Title: Randomness Compression in Networks

Speakers: Yukari Uchibori

Series: Quantum Foundations

Date: August 13, 2019 - 3:30 PM

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Abstract: Randomness is a valuable resource in both classical and quantum networks and we wish to generate desired probability distributions as cheaply as possible. If we are allowed to slightly change the distribution under some tolerance level, we can sometimes greatly reduce the cardinality of the randomness or the dimension of the entanglement. By studying statistical inequalities, we show how to upper bound of the amount of randomness required for any given classical network and tolerance level. We also present a problem we encounter when compressing the randomness in a quantum network.

Randomness Compression in Networks

Yukari Uchibori
Supervisors: Jamie Sikora, Anurag Anshu

Perimeter Institute for Theoretical Physics
August 13th 2019

Compressing The Randomness in Classical Networks and Deriving A New Statistical Inequality

Yukari Uchibori

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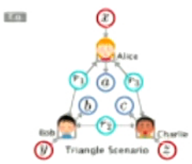
Problem

Classical Networks

Input Randomness Output



Q. How can we compress the randomness while keeping $\mathbb{P}(\text{output}|\text{input})$ (approximately) the same for a given classical network?

Motivation Randomness is a resource in a classical network and we wish to generate these probabilities as cheaply as possible.

Each party (e.g. Alice, Bob, or Charlie) measures an output for a given input and randomness. In a classical network, the source of randomness is a hidden variable.

In all classical networks, $\mathbb{P}(\text{output} \mid \text{input})$ can be expressed in the form of the summation over all randomness sources.

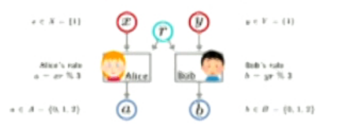
Ex. Triangle scenario

$\mathbb{P}(\text{output} \mid \text{input}) = \mathbb{P}(abc \mid xyz)$

$= \sum_{r_1} \sum_{r_2} \sum_{r_3} \mathbb{P}(r_1)\mathbb{P}(r_2)\mathbb{P}(r_3)\mathbb{P}(a \mid xr_1r_3)\mathbb{P}(b \mid yr_1r_2)\mathbb{P}(c \mid zr_2r_3)$

One reason to study classical networks is to see which probabilities $\mathbb{P}(\text{output} \mid \text{input})$ are possible to attain. This is important when we wish to study the quantum version and to test the effects of entanglement (future work).

Simple Example (Bell Scenario)



Source of randomness

$r \in R = \{0, 1, 2\}$, $|R| = 3$

$$\mathbb{P}(r) = \begin{cases} 0.04 & \text{for } r = 0 \\ 0.04 & \text{for } r = 1 \\ 0.02 & \text{for } r = 2 \end{cases} \rightarrow \mathbb{P}(\text{output} \mid \text{input}) = \mathbb{P}(ab|xy) = \begin{cases} 0.94 & \text{for } a = b = 0 \\ 0.04 & \text{for } a = b = 1 \\ 0.02 & \text{for } a = b = 2 \\ 0.00 & \text{otherwise} \end{cases}$$

Exact Case We cannot compress the cardinality of the randomness without changing $\mathbb{P}(\text{output} \mid \text{input})$.

Approximate Case We can compress R to $R' \subseteq R$ if we can slightly change $\mathbb{P}(\text{output} \mid \text{input})$ under tolerance level ϵ.

New source of randomness

$r \in R' = \{0, 1\}$, $|R'| = 2$

$$\mathbb{P}(r) = \begin{cases} 0.95 & \text{for } r = 0 \\ 0.05 & \text{for } r = 1 \end{cases} \rightarrow \mathbb{P}(\text{output} \mid \text{input}) = \mathbb{P}(ab|xy) = \begin{cases} 0.95 & \text{for } a = b = 0 \\ 0.05 & \text{for } a = b = 1 \\ 0.00 & \text{for } a = b = 2 \\ 0.00 & \text{otherwise} \end{cases}$$

We have $\| \mathbb{P}(\text{output} \mid \text{input}) - \tilde{\mathbb{P}}(\text{output} \mid \text{input}) \|_\infty \leq \epsilon = 0.02$ but $\tilde{\mathbb{P}}$ uses less randomness.

? How can we generalize this idea for any given network and tolerance level?

To solve this problem for the general case, we derived a new statistical inequality by modifying the Chernoff Bound.

Chernoff Bound (Chernoff, 1952; Hoeffding, 1963)

Let $D_1, D_2, \ldots, D_n$ be independent random variables where $\forall i, 0 \leq D_i \leq 1$ and $\mu = E(D_i)$. Define $D = \sum_{i=1}^n D_i$. Then $\forall \delta > 0$, we have

$$\mathbb{P}\left(\left|\frac{D}{n} - \mu\right| > \delta\right) \leq 2e^{-\delta^2 n}$$

We modify this...

Multivariate Chernoff Bound (NEW)

Let $r_1, r_2, \ldots, r_m$ be independent random variables and make n observations on each variable:

$$\begin{matrix} r_1^{(1)} & r_1^{(2)} & \cdots & r_1^{(n)} \\ r_2^{(1)} & r_2^{(2)} & \cdots & r_2^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ r_m^{(1)} & r_m^{(2)} & \cdots & r_m^{(n)} \end{matrix}$$

Define $D_{\mathbf{i}} = f\left(r_1^{(i_1)}, r_2^{(i_2)}, \ldots, r_m^{(i_m)}\right)$ with $\mathbf{i} = (i_1, i_2, \ldots, i_m)$ where $\forall \mathbf{i}, 0 \leq D_{\mathbf{i}} \leq 1$ and $\mu = E(D_{\mathbf{i}})$. Also define $D = \sum_{i_1=1}^n \sum_{i_2=1}^n \cdots \sum_{i_m=1}^n D_{\mathbf{i}}$. Then $\forall \delta \in [0, 1]$, we have

$$\mathbb{P}\left(\left|\frac{D}{n^m} - \mu\right| > \delta\right) \leq e^{-\frac{n}{4m^2}\left(\frac{\delta}{2}\right)^{m+1}}$$

Remarks

- To maintain the structure of the network, the Multivariate Chernoff Bound is allowed to have variables sharing sources of randomness while the Chernoff Bound can have independent variables only.
- The Multivariate Chernoff Bound tells us how many samples we need from each randomness source for a desired probability.
- The derivation of the Multivariate Chernoff Bound allows us to have a different number of samples from each randomness source.
- The proof is based on a counting argument generalizing an argument in [1].
- The Multivariate Chernoff Bound may find applications in other fields.

Acknowledgements

This opportunity was supported by Perimeter Institute for Theoretical Physics and funded by Mike and Laura Serbinis. I thank my supervisors Jamie Sikora and Anurag Anshu for continued guidance on this project.

References

[1] A. Anshu, New J. Phys 18, 083011 (2016).
[2] J. S. Bell, Physics 1 195 (1964).
[3] H. Chernoff, Ann. Math. Stat 23, 493-509 (1952).
[4] W. Hoeffding, J. Am. Stat. Assoc 58, 13-30 (1963).
[5] D. Rosset, N. Gisin, and E. Wolfe, QUANTUM INF COMPUT 18, 0910-0926 (2018).

Solution

A. Suppose we have a network with h inputs, h outputs (n_x choices of input per source, n_a choices of output per source), and m randomness sources. For tolerance level $\epsilon > 0$, using the Multivariate Chernoff Bound, we prove that it is sufficient to have the cardinality of each randomness source to be at most

$$\left\lceil 3hm^2\left(\frac{2}{\epsilon}\right)^{m+1} \ln(n_x n_a) \right\rceil$$

Bound for Approximate Case

How Good Is Our Solution?

For the general case, if the tolerance level is set to $\epsilon = 0$, the previous study in [5] determined that the cardinality of each randomness source required for any m is bounded above by $(n_x n_a)^h - 1$

Bound for Exact Case

Fig1 Comparison of our bound with the bound for the exact case

(a) $h = 3$, $\epsilon = 0.05$ (b) $m = 1$, $\epsilon = 0.05$

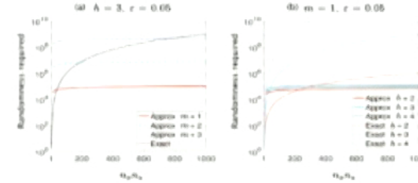
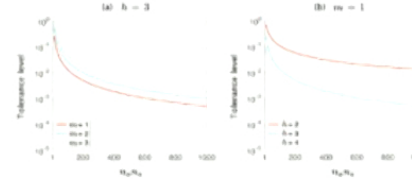


Fig2 Range of tolerance levels where our bound gives a compression

(a) $h = 3$ (b) $m = 1$



Summary of Our Solution

- Our bound performs better than the bound for the exact case for larger input and output sets (when n_x, n_a, or h is large).
- Our bound performs especially well for values of m.

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Outline

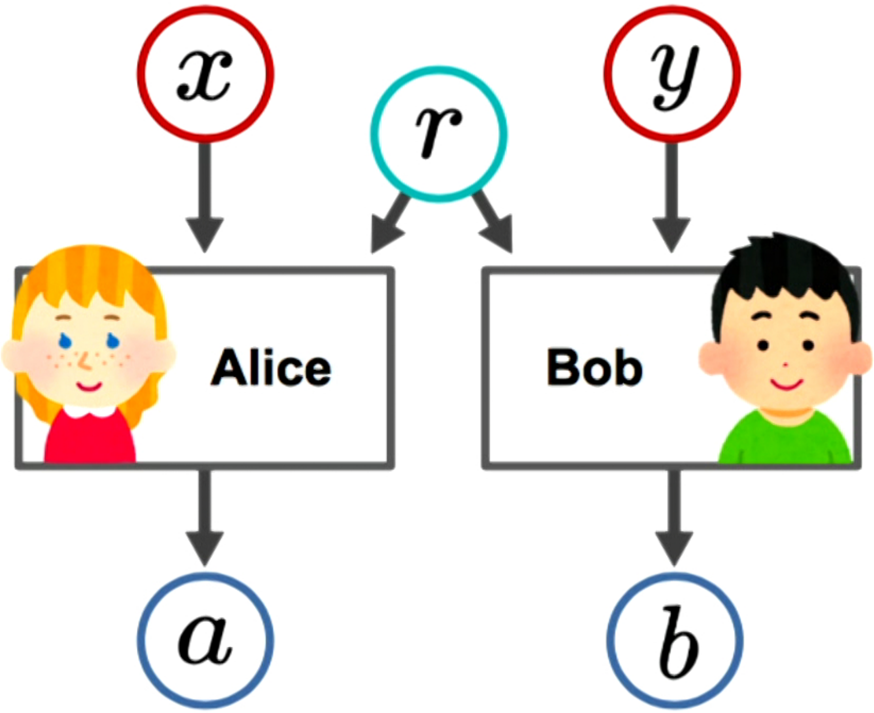
- Three types of networks
- The goal of my summer project

- Chernoff Bound & solution for the simplest classical network
- Multivariate Chernoff Bound & solution for the general classical network
- Summary for classical networks

- Solution for quantum networks
- Summary for quantum networks

- Next steps

Classical Network (Bell Scenario)



1. Randomly generate r with $\mathbb{P}(r)$
2. Alice generates a with $\mathbb{P}(a \mid xr)$
   Bob generates b with $\mathbb{P}(b \mid yr)$
3. Combining Alice and Bob, we have a joint distribution,
   $\mathbb{P}\big(\text{output} \mid \text{input}\big) = \mathbb{P}(ab \mid xy)$

Bell (1964)

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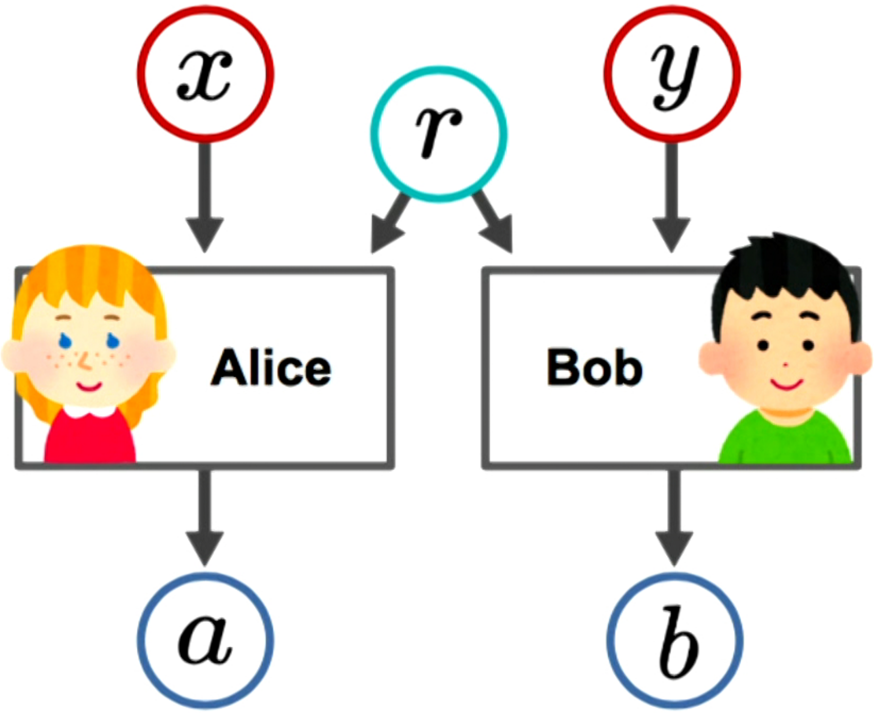

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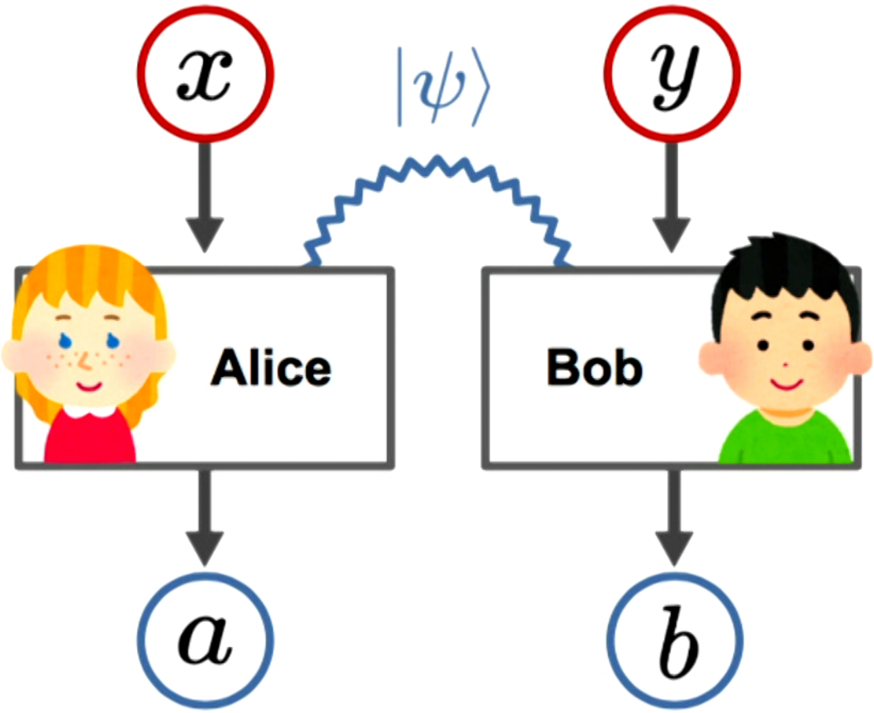
Classical Randomness

$$\mathbb{P}(ab \mid xy)$$
$$= \Sigma_r \mathbb{P}(r)\mathbb{P}(a \mid xr)\mathbb{P}(b \mid yr)$$

Bell (1964)

Quantum Network (Bell Scenario)



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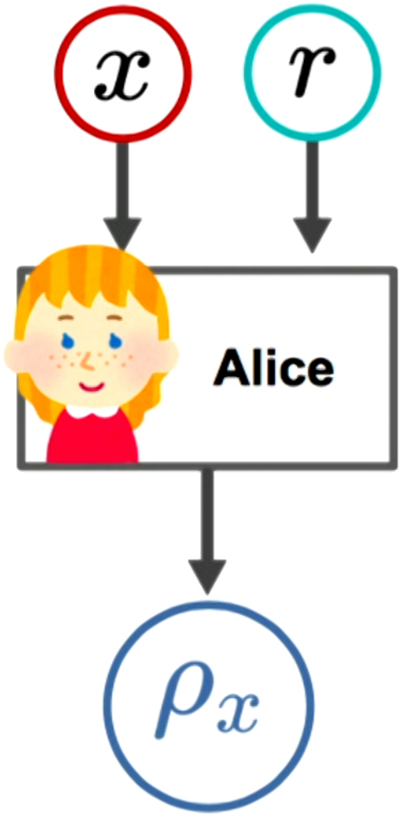
Quantum Entanglement

$$\mathbb{P}(ab \mid xy) = \langle\psi|A_a^x \otimes B_b^y|\psi\rangle$$

Bell (1964)

