Title: Spins on a Kagome Lattice: Topological Magnons

Speakers: Ranjani Seshadri

Series: Condensed Matter

Date: August 06, 2019 - 3:30 PM

URL: http://pirsa.org/19080076

Abstract: In this talk, I will focus on topological aspects and edge states of a spin system on a Kagome lattice. with the anisotropic XXZ and Dzyaloshinskii-Moriya interaction (DMI). I will begin with the rich phase diagram in the classical limit arising as a result of the interplay of the two interaction strengths, followed by a spin-wave analysis in some of these phases. These spin-waves (or magnons) are studied using the Holstein-Primakoff transformations.   In the ferromagnetic phase in which all the spins point along the +z or -z direction the bulk bands are separated from each other by finite energy gaps. Finding the Chern numbers here one finds that, there are four topologically distinct phases sharing the same ground state spin-configuration. Hence an infinite strip of the system hosts robust edge states which are directly related to the Chern number of the bands. The other phases also are found to have edge modes. However, these are not topologically protected because of the gapless nature of the energy dispersion.

Spin waves on a Kagome lattice

Topological Magnons

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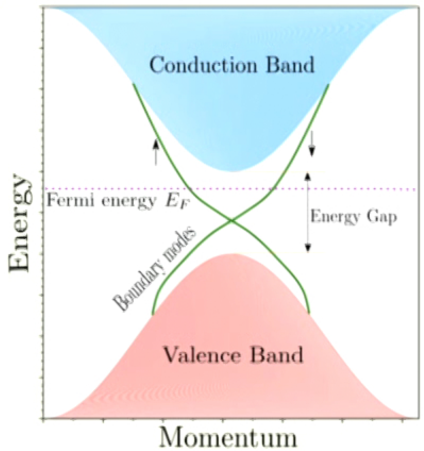
6th August 2019

Phys. Rev. B. 97, 134411 (2018)

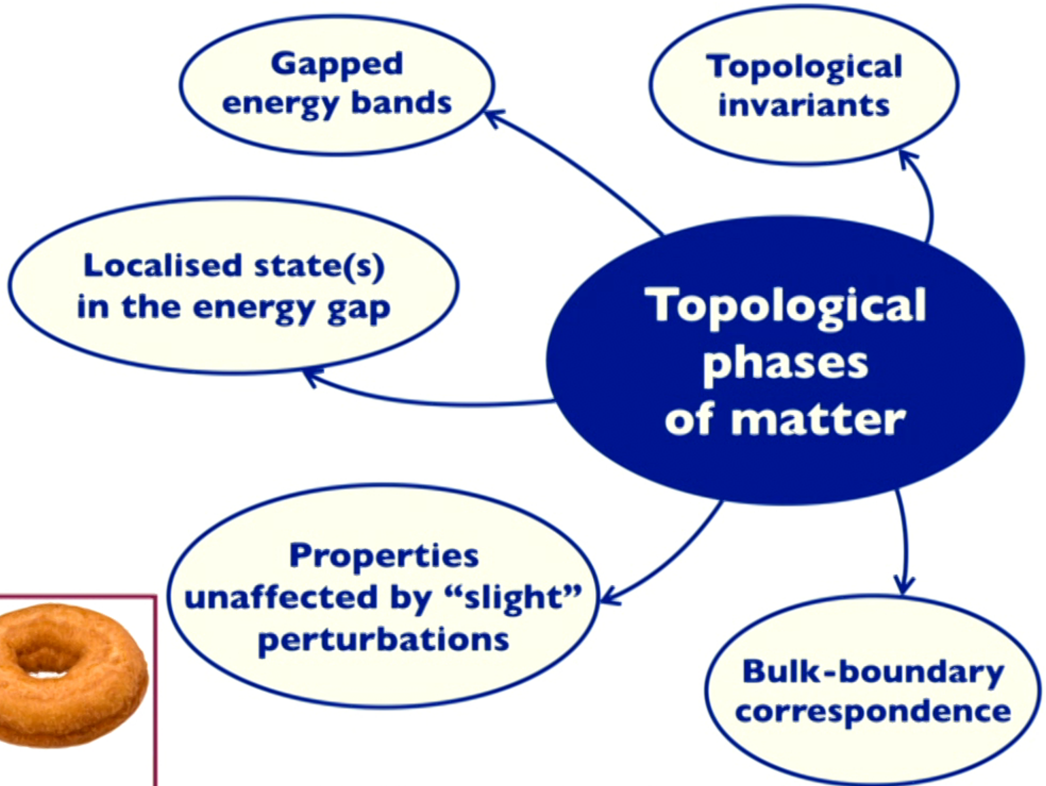
Topological magnons in a Kagome lattice spin system with XXZ and Dzyaloshinsky-Moriya interactions

Ranjani Seshadri and Diptiman Sen
Centre for High Energy Physics, Indian Institute of Science, Bengaluru 560 012, India
(Dated: December 7, 2017)

Background



Conduction Band
Fermi energy E_F
Energy Gap
Boundary modes
Valence Band
Energy
Momentum



Gapped energy bands
Topological invariants
Localised state(s) in the energy gap
Topological phases of matter
Properties unaffected by "slight" perturbations
Bulk-boundary correspondence

'Donut' break me

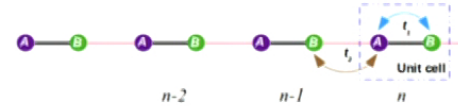
Edge
Bulk
Edge

Some standard examples …

In 1-dimension → **Su-Schrieffer-Heeger model**

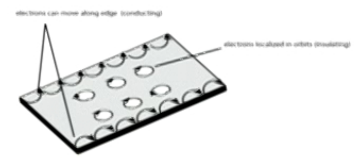
(polyacetylene chain)

Zero-dimensional end modes



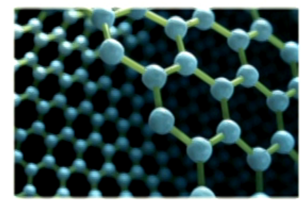
In 2-dimensions → **Quantum Hall effect**

One-dimensional edge modes



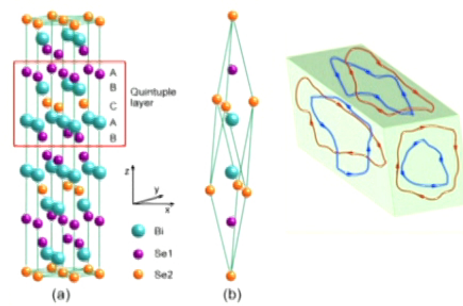
In 2-dimensions → **Quantum spin Hall effect**

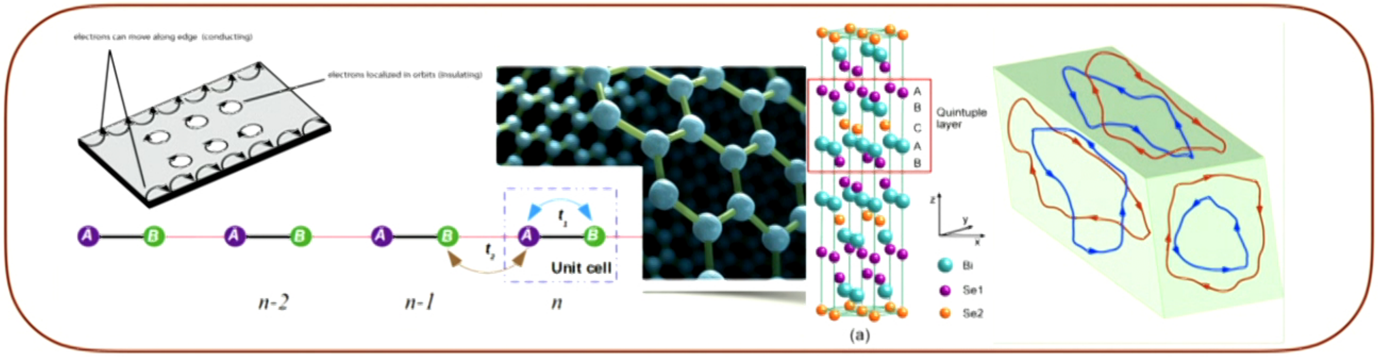
(eg. spin-orbit coupling in graphene, CdTe/HgTe/CdTe quantum wells)



