

Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 4

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

Date: September 06, 2019 - 10:45 AM

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YESTERDAY

- WE INTRODUCED CURVED METRIC

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

- DERIVED GEODESIC EQUATION

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} (g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu})$$

CHRISTOFFEL
SYMBOLS

EXAMPLE: RINDLER FRAME

UNIFORMLY ACCELERATED OBSERVER

SWEEPS TRAJECTORY

$$t = \frac{1}{a} \sinh(a\tau)$$

$$x = \frac{1}{a} \cosh(a\tau)$$

PROPER TIME
OF OBSERVER

EXAMPLE: RINDLER FRAME

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$$a^\mu = \frac{dx^\mu}{d\tau}$$

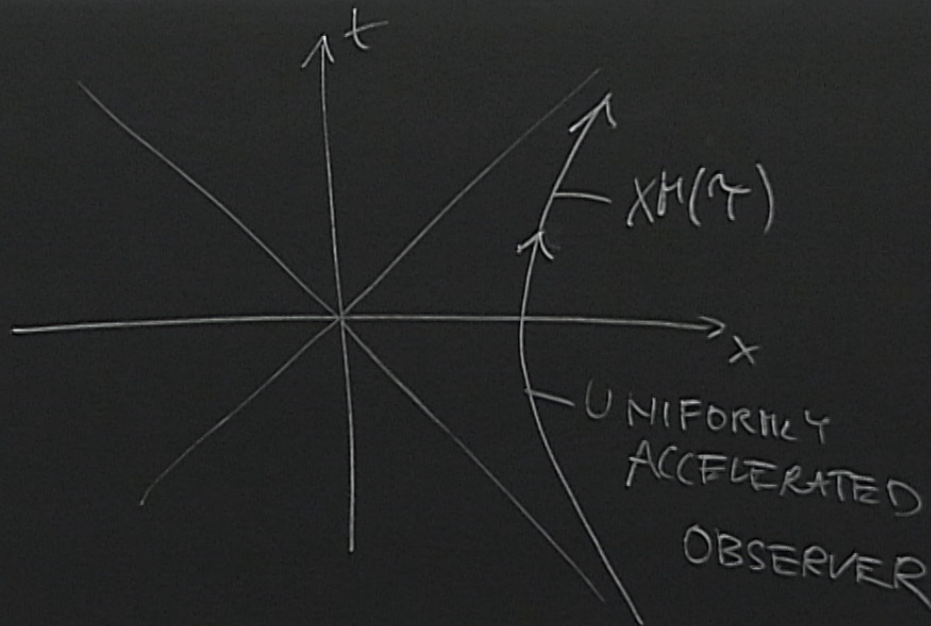
PROPER TIME
OF OBSERVER

$$a = \sqrt{\eta_{\mu\nu} a^\mu a^\nu} = \text{CONST}$$

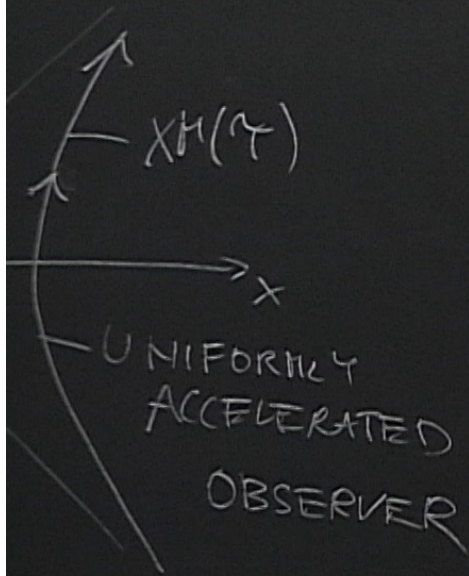
D OBSERVER

γ
 γ
 γ

PER TIME
OBSERVER

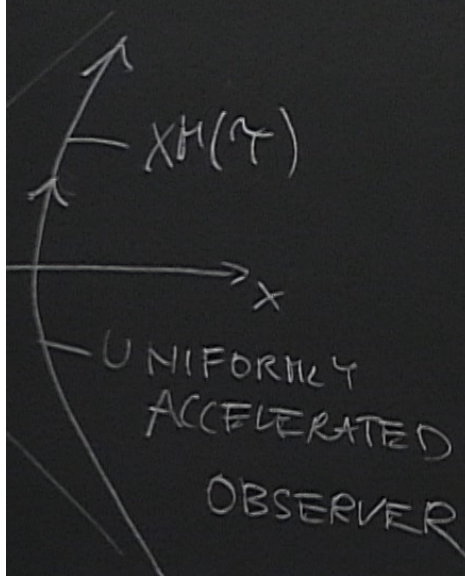


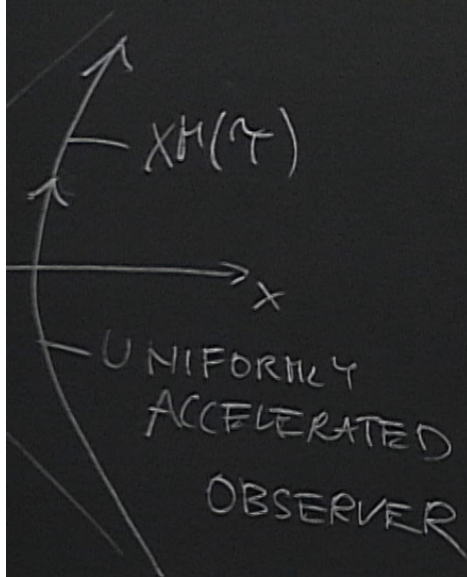
LET'S INTRODUCE A COORDINATE SYSTEM ASSOCIATED
WITH THIS OBSERVER



LET'S INTRODUCE A COORDINATE SYSTEM ASSOCIATED
WITH THIS OBSERVER. RINDLER COORDINATES

$$(T, X) : \begin{cases} t = \left(\frac{1}{a} + X\right) \sinh(aT) \\ x = \left(\frac{1}{a} + X\right) \cosh(aT) \end{cases}$$





LET'S INTRODUCE A COORDINATE SYSTEM ASSOCIATED
WITH THIS OBSERVER RINDLER COORDINATES

$$(T, X) : \begin{cases} t = \left(\frac{1}{a} + X\right) \sinh(aT) \\ x = \left(\frac{1}{a} + X\right) \cosh(aT) \end{cases}$$

OBSERVER: $X=0, T=\tau$

CURVED METRIC:

$$ds^2 = -dt^2 + dx^2 = \left| \begin{array}{l} dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial X} dX \\ dx = \frac{\partial x}{\partial T} dT + \frac{\partial x}{\partial X} dX \end{array} \right|$$

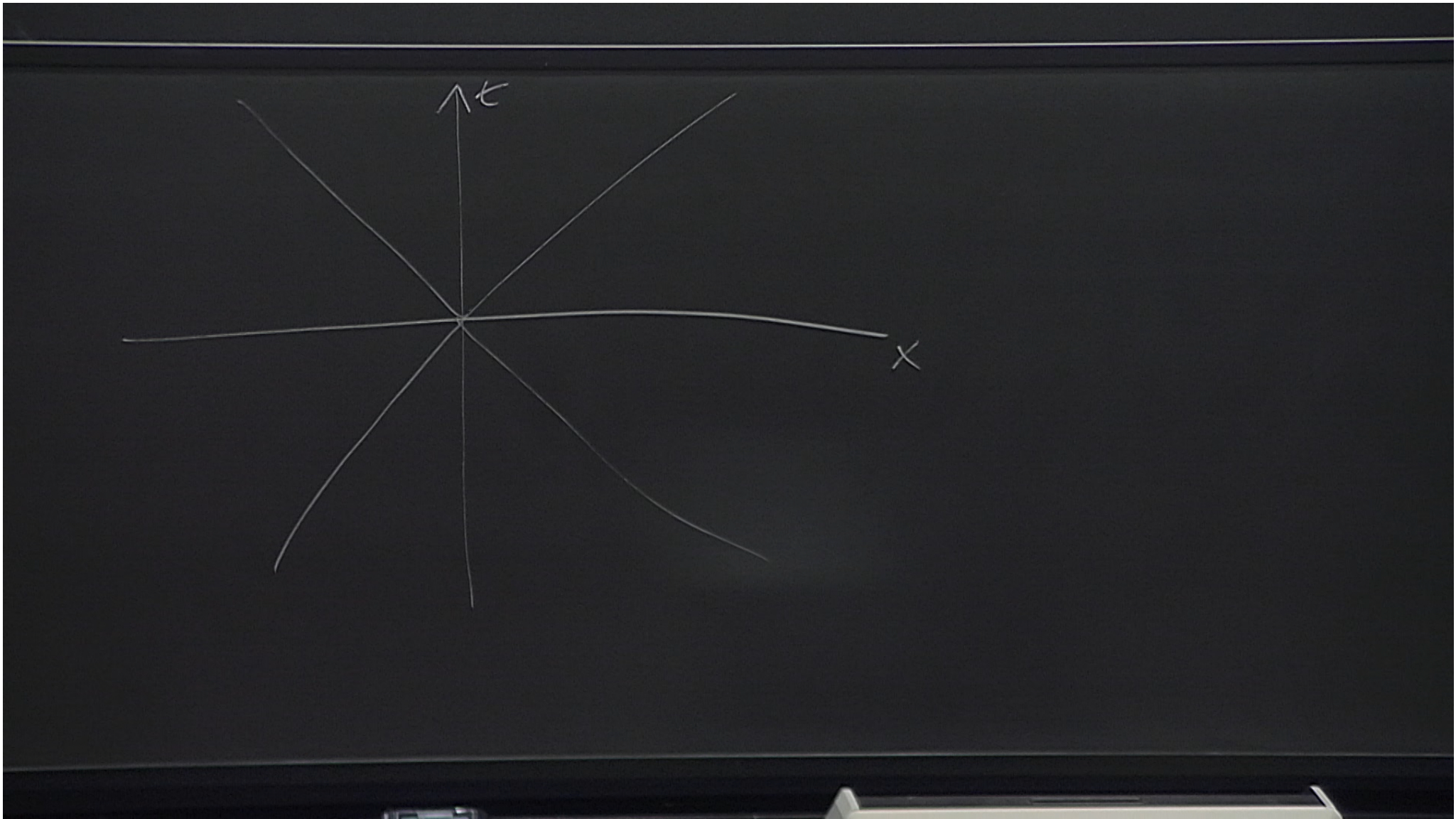
CURVED METRIC:

$$ds^2 = -dt^2 + dx^2 =$$

$$dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial X} dX$$

$$dx = \frac{\partial x}{\partial T} dT + \frac{\partial x}{\partial X} dX$$

$$ds^2 = -(1+aX)^2 dT^2 + dX^2$$



• CURVED METRIC:

$$ds^2 = -dt^2 + dx^2 = \begin{vmatrix} dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial X} dX \\ dx = \frac{\partial x}{\partial T} dT + \frac{\partial x}{\partial X} dX \end{vmatrix}$$

$$ds^2 = - (1 + aX)^2 dT^2 + dX^2$$

IN ORDER METRIC IS NON-DEGENERATE WE HAVE
TO RESTRICT THE RANGE OF COORDS.

