Title: General Relativity for Cosmology - Lecture 2

Speakers: Achim Kempf

Collection: General Relativity for Cosmology (Kempf)

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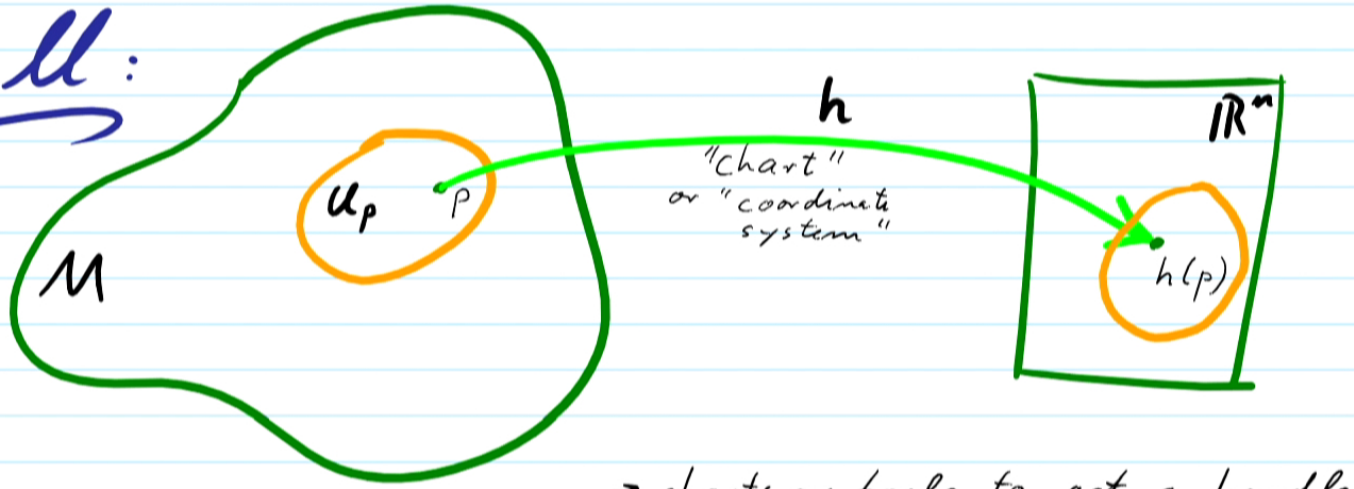
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1 of 13
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GR for Cosmology, Achim Kempf

Lecture 2

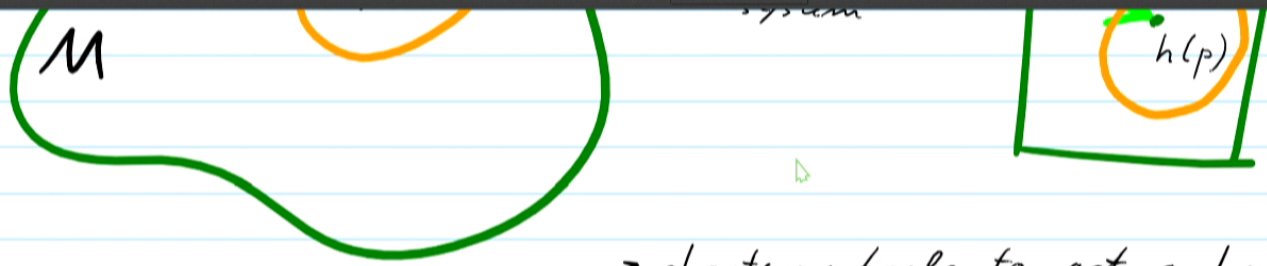
Note Title

Recall:



→ charts are tools to get a handle at the otherwise nameless abstract points of the manifold.

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→ charts are tools to get a handle at the otherwise nameless abstract points of the manifold.

