Title: Algebras of Interfaces

Speakers: Sergei Gukov

Collection: Boundaries and Defects in Quantum Field Theory

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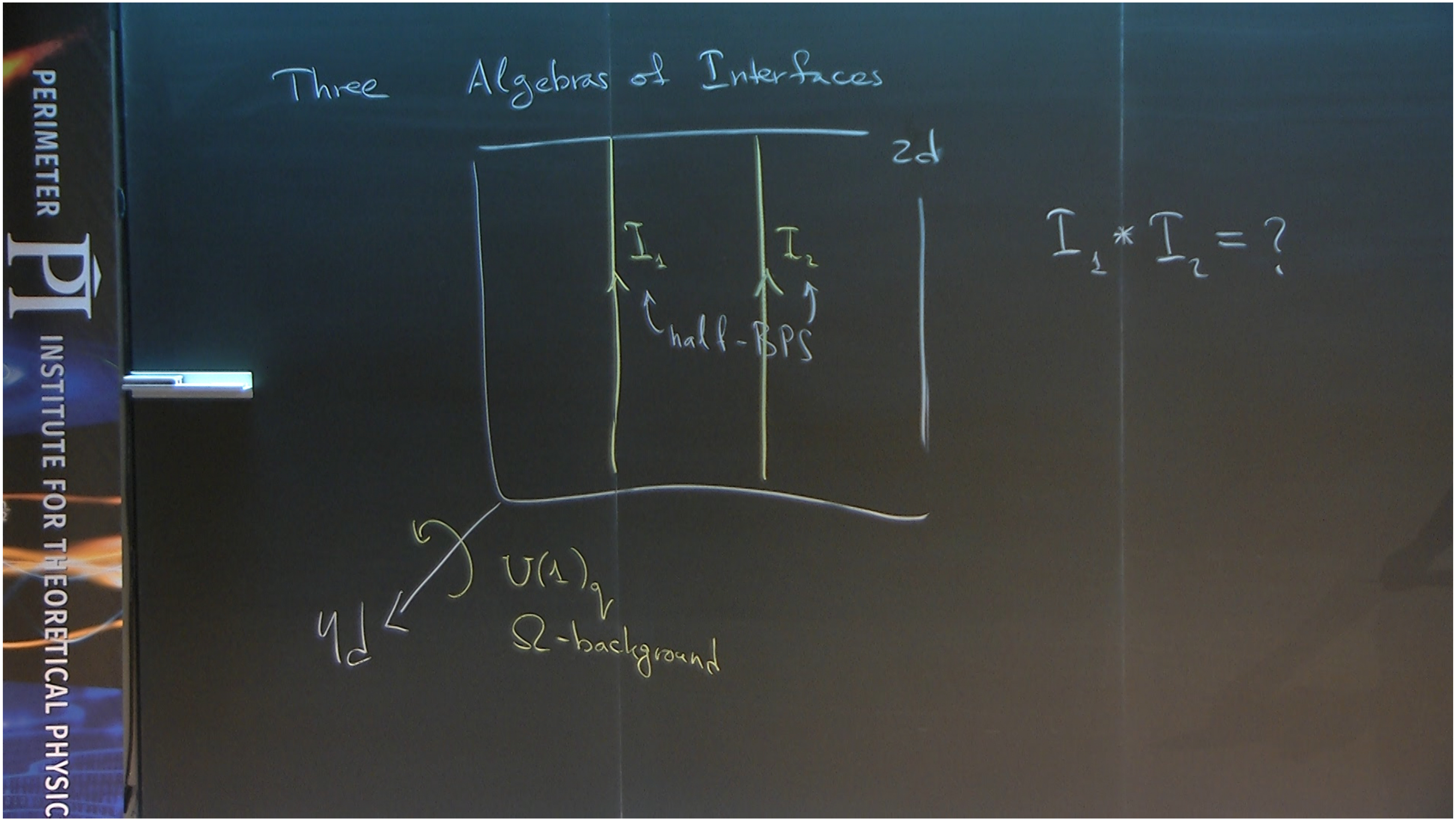