• CURVED METRIC:

$$ds^2 = -dt^2 + dx^2 = \begin{vmatrix} dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial X} dX \\ dx = \frac{\partial x}{\partial T} dT + \frac{\partial x}{\partial X} dX \end{vmatrix}$$

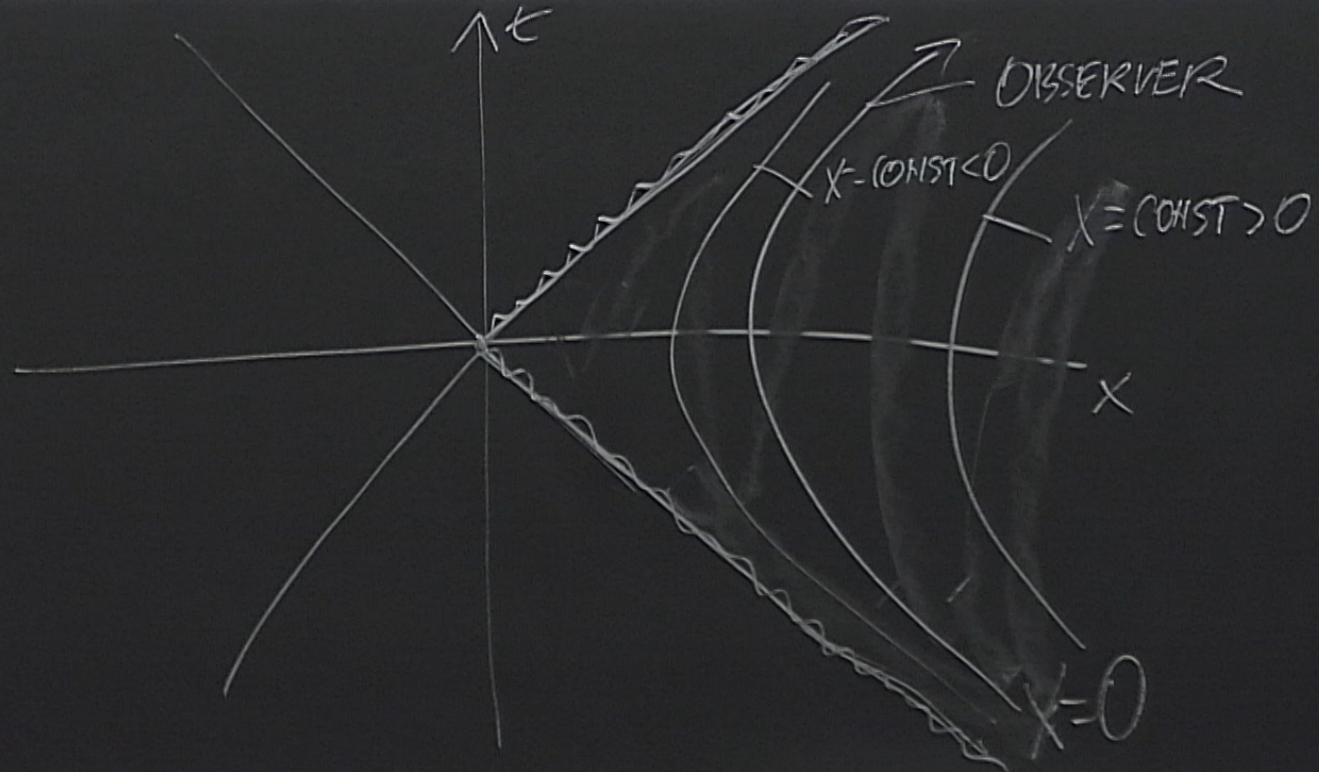
$$ds^2 = - (1+aX)^2 dT^2 + dX^2$$

IN ORDER METRIC IS NON-DEGENERATE WE HAVE TO RESTRICT THE RANGE OF COORDS.