Classical - Quantum Network



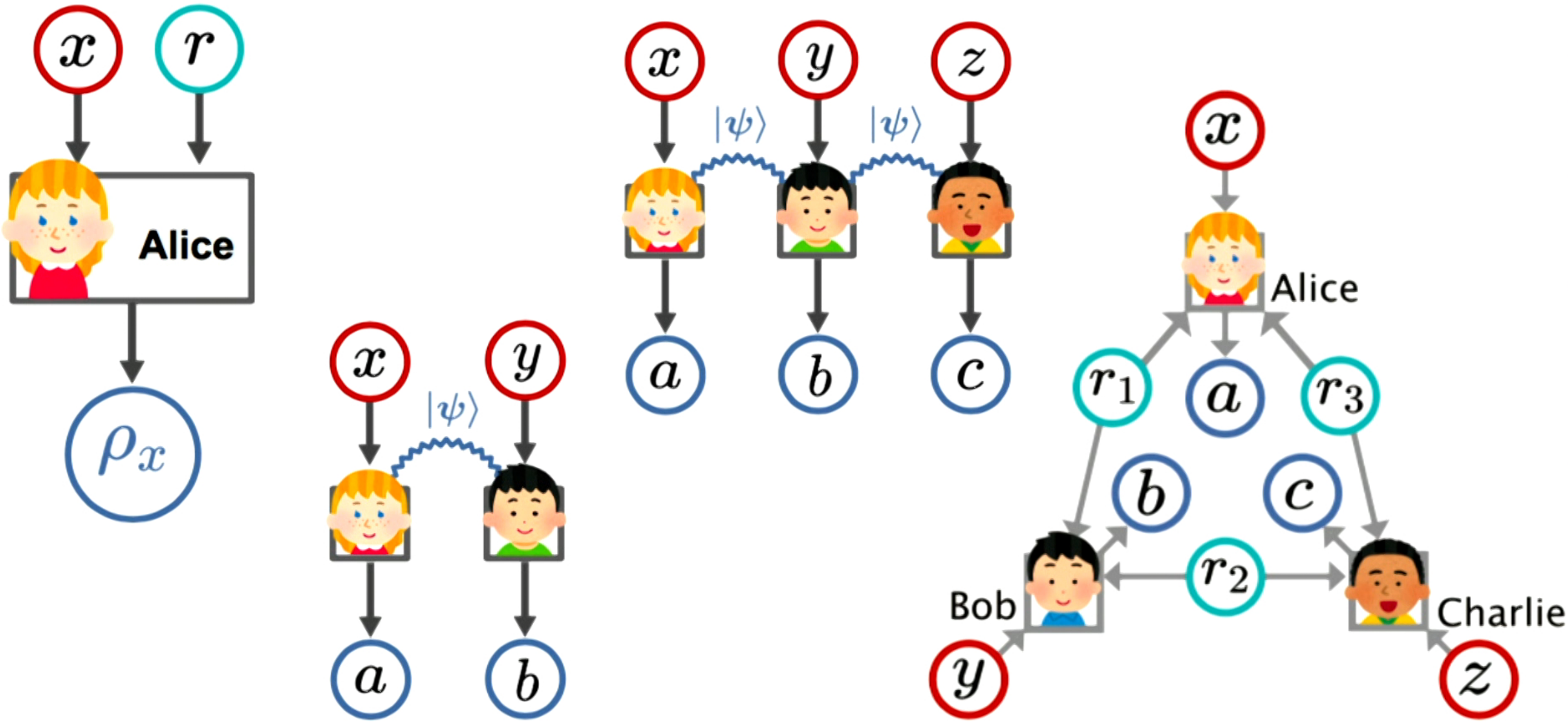
1. Alice prepares ρ_{xr} with $\mathbb{P}(r)$

2. Alice will have an output, quantum state, ρ_x

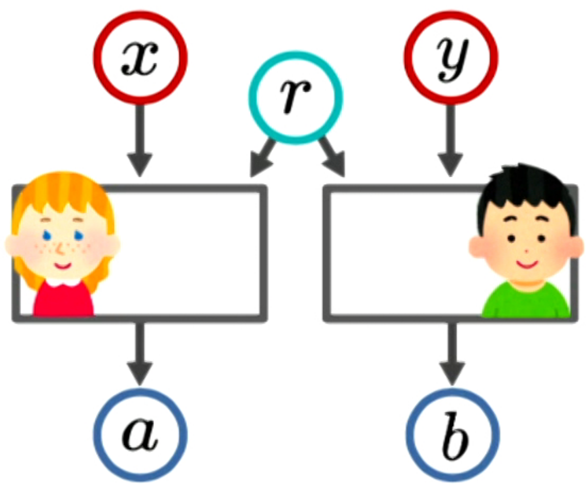
Quantum State

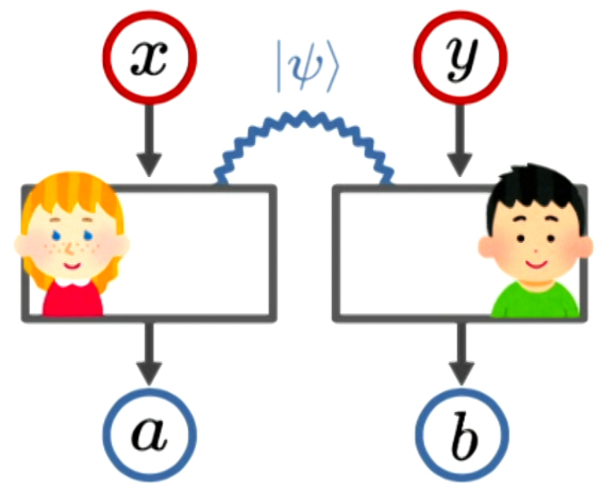
$$\rho_x = \sum_r \mathbb{P}(r)\rho_{xr}$$

Various Structure of Networks



Why Do We Study These Networks?





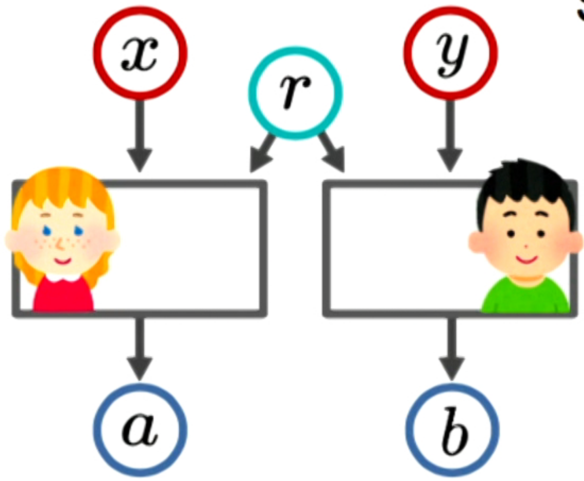
Why Do We Study These Networks?

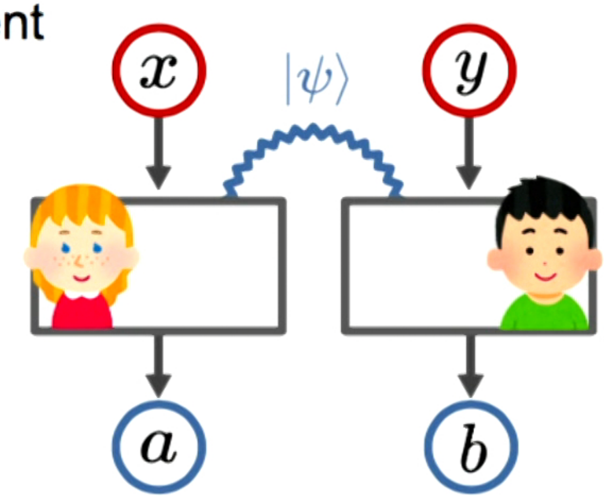
To study quantumness

What is possible and not possible classically and quantumly?

Compare the classical case and quantum case

Study effects of quantum entanglement





For a given network and $\mathbb{P}(\text{output} \mid \text{input})$,
can we compress the cardinality (dimension) of randomness (entanglement) $|R|$ required?

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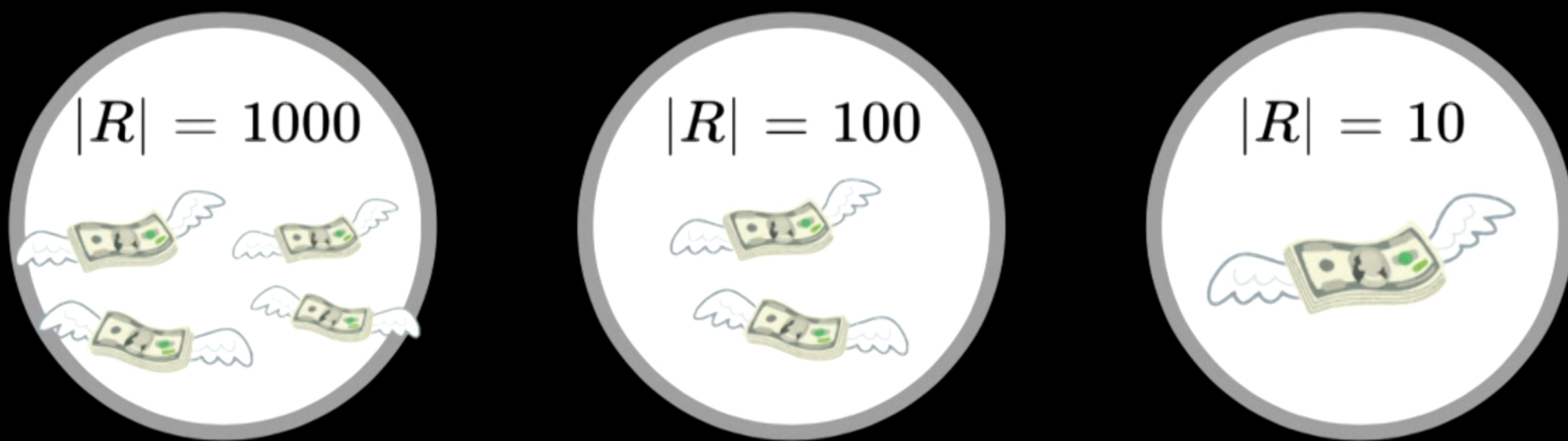
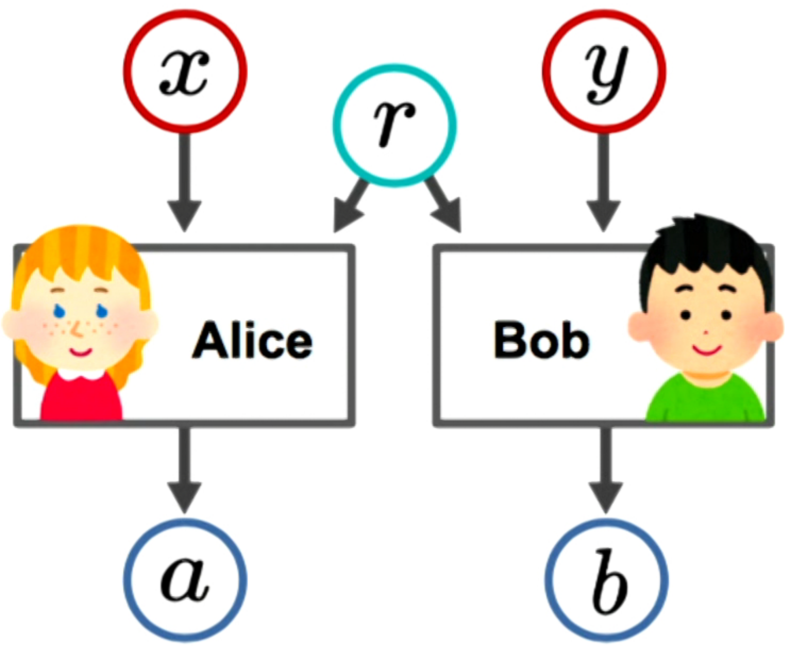


Simple Example 1 in Classical Network - Exact Case

$x \in X = \{1\}$ $\qquad$ $y \in Y = \{1\}$



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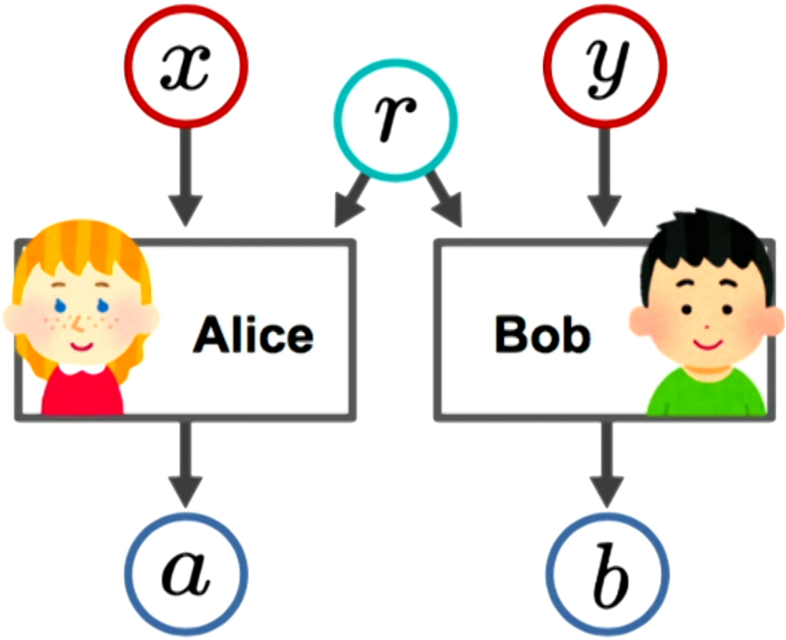

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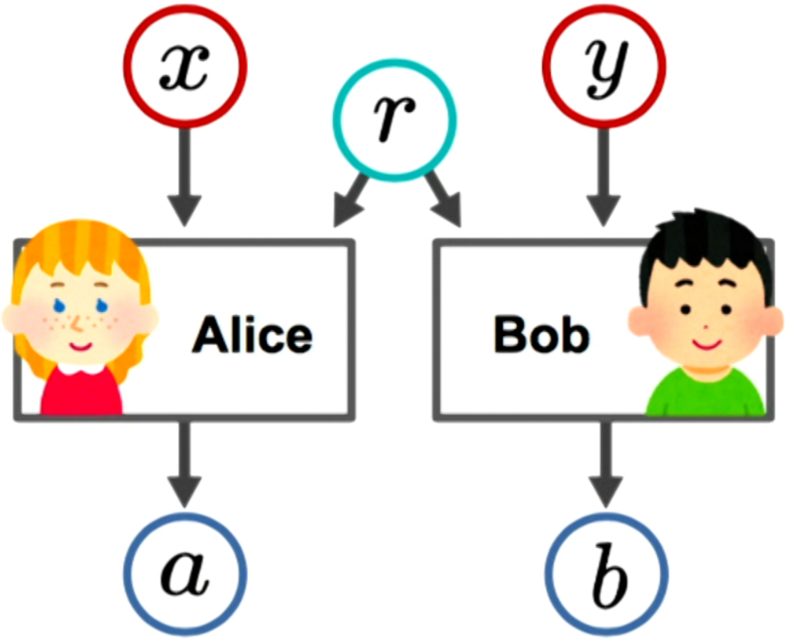
$r \in R = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$



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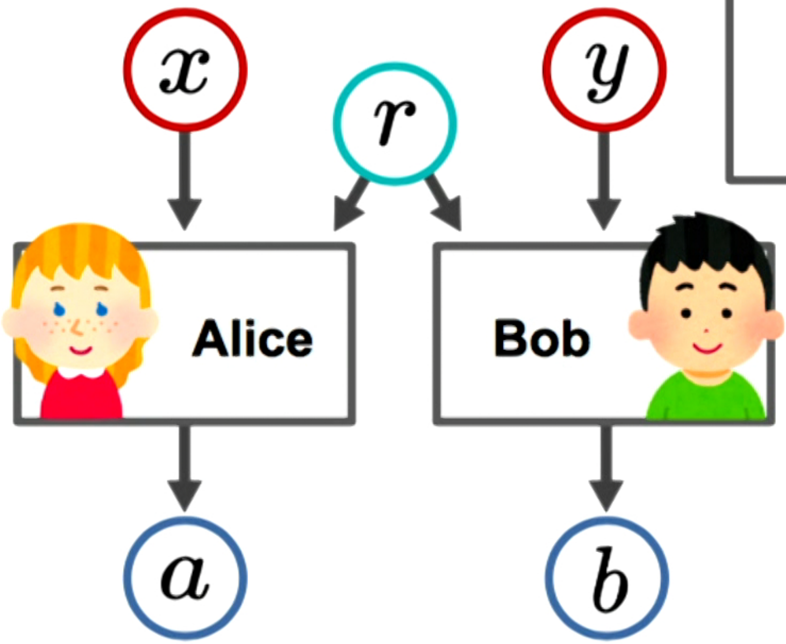
$r \in R = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $\forall r \in R, \mathbb{P}(r) = \frac{1}{10}$ $|R| = 10$



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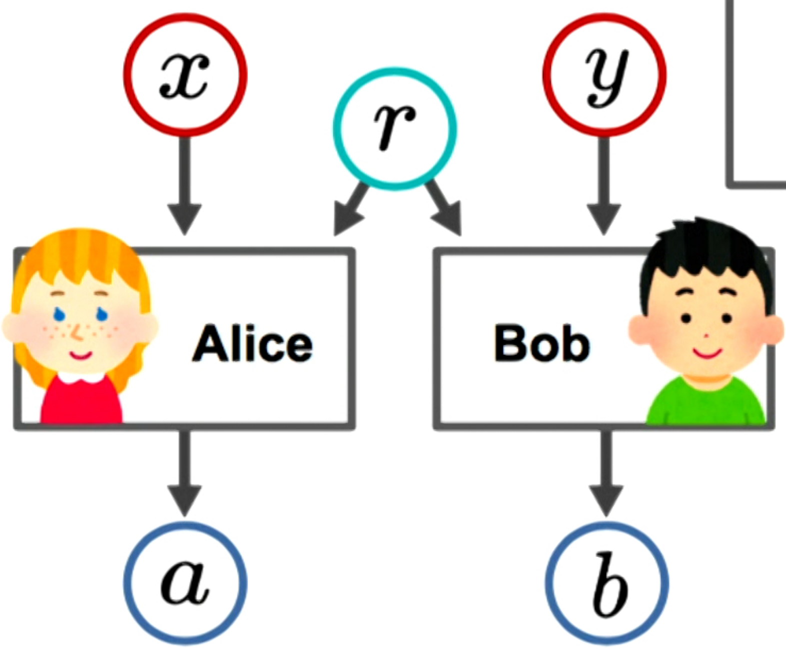