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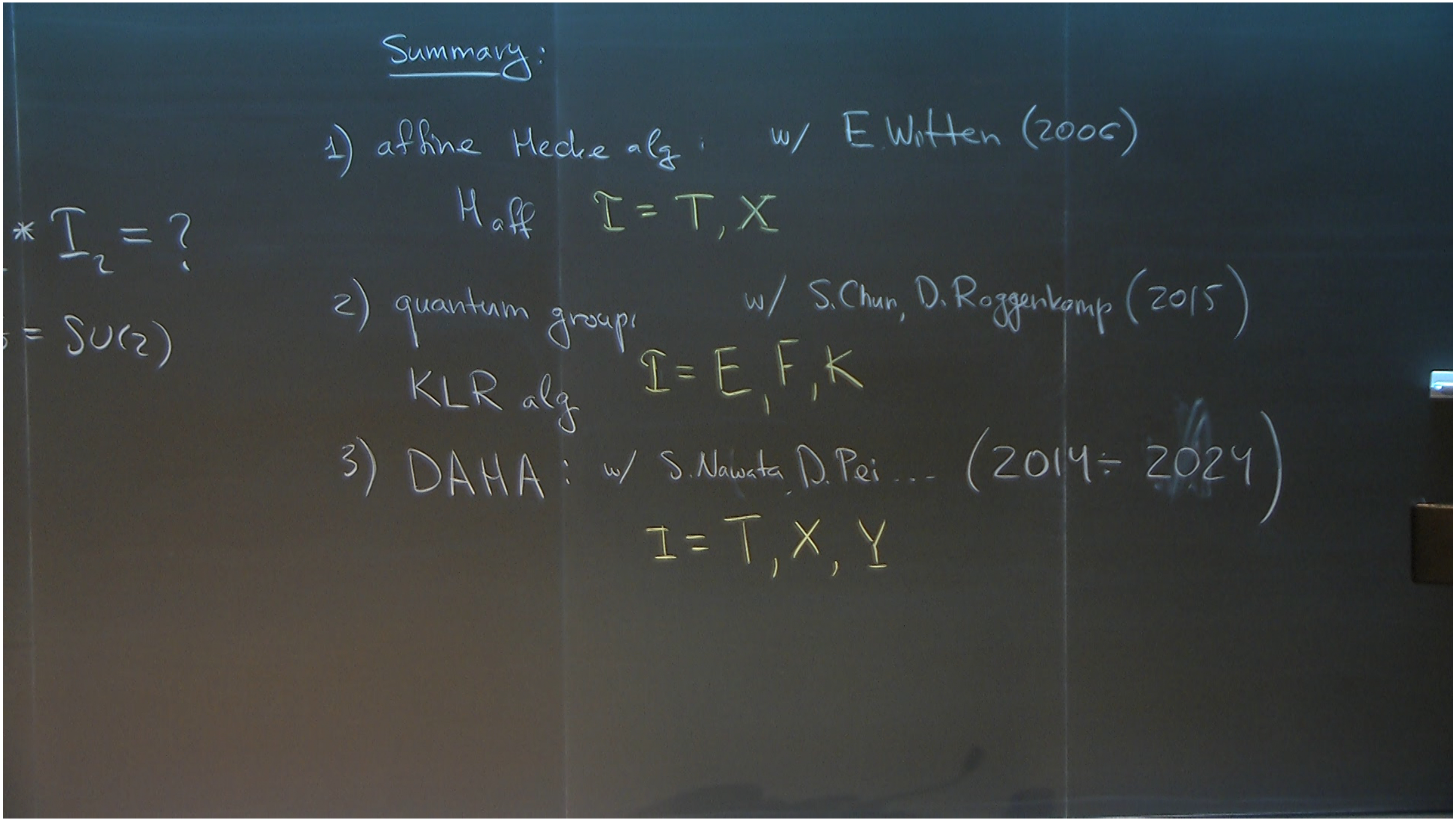
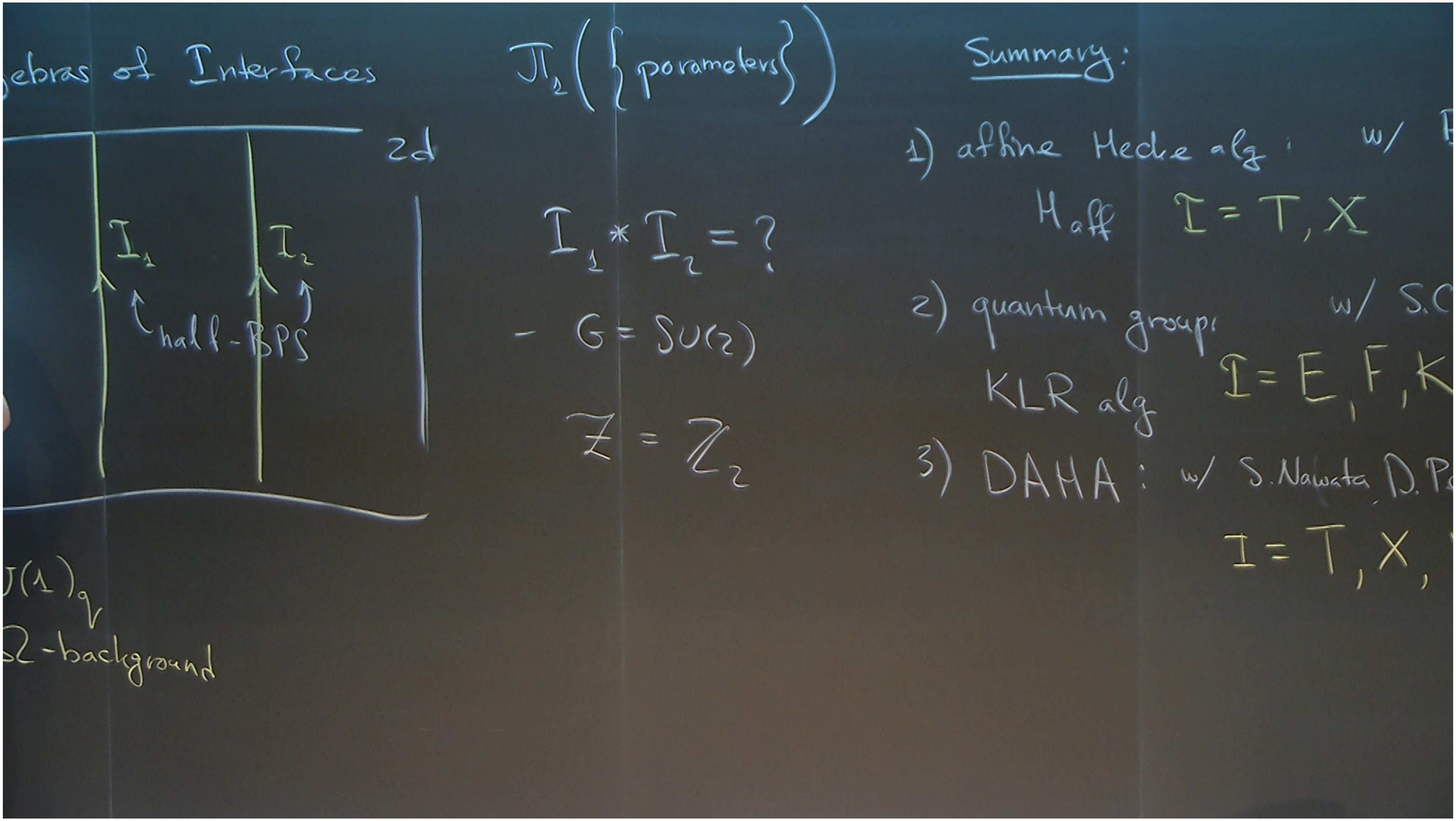
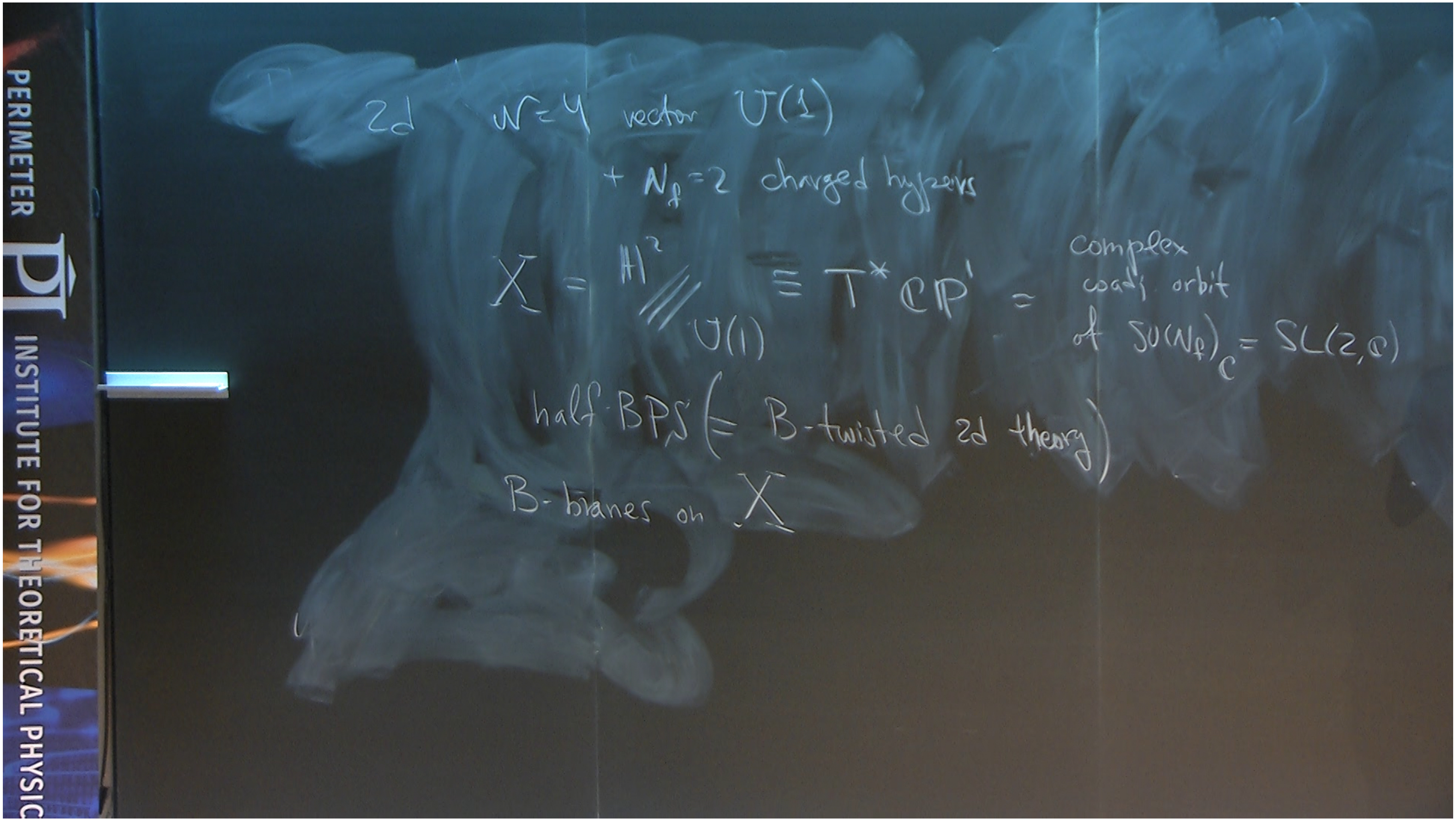
Three Algebras of Interfaces
2d
I_1
I_2
half-BPS
$U(1)_q$
Ω-background
4d
$I_1 * I_2 = ?$