$$T \in (-\infty, \infty), X \in \left(-\frac{1}{a}, \infty\right)$$

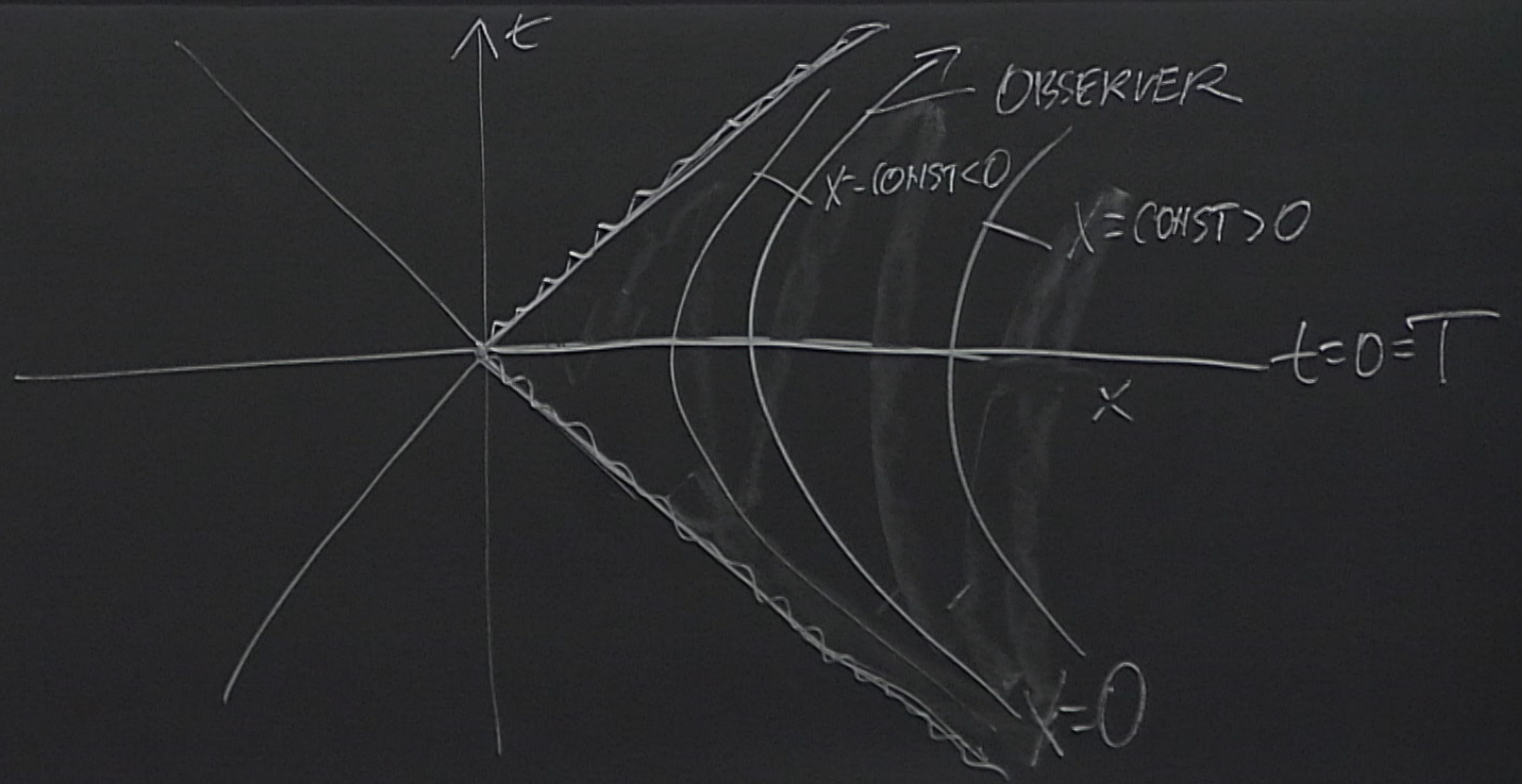
$$\frac{dx}{dt}$$

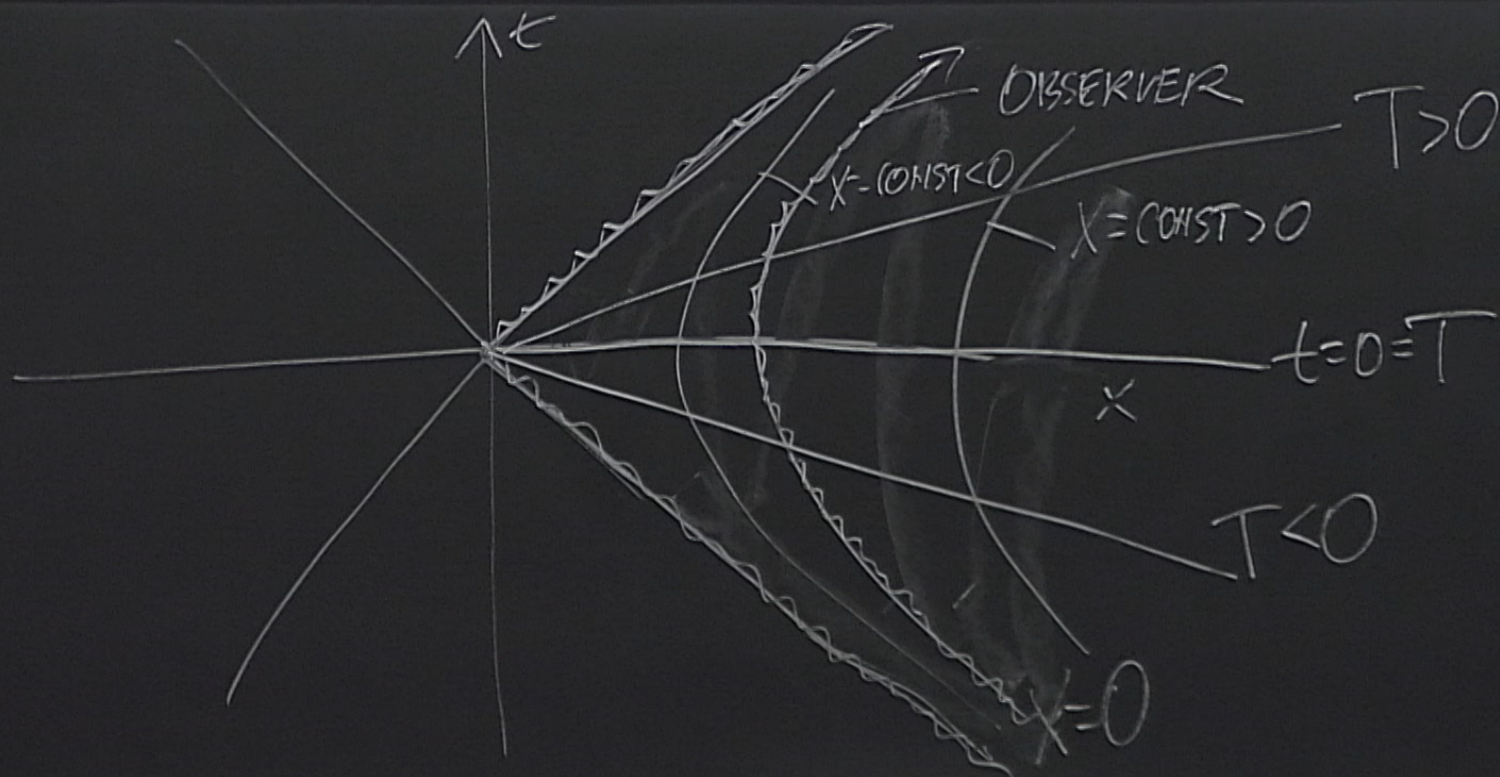
WE HAVE

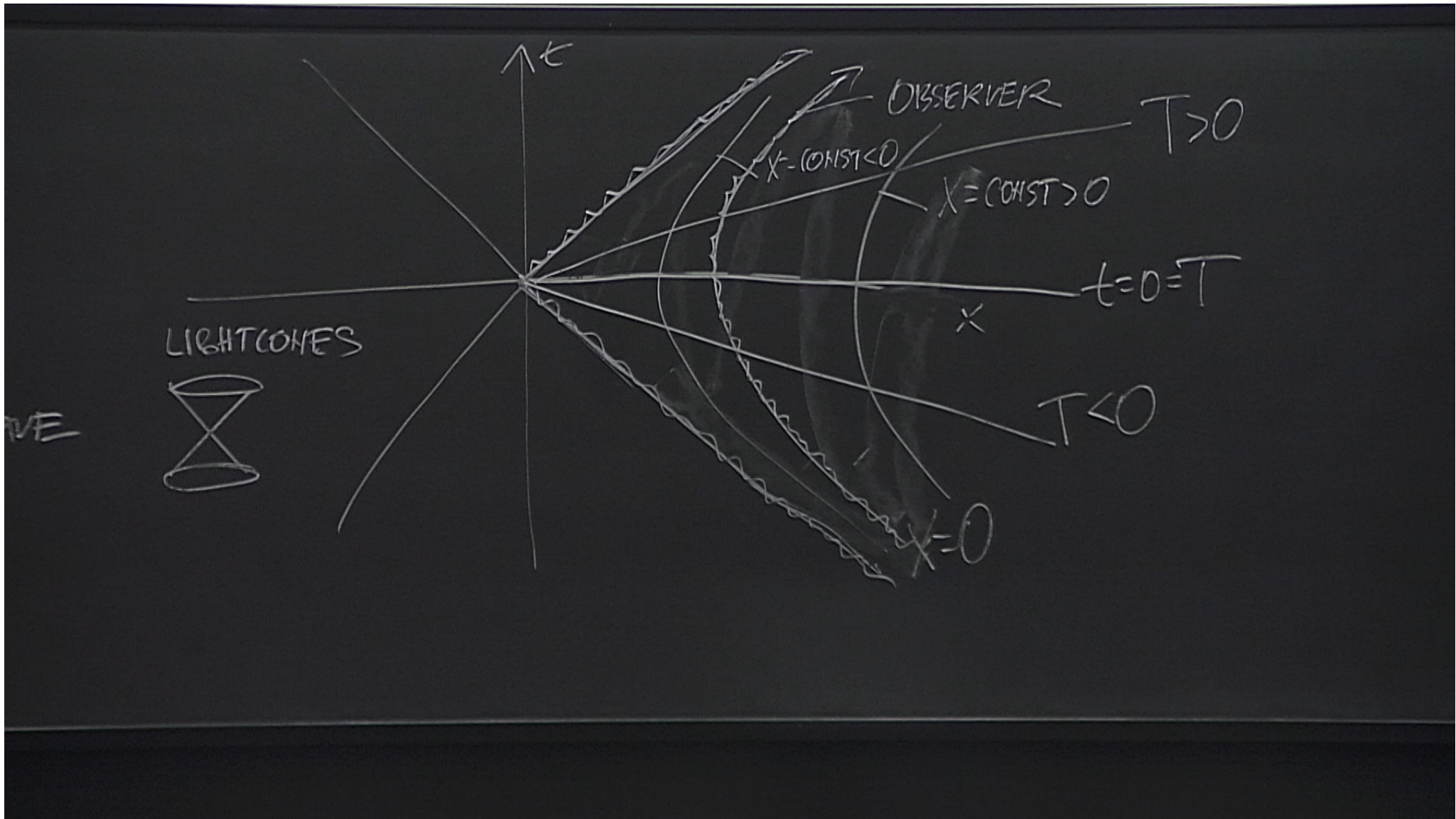


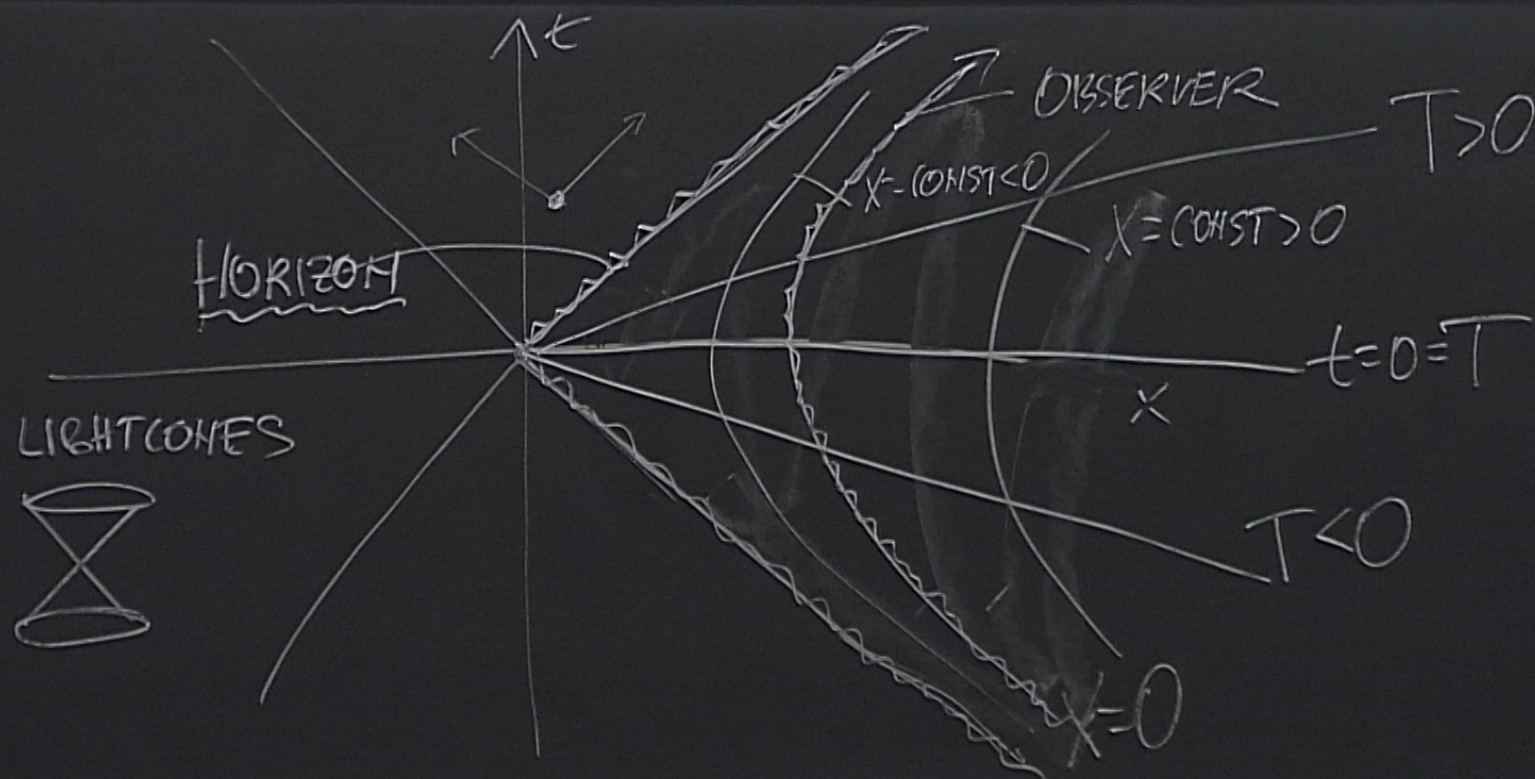
