Title: Nonsingular transition from an evaporating black hole to a white hole

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Abstract: A "quasi-classical" picture of the transition from an evaporating black hole to a white hole is described, which is based on a resolution of the Schwarzschild singularity suggested by loop quantum gravity. All quantum information trapped by the black hole is eventually released from the white hole, without any Cauchy horizons, consistent with unitarity. The effective stress-energy tensor suggests that inflow of negative energy associated with Hawking "partners" in the interior of the black hole becomes, at least initially, an outflow of negative energy from the white hole. Alternative scenarios for the further evolution of the white hole and their implications will be discussed.

Nonsingular transition from an evaporating black hole to a white hole

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Hawking radiation

Hawking's landmark result from 1974-1975 showed that in the semi-classical approximation black holes radiate with a quasi-thermal spectrum at the Hawking temperature (units G=c=1)

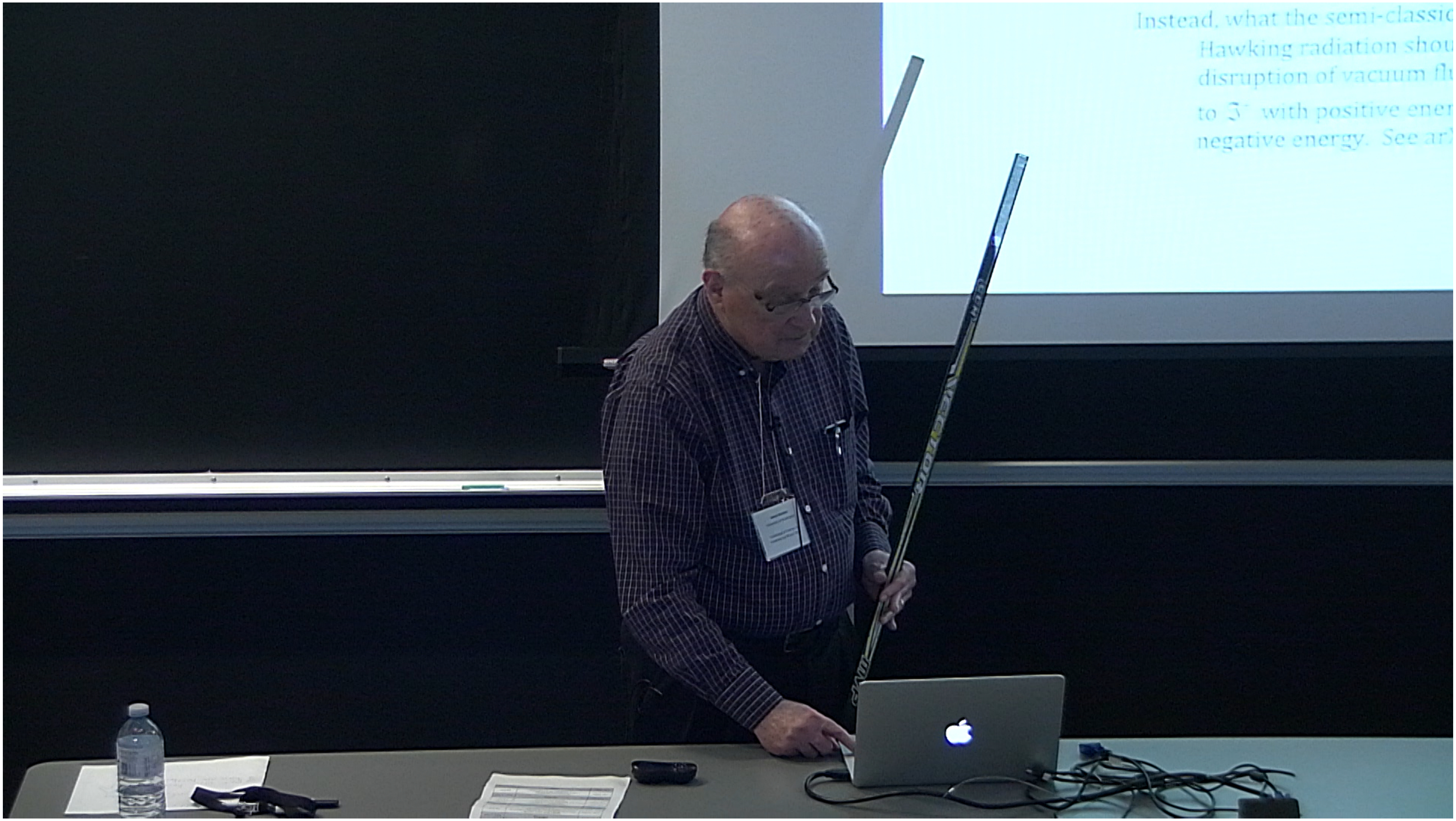
$$T_{\mathrm{H}} = \kappa m_{\mathrm{p}}^2 / 2\pi .$$