Summary:
1) affine Hecke alg: w/ E. Witten (2006)
H aff I = T, X
* I_2 = ?
= SU(2)
2) quantum group: w/ S. Chun, D. Roggenkamp (2015)
KLR alg I = E, F, K
3) DAHA: w/ S. Nawata, D. Pei ... (2014 ÷ 2024)
I = T, X, Y


ebras of Interfaces
2d
I1
I2
half-BPS
U(1)q
Ω-background
π2({parameters})
I1 * I2 = ?
- G = SU(2)
Z = Z2
Summary:
1) affine Hecke alg: w/
Haff I = T, X
2) quantum group: w/ S.
KLR alg I = E, F, K
3) DAHA: w/ S. Nawata, D. P
I = T, X,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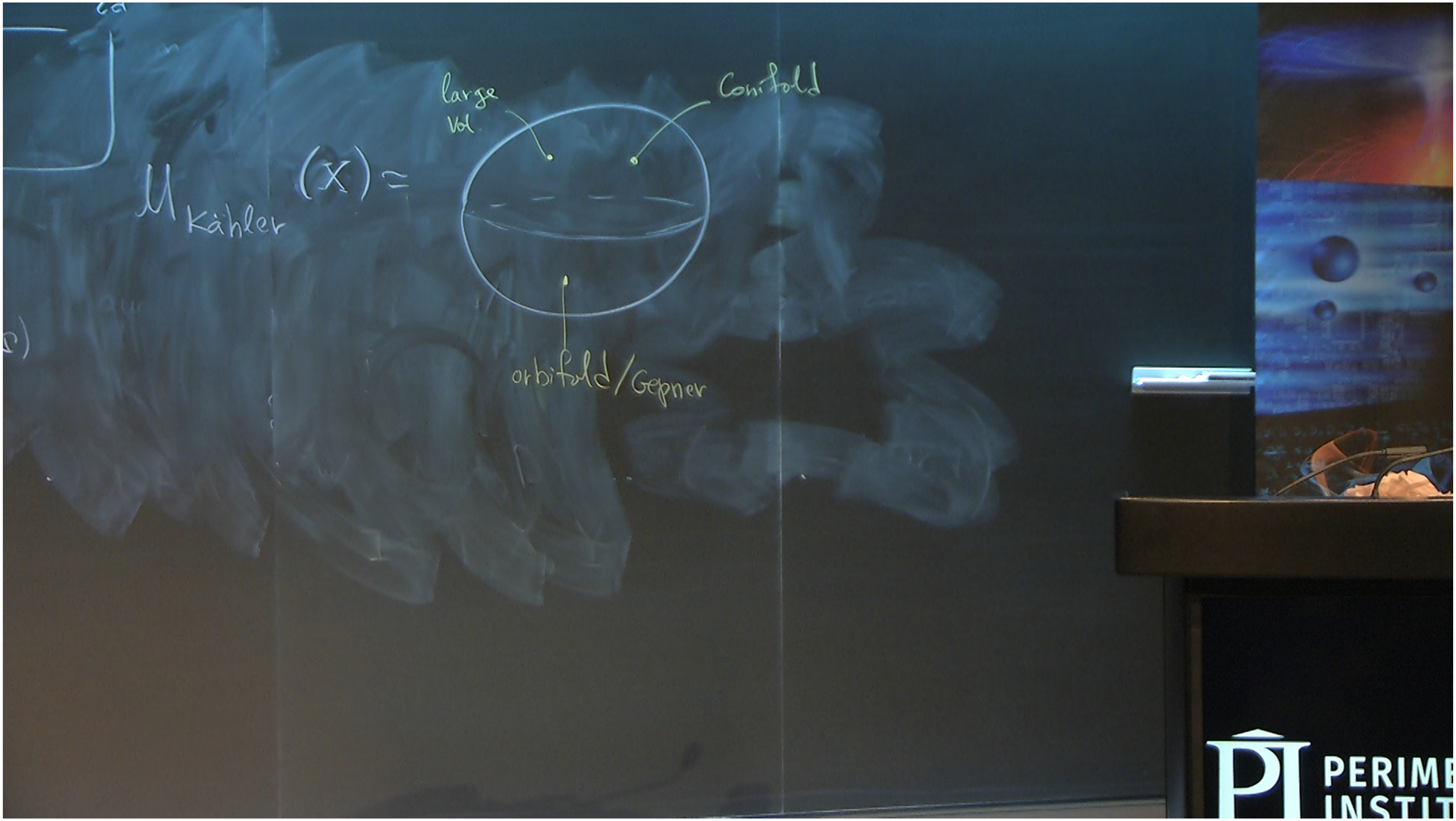
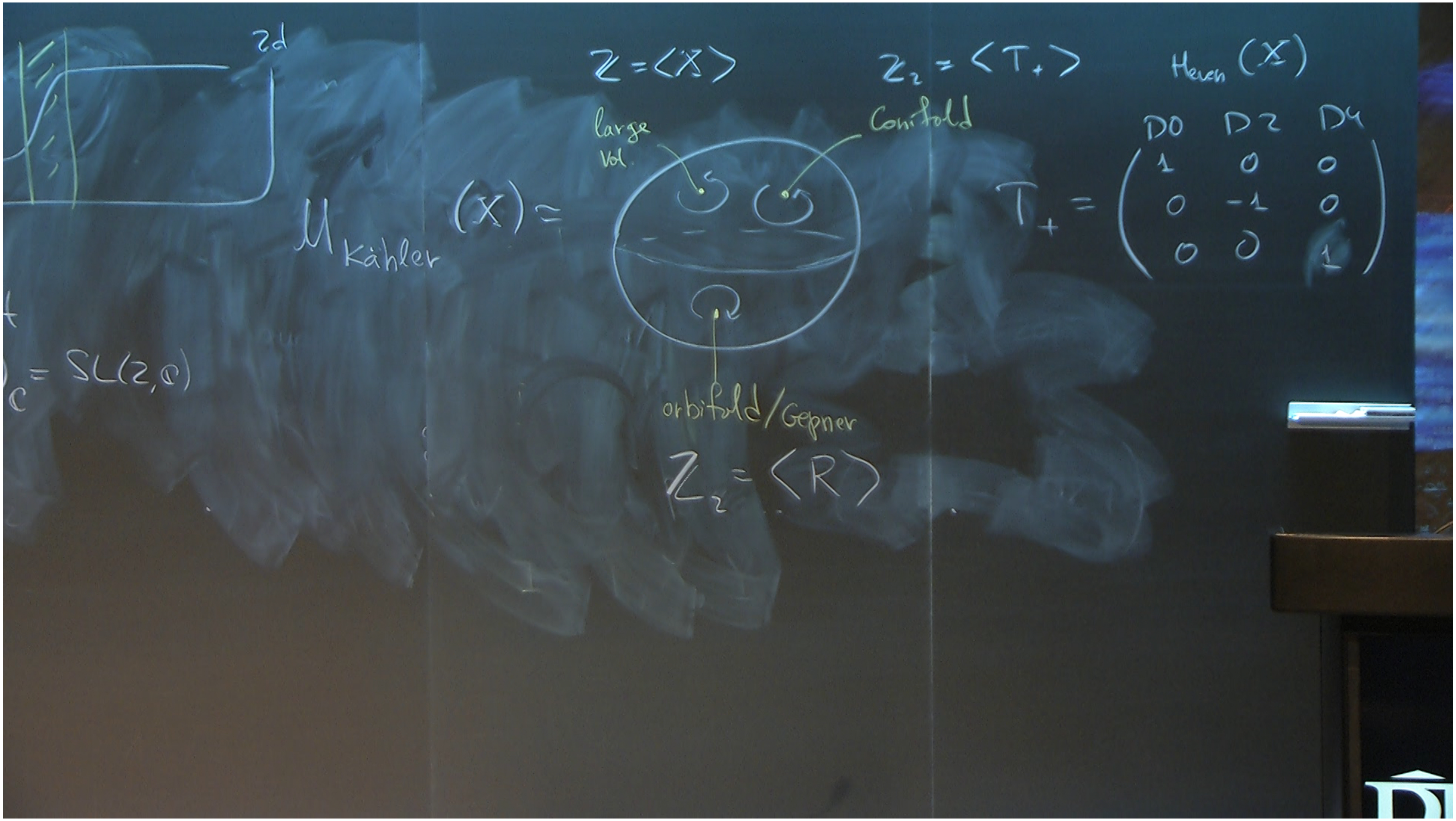
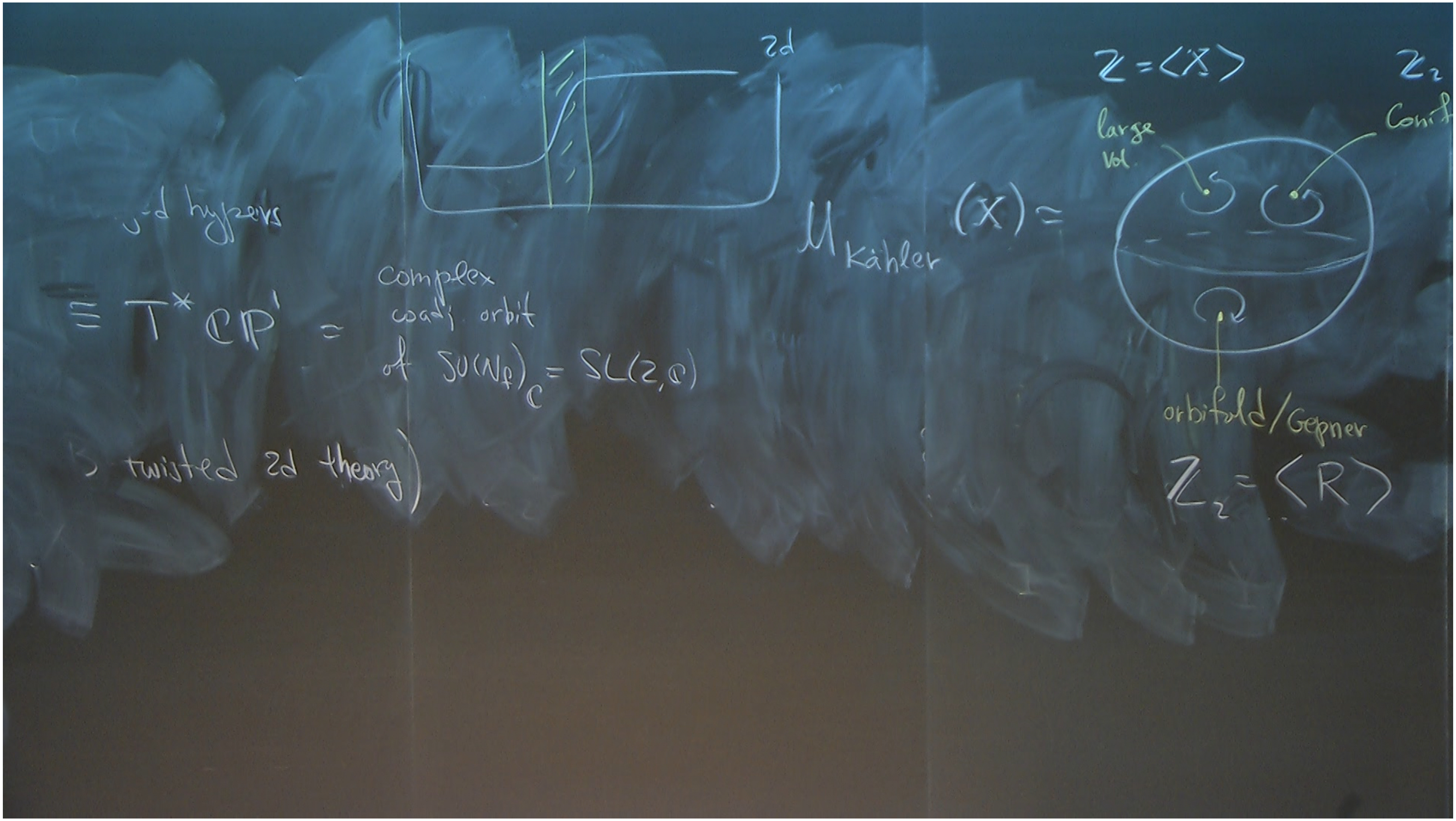
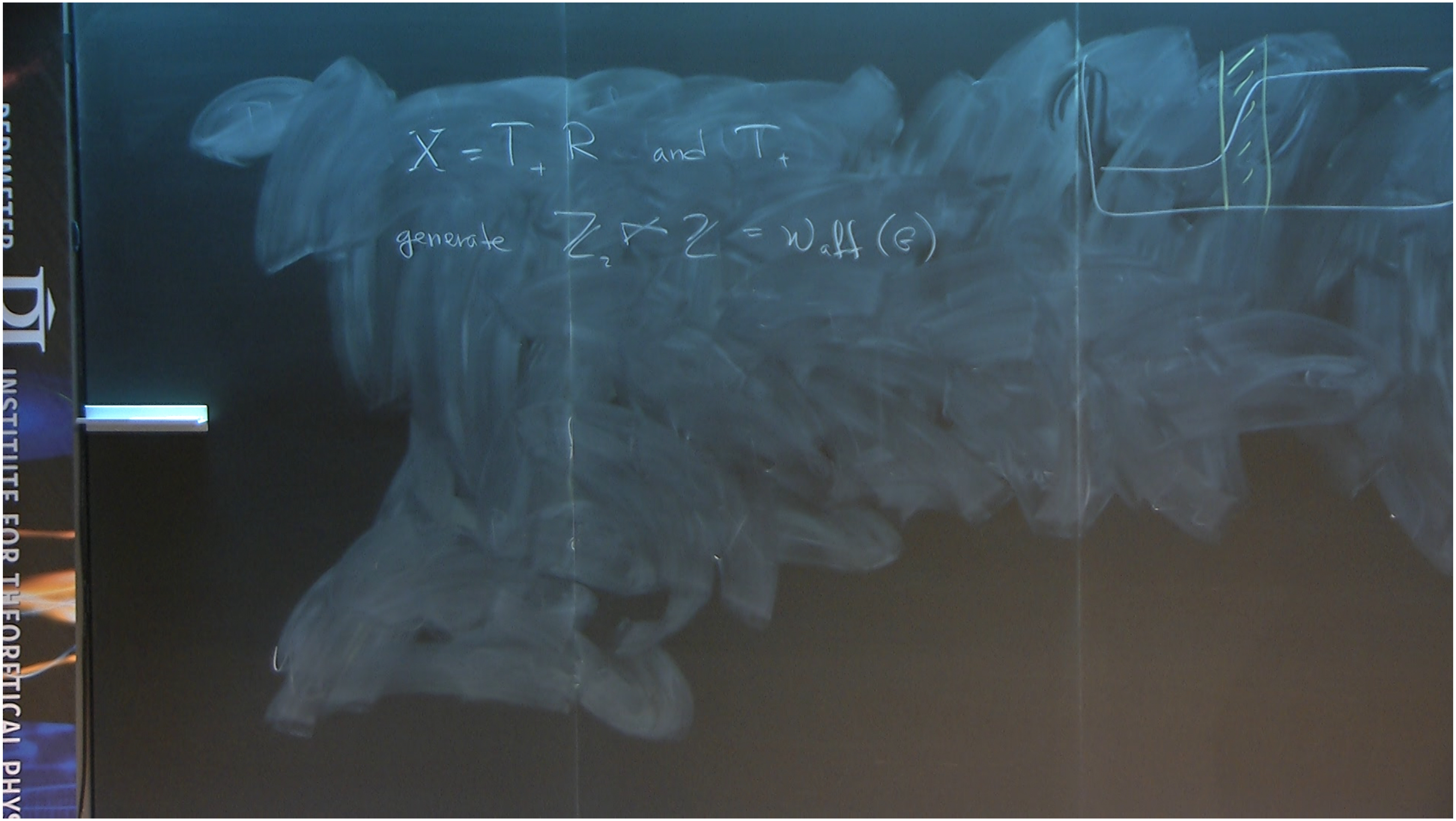
2d $\mathcal{N}=4$ vector $U(1)$
$+ N_f = 2$ charged hypers
$X = \mathbb{H}^2 /\!/\!/ U(1) \equiv T^*\mathbb{CP}^1 =$
complex coadj. orbit
of $SU(N_f)_{\mathbb{C}} = SL(2,\mathbb{C})$
half-BPS (= B-twisted 2d theory)
B-branes on X


