Title: PSI 2019/2020 - Statistical Physics - Lecture 3

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Statistical Physics (Kubiznak)

Date: August 28, 2019 - 9:00 AM

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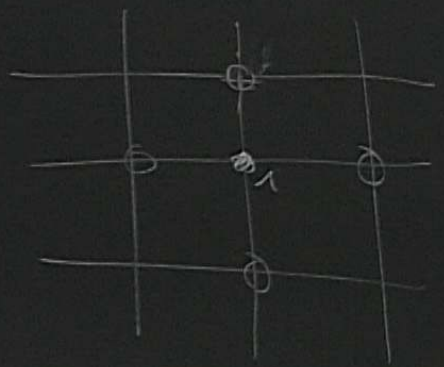
PHASE TRANSITIONS

CANONICAL EXAMPLE: ISING MODEL (1920)

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j$$

= MODEL OF FERROMAGNETISM

DISCRETE MODEL



d-DIMENSIONAL CUBIC LATTICE

SPIN VARIABLES

$\sigma_i = \pm 1$

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= MODEL OF FERROMAGNETISM

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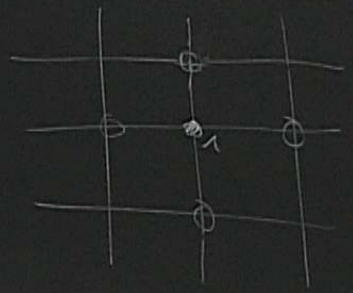


d-DIMENSIONAL CUBIC LATTICE

SPIN VARIABLES

$\sigma_i = \pm 1$

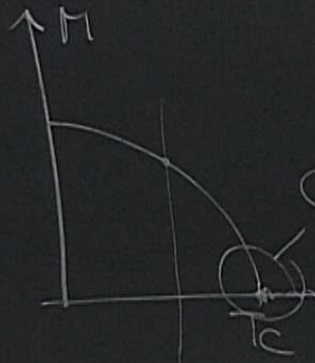
$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j$$

ADMITS 2 PHASES

$$H = -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j$$

$$M = \langle \sigma_i \rangle = \frac{\sum \sigma_i}{N} \begin{cases} 0 & \text{PM} \\ \neq 0 & \text{FM} \end{cases}$$

ADMITS 2 PHASES



$T = 0$ ... ALL SPINS ARE ALIGNED

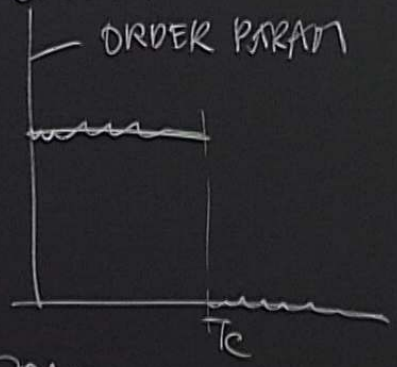
$T > 0$ ... THERMAL FLUCTUATIONS

CRITICAL POINT ... MAGNETIZATION IS CONTINUOUS

$\approx$

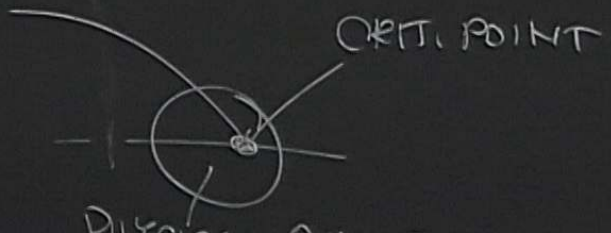
ORDER PARAMETER ... CONTINUOUS PT (2ND ORDER)

1ST-ORDER



(COMMON, BUT NOT NICE)

CONTINUOUS PTS ARE NICE
(UNIVERSAL BEHAVIOR)



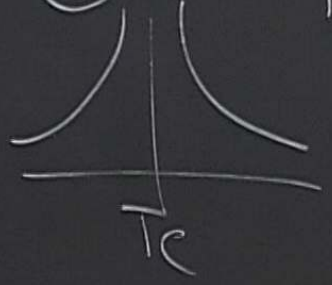
PHYSICAL QUANTITIES
BEHAVE THE "SAME WAY"
(CHARACTERIZED BY
CRITICAL EXPONENTS)

EG. SPECIFIC H

$C \propto |t$

EG. SPECIFIC HEAT

$C \propto |t|^{-\alpha}$, $t = \frac{T - T_c}{T_c}$



COMPLETELY DIFFERENT SYSTEMS MAY HAVE THE SAME CRITICAL EXPONENTS .. IN THE SAME UNIVERSILITY CLASS.

CRITICAL EXPONENTS

REMARKS:

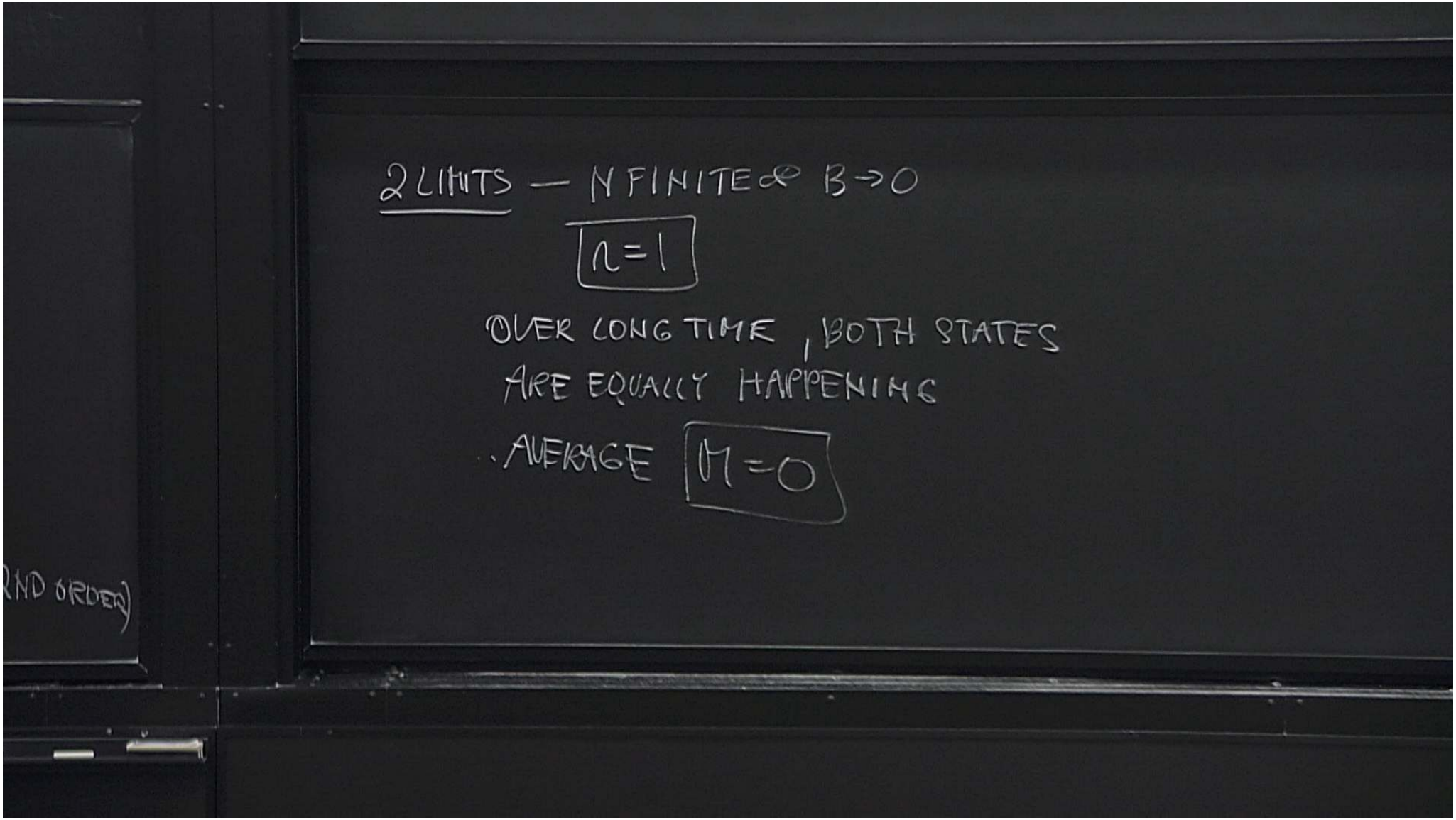
- NOTE THE NON-ANALYTIC BEHAVIOR OF M
- SYMMETRY $G_i \to -G_i$ .. $\mathbb{Z}_2$. SYMMETRY.
  .. SPONTANEOUSLY BROKEN IN FM PHASE
  ⇔
  ORDER PARAMETER