The surface gravity $\kappa = 1/4M$ for Schwarzschild. The luminosity has the form

$$L_{\mathrm{H}} = A_{\mathrm{h}} \sigma T_{\mathrm{H}}^4 \sum_s c_s \sim m_{\mathrm{p}}^2 / \left(2M\right)^2 .$$

Is Hawking radiation be created by pair creation/quantum tunneling within a Planck length of the horizon? If so, it should be apparent in the semi-classical stress-energy tensor. But as shown conclusively by Levi and Ori (2015) there is an inflow of negative energy, not an outflow of positive energy, near the horizon.

Instead, what the semi-classical stress-energy tensor suggestss is that Hawking radiation should be considered the result of tidal disruption of vacuum fluctuations around $r \sim 3M$, with part going to $\mathfrak{I}^+$ with positive energy and part going into the black hole with negative energy. See arXiv:1808.08638.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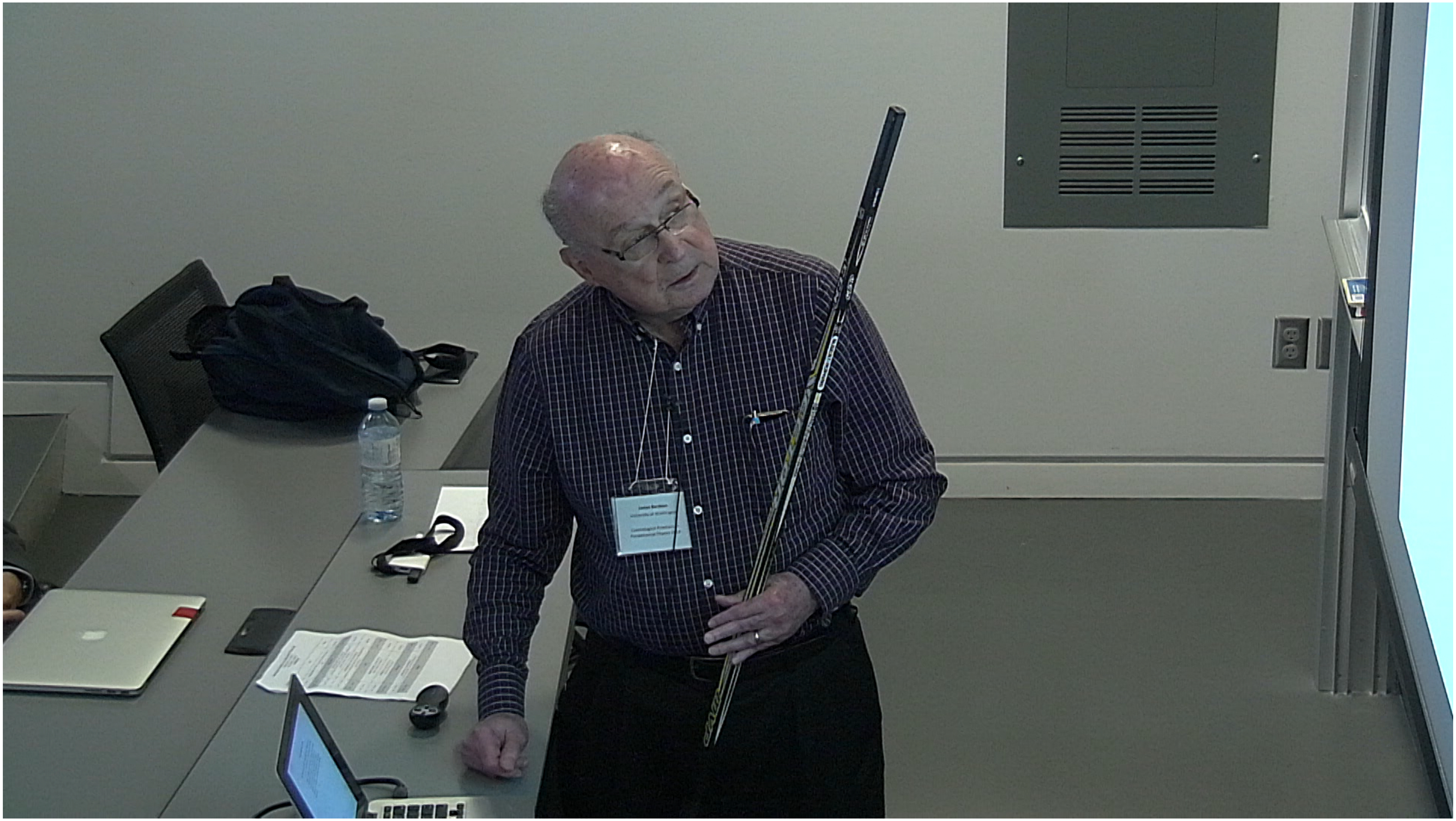

