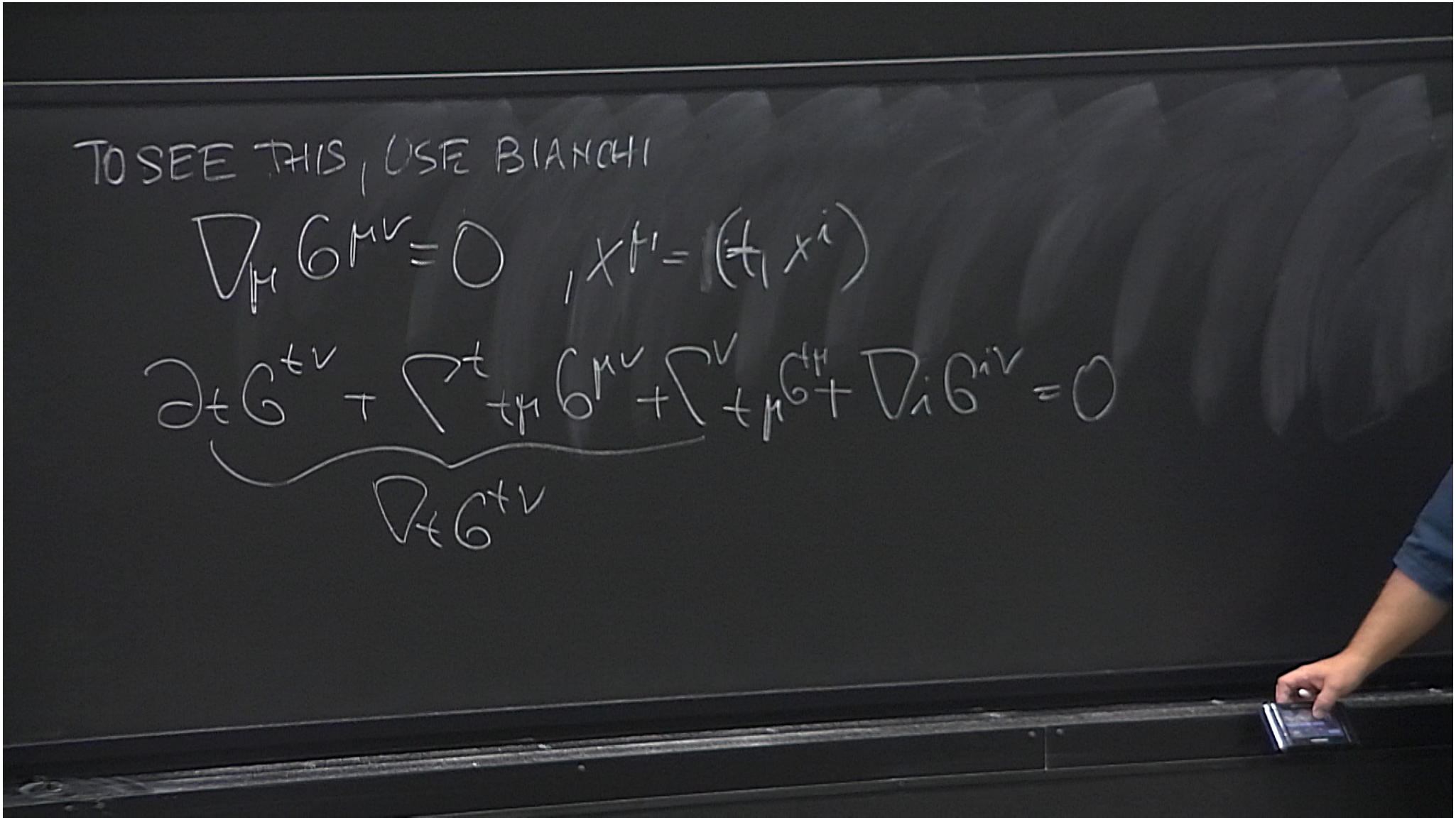
$$0 = D_\mu T^{\mu\nu}$$

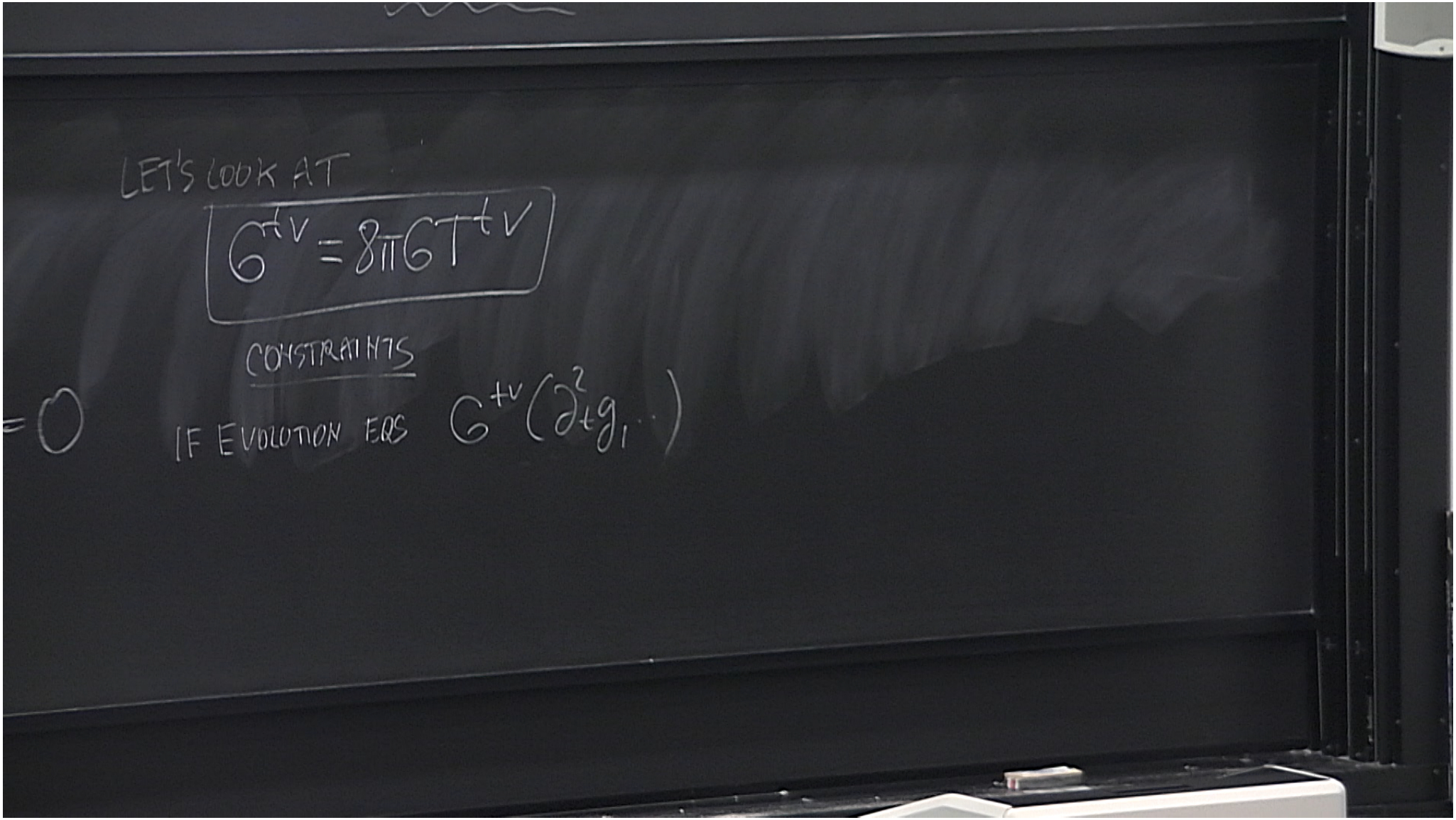
$T^{\mu\nu}$ CONSERVATION

