Title: Geometric Extremization for AdS/CFT and Black Hole Entropy

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Abstract: We consider supersymmetric $AdS_3\times Y_7$ solutions of type IIB supergravity dual to N=(0,2) SCFTs in d=2, as well as $AdS_2\times Y_9$ solutions of D=11 supergravity dual to N=2 supersymmetric quantum mechanics, some of which arise as the near horizon limit of supersymmetric, charged black hole solutions in AdS_4. The relevant geometry on Y_{2n+1}, $n\ge 3$ was first identified in 2005-2007 and around that time infinite classes of explicit examples solutions were also found but, surprisingly, there was little progress in identifying the dual SCFTs. We present new results which change the status quo. For the case of Y_7, a variational principle allows one to calculate the central charge of the dual SCFT without knowing the explicit metric. This provides a geometric dual of c-extremization for d=2 N=(0,2) SCFTs analogous to the well known geometric duals of a-maximization of d=4 N=1 SCFTs and F-extremization of d=3 N=2 SCFTs in the context of Sasaki-Einstein geometry. In the case of Y_9 a similar variational principle can be used to obtain properties of the dual N=2 quantum mechanics as well as the entropy of a class of supersymmetric black holes in AdS_4 thus providing a geometric dual of I-extremization.

Geometric Extremization for AdS/CFT and Black Hole Entropy

Jerome Gauntlett

Chris Couzens, JPG, Dario Martelli, James Sparks

JPG, Dario Martelli, James Sparks x 2

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SCFTs with abelian R-symmetry

$$\{Q,Q\} \sim P \qquad\qquad \{S,S\} \sim K$$

$$\{Q,S\} \sim M + D + R$$

The R-symmetry encodes important exact results for physical observables. E.g.

- For chiral primary operators $\Delta(\mathcal{O}) = nR(\mathcal{O})$
- Central charges/free energies can be obtained from R

Furthermore, the R-symmetry can be obtained by variational techniques.

$\boxed{\mathcal{N}=1, d=4}$ a-maximization [Intriligator,Wecht 03]

$$a(R_T) = \frac{9}{32} Tr R_T^3 - \frac{3}{32} Tr R_T$$ and $$a = a(R_*)$$

$\boxed{\mathcal{N}=2, d=3}$ F-extremization [Jafferis 10]

$$F_{S^3}(R_T)$$ and $$F_{S^3} = F_{S^3}(R_*)$$

$\boxed{\mathcal{N}=(0,2), d=2}$ c-extremization [Benini,Bobev 12]

$$c_R(R_T) = 3 Tr \gamma_3 R_T^2 \qquad c_R = c_R(R_*)$$

$\boxed{\mathcal{N}=2, d=1}$

Is there a general extremization principle for susy QM??

$\mathscr{I}$-extremization conjecture: [Benini,Hristov,Zaffaroni 15]

Compactify $\mathcal{N}=2, d=3$ SCFT on Riemann surface Σ_g with a topological twist - flows to SQM in IR

"Topologically twisted index" on $S^1 \times \Sigma_g$ [Benini,Zaffaroni 15] can be obtained via an extremization principle in large N limit

Also: if the N=2 SCFT has an AdS_4 dual, the on-shell result for the index is conjectured to give entropy of susy magnetically charged AdS_4 black holes with $AdS_2 \times \Sigma_g$ near horizon

Provides a microscopic state counting derivation of the Bekenstein-Hawking entropy for asymptotically AdS black holes!

Holographic dual for these extremization principles?

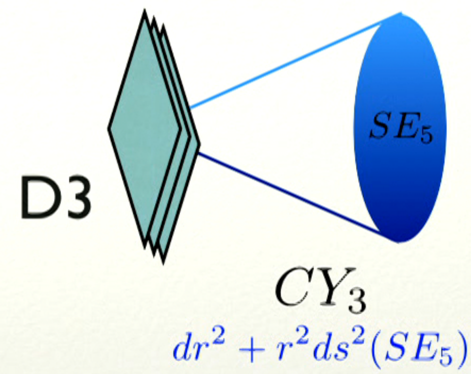
Well established in the context of Sasaki-Einstein solutions:

Type IIB

$$ds^2_{10} = L^2[ds^2(\mathrm{AdS}_5) + ds^2(SE_5)]$$

$$F_5 = -L^4\left[vol_{\mathrm{AdS}_5} + vol_{SE_5}\right]$$

Dual to N=1SCFT in d=4

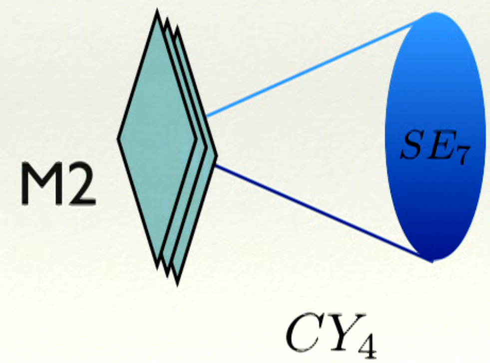


D=11

$$ds^2_{11} = L^2[ds^2(AdS_4) + ds^2(SE7)]$$

$$G = L^3 vol_{\mathrm{AdS}_4}$$

Dual to N=2 SCFT in d=3



Fact: SE have canonical Killing vector ξ

Type IIB

$$\boxed{\begin{aligned} ds^2_{10} &= L^2 e^{-B/2}[ds^2(\mathrm{AdS}_3) + ds^2(Y_7)] \\ F_5 &= -L^4\,[vol_{\mathrm{AdS}_3} \wedge F + *_7 F] \end{aligned}}$$

[Kim 05]

Dual d=2 SCFT has (0,2) supersymmetry

D=11

$$\boxed{\begin{aligned} ds^2_{11} &= L^2 e^{-2B/3}\left[ds^2(\mathrm{AdS}_2) + ds^2(Y_9)\right] \\ G_4 &= L^3 vol_{\mathrm{AdS}_2} \wedge F \end{aligned}}$$

[Kim,Park 06]

Dual SCQM has 2 supersymmetries with R-symmetry

Also can arise as near horizon limits of magnetically charged supersymmetric black holes in $AdS_4 \times SE_7$ (at least)

Both geometries special cases of GK geometry $\boxed{(Y_{2n+1}, B, F)}$

$\mathcal{N}=1, d=4$ SCFT dual to $AdS_5 \times SE_5$: $a \propto \frac{1}{Vol(SE_5)}$

$\mathcal{N}=2, d=3$ SCFT dual to $AdS_4 \times SE_7$: $F_{S^3} \propto \frac{1}{\sqrt{Vol(SE_7)}}$

R can be obtained using volume minimization: [Martelli,Sparks,Yau 05]

Go off-shell: Consider Sasaki metrics (cone is Kahler)

Extremize $Vol(Sas)(\xi)$

Very useful for identifying dual SCFTs!

Here: geometric duals for

c-extremization for AdS_3 solutions dual to $\mathcal{N}=(0,2), d=2$

New principle for AdS_2 solutions dual to $\mathcal{N}=2, d=1$

Includes a dual of $\mathscr{I}$- extremization as a special case and hence microstates of AdS4 black holes

GK Geometry $(Y_{2n+1}, B, F = dA)$ [Gauntlett,Kim 07]

Action:

$$\boxed{S = \int_{Y_{2n+1}} e^{(1-n)B} \left[R_{2n+1} - \frac{2n}{(n-2)^2} + \frac{n(2n-3)}{2} (dB)^2 + \frac{1}{4} e^{2B} F^2 \right] vol_{2n+1}}$$

Supersymmetry - existence of certain Killing spinors

Fact:

Supersymmetry + $d\left[e^{(3-n)B} *_{2n+1} F \right] = 0$

implies all equations of motion are satisfied

GK Geometry (Y_{2n+1}, B, F) $n \geq 3$

Supersymmetry implies:

- Killing vector ξ (R-symmetry) $||\xi||^2 = 1$
- Define one-form η dual to Killing vector: $\xi^a \eta_a = 1$
- Local coordinates $\xi = \frac{1}{c}\partial_z$ and $\eta = c(dz + P)$ $c = \frac{1}{2}(n-2)$
- In general

$$ds^2_{2n+1} = \eta^2 + e^B ds^2_{2n} \longleftarrow \text{Kahler } J, \rho$$

$$\rho_{ij} = J_i{}^k R_{kj}$$

$$\boxed{d\eta = c\rho} \qquad \boxed{e^B = \frac{c^2}{2} R > 0} \qquad \boxed{F = -\frac{1}{c} J + d\left(e^{-B}\eta\right)}$$

- Supersymmetric solution if gauge equation of motion holds:

$$d\left[e^{(3-n)B} *_{2n+1} F\right] = 0 \qquad \Leftrightarrow \qquad \boxed{\Box R = \frac{1}{2} R^2 - R_{ij} R^{ij}}$$

Off shell GK Geometry

Continue off-shell: just demand a "supersymmetric geometry"

