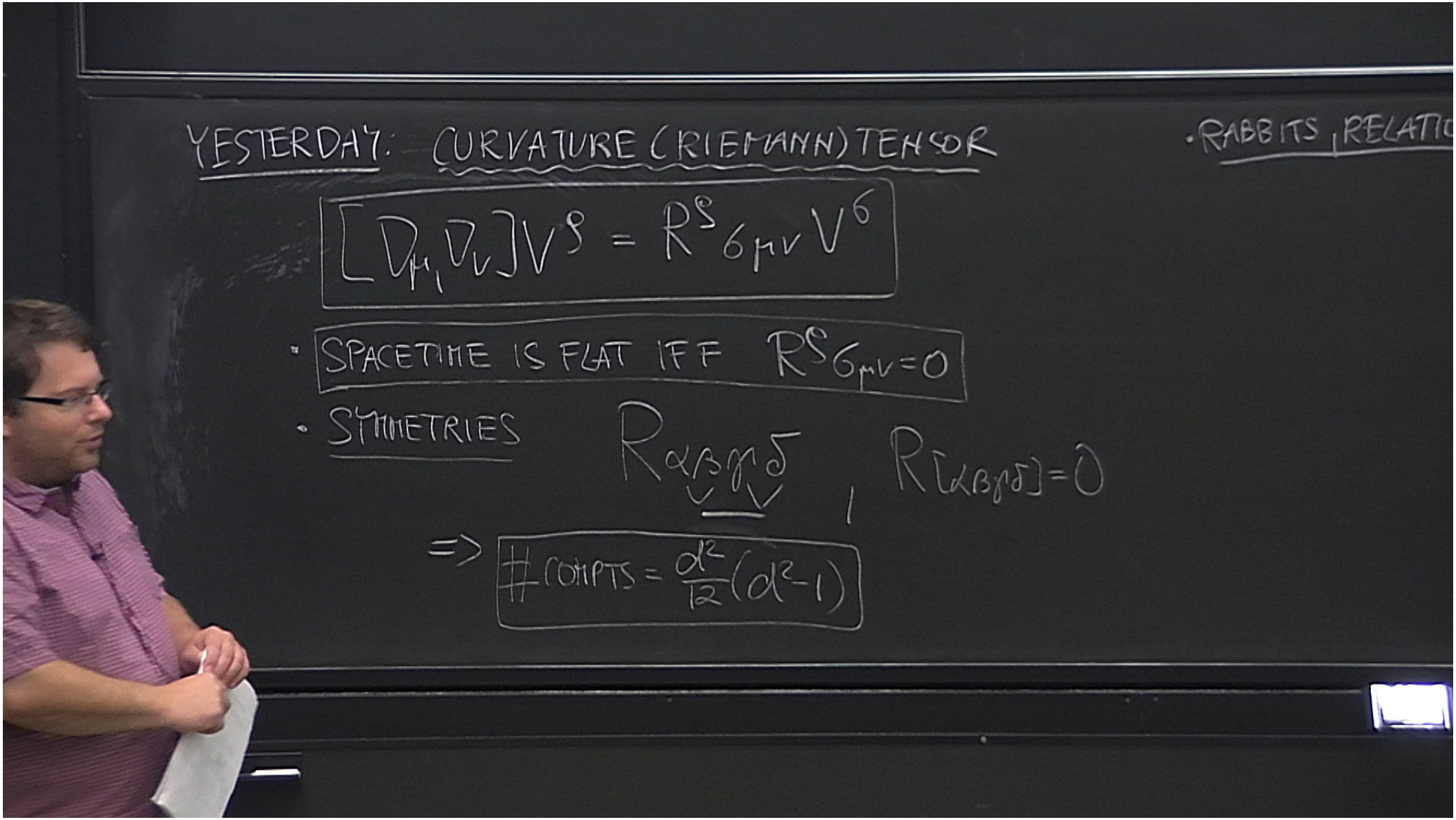
Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 8

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

Date: September 11, 2019 - 10:45 AM

URL: http://pirsa.org/19090035


YESTERDAY: CURVATURE (RIEMANN) TENSOR
$[D_\mu, D_\nu]V^\sigma = R^\sigma{}_{\rho\mu\nu}V^\rho$
• SPACETIME IS FLAT IFF $R^\sigma{}_{\rho\mu\nu}=0$
• SYMMETRIES $R_{\alpha\beta\gamma\delta}$, $R_{[\alpha\beta\gamma\delta]}=0$
$\Rightarrow$ #COMPTS $= \frac{d^2}{12}(d^2-1)$

- RABBITS, RELATIONS, AND KINS OF RIEMANN FAMILY
  - RICCI TENSOR $R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$ ... $\frac{d(d+1)}{2}$ COMPTS.

OMPTS.

- RABBITS, RELATIONS, AND KINS OF RIEMANN FAMILY

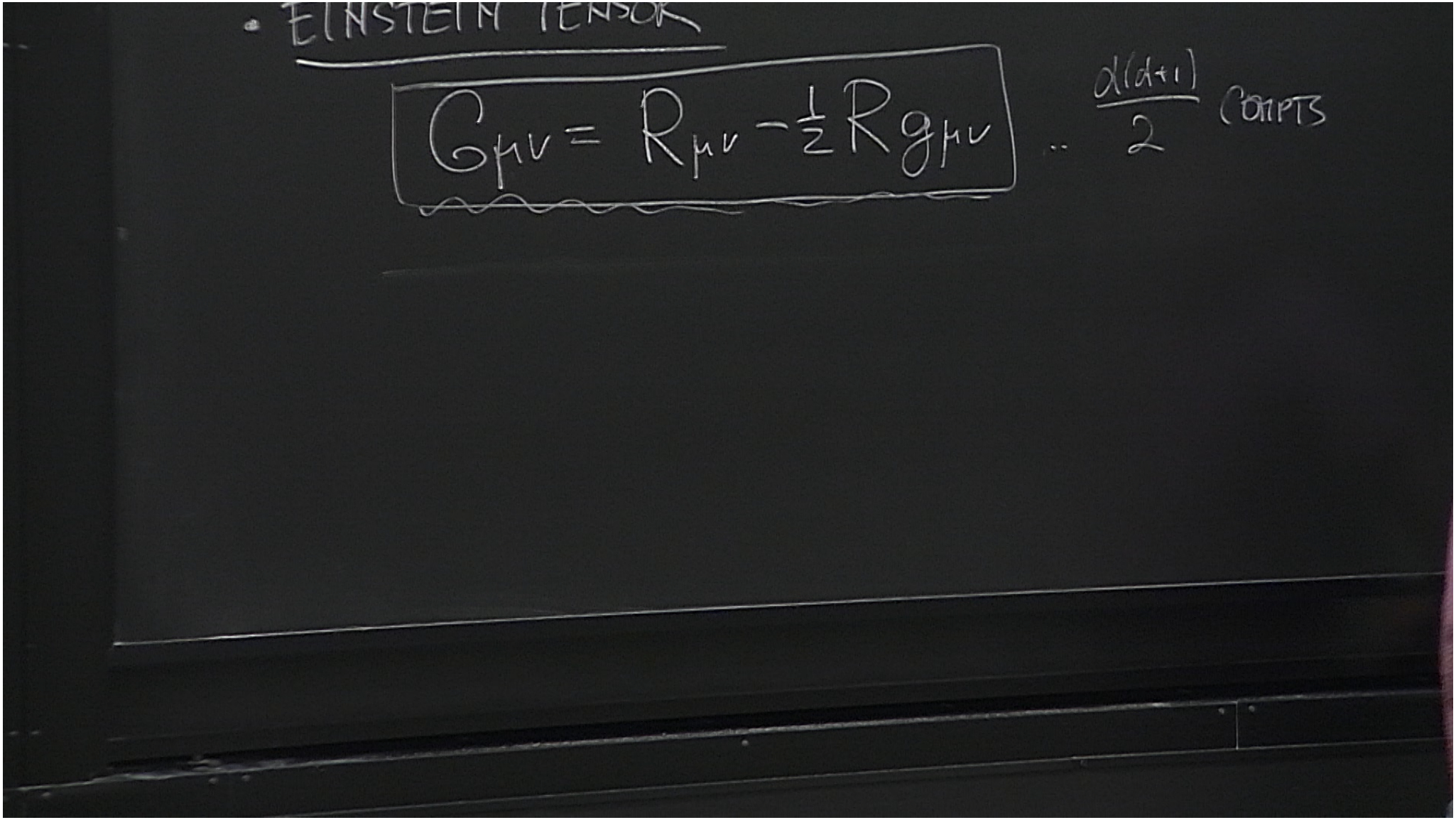
- RICCI TENSOR $\boxed{R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}}$ ... $\frac{d(d+1)}{2}$ COMPTS.

NOTE THAT SINCE $\text{RIEM} \sim \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$ ... NO EXPLICIT DEPENDANCE ON g

$\Rightarrow R_{\mu\nu}(\Gamma, g)$

- RICCI SCALAR $\boxed{R = R_{\mu\nu} g^{\mu\nu}}$ $R(\Gamma, g)$ ... 1 COMPT.

PTS.


• EINSTEIN TENSOR
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$
.. $\frac{d(d+1)}{2}$ COMPTS

