

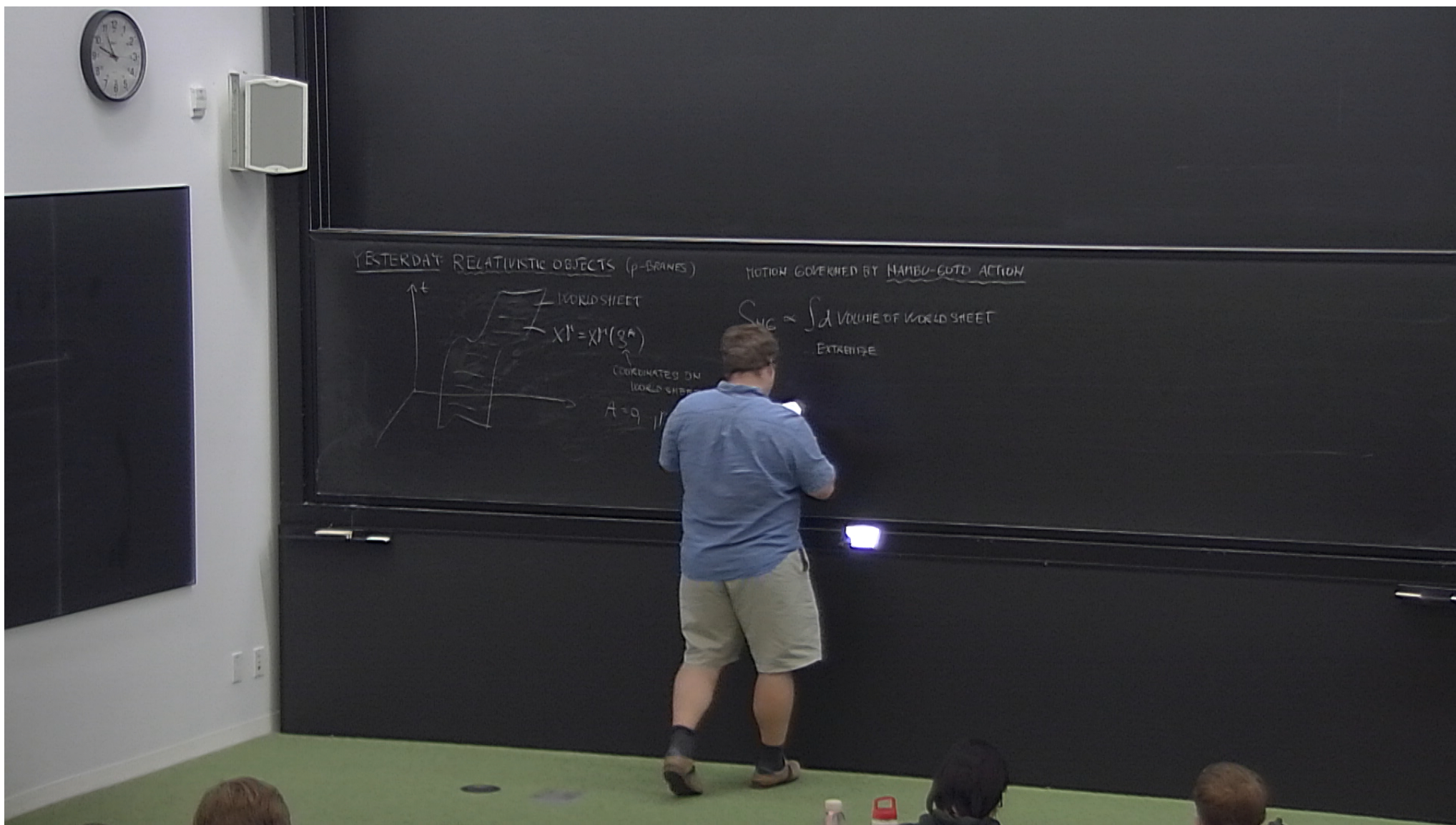
Title: PSI 2019/2020 - Relativity (Kubiznak) - Lecture 2

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Relativity (Kubiznak)