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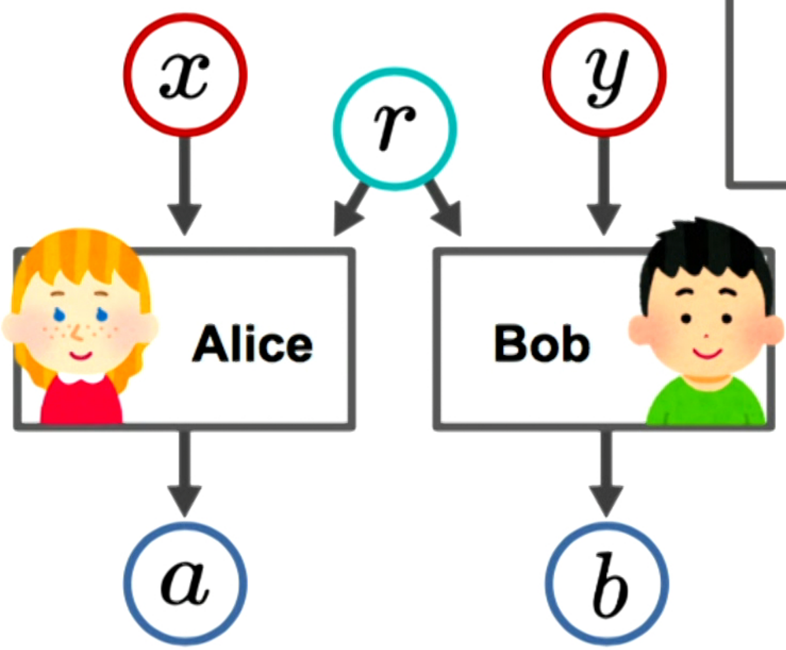

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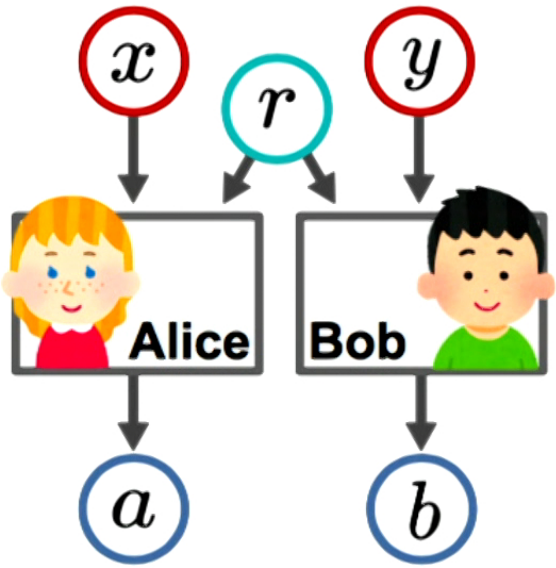

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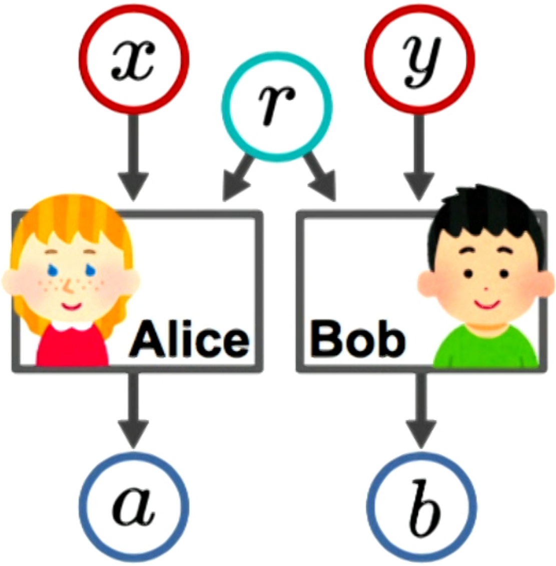

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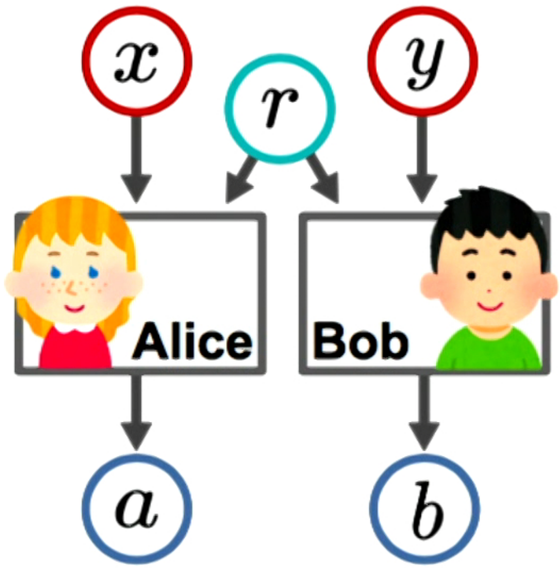

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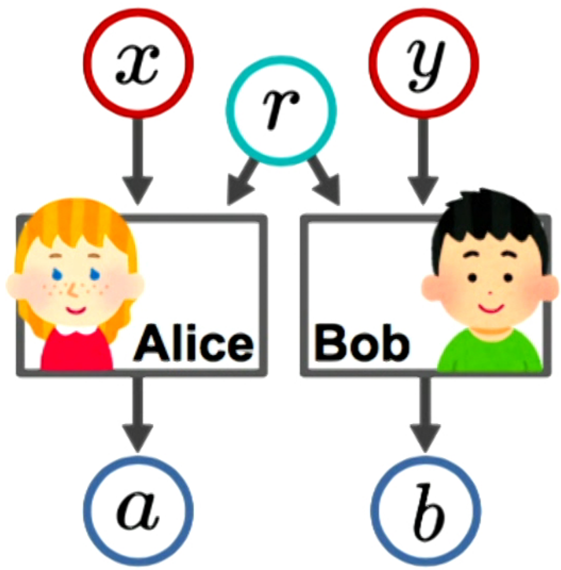

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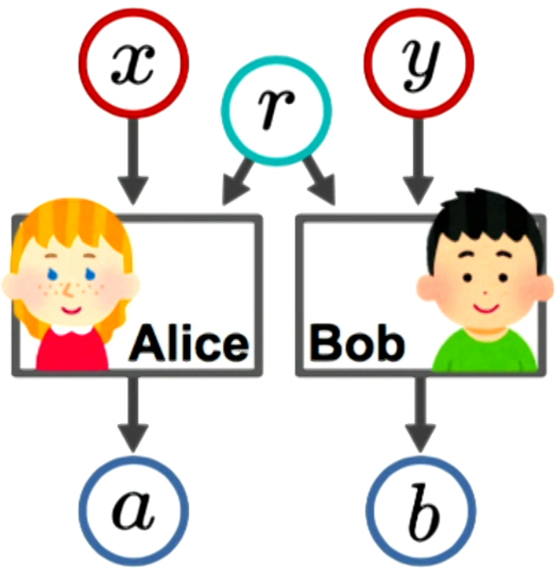

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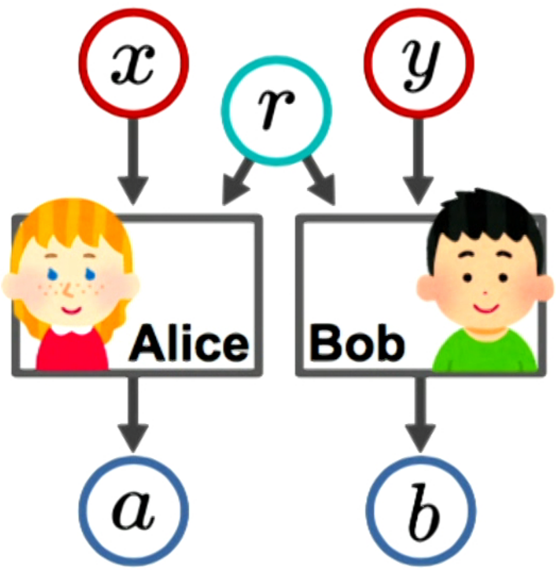

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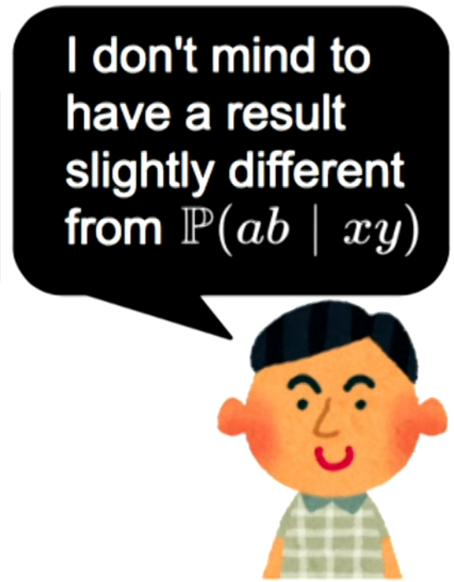

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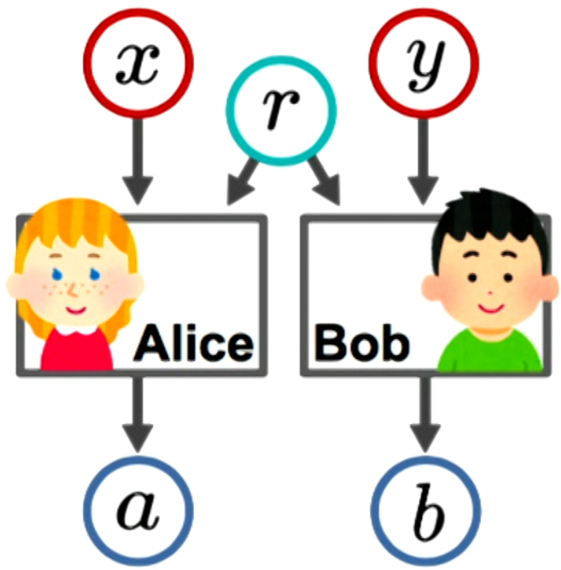

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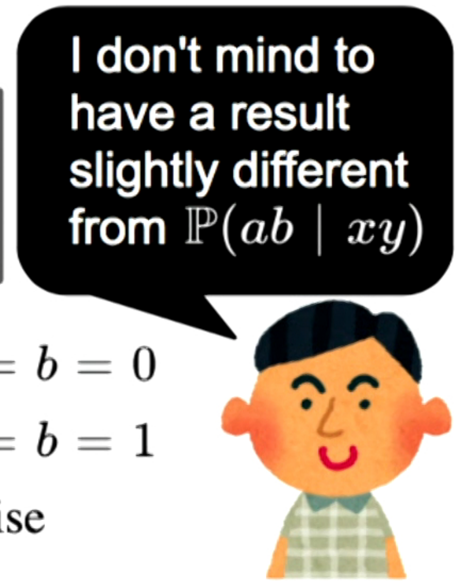

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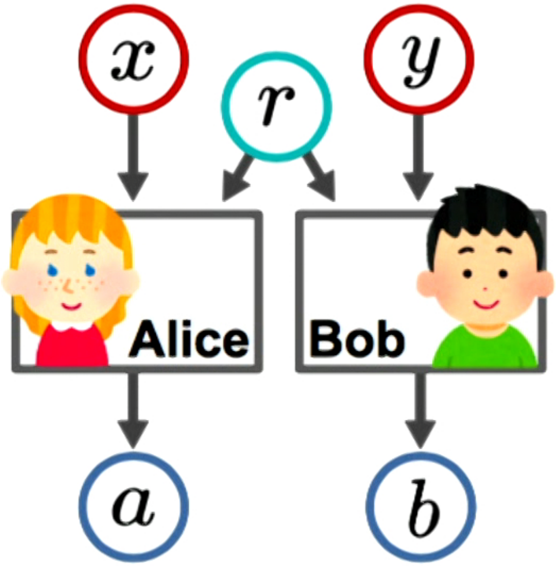
Simple Example 2 in Classical Network - Approximate Case



$x \in X = \{1\} \quad y \in Y = \{1\}$

$a \in A = \{0, 1, 2\}$

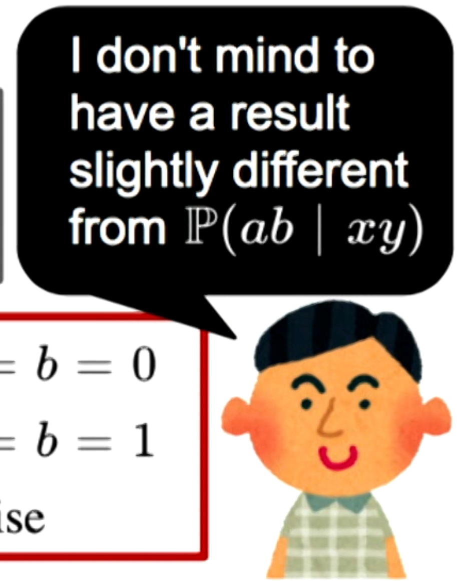
$b \in B = \{0, 1, 2\}$

$r \in R = \{0, 1, \cancel{2}\} \qquad |R| = \cancel{3}$

$$\mathbb{P}(r) = \begin{cases} 0.95 & \text{for } r = 0 \\ 0.05 & \text{for } r = 1 \\ \cancel{0.02} & \cancel{\text{for } r = 2} \end{cases}$$

Alice's Rule is
$a = xr \,\%\, 3$

Bob's Rule is
$b = yr \,\%\, 3$



$$\mathbb{P}(ab \mid xy) = \begin{cases} 0.94 & \text{for } a = b = 0 \\ 0.04 & \text{for } a = b = 1 \\ 0.02 & \text{for } a = b = 2 \\ 0.00 & \text{otherwise} \end{cases}$$

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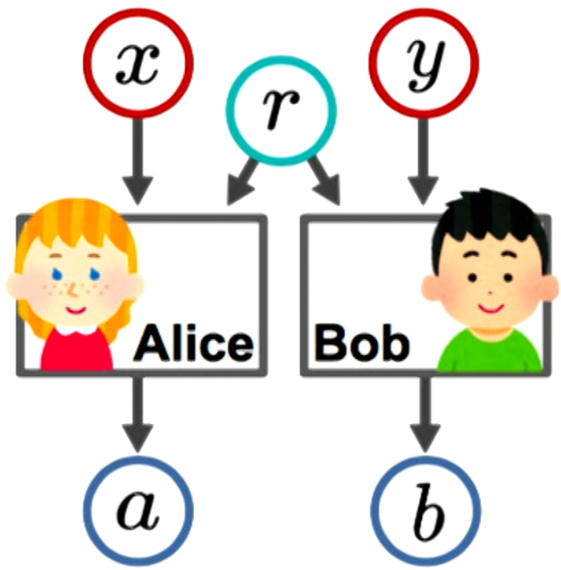

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I don't mind to have a result slightly different from $\mathbb{P}(ab \mid xy)$

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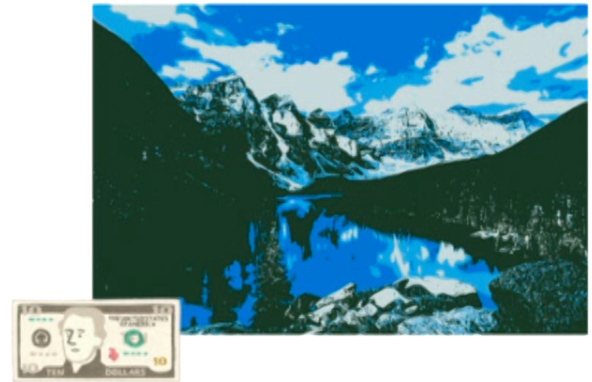
The Goal of The Summer Project

$20

$10

$0.3



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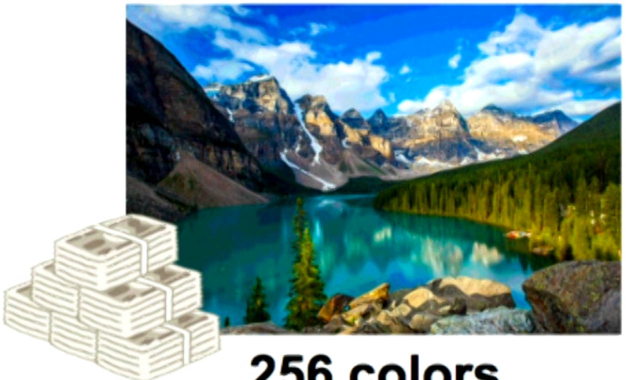
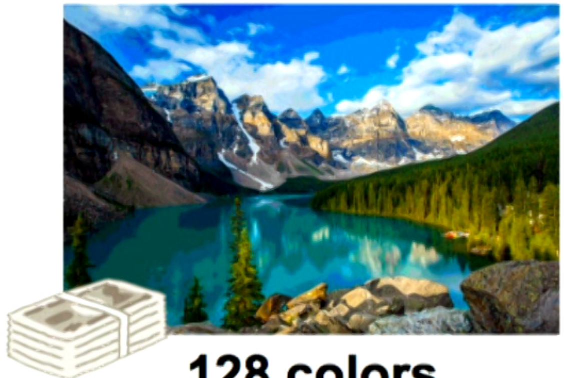


256 colors

$10



128 colors

$0.3

4 colors

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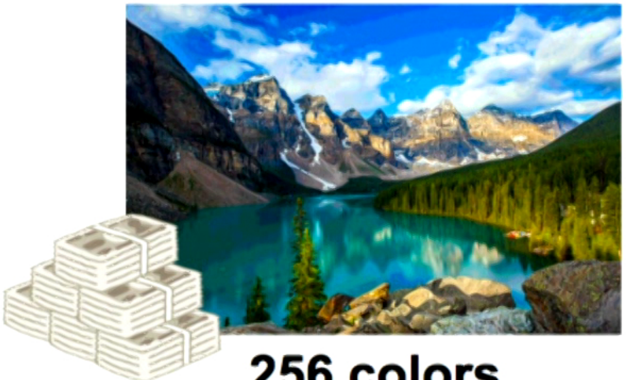

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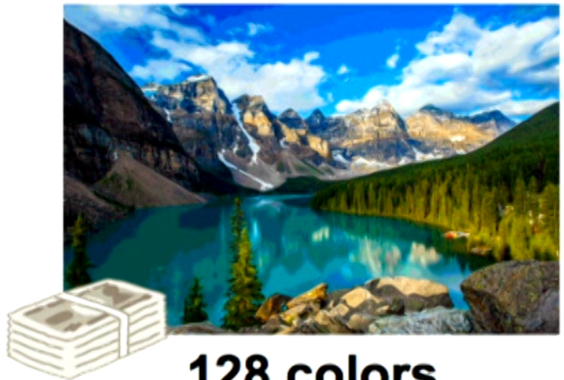

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We do not need $\left|\left|\mathbb{P}(\text{output}|\text{input}) - \widehat{\mathbb{P}}(\text{output}|\text{input})\right|\right|_\infty = 0.$

Instead, we want $\left|\left|\mathbb{P}(\text{output}|\text{input}) - \widehat{\mathbb{P}}(\text{output}|\text{input})\right|\right|_\infty \leq \varepsilon$

for given tolerance ε. So, we can reduce the dimension of R

by making such $\widehat{\mathbb{P}}(\text{output}|\text{input})$.

