Title: 3d Abelian Gauge theories at the Boundary

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Abstract: A four-dimensional abelian gauge theory can be coupled to a 3d CFT with a U(1) symmetry living on a boundary. This coupling gives rise to a continuous family of boundary conformal field theories (BCFTs) parametrized by the gauge coupling \tau and by the choice of the CFT in the decoupling limit. Upon performing an Electric-Magnetic duality in the bulk and going to the decoupling limit in the new frame, one finds a different 3d CFT on the boundary, related to the original one by Witten's SL(2, Z) action. In particular the cusps on the real \tau axis correspond to the 3d gauging of the original CFT. We study general properties of this family of BCFTs. We show how to express bulk one and two-point functions, and the hemisphere free-energy, in terms of the two-point functions of the boundary electric and magnetic currents. Finally, upon assuming particle-vortex duality (and its fermionic version), we show how to turn this machinery into a powerful computational tool to study 3d gauge theories.

# 3d Abelian Gauge Theories at the Boundary

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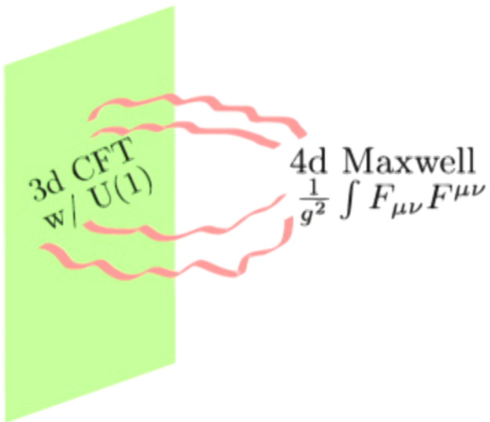
## Overview

3d Abelian Gauge Theories (in their putative IR conformal phase) as conformal boundary conditions for 4d Maxwell




1. Family of conformal boundary conditions $B(\tau, \bar{\tau})$ parametrized by the gauge coupling $\tau$.
2. Decoupling limit: free Maxwell in the bulk + 3d CFT with $U(1)$ global symmetry at the boundary
3. EM duality $\rightsquigarrow$ different decoupling: new CFT with 3d gauge fields coupled to $U(1)$

# Motivations

1. "Simple" example of BCFT. Tools from the CFT side (bootstrap) and tools from the gauge-theory side (action of the EM duality on the boundary theories).
2. CM application: similarity with EFT of graphene [Son], relations with FQHE [Son]
3. In conjunction with 3d dualities: additional computational tool for 3d CFTs, alternative to $\epsilon$ expansion or large $N_f$
   - Energy operator and $F_{S^3}$ of the $O(2)$ model
   - New prediction for $F_{S^3}$ of large $N_f$ $QED_3$

# Free conformal b.c. for Maxwell on $\mathbb{R}^3 \times \mathbb{R}_+$

Coordinates: $x = (x^a, y \geq 0)$, $F = dA$

$$S = -\frac{i}{8\pi}\int_{y\geq 0} \mathrm{d}^4x \left[\tau (F^-)^2 - \bar{\tau}(F^+)^2\right], \quad \tau = \frac{\theta}{2\pi} + \frac{2\pi i}{g^2} ,$$

b.c. obtained from vanishing of the boundary term

$$\delta S_\partial \propto \int_{y=0} \mathrm{d}^3x \; \delta A^a (\tau F^-_{ya} - \bar{\tau} F^+_{ya})\big|_{y=0} .$$

In terms of the boundary currents

$$\boxed{2\pi i\, \hat{J}_a \equiv (\tau F^-_{ya} - \bar{\tau} F^+_{ya})\big|_{y=0} , \quad 2\pi i\, \hat{I}_a \equiv (F^-_{ya} - F^+_{ya})\big|_{y=0} .}$$

Conformal boundary conditions

$$\begin{array}{lll} \text{Dirichlet} & \hat{J}_a = \text{free} & \hat{I}_a = 0 , \\ \text{Neumann} & \hat{J}_a = 0 & \hat{I}_a = \text{free} , \end{array}$$

The BCFT is just a trivial MFT for the non-zero boundary current.

# EM duality action on free b.c

$SL(2,\mathbb{Z})$ duality group $\tau \to \frac{a\tau+b}{c\tau+d}$, $a,b,c,d \in \mathbb{Z}$ s.t. $ad-bc=1$ induces action on b.c, since

$$\begin{pmatrix} \hat{J} \\ \hat{I} \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \hat{J} \\ \hat{I} \end{pmatrix}$$

most general free conformal b.c: "$(p,q)$"

$$p\hat{J}_a + q\hat{I}_a = 0 \ , p,q \in \mathbb{Z}$$

The BCFT is just a trivial MFT for the non-zero boundary current.