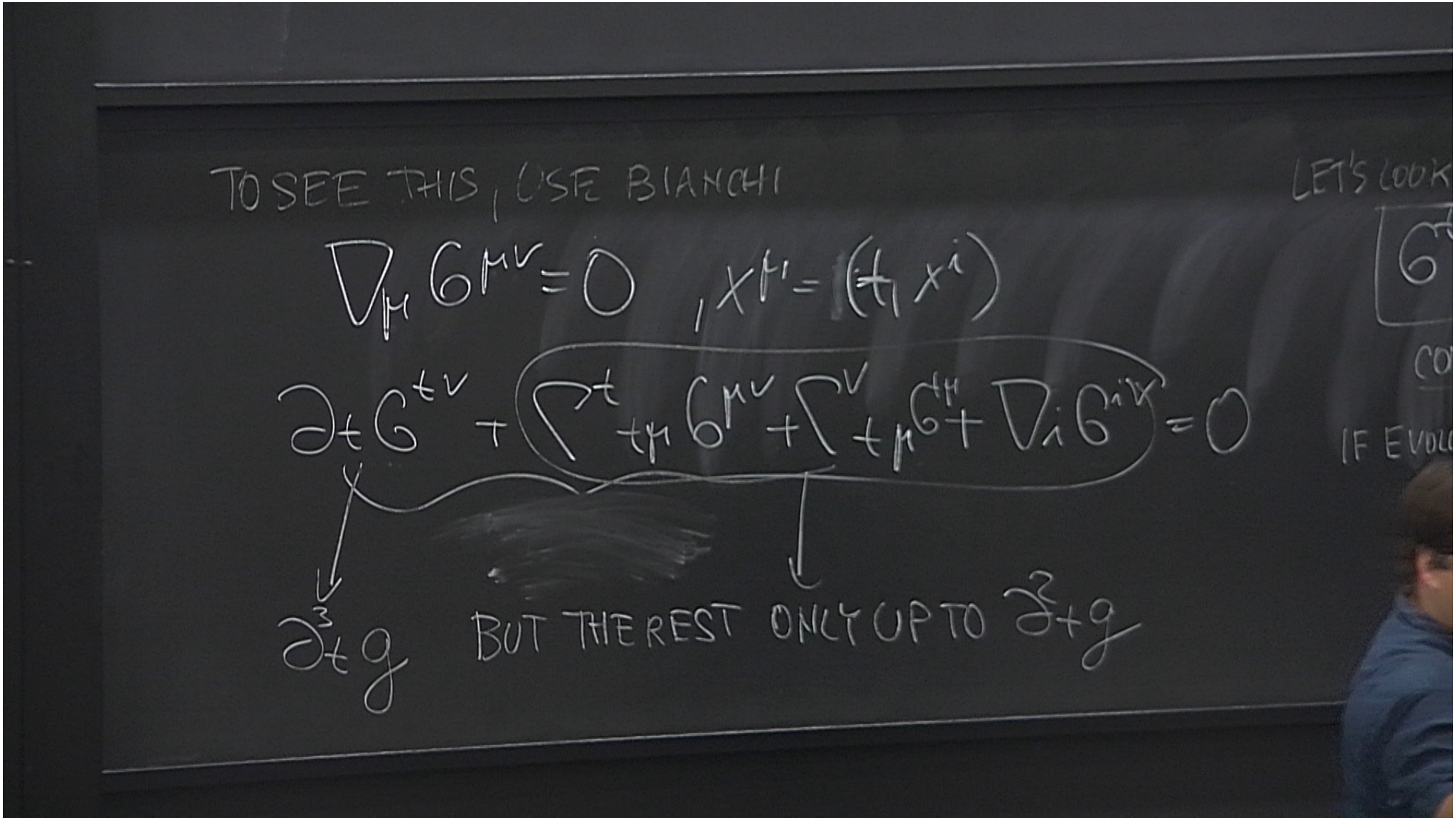
$\simeq$ EQ. OF MOTION FOR MATTER

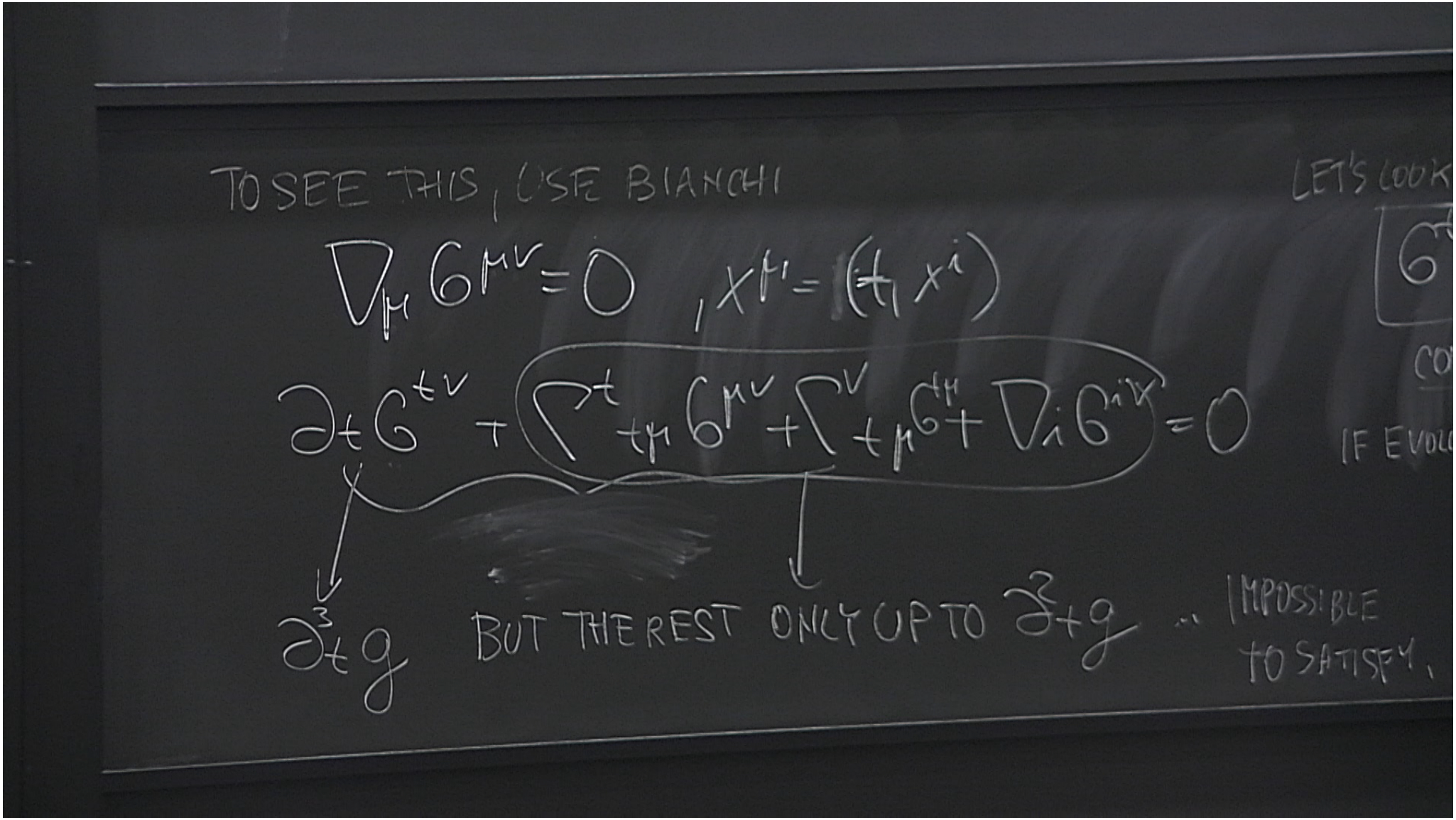

DOUBLE AGENT ROLE OF EINSTEIN EQS.
• MATTER DETERMINES HOW THE SPACE IS CURVED (FIELD EQS FOR METRIC)
• DETERMINE HOW MATTER MOVES IN CURVED SPACE (EQS OF MOTION FOR MATTER)