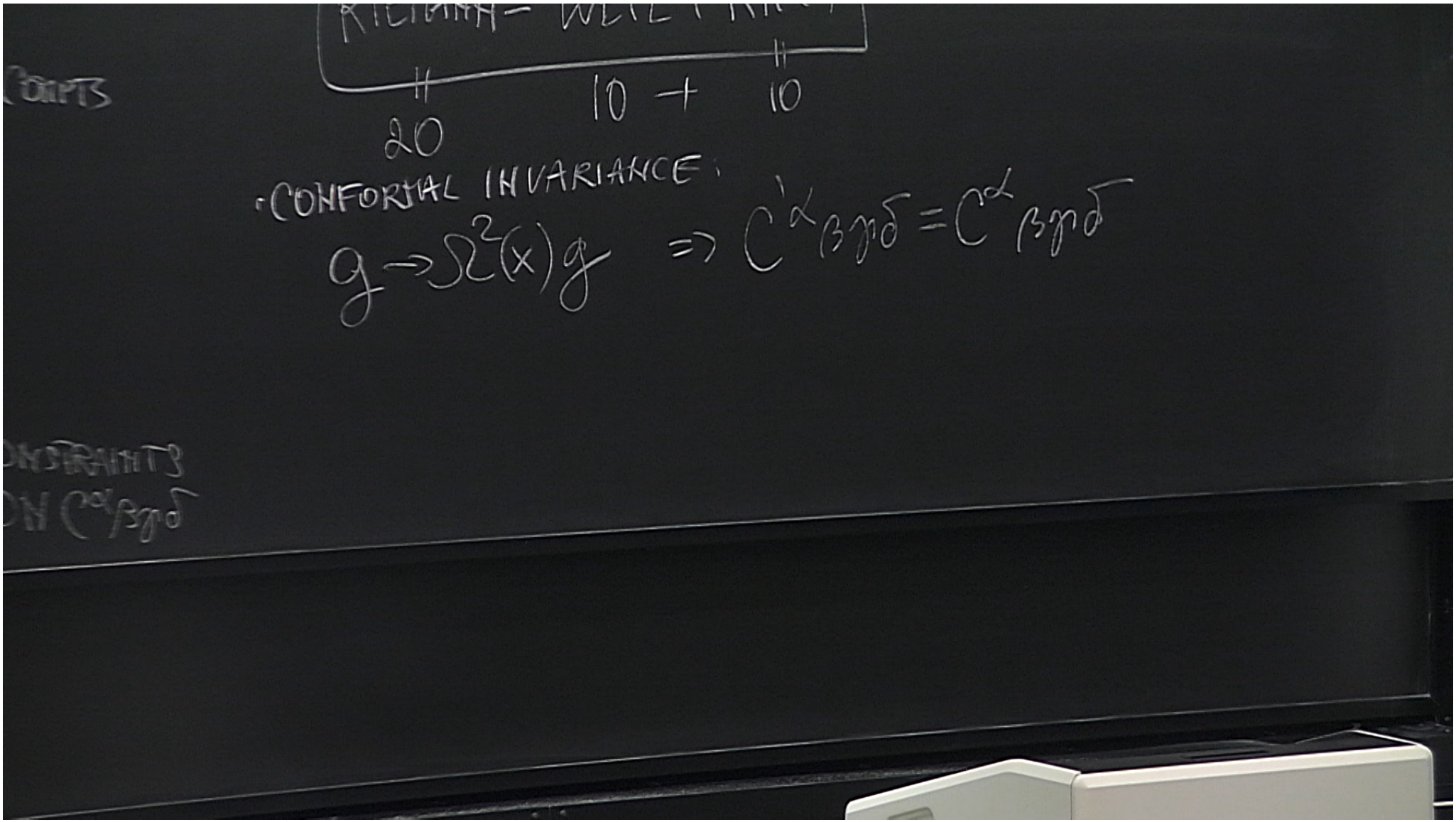
- EINSTEIN TENSOR

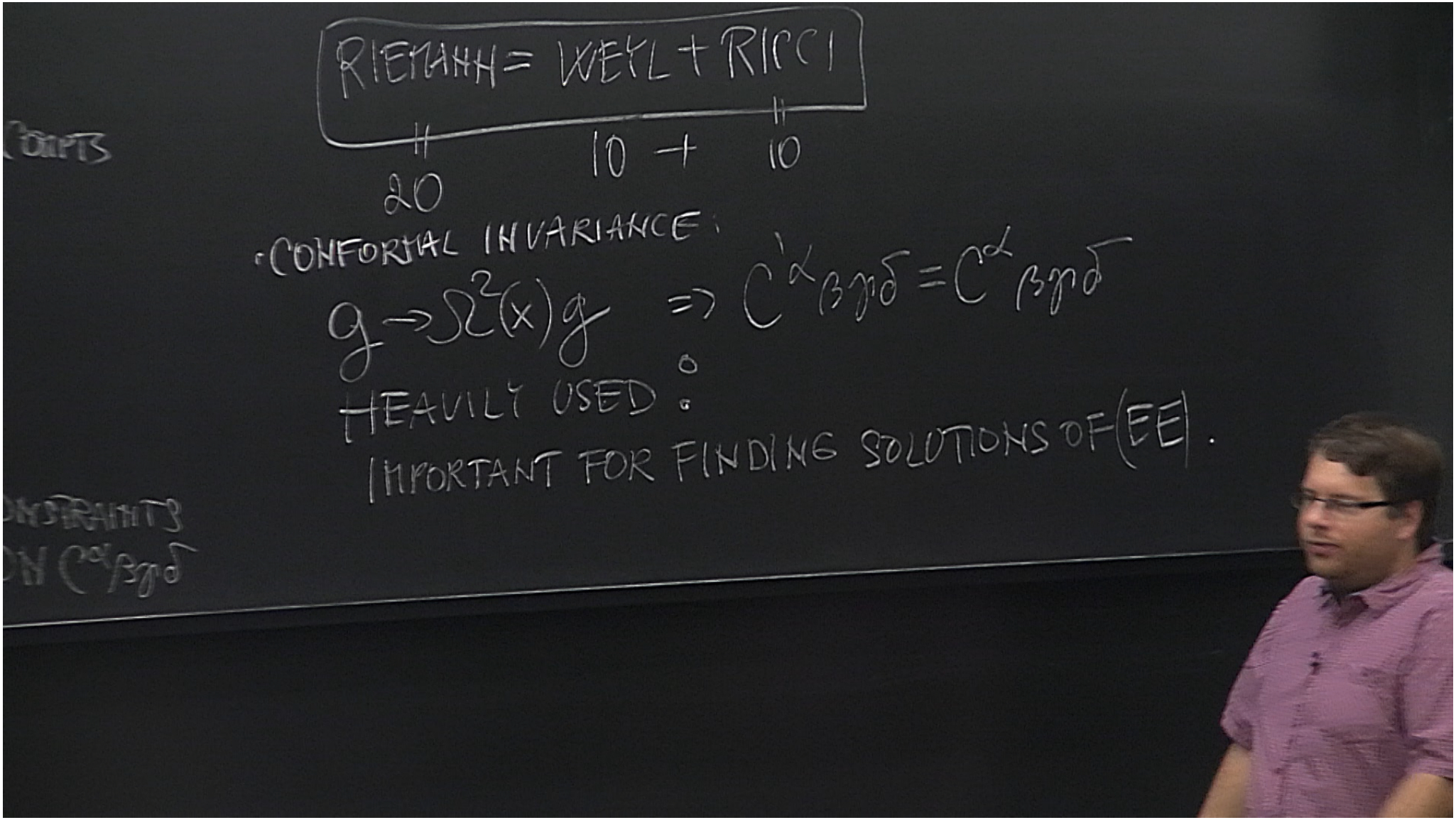
$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad \cdots \quad \frac{d(d+1)}{2} \text{ COMPTS}$$

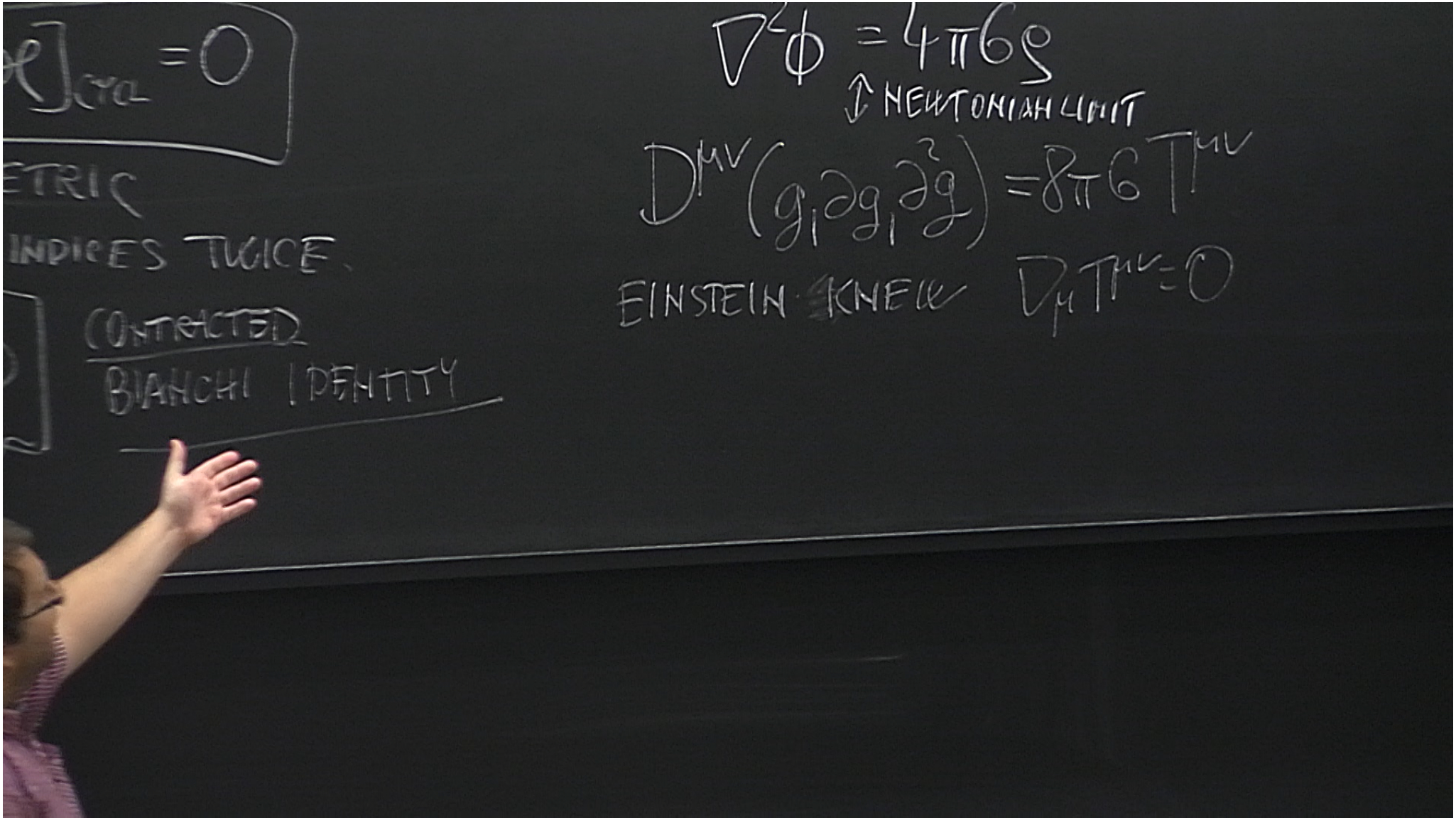
- WEYL TENSOR $C^{\alpha}{}_{\beta\gamma\delta}$