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The energy conditions required for an *event* horizon, while plausible classically, can be violated in quantum field theory. Also, quantum backreaction may resolve the singularities in black hole interiors that are inevitable according to classical singularity theorems.

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I will discuss the possibility that the interior of the black hole evolves smoothly into the interior of a white hole, with eventual release of all trapped quantum information across the white hole horizon, in the context of a particular type of singularity resolution suggested by loop quantum gravity.

Basic assumptions

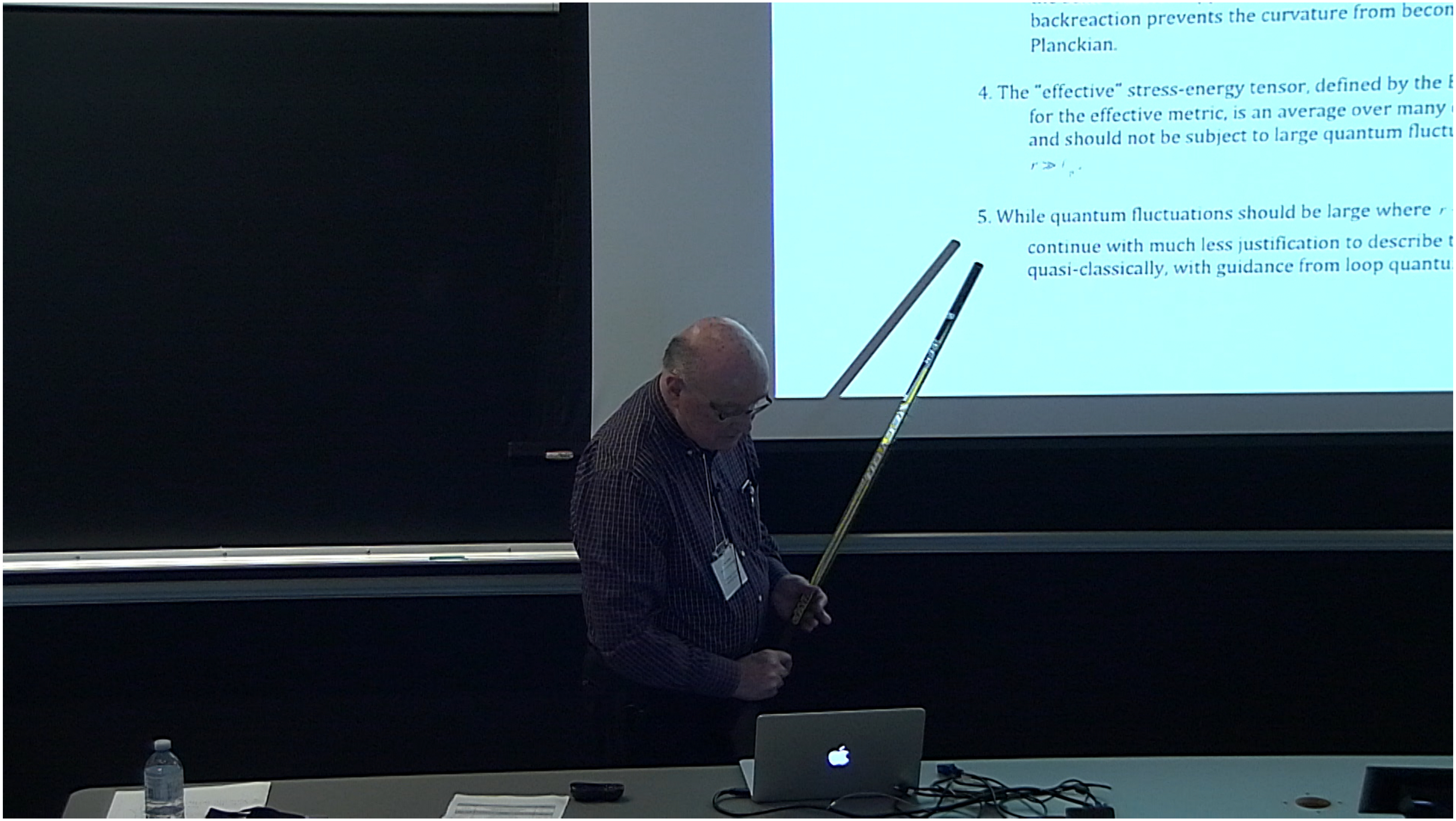
1. Spherical symmetry

2. Quantum corrections are very small in the vicinity of the horizon of a large black hole, and are in accord with semi-classical theory.