(doesn't extremize action S)

$$\xi^a \eta_a = 1 \qquad \eta = c(dz + P) \qquad d\eta = c\rho$$

$$ds^2_{2n+1} = \eta^2 + e^B ds^2_{2n} \longleftarrow \text{Kahler}$$

$$\boxed{d\eta = c\rho} \qquad \boxed{e^B = \frac{c^2}{2} R > 0} \qquad \boxed{F = -\frac{1}{c} J + d\left(e^{-B}\eta\right)}$$

$$\cancel{d\left[e^{(3-n)B} *_{2n+1} F\right] = 0 \quad \Rightarrow \quad \boxed{\Box R = \frac{1}{2} R^2 - R_{ij} R^{ij}}}$$

Off-shell GK Geometry

- Consider cone metric on $C(Y_{2n+1}) \equiv \mathbb{R}_{>0} \times Y_{2n+1}$

$$ds^2_{2n+2} = dr^2 + r^2 ds^2(Y_{2n+1})$$

- Cone has an integrable complex structure
- R symmetry vector ξ is holomorphic
- No-where vanishing (n+1,0) form Ψ so $c_1 = 0$ and

$$d\Psi = 0 \qquad \mathcal{L}_\xi \Psi = \frac{i}{c}\Psi \qquad c = \tfrac{1}{2}(n-2)$$

Off-shell GK geometry and extremization - part I

Consider supersymmetric geometries on Y_{2n+1}

$$ds^2_{2n+1} = \eta^2 + e^B ds^2_{2n}$$

$$\boxed{d\eta = c\rho} \qquad \boxed{e^B = \frac{c^2}{2}R > 0} \qquad \boxed{F = -\frac{1}{c}J + d\left(e^{-B}\eta\right)}$$

Fix Y_{2n+1} and the complex cone $C(Y_{2n+1})$

Choose holomorphic $\xi \neq 0$ with $\mathcal{L}_\xi \Psi = \frac{i}{c}\Psi$

as well as a Kahler metric ds^2_{2n}

- To get a supersymmetric solution: need to vary action S

$$S_{\text{SUSY}} = \int_{Y_{2n+1}} \eta \wedge \rho \wedge \frac{J^{n-1}}{(n-1)!}$$

$$= S_{\text{SUSY}}(\xi, [J]) \qquad \text{Nice!} \qquad [J] \in H^2_B(\mathcal{F}_\xi)$$

Off-shell GK geometry and extremization - part II

Need to consider flux quantization

$$\frac{1}{(2\pi\ell_s)^4 g_s}\int_{\Sigma_A} F_5 = N_A \qquad \text{or} \qquad \frac{1}{(2\pi\ell_p)^6}\int_{\Sigma_A} *_{11}G = N_A$$

Requires $dF_5 = 0$ or $d *_{11} G_4 = 0$

But for a supersymmetric geometry, this requires

$\Box R = \frac{1}{2}R^2 - R_{ij}R^{ij}$ and puts solution on shell!?

Off shell geometry for extremization - part III

If Σ_A can all be represented by cycles tangent to ξ

then sufficient to impose integral of $\Box R = \frac{1}{2}R^2 - R_{ij}R^{ij}$

$$\int_{Y_{2n+1}} \eta \wedge \rho^2 \wedge \frac{J^{n-2}}{(n-2)!} = 0$$ "topological constraint"

Flux quantization conditions are then given by

$$\int_{\Sigma_A} \eta \wedge \rho \wedge \frac{J^{n-2}}{(n-2)!} = \begin{cases} \frac{2(2\pi \ell_s)^4 g_s}{L^4} N_A\,, & n = 3 \\ \frac{(2\pi \ell_p)^6}{L^6} N_A\,, & n = 4 \end{cases}$$

Off-shell GK geometry and extremization - part II

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Geometric extremal problem- Summary I

[Couzens,Gauntlett, Martelli,Sparks 18]

- Consider a supersymmetric geometry on Y_{2n+1}
- Complex cone $C(Y_{2n+1})$ with (n+1,0) form Ψ
- Choose holomorphic $\xi \neq 0$ and $\mathcal{L}_\xi \Psi = \frac{i}{c}\Psi$
- Choose basic class $[J]$ for transverse Kahler metric
- Impose constraint: $\int_{Y_{2n+1}} \eta \wedge \rho^2 \wedge \frac{J^{n-2}}{(n-2)!} = 0$
- Impose flux quantization: $\int_{\Sigma_A} \eta \wedge \rho \wedge \frac{J^{n-2}}{(n-2)!} = \begin{cases} \frac{2(2\pi\ell_s)^4 g_s}{L^4} N_A\,, & n = 3 \\ \frac{(2\pi\ell_p)^6}{L^6} N_A\,, & n = 4 \end{cases}$
- Extremise action: $S_{\mathrm{SUSY}}(\xi, [J]) = \int_{Y_{2n+1}} \eta \wedge \rho \wedge \frac{J^{n-1}}{(n-1)!}$