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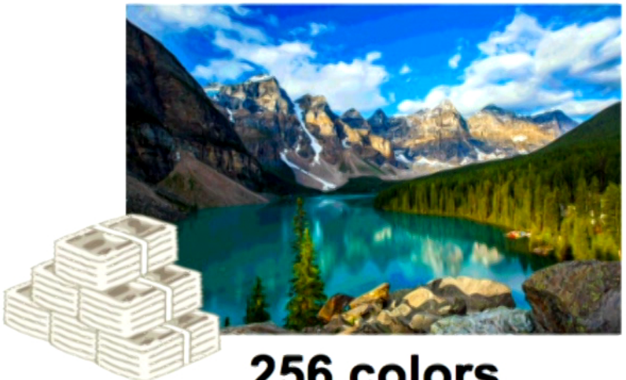

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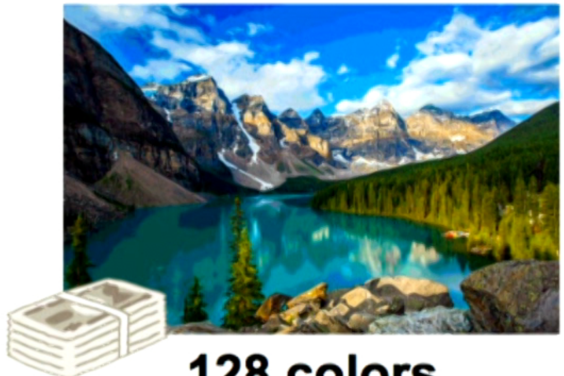

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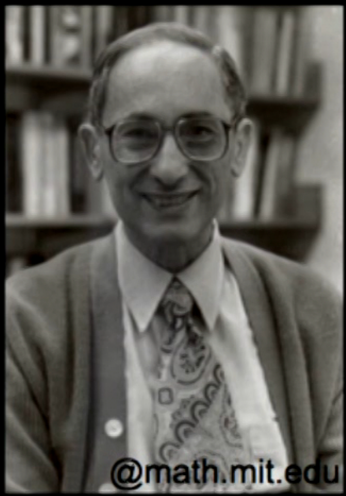
HOW?

For the classical network, using the power of statistics! Chernoff Bound



Herman Chernoff

Our Solution for The Simplest Classical Network

Chernoff Bound (Chernoff, 1952; Hoeffding, 1963)

Let $r^{(1)}, r^{(2)}, \ldots, r^{(n)}$ be independent random variables. Also let $D_i \equiv f\left(r^{(i)}\right)$

where $\forall i, 0 \leq D_i \leq 1$ and $\mu = E(D_i)$. Define $D \equiv \Sigma_{i=1}^{n} D_i$.

Then $\forall \varepsilon > 0$, we have

$$\mathbb{P}\left(\left|\frac{D}{n} - \mu\right| > \varepsilon\right) \leq 2e^{-n\varepsilon^2}$$

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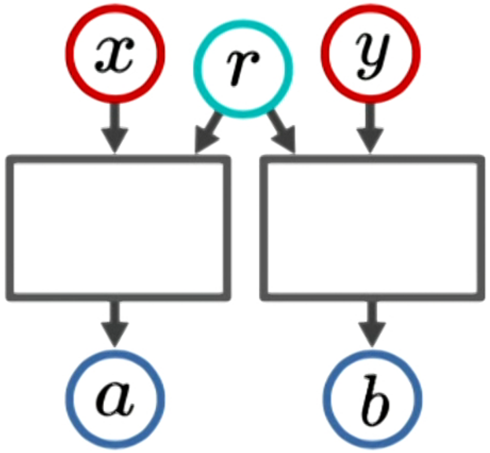

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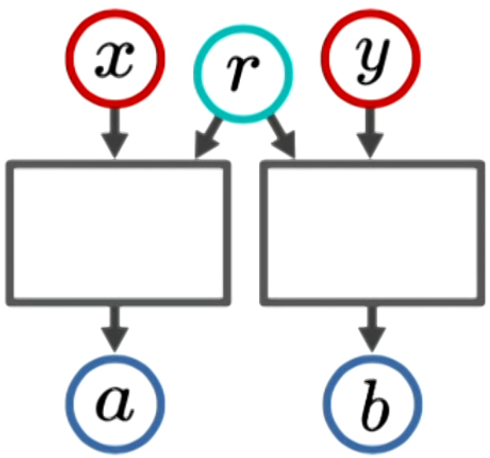

$$\mathbb{P}\left(ab \mid xyr^{(i)}\right) = D_i$$

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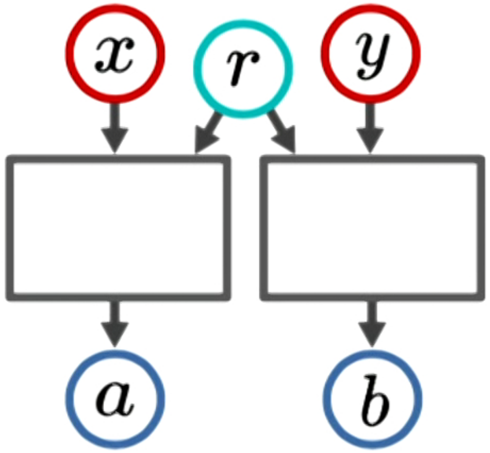

$$\mathbb{P}\left(ab \mid xyr^{(i)}\right) = D_i$$

$$\widehat{\mathbb{P}}(ab \mid xy) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{P}\left(ab \mid xyr^{(i)}\right)$$

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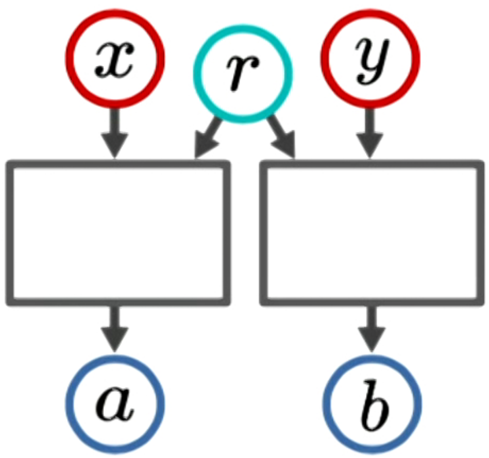

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n to get $\widehat{\mathbb{P}}(ab \mid xy)$ ε close to $\mathbb{P}(ab \mid xy)$

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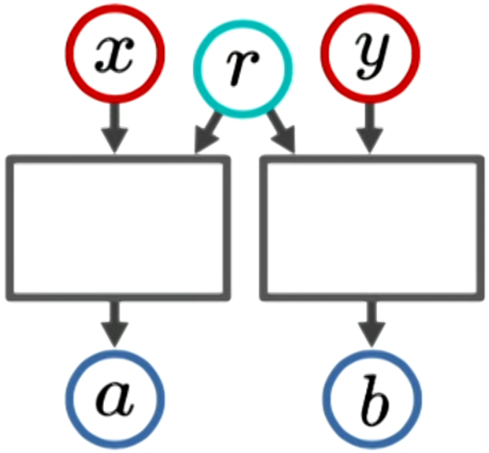
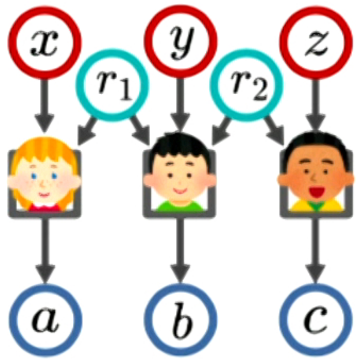
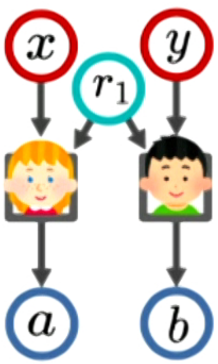


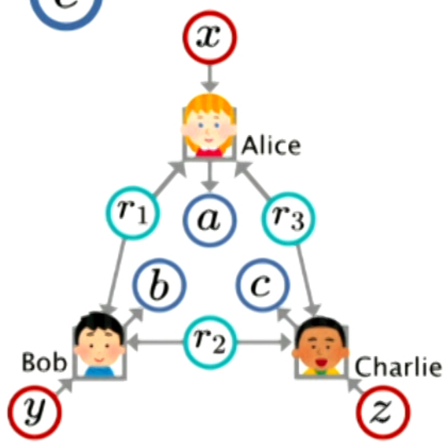
The cardinality of the randomness required is bounded as

$$|R| < \left\lceil \frac{1}{\varepsilon^2} \ln(2 n_x n_y n_a n_b) \right\rceil$$

Our Solution for The General Classical Network







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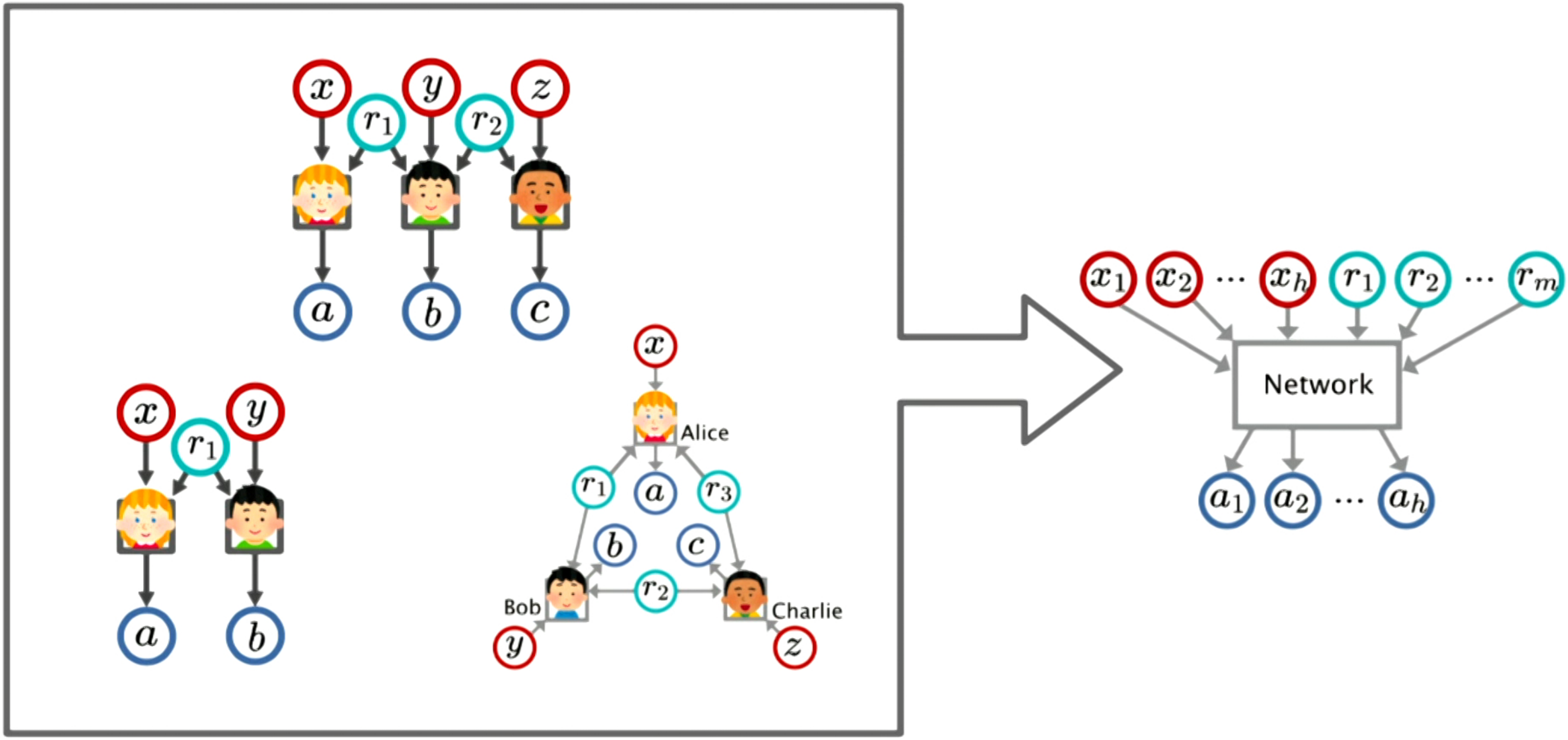

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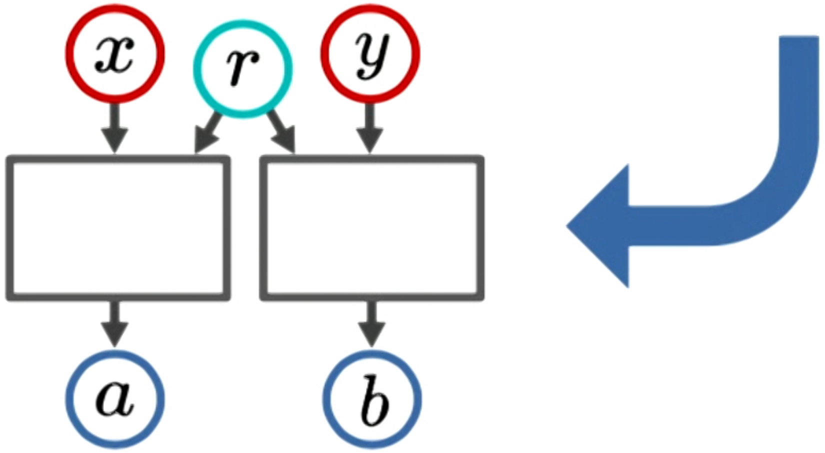

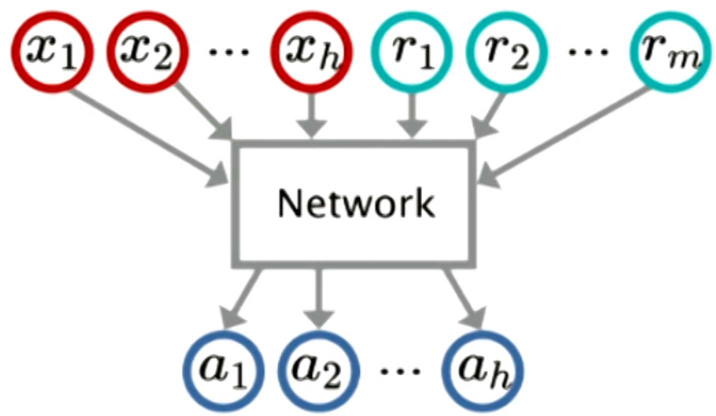

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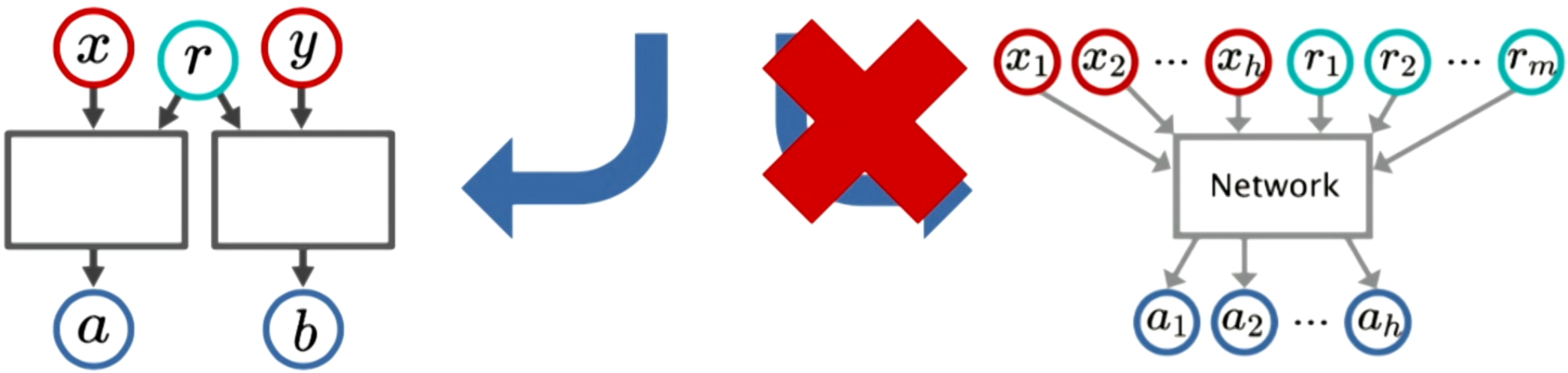


Deriving Multivariate Chernoff Bound

Let $r_1, r_2, \ldots, r_m$ be independent random variables and make n observations on each variable.

$$\begin{matrix} r_1^{(1)} & r_1^{(2)} & \ldots & r_1^{(n)} \\ r_2^{(1)} & r_2^{(2)} & \ldots & r_2^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ r_m^{(1)} & r_m^{(2)} & \ldots & r_m^{(n)} \end{matrix}$$

Define $D_{\mathrm{i}} \equiv f\left(r_1^{(i_1)}, r_2^{(i_2)}, \ldots, r_m^{(i_m)}\right)$ with $\mathrm{i} = (i_1, i_2, \ldots, i_m)$ where $\forall \mathrm{i}, 0 \leq D_{\mathrm{i}} \leq 1$

and $\mu = E(D_{\mathbf{i}})$. Also define $D \equiv \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} \cdots \sum_{i_m=1}^{n} D_{\mathrm{i}}$.