3. A "quasi-classical" picture of the evolution of the geometry, with small fluctuations about an effective metric, incorporating quantum backreaction, even where the curvature becomes Planckian and the semi-classical approximation breaks down. Quantum backreaction prevents the curvature from becoming super-Planckian.

4. The "effective" stress-energy tensor, defined by the Einstein tensor for the effective metric, is an average over many quantum modes and should not be subject to large quantum fluctuations as long as $r \gg \ell_p$.

5. While quantum fluctuations should be large where $r \sim \ell_p$, I will continue with much less justification to describe the geometry quasi-classically, with guidance from loop quantum gravity.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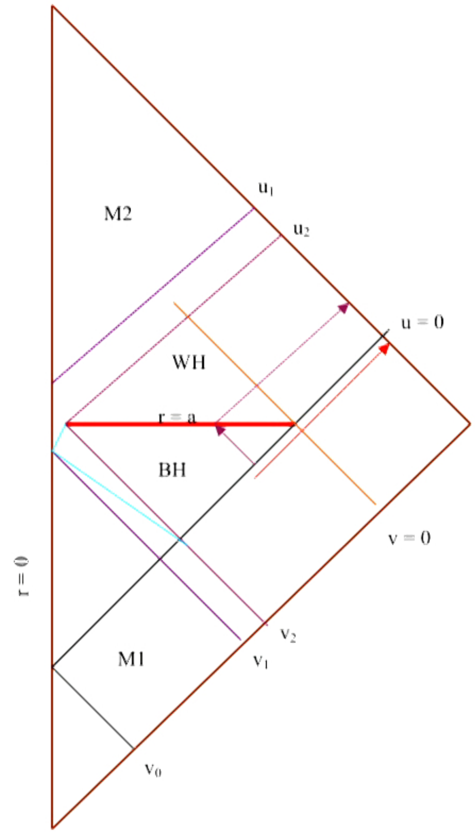
Basic scenario