Problem:

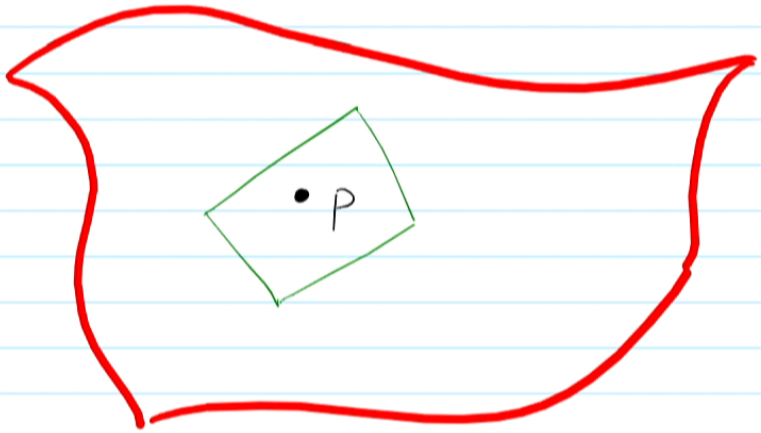
How to define the abstract "Tangent space, $T_p(M)$," to a diffable mfld at a point p?

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Intuition :



⇝ Proper definition should imply :

An n-dim mfld possesses for every point p an n-dim vector space

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3 equivalent definitions of $T_p(M)$:

1. "Algebraic" definition of $T_p(M)$:

Most powerful b/c no need for coordinates

Idea:
- A tangent vector = directional derivative,
- Derivatives definable through Leibniz rule:

$$(fg)' = f'g + fg'$$

2. "Physicist" definition of $T_p(M)$:

Idea: The elements of $T_p(M)$ are

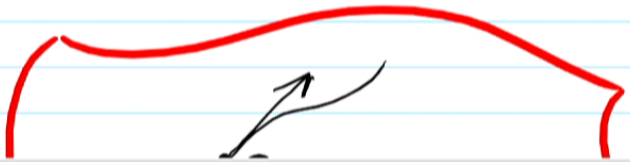
2. "Physicist" definition of $T_p(M)$:

Idea: The elements of $T_p(M)$ are to be vectors ⇒ recognizable by how their components change with charts.

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3. "Geometric" definition of $T_p(\mathcal{M})$:

Idea: The elements of $T_p(\mathcal{M})$ are to be actual tangent vectors of one-dim. paths in the manifold, that pass through p.

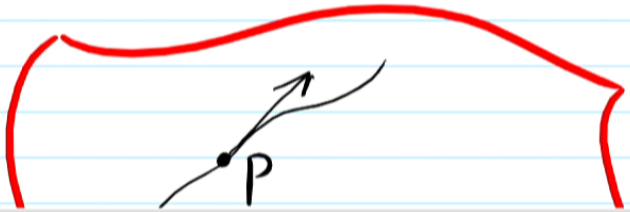


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→ we will do all 3:

1. Algebraic definition of $T_p(M)$

Idea: a) A tangent vector = directional derivative,

b) Derivatives definable through Leibniz rule:

$$(fg)' = f'g + fg'$$

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Key example: $\mathcal{M} = \mathbb{R}^n$

a) The tangent vectors ξ at a point p are identified with the directional 1st derivatives:

$$\xi = \sum_{i=1}^{n} \xi_i \cdot \frac{\partial}{\partial x^i}\bigg|_{x=p}$$

b) Thus, tangent vectors at p should be those maps

$$\xi : f \to \xi(f) = \sum_{i=1}^{n} \xi_i \cdot \frac{\partial}{\partial x^i} \cdot f(x)\bigg|_{x=p}$$

which obey the "Leibniz rule" at p:

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which obey the "Leibniz rule" at p:

"$$\xi(fg) = \xi(f)\, g + f\, \xi(g)\Big|_{\text{at } p}$$"

Q. How to express the local nature of $\xi \in T_p(M)$ ...?

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"$$\xi(fg) = \xi(f)\, g + f\, \xi(g)\Big|_{at\ p}$$"

Q: How to express the local nature of $\xi \in T_p(M)$ properly?