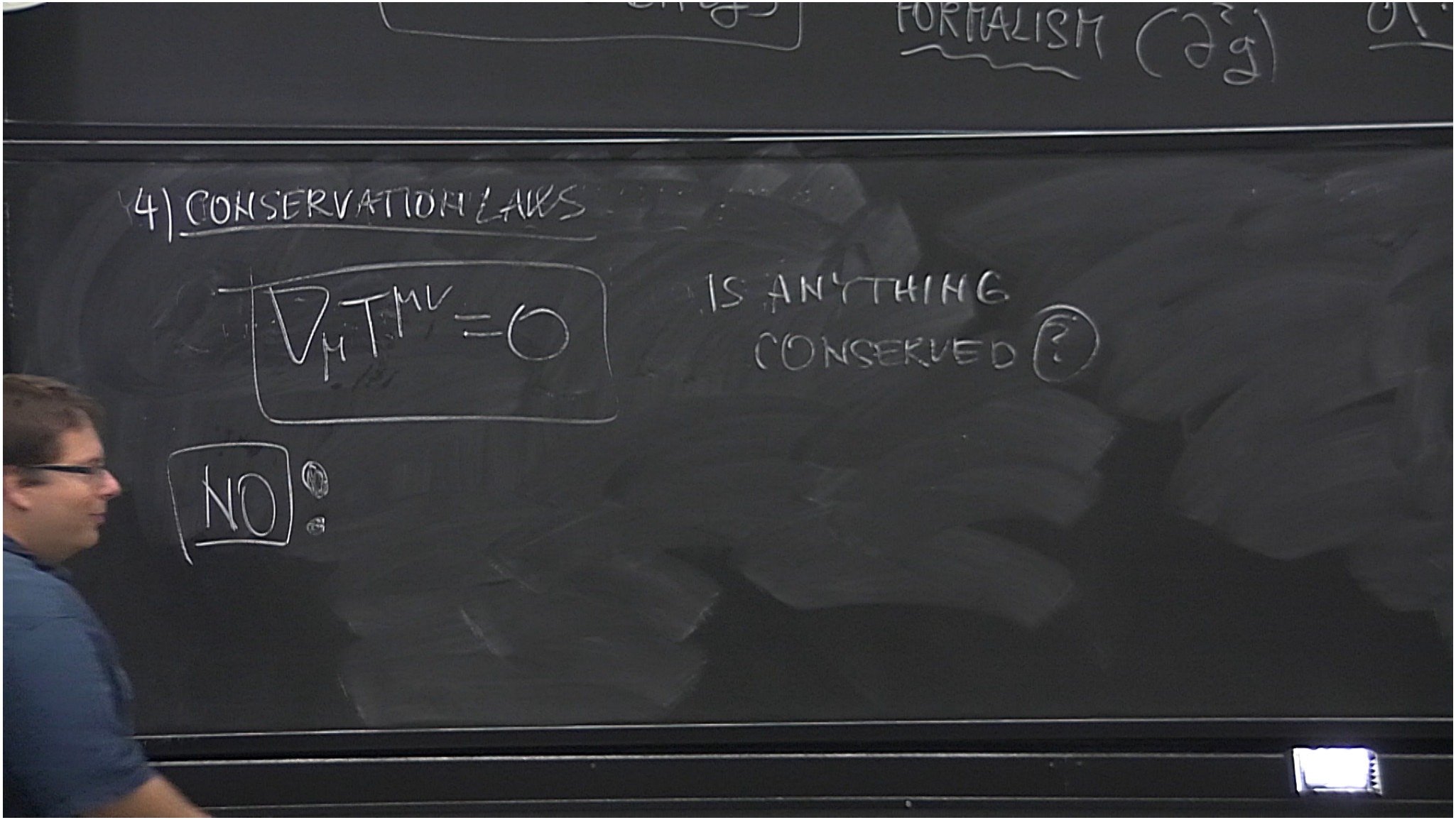

DOUBLE AGENT ROLE OF EINSTEIN EQS.
• MATTER DETERMINES HOW THE SPACE IS CURVED (FIELD EQS FOR METRIC)
• DETERMINE HOW MATTER MOVES IN CURVED SPACE (EQS OF MOTION FOR MATTER)
3) HOW MANY EQS ?
(EE) ... 10 EQUATIONS
ONLY 6 ARE EVOLUTION EQS ($\partial_t^2 g \ldots$)
4 ARE CONSTRAINTS


TO SEE THIS, USE BIANCHI
$\nabla_\mu G^{\mu\nu} = 0$, $x^\mu = (t, x^i)$
$\partial_t G^{tv} + \Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu} = 0$
$\nabla_t G^{tv}$


LET'S LOOK AT
$G^{\mu\nu} = 8\pi G T^{\mu\nu}$
CONSTRAINTS
IF EVOLUTION EQS $G^{\mu\nu}(\partial^2 + g_1 \ldots)$
$= 0$


TO SEE THIS, USE BIANCHI
$\nabla_\mu G^{\mu\nu} = 0$, $x^\mu = (t, x^i)$
$\partial_t G^{t\nu} + \Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu} = 0$
$\partial_t^3 g$
BUT THE REST ONLY UP TO $\partial_t^2 g$
LET'S LOOK
IF EVOL


TO SEE THIS, USE BIANCHI
$\nabla_\mu G^{\mu\nu} = 0$, $x^\mu = (t, x^i)$
$\partial_t G^{tv} + \left(\Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu}\right) = 0$
$\partial_t^3 g$
BUT THE REST ONLY UP TO $\partial_t^2 g$
.. IMPOSSIBLE TO SATISFY.
LET'S LOOK
IF EVOL


FORMALISM
4) CONSERVATION LAWS
$\nabla_\mu T^{\mu\nu} = 0$
IS ANYTHING CONSERVED ?
NO !

WE UNDERSTAND IT AS

4) CONSERVATION LAWS

$\nabla_\mu T^{\mu\nu} = 0$

IS ANYTHING CONSERVED ?