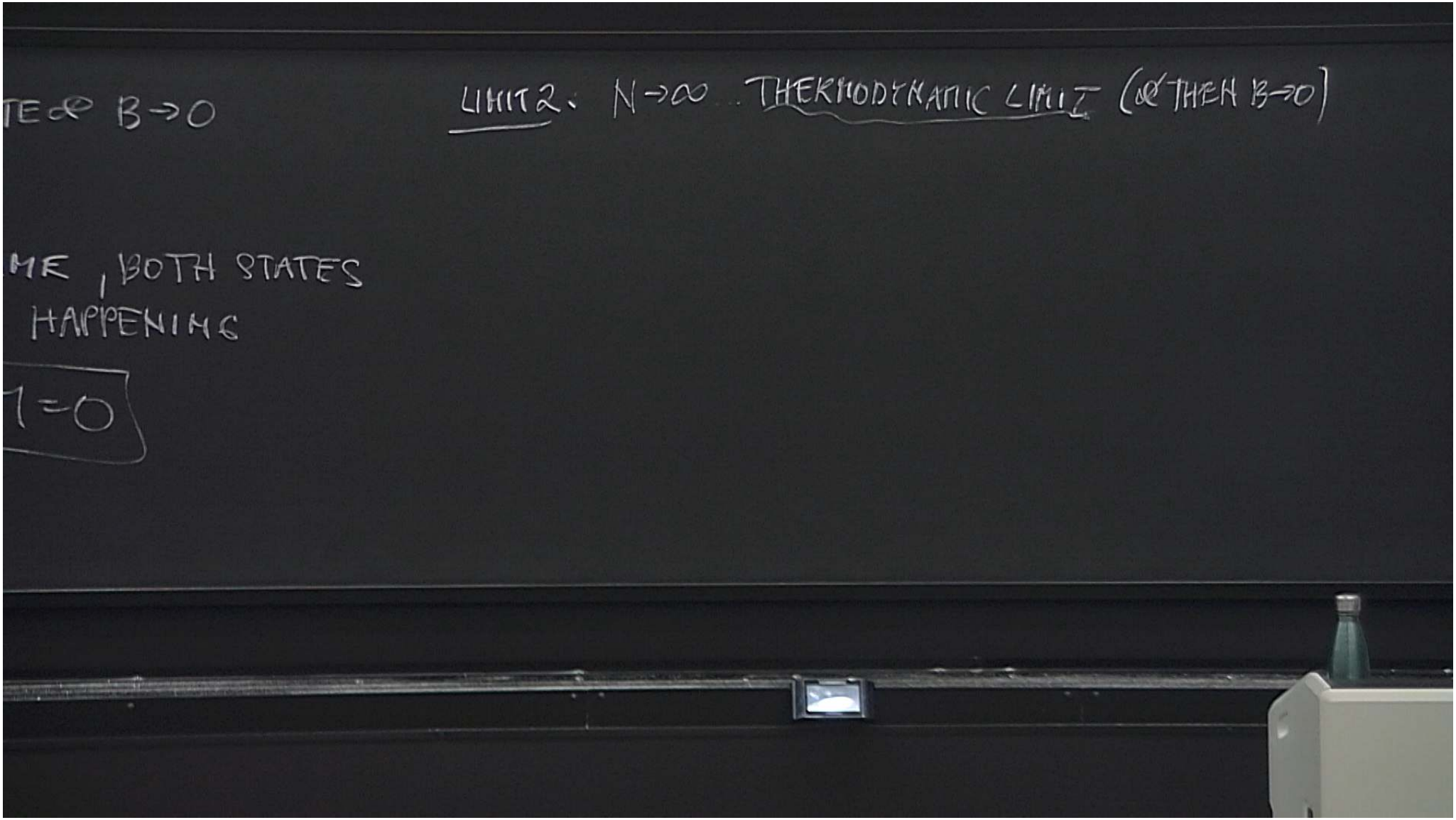
EXERCISE. SWITCH ON MAGNETIC FIELD B & CONSIDER FINITE SIZE SYSTEM (N. FINITE)