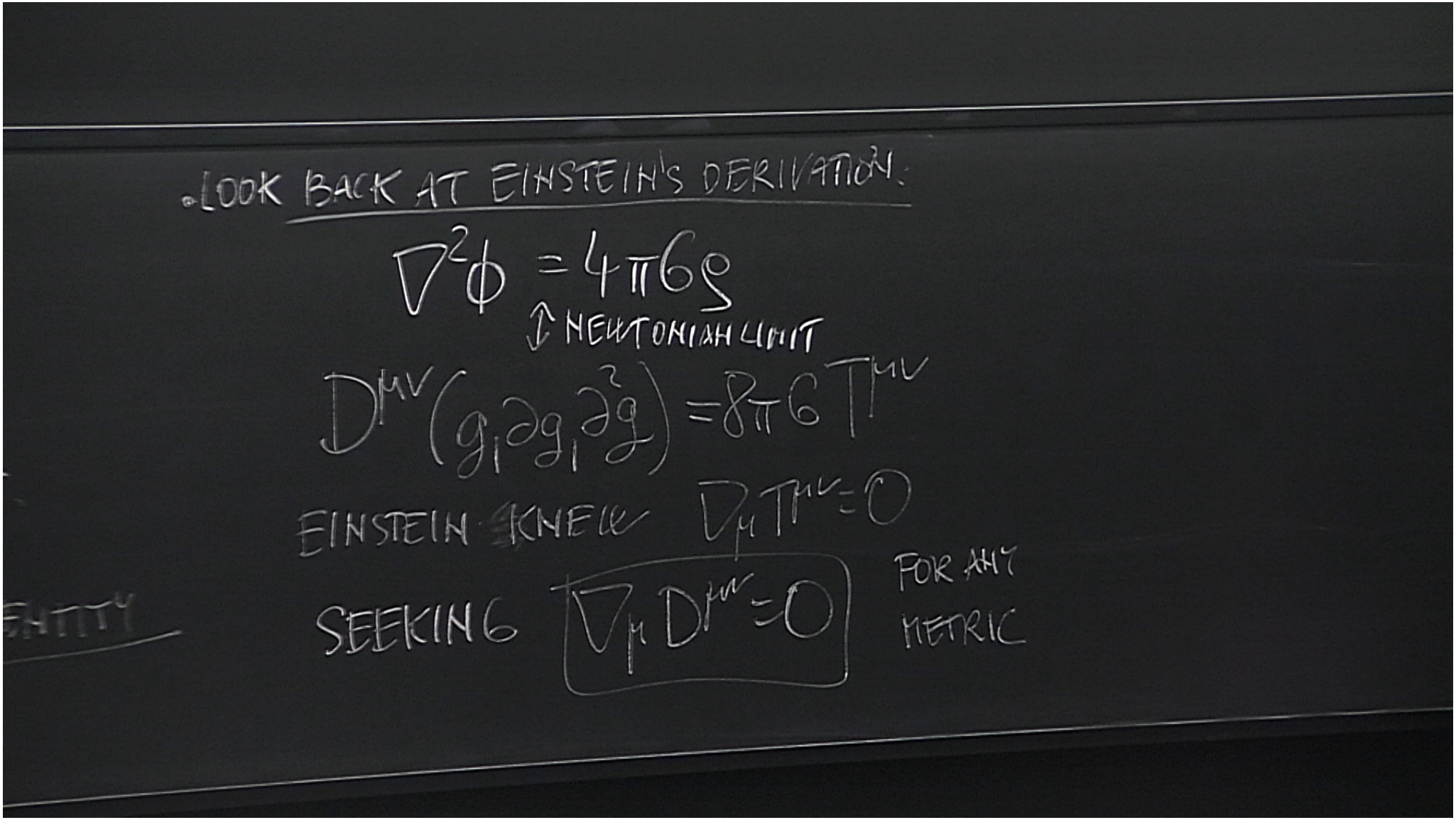
SAME SYMMETRIES AS RIEMANN

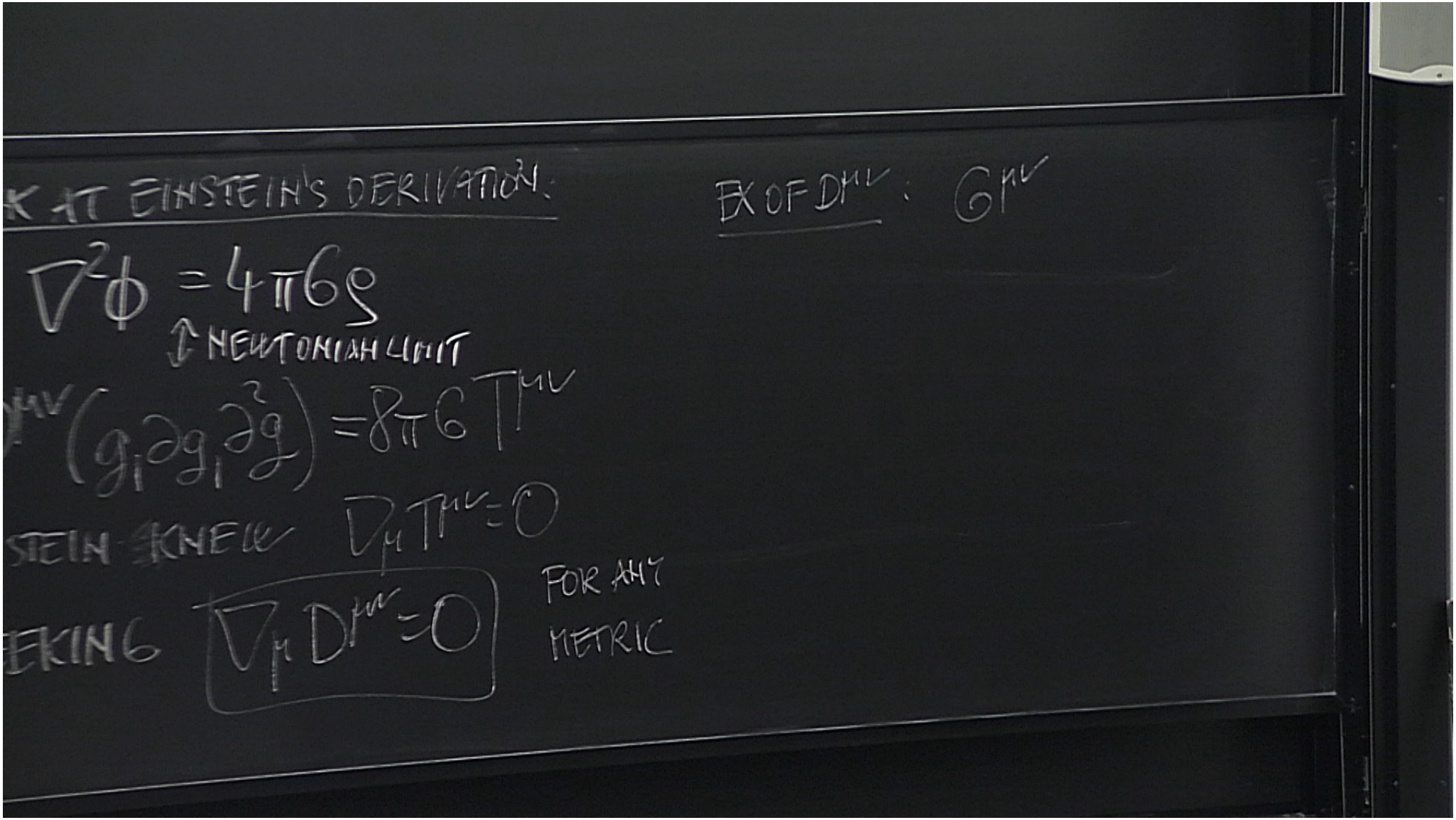
$$C_{\mu\nu} = C^{\alpha}{}_{\mu\alpha\nu} = 0 \quad \cdots \quad \frac{d(d+1)}{2} \text{ CONSTRAINTS ON } C^{\alpha}{}_{\beta\gamma\delta}$$

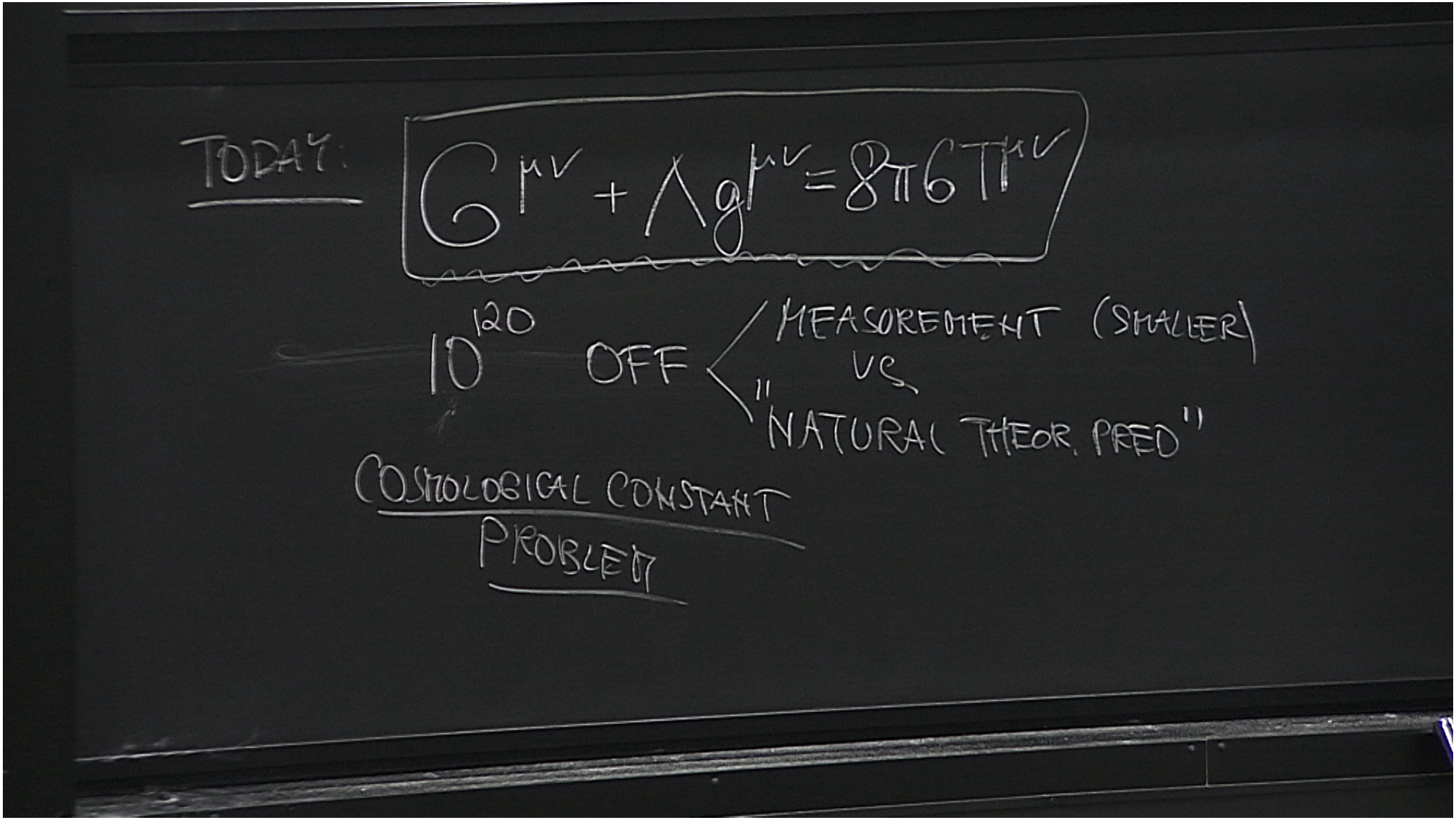

10 + 10
20
• CONFORMAL INVARIANCE:
$g \to \Omega^2(x) g \Rightarrow C'^{\alpha}{}_{\beta\gamma\delta} = C^{\alpha}{}_{\beta\gamma\delta}$

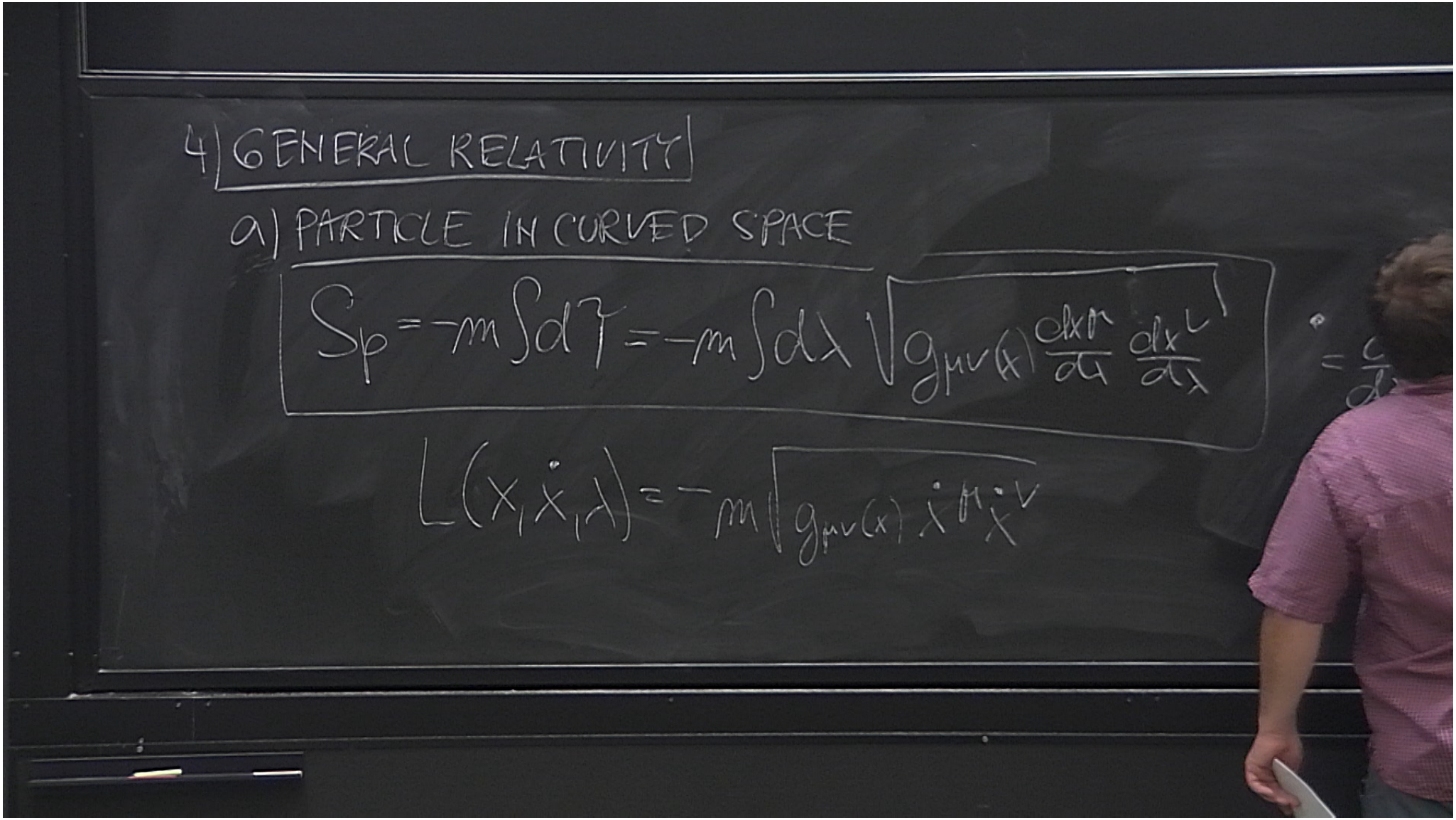

RIEMANN = WEYL + RICCI
20
10 + 10
• CONFORMAL INVARIANCE:
$g \to \Omega^2(x) g \Rightarrow C^{\alpha}{}_{\beta\gamma\delta} = C^{\alpha}{}_{\beta\gamma\delta}$
HEAVILY USED.
IMPORTANT FOR FINDING SOLUTIONS OF (EE).