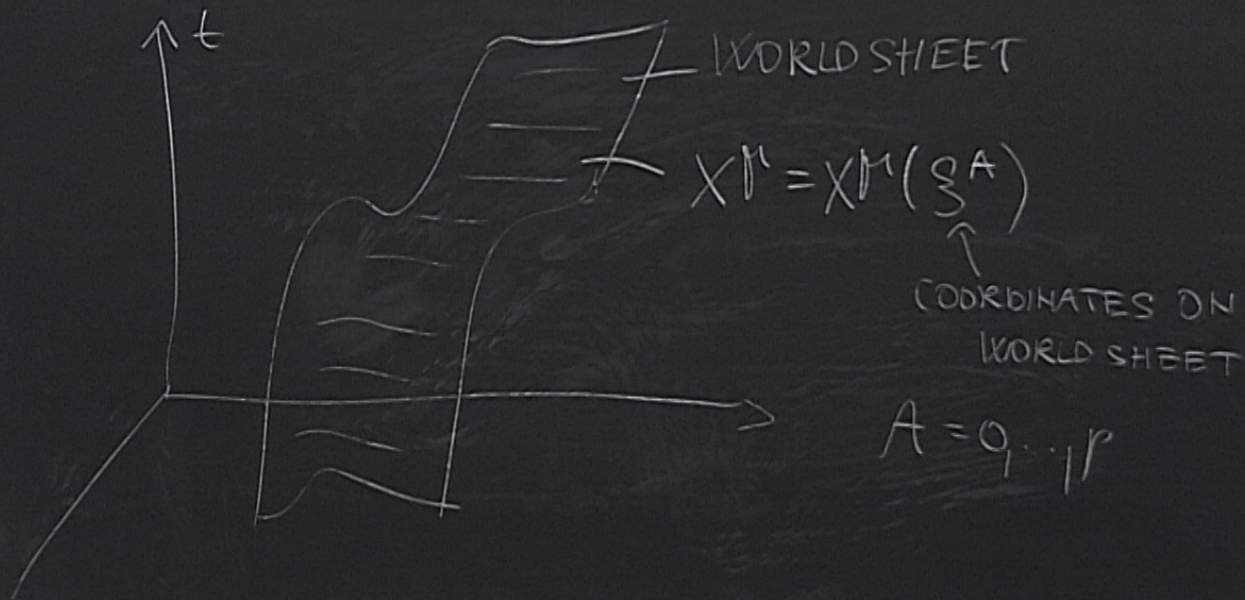
Date: September 04, 2019 - 10:45 AM

URL: <http://pirsa.org/19090030>



YESTERDAY: RELATIVISTIC OBJECTS (p-BRANES)

MOTION GOVE



$$S_{NG} \propto \int d\tau d\sigma$$

(p-BRANES)

MOTION GOVERNED BY Nambu-Goto ACTION

$$S_{NG} \propto \int d \text{ VOLUME OF WORLD SHEET}$$

...EXTREMIZE WORLDSHEET VOLUME

(p-BRANES)

MOTION GOVERNED BY Nambu-Goto ACTION

SPEC. PARTICLE

$$S_{NG} \propto \int d \text{ VOLUME OF WORLD SHEET}$$

... EXTREMIZE WORLDSHEET VOLUME

INDUCED METRIC ON WORLDSHEET

$$\eta_{AB} = \eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^A} \frac{\partial x^\nu}{\partial \xi^B}$$

$$S_{NG} \propto \int d\xi^0 \sqrt{-\det \eta_{AB}}$$

HEET

(ξ^A)

↑
COORDINATES ON
WORLD SHEET

$t = 0 \dots p$

ACTION

SPEC. PARTICLE $\{A = \int \lambda\}$

$$\gamma_{AB} \sim \gamma_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$$

$$S_{NG} = -mc^2 \int d\lambda \sqrt{-\gamma_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$

MOTION GOVERNED BY Nambu-Goto Action

$$S_{NG} \propto \int d \text{ VOLUME OF WORLD SHEET}$$

... EXTREMIZE WORLD SHEET VOLUME

INDUCED METRIC ON WORLD SHEET

$$g_{AB} = \eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^A} \frac{\partial x^\nu}{\partial \xi^B}$$

$$S_{NG} \propto \int d\xi^1 \sqrt{-\det g_{AB}}$$

SPEC. PARTICLE $\xi^A = \{\lambda\}$

$$\eta_{\mu\nu} \sim \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$$

$$S_{NG} = -m_0 c^2 \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$

GOVERNED BY Nambu-Goto Action

1 VOLUME OF WORLD SHEET

EXTREMIZE WORLD SHEET VOLUME

METRIC ON WORLD SHEET

$$g_{AB} = \eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^A} \frac{\partial x^\nu}{\partial \xi^B}$$

$$S = - \int d\xi^0 d\xi^1 \sqrt{-\det g_{AB}}$$

SPEC. PARTICLE $\xi^A = \{\tau, \sigma\}$

$$\dot{x}^\mu \sim \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$$

$$S_{NG} = -m_0 c^2 \int d\tau \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}$$

• NICE GEOMETRICAL MEANING

• PROBLEMS $\leftarrow$ WORKS ONLY FOR $m_0 \neq 0$
COMPLICATED (σ)

GOVERNED BY Nambu-Goto Action

Δ VOLUME OF WORLD SHEET

EXTREMIZE WORLD SHEET VOLUME

METRIC ON WORLD SHEET

$$g_{AB} = \eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^A} \frac{\partial x^\nu}{\partial \xi^B}$$
$$\propto \int d\xi^1 \sqrt{-\det g_{AB}}$$

SPEC. PARTICLE $\xi^A = \{\tau, \sigma\}$

$$\eta_{\mu\nu} \sim \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$$

$$S_{NG} = -m_0 c^2 \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$

• NICE GEOMETRICAL MEANING

• PROBLEMS $\leftarrow$ WORKS ONLY FOR $m_0 \neq 0$

COMPLICATED ($\sqrt{\quad}$)

... PROBLEM FOR QUANTIZATION

CAN CONSIDER POLTAKOV ACTION INSTEAD

$$S_{\text{PARTICLE}} = \frac{c^2}{2} \int \left(\frac{1}{e} \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} - m_0^2 e \right) d\lambda$$