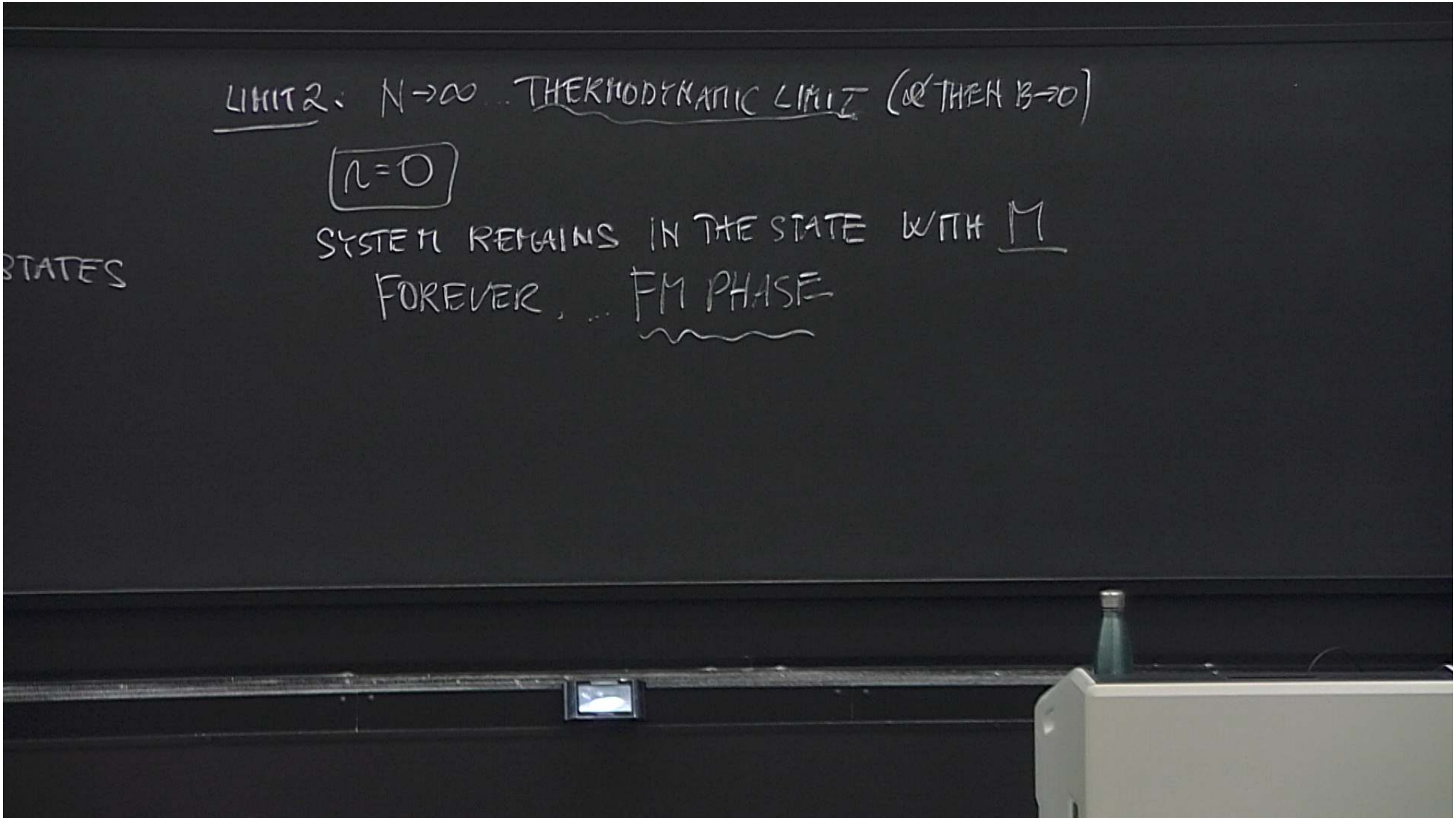
$$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - B\sum_i \sigma_i$$

PROBABILITY OF FINDING ALL SPINS UP VS. DOWN: (B>0)

$$R = \frac{P_{H<0}}{P_{H>0}} \overset{\text{BOLTZMANN}}{=} \frac{e^{-NBM/T}}{e^{+NBM/T}} = e^{-2NBM/T}$$


2 LIMITS — N FINITE & B → 0
n = 1
OVER LONG TIME, BOTH STATES
ARE EQUALLY HAPPENING
AVERAGE M = 0
RND ORDER)


TE & B→0
LIMIT 2. N→∞ THERMODYNAMIC LIMIT (& THEN B→0)
ME, BOTH STATES
HAPPENING
=0


LIMIT 2. N→∞ THERMODYNAMIC LIMIT (& THEN B→0)
h=0
SYSTEM REMAINS IN THE STATE WITH M
FOREVER ... FM PHASE
STATES

LIMIT 2. $N \to \infty$ THERMODYNAMIC LIMIT (α THEN $\beta \to 0$)

$h = 0$

SYSTEM REMAINS IN THE STATE WITH M FOREVER. ... FM PHASE

STATEMENT: PHASES & PTS ARE SHARPLY DEFINED FOR INFINITE SYSTEMS (THERMODYNAMIC LIMIT)

REAL MACROSCOPIC SYSTEMS LONG TIMESCALES.

MEAN FIELD THEORY APPROX

IDEA: REPLACE MANY BODY INTERACTIONS WITH 1-BODY INTERACTION WITH AN MEAN FIELD (PRODUCED BY OTHER BODIES)

IN PRACTICE. EXPANSION OF $\hbar$ IN FLUCTUATIONS

→ LEADING ORDER (NO FLUCTUATIONS)

EX: ISING

$\sigma_i = \underbrace{\sigma_i - \langle\sigma_i\rangle}_{\varepsilon \ldots \text{FLUCTUATION}} + \langle\sigma_i\rangle$

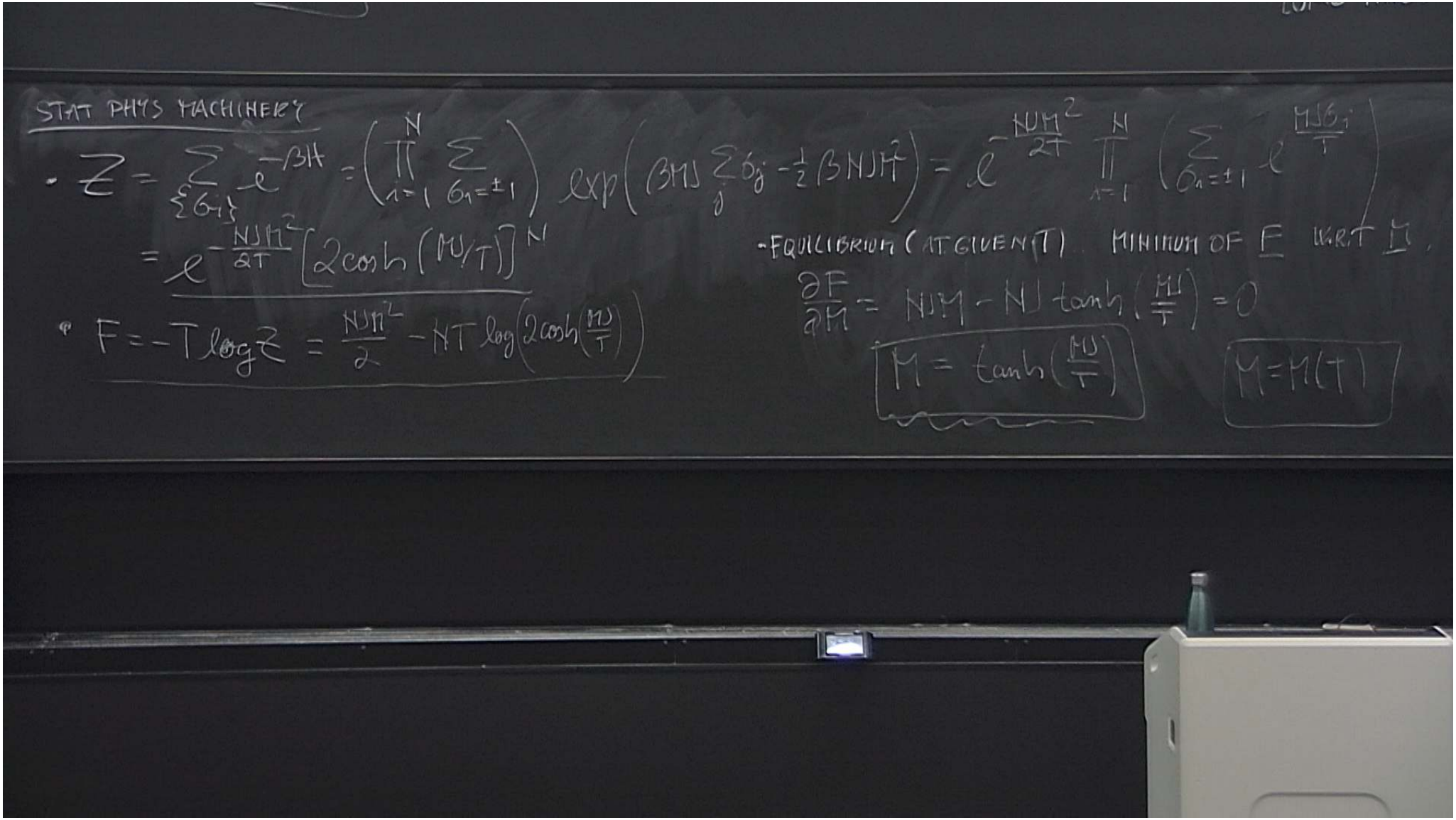
$\langle\sigma\rangle = \frac{1}{N}\sum_j \sigma_j = M$