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Q: How to express the local nature of $\xi \in T_p(M)$ properly?

A: ξ acts on function germs, not on functions.

Def:

- Assume M, N are diffable mflds and $p \in M$.
- We say that two differentiable functions ϕ, ψ are germ-equivalent about p if in a neighborhood ($\leftarrow$ i.e. an open set) $U \subset M$ of p:

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$$\phi(q) = \psi(q) \quad \forall q \in U$$

- Each such equivalence class of functions is called a **germ at p**.
- Then, the "germ" of ϕ at p, denoted $\overline{\phi}_p$, is the equivalence class of all functions ψ which are identical to ϕ in

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- We say that two differentiable functions ϕ, ψ are germ-equivalent about p if in a neighborhood ($\mathcal{U} \subset \mathcal{M}$ i.e. an open set) of p:

$$\phi(q) = \psi(q) \quad \forall q \in \mathcal{U}$$

- Each such equivalence class of functions is called a **germ at p.**

- Then, the "germ" of ϕ at p, denoted $\overline{\phi}_p$, is the equivalence class of all functions ψ which are identical to ϕ in some neighborhood of p:

$$\psi \in \overline{\phi}_p \text{ if } \exists\ \mathcal{U}_p \ (\text{some open neighborhood of } p \text{ in } \mathcal{M}) \ \forall q \in \mathcal{U}_p : \phi(q) = \psi(q)$$

("there exists")

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- We say that two differentiable functions ϕ, ψ are germ-equivalent about p if in a neighborhood ($\mathcal{U} \subset \mathcal{M}$ of p: ← i.e. an open set)

$$\phi(q) = \psi(q) \quad \forall q \in \mathcal{U}$$

- Each such equivalence class of functions is called a **germ at p.**

- Then, the "germ" of ϕ at p, denoted $\overline{\phi}_p$, is the equivalence class of all functions ψ which are identical to ϕ in some neighborhood of p:

$\psi \in \overline{\phi}_p$ if $\exists\ \mathcal{U}_p$ ← some open neighborhood of p in $\mathcal{M}$. $\forall q \in \mathcal{U}_p : \phi(q) = \psi(q)$

↖ "there exists"

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Notice: Assume $\phi : M \to N$ is diffable at $p \in M$.

Then all $\Psi \in \mathcal{F}_p$ possess the same first derivative at p.

For example:

Consider germs of scalar functions f:

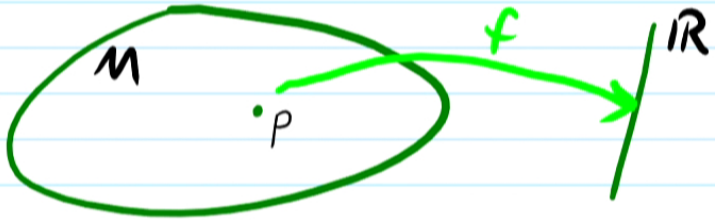


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For example:

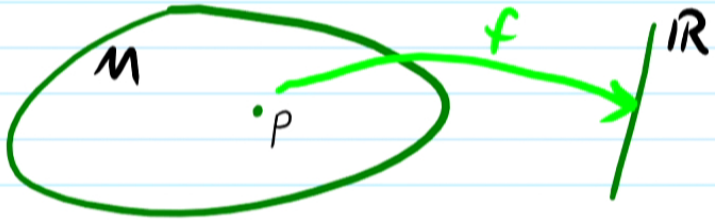
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, i.e.: $f : M \to \mathbb{R}$

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5 of 13
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Note:

- To specify a germ, it suffices to specify any arbitrary one of its functions.
- The set of all germs at p is denoted $\underline{\mathcal{F}(p)}$.