- "LOOKS LIKE HO" ... WE CAN QUANTIZE IT.
- PRICE TO PAY : INTRODUCED EXTRA FIELD $e = e(\lambda)$

CAN CONSIDER POLTAKOV ACTION INSTEAD

$$S_{\text{PARTICLE}} = \frac{c^2}{2} \int \left(\frac{1}{e} \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} - m_0^2 e \right) d\lambda$$

- "LOOKS LIKE HO" ... WE CAN QUANTIZE IT.
- PRICE TO PAY: INTRODUCED EXTRA FIELD $e = e(\lambda)$
... LAGRANGE MULTIPLIER (NO $\frac{de}{d\lambda}$ ANYWHERE)

δe :

$$e = \frac{1}{m_0} \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \quad \dots \text{PLUG BACK}$$

... NG

- ACTIONS ARE CLASSICALLY EQUIVALENT.
- MOREOVER WE CAN TAKE $m_0 = 0$,

$$S_{\text{MASSLESS}} = \frac{c^2}{2} \int \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda$$

- POLYAKOV ACTION

• ACTIONS ARE CLASSICALLY EQUIVALENT.

• MOREOVER WE CAN TAKE $m_0 = 0$,

$$S_{\text{MASSLESS}} = \frac{c^2}{2} \int \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda$$

• POLYAKOV ACTION FOR A STRING

$$S \propto \int d^2\xi \sqrt{-h} h^{AB} \tilde{m}_{AB}$$

INDUCED METRIC
AUXILIARY METRIC

$$h = \det h_{AB}$$

- ACTIONS ARE CLASSICALLY EQUIVALENT.
- MOREOVER WE CAN TAKE $m_0 = 0$,

$$S_{\text{MASSLESS}} = \frac{c^2}{2} \int \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda$$

- POLYAKOV ACTION FOR A STRING

$$S \propto \int d^2\xi \sqrt{-h} h^{AB} \gamma_{AB}$$

AUXILIARY METRIC

INDUCED METRIC

$$h = \det h_{AB}$$

MASSLESS STRING

NOTE: SCALAR FIELD ACTION (d-DIMS)

$$S_\phi = \int d^d x \left(-\frac{1}{2} (\nabla \phi)^2 + \dots \right)$$

CURVED SPACE . . η_{AB} . . CURVED METRIC

$$S_\phi = \int d^d x \sqrt{-h} \left(-\frac{1}{2} h^{AB} \nabla_A \phi \nabla_B \phi \right)$$

LOOK AT STRING ACTION:

$$S \propto \int d\tau d\sigma \sqrt{-h} h^{AB} \eta_{\mu\nu} \nabla_A X^\mu \nabla_B X^\nu$$

$$(\nabla_A \psi \nabla_B \psi)$$

LOOK AT STRING ACTION:

$$S \propto \int d\tau \int d\sigma \sqrt{-h} h^{AB} \left(\eta_{\mu\nu} \partial_A X^\mu \partial_B X^\nu \right) \\ - \underbrace{\partial_A X^0 \partial_B X^0 + \partial_A X^1 \partial_B X^1}_{\text{4 SCALAR FIELDS } X^\mu \text{ THAT LIVE IN } \underline{2D}} + \dots$$

LOOK AT STRING ACTION:

$$S \propto \int d\tau \int d\sigma \sqrt{-h} h^{AB} \left(\eta_{\mu\nu} \partial_A X^\mu \partial_B X^\nu \right) \\ - \underbrace{\partial_A X^0 \partial_B X^0 + \partial_A X^1 \partial_B X^1} + \dots$$

4 SCALAR FIELDS X^μ THAT LIVE IN 2D.

IN 2D FREE SCALAR FIELD HAS CONFORMAL SYMMETRY

2D CFT

2) FIRST LOOK AT GR

WE WANT TO INCLUDE
GRAVITATIONAL FIELD

a) CONCEPTUAL PATH TO THEORY

2) FIRST LOOK AT GR

WE WANT TO INCLUDE
GRAVITATIONAL FIELD

a) CONCEPTUAL PATH TO THEORY

INGREDIENT 1 : CONSISTENT FIELD THEORY OF
GRAVITY WITH SPECIAL RELATIVITY

IN PARTICULAR GET RID OF ACTION AT DISTANCE

REHIND: MAXWELL THEORY

FIELDS: $(\Phi, A) = A^\mu$

MAXWELL: $\square A^\mu = -4\pi J^\mu$

$$\partial_\mu A^\mu = 0$$

EQ OF MOTION: $\frac{dp}{dt} = q(E + v \times B)$

REVIEW: MAXWELL THEORY

FIELDS: $(\Phi, A) = A^\mu$

MAXWELL: $\square A^\mu = -4\pi J^\mu$

$\partial_\mu A^\mu = 0 \quad \checkmark$

EQ OF MOTION: $\frac{dp}{dt} = q(E + v \times B)$

ELECTROSTATIC LIMIT ($A \rightarrow 0, \partial_t \rightarrow 0$)