Then $\forall \varepsilon \in [0, 1]$,

$$\mathbb{P}\left(\left|\frac{D}{n^m} - \mu\right| > \varepsilon\right) \leq \textbf{?????}$$

$$] = \mathbb{E}\left[\left(\sum_{i_1=1}\sum_{i_2=1}\cdots\sum_{i_m=1} Z(\vec{i}) - N\mu\right)^t\right]$$

$$= \mathbb{E}\left[\left(\sum_{i_1=1}^{n_1}\sum_{i_2=1}^{n_2}\cdots\sum_{i_m=1}^{n_m}\left(Z(\vec{i}) - \mu\right)\right)^t\right]$$

$$= \mathbb{E}\left[\left(\sum_{\vec{i}\in I}\left(Z(\vec{i}) - \mu\right)\right)^t\right]$$

$$= \mathbb{E}\left[\sum_{\vec{i}_1\in I}\left(Z(\vec{i}_1) - \mu\right)\sum_{\vec{i}_2\in I}\left(Z(\vec{i}_2) - \mu\right)\cdots\sum_{\vec{i}_t\in I}\left(Z(\vec{i}_t) - \mu\right)\right]$$

$$= \sum_{\vec{i}_1\in I}\sum_{\vec{i}_2\in I}\cdots\sum_{\vec{i}_m\in I}\mathbb{E}\left[\left(Z(\vec{i}_1) - \mu\right)\left(Z(\vec{i}_2) - \mu\right)\cdots\left(Z(\vec{i}_t) - \mu\right)\right]$$

$$= \sum_{\vec{i}_1\in I}\sum_{\vec{i}_2\in I}\cdots\sum_{\vec{i}_m\in I} E\left(\vec{i}_1, \vec{i}_2, \ldots, \vec{i}_t\right)$$

$$\mathbb{E}[(Z - N\mu)^t] \leq \left[\frac{(2N)^{m+1}mt}{m+1}\sum_{j=1}^{m}\frac{1}{n_j}\right]^{\frac{1}{m+1}t}.$$

$$\mathbb{E}[Z] = \mathbb{E}\left[\sum_{i_1=1}^{n_1}\sum_{i_2=1}^{n_2}\cdots\sum_{i_m=1}^{n_m} Z(\vec{i})\right]$$

$$= \sum_{i_1=1}^{n_1}\sum_{i_2=1}^{n_2}\cdots\sum_{i_m=1}^{n_m}\mathbb{E}[Z(\vec{i})]$$

$$n_1 \quad n_2 \quad n_m$$

$$\frac{\mathbb{E}[(Z - N\mu)^t]}{(N\delta)^t} \geq \mathbb{P}((Z - N\mu)^t > (N\delta)^t)$$

$$= \mathbb{P}(|Z - N\mu| > N\delta)$$

$$= \mathbb{P}\left(\left|\frac{Z}{N} - \mu\right| > \delta\right).$$

$$m(t-p) = m\left(t - \left\lfloor\frac{m}{m+1}t\right\rfloor - h\right)$$

$$= m\left(\frac{m+1}{m+1}t - \left\lfloor\frac{m}{m+1}t\right\rfloor - h\right)$$

$$= m\left(\frac{1}{m+1}t + \frac{m}{m+1}t - \left\lfloor\frac{m}{m+1}t\right\rfloor - h\right)$$

$$< m\left(\frac{1}{m+1}t\right)$$

$$\leq p.$$

$$E\left(\vec{i}_1, \vec{i}_2, \ldots, \vec{i}_t\right) = \mathbb{E}\left[\left(Z(\vec{i}_1) - \mu\right)\left(Z(\vec{i}_2) - \mu\right)\cdots\left(Z(\vec{i}_t) - \mu\right)\right].$$

$$\left|\frac{Z}{N} - \mu\right| > \delta\Bigg) \leq \frac{\mathbb{E}[(Z - N\mu)^t]}{(N\delta)^t}$$

$$\leq \frac{1}{N^t\delta^t}\left[\frac{(2N)^{m+1}mt}{m+1}\sum_{j=1}^{m}\frac{1}{n_j}\right]^{\frac{1}{m+1}t} \text{ by Lemma (3.6) with}$$

$$\leq \left[\frac{2^{m+1}mt\sum_{j=1}^{m}\frac{1}{n_j}}{(m+1)\delta^{m+1}}\right]^{\frac{t}{m+1}}$$

$$\leq \left(\frac{1}{3}\right)^{\frac{t}{m+1}} \text{ by Eq. (24)}$$

$$\leq \left(\frac{1}{3}\right)^{\frac{1}{m+1}\left\lfloor\frac{(m+1)\delta^{m+1}}{3(2^{m+1})m\sum_{j=1}^{m}\frac{1}{n_j}}\right\rfloor} \text{ by I}$$

$$\leq e^{-\frac{1}{m+1}\left\lfloor\frac{(m+1)\delta^{m+1}}{3(2^{m+1})m\sum_{j=1}^{m}\frac{1}{n_j}}\right\rfloor}.$$

$$\vec{i} = (i_1, i_2, \ldots, i_m) \in \{1, 2, \ldots, n_1\} \times \{1, 2, \ldots, n_2\} \times \cdots \times \{1, 2, \ldots, n_m\}$$

$$E\left(\vec{i}_1, \cdots, \vec{i}_t\right) = \mathbb{E}\left[\left(Z(\vec{i}_1) - \mu\right)\cdots\left(Z(\vec{i}_j) - \mu\right)\cdots\left(Z(\vec{i}_t) - \mu\right)\right]$$

$$= \mathbb{E}\left[Z(\vec{i}) - \mu\right]\mathbb{E}\left[\left(Z(\vec{i}_1) - \mu\right)\cdots\left(Z(\vec{i}_t) - \mu\right)\right]$$

$$= \left(\mathbb{E}\left[Z(\vec{i}_j)\right] - \mu\right)\mathbb{E}\left[\left(Z(\vec{i}_1) - \mu\right)\cdots\left(Z(\vec{i}_t) - \mu\right)\right]$$

$$= (\mu - \mu)\mathbb{E}\left[\left(Z(\vec{i}_1) - \mu\right)\cdots\left(Z(\vec{i}_t) - \mu\right)\right]$$

$$= 0.$$

$$t = 2\left\lfloor\frac{(m+1)\delta^{m+1}}{6(2^{m+1})m\sum_{j=1}^{m}\frac{1}{n_j}}\right\rfloor \leq \frac{m+1}{m\sum_{j=1}^{m}\frac{1}{n_j}} \text{ for } \delta \in [0, 1],$$

$$\left\{\begin{array}{l}\text{the number of all}\\ \text{possible Bad tuples}\end{array}\right\} \leq \left\{\begin{array}{l}\text{the number of distinct}\\ \text{strings we can obtain}\end{array}\right\} \cdot \left\{\begin{array}{l}\text{the number of I}\\ \text{leading to each}\end{array}\right.$$

$$\leq 2^t \cdot N^p \cdot \left[p\sum_{j=1}^{m}\frac{N}{n_j}\right]^{t-p} \text{ by Eq. (39, 40)}$$

$$\leq 2^t \cdot \left[\frac{1}{p\sum_{j=1}^{m}\frac{1}{n_j}}\right]^p\left[p\sum_{j=1}^{m}\frac{N}{n_j}\right]^t$$

$$\leq 2^t \cdot \left[\frac{1}{p\sum_{j=1}^{m}\frac{1}{n_j}}\right]^{\frac{m}{m+1}t}\left[p\sum_{j=1}^{m}\frac{N}{n_j}\right]^t \text{ by Eq. (41, 43}$$

$$\leq 2^t \cdot \left[pN^{m+1}\sum_{j=1}^{m}\frac{1}{n_j}\right]^{\frac{1}{m+1}t}$$

$$< \left[\frac{(2N)^{m+1}mt}{}\sum^{m}\frac{1}{}\right]^{\frac{1}{m+1}t} \text{ by Eq. (41).}$$

Multivariate Chernoff Bound

Let $r_1, r_2, \ldots, r_m$ be independent random variables and make n observations on each variable.

$$\begin{matrix} r_1^{(1)} & r_1^{(2)} & \cdots & r_1^{(n)} \\ r_2^{(1)} & r_2^{(2)} & \cdots & r_2^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ r_m^{(1)} & r_m^{(2)} & \cdots & r_m^{(n)} \end{matrix}$$

The derivation also allows us to have different number of observations on each variable.

Define $D_{\mathrm{i}} \equiv f\left(r_1^{(i_1)}, r_2^{(i_2)}, \ldots, r_m^{(i_m)}\right)$ with $\mathrm{i} = (i_1, i_2, \ldots, i_m)$ where $\forall \mathrm{i}, 0 \leq D_{\mathrm{i}} \leq 1$

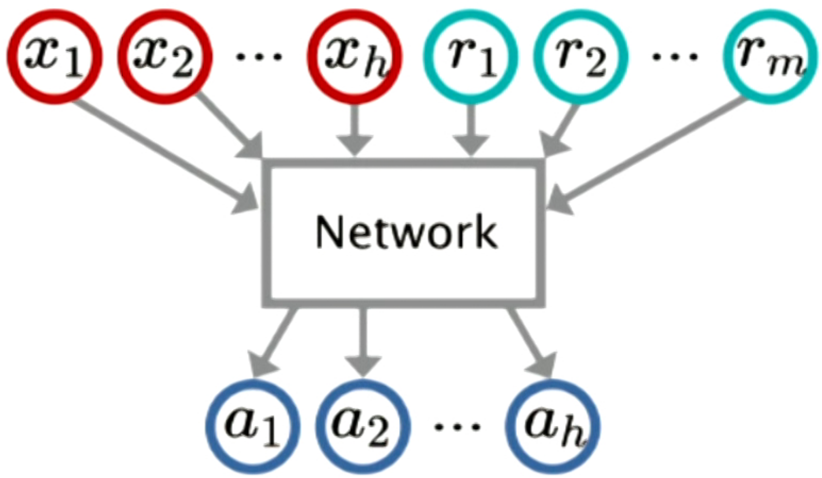
and $\mu = E(D_{\mathbf{i}})$. Also define $D \equiv \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} \cdots \sum_{i_m=1}^{n} D_{\mathrm{i}}$.

Then $\forall \varepsilon \in [0, 1]$,

$$\mathbb{P}\left(\left|\frac{D}{n^m} - \mu\right| > \varepsilon\right) \leq e^{-\frac{n}{6m^2}\left(\frac{\varepsilon}{2}\right)^{m+1}}$$

Our Solution for The General Classical Network

Using Multivariate Chernoff Bound, we prove that it is sufficient to have the cardinality of each randomness source to be at most



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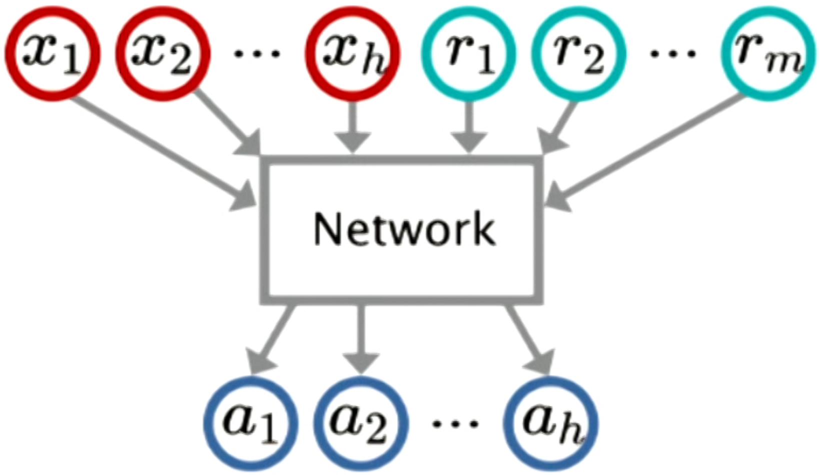

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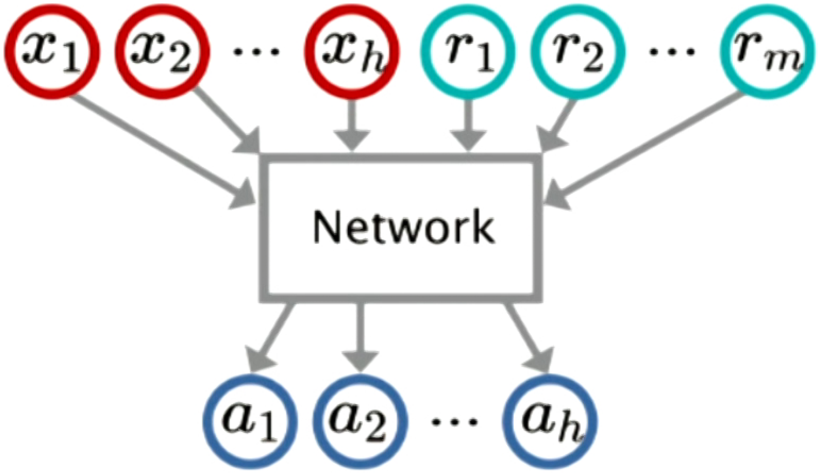

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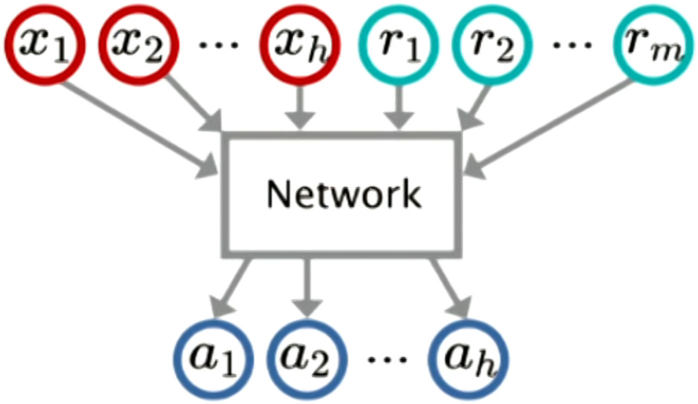

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Bound for Approximate Case

For the general case, if the tolerance level is set to $\varepsilon = 0$, the previous study in [1] determined that the cardinality of each randomness source required for any m is bounded above by $\left(n_x n_a\right)^h + 1$

Bound for Exact Case

[1] D. Rosset, N. Gisin, and E. Wolfe, QUANTUM INF COMPUT 18, 0910-0926 (2018)

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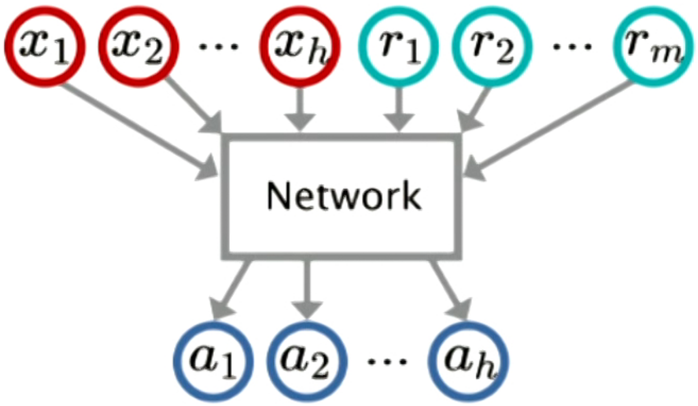

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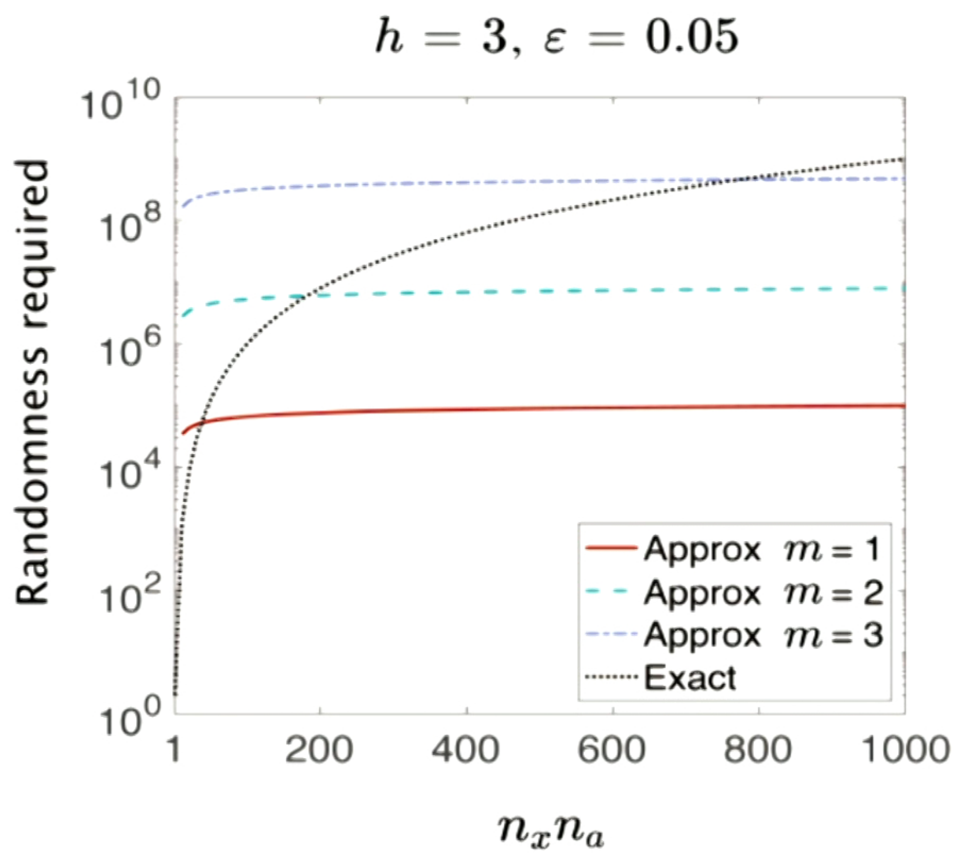