$\frac{dx}{dt}$

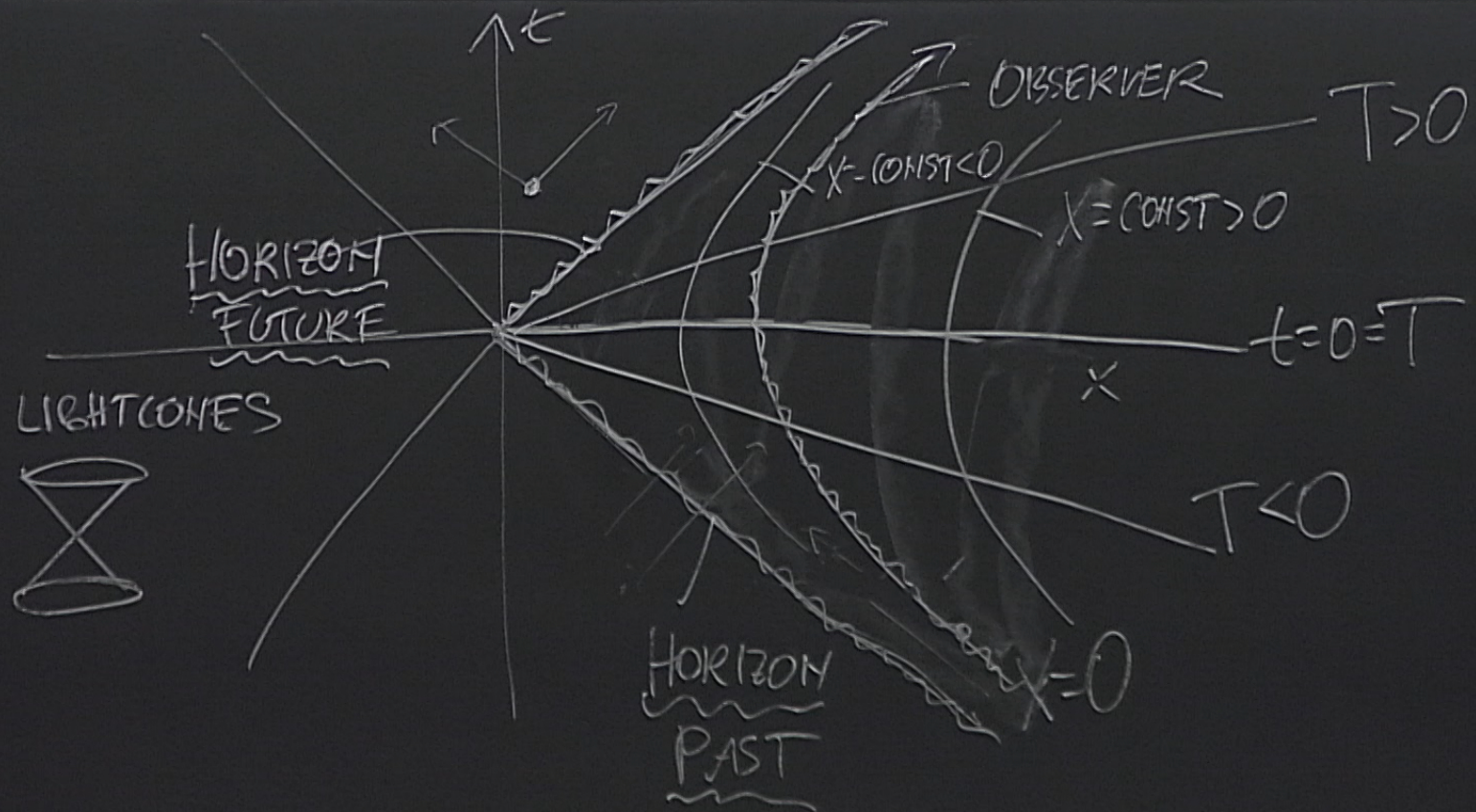
WE HAVE

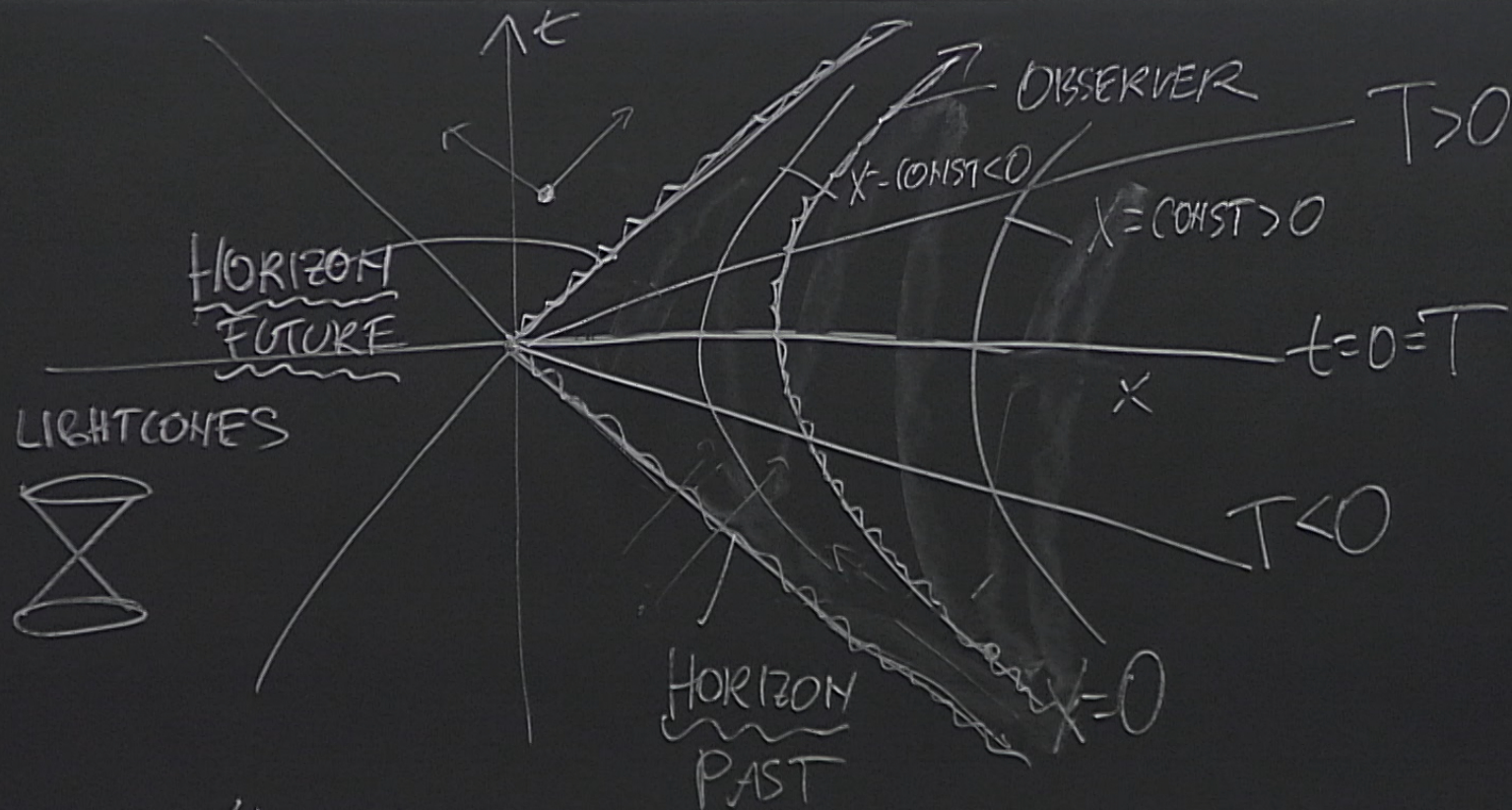






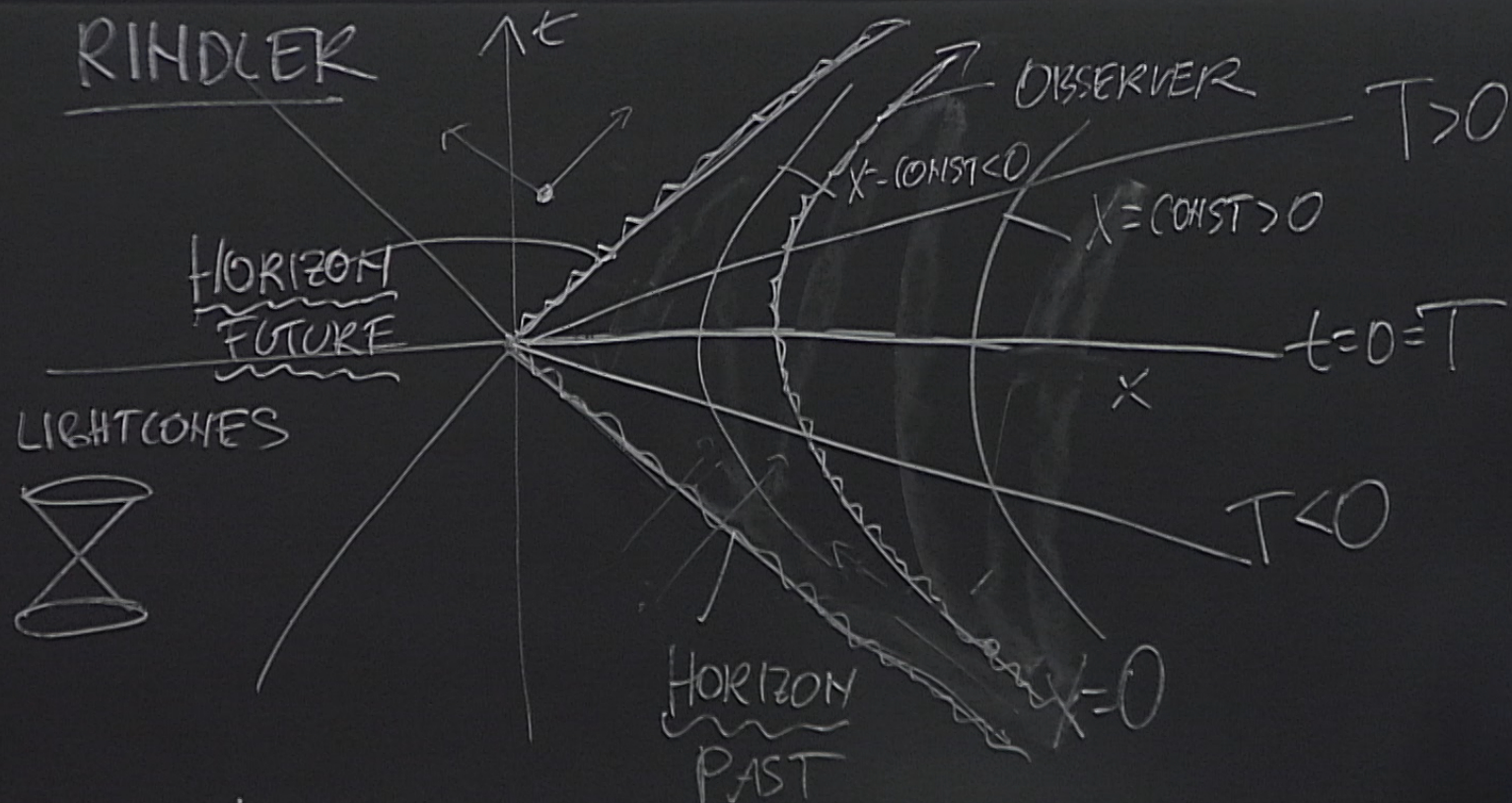




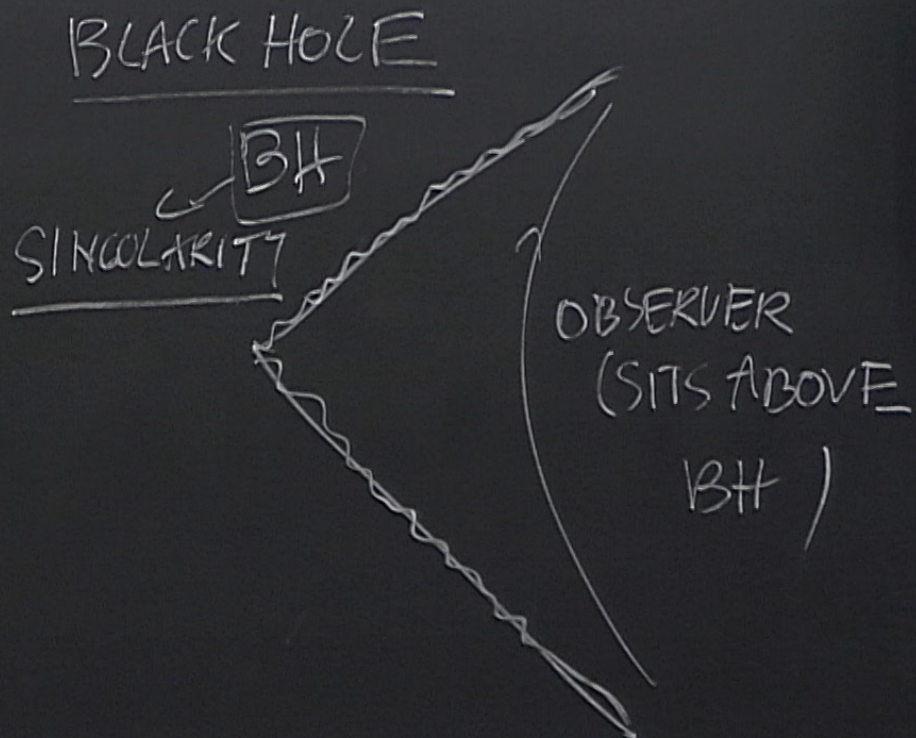
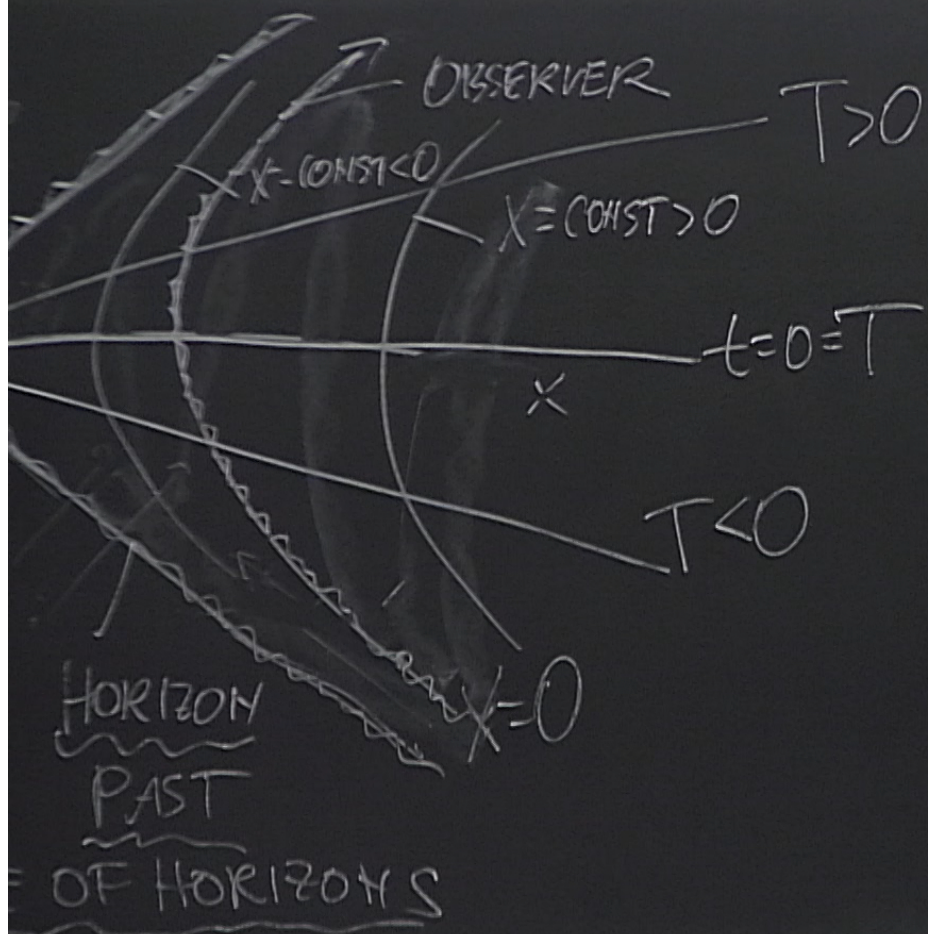