$\sigma_i\sigma_j = (\underbrace{\sigma_i - M}_{\varepsilon} + M)(\underbrace{\sigma_j - M}_{\varepsilon} + M) = \underbrace{(\sigma_i - M)(\sigma_j - M)}_{\varepsilon^2 \ldots \text{NEGLECT}} + M(\sigma_i + \sigma_j) - M^2$

$H = -\frac{1}{2}\sum_{ij}\left(M(\sigma_i + \sigma_j) - M^2\right)J_{ij}$; $J_i = \sum_j J_{ij} = J$ INDEP OF i (LATTICE IS UNIFORM)

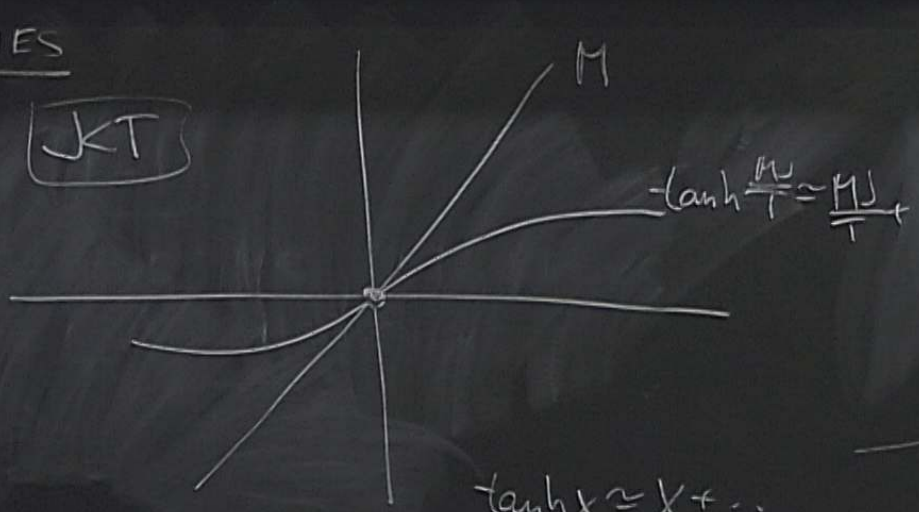
$$\boxed{H = \tfrac{1}{2}M^2 JN - \sum_i M\sigma_i J}$$

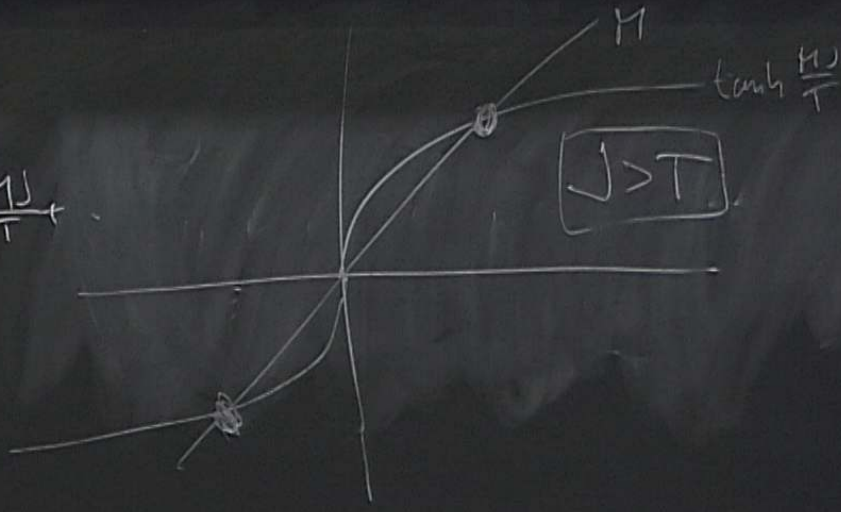
PROBABILITY OF ... $n = \frac{P_{H<0}}{P_{H>0}}$ BOLTZMANN $\frac{e^{-NBM/T}}{e^{+NBM/T}} = e^{-2NBM/T}$


STAT PHYS MACHINERY
$\mathcal{Z} = \sum_{\{\sigma_i\}} e^{-\beta H} = \left(\prod_{i=1}^{N} \sum_{\sigma_i=\pm 1}\right) \exp\left(\beta m J \sum_j \sigma_j - \frac{1}{2}\beta N J m^2\right) = e^{-\frac{NJm^2}{2T}} \prod_{i=1}^{N} \left(\sum_{\sigma_i=\pm 1} e^{\frac{mJ\sigma_i}{T}}\right)$
$= e^{-\frac{NJm^2}{2T}} \left[2\cosh\left(mJ/T\right)\right]^N$
$F = -T \log \mathcal{Z} = \frac{NJm^2}{2} - NT \log\left(2\cosh\left(\frac{mJ}{T}\right)\right)$
- EQUILIBRIUM (AT GIVEN T) MINIMUM OF F W.R.T m.
$\frac{\partial F}{\partial m} = NJm - NJ \tanh\left(\frac{mJ}{T}\right) = 0$
$m = \tanh\left(\frac{mJ}{T}\right)$
$m = m(T)$