In 3-dimensions → **Bismuth selenide, bismuth telluride etc…**

Two-dimensional surface states

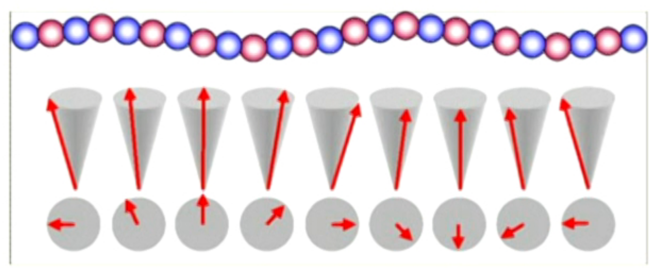




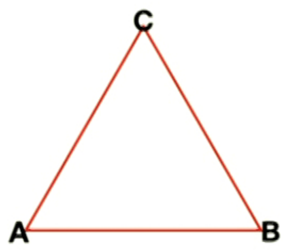
All these examples involve electronic systems

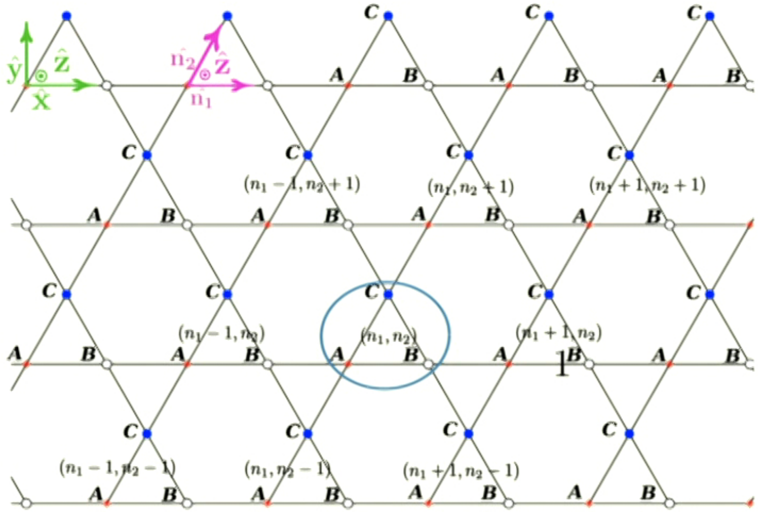
Magnons

- Relatively new entrant in the study of topological systems
- Interacting spins on a lattice
- ***Collective excitations*** of spins on a lattice
- Follow ***Bose-Einstein*** statistics
- Spin-wave theory

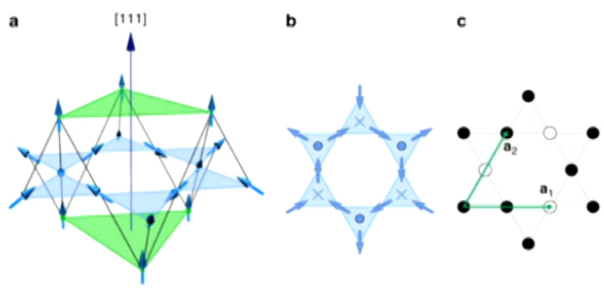


Kagome Lattice



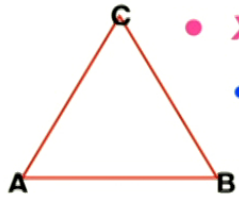


- *Triangular lattice* with a *triangular unit cell*
- Three sites per unit cell "***A***", "***B***" and "***C***"
- Four nearest neighbours for each site
- Each A has two B neighbours and two C neighbours



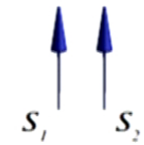
- Spin with magnitude "S" at each site
- Allow for nearest neighbour interactions

Interactions



- ***XXZ interaction***
  - Anisotropy

$$H_{\Delta} = -\sum_{\substack{\langle \vec{n}\vec{n}' \rangle \\ \alpha = x,y,z}} J_{\alpha}\left(A^{\alpha}_{\vec{n}} B^{\alpha}_{\vec{n}'} + B^{\alpha}_{\vec{n}} C^{\alpha}_{\vec{n}'} + C^{\alpha}_{\vec{n}} A^{\alpha}_{\vec{n}'}\right)$$



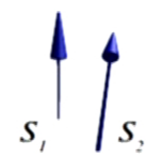
$$J_{\alpha} = \begin{cases} J & \text{if} \quad \alpha = x \ \text{or} \ y, \\ \Delta J & \text{if} \quad \alpha = z, \end{cases}$$

$J > 0$ i.e. interaction for x and y components is ferromagnetic

$\Delta = 1$ => *Isotropic case* i.e ferromagnetic Heisenberg model

- ***Dzyaloshinskii-Moriya Interaction (DMI)***
  - Cross product terms
  - Here only out of place component
    - True only for a perfect Kagome system
    - For systems with lower symmetry, there can be a small in-plane component of the DMI
  - Gives a ***"handedness"*** to the system
  - Here, anticlockwise, i.e. A to B to C within a unit cell
  - Invariant under arbitrary spin rotations on the x-y plane (i.e. about z-axis)

$$H_{DM} = D\,\hat{z} \cdot \sum_{\langle \vec{n}\vec{n}' \rangle} \left(\vec{A}_{\vec{n}} \times \vec{B}_{\vec{n}'} + \vec{B}_{\vec{n}} \times \vec{C}_{\vec{n}'} + \vec{C}_{\vec{n}} \times \vec{A}_{\vec{n}'}\right)$$



$$H = H_{\Delta} + H_{DM}$$

Competition between the XXZ and DMI term

Plan of the Talk

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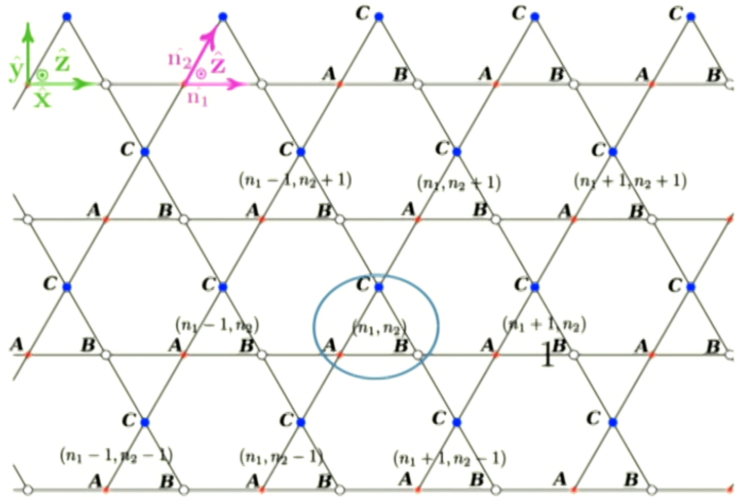
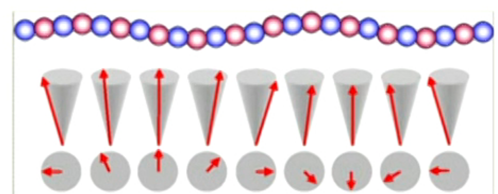
- Spins on a Kagome Lattice
  - XXZ + DM interaction
  - Ground state configuration
  - Phase diagram
- Spin wave analysis in "Phase I"
  - Holstein - Primakoff Transformation
  - Magnon Spectrum
  - Chern numbers and Edge Modes
  - Thermal Hall effect
- Spin waves in other phases
- Conclusions and future work