$$\Delta \Phi = -4\pi \rho$$
$$\frac{dp}{dt} = -q \nabla \Phi$$

REVIEW: MAXWELL THEORY

FIELDS: $(\Phi, A) = A^\mu$

MAXWELL: $\square A^\mu = -4\pi j^\mu$

$\partial_\mu A^\mu = 0 \quad \checkmark$

EQ OF MOTION: $\frac{dp}{dt} = q(E + v \times B)$

ELECTROSTATIC LIMIT ($A=0, \partial_t \rightarrow 0$)

$$\Delta \Phi = -4\pi q$$
$$\frac{dp}{dt} = -q \nabla \Phi$$

COMPARE TO NEWTON ... NEWTONS POTENTIAL Φ_N

$$\Delta \Phi_N = 4\pi G \rho_m$$
$$\frac{dp}{dt} = -m \nabla \Phi$$

REVIEW: MAXWELL THEORY

FIELDS: $(\Phi, A) = A^\mu$

MAXWELL: $\square A^\mu = -4\pi J^\mu$

$\partial_\mu A^\mu = 0 \quad \checkmark$

EQ OF MOTION: $\frac{dp}{dt} = q(E + v \times B)$

ELECTROSTATIC LIMIT ($A \rightarrow 0, \partial_t \rightarrow 0$)

$$\Delta \Phi = -4\pi \rho$$
$$\frac{dp}{dt} = -q \nabla \Phi$$

COMPARE TO NEWTON ... NEWTON'S POTENTIAL Φ_N

$$\Delta \Phi_N = 4\pi G \rho_m$$
$$\frac{dp}{dt} = -m \nabla \Phi$$

NEWTON'S THEORY



2) FIRST LOOK AT GR

WE WANT TO INCLUDE
GRAVITATIONAL FIELD

REHIND: MAXWELL

FIELDS: $(\Phi, \mathbf{A})$

MAXWELL: $\square A$

∂_μ

EQ OF MOTION:

a) CONCEPTUAL PATH TO THEORY

INGREDIENT 1: CONSISTENT FIELD THEORY OF
GRAVITY WITH SPECIAL RELATIVITY

IN PARTICULAR GET RID OF ACTION AT DISTANCE

CAN WE WRITE MAXWELL-LIKE THEORY FOR GRAVITY?

INGREDIENT 2. RELATIVITY OF MOTION

HISTORICALLY SPACE $\left\{ \begin{array}{l} \text{ABSOLUTE ENTITY} \\ \text{RELATION .. ADVANCEMENT RELATION} \\ \text{BETWEEN OBJECTS} \\ \text{(A.GREEKS)} \end{array} \right.$

INGREDIENT 2. RELATIVITY OF MOTION

HISTORICALLY: SPACE $\left\{ \begin{array}{l} \text{ABSOLUTE ENTITY (NEWTON)} \\ \text{RELATION .. ADVANCEMENT RELATION} \\ \text{BETWEEN OBJECTS} \\ \text{(A. GREEKS)} \end{array} \right.$

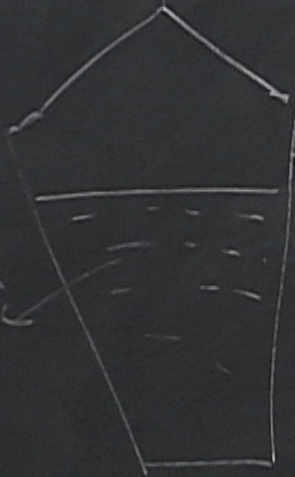
INGREDIENT 2. RELATIVITY OF MOTION

HISTORICALLY SPACE $\left\{ \begin{array}{l} \text{ABSOLUTE ENTITY (NEWTON)} \\ \text{RELATION .. ADVANCEMENT RELATION} \\ \text{BETWEEN OBJECTS} \\ \text{(A.GREEKS)} \end{array} \right.$