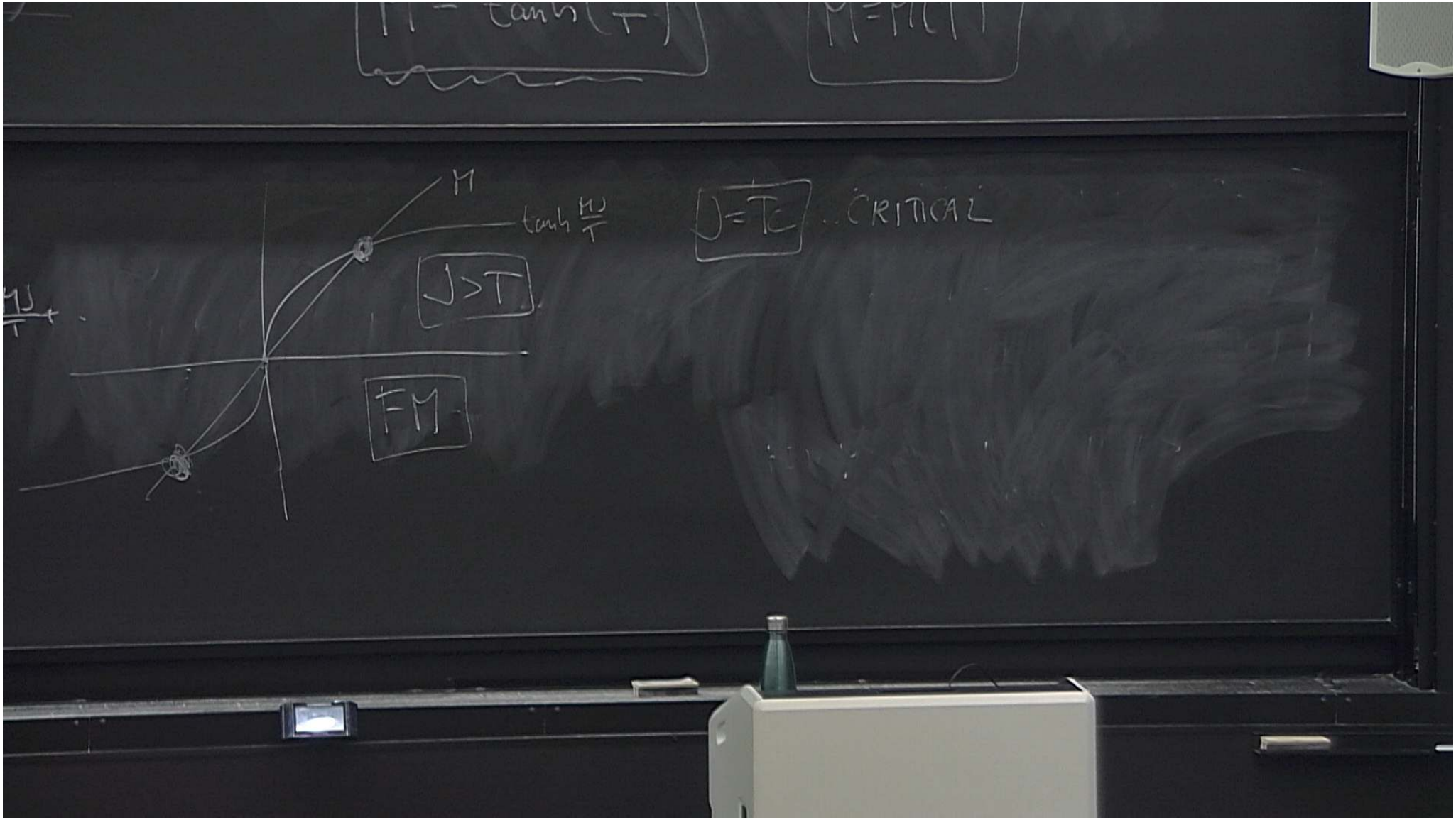
2 POSSIBILITIES

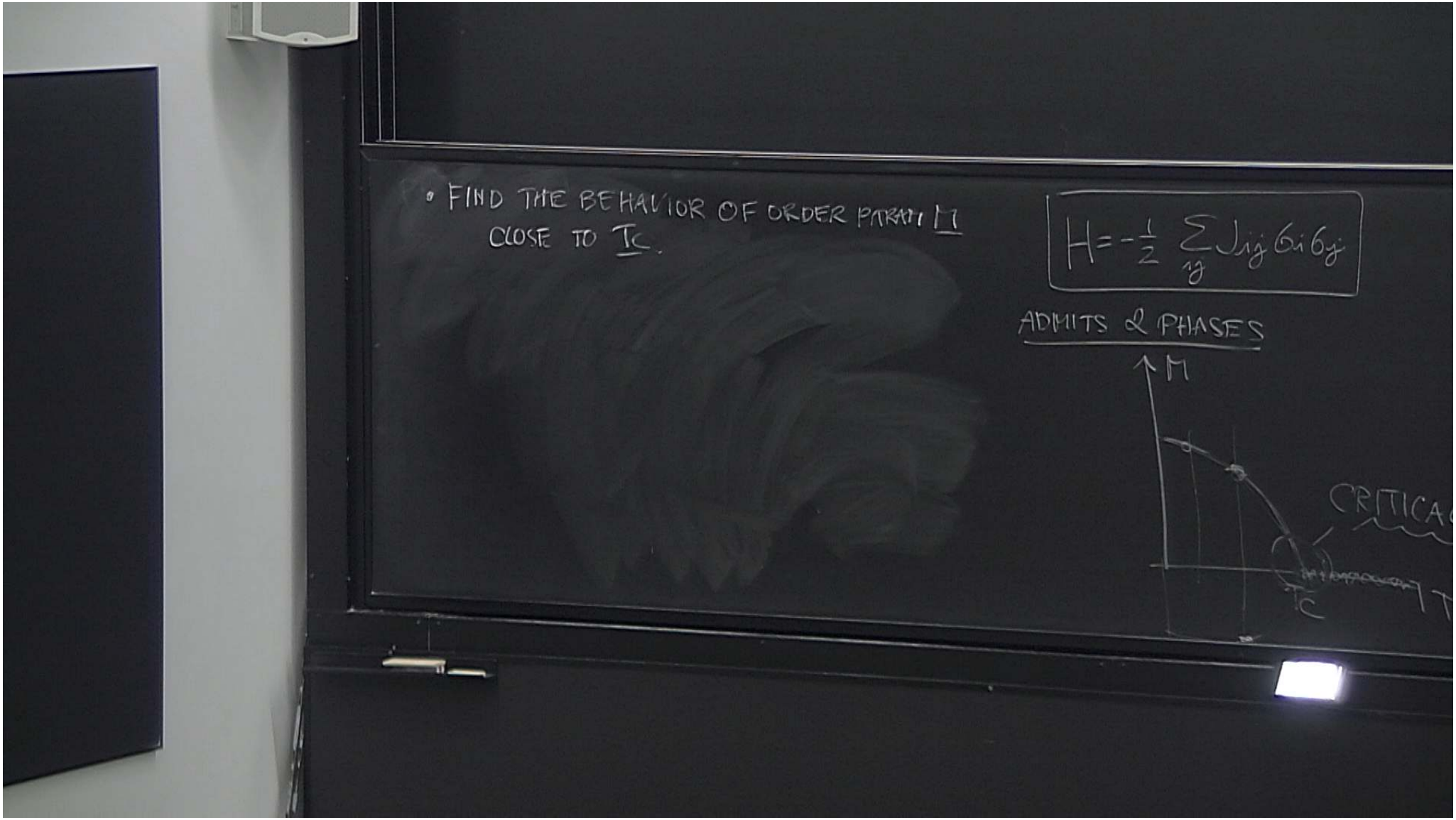
J<T





J>T


M
tanh MJ/T
J>T
FM
J=Tc
..CRITICAL


• FIND THE BEHAVIOR OF ORDER PARAM M
CLOSE TO Tc.
H = -½ Σ Jij σi σj
ij
ADMITS 2 PHASES
M
CRITICA