Requires additional topological dof on the boundary
[Witten][Gaiotto-Witten][Tikhonov-Kapustin]

# Interacting b.c. for Maxwell on $\mathbb{R}^3 \times \mathbb{R}_+$

Start with a 3d CFT with a $U(1)$ global symmetry at the boundary and 4d Maxwell with Neumann b.c.

Around $\tau = \infty$ couple them by "weakly" gauge $U(1)$ with the boundary value $A_a$ of the 4d gauge field

$$\int_{y=0} \mathrm{d}^3 x \ \hat{J}^a_{\mathrm{CFT}} A_a + \text{seagulls} \ .$$

Due to edge modes: " Modified Neumann b.c"

$$\hat{J}^a = \hat{J}^a_{\mathrm{CFT}}$$

Defines interacting set of correlators at the boundary: 2pt functions of $\hat{J}^a, \hat{I}^a$ are non-trivial functions of $\tau, \bar{\tau}$.

At $\tau = \infty$ $\hat{I}_a$ decouples and we recover the original 3d CFT + MFT of $\hat{I}_a$

## A family of BCFTs

Conformal b.c.: make sure this coupling preserves the boundary conformal symmetry:

- Gauge coupling $\tau$ is the coefficient of a bulk operator. Boundary interactions cannot renormalise it (Locality)

  As interactions localized on the boundary, $\tau$ is exactly marginal. Two loop check [Teber] and more general argument based on Ward identities [Herzog-Huang][Dudal-Mizher-Pais]
- Assume we can set the boundary couplings to their critical values$^{(*)}$

$$\Longrightarrow \text{Continuous family of BCFTs } B(\tau, \bar{\tau})$$

$^{(*)}$ Possible obstructions as we move in $\tau$ plane: emergence of a condensate, boundary operator crossing marginality,...

"Integrate" out the bulk: $B(\tau, \bar{\tau})$ is a conformal manifold of 3d CFTs, generically without a stress tensor

# Decoupling Limits

Approach a "local" 3d CFT when bulk decouples

- The " original" decoupling:

$$\tau \to \infty, \quad B(\tau,\bar\tau) \to \hat{I}_a \text{ MFT } + \underbrace{\text{3d CFT}}_{T_{0,1}}$$

- Cusp on the real axis: $\tau \to -\frac{q}{p}$, $q, p \in \mathbb{Z}$ (this is $\tau' \to \infty$)

$$B(\tau,\bar\tau) \to (p\hat{J}_a + q\hat{I}_a) \text{ MFT } + \underbrace{\text{3d CFT'}}_{T_{p,q}}$$

In general $T_{p,q}$ can be obtained from $T_{0,1}$ by $SL(2,\mathbb{Z})$ action on 3d CFTs with a $U(1)$ symmetry [Witten]

$$S : \tau \to -\frac{1}{\tau} \qquad \text{3d gauging } U(1) \ \& \ U(1)' = U(1)_{\text{top}}$$

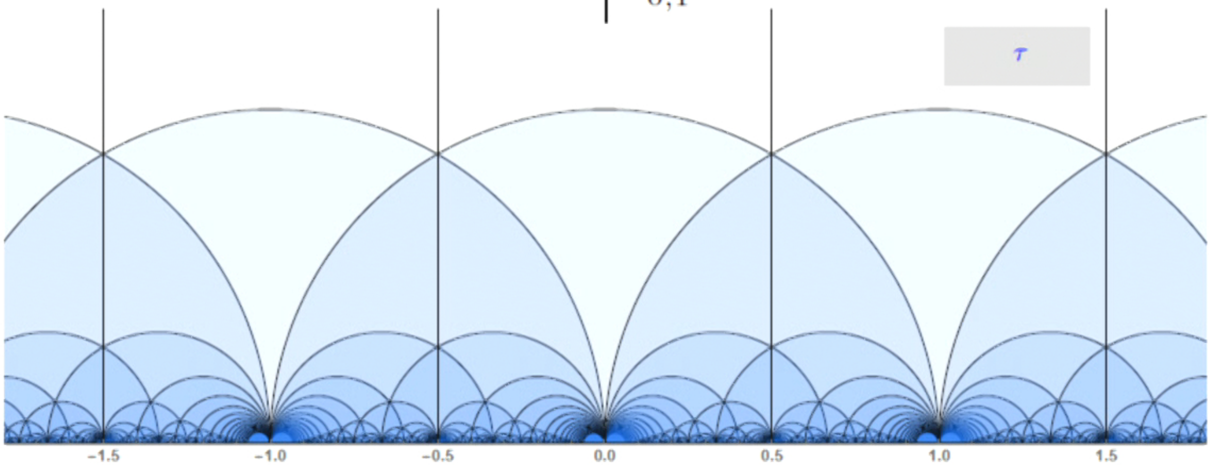
$$T : \tau \to \tau + 1 \qquad \text{CS contact term for the global } U(1)$$