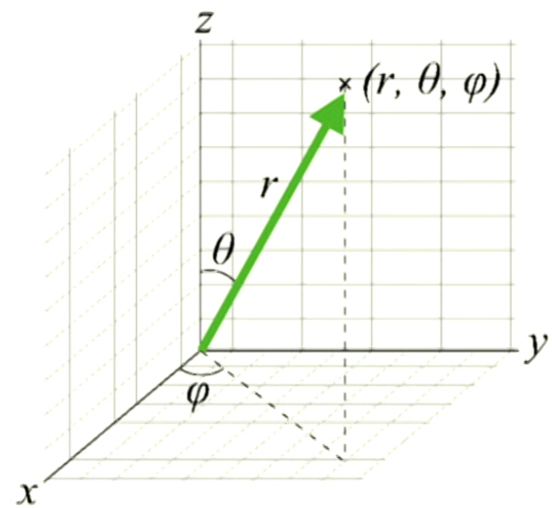


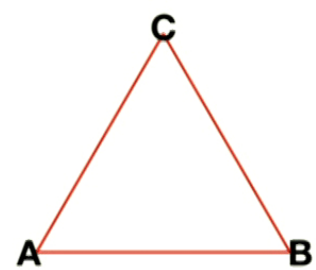
Ground state

- In the classical limit, large S
- At each site $\vec{A}_{\vec{n}}^2 = \vec{B}_{\vec{n}}^2 = \vec{C}_{\vec{n}}^2 = S(S+1)\hbar^2$
- The spin at each lattice site is a vector of length "S" and has a polar angle and an azimuthal angle
- Choose spin at ***C*** on the *x-z* plane: $\phi_C = 0$ (because only relative orientations of spin matter)

$$
\begin{aligned}
\vec{A}_{\vec{n}} &= S\big(\sin\theta_A \cos\phi_A, \sin\theta_A \sin\phi_A, \cos\theta_A\big),\\
\vec{B}_{\vec{n}} &= S\big(\sin\theta_B \cos\phi_B, \sin\theta_B \sin\phi_B, \cos\theta_B\big),\\
\vec{C}_{\vec{n}} &= S\big(\sin\theta_C, 0, \cos\theta_C\big),
\end{aligned}
$$

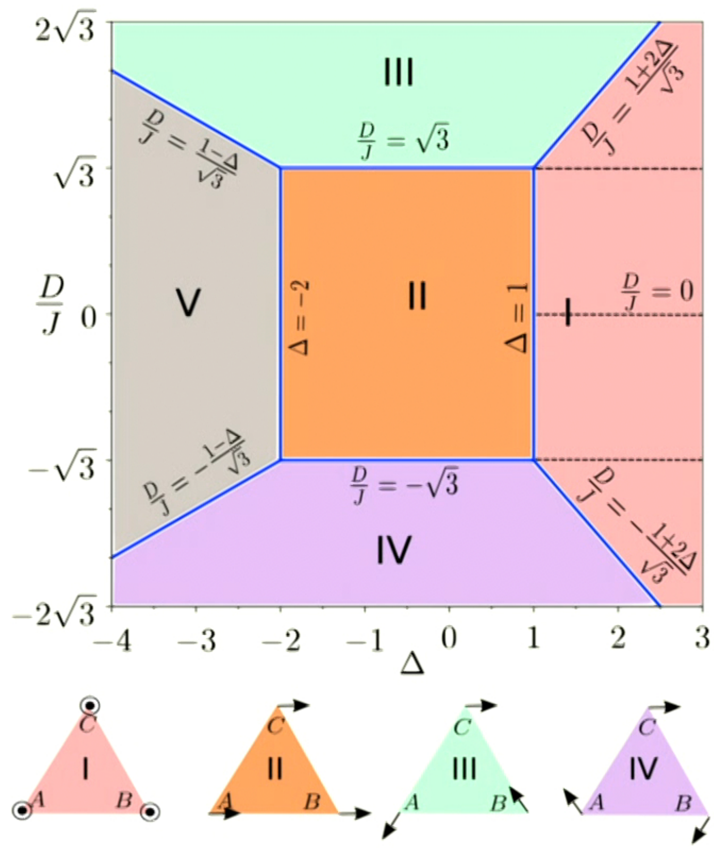
- Minimize $H = H_\Delta + H_{DM}$ with respect to five parameters $p_i = \theta_A, \theta_B, \theta_C, \phi_A, \phi_B$
- Numerically find the minimum energy configuration for various values of the parameters Δ and D/J





Phase Diagram

- **Phase I:** All spins point along **+z direction** (collinear, ferromagnetic)
- **Phase II:** All spins point along **+x direction** (collinear, ferromagnetic)
- **Phase III:** Neighbouring spins **rotated by 120°** in the **clockwise** direction
- **Phase IV:** Neighbouring spins **rotated by 120°** in the **anticlockwise** direction
- **Phase V:** In general, the spins are **non-coplanar** and the configuration is different for each Δ and D/J.



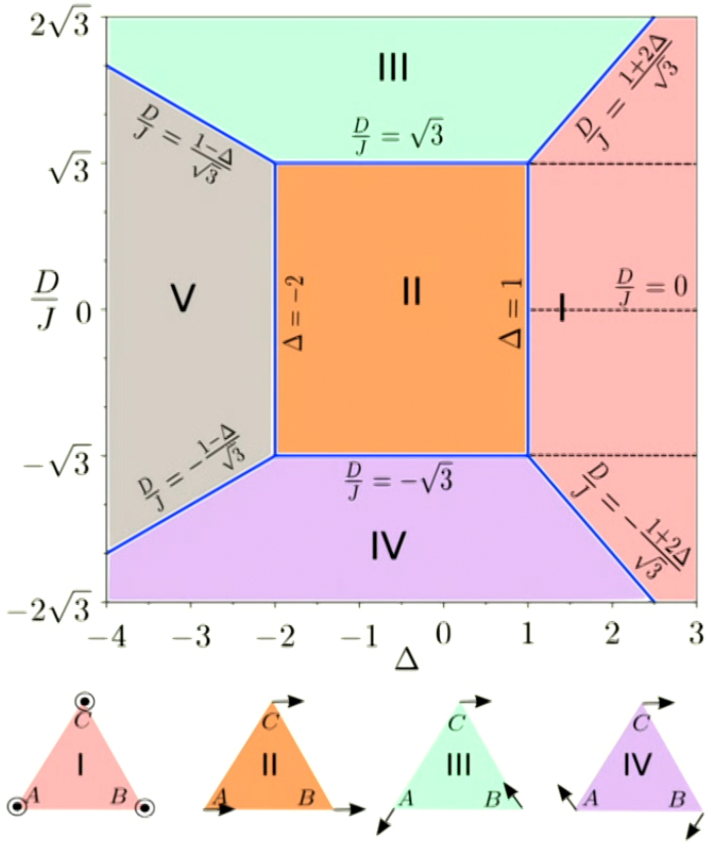
Phase Diagram

Phase Boundaries

- Phase transition (P.T) lines: Ground state energies of adjacent phases equal
- All P.T. between I-IV : **First order**
- Derivative of ground state energy discontinuous i.e. approaches different values from either sides of the line
- ***Example*:** I-II Phase boundary (Δ=1 line)
  - Ferromagnetic state

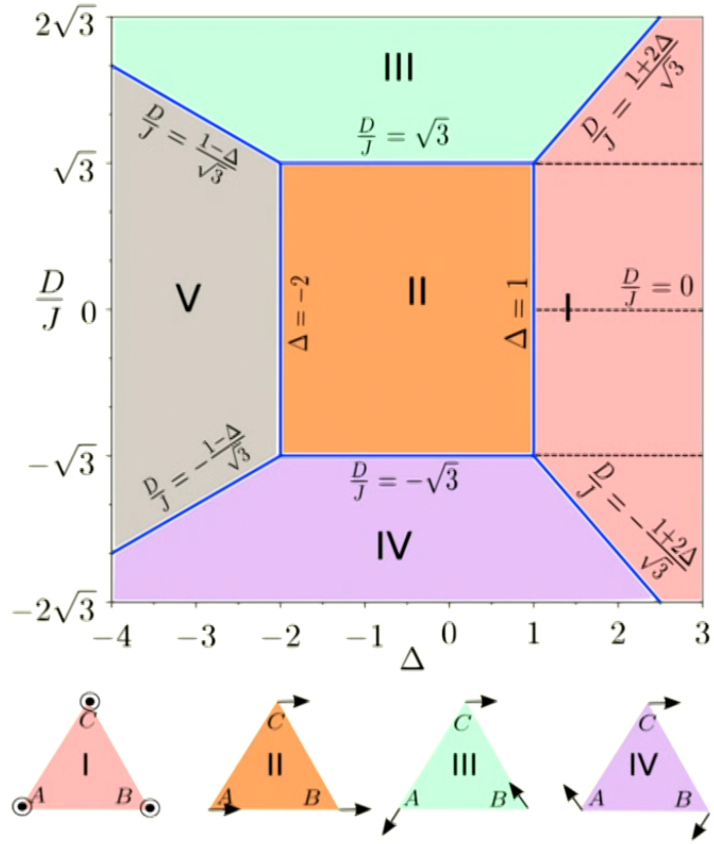
$$E(\theta) \;=\; -3JS^2(\Delta\cos^2\theta + \sin^2\theta)$$