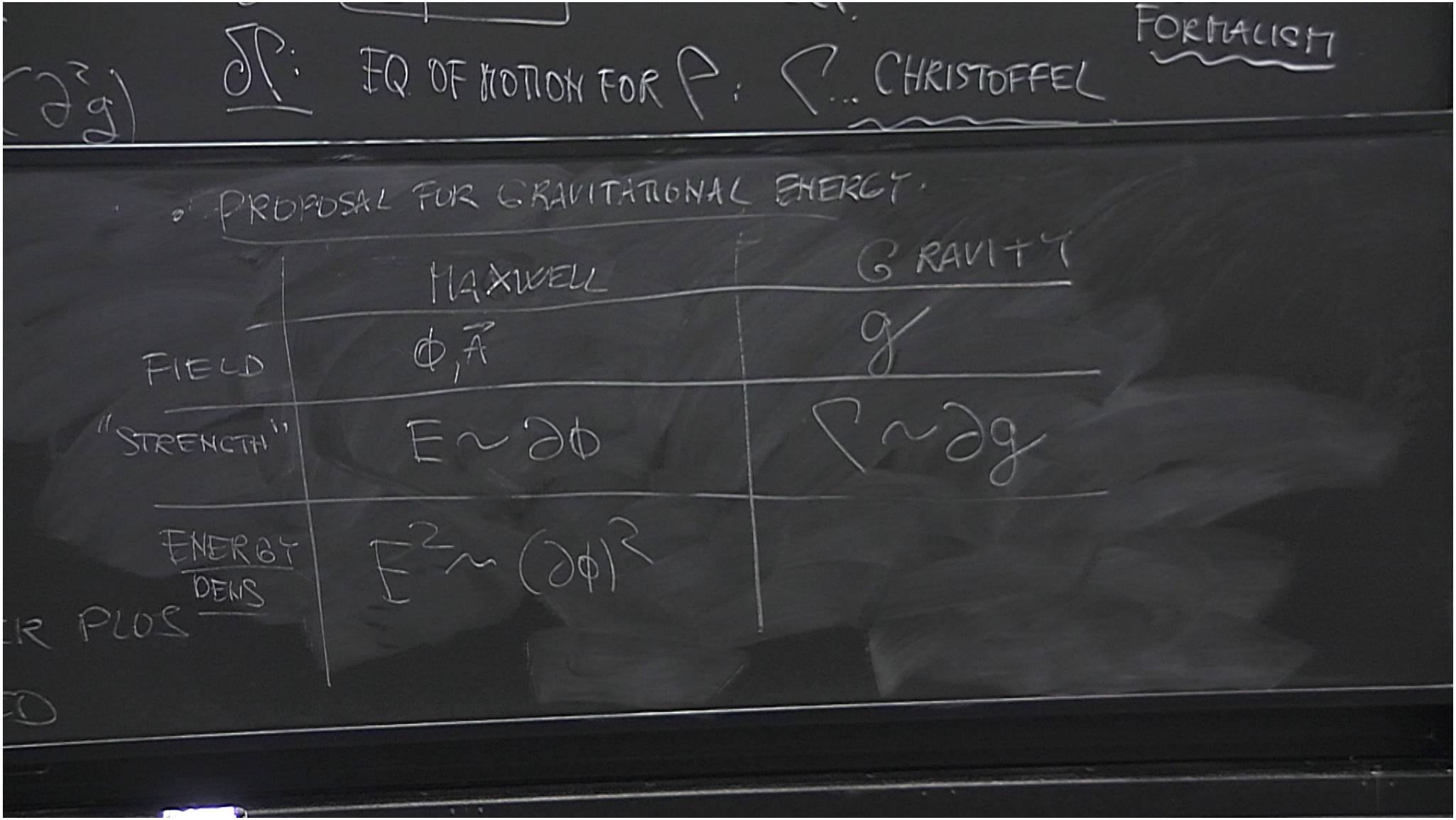
NO

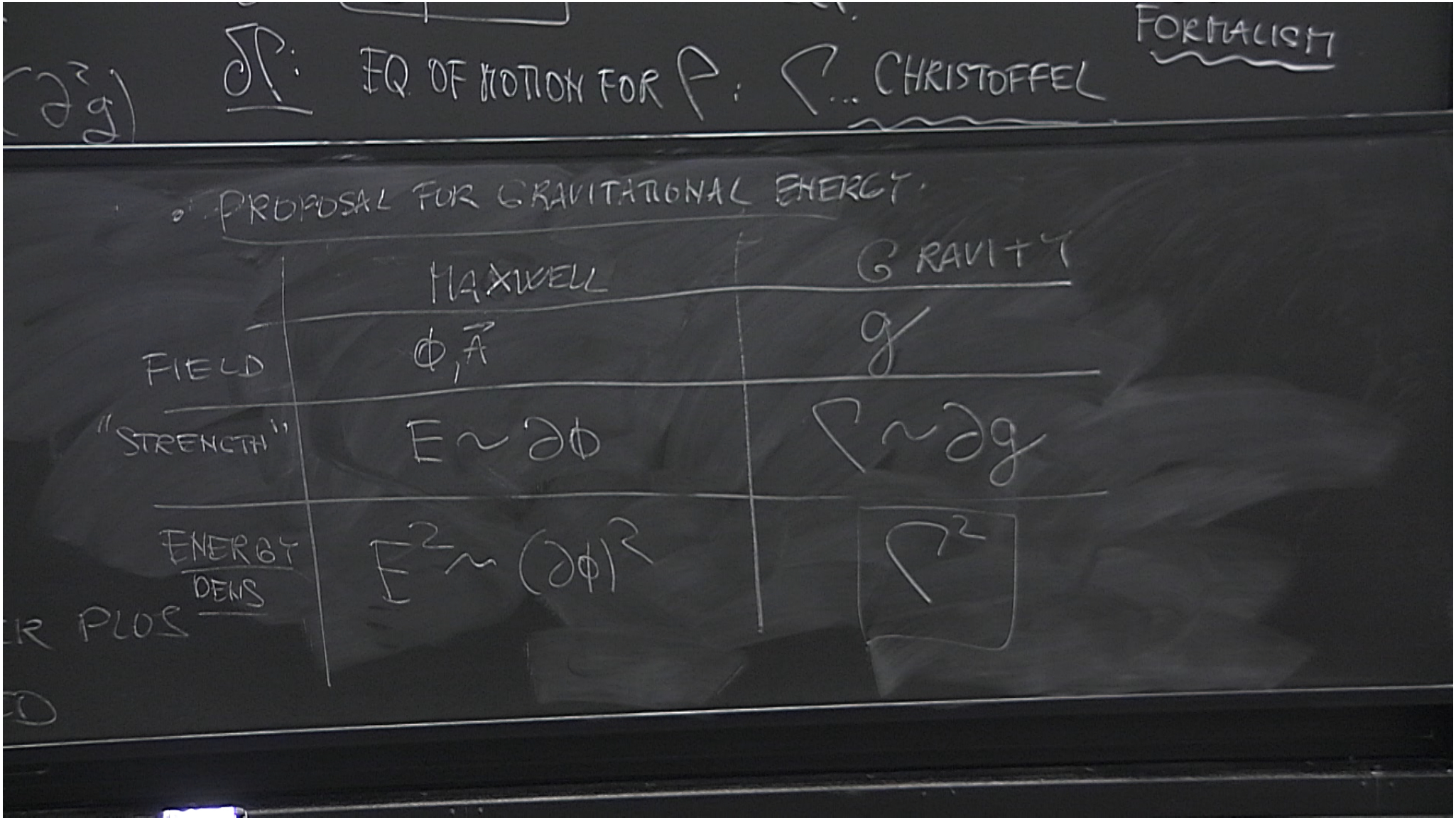
STOKES THEOREM DOES NOT EXIST FO $T^{\mu\nu}$ (SYMMETRIC)