$\rightsquigarrow$ $T_{p,q}$ is a 3d Abelian gauge theory with a $U(1)$ global symmetry

# Application

$B(\tau, \bar{\tau})$ interpolates between data of $\infty$-ly many 3d gauge theories $T_{p,q}$ with a $U(1)$ global symmetry.

$\uparrow T_{0,1}$

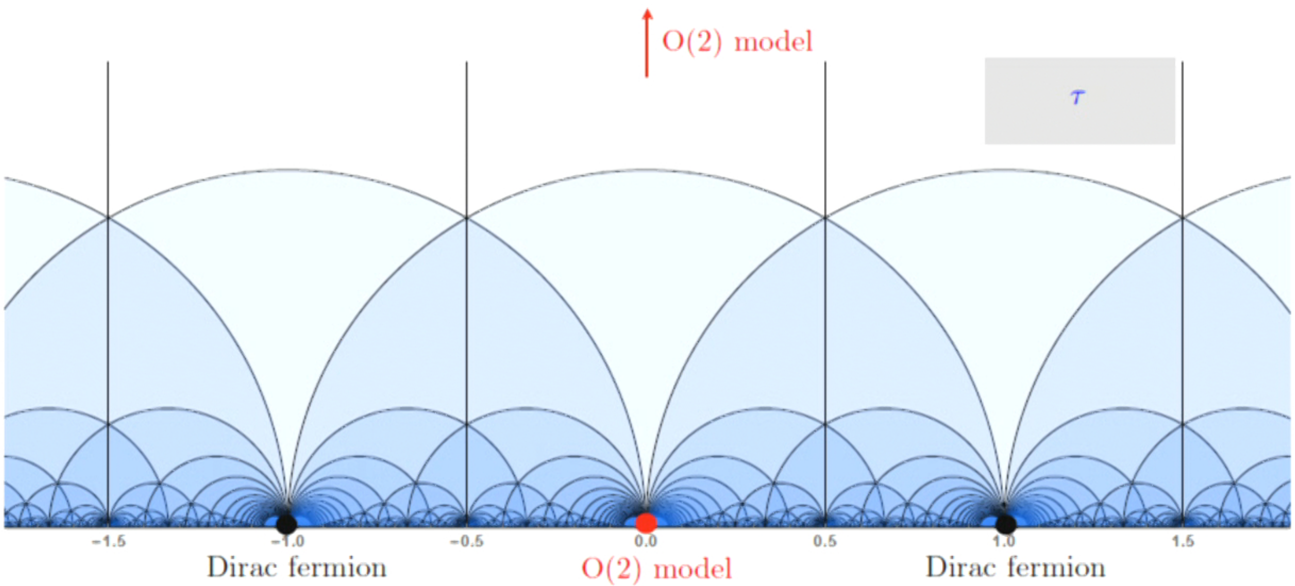



$$T_{p,q} \ , \quad (p,q) \neq (0,1)$$

If $T_{0,1}$ is known $\rightsquigarrow$ conformal perturbation theory around $\tau = \infty$ and extrapolate to $\tau = -\frac{q}{p}$ to compute data of $T_{p,q}$.