  - For Δ<1, min(E) = $E_0 = -3JS^2$ when $\theta = \pi/2$
  - For Δ>1, min(E) = $E_0 = -3JS^2\Delta$ when $\theta = 0$ or π



Phase Diagram

Now that the ground state configuration is established in these different phases, we study the spin-waves in some of these phases



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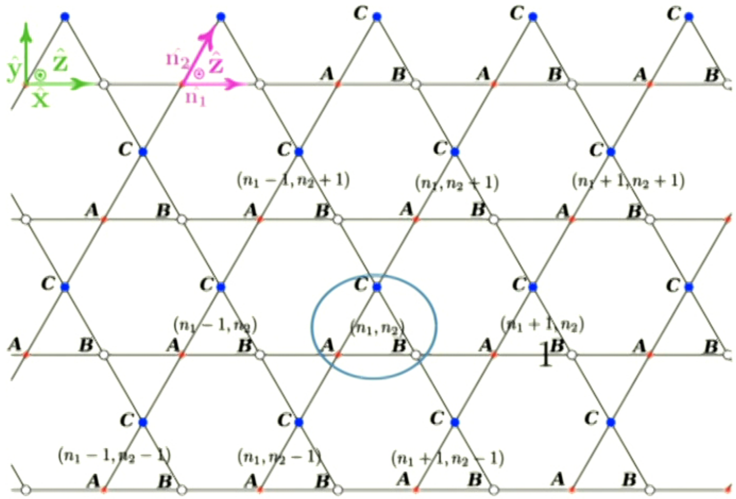
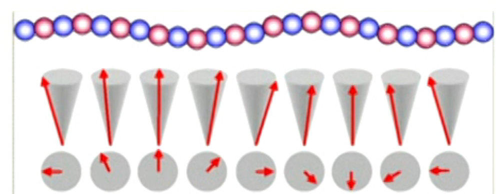




Holstein-Primakoff Transformation

Example : 1-D Heisenberg model

$$H = -J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j - \sum_i \vec{h} \cdot \vec{S}_i$$

- Ferromagnetic ground state (J>0) : $|GS\rangle$
- All spins aligned parallel (+z, say)
- In the ground state all the spins have maximum

  spin projection, say "s" along the z-direction

$$S_i^z \left| GS \right\rangle = s \left| GS \right\rangle, \quad \forall i.$$

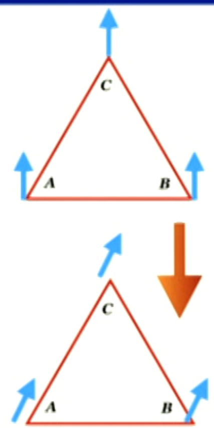
- Ground state energy

$$H \left| GS \right\rangle = (-J s^2 - hs)\, n \left| GS \right\rangle.$$

- Guess for the first excited state : $\cancel{S_{i_0}^{-} \left| GS \right.}\rangle$. **Not an eigenstate**

- *Correct eigenstate*: superposition of all possible states with the spin projection reduced.
- Obtained by using the HP transformations which map the spin eigenstates to a quantum harmonic oscillator-like states

Phase I: Spin-Wave Analysis



Ground State

↓

Holstein-Primakoff (HP) Transformation

Map spin operators to harmonic oscillator-like states

$$\begin{aligned} A^z_{\vec{n}} &= S - a^\dagger_{\vec{n}} a_{\vec{n}}, \\ A^+_{\vec{n}} &= A^x_{\vec{n}} + iA^y_{\vec{n}} \simeq \sqrt{2S}\, a_{\vec{n}} \\ A^-_{\vec{n}} &= A^x_{\vec{n}} + iA^y_{\vec{n}} \simeq \sqrt{2S}\, a^\dagger_{\vec{n}} \end{aligned}$$

↓

Hamiltonian (Real Space)

$$\begin{aligned} H &= H_\Delta + H_{DM}, \qquad (1) \\ H_\Delta &= - \sum_{\substack{\langle \vec{n}\vec{n}' \rangle \\ \alpha = x,y,z}} J_\alpha (A^\alpha_{\vec{n}} B^\alpha_{\vec{n}'} + B^\alpha_{\vec{n}} C^\alpha_{\vec{n}'} + C^\alpha_{\vec{n}} A^\alpha_{\vec{n}'}), \\ H_{DM} &= D\, \hat{z} \cdot \sum_{\langle \vec{n}\vec{n}' \rangle} (\vec{A}_{\vec{n}} \times \vec{B}_{\vec{n}'} + \vec{B}_{\vec{n}} \times \vec{C}_{\vec{n}'} + \vec{C}_{\vec{n}} \times \vec{A}_{\vec{n}'}), \end{aligned}$$

↓

Fourier transform (k-space)

$$(a, b, c)_{\vec{n}} = \sum_{\vec{k}} (a, b, c)_{\vec{k}}\, e^{i\vec{k}\cdot\vec{n}}$$

↓

Retain terms only upto 2nd order in a, a† etc

Phase I: Spin-Wave Analysis

Hamiltonian (momentum space)

- Number conserving Hamiltonian: Directly diagonalising gives the energy spectrum
- Anisotropy Δ appears only as a overall energy shift (only diagonal terms)
- Energy spectrum depends only on the magnitude of DMI strength (symmetric under D to -D)
- Three bands in the spectrum

$$H = \sum_{\vec{k}} \begin{pmatrix} a^\dagger_{\vec{k}} & b^\dagger_{\vec{k}} & c^\dagger_{\vec{k}} \end{pmatrix} h(\vec{k}) \begin{pmatrix} a_{\vec{k}} \\ b_{\vec{k}} \\ c_{\vec{k}} \end{pmatrix}, \qquad (6)$$

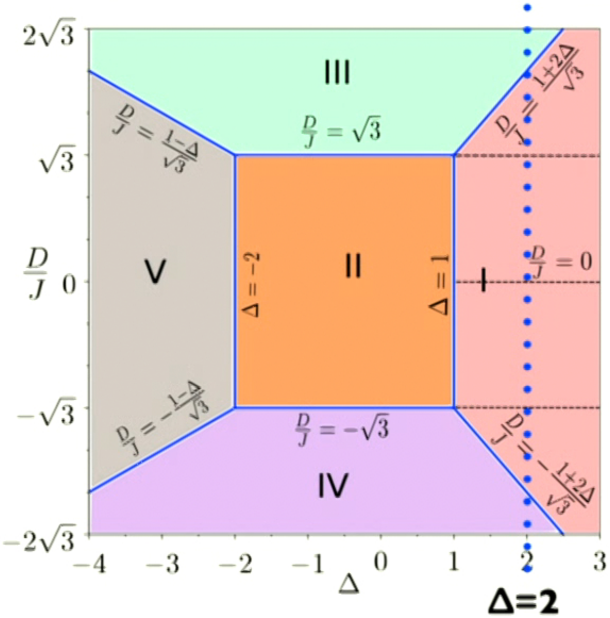
where the 3×3 matrix $h(\vec{k})$ has the form

$$h(\vec{k}) = JS \begin{pmatrix} 4\Delta & -\mathcal{D}f(-k_1) & -\mathcal{D}^* f(-k_2) \\ -\mathcal{D}^* f(k_1) & 4\Delta & -\mathcal{D}f(k_1 - k_2) \\ -\mathcal{D}f(k_2) & -\mathcal{D}^* f(k_2 - k_1) & 4\Delta \end{pmatrix} \qquad (7)$$