Note:

- One has for all $c \in \mathbb{R}$ and $f, g \in \mathcal{F}(p)$:

$$\overline{c \cdot f} = c\,\bar{f} \qquad (a)$$

$$\overline{f \cdot g} = \bar{f}\,\bar{g} \qquad (b)$$

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$$\overline{f \cdot g} = \bar{f}\,\bar{g} \qquad (b)$$

$$\overline{f + g} = \bar{f} + \bar{g} \qquad (c)$$

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5 of 13
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$$\overline{f+g} = \bar{f} + \bar{g} \qquad (c)$$

$\Rightarrow$ $\mathcal{F}(p)$ obeys the axioms of an associative algebra.

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Finally: Algebraic definition of $T_p(M)$

Recall idea: The elements of $T_p(M)$ are to be 1st derivatives $\Rightarrow$ definable by Leibniz rule.

Definition: The tangent space $T_p(M)$ is the set of "derivations" of $\mathcal{F}(p)$, i.e. the set of linear maps $\xi : \mathcal{F}(p) \to \mathbb{R}$ which obey:

$$\xi(\tilde{f}\tilde{g}) = \xi(\tilde{f})\tilde{g}(p) + \tilde{f}(p)\xi(\tilde{g})$$

by Leibniz rule.

Definition: The tangent space $T_p(M)$ is the set of "derivations" of $\mathcal{F}(p)$, i.e. the set of linear maps $\xi : \mathcal{F}(p) \to \mathbb{R}$ which obey:

$$\xi(\bar{f}_p \bar{g}_p) = \xi(\bar{f}_p)\cdot \bar{g}_p(p) + \bar{f}_p(p)\,\xi(\bar{g}_p)$$

$\bar{g}_p(p) = g(p)$, $\bar{f}_p(p) = f(p)$

remember this (*)

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6 of 13
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Remark:

- this definition is abstract enough not only for arbitrary diffable mflds!
- this definition (as derivations of the algebra of functions) is also suitable for "Noncommutative Geometry": There, (Quantum Gravity) the algebra of functions $F(\rho)$ is noncommutative.

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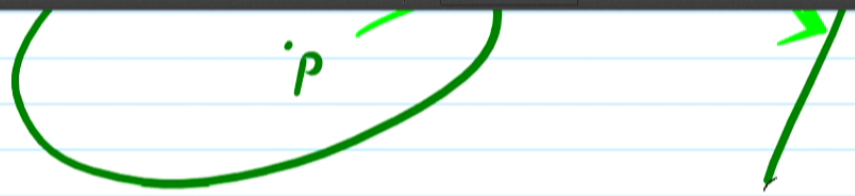
First example: a constant function, c, and its germ $\bar{c}$.



$c(x) := c$ and c is a constant: $c \in \mathbb{R}$

Then: $\xi(\bar{c}) = 0$ for all $\xi \in T_p(M)$

Proof: $\xi(\bar{c}) = c\,\xi(1) = c\,\xi(1 \cdot 1) = c(\xi(1)1 + 1\xi(1))$ (Leibnit rule)



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Proof: $\xi(\bar{c}) = c\,\xi(1) = c\,\xi(1 \cdot 1) = c(\xi(1)1 + 1\xi(1))$ (Leibniz rule)

$= 2c\,\xi(1) \Longrightarrow \xi(\bar{c}) = 0$ ✓

every $\xi \in T_p(M)$ is of the form:

$$\xi = \sum_{i=1}^{n} \xi^i \frac{\partial}{\partial x^i}\Big|_{x=p}$$

Proof:

- We choose p to have coordinates $x = (0, 0, \ldots)$.
- Assume $\xi \in T_p(M)$ and $\bar{f} \in \mathcal{F}(p)$.