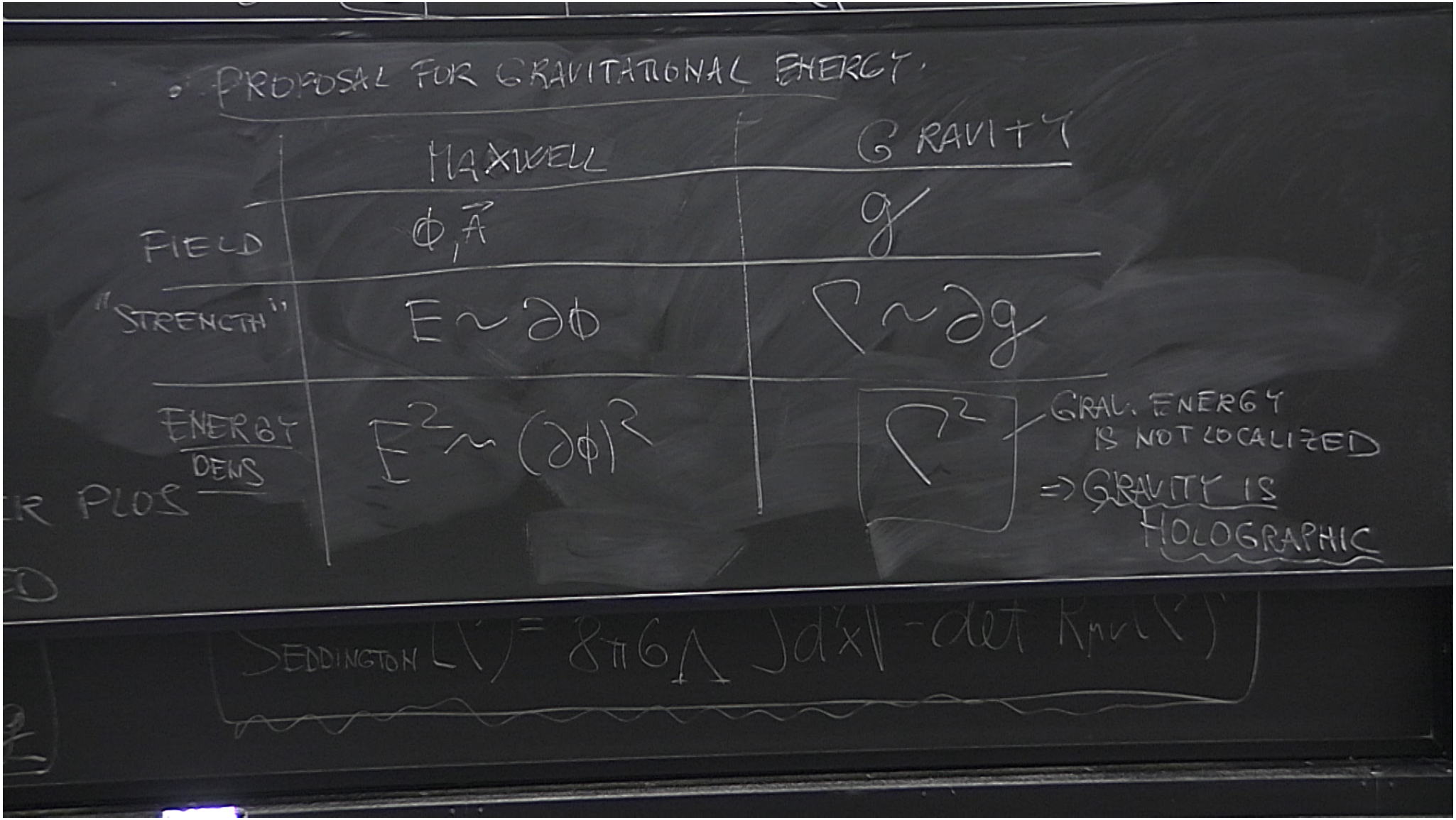
ONLY THE TOTAL ENERGY OF MATTER PLUS GRAV. FIELD CAN BE CONSERVED

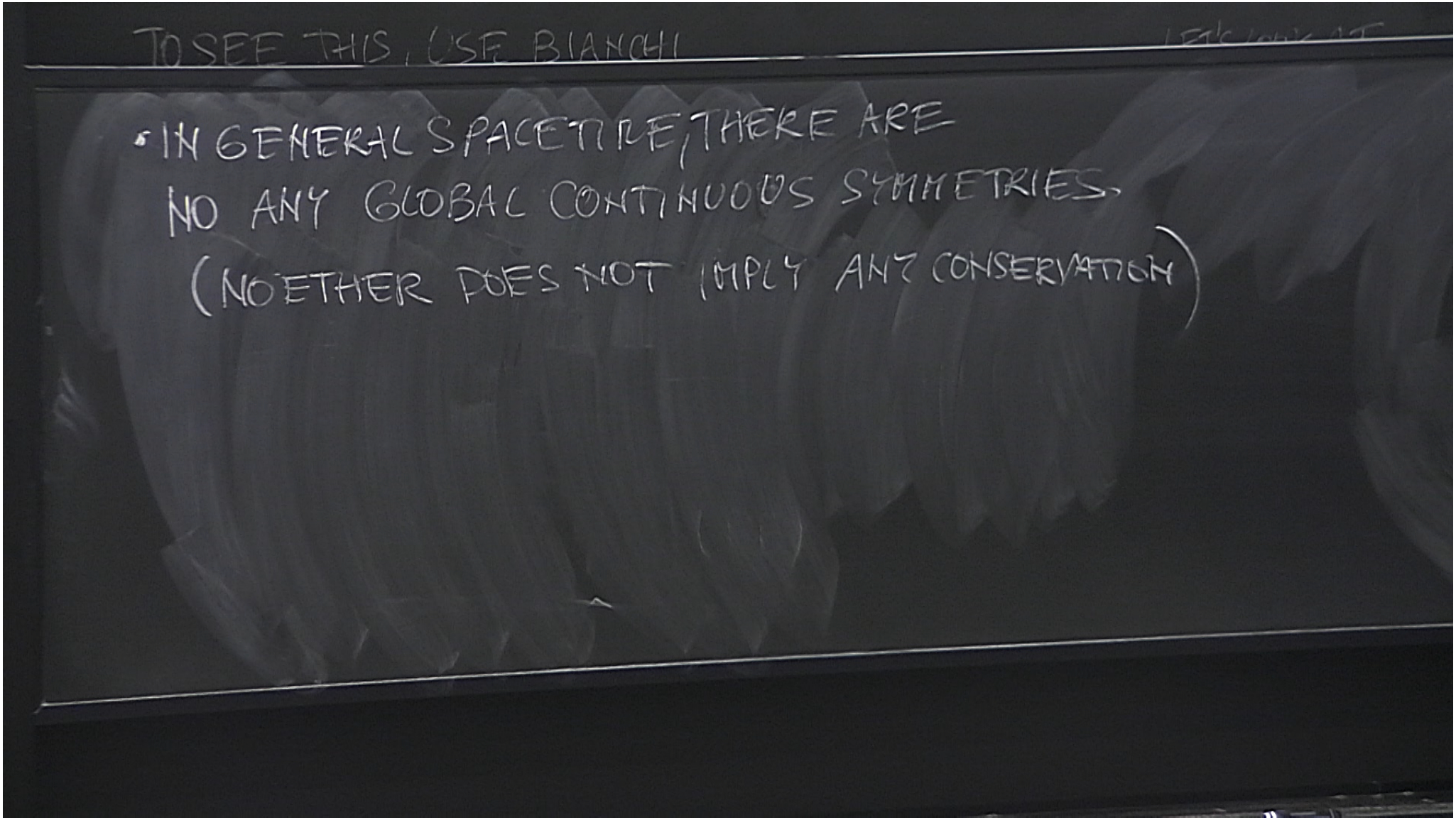
THE ONLY POSSIBILITY (UP TO TOP. TERMS)