NEWTON INTRODUCED ABSOLUTE SPACE

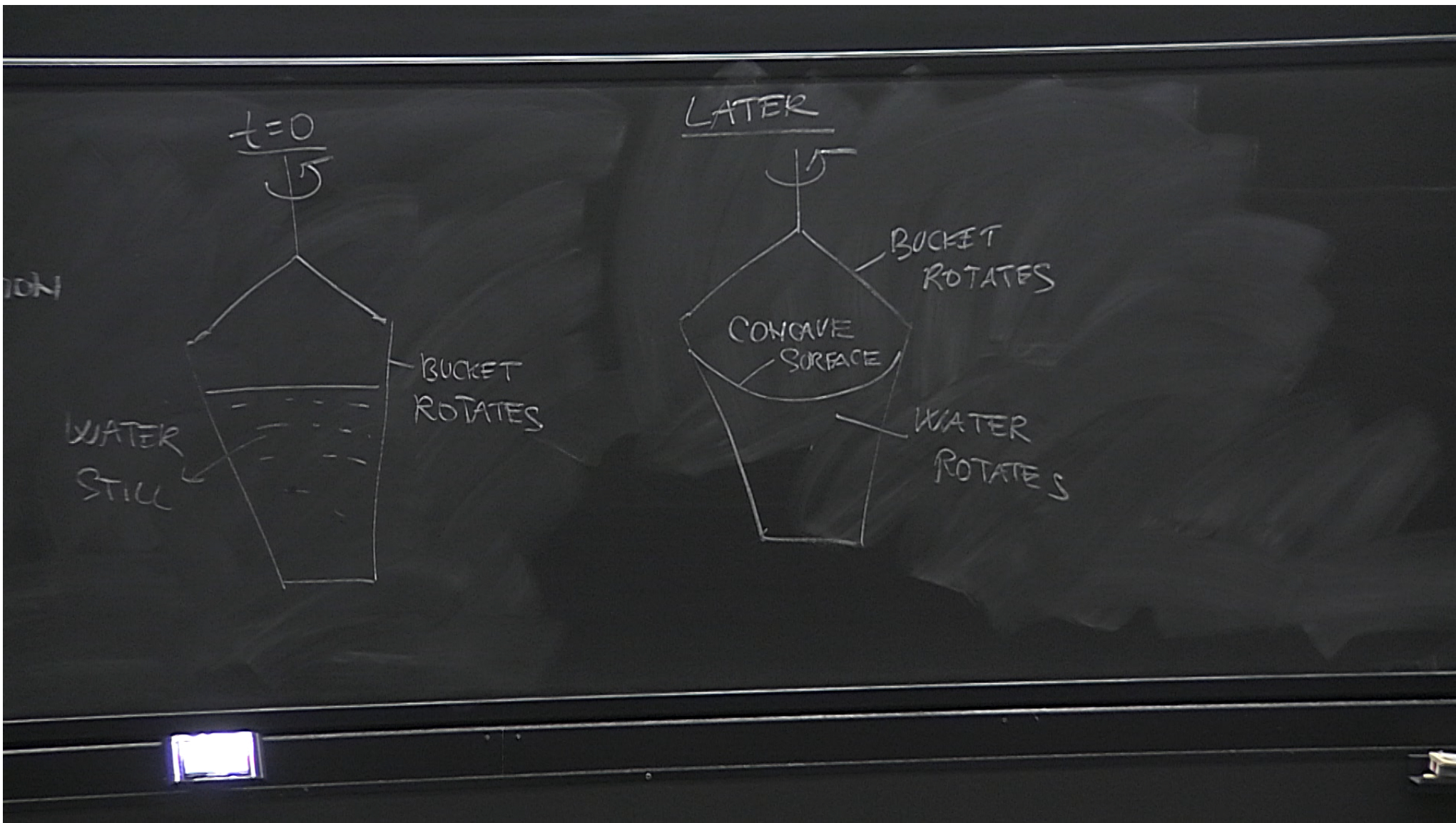
BASED ON THE FOLLOWING NEWTON'S BUCKET
EXPERIMENT

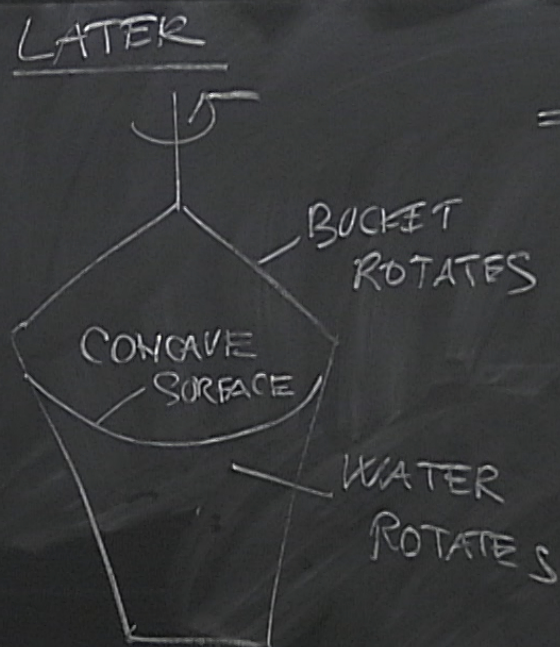
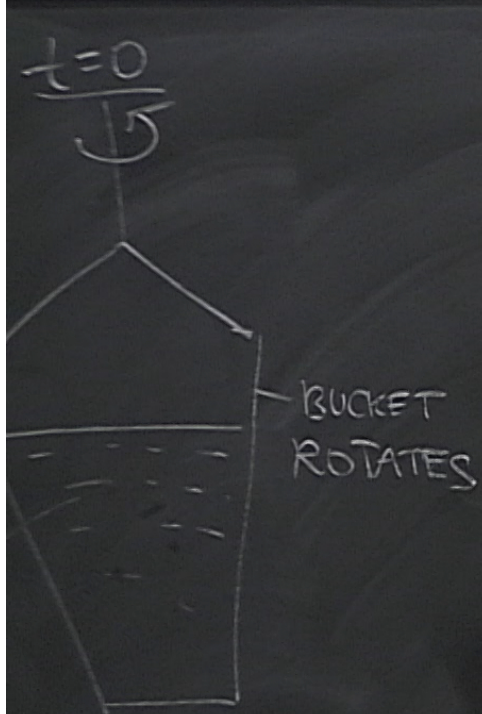
$t=0$
5