Notation: $h_{,i}(a^1, \ldots, a^n) := \frac{\partial}{\partial a^i} h(a^1, \ldots, a^n)$

Then:

(Note: these are not 3 numbers! These are 3 function germs, i.e.; 3 equivalence classes of functions;)

germ of a constant function

$$\S(\overline{f(x)}) = \S(\overline{f(0)} + \overline{f(x)} - \overline{f(0)})$$

a constant function

$$\overset{(cc)}{=} \S\left(\overline{f(0)} + \int_0^1 \frac{d}{dt} \bar{f}(tx^1, \ldots, tx^n)\, dt\right)$$

$$\overset{(b)}{=} \S(\overline{f(0)}) + \S\left(\int_0^1 \sum_{i=1}^n \frac{\partial \bar{f}(tx^1, \ldots, tx^n)}{d(tx^i)}\, d(\overline{tx^i})\, dt\right)$$

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7 of 13
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$\bar{f}$ a constant function

$$\overset{(c)}{=} \xi\left(\overline{f(0)} + \int_0^1 \frac{d}{dt}\bar{f}(tx^1, \ldots, tx^n)\, dt\right)$$

$$\overset{(b)}{=} \underbrace{\xi(\bar{f}(0))}_{\| \; 0} + \xi\left(\int_0^1 \sum_{i=1}^n \frac{\partial \bar{f}(tx^1, \ldots, tx^n)}{\partial(tx^i)} \frac{d(\overline{tx^i})}{dt}\, dt\right)$$

$$= \xi\left(\int_0^1 \sum_{i=1}^n \bar{f}_{,i}(tx^1, \ldots, tx^n)\, \bar{x}^i\, dt\right)$$

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8 of 13 280%

$$= \xi\left(\int_0^1 \sum_{i=1}^n \bar{f}_{,i}(tx^1,\dots,tx^n)\,\bar{x}^i\,dt\right)$$

Linearity of $\xi \Rightarrow$

$$= \sum_{i=1}^n \xi\left(\int_0^1 \bar{f}_{,i}(tx^1,\dots,tx^n)dt \cdot \bar{x}^i\right)$$

Leibniz rule $\Rightarrow$

$$= \sum_{i=1}^n \xi\left(\int_0^1 \bar{f}_{,i}(tx^1,\dots,tx^n)dt\right) \cdot \overbrace{\bar{x}^i\Big|_{x=p=0}}^{=0}$$

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(3 function germs, i.e., 3 equivalence classes of functions.)

germ of a constant function

$$\xi(\overline{f(x)}) = \xi\left(\overline{f(0)} + \overline{f(x)} - \overline{f(0)}\right)$$

$\overline{f(0)}$: a constant function

$$\overset{(c)}{=} \xi\left(\overline{f(0)} + \int_0^1 \frac{d}{dt}\, \bar{f}(tx^1, \ldots, tx^n)\, dt\right)$$

$$\overset{(b)}{=} \underbrace{\xi(\overline{f(0)})}_{0} + \xi\left(\int_0^1 \sum_{i=1}^{n} \frac{\partial \bar{f}(tx^1, \ldots, tx^n)}{\partial(tx^i)} \frac{d(\overline{tx^i})}{dt}\, dt\right)$$

$$= \xi\left(\int_0^1 \sum_{i=1}^{n} \bar{f}_{,i}(tx^1, \ldots, tx^n)\, \bar{x}^i\, dt\right)$$

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Linearity of $\xi \Rightarrow$

$$= \sum_{i=1}^{n} \xi\left(\int_0^1 \bar{f}_{,i}(t x^1, \ldots, t x^n)\, dt \cdot \bar{x}^i \right)$$

Leibniz rule $\Rightarrow$

$$= \sum_{i=1}^{n} \xi\left(\int_0^1 \bar{f}_{,i}(t x^1, \ldots, t x^n)\, dt \right) \cdot \overbrace{\bar{x}^i}^{=0} \Big|_{x=p=0}$$

$$+ \sum_{i=1}^{n} \left(\int_0^1 \bar{f}_{,i}(t x^1, \ldots, t x^n)\, dt \right) \Big|_{x=0=0} \cdot \xi(\bar{x}^i)$$

(Remember from ## above)

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Linearity of $\xi \Rightarrow$

$$= \sum_{i=1}^{n} \xi\left(\int_0^1 \bar{f}_{,i}(tx^1, \ldots, tx^n)\,dt \cdot \bar{x}^i \right)$$