Geometric extremal problem- Summary II

For $\boxed{AdS_3 \times Y_7}$ define

$$\mathscr{Z} = \frac{3L^8}{(2\pi)^6 g_s^2 \ell_s^8} S_{\text{susy}} \qquad \text{and} \qquad \mathscr{Z}|_{\text{on-shell}} = c_{\text{sugra}} = \frac{3L}{2G_3}$$

For $\boxed{AdS_2 \times Y_9}$ define "entropy function"

$$\mathscr{S} = \frac{4\pi L^9}{(2\pi)^8 \ell_p^9} S_{\text{susy}} \qquad \text{and} \qquad \mathscr{S}|_{\text{on-shell}} = \frac{1}{4G_2}$$

Generically expect $\mathscr{S}|_{\text{on-shell}} = \ln Z, \quad \mathscr{I}$

For black hole horizons $\mathscr{S}|_{\text{on-shell}} = S_{BH}$

Special Cases and Toric Geometry

Type IIB $AdS_3 \times Y_7$ with $\boxed{Y_5 \hookrightarrow Y_7 \to \Sigma_g}$

Physical picture:

- Start with $AdS_5 \times Y_5$ and SE metric on Y_5

  Dual to d=4 N=1 SCFT

  Isometries of Y_5 give rise to global (mesonic) symmetries

  3-cycles of Y_5 give rise to global (baryonic) symmetries

- Compactify d=4 SCFT on Σ_g and switch on magnetic fluxes for the global symmetries (including topological twist)

- IF we flow to d=2 SCFT in IR then expect it is dual to $AdS_3 \times Y_7$ with Y_7 fibred as above

Special Cases and Toric Geometry

Type IIB $AdS_3 \times Y_7$ with $\boxed{Y_5 \hookrightarrow Y_7 \rightarrow \Sigma_g}$

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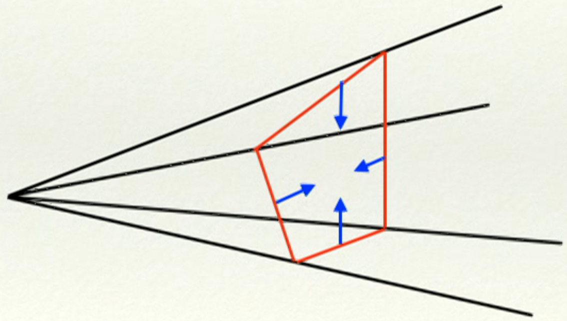
Focus on $AdS_5 \times Y_5$ with the complex cone $C(Y_5)$ admitting a toric Kahler cone metric: [Martelli,Sparks,Yau 05]

- Three holomorphic Killing vectors ∂_{φ_i} generate $U(1)^3$
- Can introduce three moment map coordinates y_i which span a polyhedral cone $\mathcal{C} \subset \mathbb{R}^3$ with d facets specified by inward pointing normal vectors $\vec{v}_a \in \mathbb{Z}^3$



$\vec{v}_a$ specifies which $U(1)$ collapses along that facet

- The extremization problem for the $AdS_3 \times Y_7$ solutions with $Y_5 \hookrightarrow Y_7 \to \Sigma_g$ becomes algebraic in the $\vec{v}_a$!

Master volume

[Gauntlett,Martelli,Sparks 18]

$$\mathcal{V}(\vec{b};\{\lambda_a\}) = \frac{(2\pi)^3}{2}\sum_{a=1}^{d}\lambda_a \frac{\lambda_{a-1}(\vec{v}_a,\vec{v}_{a+1},\vec{b}) - \lambda_a(\vec{v}_{a-1},\vec{v}_{a+1},\vec{b}) + \lambda_{a+1}(\vec{v}_{a-1},\vec{v}_a,\vec{b})}{(\vec{v}_{a-1},\vec{v}_a,\vec{b})(\vec{v}_a,\vec{v}_{a+1},\vec{b})}$$