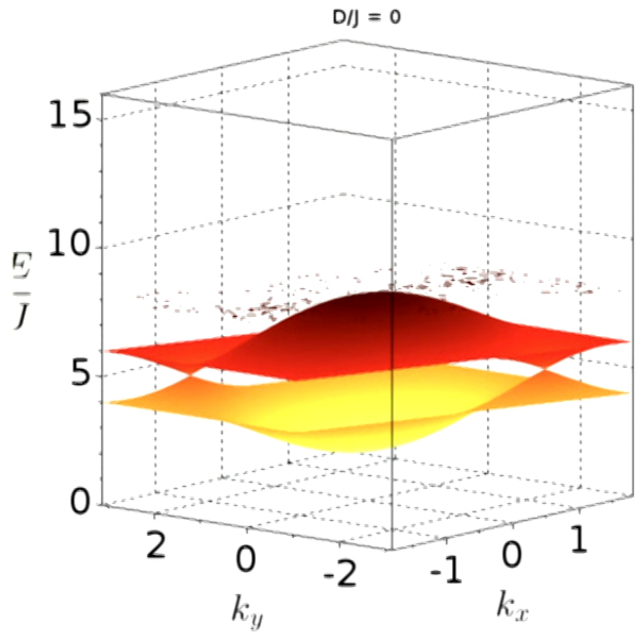
with

$$\begin{aligned} \mathcal{D} &= 1 + \frac{iD}{J}, \\ f(k) &= 1 + e^{ik} = f^*(-k). \end{aligned} \qquad (8)$$

Phase 1: Magnon spectrum

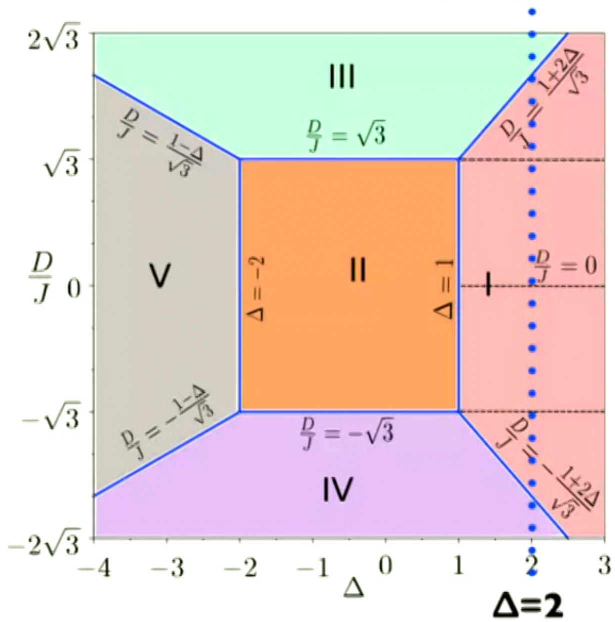


- Vary D/J (the DMI strength), keeping the Δ=2
- Stay within Phase I ($-5/\sqrt{3} < D < 5/\sqrt{3}$)
- Single Brillouin zone
- Three values of D/J where gap closes within this phase



- Bottom band : 1
- Middle band : 2
- Top band : 3

Phase I: Magnon spectrum



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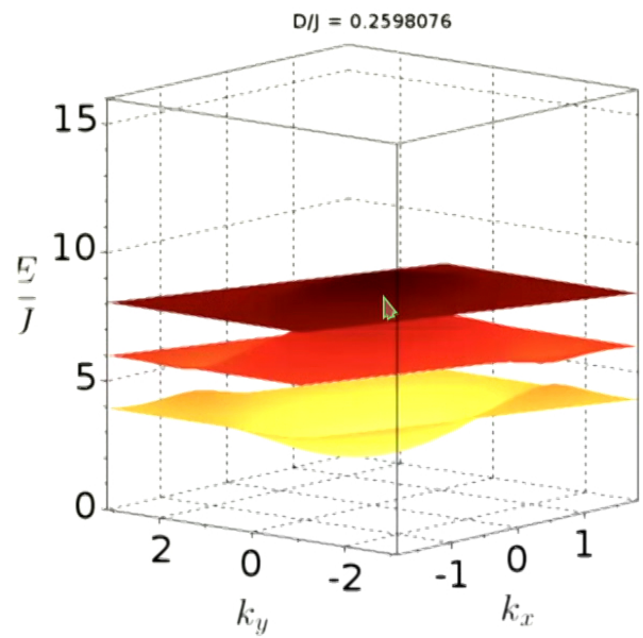

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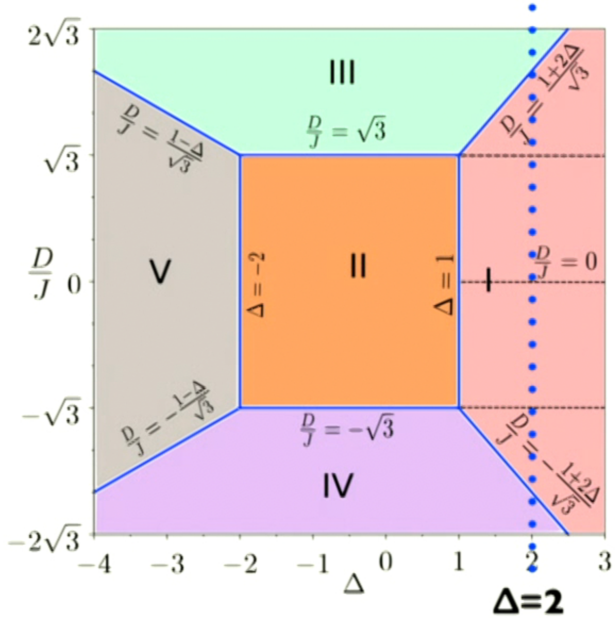

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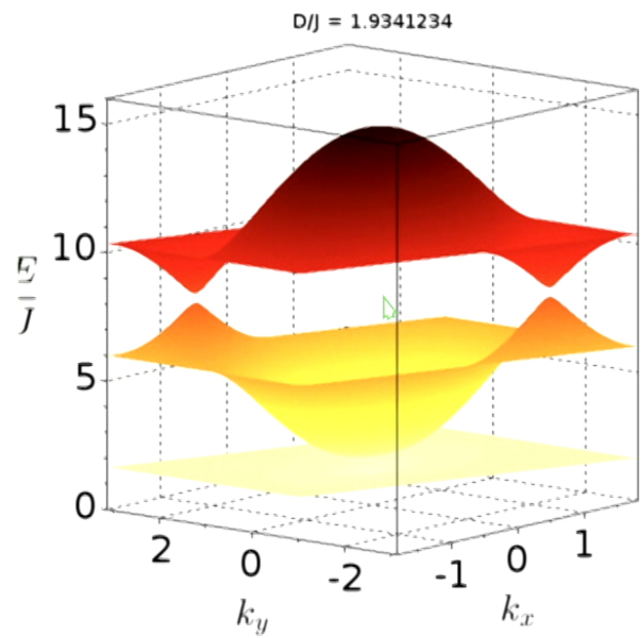

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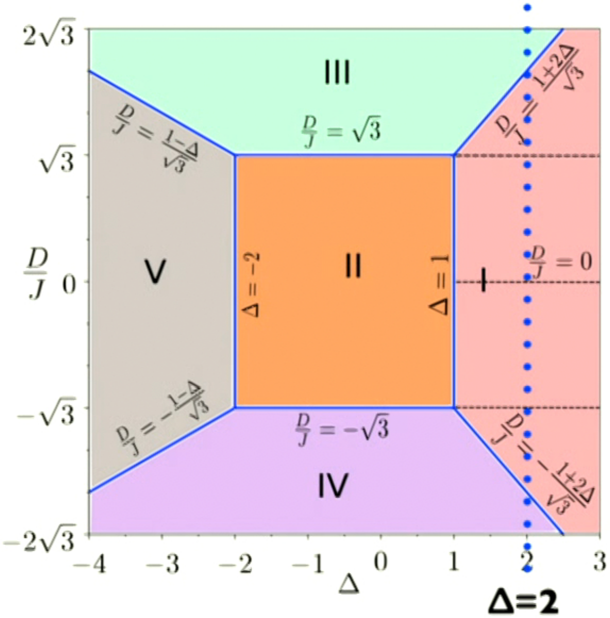
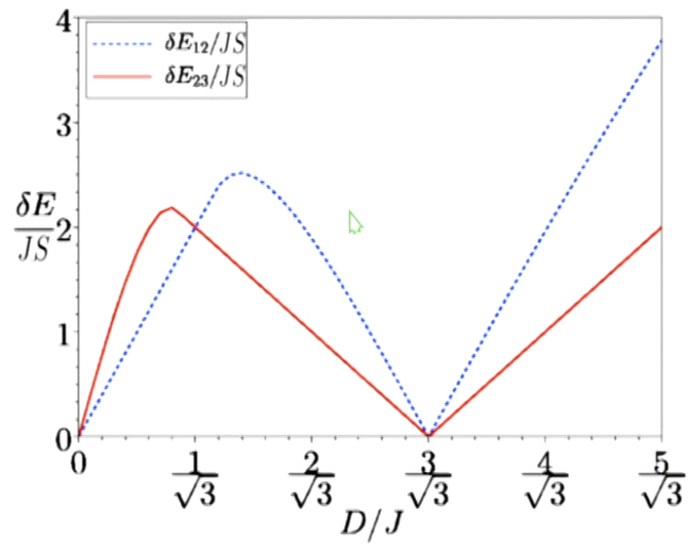