Leibniz rule $\Rightarrow$

$$= \sum_{i=1}^{n} \xi\left(\int_0^1 \bar{f}_{,i}(tx^1, \ldots, tx^n)\,dt \right) \cdot \overbrace{\bar{x}^i\big|_{x=p=0}}^{=0}$$

$$+ \sum_{i=1}^{n} \left(\int_0^1 \bar{f}_{,i}(tx^1, \ldots, tx^n)\,dt \right)\Big|_{x=p=0} \cdot \xi(\bar{x}^i)$$

(Remember from (*) above)

$$\sum^{n} \; (-1) \int_0^1 \bar{f}\,(\,\ldots\,)\,dt$$

$$= \sum_{i=1} \xi(x^i) \int_0 f_{,i}(0,\dots,0)\, dt$$

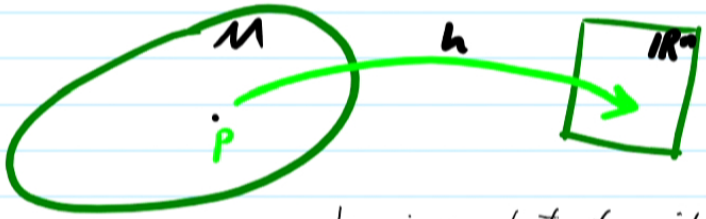
$$\left(\int_0^1 c\, dt = c \Rightarrow \right)$$

$$= \sum_{i=1}^{n} \xi(\bar{x}^i) \frac{\partial}{\partial x^i} f(x^1, \dots, x^n)\Big|_{x=p=0}$$

$\Rightarrow$ Indeed, every $\xi \in T_p(M)$ is of the form

$$\xi = \sum \xi^i \partial_i \qquad (T)$$

- **Recall:**



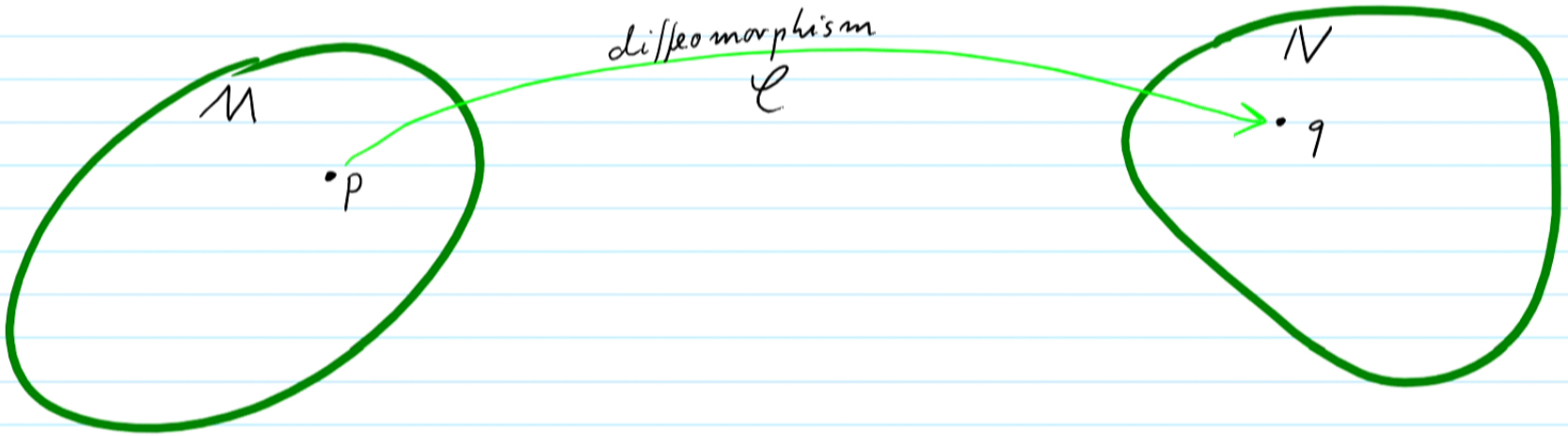
h gives abstract points a name, i.e. makes them concrete.