$\mathcal{M}_{\text{Kähler}}(X) =$
large Vol
Conifold
orbifold/Gepner


2d
Z = <X>
Z2 = <T+>
Heven (X)
large Vol.
Conifold
D0 D2 D4
MKähler (X) ≃
T+ =
1 0 0
0 -1 0
0 0 1
orbifold/Gepner
Z2 = <R>
= SL(2,C)


2d
Z = ⟨X⟩
Z2
large vol.
Conif
d hypers
≡ T*CP1 = complex coadj. orbit of SU(Nf)C = SL(2,C)
M Kähler (X) =
orbifold/Gepner
twisted 2d theory)
Z2 = ⟨R⟩


X = T+ R and T+
generate Z2 ⋉ Z = Waff (G)
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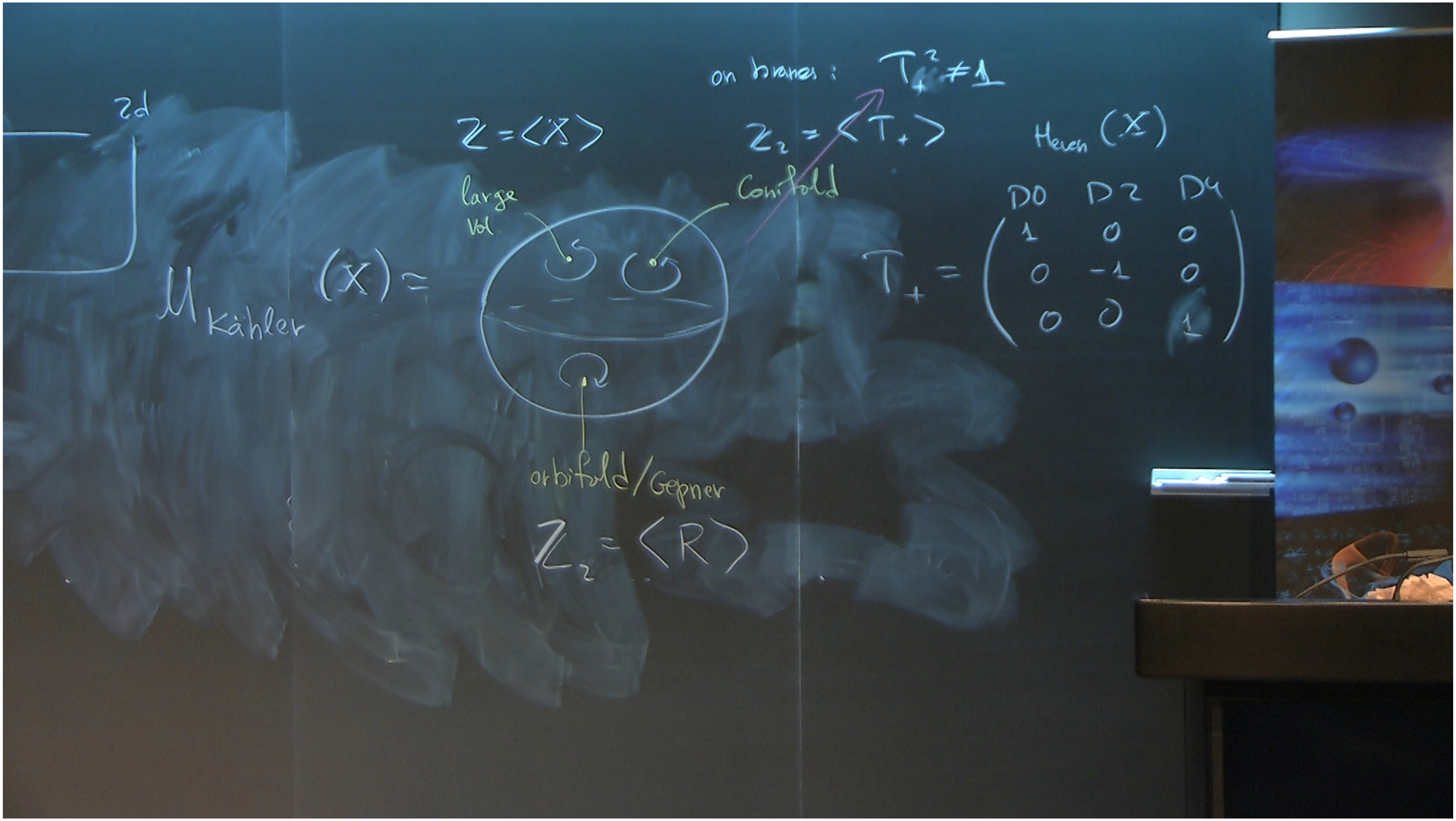
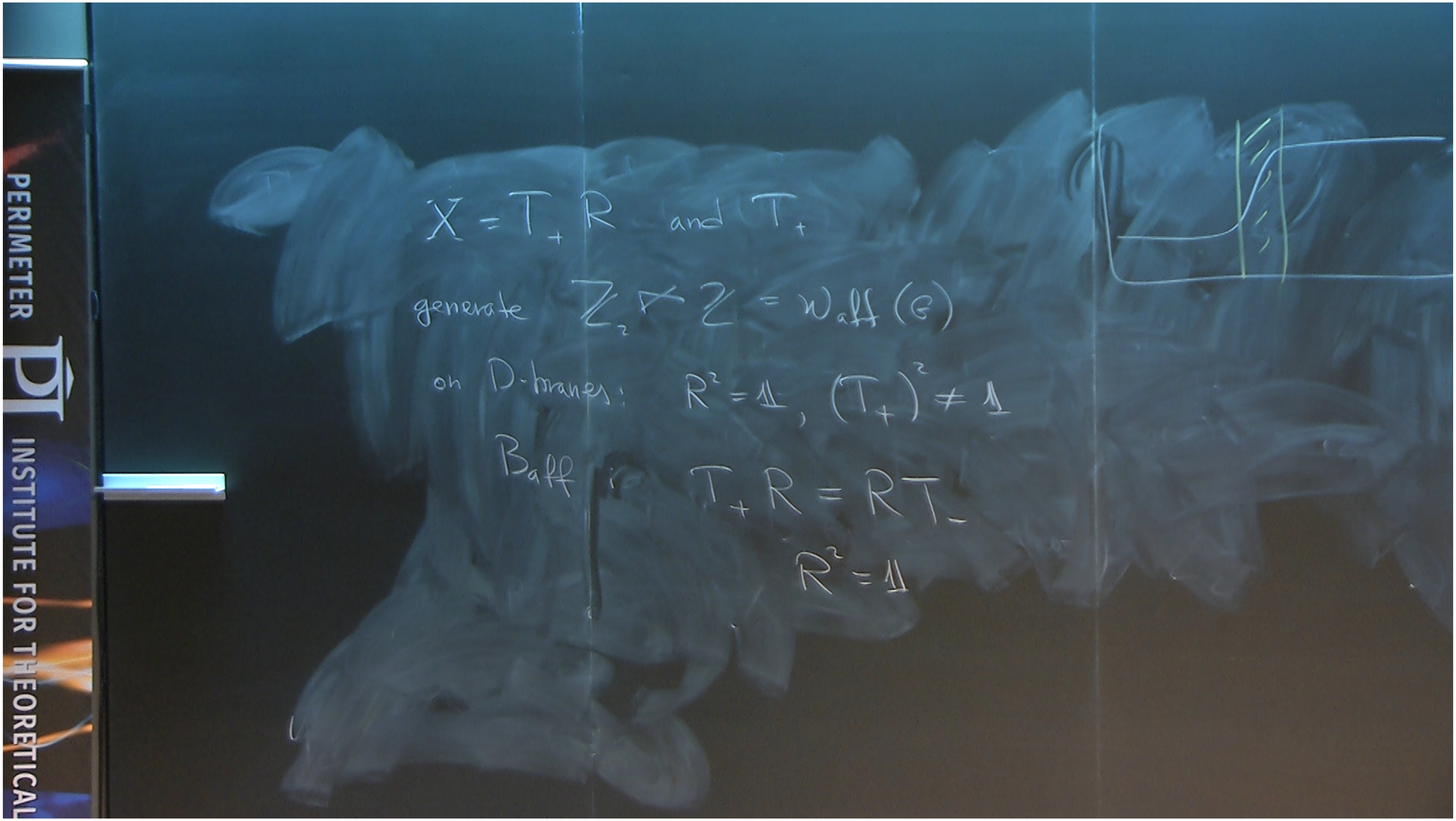