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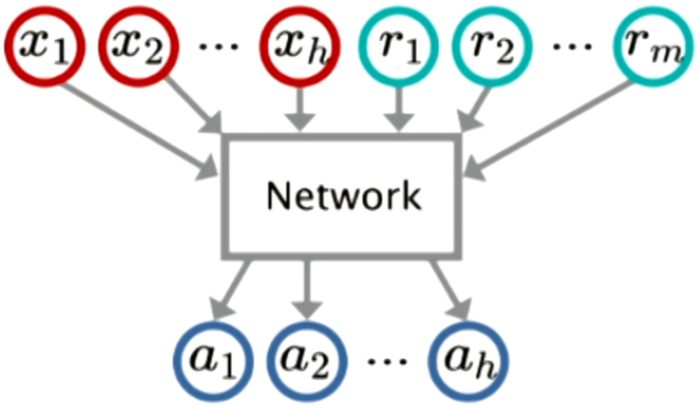

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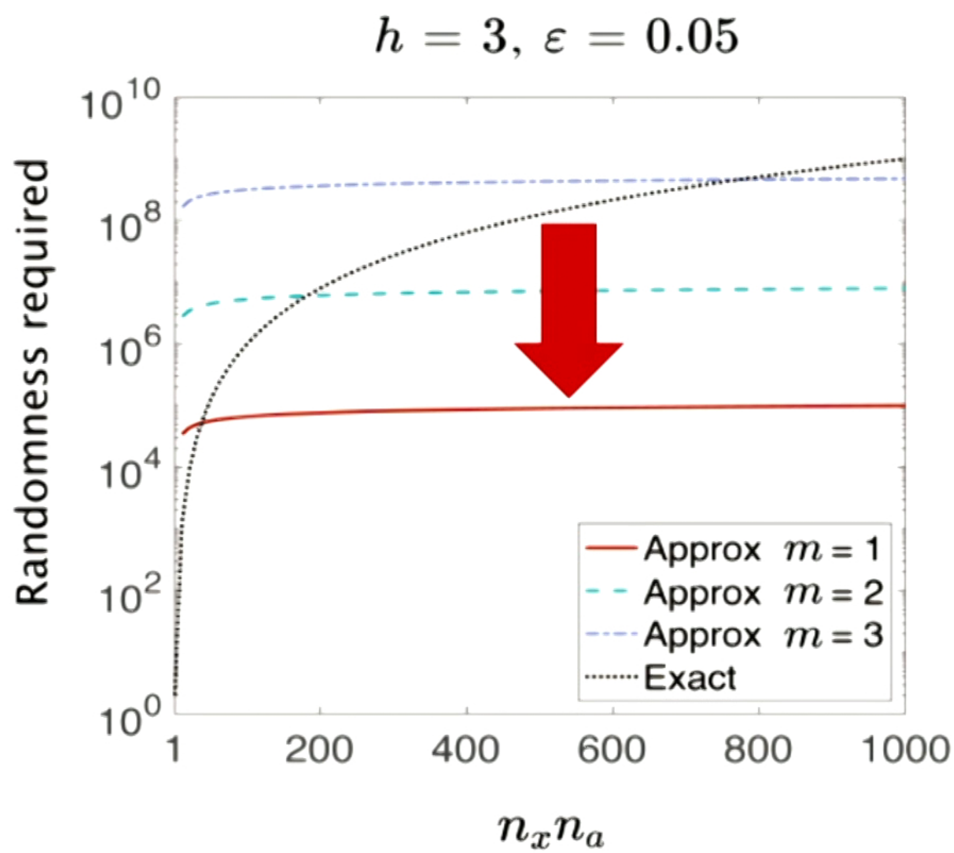

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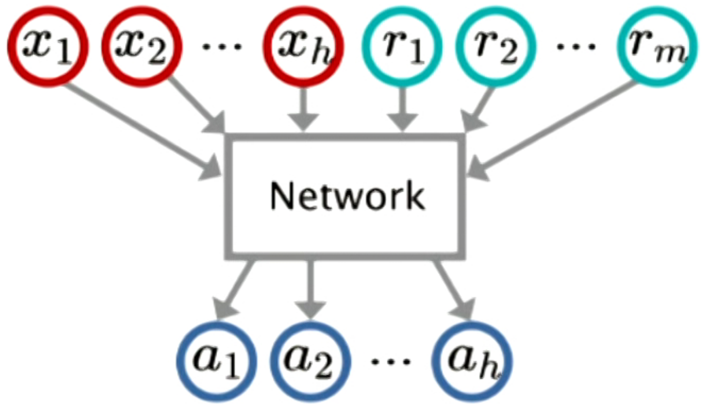

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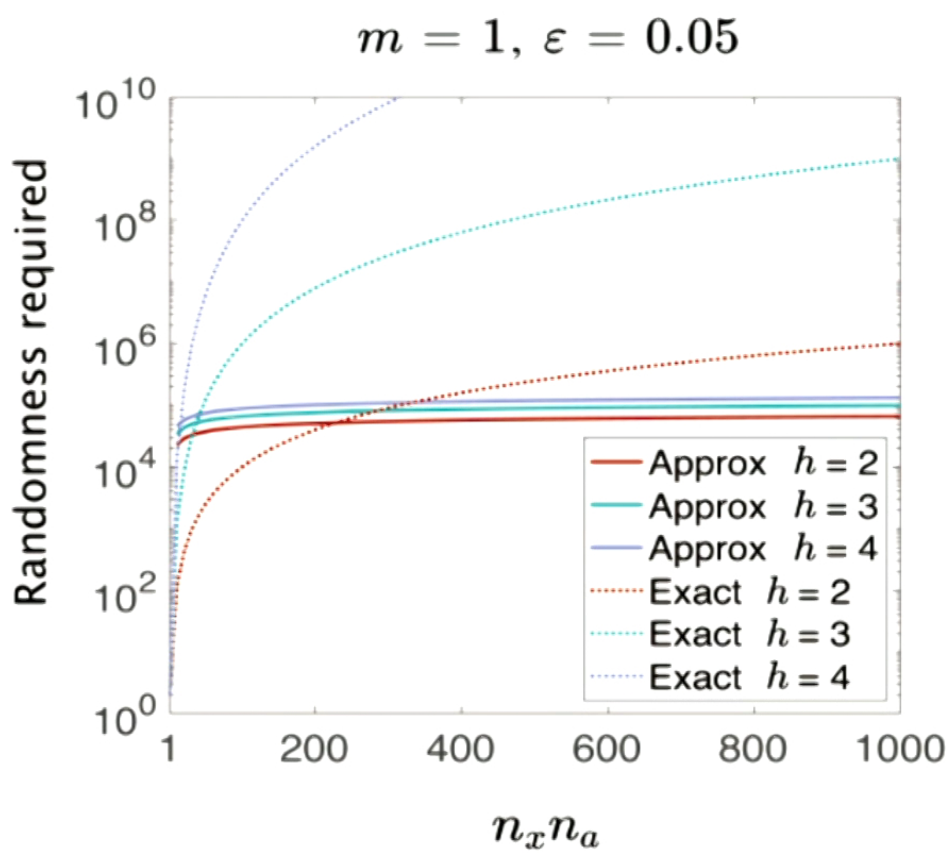

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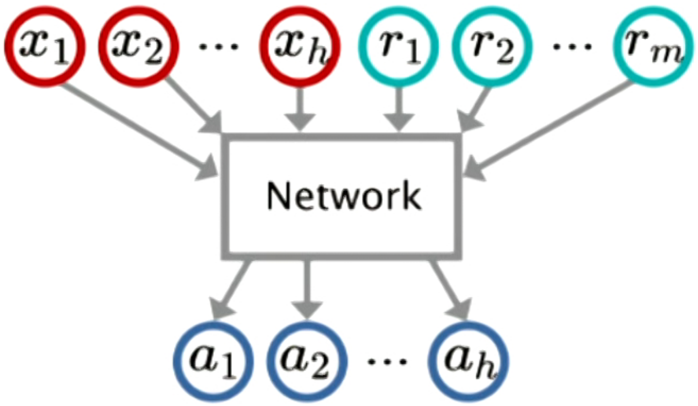

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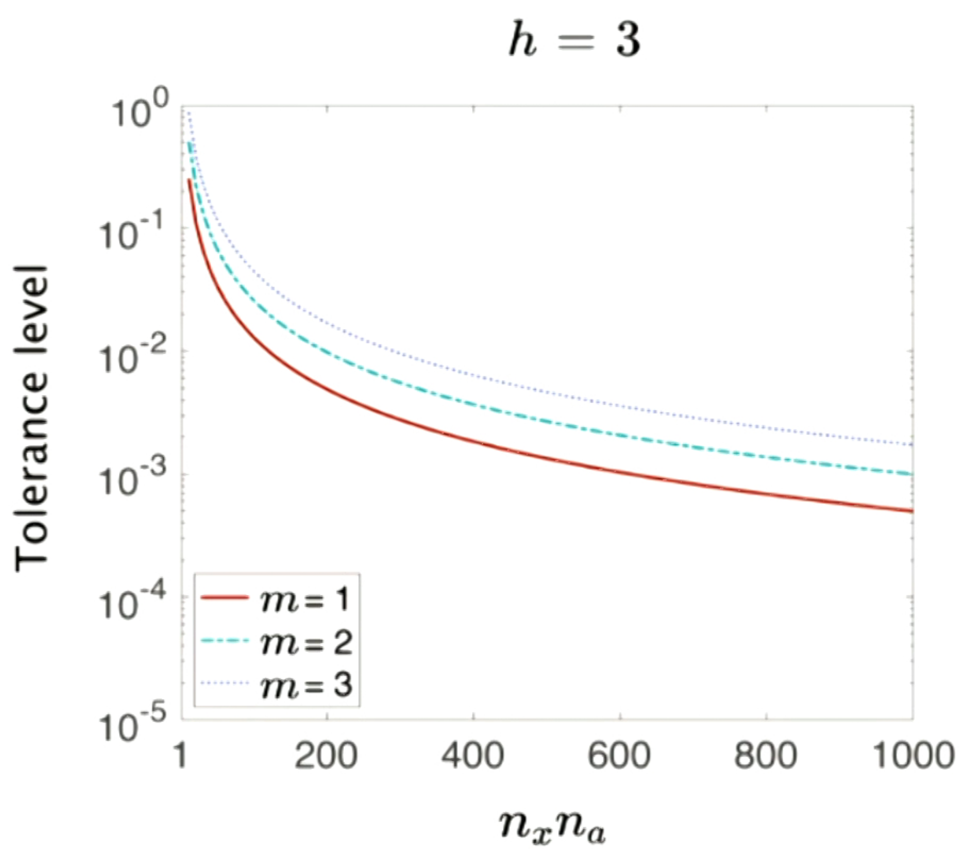

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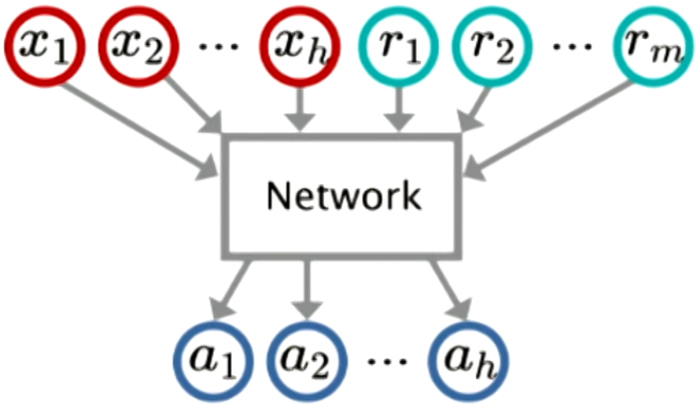

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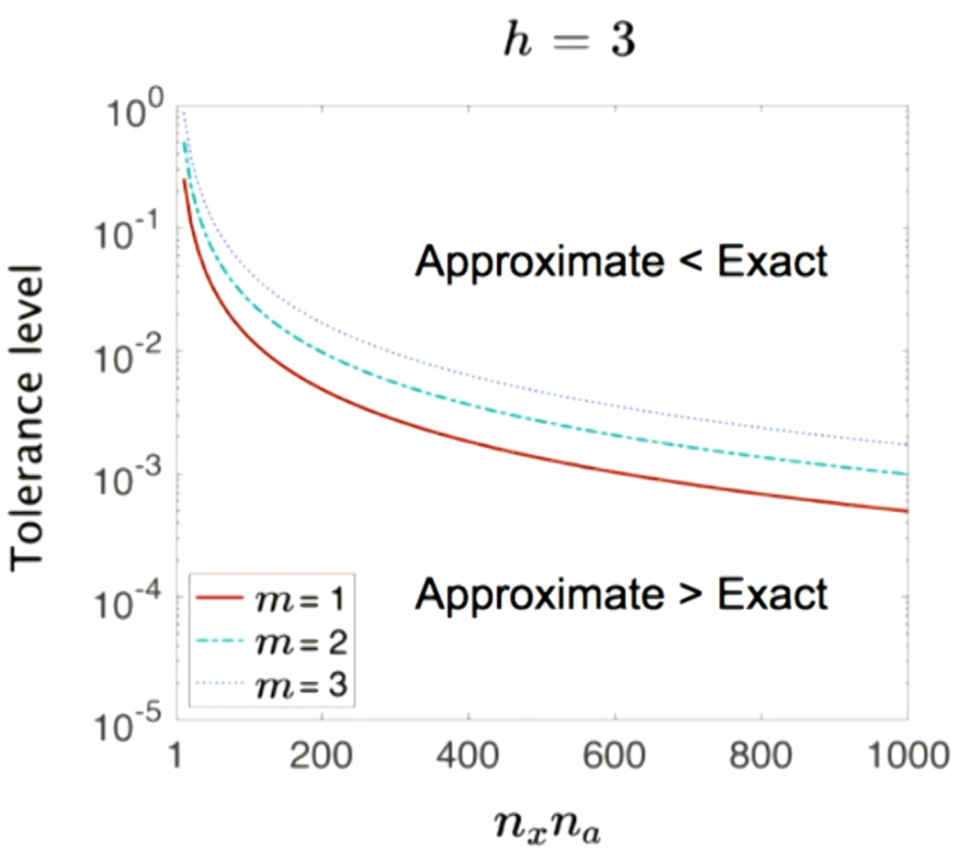

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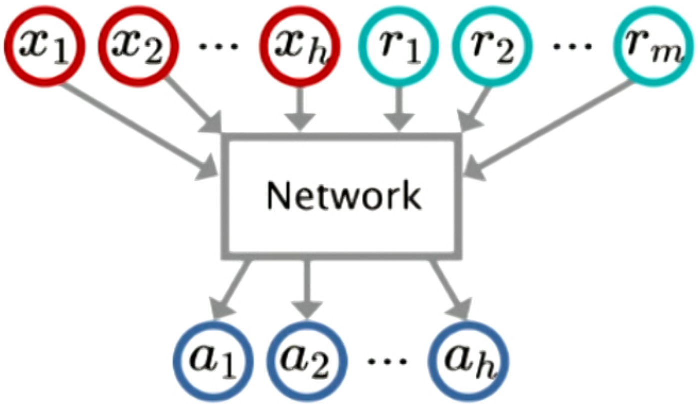

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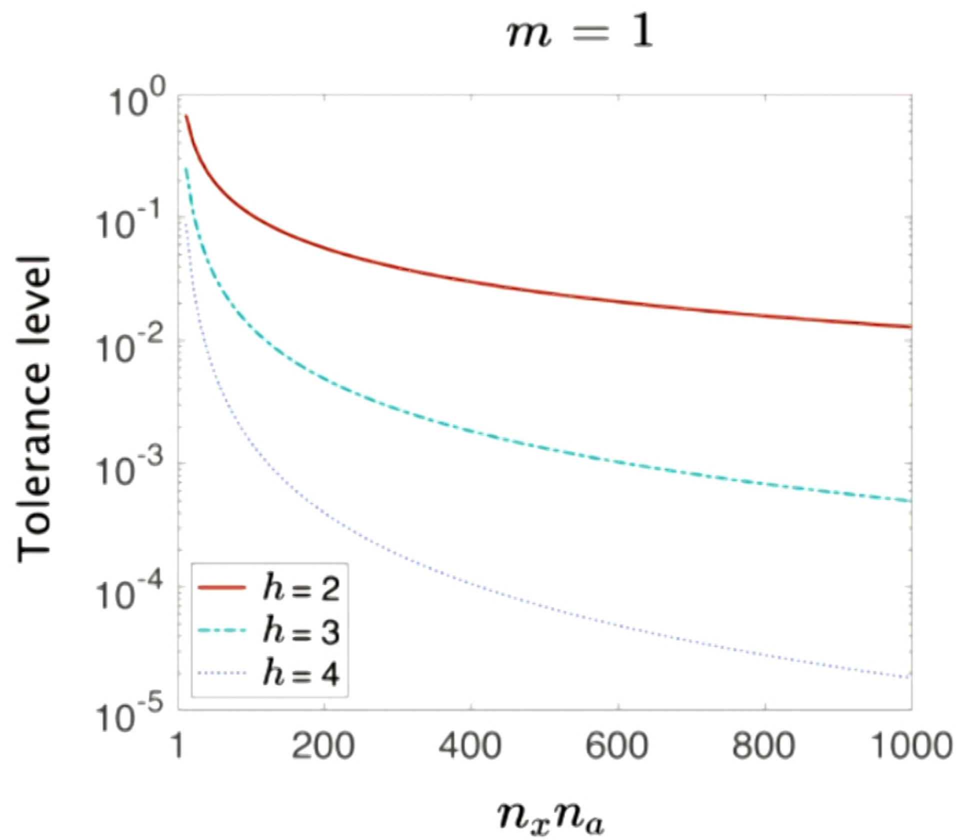
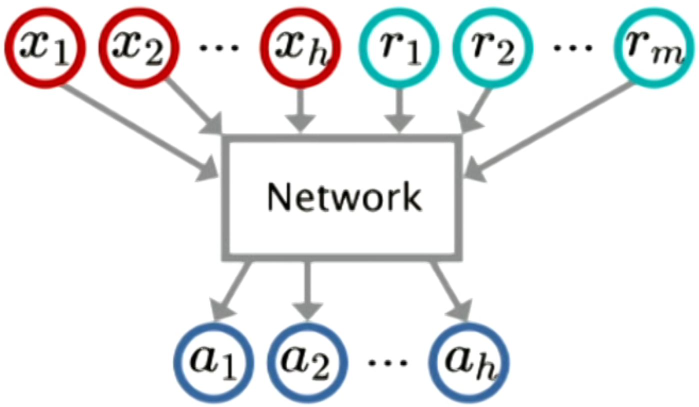


Very Recent Progress



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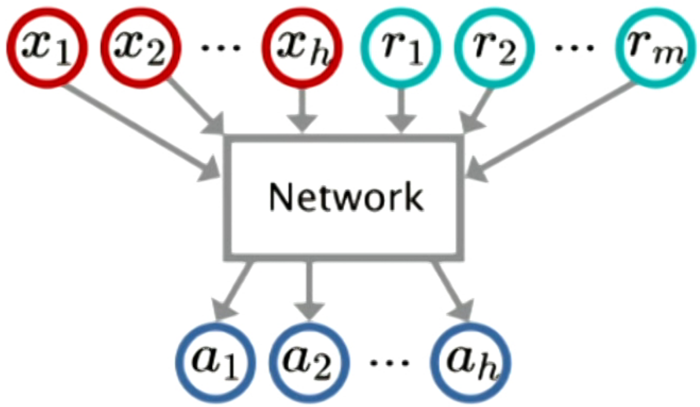

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By compressing randomness sources one by one using Chernoff Bound

$$\left\lceil \left(\frac{m}{\varepsilon}\right)^2 \ln(2m n_x n_a) \right\rceil$$

Improved!