- **Problem:** How to make abstract $\xi \in T_p(M)$ concrete?

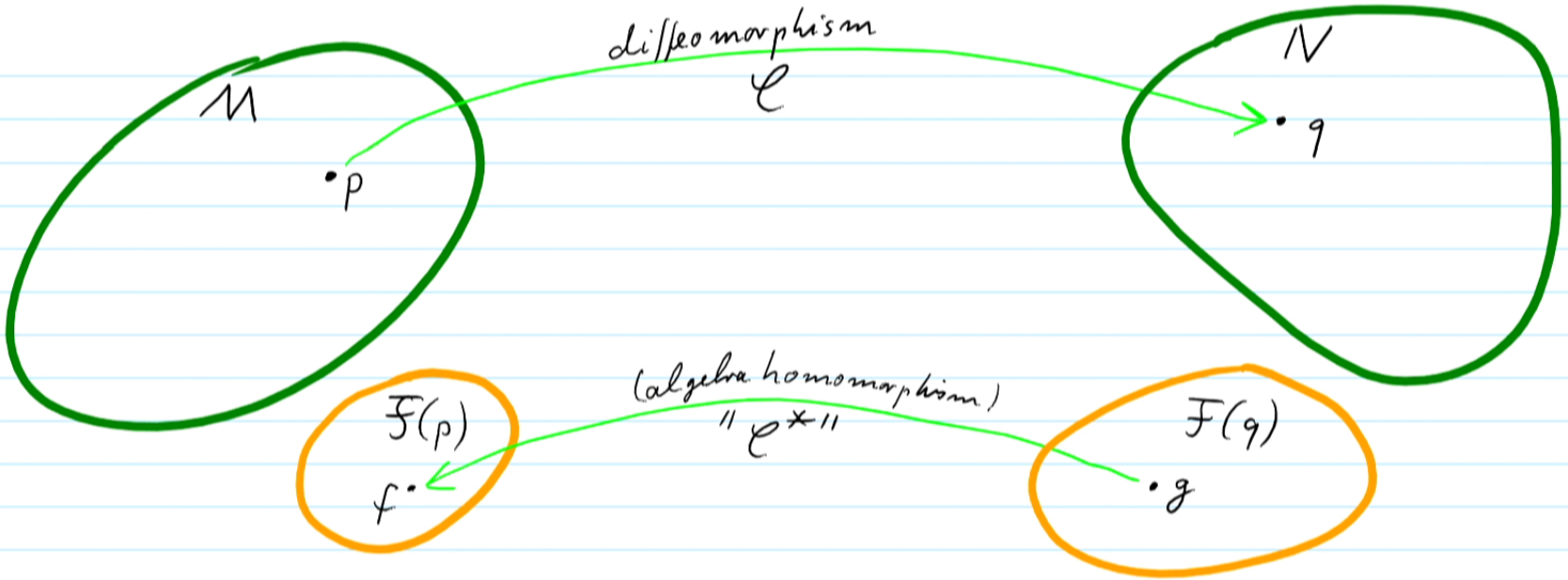
- **Solution:** Make use of charts in clever way!

Preparation: $T_p(M)$ and Diffeomorphisms.

Consider two diffable manifolds, M and N:

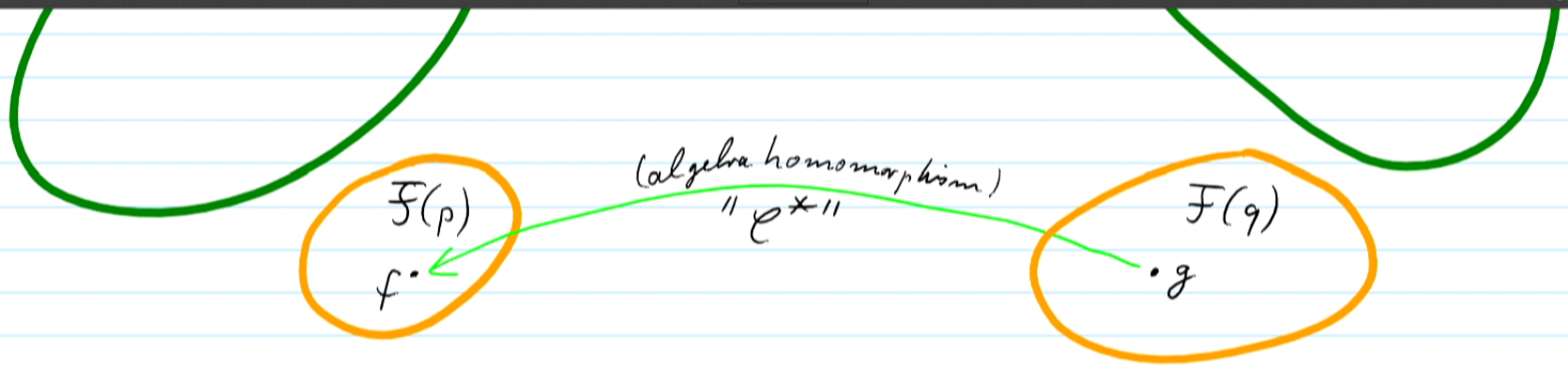


Note: If $N = \mathbb{R}^n$, then $\mathcal{C}$ is a chart.



Here: $\mathcal{F}(q)$ and $\mathcal{F}(p)$ are algebras of function (germs).

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$\varphi^*: g \to f = \varphi^*(g)$ with $f(x) = g(\varphi(x))$ $\forall x \in M$

i.e.: $f = \varphi^*(g) = g \circ \varphi$ (+)

- Namely: $\eta = \xi \circ \varphi^*$

i.e.: $\eta(g) = \xi(\varphi^*(g))$

- From (+) =>

$$\eta(g) = \xi(g \circ \varphi)$$

The crucial special case:

- $N = \mathbb{R}^n$ (with $n = \dim(N)$)

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11 of 13
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11 of 13 280%

The crucial special case:

- $N = \mathbb{R}^n$ (with $n = \dim(N)$)
- $\mathcal{C}$ is invertible
- ($\Rightarrow$ $\mathcal{C}^*$ is algebra isomorphism)
- $\Rightarrow T_p\mathcal{C}$ is vector space isomorphism

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ENG 5:11 PM 2019-09-10

- Given a chart φ, every abstract point $p \in M$ has a concrete image $\varphi(p) \in \mathbb{R}^n$, **and**:
- Every abstract vector $\xi \in T_p(M)$ has a concrete image $\eta \in T_{\varphi(p)}(\mathbb{R}^n)$ namely:

$$\eta = T_p\varphi(\xi)$$

($\varphi(p) = q$)

- The image η is concrete because

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$\varphi(p) = q$

- The image η is concrete because

form (we showed this):

$$\eta = \sum_{i=1}^{n} \eta^i \frac{\partial}{\partial x^i}\Big|_{x=q}$$

concrete numbers.

Conversely: (and very conveniently)

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12 of 13 — 280%

Conversely: (and very conveniently)

- Assuming a fixed $\mathcal{C}$, any choice of a $q = (x^1, \ldots, x^n)$ denotes a $p \in M$ and any choice of a $(y^1, \ldots, y^n)$ (some numbers) denotes a $\xi \in T_p(M)$.

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13 of 13
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- E.g. $\eta = \frac{\partial}{\partial x^i}\Big|_{x=q}$ is the image of some abstract $\xi \in T_p(M)$, for fixed γ.

Notation: $\xi = "\frac{\partial}{\partial x^i}\Big|_{x=p}"$

↑ symbolic notation

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13 of 13 — 280%

Next:

If we hold p and $\xi \in T_p(M)$ fixed, how do the numbers $(x^1, \ldots, x^n)$ and $(y^1, \ldots, y^n)$ change when we change the chart? → Physicists' def of $T_p(M)$

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ENG 5:17 PM 2019-09-10