2d
Z = ⟨X⟩
on branes : T+² ≠ 1
Z2 = ⟨T+⟩
Heven (X)
large vol.
Conifold
D0 D2 D4
MKähler (X) =
T+ = (1 0 0 / 0 -1 0 / 0 0 1)
orbifold/Gepner
Z2 = ⟨R⟩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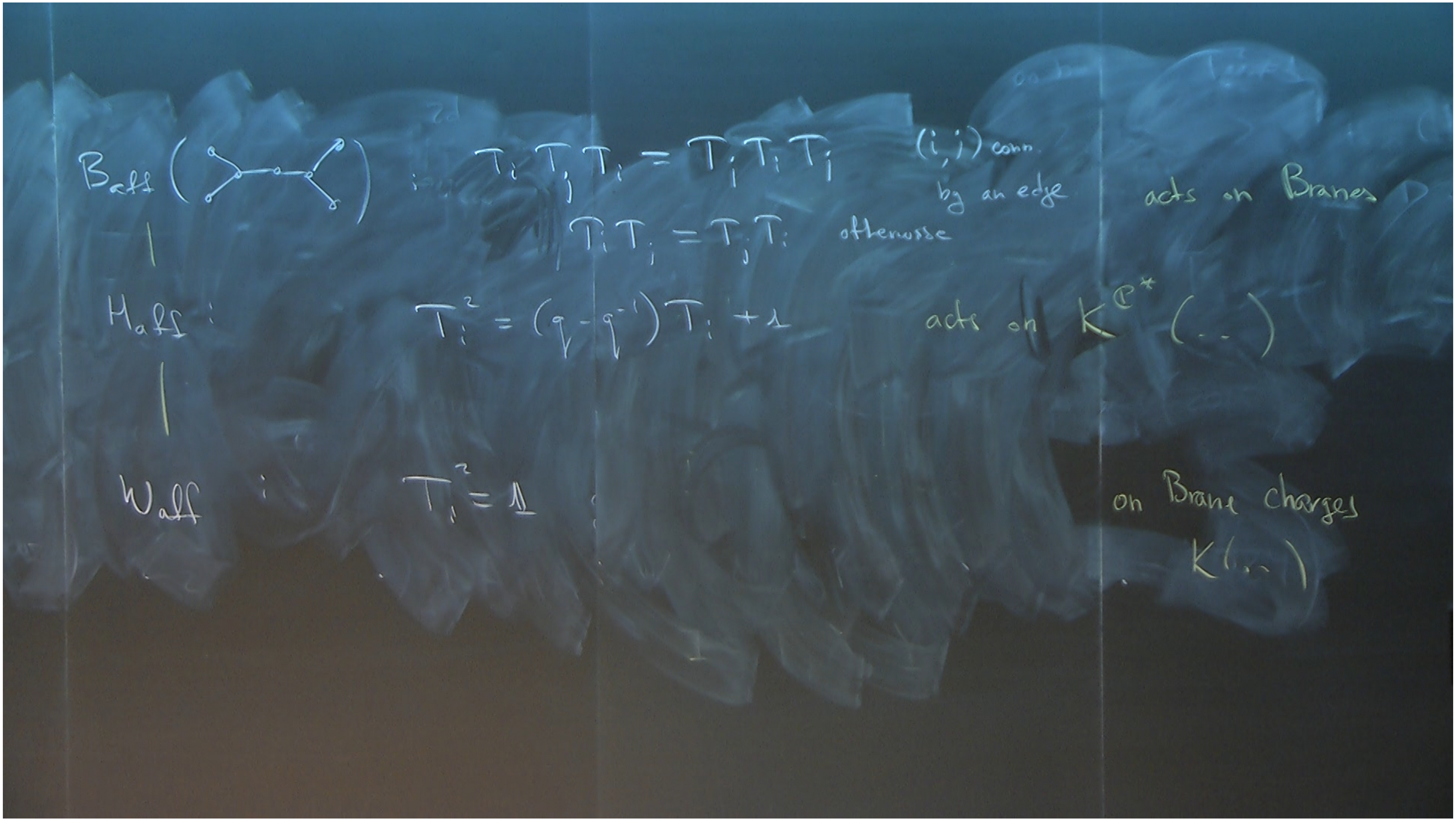
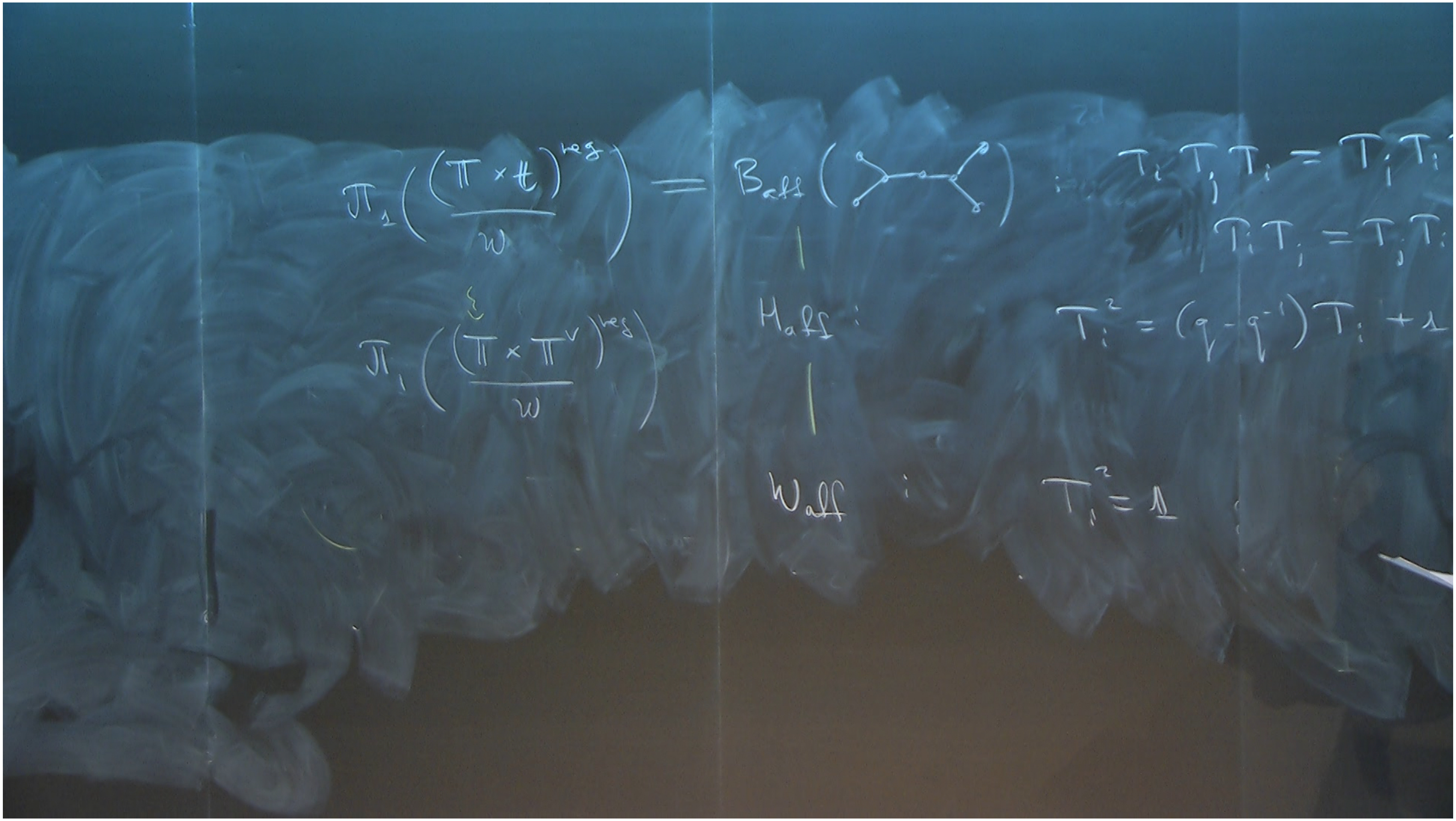
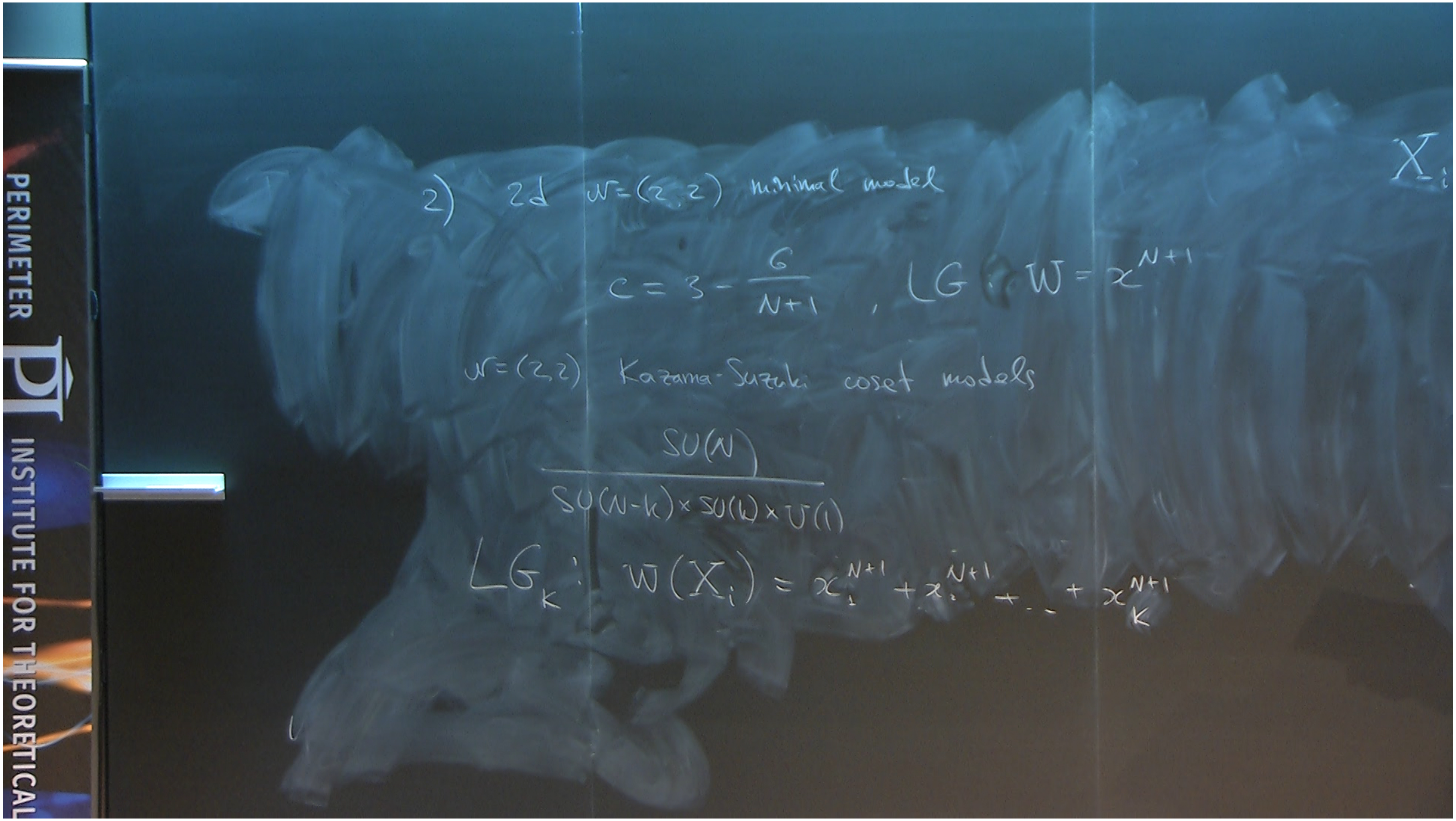
$X = T_+ R$ and T_+
generate $\mathbb{Z}_2 \ltimes \mathbb{Z} = W_{aff}(G)$
on D-branes: $R^2 = 1$, $(T_+)^2 \neq 1$
B_{aff} : $T_+ R = R T_-$
$R^2 = 1$


B_{aff} ( ) : $T_i T_j T_i = T_j T_i T_j$ (i, j) conn. by an edge
$T_i T_j = T_j T_i$ otherwise
acts on Branes
H_{aff} : $T_i^2 = (q - q^{-1}) T_i + 1$
acts on $K^{\mathbb{C}^*}(..)$
W_{aff} : $T_i^2 = 1$
on Brane charges
$K(..)$