BUCKET
ROTATES

WATER
STILL





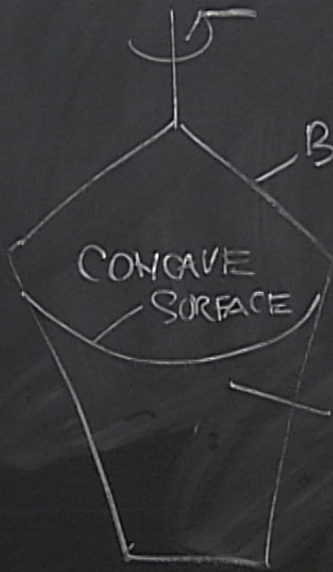
$\Rightarrow$ EXISTENCE OF
ABSOLUTE SPACE

10
5



BUCKET
ROTATES

LATER

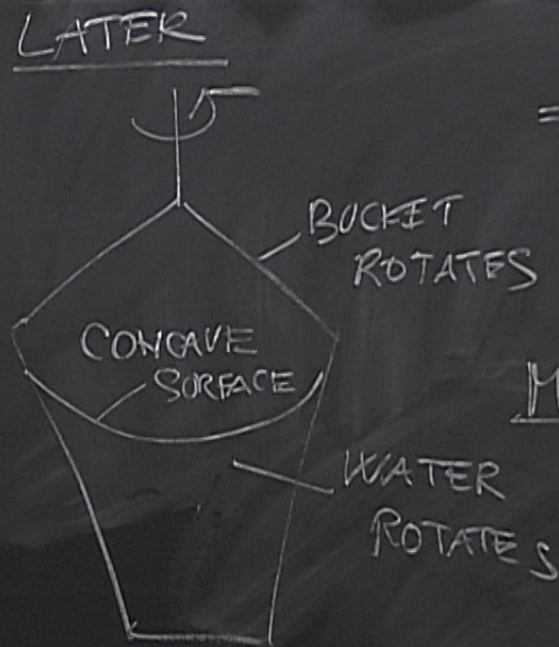
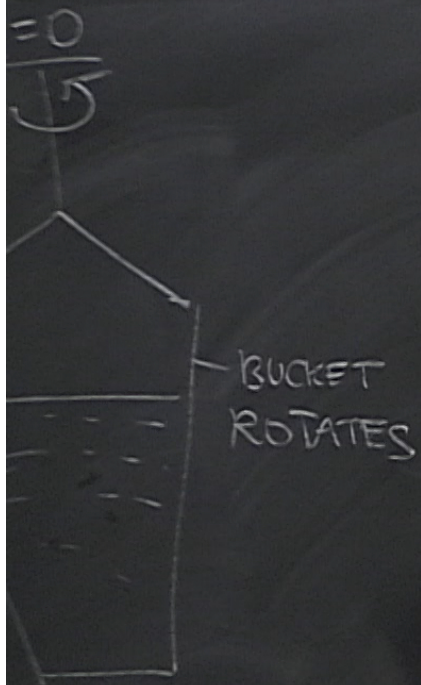


BUCKET
ROTATES

WATER
ROTATES

=> EXISTENCE OF
ABSOLUTE SPACE
TOOK 300 YEARS

MACH:



$\Rightarrow$ EXISTENCE OF ABSOLUTE SPACE

TOOK 300 YEARS

MACH: MOTION W.R.T THE
MATTER CONTENT OF
UNIVERSE (DISTANT)
STARS

GRAVITY WITH SPEED
IN PARTICULAR GET RID OF ACTION AT DISTANCE

EINSTEIN: LIKES RELATIVE

HATES ACTION AT DISTANCE

RELATIVE MOTION W.R.T GRAV. FIELD
(LOCAL)

GRAVITY WITH SPECIAL RELATIVITY
IN PARTICULAR GET RID OF ACTION AT DISTANCE

EINSTEIN: LIKES RELATIVE

HATES ACTION AT DISTANCE

RELATIVE MOTION W.R.T GRAV. FIELD
(LOCAL)

INERTIAL ACCELERATION $\approx$ GRAV. FORCE

GRAVITY DETERMINES INERTIAL FRAME

GRAVITY WITH SPEED
IN PARTICULAR GET RID OF ACTION AT DISTANCE

EINSTEIN: LIKES RELATIVE

HATES ACTION AT DISTANCE

RELATIVE MOTION W.R.T GRAV. FIELD
(LOCAL)

INERTIAL ACCELERATION $\approx$ GRAV. FORCE

GRAVITY DETERMINES INERTIAL FRAME

$\Rightarrow$ CANNOT HAVE ANOTHER MAXWELL-LIKE
THEORY IN FLAT SPACE

b) PRINCIPLE OF EQUIVALENCE

$$F = m_I a = m_g g$$

INERTIAL MASS

GRAVITATIONAL MASS

$$m_I = m_g$$

(EXPERIMENTS)