WHICH YIELDS 2ND-ORDER EOS FOR


$(\partial^2 g)$
$\partial\Gamma$: EQ OF MOTION FOR Γ: Γ... CHRISTOFFEL
FORMALISM
PROPOSAL FOR GRAVITATIONAL ENERGY.
MAXWELL
GRAVITY
FIELD
$\phi, \vec{A}$
g
"STRENGTH"
$E \sim \partial\phi$
$\Gamma \sim \partial g$
ENERGY DENS
$E^2 \sim (\partial\phi)^2$
R PLUS
D


FORMALISM
EQ OF MOTION FOR Γ: Γ... CHRISTOFFEL
PROPOSAL FOR GRAVITATIONAL ENERGY.
MAXWELL
GRAVITY
FIELD
$\phi, \vec{A}$
g
"STRENGTH"
$E \sim \partial\phi$
$\Gamma \sim \partial g$
ENERGY DENS
$E^2 \sim (\partial\phi)^2$
Γ^2


PROPOSAL FOR GRAVITATIONAL ENERGY.
MAXWELL
GRAVITY
FIELD
$\phi, \vec{A}$
g
"STRENGTH"
$E \sim \partial\phi$
$\Gamma \sim \partial g$
ENERGY DENS
$E^2 \sim (\partial\phi)^2$
Γ^2
GRAV. ENERGY IS NOT LOCALIZED
=> GRAVITY IS HOLOGRAPHIC
SEDDINGTON $L(V) = 8\pi G \Lambda \int d^3x \sqrt{-\det K_{\mu\nu}(S)}$