π1((T × t)^reg / W) = Baff( ) : Ti Tj Ti = Tj Ti
Ti Tj = Tj Ti
π1((T × T^∨)^reg / W)
Haff : Ti^2 = (q - q^-1) Ti + 1
Waff : Ti^2 = 1


2) 2d N=(2,2) minimal model
c = 3 - 6/(N+1) , LG : W = x^{N+1}
N=(2,2) Kazama-Suzuki coset models
SU(N) / SU(N-k) × SU(k) × U(1)
LG_k : W(X_i) = x_1^{N+1} + x_2^{N+1} + ... + x_k^{N+1}
X_i
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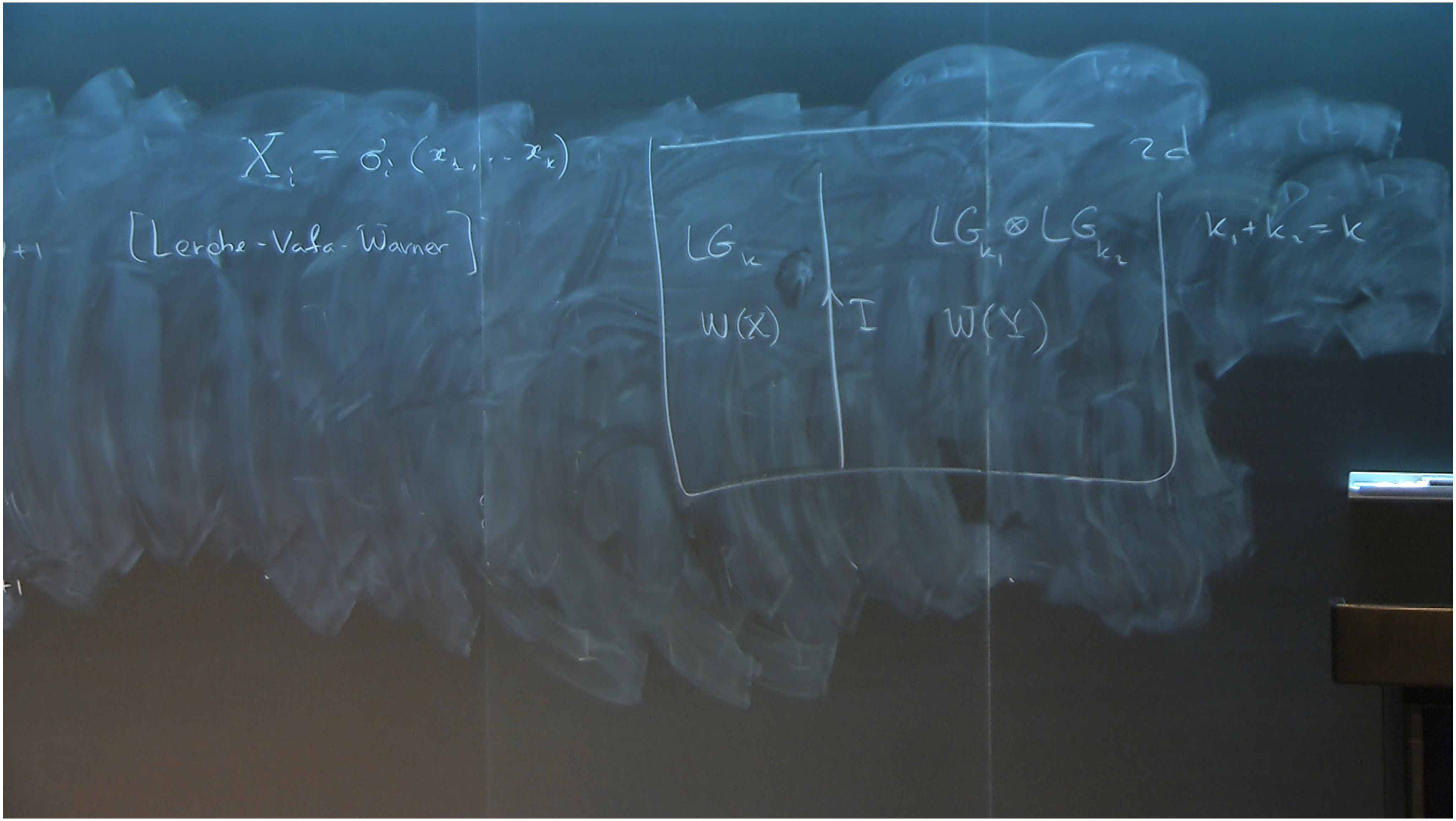
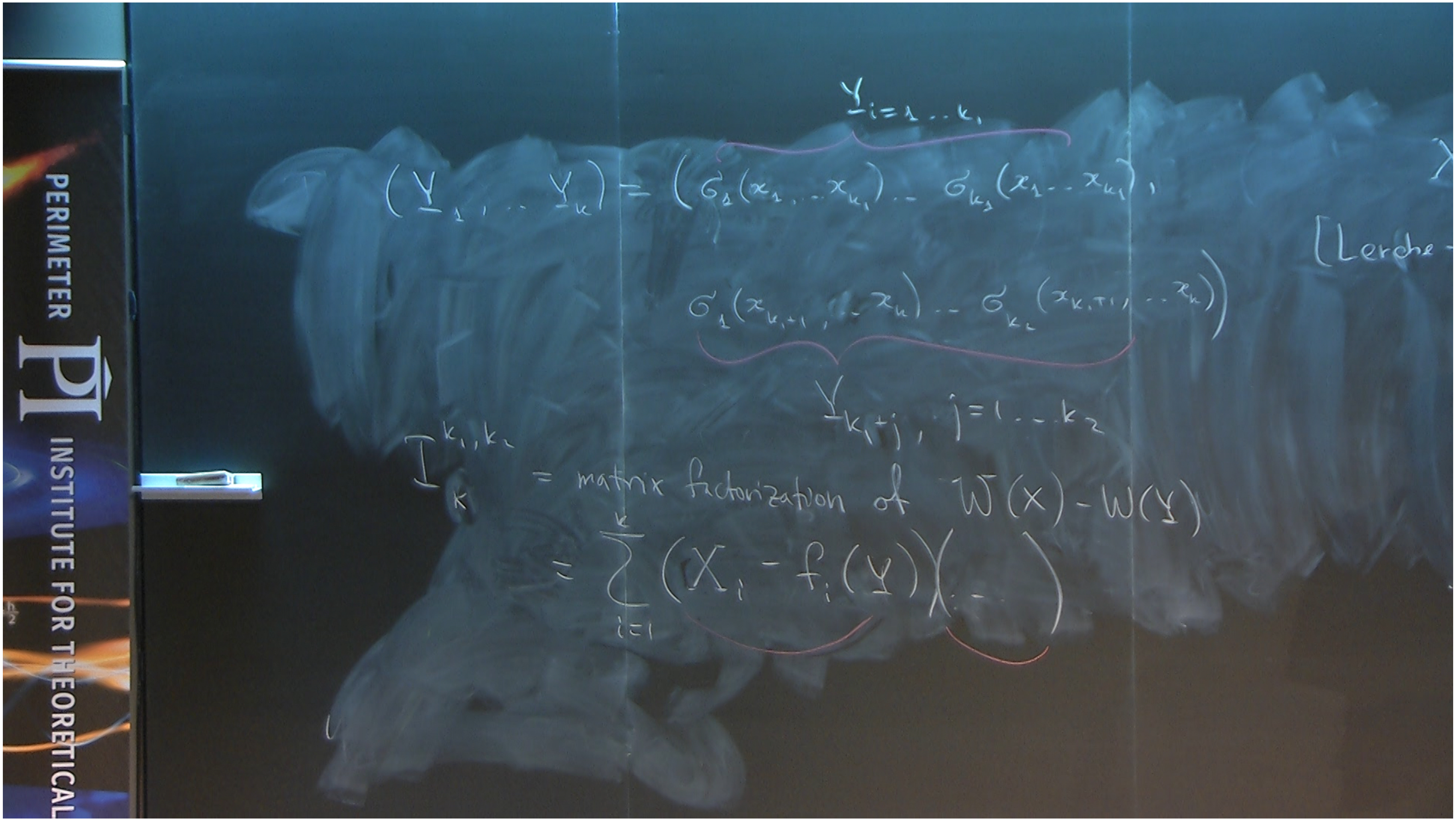