$\nabla^2\phi = 4\pi G\rho$
NEWTONIAN LIMIT
$D^{\mu\nu}(g, \partial g, \partial^2 g) = 8\pi G T^{\mu\nu}$
EINSTEIN KNEW $D_\mu T^{\mu\nu} = 0$
$]_{CYCL} = 0$
ETRIC
INDICES TWICE.
CONTRACTED
BIANCHI IDENTITY


LOOK BACK AT EINSTEIN'S DERIVATION:
$\nabla^2\phi = 4\pi G\rho$
NEWTONIAN LIMIT
$D^{\mu\nu}(g, \partial g, \partial^2 g) = 8\pi G T^{\mu\nu}$
EINSTEIN KNEW $\nabla_\mu T^{\mu\nu} = 0$
SEEKING $\nabla_\mu D^{\mu\nu} = 0$
FOR ANY METRIC


K AT EINSTEIN'S DERIVATION:
$\nabla^2\phi = 4\pi G\rho$
NEWTONIAN LIMIT
$D^{\mu\nu}(g, \partial g, \partial^2 g) = 8\pi G T^{\mu\nu}$
STEIN-KNELL $\nabla_\mu T^{\mu\nu} = 0$
EKING $\nabla_\mu D^{\mu\nu} = 0$
FOR ANY METRIC
EX OF $D^{\mu\nu}$: $G^{\mu\nu}$


TODAY:
$G^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu}$
10^{120} OFF
MEASUREMENT (SMALLER)
VS.
"NATURAL THEOR. PRED"
COSMOLOGICAL CONSTANT
PROBLEM


4| GENERAL RELATIVITY
a) PARTICLE IN CURVED SPACE
$S_p = -m\int d\tau = -m\int d\lambda \sqrt{g_{\mu\nu}(x)\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$
$L(x,\dot{x},\lambda) = -m\sqrt{g_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu}$

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{x}^\alpha}\right) - \frac{\partial L}{\partial x^\alpha} = 0$$

$$\frac{\partial L}{\partial x^\alpha} = \frac{m\, g_{\mu\nu,\alpha}\, \dot{x}^\mu \dot{x}^\nu}{2\sqrt{\quad}}$$

$$\frac{\partial L}{\partial \dot{x}^\alpha} = \frac{m\, g_{\mu\alpha}\, \dot{x}^\mu}{\sqrt{\quad}}$$

$$\frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}^\alpha}\right) - \frac{\partial L}{\partial x^\alpha} = 0$$

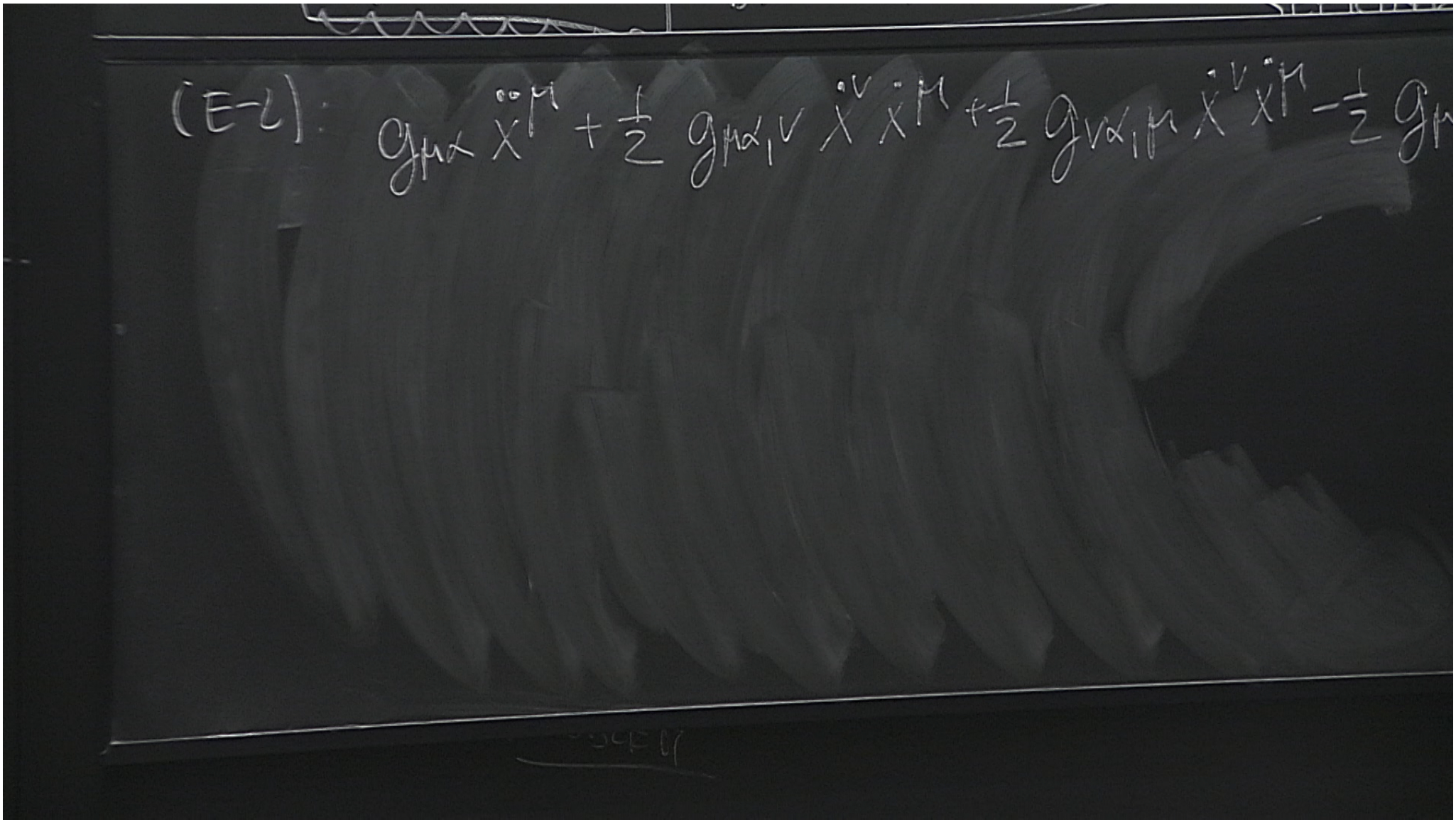
$$\frac{\partial L}{\partial x^\alpha} = \frac{m\, g_{\mu\nu,\alpha}\, \dot{x}^\mu \dot{x}^\nu}{2\sqrt{\;}}$$

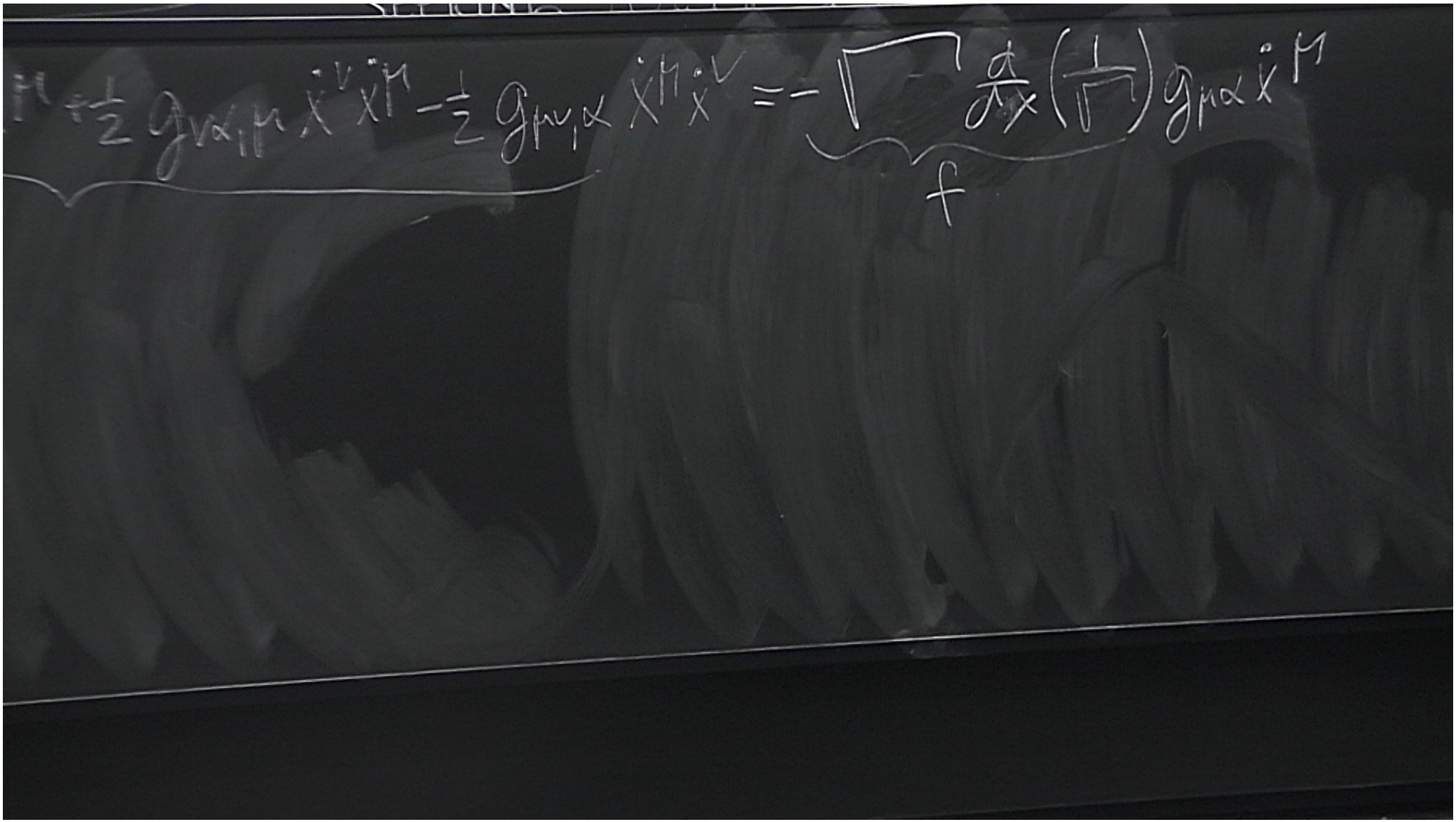
$$\frac{\partial L}{\partial \dot{x}^\alpha} = \frac{m\, g_{\mu\alpha}\, \dot{x}^\mu}{\sqrt{\;}}$$

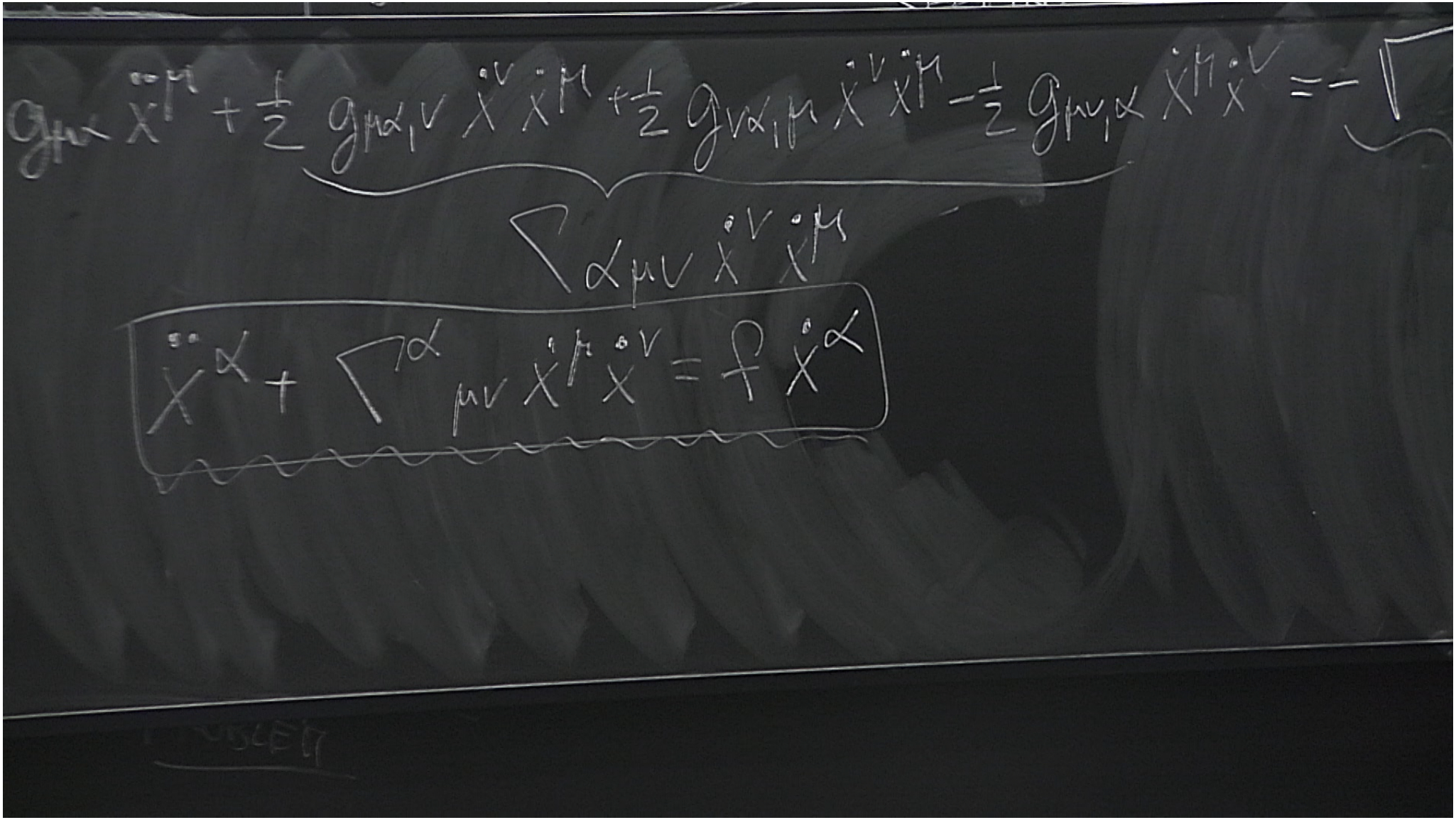
$$\frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}^\alpha}\right) = \frac{d}{d\lambda}\left(\sqrt{\;}\right) m g_{\mu\alpha} \dot{x}^\mu$$