- D/J = 0
  - Bands 1 and 2 touch at "**Dirac points**"
    - Two such points per Brillouin zone $\vec{k} = \pm(2\pi/(3\sqrt{3}), -2\pi/3)$
  - Bands 2 and 3 touch at "**Γ point**" $\vec{k} = (0, 0)$
- D/J = √3
  - Band touching points switched



Phase I: Chern numbers

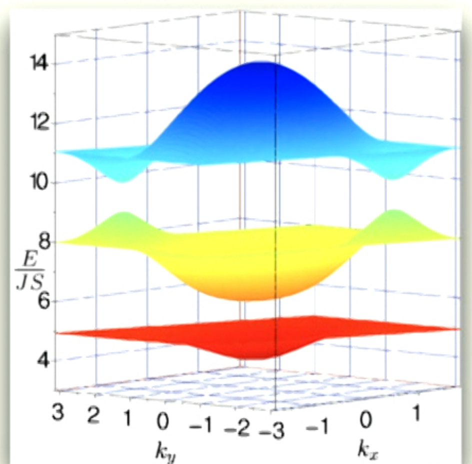
Every time the energy gap closes and reopens:
Change in topological character of the bands

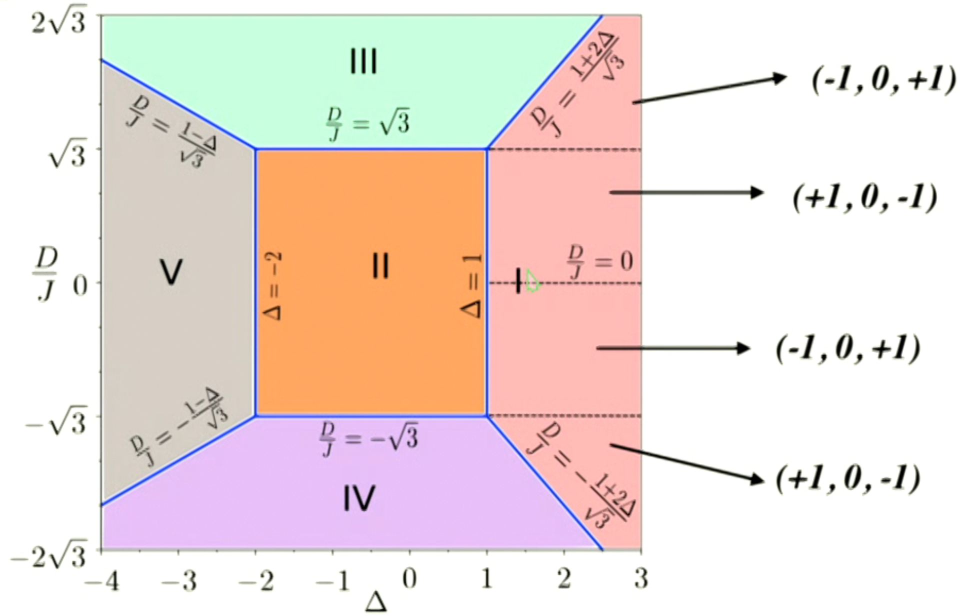
- **Berry Curvature** for i^{th} band

$$\Omega_i(\vec{k}) = \mathrm{i} \sum_{j \neq i} \frac{(\psi_i^\dagger(\vec{k}) \vec{\nabla}_{\vec{k}} H(\vec{k}) \psi_j(\vec{k})) \times (\psi_j^\dagger(\vec{k}) \vec{\nabla}_{\vec{k}} H(\vec{k}) \psi_i(\vec{k}))}{[E_i(\vec{k}) - E_j(\vec{k})]^2}$$

- **Chern number** :

$$C_i = \frac{1}{2\pi} \int_{BZ} d^2k \;\; \Omega_i(\vec{k})$$



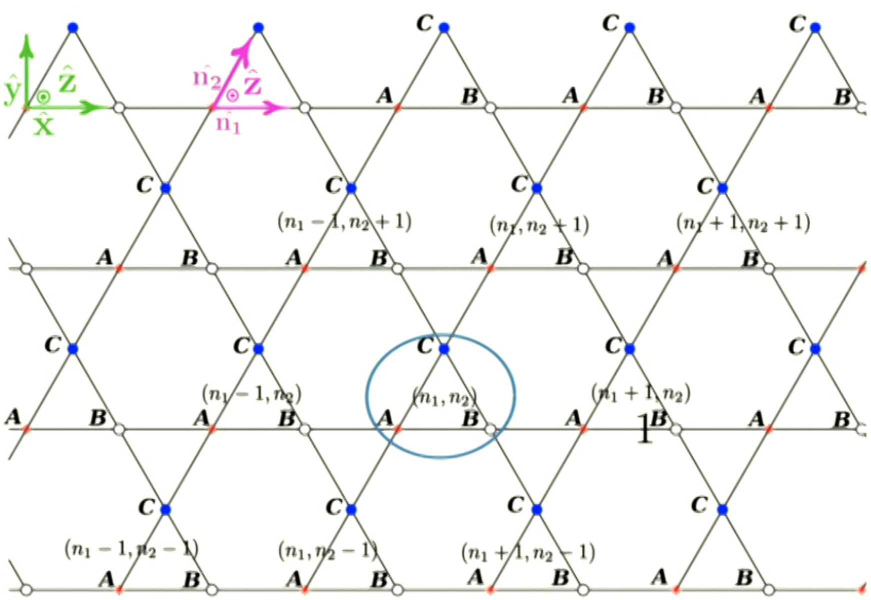


Phase 1: Edge modes

So What ….. ?

Chern Numbers closely related to the edge modes the system $\nu_i = \left|\sum_{j\le i} C_j\right|$

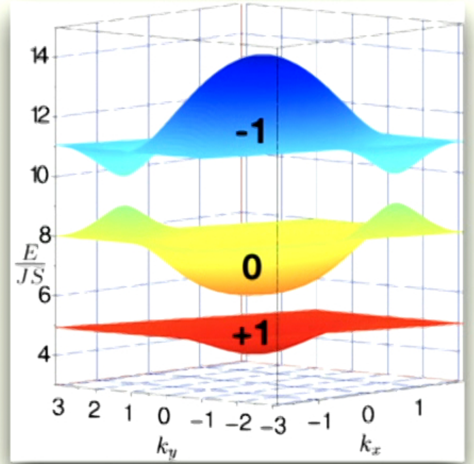
To study edge modes: Look at a ribbon of a finite width

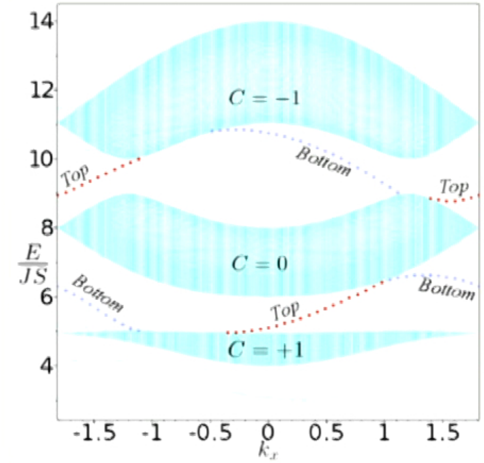


- Top edge :"zig-zag"
- Bottom edge : straight
- Only x-momentum is a good quantum number
- Ny = 150 sites along y
- Diagonalize 3Ny x 3Ny Hamiltonian