The collapsing matter that formed the black hole bounces quickly as measured by advanced time and local proper time. Outside the matter the circumferential radius has a minimum value $r = a$ on a spacelike hypersurface separating the interior of the black hole from the interior of a white hole to its future, This is an old idea (e. g., Ashtekar and Bojowald). Various versions have been discussed by Rovelli and collaborators. I will discuss how this might work for an evaporating black hole with a finite lifetime, as described by an evolving quasi-classical geometry.


M2
u_1
u_2
$u = 0$
WH
r = a
BH
$v = 0$
r = 0
v_2
M1
v_1
v_0

An ansatz for $g_{\theta\theta} \equiv r^2$ from LQG

Ashtekar, Olmedo, and Singh (2018) for a Kruskal black hole in LQG:

$$r^2 = \left(2Me^T\right)^2 + \frac{1}{4}\frac{\left(\gamma L_0 \delta_c\right)^2 M^2}{\left(2Me^T\right)^2}.$$

Here M is the fixed black hole mass, γ, L_0, and δ_c are standard LQG parameters, and T is a coordinate.

Rewrite this in terms of a coordinate z as $r^2 = z^2 + a^2$, with

$$z = -2Me^T + \frac{\gamma L_0 \delta_c}{4e^T}, \quad a^2 = \gamma L_0 \delta_c M.$$

The coordinate z ranges from $-\infty$ far outside the black hole to $+\infty$ far outside the white hole.

A regular metric for the black hole in terms of coordinate z and an advanced null coordinate v has the form

$$ds^2 = -e^{2\psi_v} g^{zz} dv^2 - 2e^{\psi_v} dv dz + \left(z^2 + a^2\right) d\Omega^2.$$

The effective stress-energy tensor for the black hole

The Misner-Sharp mass function m is defined by

$$1-2m/r=\nabla_\alpha r\nabla^\alpha r=\frac{z^2}{r^2}g^{zz}.$$

The effective stress-energy tensor is completely determined given $g^{zz}(z,v)$ and $\psi_v(z,v)$. From $g^{zz}(z,v)$,

$$8\pi T_v^v=-\frac{2}{r^2}\left(\frac{\partial m}{\partial r}\right)_v=-\frac{1}{r^2}\left[1-\left(1+\frac{a^2}{r^2}\right)g^{zz}-z\left(\frac{\partial g^{zz}}{\partial z}\right)_v\right],$$

$$8\pi T_v^z=\frac{2}{zr}\left(\frac{\partial m}{\partial v}\right)_z=-\frac{z}{r^2}\left(\frac{\partial g^{zz}}{\partial v}\right)_z,$$

and from $\psi_v(z.v)$,

$$8\pi e^{\psi_v}T_v^z=2\left[\frac{a^2}{r^4}-\frac{z}{r^2}\left(\frac{\partial\psi_v}{\partial z}\right)_v\right].$$

Then $T_z^z\equiv T_v^v-g^{zz}e^{\psi_v}T_z^v$, and the transverse stress $T_\theta^\theta\left(=T_\varphi^\varphi\right)$ follows from conservation of the stress-energy tensor.

The transition to the white hole

Trapped surfaces in the black hole at $z<0$ become anti-trapped surfaces in the white hole for $z>0$, and $z=0$ must be a spacelike hypersurface, i.e., $g^{zz}(0,v)\leq 0$, with $g^{zz}=0$ only at the endpoint of black hole evaporation, by definition at $v=0$.

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Appropriate coordinates for the white hole are *retarded* null coordinates (z,u), u constant along outgoing radial null geodesics, since advanced coordinates are singular at the white hole horizon. The metric in retarded coordinates is

$$ds^2=-e^{2\psi_u}g^{zz}du^2-2e^{\psi_u}dudz+r^2d\Omega^2,$$

if u is defined such that $u=-v$ at $z=0$, with u as well as v increasing to the future.