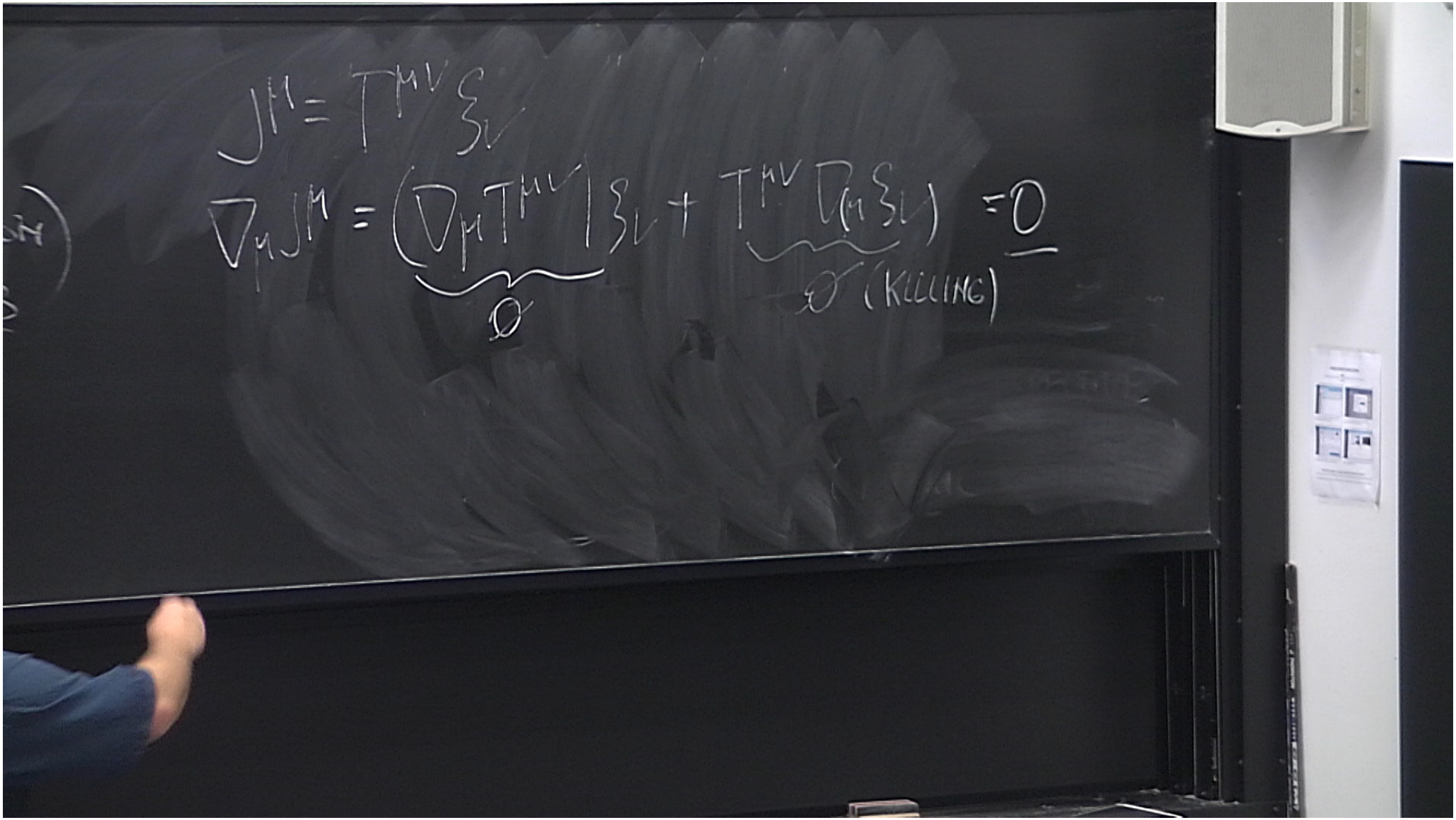

TO SEE THIS, USE BIANCHI
• IN GENERAL SPACETIME, THERE ARE
NO ANY GLOBAL CONTINUOUS SYMMETRIES.
(NOETHER DOES NOT IMPLY ANY CONSERVATION)

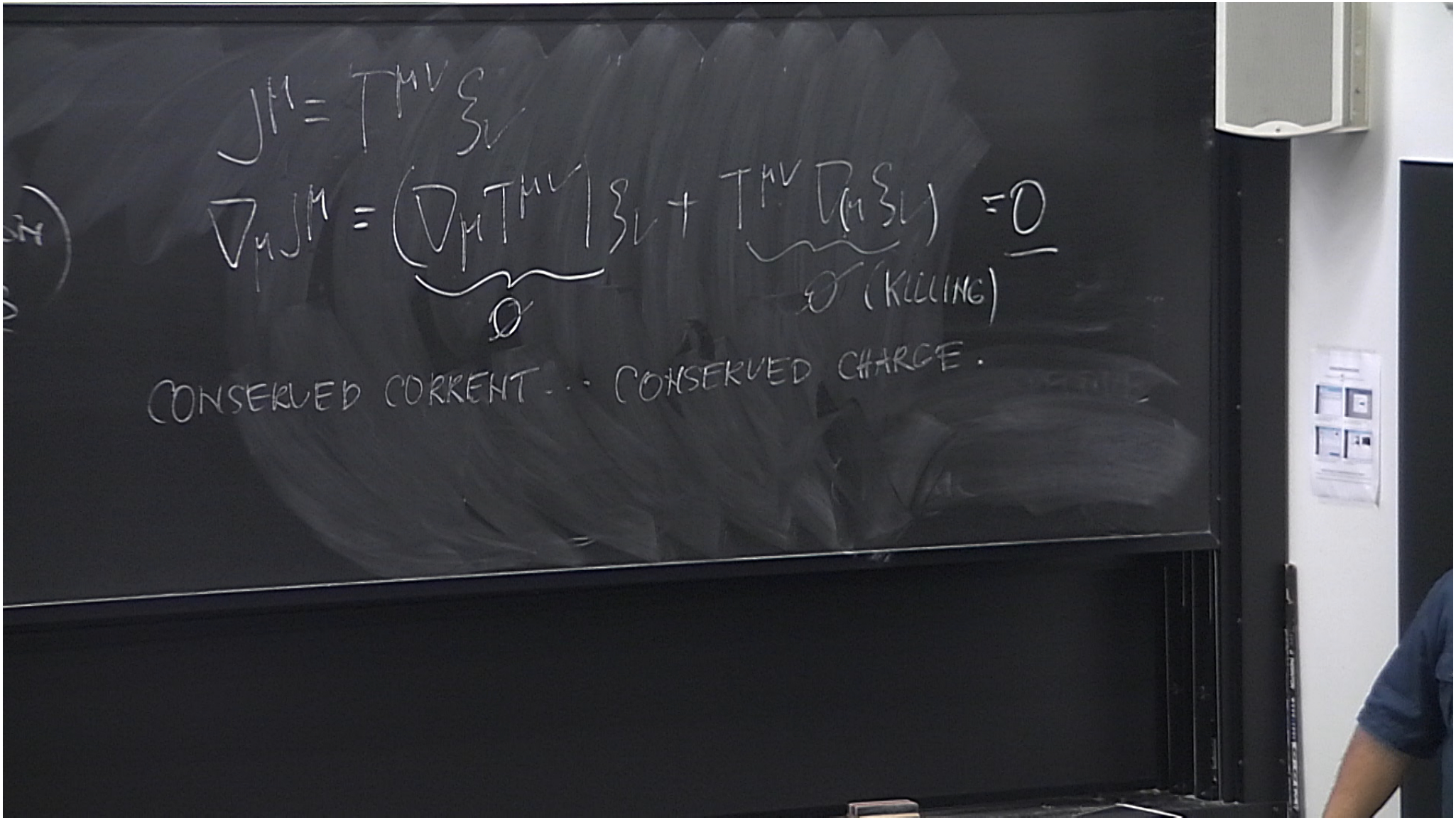
- IN GENERAL SPACETIME, THERE ARE NO ANY GLOBAL CONTINUOUS SYMMETRIES.
  (NOETHER DOES NOT IMPLY ANY CONSERVATION)
  SYMMETRIES CORRESPOND TO KILLING VECTORS

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$

$J^\mu =$

$\nabla_\mu J^$


$J^\mu = T^{\mu\nu}\xi_\nu$
$\nabla_\mu J^\mu = (\nabla_\mu T^{\mu\nu})\xi_\nu + T^{\mu\nu}\nabla_{(\mu}\xi_{\nu)} = 0$
$= 0$
(KILLING)


$J^\mu = T^{\mu\nu}\xi_\nu$
$\nabla_\mu J^\mu = (\nabla_\mu T^{\mu\nu})\xi_\nu + T^{\mu\nu}\nabla_{(\mu}\xi_{\nu)} = 0$
$= 0$
(KILLING)
CONSERVED CURRENT ⋯ CONSERVED CHARGE.