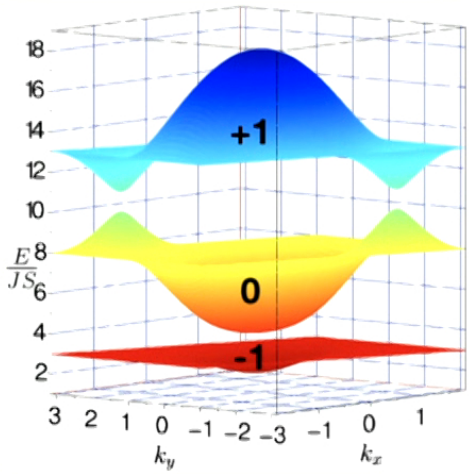
Phase I: Edge modes

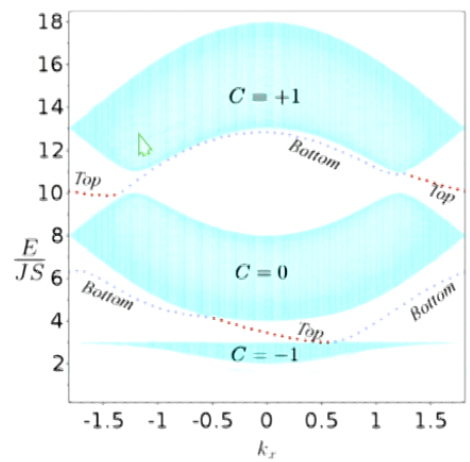
$D/J = 2/\sqrt{3}$





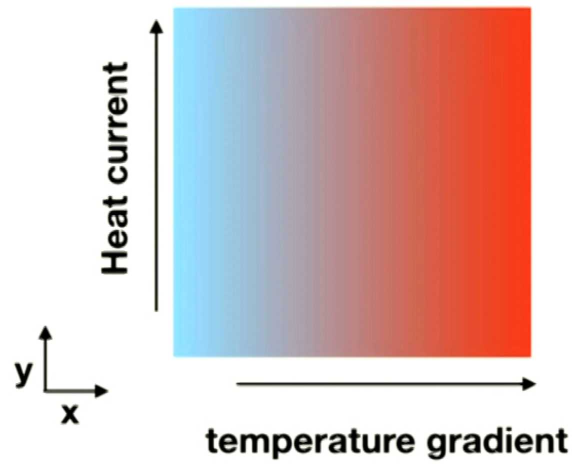
$D/J = 4/\sqrt{3}$





Phase I: Thermal Hall effect

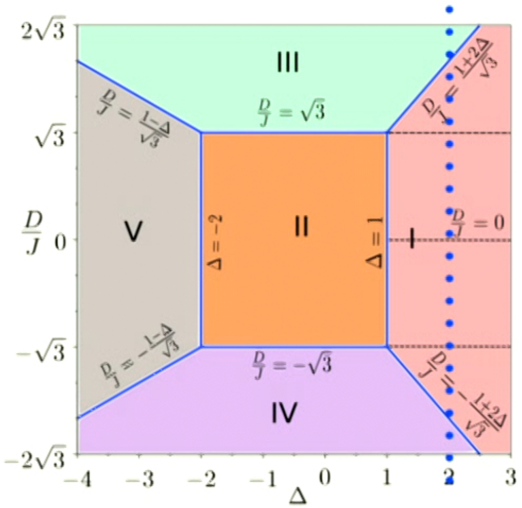
Spatial variation of temperature results in a heat current in the perpendicular direction



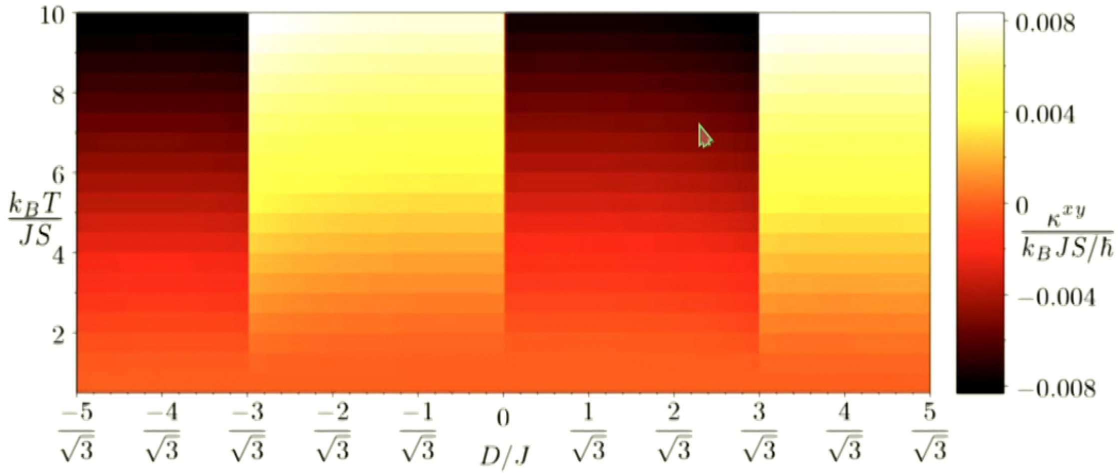
- Thermal Hall conductance $\kappa^{xy} = -\frac{k_B^2 T}{4\pi^2\hbar} \sum_i \int_{BZ} d^2k \; c_2(\rho_i(\vec{k})) \; \Omega_i(\vec{k})$
- Where $c_2(x) = (1+x)\left(\ln\frac{1+x}{x}\right)^2 - (\ln x)^2 - 2\mathrm{Li}_2(-x)$
- Dilogarithm function $\mathrm{Li}_2(z) = -\int_0^z du \, \frac{\ln(1-u)}{u}$
- Bose distribution function $\rho_i(\vec{k}) = \frac{1}{e^{E_i(\vec{k})/(k_B T)} - 1}$

Ref: Matsumoto and Murakami, Phys. Rev. B 84 184406

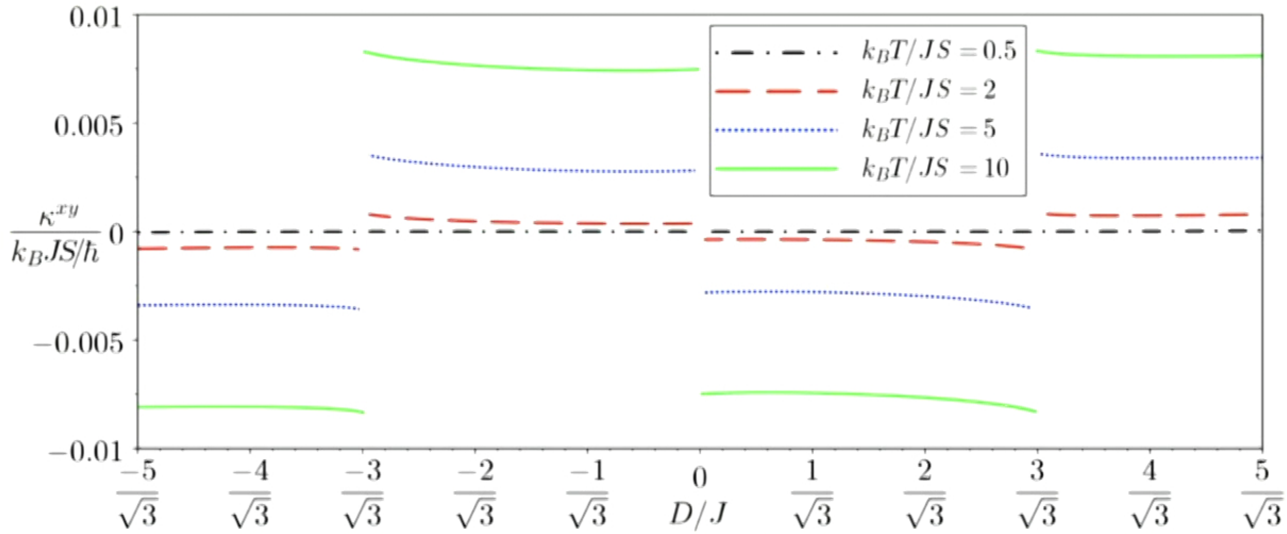
Phase I: Thermal Hall effect



- Vary D/J (the DMI strength), keeping the Δ=2
- Stay within Phase I ($-5/\sqrt{3} < D < 5/\sqrt{3}$)
- Every time the gap closing point is crossed: sudden change in the *Hall conductance*