CONSEQUENCE 4: EXISTENCE OF HORIZONS

RINDLER



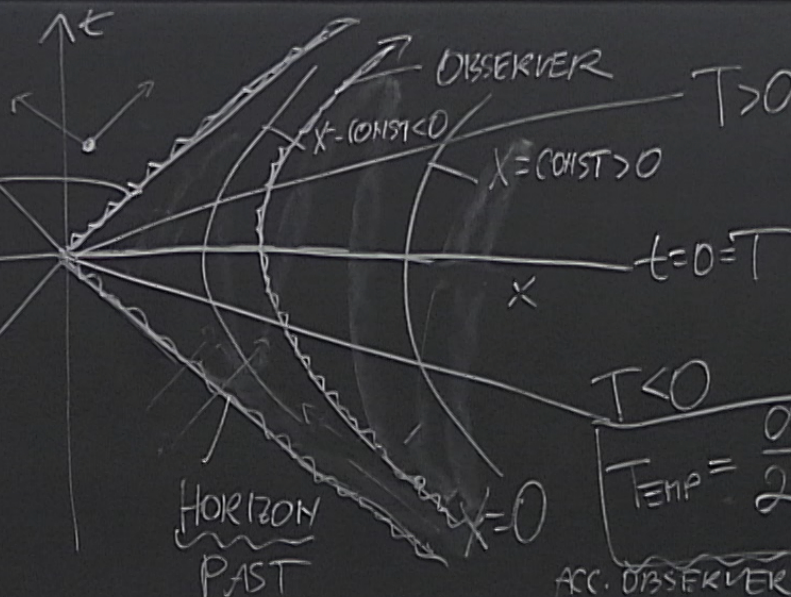
CONSEQUENCE 4: EXISTENCE OF HORIZONS



RINDLER

HORIZON
FUTURE

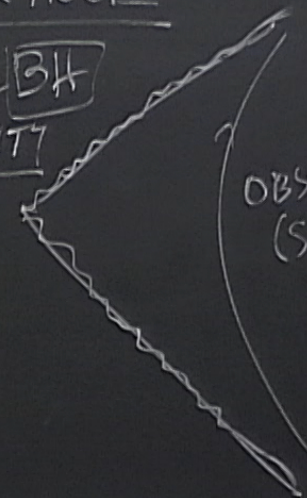
LIGHTCONES



CONSEQUENCE 4: EXISTENCE OF HORIZONS

BLACK HOLE

SINGULARITY

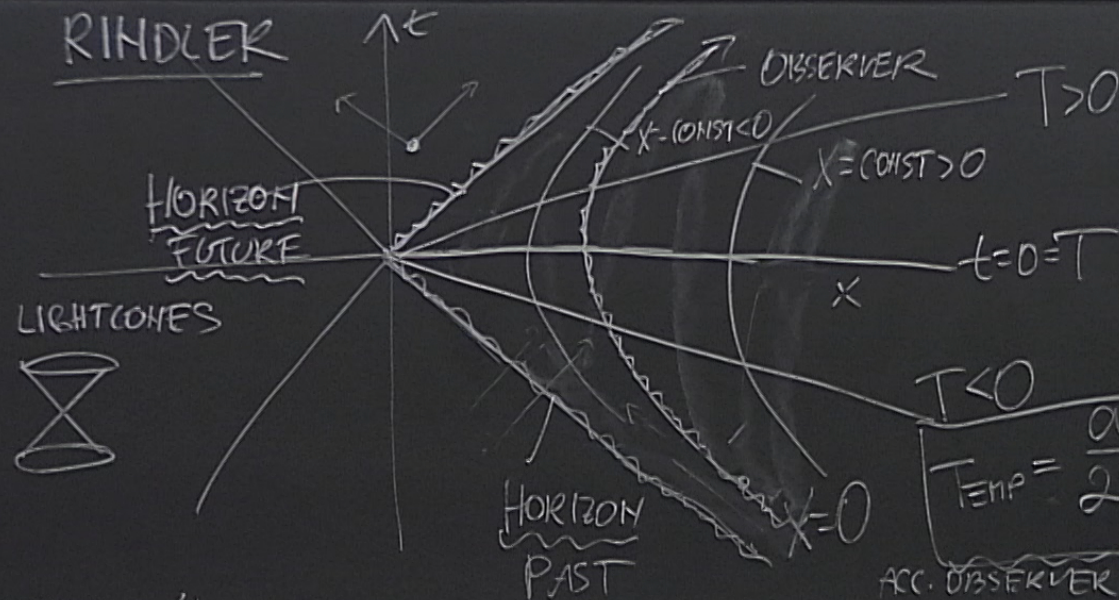


$$T < 0$$

$$T_{\text{TEMP}} = \frac{\alpha}{2\pi}$$

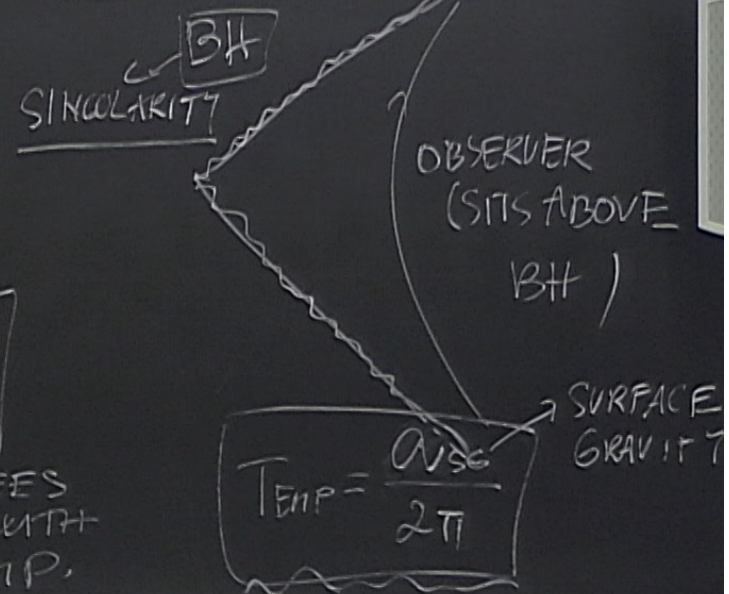
ACC. OBSERVER SEES
BLACK BODY RAD WITH
THIS TEMP.

RINDLER



CONSEQUENCE 4: EXISTENCE OF HORIZONS

BLACK HOLE



ACC. OBSERVER SEES BLACK BODY RAD WITH THIS TEMP.

OBSERVER
 $T > 0$
 $X = \text{CONST} > 0$
 $t = 0 = T$
 x

$T < 0$
 $T_{\text{EMP}} = \frac{a}{2\pi}$
 $x = 0$
 ACC. OBSERVER SEES
 BLACK BODY RAD WITH
 THIS TEMP.

BLACK HOLE

BH
SINGULARITY

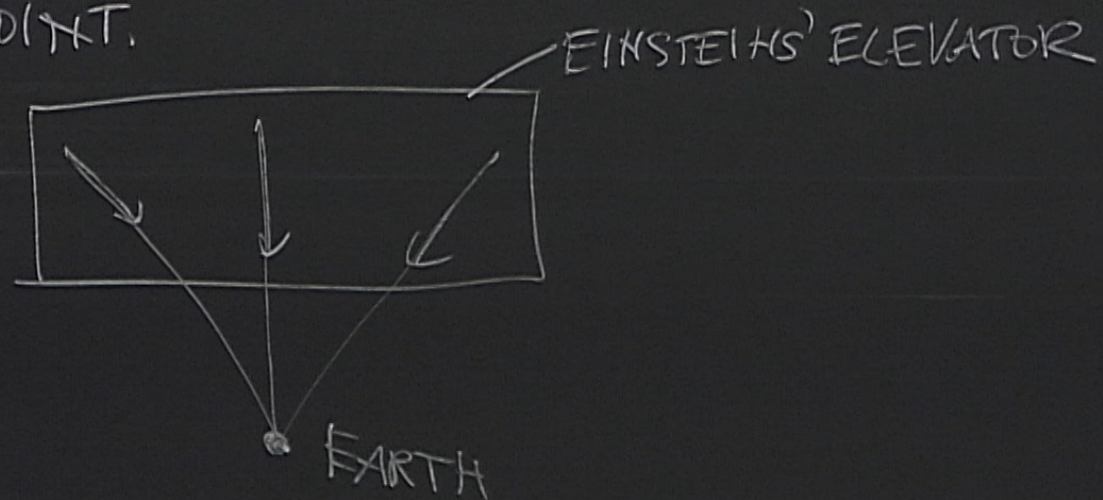
OBSERVER
 (SITS ABOVE
 BH)

$T_{\text{EMP}} = \frac{a_{\text{SG}}}{2\pi}$
 SURFACE
 GRAVITY.

a) TRUE GRAVITATIONAL FIELD

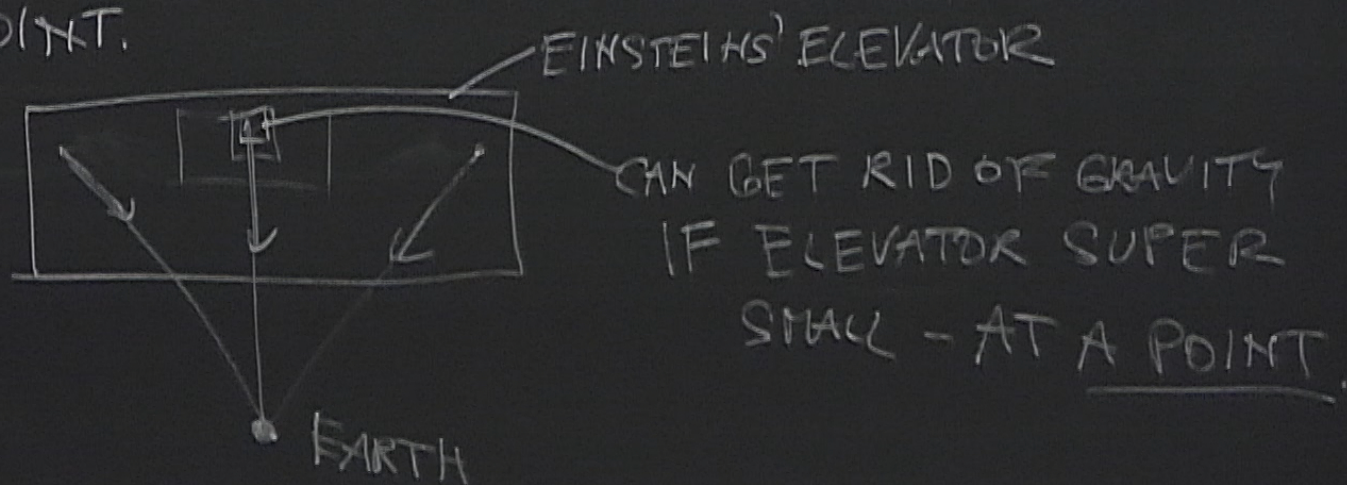
GENERAL GRAV. FIELD (IN HOP, TIME VARYING)

CANNOT BE FULLY MIMICKED BY ACCELERATION
EXCEPT AT A POINT.



d) TRUE GRAVITATIONAL FIELD

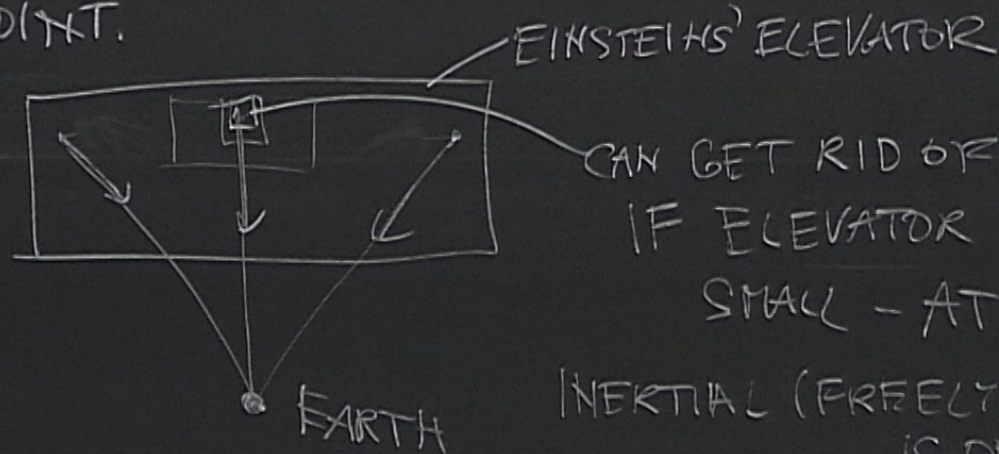
GENERAL GRAV. FIELD (IN HON, TIME VARYING)
CANNOT BE FULLY MIMICKED BY ACCELERATION
EXCEPT AT A POINT.



a) TRUE GRAVITATIONAL FIELD

GENERAL GRAV. FIELD (IN HON, THE VARYING)

CANNOT BE FULLY MIMICKED BY ACCELERATION
EXCEPT AT A POINT.