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    - MCB is allowed to have variables sharing sources of randomness while the General Chernoff Bound can have only independent variables.
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- For the classical network, using MCB, **we bounded the cardinality of randomness required for the approximate case**.
    - Our bound performs better than the bound for the exact case for larger input and output sets.
    - Our bound performs especially well for smaller number of randomness sources.

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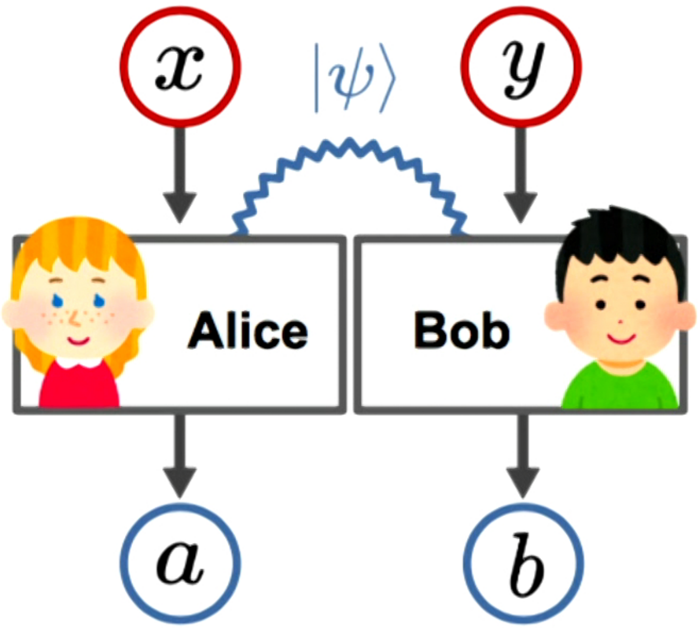

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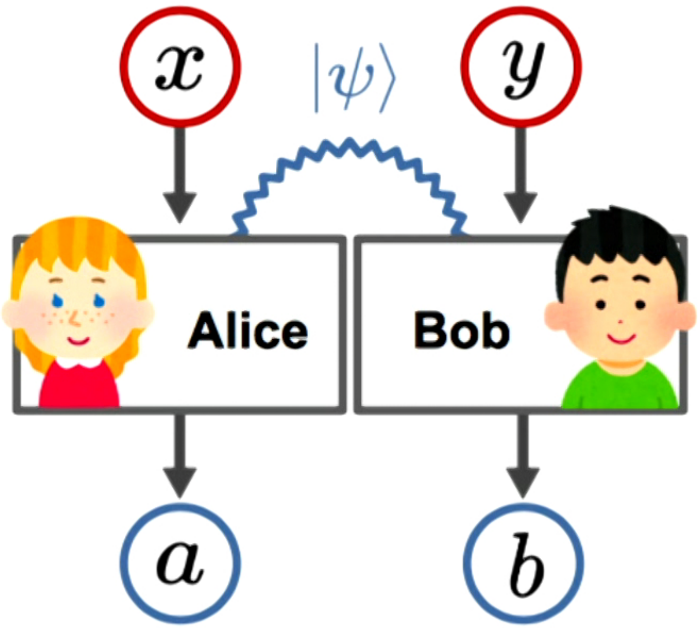

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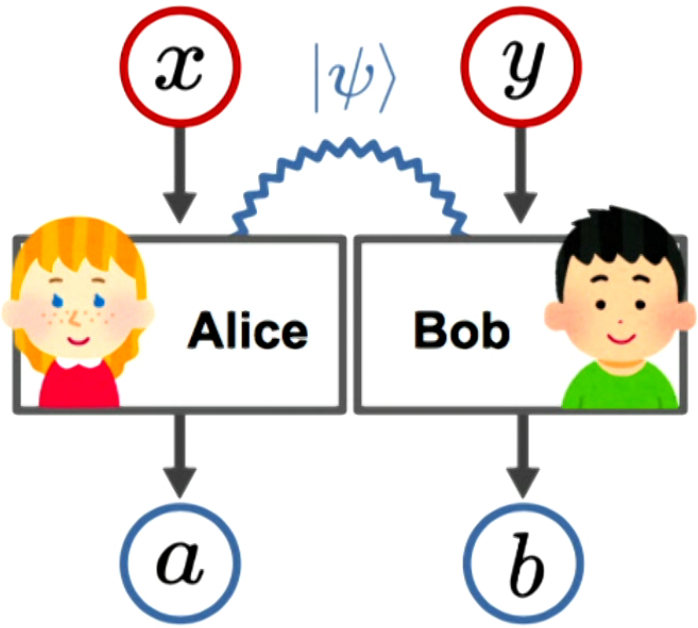

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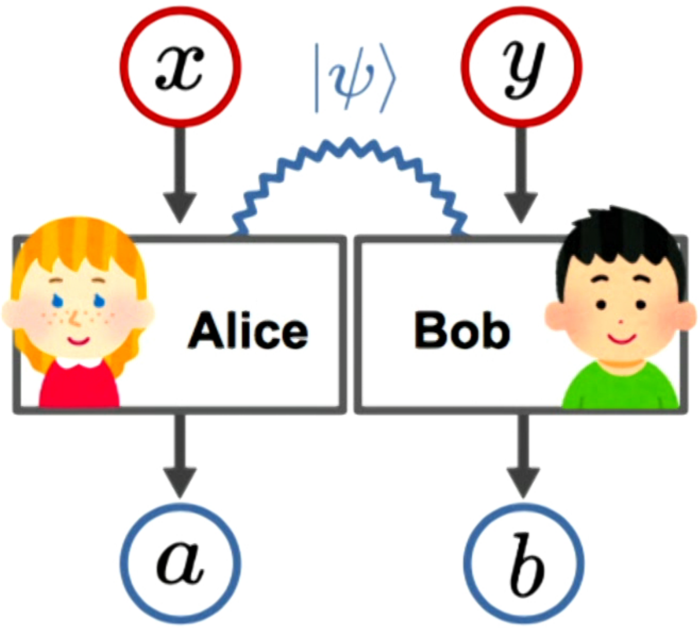

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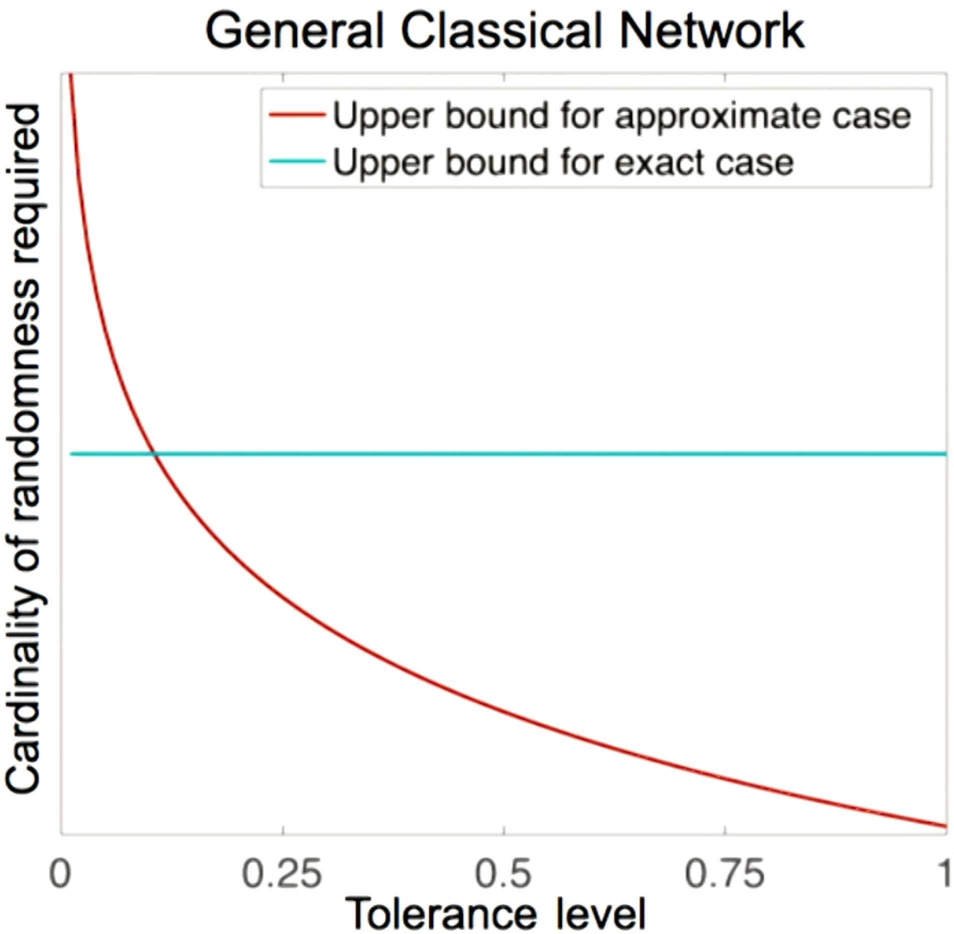

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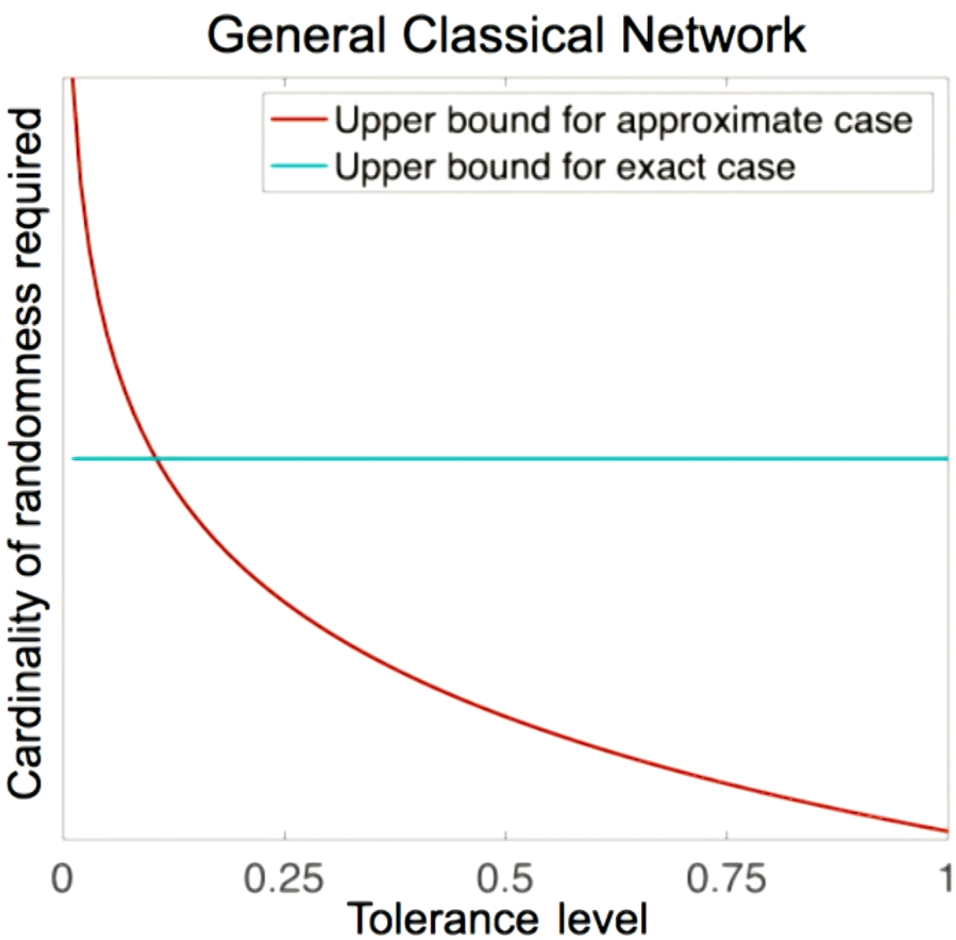

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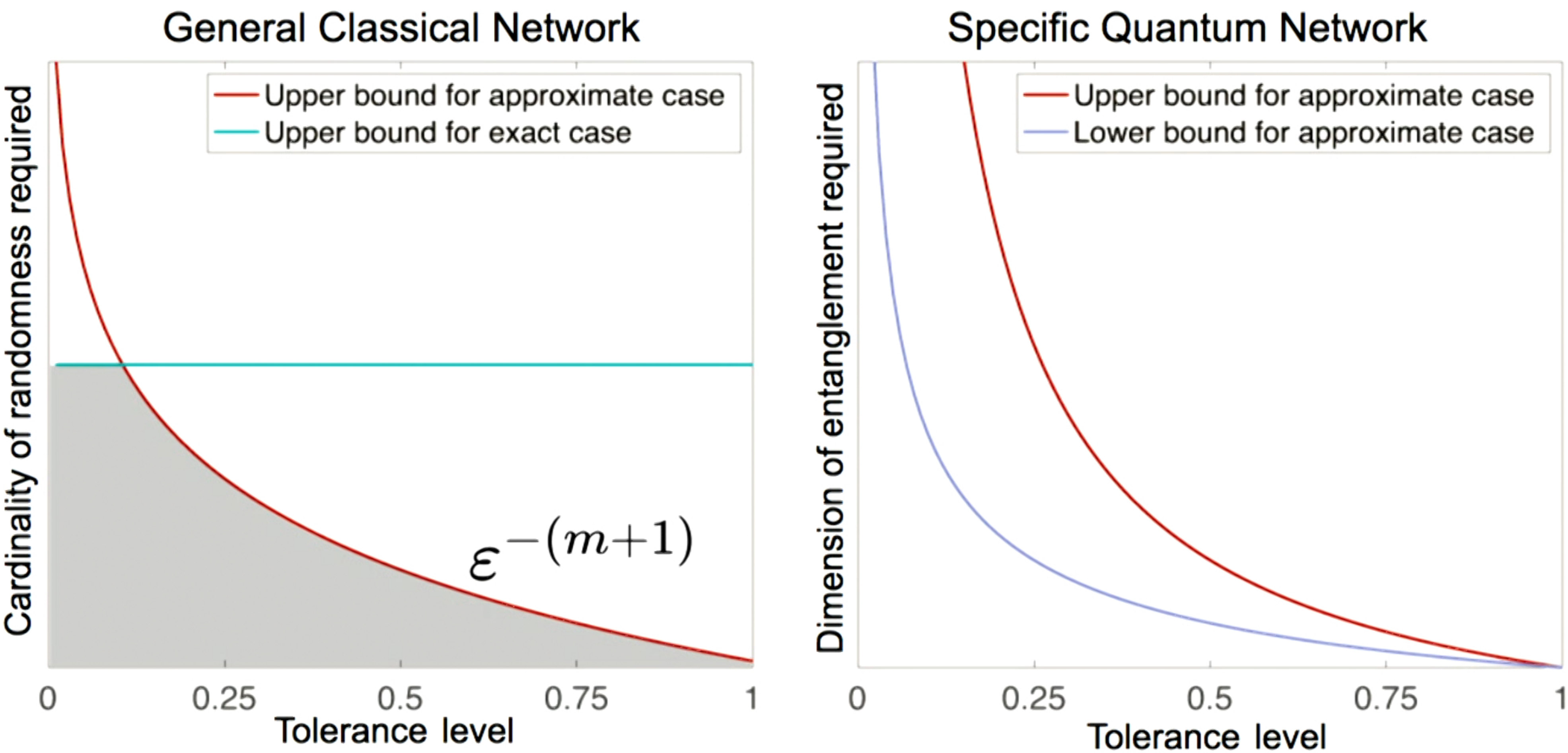