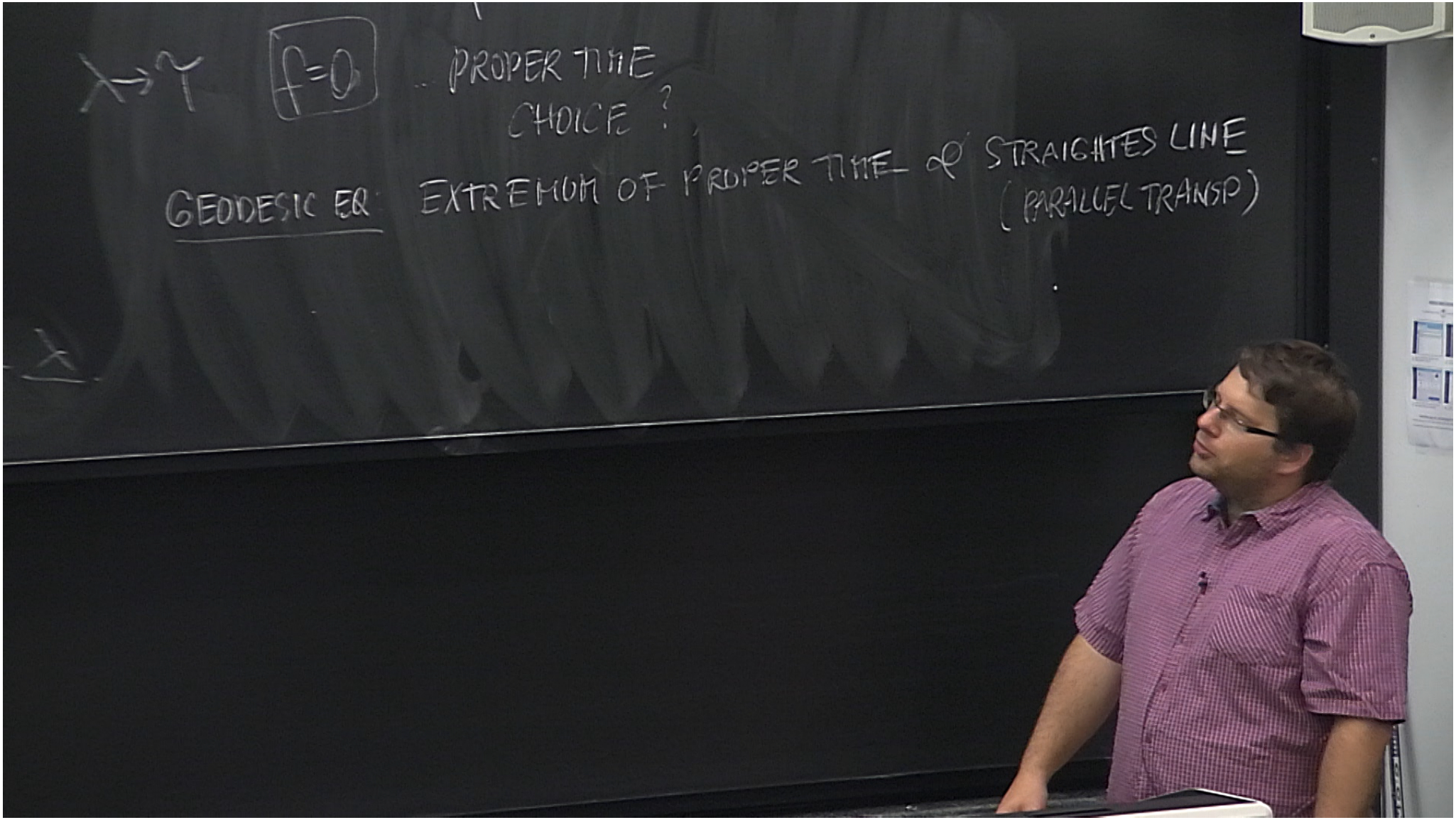
$$+ \frac{m}{\sqrt{\;}}\left(g_{\mu\alpha,\nu}\, \dot{x}^\nu \dot{x}^\mu + g_{\mu\alpha}\, \ddot{x}^\mu\right)$$

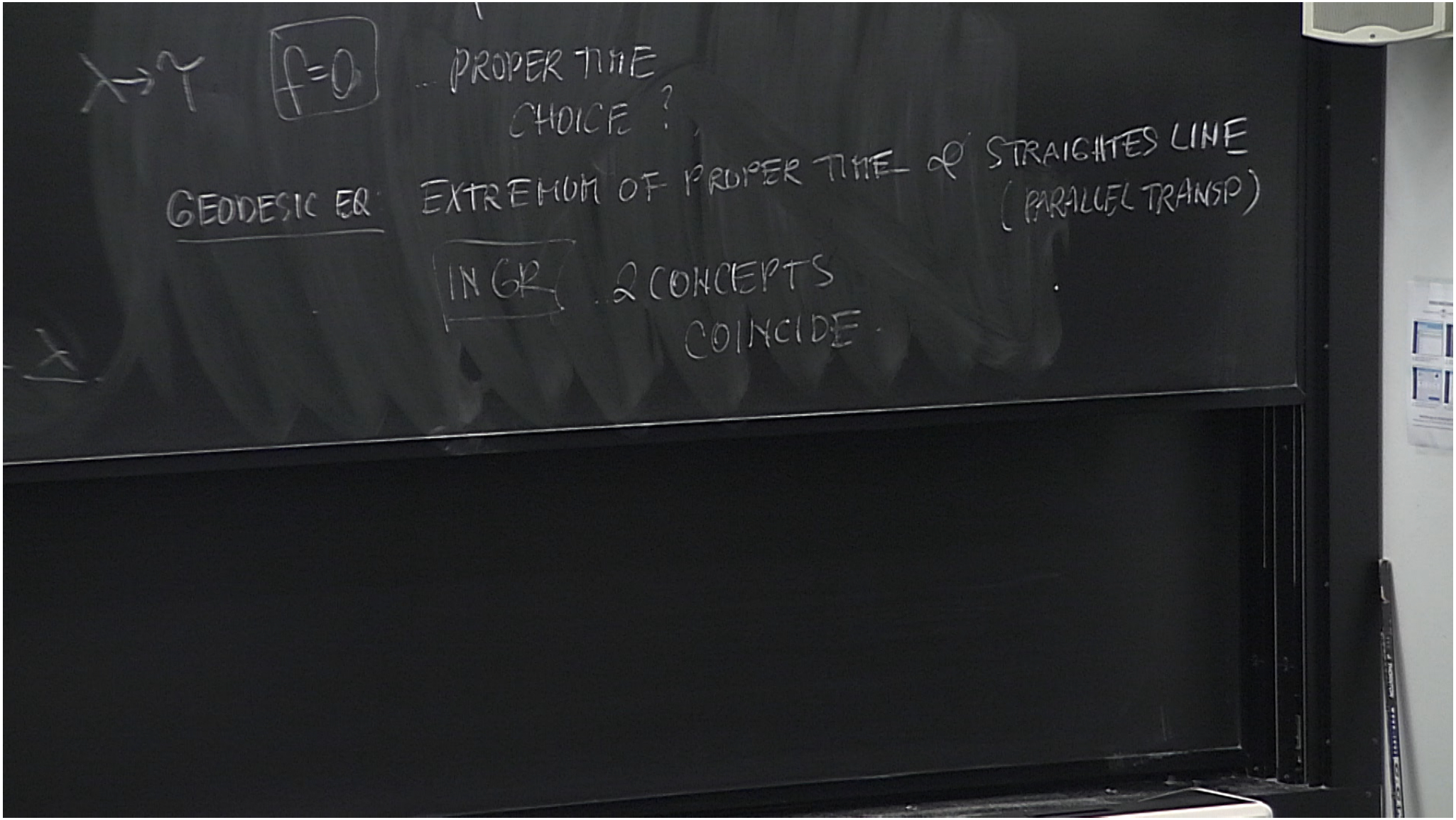
MAGIC $1 = \frac{1}{2} + \frac{1}{2}$


(E-L): $g_{\mu\alpha}\ddot{x}^{\alpha} + \frac{1}{2} g_{\mu\alpha,\nu} \dot{x}^{\nu} \dot{x}^{\alpha} + \frac{1}{2} g_{\nu\alpha,\mu} \dot{x}^{\nu} \dot{x}^{\alpha} - \frac{1}{2} g_{\nu}$


$\Gamma^{\mu}_{\nu\alpha} + \frac{1}{2} g_{\nu\alpha,\mu} \dot{x}^{\nu} \dot{x}^{\alpha} - \frac{1}{2} g_{\mu\nu,\alpha} \dot{x}^{\nu} \dot{x}^{\alpha} = -\underbrace{\sqrt{-} \frac{d}{dx} \left(\frac{1}{\sqrt{-}}\right)}_{f} g_{\mu\alpha} \ddot{x}^{\alpha}$


$g_{\mu\nu}\ddot{x}^{\mu} + \frac{1}{2} g_{\mu\alpha,\nu}\dot{x}^{\nu}\dot{x}^{\mu} + \frac{1}{2} g_{\mu\alpha,\mu}\dot{x}^{\nu}\dot{x}^{\mu} - \frac{1}{2} g_{\mu\nu,\alpha}\dot{x}^{\mu}\dot{x}^{\nu} = -$
$\Gamma_{\alpha\mu\nu}\dot{x}^{\nu}\dot{x}^{\mu}$
$\ddot{x}^{\alpha} + \Gamma^{\alpha}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = f\dot{x}^{\alpha}$

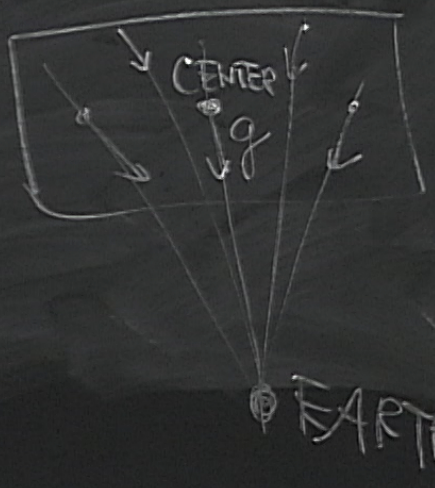

λ→γ
f=0
PROPER TIME
CHOICE ?
GEODESIC EQ
EXTREMUM OF PROPER TIME ⇔ STRAIGHTEST LINE
(PARALLEL TRANSP)


λ→Y
f=0
PROPER TIME
CHOICE ?
GEODESIC EQ
EXTREMUM OF PROPER TIME ⇔ STRAIGHTEST LINE
(PARALLEL TRANSP)
INGR
2 CONCEPTS
COINCIDE

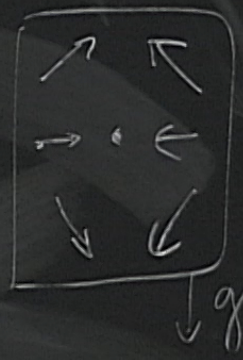
b) GEODESIC DEVIATION EQUATION

Q: HOW TO QUANTIFY STRENGTH OF GRAVITY?