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The transformation between advanced and retarded coordinates leaves g^{zz} unchanged, and $e^{\psi_u}=-\left(\partial v/\partial u\right)_z e^{\psi_v}$, with $e^{\psi_u}=e^{\psi_v}$ at $z=0$.

The flow of energy from the black hole to the white hole

Consider the physical components of the stress-energy tensor in an orthonormal frame in which the radial basis vector is tangent to a spacelike constant-z hypersurface pointing outward from the surface of the shell, as appropriate near the black hole to white hole transition, and a future-directed time basis vector.

In terms of advanced coordinate components

$$E = -\left(-g^{zz}\right)^{-1} e^{-\psi_v} T_v^z - T_z^z = -F - T_z^z , \quad P_r = -F + T_v^v .$$

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$$E = -\left(-g^{zz}\right)^{-1} e^{-\psi_v} T_u^z - T_z^z = +F - T_z^z , \quad P_r = +F + T_v^v .$$

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The energy flux F is positive in the interior of the black hole, but goes to 0 at $z = 0$ linearly in z, and becomes negative in a smooth transition to the white hole. In the interior of the black hole T_v^z contributes negatively to the energy density and radial stress, corresponding to the inflow of negative energy. In the white hole $F < 0$ implies inflow of positive energy or outflow of negative energy.

Outflow of negative energy across the white hole horizon increases the mass of the white hole.

End of the black hole and beginning of the white hole

Very near the end of the lifetime of the black hole the constraints on g^{zz} imply it should have the form

$$g^{zz} \cong -A(v/a)^2 + B(z/a)^2,$$

The gauge freedom of adding an arbitrary function of v to ψ_v makes it possible to set $\psi_v(0,v) \cong 0 + C(z/a)^2$. A, B, and C are positive constants of order unity.

On the white hole side, in retarded coordinates, just replace v by u.

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The effective stress-energy tensor is perfectly regular near $z = v = 0$,

$$8\pi a^4 T_v^v \cong -a^2 - 2Av^2 + 4Bz^2, \ 8\pi a^4 T_v^z \cong 2Avz,$$

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Negative energy associated with Hawking "partners" flows out across the white hole horizon to $\Im^+$, since $T_u^z > 0$ for $u > 0$.

Alternatives for the evolution of the white hole

1. The flow of negative energy out of the white hole continues for the lifetime of the white hole, gradually increasing its mass back to the initial mass of the black hole just before the bouncing matter shell emerges and the white hole disappears. The evolution of the white hole is then roughly the time-reverse of the evolution of the black hole. But extended flow of negative energy out to $\mathfrak{I}^+$ is apparently inconsistent with quantum field theory in the asymptotic region (violates Ford-Roman minimum average energy density theorems).

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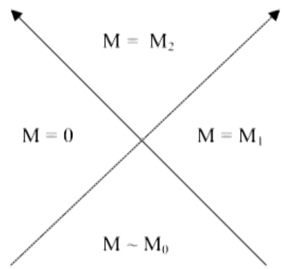
2. The outflow of negative energy across the white hole apparent horizon stops ($T_u^z \to 0$ at $g^{zz} = 0$) while the white hole still has a Planck-scale mass. The mass and radius of the white hole are frozen until the bouncing matter shell reemerges. The quantum information and negative energy of Hawking "partners" largely remain trapped inside the white hole. But how can most of the quantum information and all of the matter shell suddenly emerge from a white hole with just a Planck-scale mass and radius?

End of the white hole

Scenario 1: The shell that formed the black hole emerges from the white hole much as at went into the black hole, leaving nothing behind. All quantum information trapped by the black hole has been released.

End of the white hole

Scenario 2: most of quantum information remains inside the white hole, negative energy flows just inside the white hole horizon. Treat the interaction with the emerging matter shell as the interaction of two null shells ala Dray and 't Hooft.