X_i = σ_i(x_1, .. x_k)
[Lerche-Vafa-Warner]
2d
LG_k
LG_k1 ⊗ LG_k2
k_1 + k_2 = k
W(X)
I
W(Y)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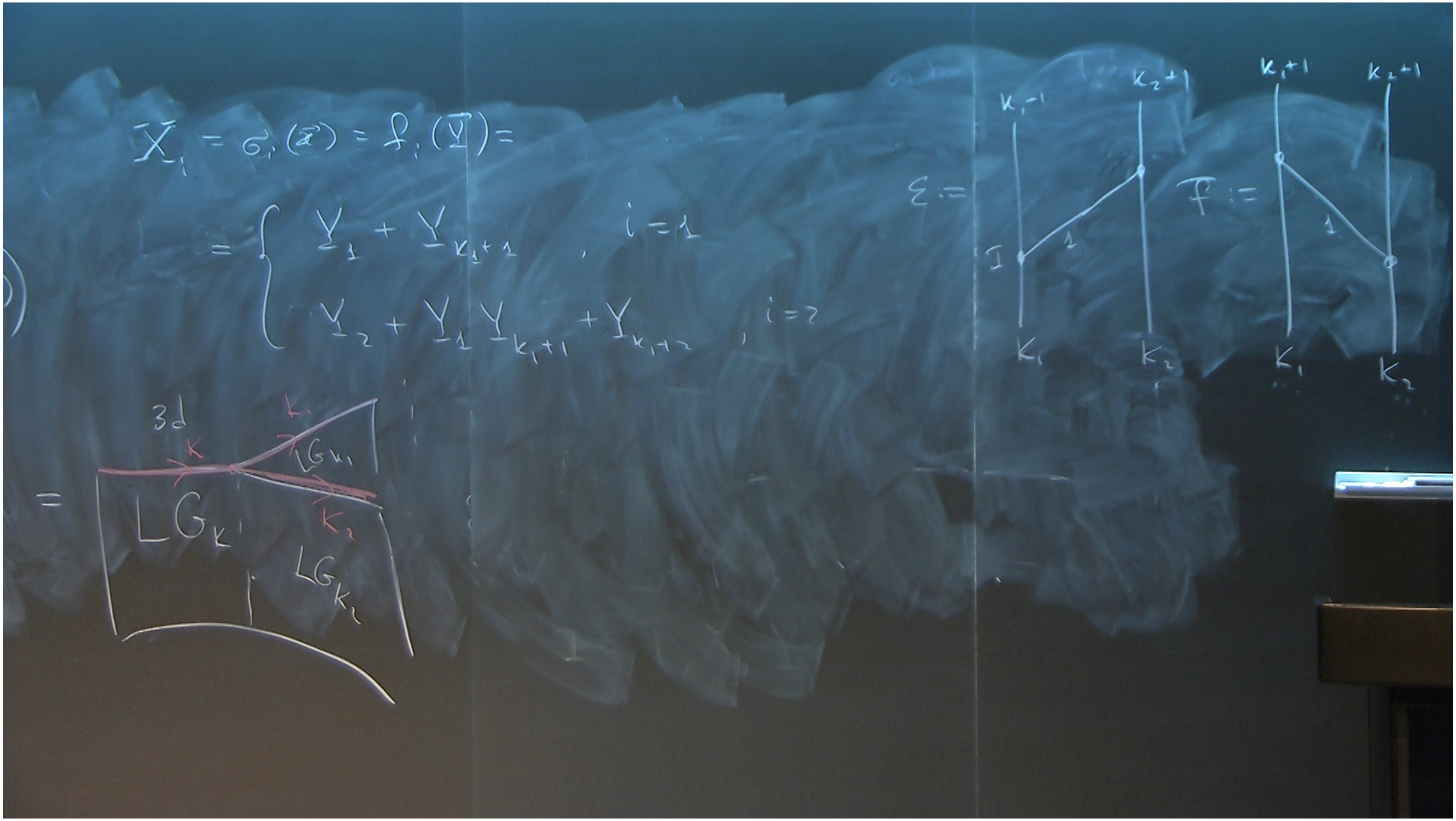
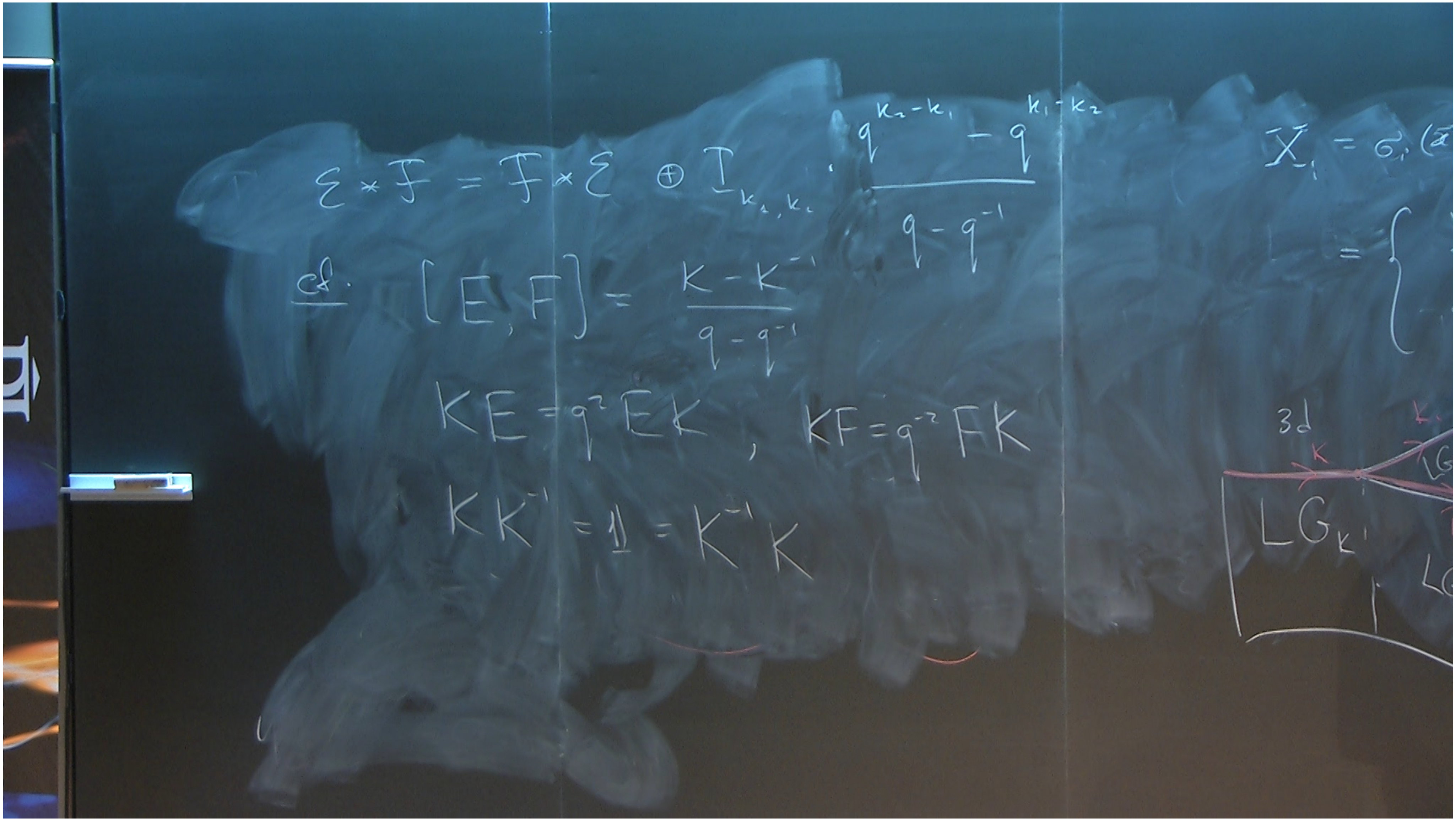
Y_{i=1..k_1}
(Y_1, .. Y_k) = (σ_1(x_1, .. x_{k_1}) .. σ_{k_1}(x_1 .. x_{k_1}),
σ_1(x_{k_1+1}, .. x_k) .. σ_{k_2}(x_{k_1+1}, .. x_k))
Y_{k_1+j}, j=1..k_2
[Lerche
I^{k_1,k_2}_k = matrix factorization of W(X) - W(Y)
= Σ_{i=1}^{k} (X_i - f_i(Y))(..)


X_i = σ_i(x) = f_i(Y) =
= Y_1 + Y_{k_1+1} , i=1
Y_2 + Y_1 Y_{k_1+1} + Y_{k_1+2} , i=2
3d
k
k_1
k_2
LG_k
LG_{k_1}
LG_{k_2}
ε :=
f :=
k_1-1
k_2+1
k_1+1
k_1
k_2
1


ℰ * ℱ = ℱ * ℰ ⊕ I_{k1,k2} · (q^{k2-k1} - q^{k1-k2}) / (q - q^{-1})
cf. [E, F] = (K - K^{-1}) / (q - q^{-1})
KE = q^2 EK , KF = q^{-2} FK
KK^{-1} = 1 = K^{-1}K
X_1 = σ_1 (
3d
K
LG_k