Tc
T

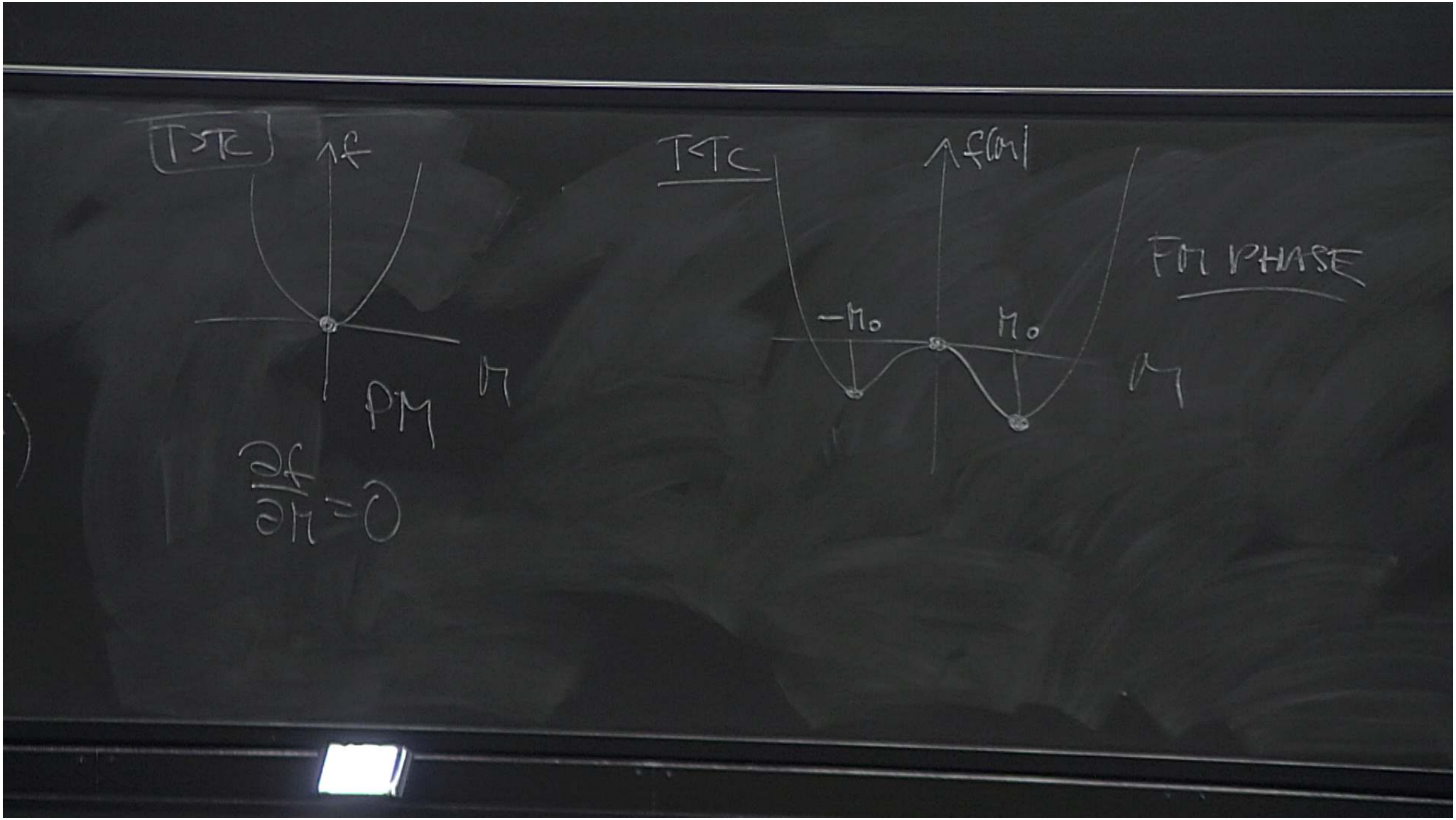
- FIND THE BEHAVIOR OF ORDER PARAM M CLOSE TO T_c. (M SMALL HERE)

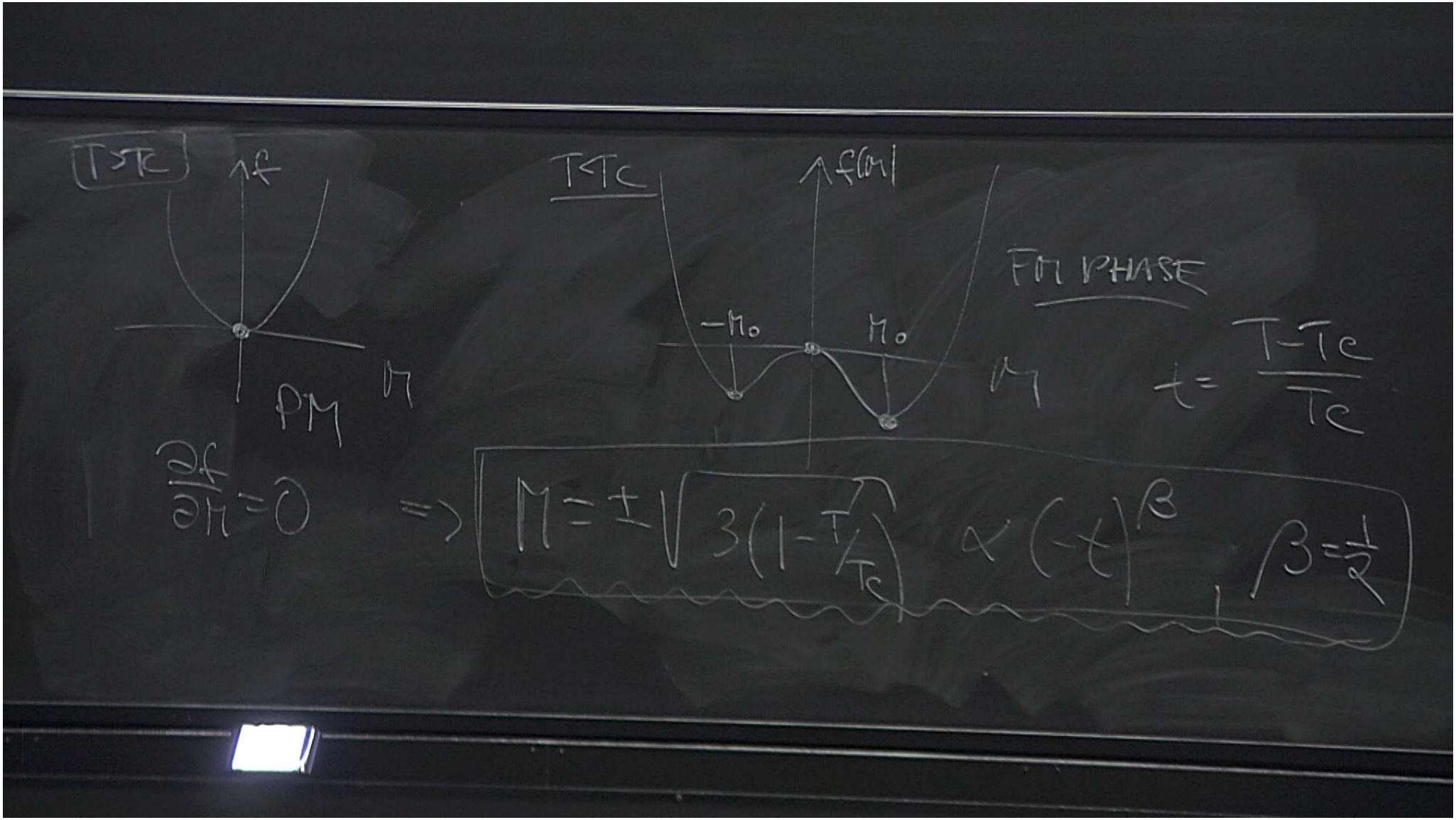
$$f = \frac{F}{N} = \frac{JM^2}{2} - T\log\left(2\cosh\left(\frac{MJ}{T}\right)\right)$$