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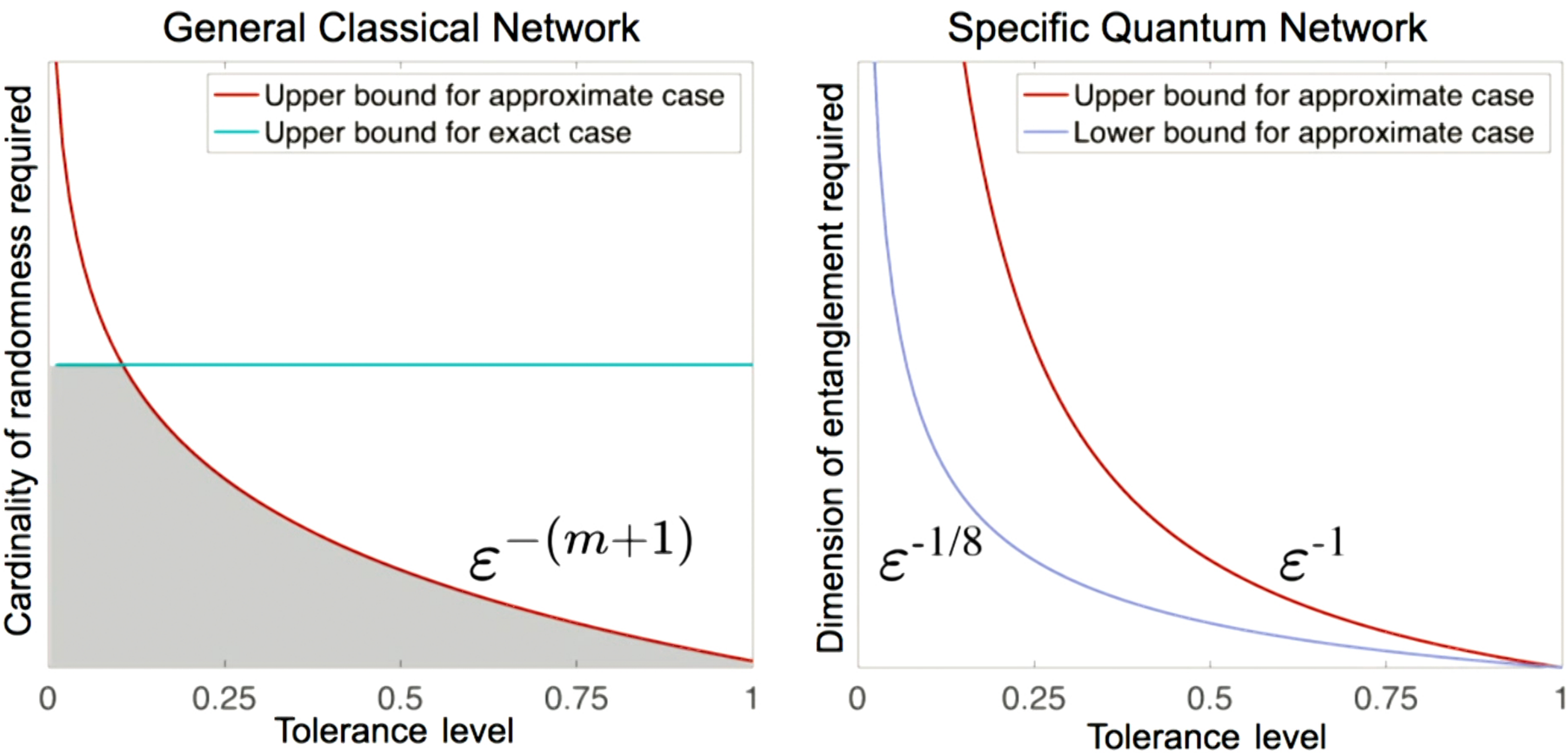

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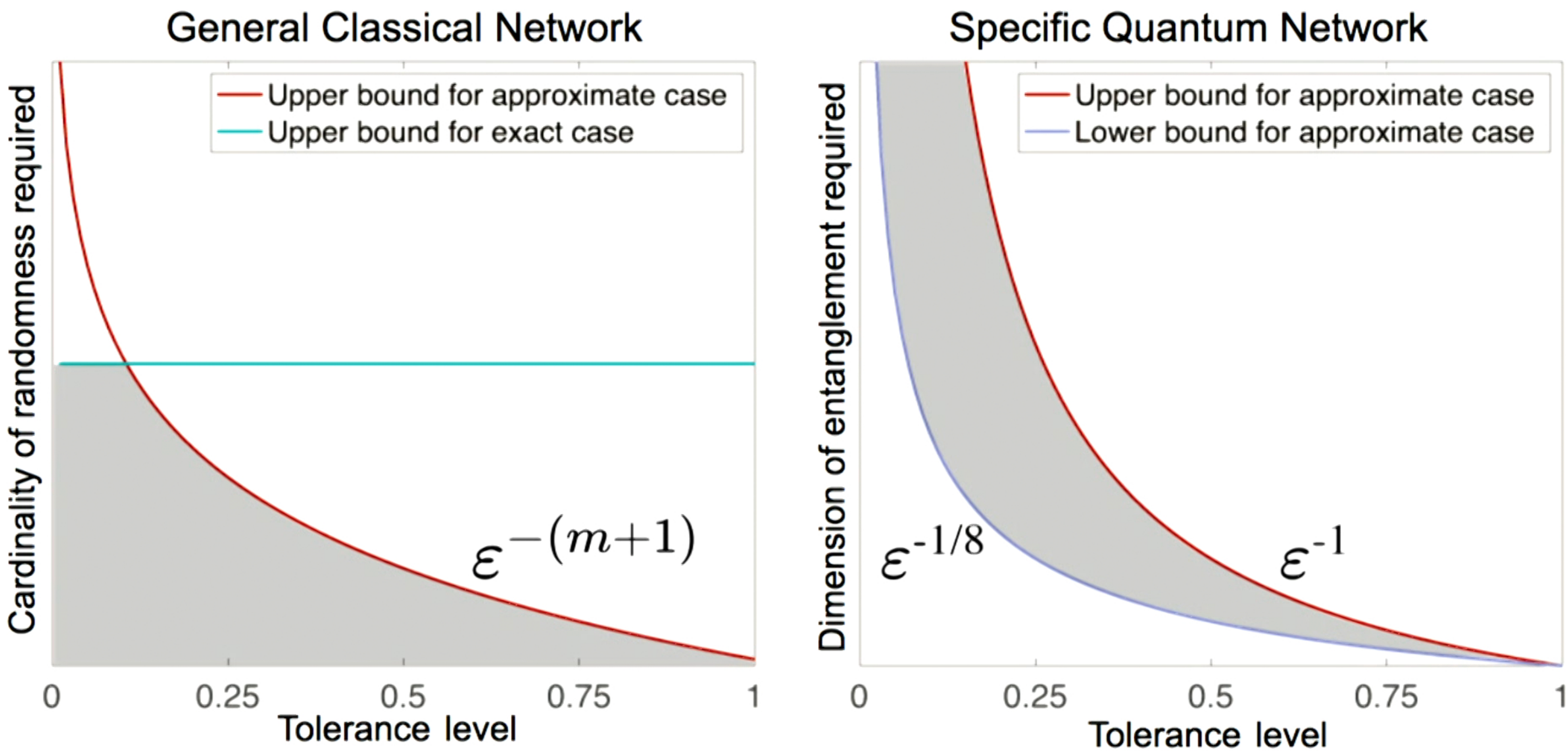

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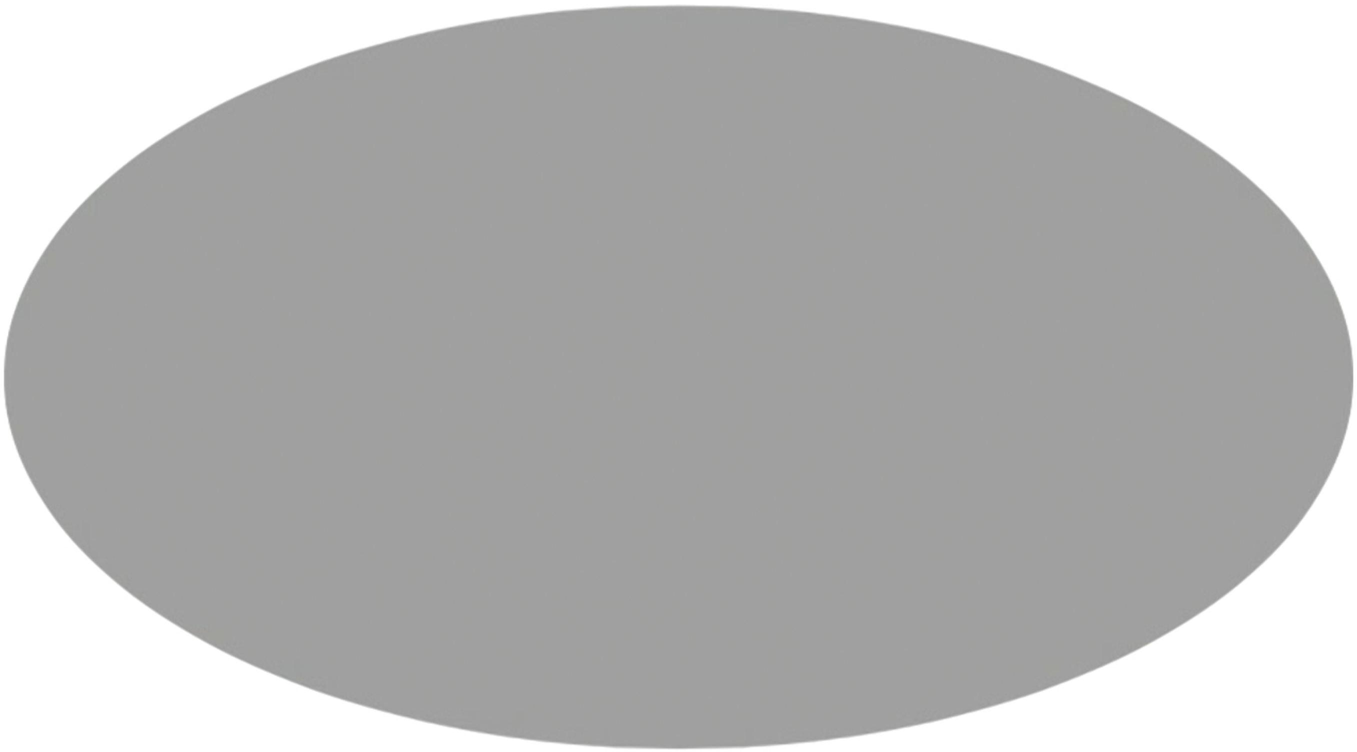

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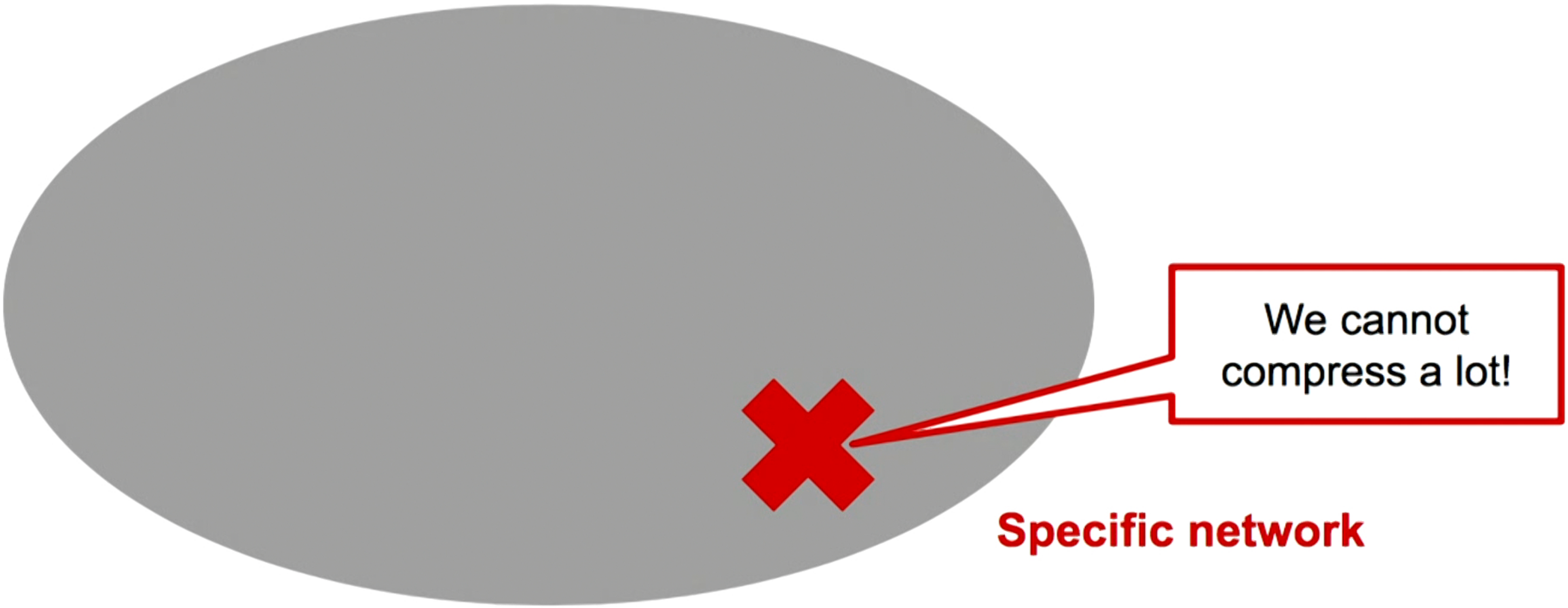

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- We **cannot compress the dimension of entanglement** in the specific quantum network so much.
  - The lower bound of the dimension required grows to infinity as the tolerance level gets smaller.
- We **cannot have the "good" solution** for the general quantum network.

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  - we have some ideas to make it tighter
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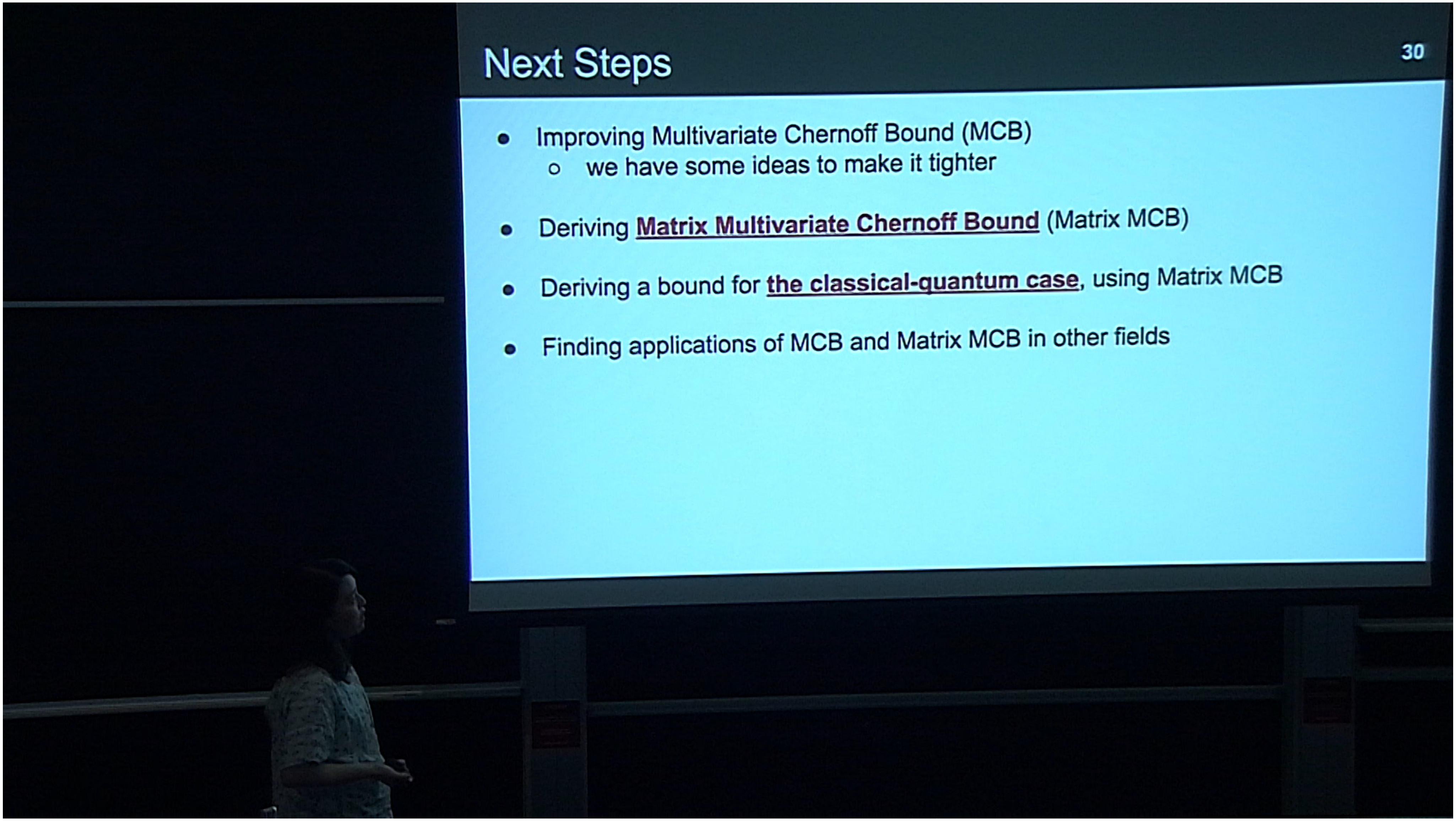
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