A: BY TIDAL FORCES



CENTRE
FREELY FALLING

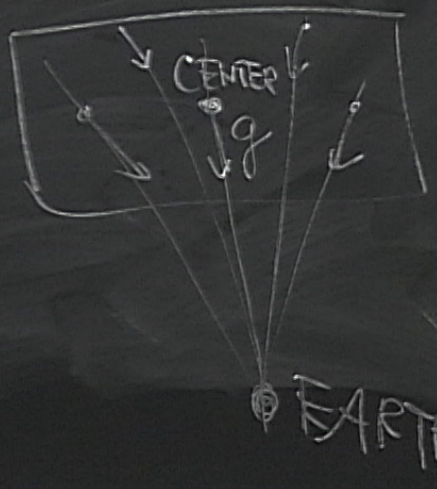


TIDAL FORCES

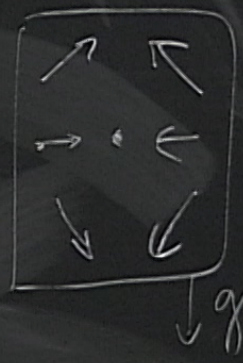
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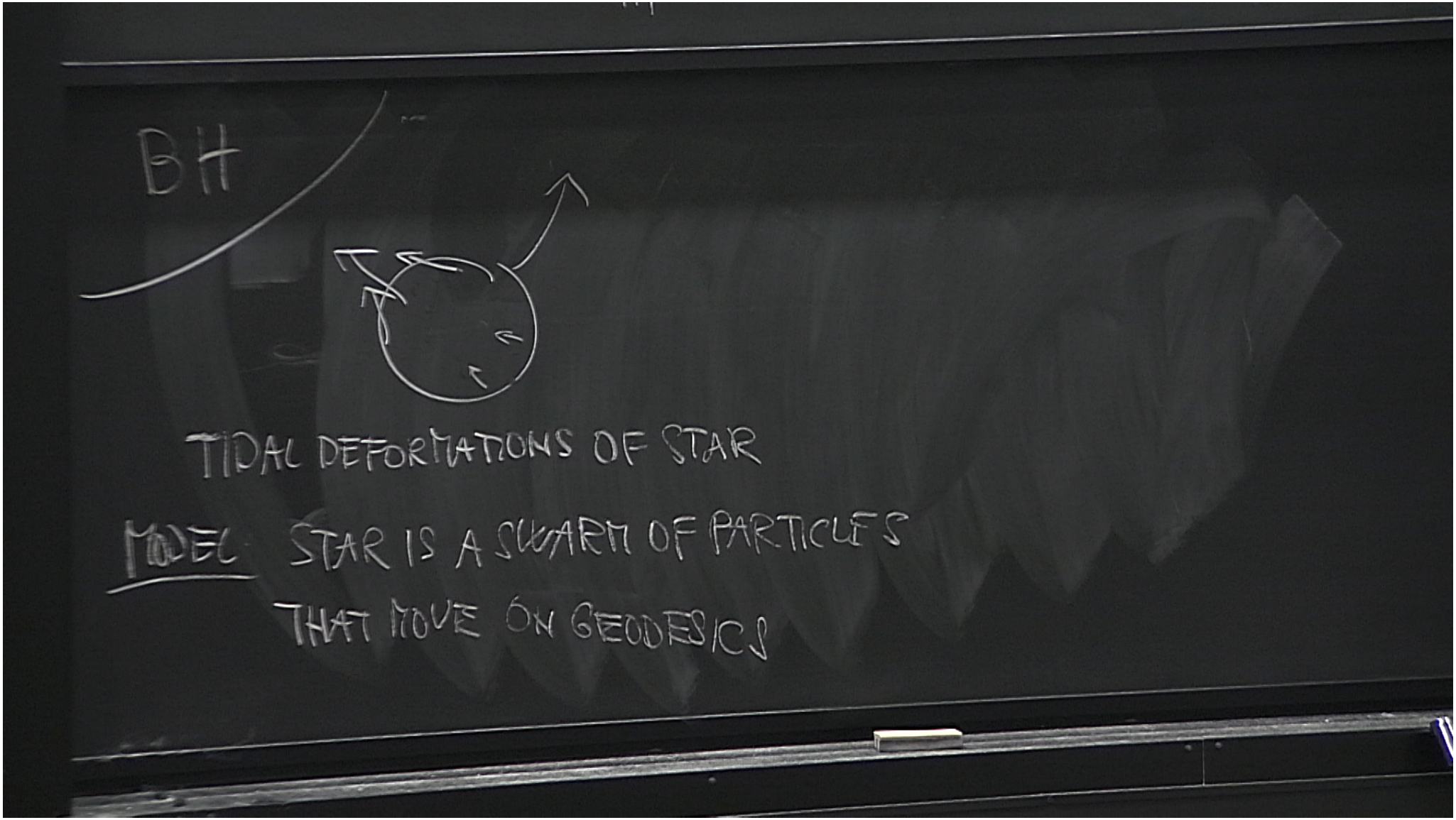
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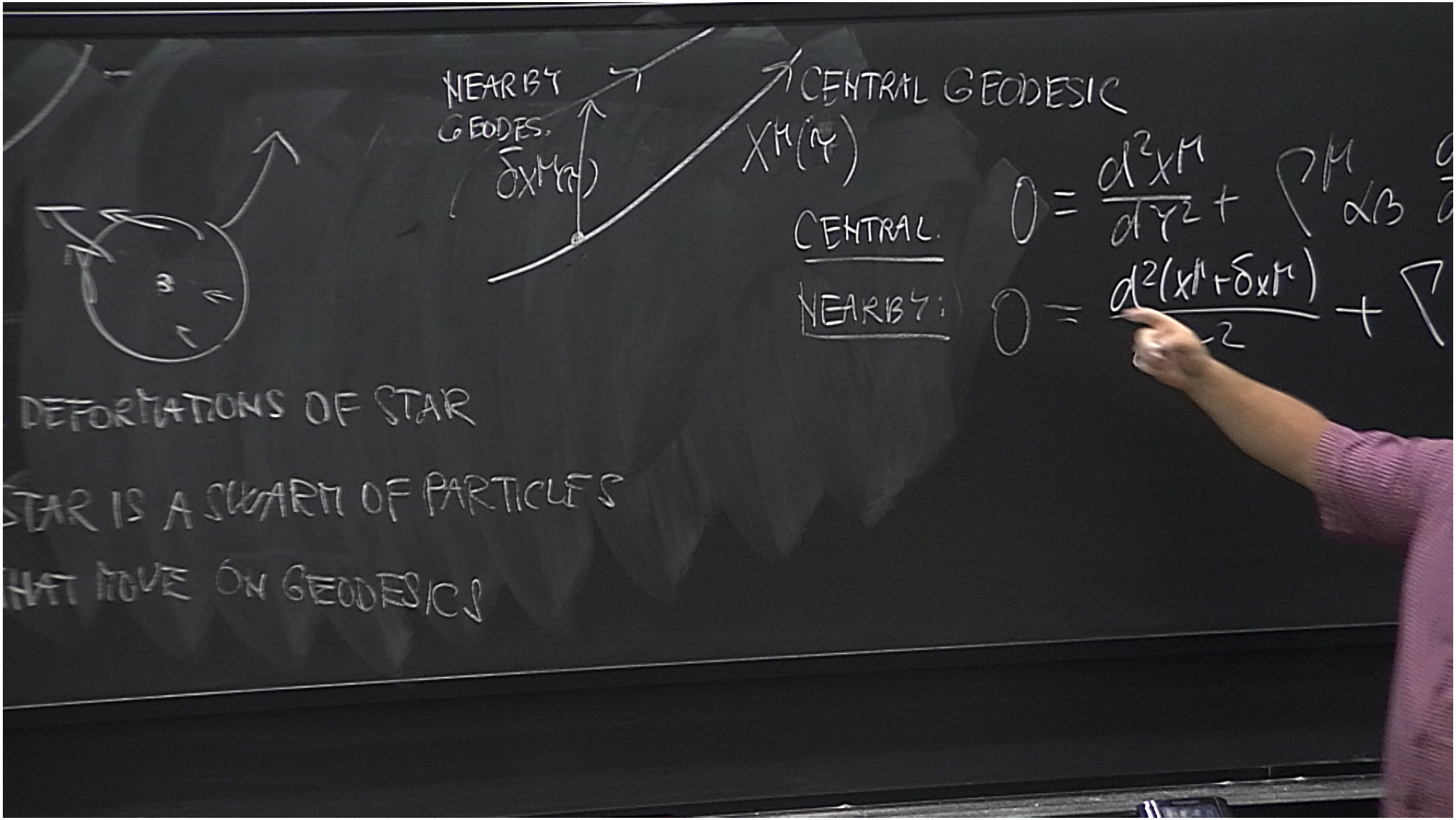