Phase I: Thermal Hall effect



- Every time the gap closing point is crossed: sudden change in the hall conductance

Plan of the Talk

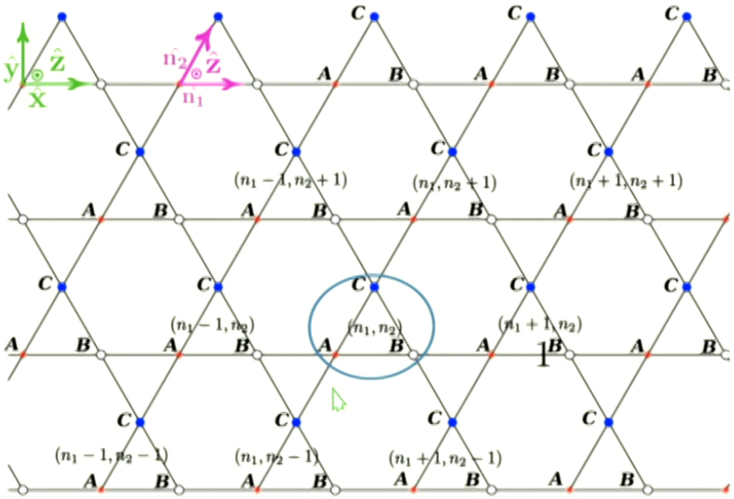
Topological magnons in a Kagome lattice spin system with XXZ and Dzyaloshinsky-Moriya interactions

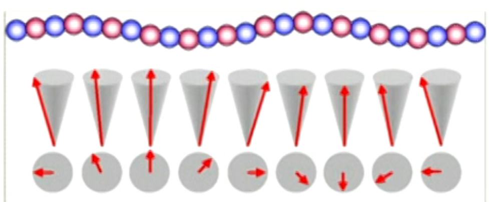
Ranjani Seshadri and Diptiman Sen

Centre for High Energy Physics, Indian Institute of Science, Bengaluru 560 012, India

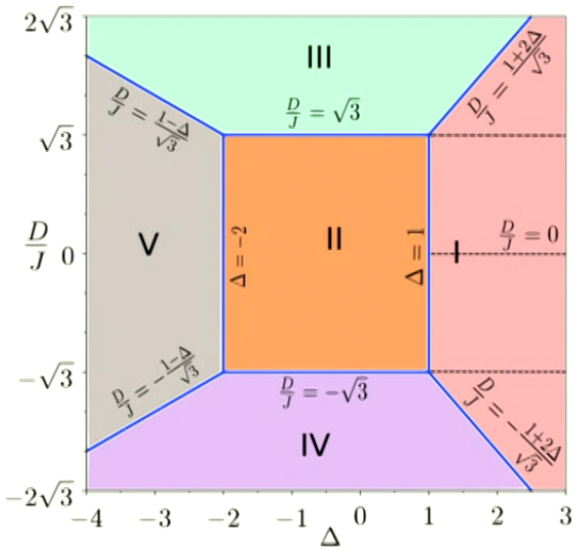
(Dated: December 7, 2017)

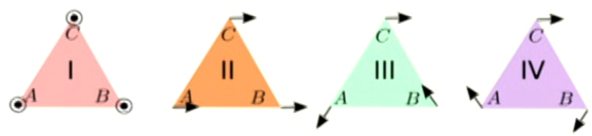
- Introduction
- Spins on a Kagome Lattice
  - XXZ + DM interaction
  - Ground state configuration
  - Phase diagram
- Spin wave analysis in "Phase I"
  - Holstein - Primakoff Transformation
  - Magnon Spectrum
  - Chern numbers and Edge Modes
  - Thermal Hall effect
- **Spin waves in other phases**
- Conclusions and future work





Phase III





- Gapless spectrum throughout this phase

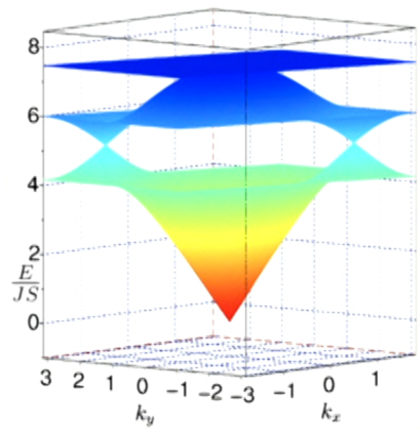
- Ground State : All spins pointing in +x-direction

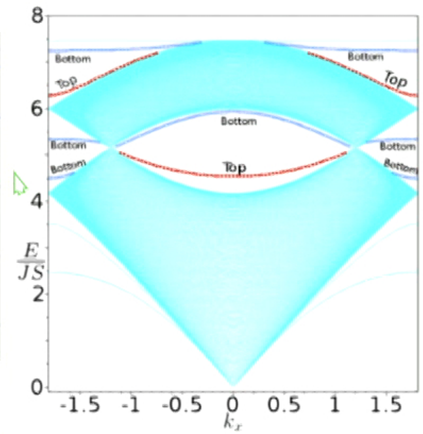
- H-P transformation :

$$A^x = S - a^\dagger a, \quad A^+ \simeq \sqrt{2S}\, a, \quad A^- \simeq \sqrt{2S}\, a^\dagger,$$
$$B^x = S - b^\dagger b, \quad B^+ \simeq \sqrt{2S}\, b, \quad B^- \simeq \sqrt{2S}\, b^\dagger,$$
$$C^x = S - c^\dagger c, \quad C^+ \simeq \sqrt{2S}\, c, \quad C^- \simeq \sqrt{2S}\, c^\dagger,$$

- Where,

$$A^\pm = A^{-\hat{n}_2} \pm i\, A^{-\hat{n}_2'},$$
$$B^\pm = B^{\hat{n}_3} \pm i\, B^{\hat{n}_3'},$$
$$C^\pm = C^y \pm i\, C^z,$$



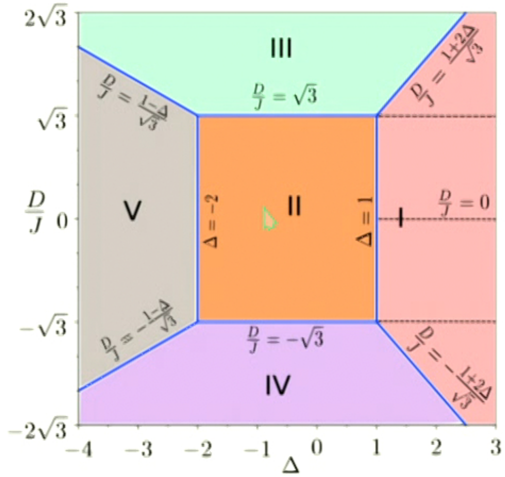


Summary

- Spins on a Kagome lattice
  XXZ + Dzyaloshinskii-Moriya interactions
- Very rich **phase diagram:** Five phases with different ground state configurations
  - Phases I-IV: spins are coplanar
  - Phase V: in general non-coplanar
- **Phases II - IV**
  - Topologically trivial
  - Always gapless gapless spectrum
- **Phase I**
  - spectrum, in general, is gapped
  - Four "sub-phases" separated by lines along which the gap closes
  - Adjacent sub-phases differ in terms of the Chern numbers, and edge modes
- Thermal hall conductance

Outlook

- More detailed analysis of phase V: more complicated bulk/edge spectrum??
- Add next-nearest-neighbor terms: significant changes in spectrum/Chern numbers??
- In-plane DMI ?



Thank You!

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Further possibilities

- More detailed analysis of phase V: more complicated bulk/edge spectrum??
- Add next-nearest-neighbour terms: significant changes in spectrum/Chern numbers?
- Other kinds of lattices?

Reference:

Ranjani Seshadri and Diptiman Sen
"Topological magnons in a Kagome lattice spin system with XXZ and Dzyaloshinsky-Moriya interactions"

Phys. Rev. B. 97, 134411 (2018)
arXiv 1711.11232