CAN GET RID OF GRAVITY
IF ELEVATOR SUPER

SMALL - AT A POINT.

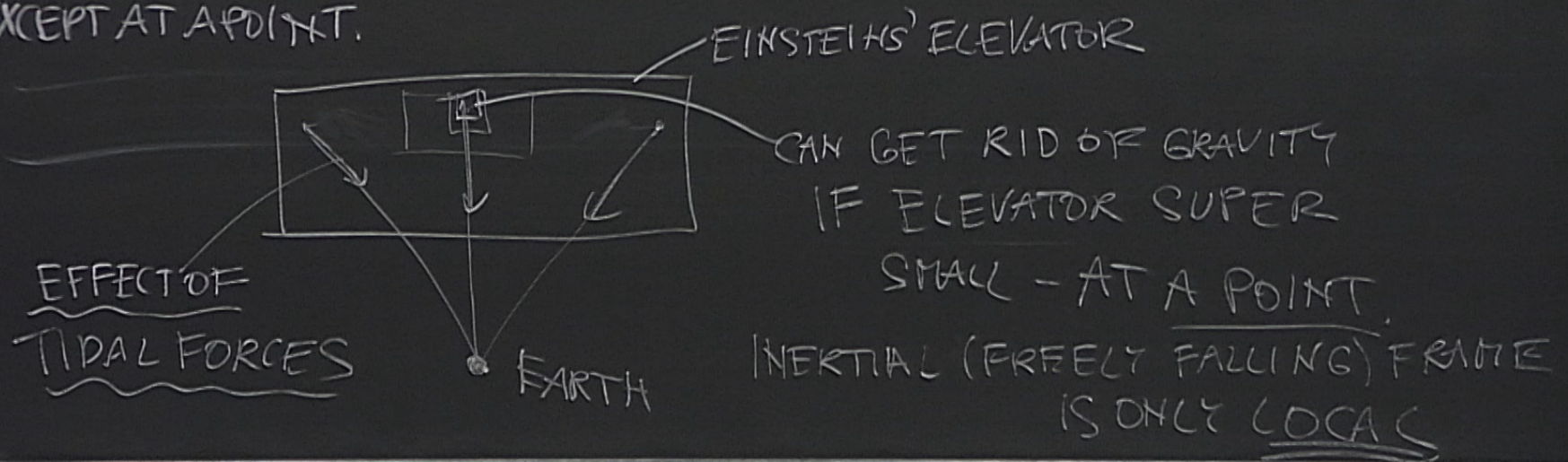
INERTIAL (FREELY FALLING) FRAME
IS ONLY LOCAL

a.k.a. generalized measurements

a) TRUE GRAVITATIONAL FIELD

GENERAL GRAV. FIELD (IN HON, THE VARYING)

CANNOT BE FULLY MIMICKED BY ACCELERATION
EXCEPT AT A POINT.



a.k.a. generalized measurements

TRUE GRAVITY.. DESCRIBED BY TIDAL FORCES.

.. WE CANNOT DO A COORDINATE TRANSF.
TO MINKOWSKI FRAME EVERYWHERE

$$g_{\mu\nu} = \frac{\partial x^\alpha}{\partial x^\mu} \frac{\partial x^\beta}{\partial x^\nu} \eta_{\alpha\beta} \quad \text{CHARACTERISTIC OF FAKE GRAV. F.}$$

ITY
ER

POINT.
(ING) FRAME
LOCAL

$$E_+ = \frac{3}{4} | +X + |$$

$$E_- = \frac{3}{4} | -X - |$$

$$V(t) = \frac{1}{2} (4 + (X+)^2 + (X-)^2)$$

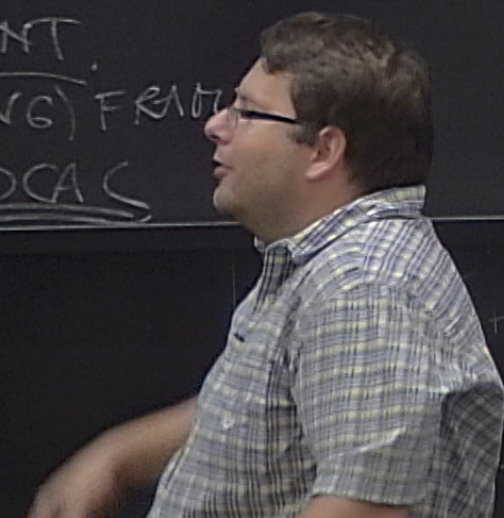
$$= \frac{3}{4} \langle 0 | +X + | 0 \rangle = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

TRUE GRAVITY.. DESCRIBED BY TIDAL FORCES.

.. WE CANNOT DO A COORDINATE TRANSF.
TO MINIKOWSKI FRAME EVERYWHERE

$$g_{\mu\nu} = \frac{\partial x^\alpha}{\partial x^\mu} \frac{\partial x^\beta}{\partial x^\nu} \eta_{\alpha\beta} \dots \text{CHARACTERISTIC OF FAKE GRAV. F.}$$

NOT POSSIBLE IN GENERAL.



YESTERDAY

- WE INTRODUCED CURVED METRIC

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad g_{\mu\nu} \text{ GENERAL } 4 \times 4 \text{ MATRIX}$$

- DERIVED GEODESIC EQUATION

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

CHRISTOFFEL
SYMBOLS

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} (g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu})$$



TRUE GRAVITY.. DESCRIBED BY TIDAL FORCES.

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NOT POSSIBLE IN GENERAL?

GRAVITATIONAL FIELD = GEOMETRY



$$F_{00} = \frac{1}{4} |0X0|$$

$$\frac{1}{4} |1X1|$$

Given $p = |0X0|$, what is $Pr(+)$

$$Pr(+)=Tr\left[\left(\frac{3}{4}|X+|\right)(|0X0|)\right]$$

2) SEARCH FOR FIELD THEORY FOR GRAVITY

	MAXWELL	NEWTON	EINSTEIN
FIELD	A^μ *	Φ_N	$g_{\mu\nu}$ *
EQ. OF MOTION	$\frac{dp^\mu}{d\tau} = e F^{\mu\nu} u_\nu$	$\frac{d\mathbf{p}}{dt} = -\nabla \Phi_N$	$\frac{dp^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} p^\alpha u^\beta = 0$
FIELD EQUATIONS	$\square A^\mu = -4\pi J^\mu$ $\partial_\mu A^\mu = 0$	$\nabla^2 \Phi_N = 4\pi G \rho$	(?)

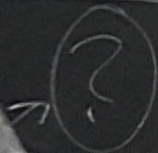
2) SEARCH FOR FIELD THEORY FOR GRAVITY

	MAXWELL	NEWTON	EINSTEIN
FIELD	A^μ *	Φ_N	$g_{\mu\nu}$ *
EQ. OF MOTION	$\frac{dp^\mu}{d\tau} = q F^{\mu\nu} u_\nu$	$\frac{d\mathbf{p}}{dt} = -\nabla \Phi_N$	$\frac{dp^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} p^\alpha u^\beta = 0$
FIELD EQUATIONS	$\square A^\mu = -4\pi J^\mu$ $\partial_\mu A^\mu = 0$	$\nabla^2 \Phi_N = 4\pi G \rho$	$\rightarrow (?)$ NEED TO LEARN DIFF. GEOM.

EINSTEIN

$$g_{\mu\nu} *$$

$$\int \alpha \beta \rho^\alpha_\mu \rho^\beta_\nu = 0$$



TO LEARN DIFF. GEOM.

SPOILER:

$$\nabla^2 \phi_H = 4\pi G \rho \quad \text{INVARIANT IN NEWTONIAN LIMIT}$$

$$-\nabla^2 g_{tt} = 8\pi G T_{tt} \quad \leftarrow \text{ENERGY MOM. TENSOR}$$

$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

$$G_{\mu\nu} = G_{\mu\nu}(g, \partial g, \partial^2 g)$$

EINSTEIN

$$g_{\mu\nu} *$$

$$\leftarrow (+2\partial_\mu) \frac{d\rho_\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} \rho^\alpha_\mu \rho^\beta_\mu = 0$$

$$\rho \rightarrow \pi (?)$$

NEED TO LEARN DIFF. GEOM.

SPOILER: $\nabla^2 \phi_H = 4\pi G \rho$ INVARIANT IN NEWTONIAN LIMIT

$$-\nabla^2 g_{tt} = 8\pi G T_{tt} \leftarrow \text{ENERGY MOM. TENSOR}$$

$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

$$G_{\mu\nu} = G_{\mu\nu}(g, \partial g, \partial^2 g)$$

$$\partial_\mu T^{\mu\nu} = 0 \rightarrow \boxed{\nabla_\mu T^{\mu\nu} = 0}$$

$$\Rightarrow \nabla_\mu G^{\mu\nu} = 0 \dots \text{CONSISTENCY} \dots \text{FOR ANY METRIC}$$

EINSTEIN

$$g_{\mu\nu} *$$

$$\leftarrow \frac{d\phi_H}{d\tau} + \Gamma_{\alpha\beta}^{\mu} p_{\mu}^{\alpha} p^{\beta} = 0$$

?

NEED TO LEARN DIFF. GEOM.