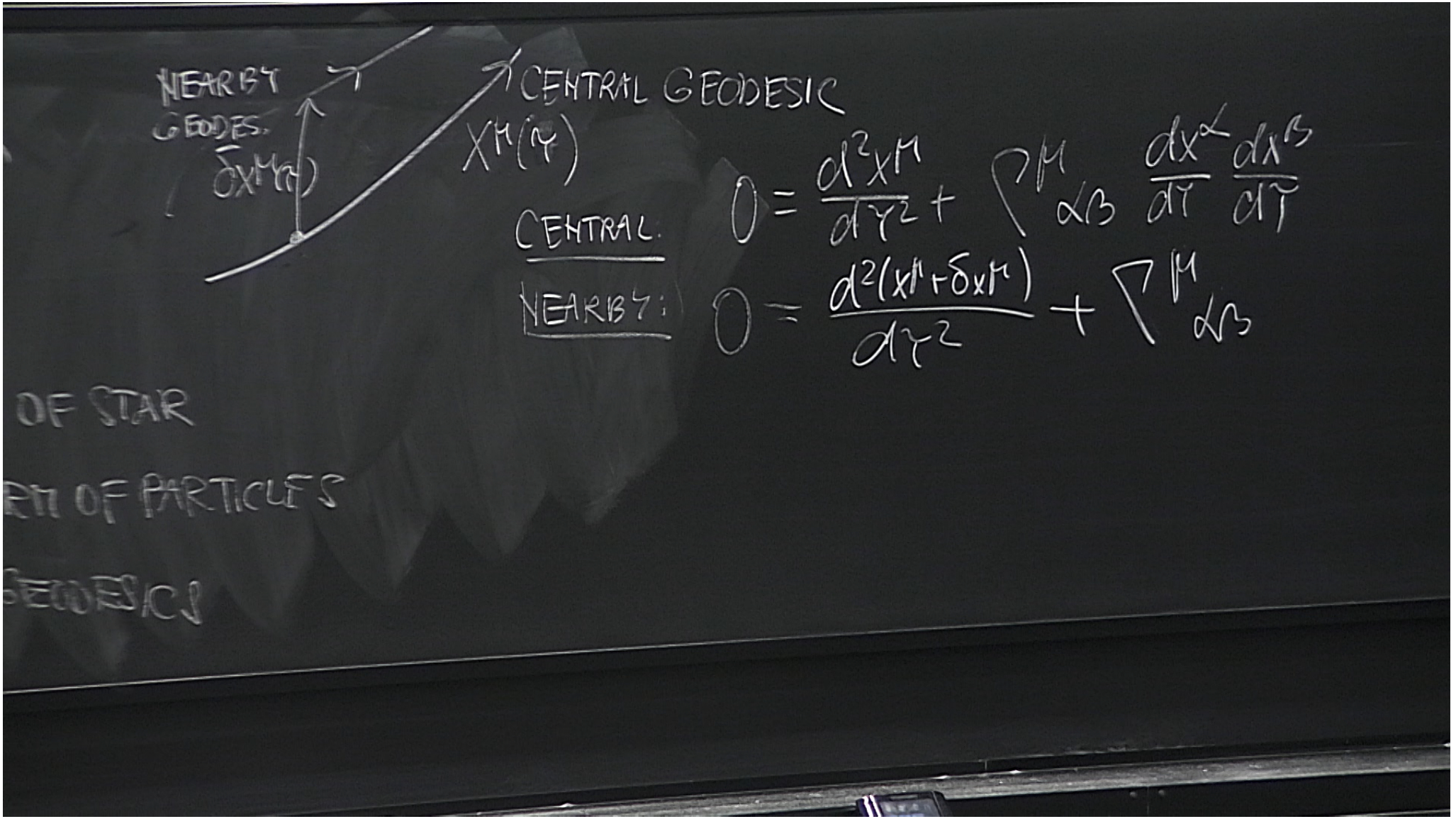
CENTRE
FREELY FALLING

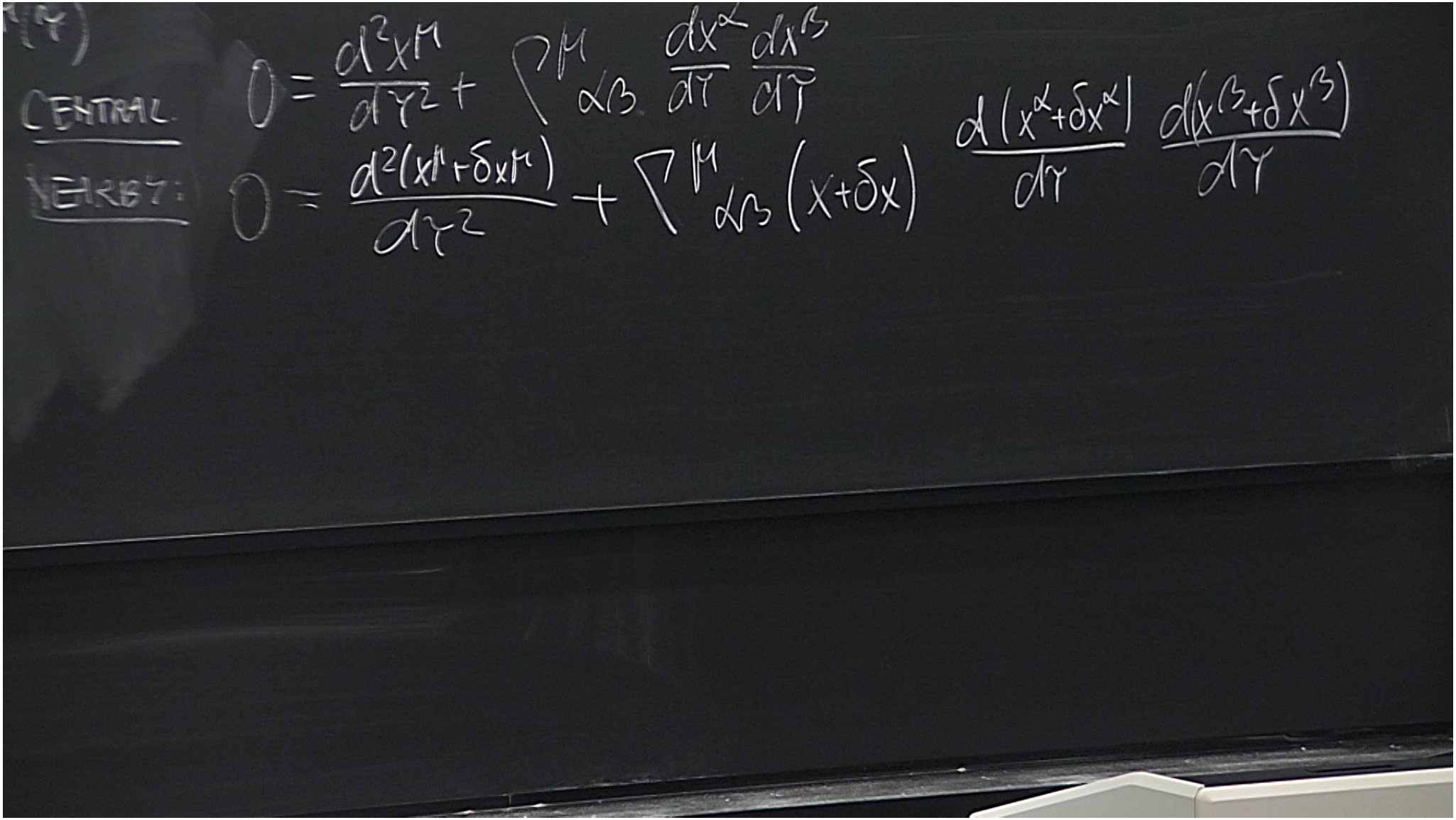


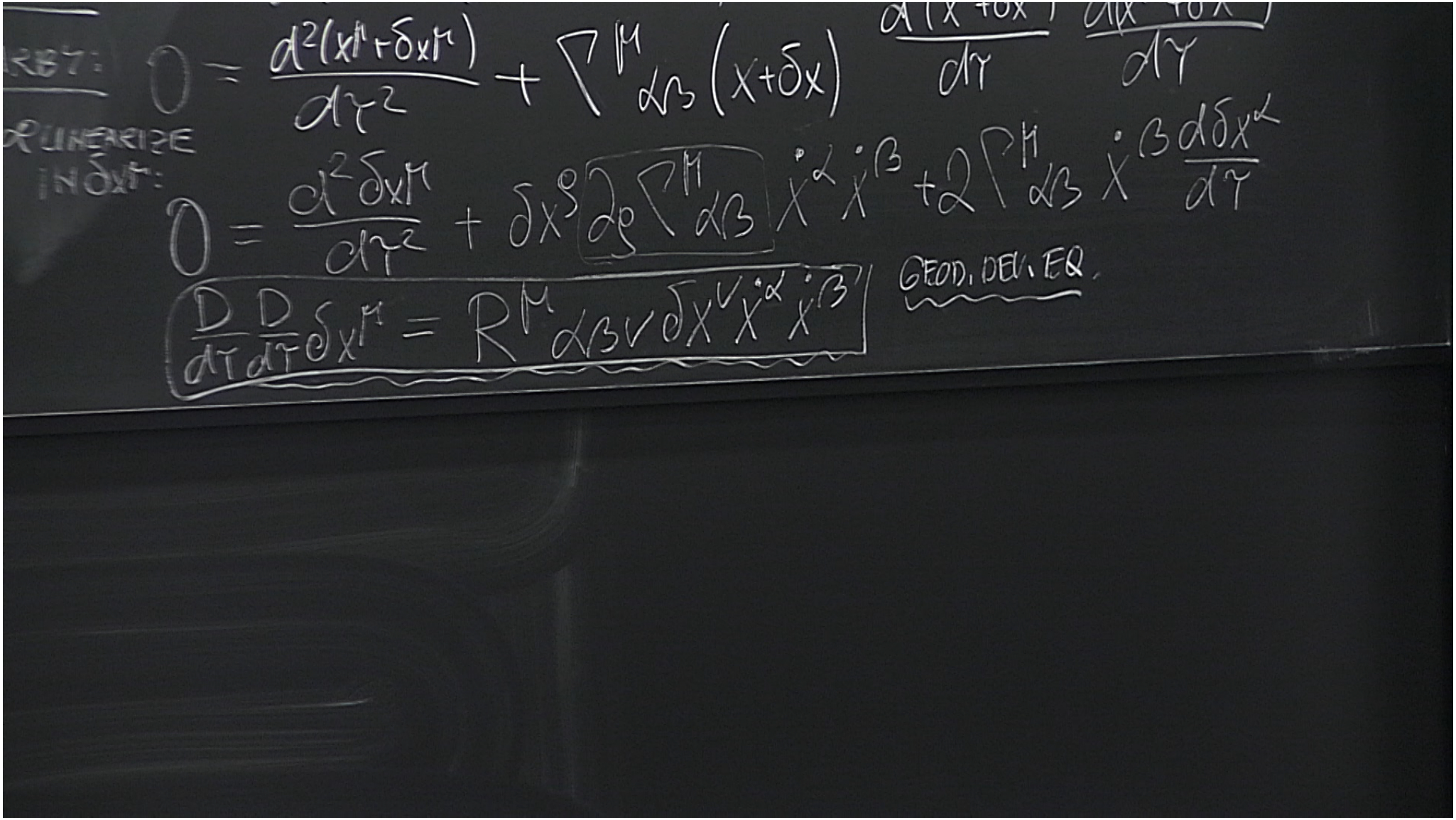
TIDAL FORCES


BH
TIDAL DEFORMATIONS OF STAR
MODEL STAR IS A SWARM OF PARTICLES
THAT MOVE ON GEODESICS


NEARBY GEODES.
$\delta x^\mu(\tau)$
CENTRAL GEODESIC
$X^\mu(\tau)$
CENTRAL:
$0 = \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}$
NEARBY:
$0 = \frac{d^2(x^\mu + \delta x^\mu)}{\quad 2} +$
DEFORMATIONS OF STAR
STAR IS A SWARM OF PARTICLES
THAT MOVE ON GEODESICS


NEARBY GEODES.
$\delta x^\mu(\tau)$
τ
CENTRAL GEODESIC
$X^\mu(\tau)$
CENTRAL:
$0 = \frac{d^2 X^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$
NEARBY:
$0 = \frac{d^2(X^\mu + \delta x^\mu)}{d\tau^2} + \Gamma^\mu_{\alpha\beta}$
OF STAR
RTH OF PARTICLES
DESICS


CENTRAL: $0 = \frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$
NEARBY: $0 = \frac{d^2(x^\mu+\delta x^\mu)}{d\tau^2} + \Gamma^\mu_{\alpha\beta}(x+\delta x)\frac{d(x^\alpha+\delta x^\alpha)}{d\tau}\frac{d(x^\beta+\delta x^\beta)}{d\tau}$


RBT:
$0 = \frac{d^2(x^\mu+\delta x^\mu)}{d\tau^2} + \Gamma^\mu_{\alpha\beta}(x+\delta x)\frac{d(x+\delta x)^\alpha}{d\tau}\frac{d(x+\delta x)^\beta}{d\tau}$
LINEARIZE IN δx^μ:
$0 = \frac{d^2\delta x^\mu}{d\tau^2} + \delta x^\rho \partial_\rho \Gamma^\mu_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta + 2\Gamma^\mu_{\alpha\beta}\dot{x}^\beta\frac{d\delta x^\alpha}{d\tau}$
$\frac{D}{d\tau}\frac{D}{d\tau}\delta x^\mu = R^\mu_{\alpha\beta\nu}\delta x^\nu \dot{x}^\alpha \dot{x}^\beta$
GEOD. DEV. EQ.

PHYSICAL MEANING OF RIEMANN ≃ DESCRIBES TIDAL FORCES

- DEVIATION EQ IS LINEAR !
  IS SUPER COMPLICATED
  IN ORDER TO SOLVE FOR $\delta x^\mu(\tau)$
  CENTRAL TRAJ. $x^\mu(\tau)$ → $g_{\mu\nu}(x(\tau))$
  → $R_{\mu\nu\rho\sigma}(x(\tau))$

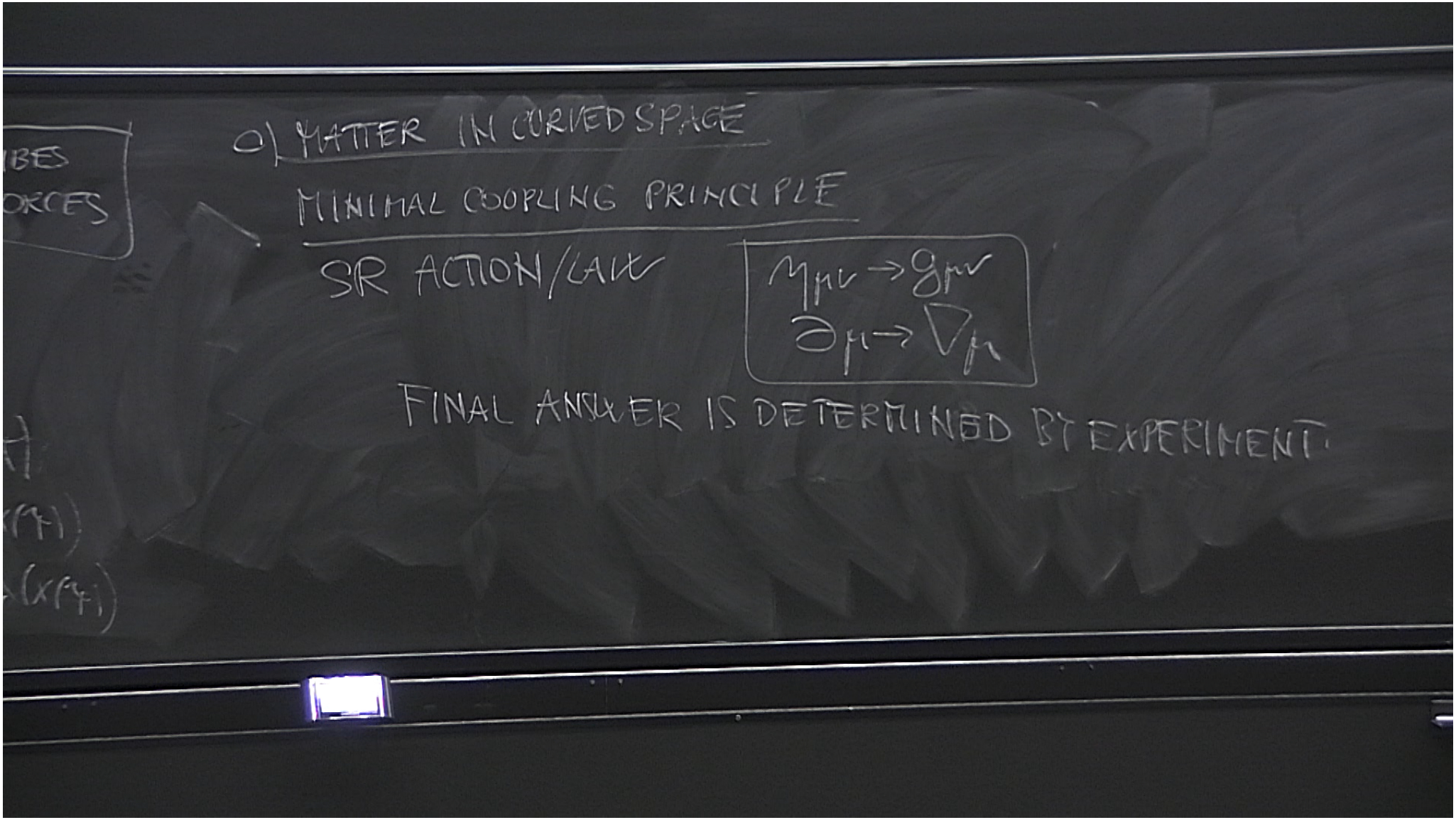
PHYSICAL MEANING OF RIEMANN ≃ DESCRIBES TIDAL FORCES