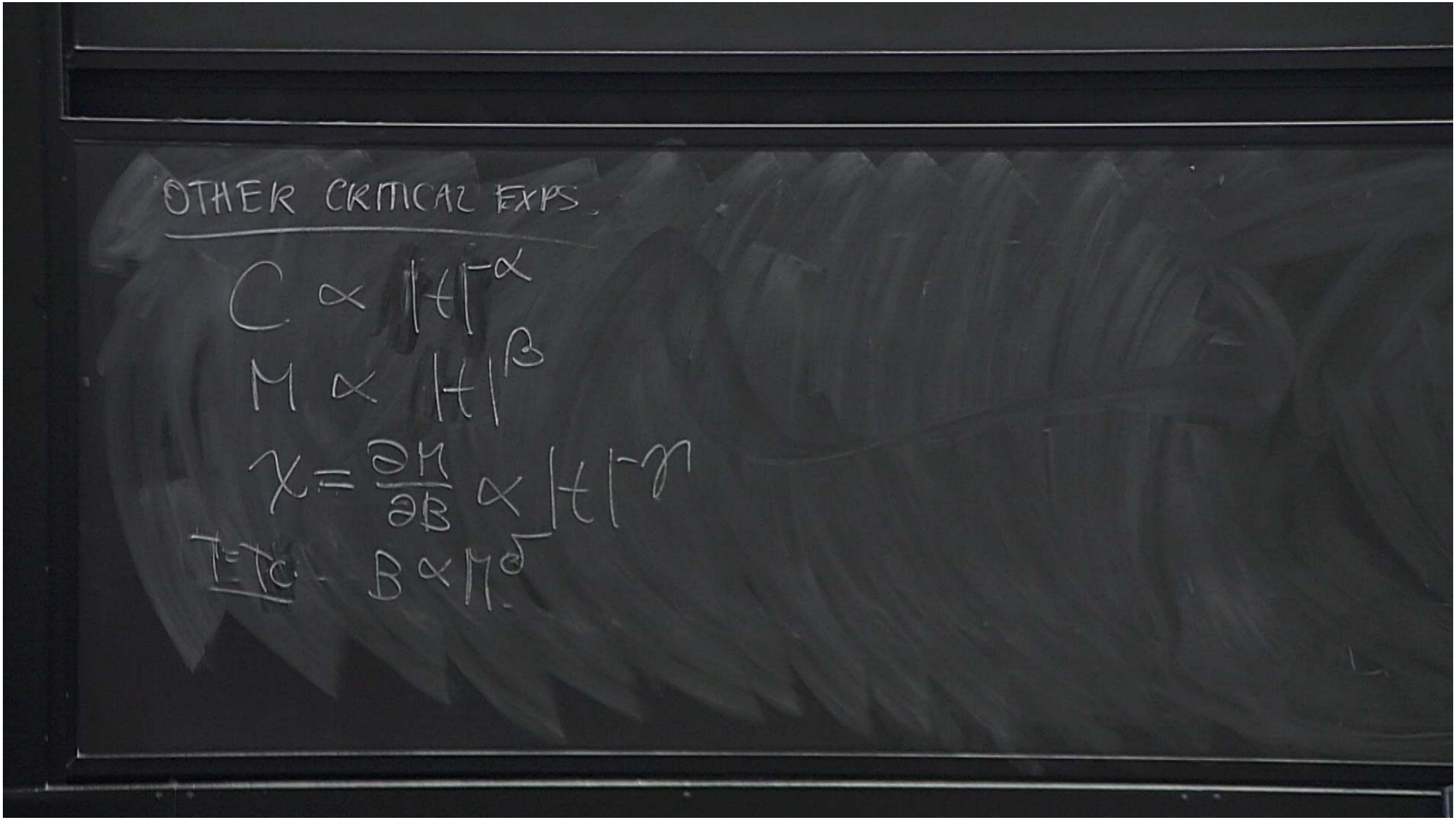
$$= \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4 + O(M^6)$$

$$H = -\frac{1}{2}\sum_{ij} J_{ij}$$

ADMITS 2 PHASES


T>Tc
f
PM
M
$\frac{\partial f}{\partial M}=0$
T<Tc
f(M)
$-M_0$
M_0
M
FM PHASE

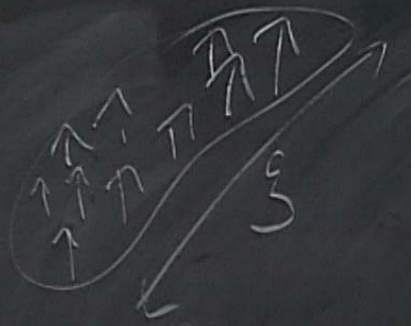

T>Tc
f
PM
M
T<Tc
f(M)
-M₀
M₀
FM PHASE
M
t = (T-Tc)/Tc
∂f/∂M = 0 ⇒ M = ± √(3(1 - T/Tc)) ∝ (-t)^β , β = 1/2