Extremization problem

$A \sim$ Kahler class for Σ_g

$$S_{\text{SUSY}}(\vec{b};\{\lambda_a\};A) = -A\sum_{a=1}^{d}\frac{\partial\mathcal{V}}{\partial\lambda_a} - 4\pi\sum_{i=1}^{3} n_i \frac{\partial\mathcal{V}}{\partial b_i}$$

$$0 = A\sum_{a,b=1}^{d}\frac{\partial^2\mathcal{V}}{\partial\lambda_a\partial\lambda_b} - 2\pi n_1\sum_{a=1}^{d}\frac{\partial\mathcal{V}}{\partial\lambda_a} + 4\pi\sum_{a=1}^{d}\sum_{i=1}^{3} n_i\frac{\partial^2\mathcal{V}}{\partial\lambda_a\partial b_i}$$

$$\frac{2(2\pi\ell_s)^4 g_s}{L^4} N = -\sum_{a=1}^{d}\frac{\partial\mathcal{V}}{\partial\lambda_a}$$

Flux on Y_5

$$\frac{2(2\pi\ell_s)^4 g_s}{L^4} M_a = \frac{1}{2\pi}A\sum_{b=1}^{d}\frac{\partial^2\mathcal{V}}{\partial\lambda_a\partial\lambda_b} + 2\sum_{i=1}^{3} n_i\frac{\partial^2\mathcal{V}}{\partial\lambda_a\partial b_i}$$

Flux on $\Sigma_g \times (\Sigma_a \subset Y_5)$

Results

- For various examples, including e.g. $Y_5 = Y^{p,q}$ [JPG,Martelli,Sparks, Waldram 04]

  can calculate $c_{\rm sugra}$ for the $AdS_3 \times Y_7$ solutions

  as a function of the geometric twists and fluxes

- Can compare with known dual quiver gauge theories using field theory c-extremization procedure

Find exact agreement (even off-shell)!

[JPG,Martelli,Sparks 19]

Conjecture: this is true for arbitrary toric Y_5

Conjecture recently proven [Hosseini,Zaffaroni 19]

Comments

This provides an identification of an infinite classes of d=4 quiver field theories compactified on Σ_g with these $AdS_3 \times Y_7$ solutions!

Caveat: provided that they both exist…

- Geometry: there can be obstructions to the existence of Y_7
- Field theory: the field theory may not flow in the IR to a SCFT of the type we are considering

Conjecture: sufficient to check $c > 0$ and $R_a > 0$

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Comments

- There is an analogous story for $AdS_2 \times Y_9$ solutions with with $Y_7 \hookrightarrow Y_9 \rightarrow \Sigma_g$ and Y_7 toric [JPG,Martelli,Sparks 19] [Hosseini,Zaffaroni 19]

- Using toric data can calculate an off shell entropy function as a function of geometric twists and fluxes

- This can be identified with the entropy of a magnetically charged black hole in $AdS_4 \times Y_7$ (provided that they exist)

- Field theory: off-shell calculation of topological index $\mathscr{I}$ for certain quiver gauge theories compactified on Σ_g calculated in [Hosseini,Zaffaroni 16

When there are no baryonic twists:

$\boxed{\text{Find exact agreement, even off-shell!}}$

[JPG,Martelli,Sparks 19]
[Hosseini,Zaffaroni 19]
[Kim,Kim19]

Summary and outlook

- Geometric dual of c-extremization for type IIB $AdS_3 \times Y_7$
- Geometric exremization for SCQM dual to D=11 $AdS_2 \times Y_9$
  - What is the field theory story? does it exist for finite N?
  - If arise as black hole horizons, get entropy via extremization
  - Which solutions can be obtained as horizons of black holes??
- Interesting sub-class of examples

$$Y_5 \hookrightarrow Y_7 \rightarrow \Sigma_g$$
$$Y_7 \hookrightarrow Y_9 \rightarrow \Sigma_g$$

  - Toric case: striking agreement with field theory results and new microstate counting of entropy of AdS4 black holes
  - Obstructions? Geometry/black hole solutions/field theory
  - Novel features arise in toric geometry - develop
  - Non-toric class?

Focus on $AdS_5 \times Y_5$ with the complex cone $C(Y_5)$ admitting a toric Kahler cone metric: [Martelli,Sparks,Yau 05]

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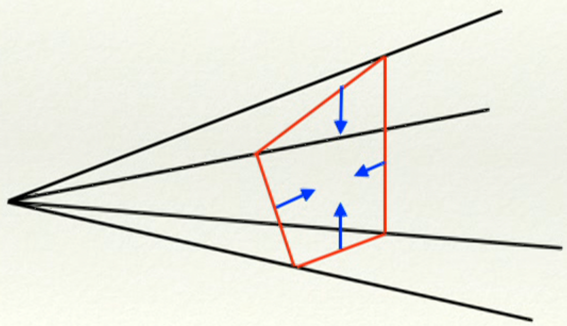

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