$\Rightarrow$

$$\boxed{a = g}$$

ALL BODIES IN A GIVEN GRAV FIELD
EXPERIENCE THE SAME ACCELERATION

VALENCE

~~$g = \frac{d^2x}{dt^2}$~~

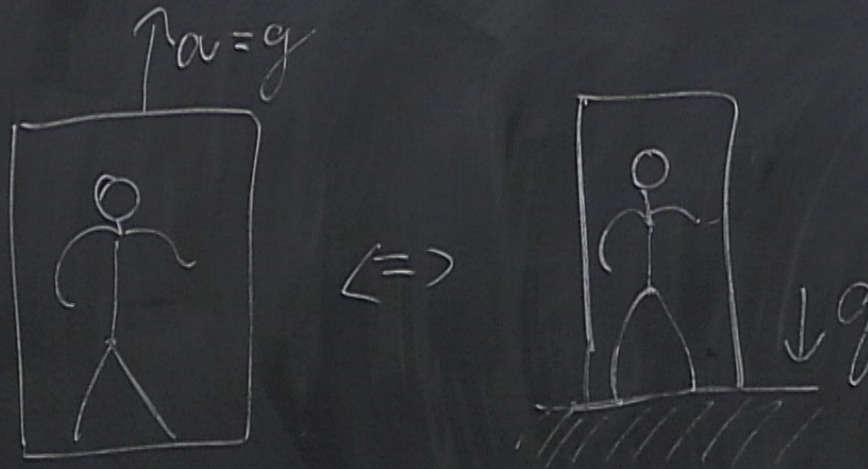
GRAVITATIONAL MASS

$a = g$

GIVEN GRAV FIELD

SAME ACCELERATION

• EINSTEIN'S ELEVATOR (MOST GENIUS IDEAS)



VALENCE

$$g = \cancel{E}$$

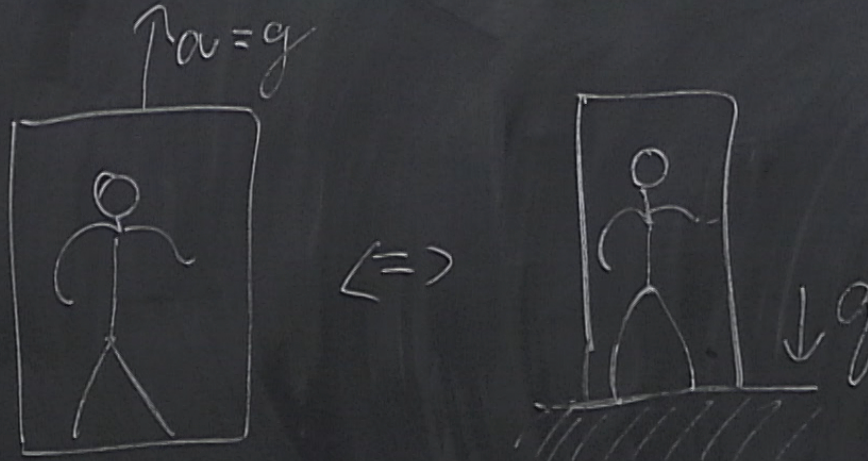
GRAVITATIONAL MASS

$$\boxed{a = g}$$

GIVEN GRAV FIELD

SAME ACCELERATION

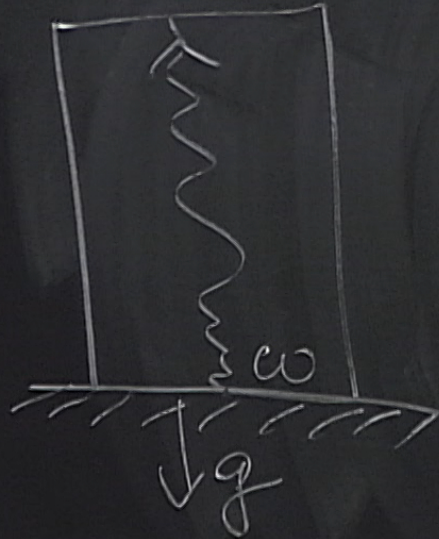
• EINSTEIN'S ELEVATOR (MOST GENIUS)



CAN MIMIC GRAV. FIELD
BY ACCELERATION

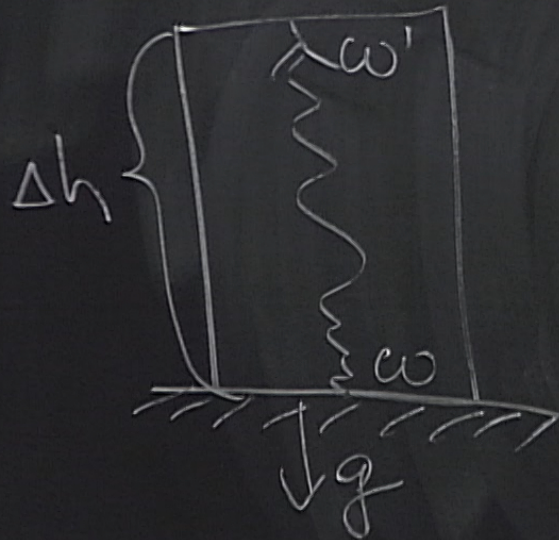
IN PARTICULAR GET RID OF ACTION AT DISTANCE