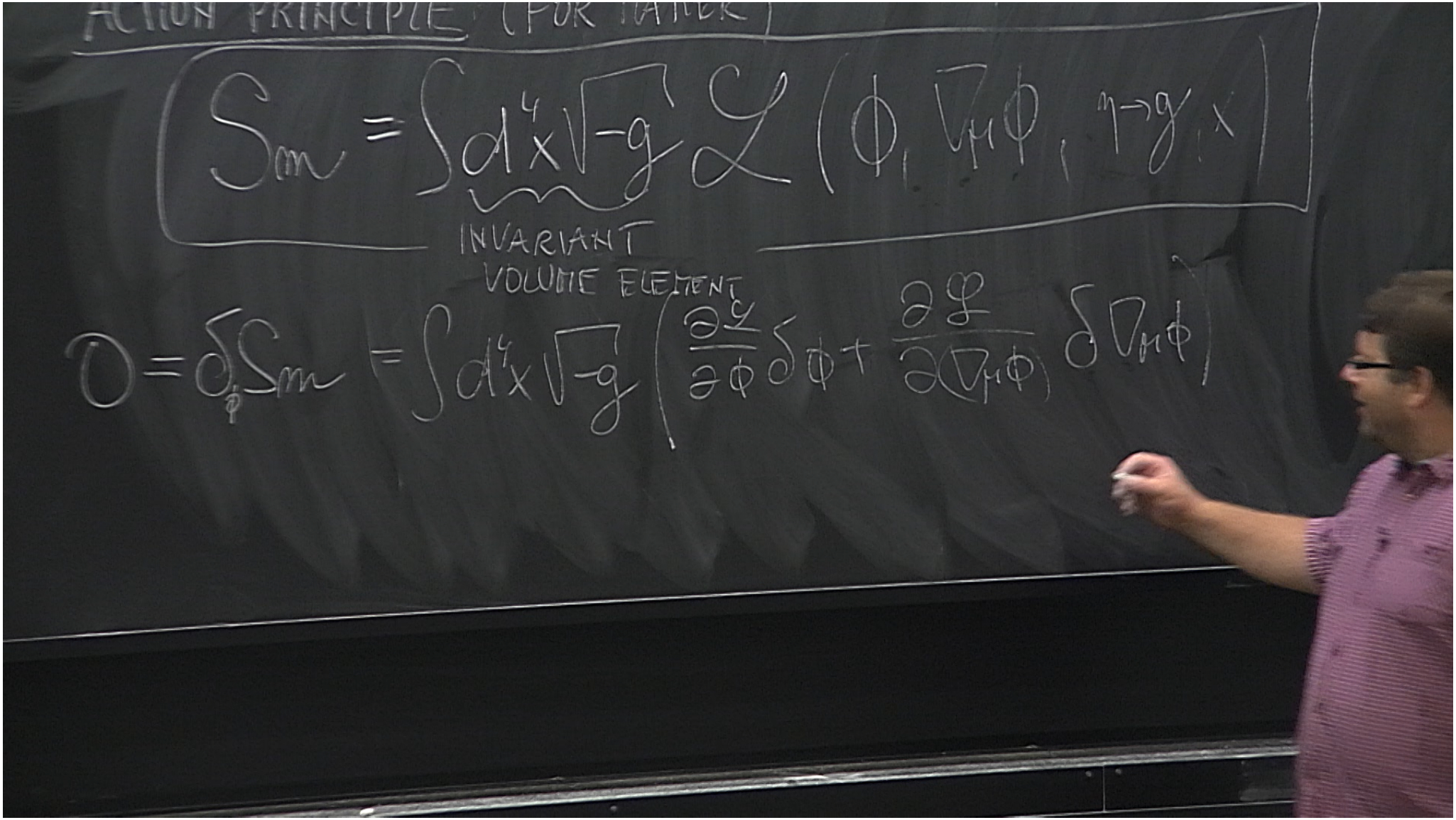
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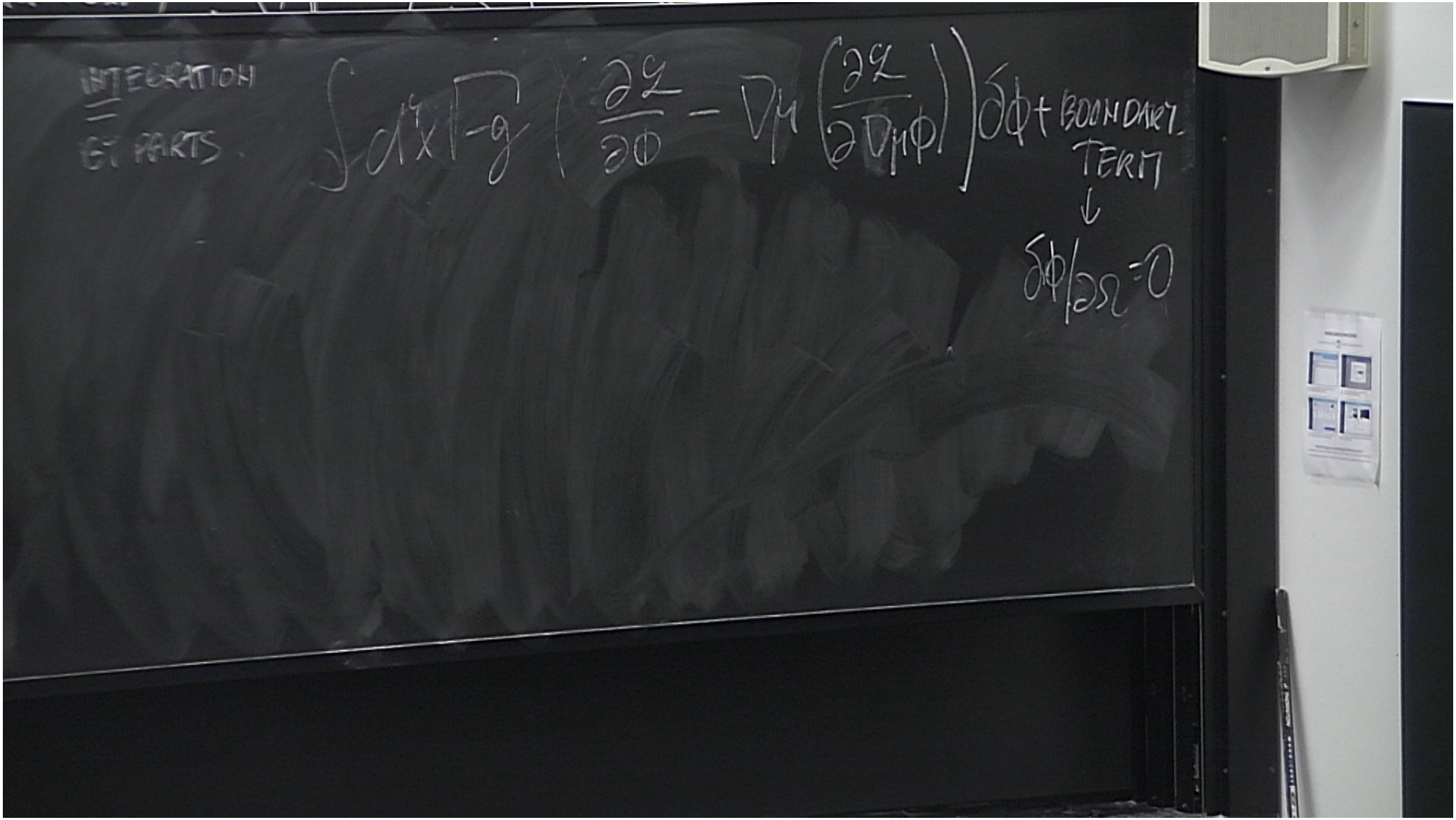
IN ORDER TO SOLVE FOR $\delta x^\mu(\tau)$

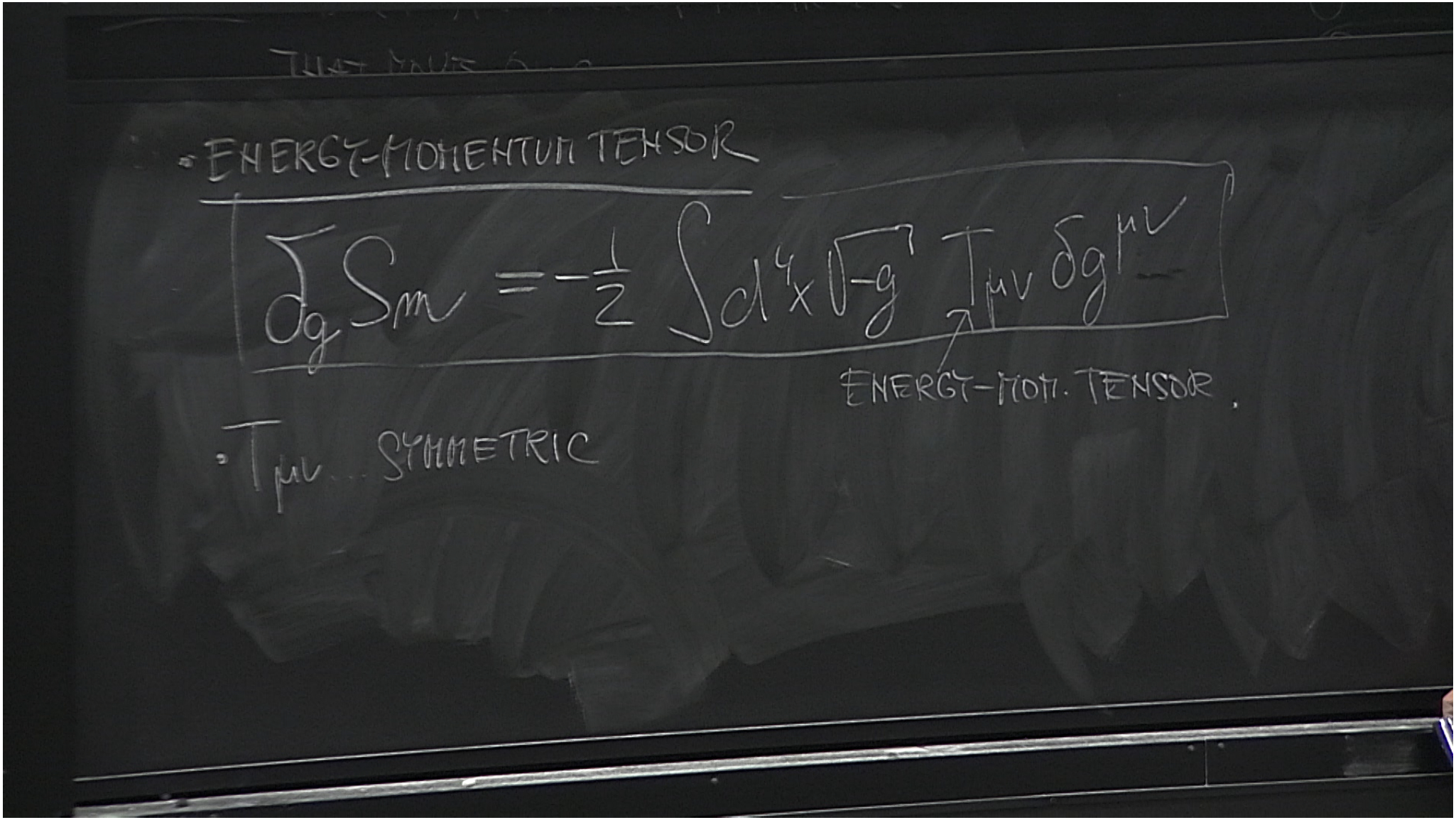
CENTRAL TRAJ. $X^\mu(\tau)$ $\rightarrow g_{\mu\nu}(x(\tau))$

$\rightarrow R_{\mu\nu\alpha\beta}(x(\tau))$


IBES
ORCES
0) MATTER IN CURVED SPACE
MINIMAL COUPLING PRINCIPLE
SR ACTION/LAW
$\eta_{\mu\nu} \to g_{\mu\nu}$
$\partial_\mu \to \nabla_\mu$
FINAL ANSWER IS DETERMINED BY EXPERIMENT.


ACTION PRINCIPLE (FOR FATHER)
$S_m = \int d^4x \sqrt{-g}\, \mathcal{L}(\phi, \nabla_\mu \phi, \eta \to g, x)$
INVARIANT VOLUME ELEMENT
$0 = \delta_\phi S_m = \int d^4x \sqrt{-g} \left(\frac{\partial \mathcal{L}}{\partial \phi} \delta\phi + \frac{\partial \mathcal{L}}{\partial (\nabla_\mu \phi)} \delta \nabla_\mu \phi \right)$


INTEGRATION BY PARTS
$\int d^4x \sqrt{-g} \left(\frac{\partial \mathcal{L}}{\partial \phi} - D_\mu \left(\frac{\partial \mathcal{L}}{\partial D_\mu \phi} \right) \right) \delta\phi$ + BOUNDARY TERM
$\delta\phi|_{\partial\Omega} = 0$


• ENERGY-MOMENTUM TENSOR
$\delta_g S_m = -\frac{1}{2}\int d^4x \sqrt{-g}\, T_{\mu\nu}\delta g^{\mu\nu}$
ENERGY-MOM. TENSOR.
• $T_{\mu\nu}$ ... SYMMETRIC