SPOILER: $\nabla^2 \phi_H = 4\pi G \rho$ INVARIANT IN NEWTONIAN LIMIT

$$-\nabla^2 g_{tt} = 8\pi G T_{tt} \leftarrow \text{ENERGY MOM. TENSOR}$$

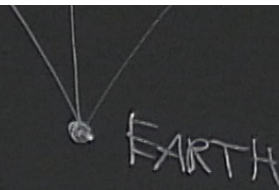
$$\boxed{G_{\mu\nu} = 8\pi G T_{\mu\nu}}$$

$$G_{\mu\nu} = G_{\mu\nu}(g, \partial g, \partial^2 g) \Rightarrow G_{\mu\nu} \text{ EINSTEIN TENSOR}$$

$$\Rightarrow \boxed{\partial_{\mu} T^{\mu\nu} = 0} \rightarrow \boxed{\nabla_{\mu} T^{\mu\nu} = 0}$$

$$\Rightarrow \nabla_{\mu} G^{\mu\nu} = 0 \dots \text{CONSISTENCY} \dots \text{FOR ANY METRIC}$$

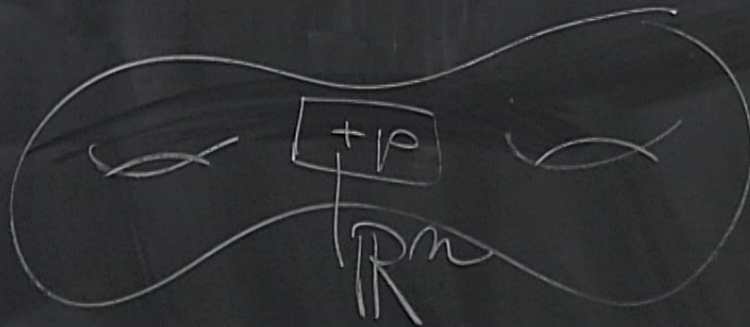
EFFECT OF
TIDAL FORCES



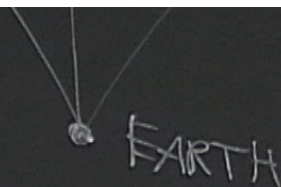
SMALL - AT A POINT
INERTIAL (FREELY FALLING
IS ONLY LOC

3) GEOMETRIC TOOLS

α) MANIFOLDS



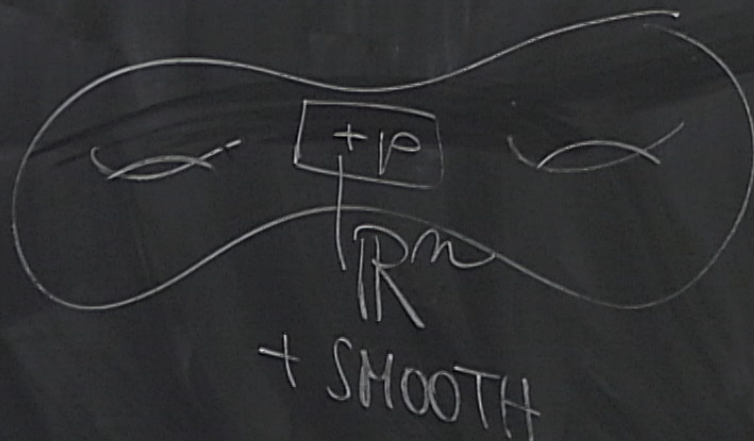
EFFECT OF
TIDAL FORCES



SMALL - AT A POINT
INERTIAL (FREELY FALLING
IS ONLY LOC

3) GEOMETRIC TOOLS

α) MANIFOLDS



SMALL - AT A POINT.

GRAVITATIONAL

LOCALLY LOOKS LIKE $\mathbb{R}^n$, BUT NOT GLOBALLY.

DEF: AN n -DIM MANIFOLD M IS A 'SET OF POINTS'
TOGETHER WITH A COLLECTION OF SUBSETS $\{O_\alpha\}$;
1) EACH $p \in M$ LIES IN AT LEAST ONE O_α .
 $\{O_\alpha\}$ COVER M .

LOCALLY LOOKS LIKE $\mathbb{R}^n$, BUT NOT GLOBALLY.

DEF. AN n -DIM MANIFOLD M IS A 'SET OF POINTS'
TOGETHER WITH A COLLECTION OF SUBSETS $\{O_\alpha\}$:

i) EACH $p \in M$ LIES IN AT LEAST ONE O_α .
 $\{O_\alpha\}$ COVER M .

ii) FOR EACH α , THERE IS 1-1, ONTO MAP $\psi_\alpha: O_\alpha \rightarrow U_\alpha$
WHICH IS AN OPEN SUBSET OF $\mathbb{R}^n$

red measurements

$$E_- = \frac{3}{4} |X|$$

BALL.

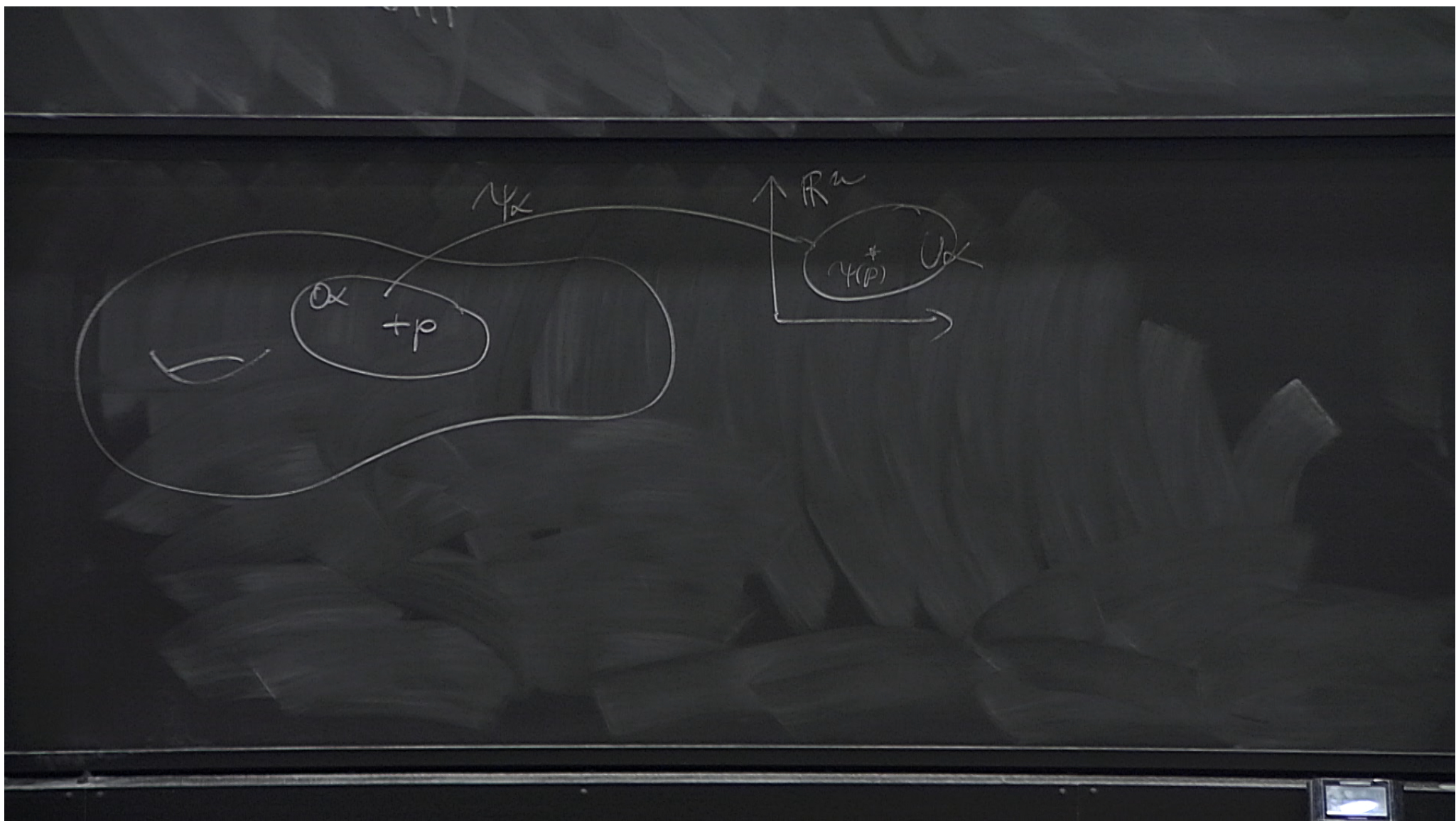
SET OF POINTS
SUBSETS $\{O_\alpha\}$:
THE O_α .

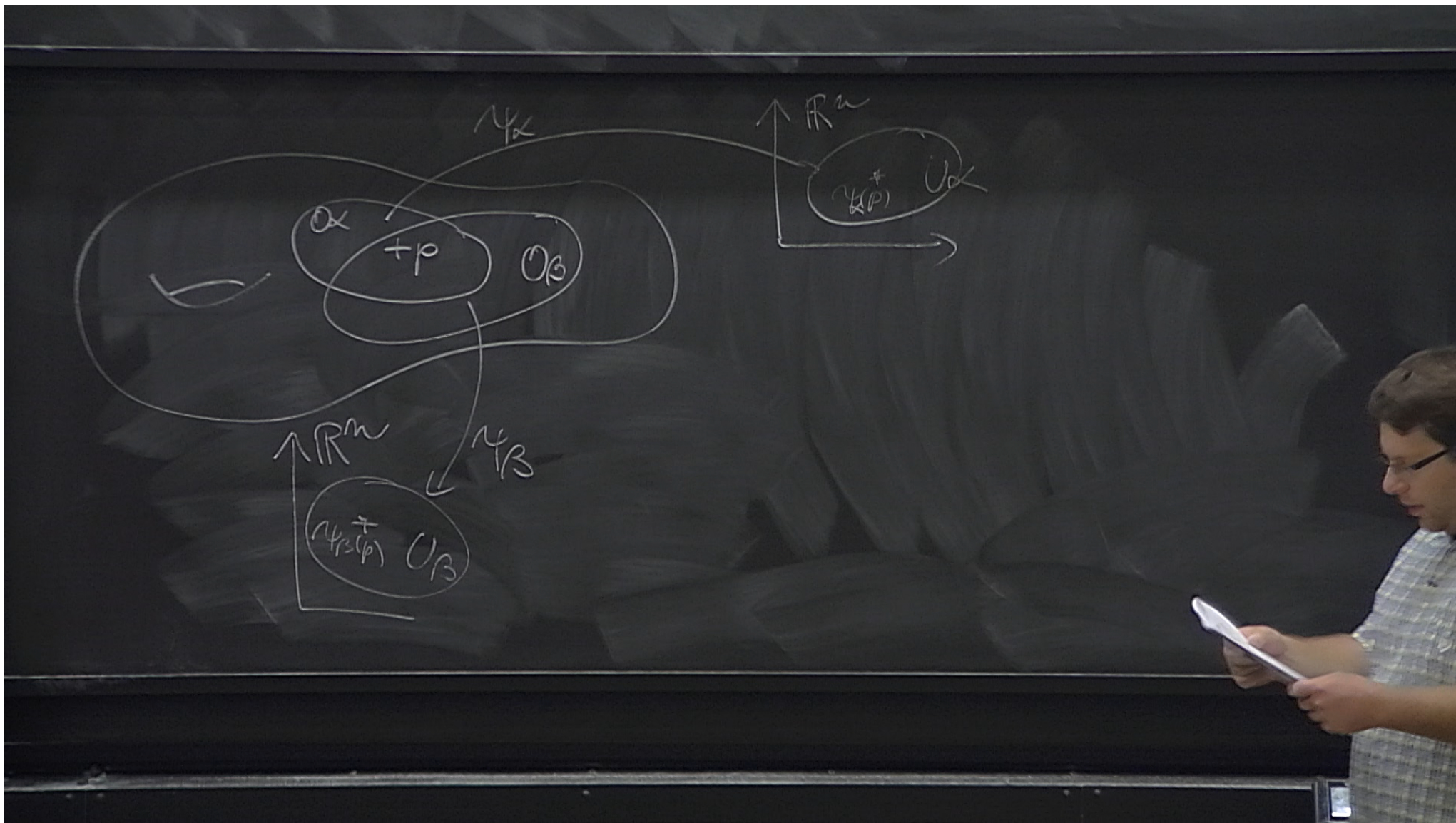
TO MAP $\psi_\alpha: O_\alpha \rightarrow U_\alpha$
 $\mathbb{R}^m$