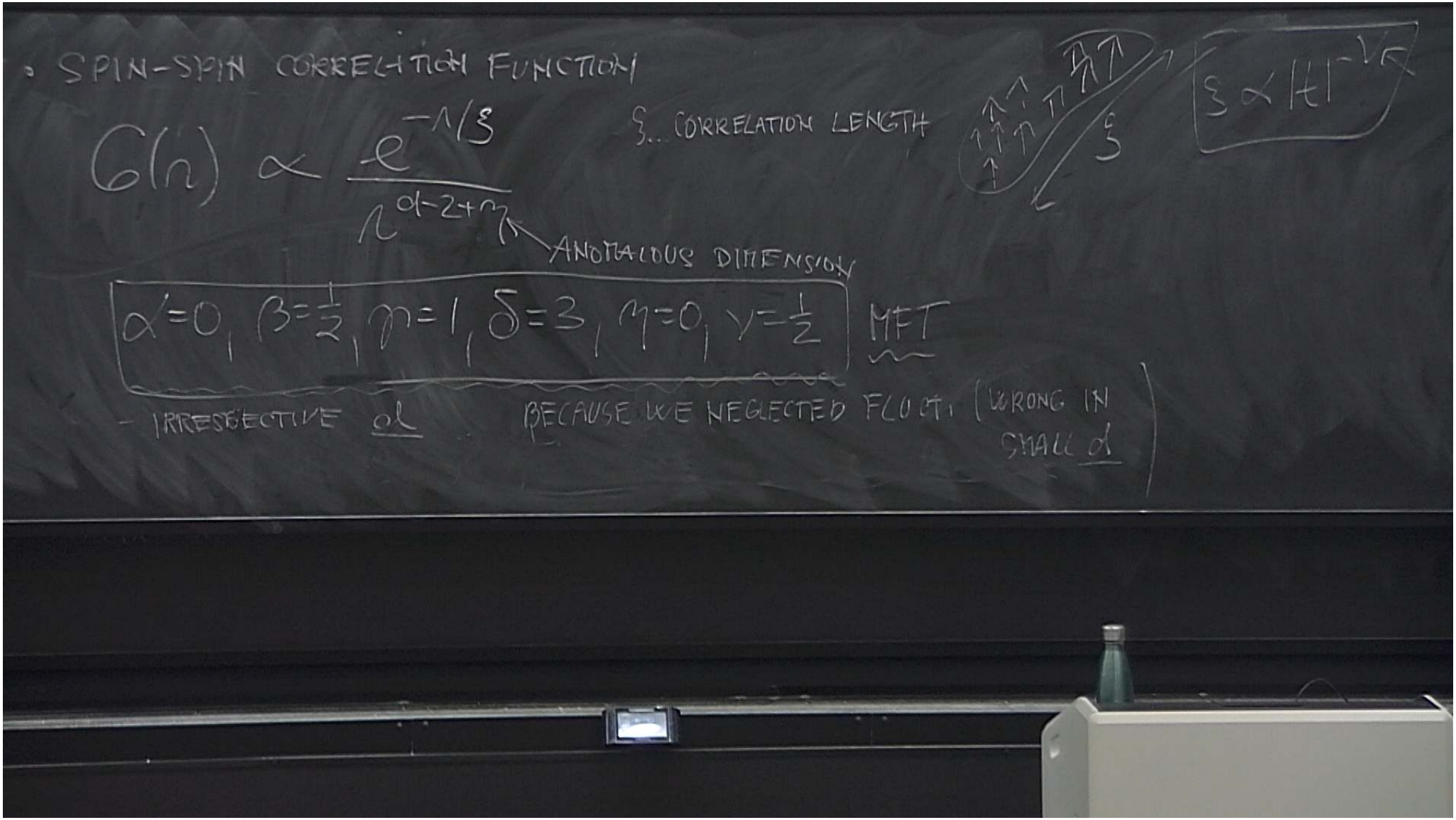

OTHER CRITICAL EXPS.
$C \propto |t|^{-\alpha}$
$M \propto |t|^{\beta}$
$\chi = \frac{\partial M}{\partial B} \propto |t|^{-\gamma}$
$T = T_c$ $B \propto M^{\delta}$

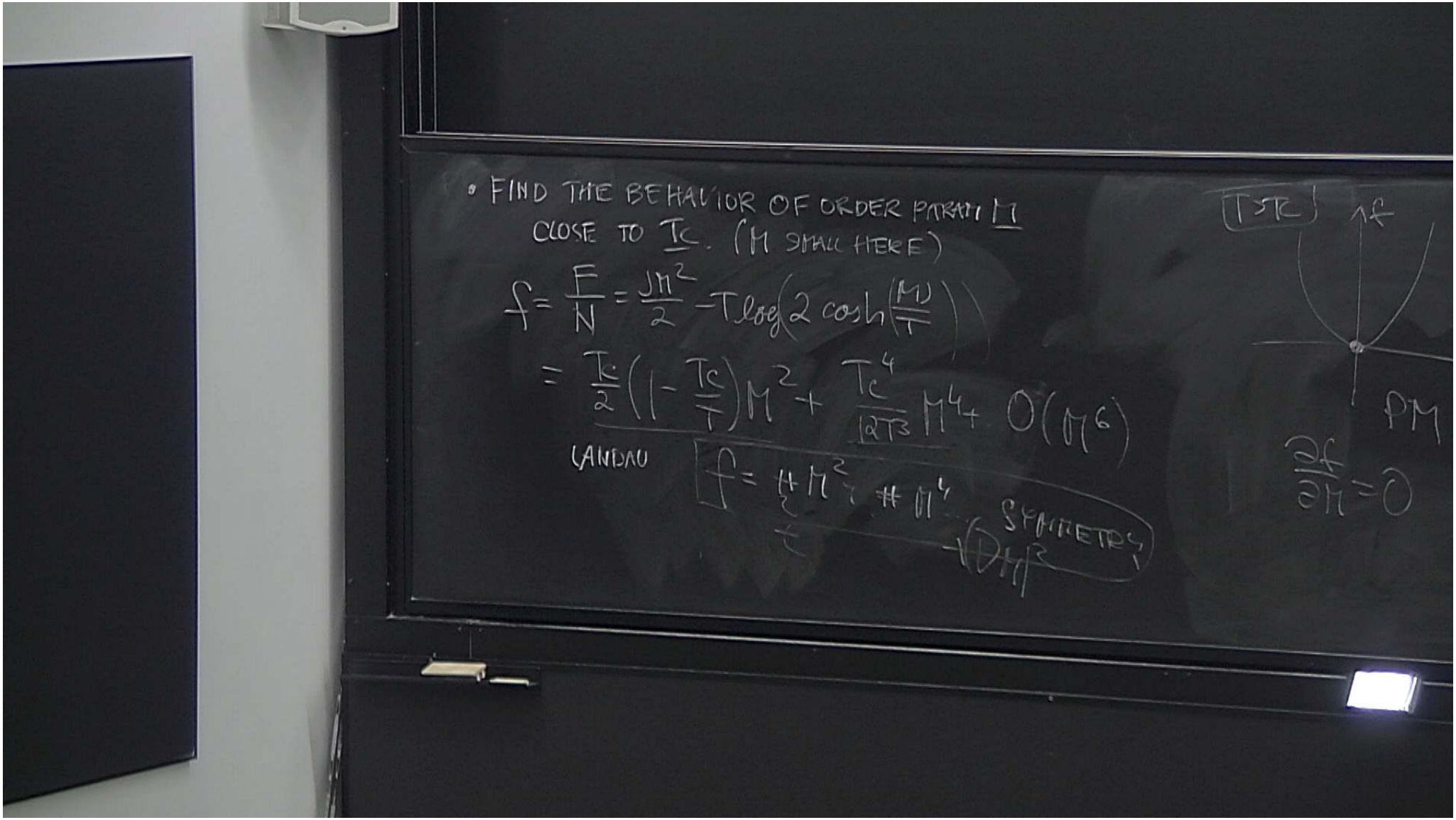
- SPIN-SPIN CORRELATION FUNCTION

$$G(r) \propto \frac{e^{-r/\xi}}{r^{d-2+\eta}}$$

ξ... CORRELATION LENGTH




• SPIN-SPIN CORRELATION FUNCTION
$G(r) \propto \frac{e^{-r/\xi}}{r^{d-2+\eta}}$
ξ.. CORRELATION LENGTH
ANOMALOUS DIMENSION
$\xi \propto |t|^{-\nu}$
$\alpha=0,\ \beta=\frac{1}{2},\ \gamma=1,\ \delta=3,\ \eta=0,\ \nu=\frac{1}{2}$ MFT
- IRRESPECTIVE of
BECAUSE WE NEGLECTED FLUCT. (WRONG IN SMALL d)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$= \frac{T_c}{2}\left(1 - \frac{T_c}{T}\right)M^2 + \frac{T_c^4}{12T^3}M^4 + O(M^6)$
LANDAU
$f = \frac{1}{2}M^2 + \#M^4$
SYMMETRY
$+DM^2$
T>Tc
f
PM
$\frac{\partial f}{\partial M} = 0$