Shells cross at $r = r_0 = 2M_1 - \varepsilon$,

$(2M_2 - r_0)(2M_0 - r_0) = (-r_0)(2M_1 - r_0) \Rightarrow 2M_2 \simeq r_0 - \varepsilon r_0 / (2M_0)$.

The ingoing shell acquires positive mass M_2 and forms a small remnant black hole. The outgoing shell ends with a positive mass $M_1 - M_2$ very small compared with M_1.

Entropy

The von Neumann entropy S_{vN} increases steadily as the black hole evaporates, while the mass and horizon area and therefore the Bekenstein-Hawking entropy S_{BH} steadily decrease. $S_{\mathrm{vN}} = S_{\mathrm{BH}}$ at the Page time, when the black hole has lost only about 1/2 of its original mass.

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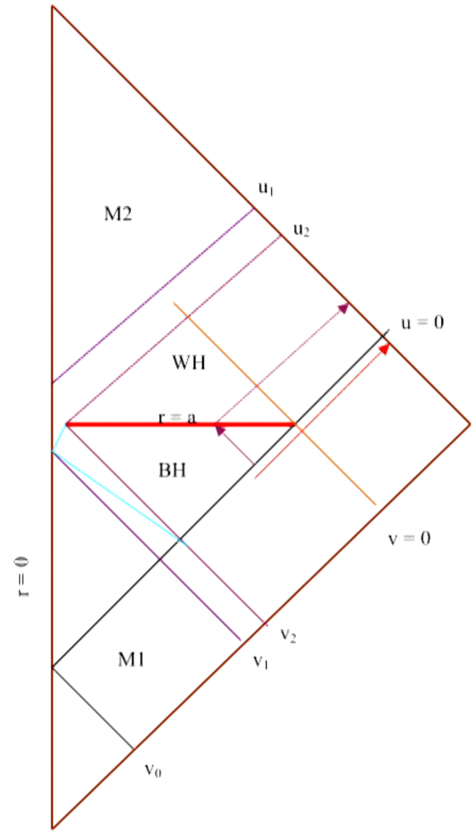
While S_{BH} is plausibly a measure of the total number of quantum degrees of freedom residing on the black hole horizon, it is apparent from the Penrose diagram for the black hole to white hole scenario that it makes absolutely no sense to demand that it be a measure of the total number of degrees of freedom including those in the interior of the black hole. The Hawking "partner" degrees of freedom cross into the white hole at a spacelike separation from the endpoint evaporation and are not part of a thermodynamic system with the black hole at the end of its lifetime. There is no reason why S_{vN} cannot become much larger than S_{BH}.

The density of quantum degrees of freedom on the $z = 0$ spacelike hypersurface never has to exceed one per Planck volume. However, there may be an entropy argument against Scenario 2 for the evolution of the white hole.


u₁
M2
u₂
u = 0
WH
r = a
BH
v = 0
r = 0
v₂
M1
v₁
v₀

Conclusions and questions

A resolution of the Schwarzschild singularity suggested by loop quantum gravity allows a black hole to evolve into a white hole, with a potential for resolving the information paradox. I have discussed in some detail how the transition works for a black hole that evaporates down to the Planck scale, without need for the quantum tunneling.

The evolution of the white hole raises serious questions, since long term emission of negative energy across the white hole horizon to $\mathfrak{I}^+$ violates theorems on minimum average energy densities proven by Ford and Roman for quantum fields in Minkowski spacetime. Some emission of negative energy just after the formation of the white hole must occur, as shown to be necessary to preserve unitarity in 2D by Bianchi and Smerlak.

Could quantum information leak out of the white hole as vacuum correlations, unaccompanied by any flow of energy?

My quasi-classical approach to the evolution of the geometry is a conjecture, and must be justified by a full treatment in quantum gravity. I have discussed one class of explicit Scenario 1 models for the evolution of the geometry over the entire spacetime in arXiv:1811.06683.