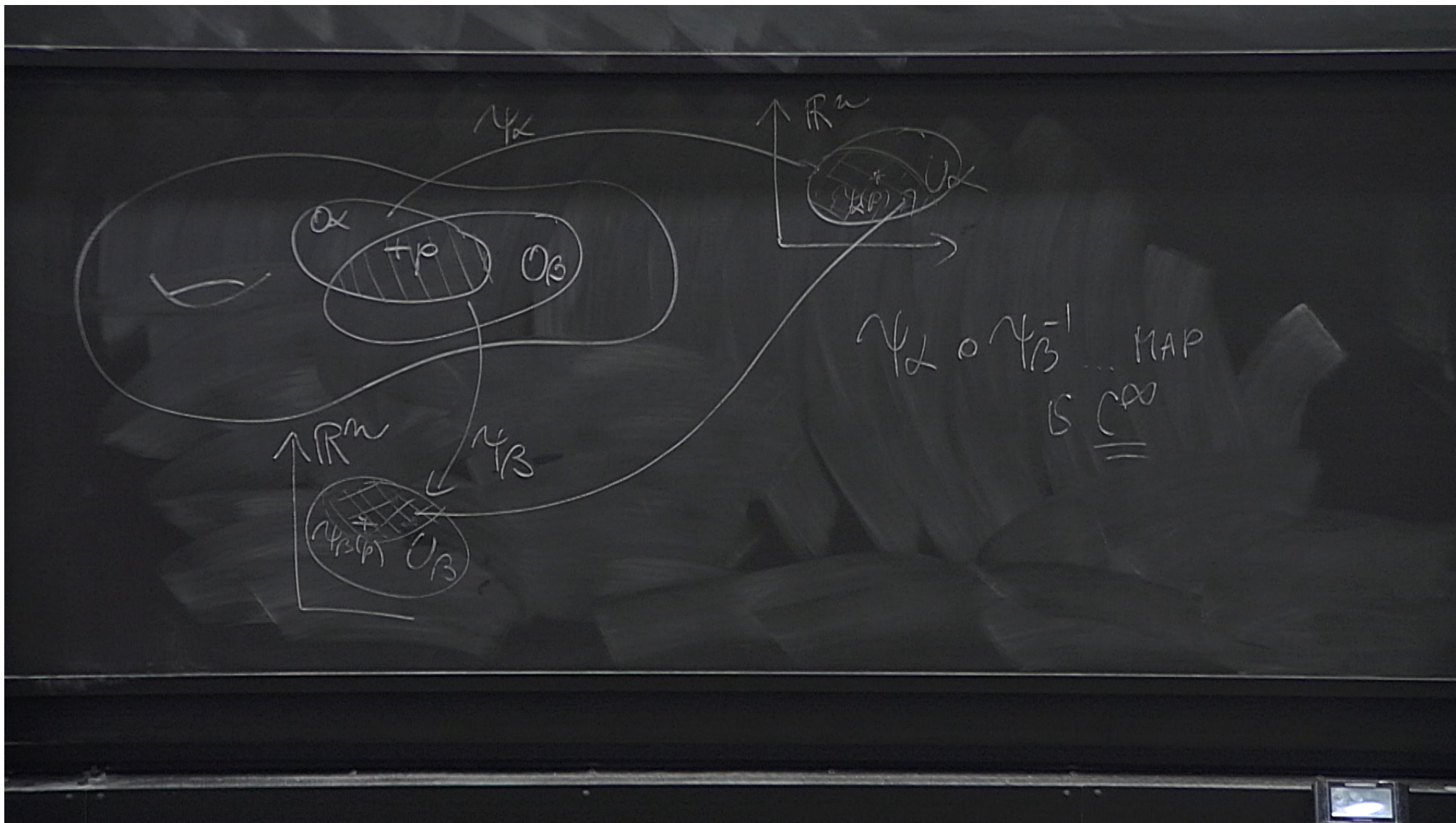
iii) IF ANY TWO SETS OF O_α & O_β
OVERLAP, $O_\alpha \cap O_\beta \neq \emptyset$, THEN MAP
 $\psi_\beta \circ \psi_\alpha^{-1}$ IS C^∞

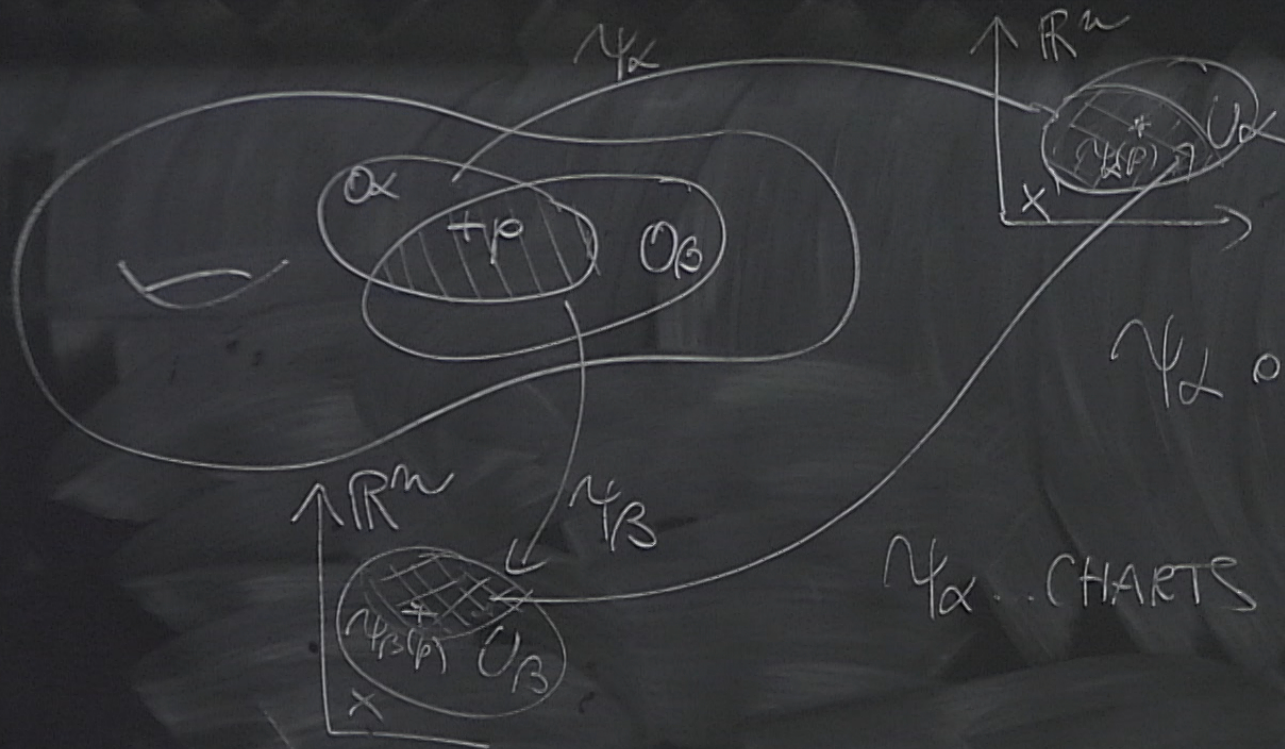
TIME CURVATURE DESCRIBED BY TENSOR FORCES







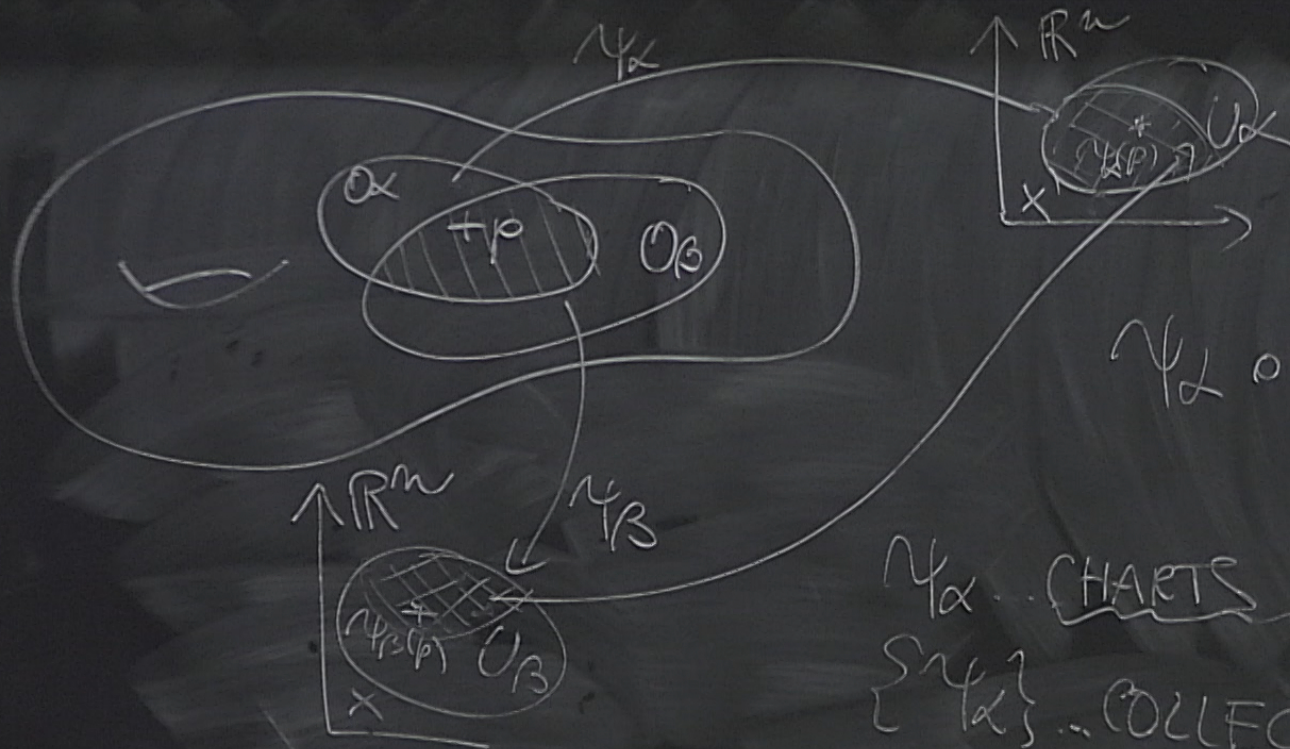




$$\psi_\alpha \circ \psi_\beta^{-1} \text{ MAP IS } \underline{\underline{C^\infty}}$$

ψ_α CHARTS (COORDINATE SYSTEMS)

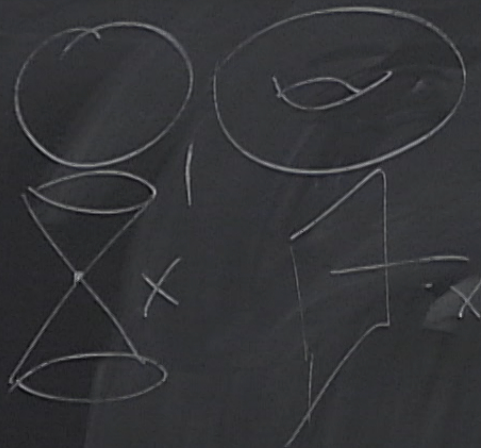
Measurements



$\psi_\alpha \circ \psi_\beta^{-1}$ MAP
IS $\mathbb{C}^\infty$

ψ_α CHARTS (COORDINATE SYSTEMS)
 $\{\psi_\alpha\}$ COLLECTION IS ATLAS

EXS:



SHAPED BY TIDAL FORCES.

A COORDINATE TRANSF.

SAME EVERYWHERE

AS B

CHARACTERISTIC

OF FAKE GRAV. F.

LO

EOMETRY

$$E_L = \frac{3}{4} |X|$$

$$= \frac{3}{4} \langle 0 | X + 1 | 0 \rangle - \frac{3}{4} \langle 2 | 8$$