CONSEQUENCE 1: GRAVITATIONAL REDSHIFT

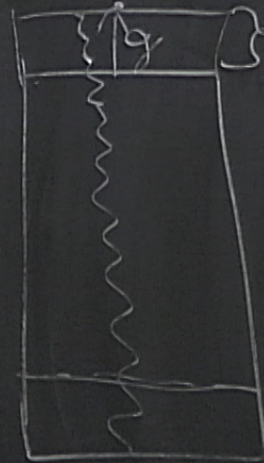


IN PARTICULAR GET RID OF ACTION AT DISTANCE

CONSEQUENCE 1: GRAVITATIONAL REDSHIFT



$\Leftrightarrow$



$$\frac{1}{2}gt^2, v=gt$$

$$ct = \Delta h + \frac{1}{2}gt^2$$

$$v = gt = g \frac{\Delta h}{c}$$

WE EXPERIENCE DOPPLER SHIFT

$$(\omega' < \omega)$$

BOTTOM: $k^\mu = \frac{\omega}{c}(1, 0, 0, 1)$ $\xrightarrow{z}$ DOPPLER

UP: $k'^\mu = \frac{\omega'}{c}(1, 0, 0, 1) = \gamma \frac{\omega}{c}(1 - \beta)(1, 0, 0, 1)$

$$\omega' = \gamma \omega (1 - \beta)$$

WE EXPERIENCE DOPPLER SHIFT

$$(\omega' < \omega)$$

BOTTOM:

$$k^\mu = \frac{\omega}{c} (1, 0, 0, 1)$$

DOPPLER

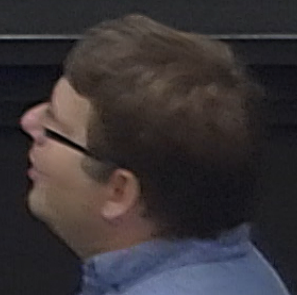
UP:

$$k^\mu = \frac{\omega'}{c} (1, 0, 0, 1)$$

$$= \gamma \frac{\omega}{c} (1 - \beta) (1, 0, 0, 1)$$

$$\omega' = \gamma \omega (1 - \beta) \approx \omega (1 - \frac{v}{c}) = \omega (1 - \frac{g \Delta h}{c^2})$$

$$Z = \frac{\omega' - \omega}{\omega} \approx - \frac{g \Delta h}{c^2}$$



WE EXPERIENCE DOPPLER SHIFT

$$(\omega' < \omega)$$

BOTTOM: $k^\mu = \frac{\omega}{c}(1, 0, 0, 1)$ ✓ DOPPLER

UP: $k^\mu = \frac{\omega'}{c}(1, 0, 0, 1) = \gamma \frac{\omega}{c}(1 - \beta)(1, 0, 0, 1)$

$$\omega' = \gamma \omega (1 - \beta) \approx \omega \left(1 - \frac{v}{c}\right) = \omega \left(1 - \frac{g \Delta h}{c^2}\right)$$

$$Z = \frac{\omega' - \omega}{\omega} = -\frac{g \Delta h}{c^2}$$

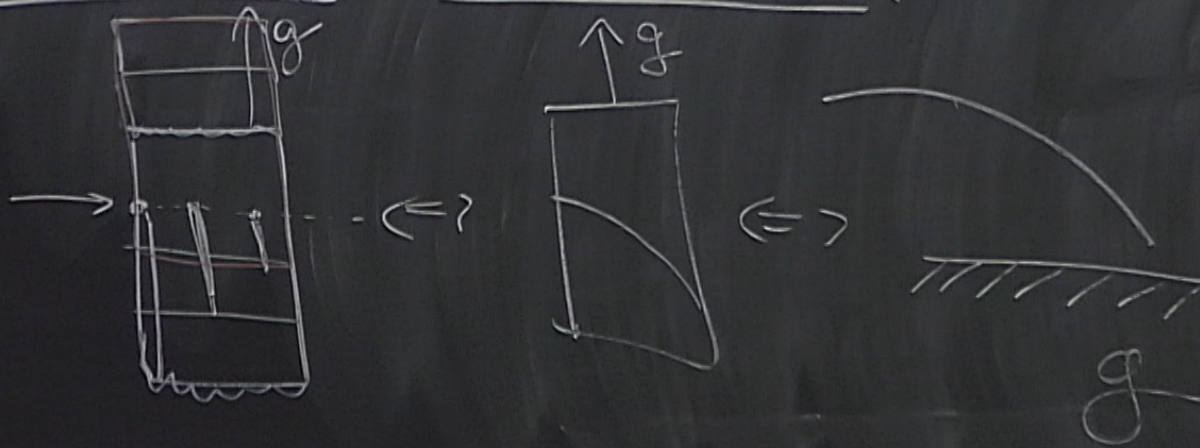
POUND & REBKA (1960)

($\Delta h = 22 \text{ m}$)

$$Z \approx 2.5 \times 10^{-15}$$

AT THE BOTTOM CLOCK GOES MORE SLOWLY?

CONSEQUENCE 2. LIGHT BENDING (CURVED SPACE)



GRAVITY CHANGES WHAT
IT MEANS FOR A PATH TO
BE STRAIGHT ⁶

GRAVITY CHANGES WHAT
IT MEANS FOR A PATH TO
BE STRAIGHT⁶
(CURVES SPACETIME)