# The Best Case scenario

$$U(1) + \phi \longleftrightarrow \phi$$
$$U(1)_{\pm 1} + \phi \longleftrightarrow \psi$$




Web of 3d dualities from different decoupling limits on $B(\tau, \bar{\tau})$

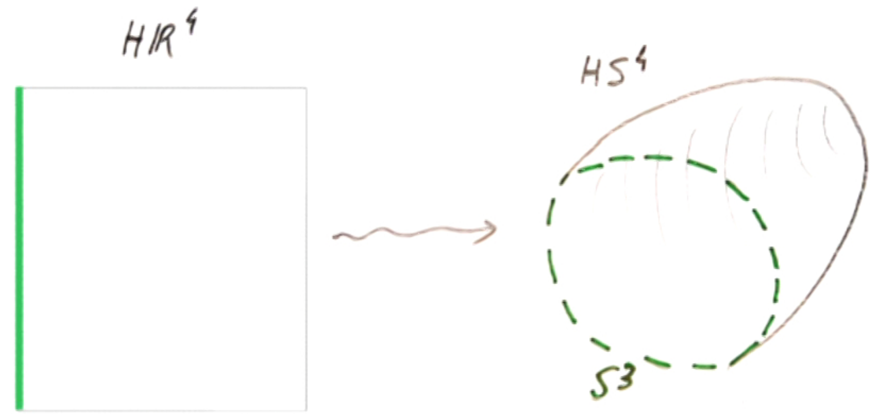
[Wang-Senthil][Seiberg-Senthil-Wang-Witten][Metlitski-Vishwanath][Hsiao-Son]

## Observables

- Scaling Dimensions / OPE coefficients of local operators on the boundary
- Current Central Charges [Hsiao-Son][Teber-Kotikov]
- Boundary Anomalies [Herzog-Huang][Herzog-Huang-Jensen]
- Boundary Free Energy (next slide)
- Endpoints of Bulk Wilson Lines

# Boundary Free Energy

$$F_\partial = -\tfrac{1}{2}\log\frac{|Z_{HS^4}|^2}{Z_{S^4}} = -\mathrm{Re}\log Z_{HS^4} + \tfrac{1}{2}\log Z_{S^4}$$ [Gaiotto]

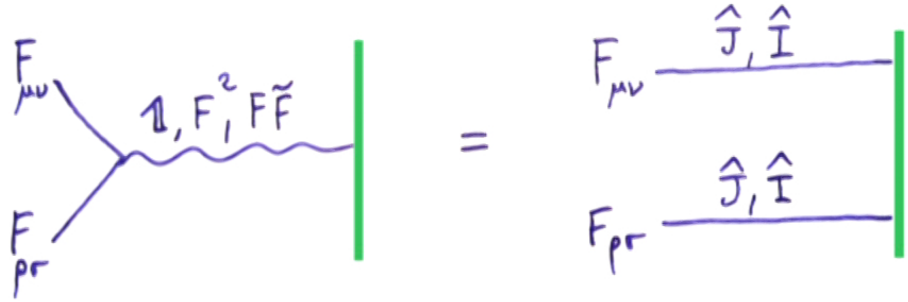



Similar to $F^{\mathrm{CFT}}_{S^3}$ (conjecturally) monotonic under boundary RG flow. In our set-up we find

$$\boxed{\frac{\partial F_\partial}{\partial \mathrm{Im}\tau} = \frac{\pi}{6}\, a_{F^2}(\tau,\bar\tau) \quad \frac{\partial F_\partial}{\partial \mathrm{Re}\tau} = \frac{\pi}{6}\, i\, a_{F\tilde{F}}(\tau,\bar\tau)}$$

Since in any BCFT $\langle \mathcal{O}(\vec{x},y)\rangle = a_{\mathcal{O}}\, y^{-\Delta_{\mathcal{O}}}$

$a_{F^2}, a_{F\tilde{F}}$ completely determined by boundary currents 2pt functions via a boundary bootstrap [Liendo-Rastelli-van Rees] reasoning




$$\boxed{a_{F^2} = 3(\pi^2 c_{\hat{I}\hat{I}} - \frac{1}{2\pi\,\mathrm{Im}\tau}), \quad a_{F\tilde{F}} = i\frac{3\pi^2}{\mathrm{Im}\tau}(c_{\hat{I}\hat{J}} - \mathrm{Re}\tau\; c_{\hat{I}\hat{I}}),}$$

$F_\partial$ fixed by current 2pt functions $\hat{I}$, $\hat{J}$ and an initial condition

## Boundary free energy and EM duality

The decoupling limit fixes the initial condition

$$F_\partial \underset{\tau\to\infty}{\sim} \underbrace{-\frac{1}{4}\log\left[\frac{2\,\mathrm{Im}\tau}{|\tau|^2}\right]}_{\text{4d free vector}} + \underbrace{F^{\mathsf{CFT}}_{0,1}}_{S^3\text{part funct}} + \mathcal{O}(|\tau|^{-1})\ .$$

Around the gauged cusp $\tau = -q/p$, $\tau' = \frac{p'\tau+q'}{p\tau+q}$

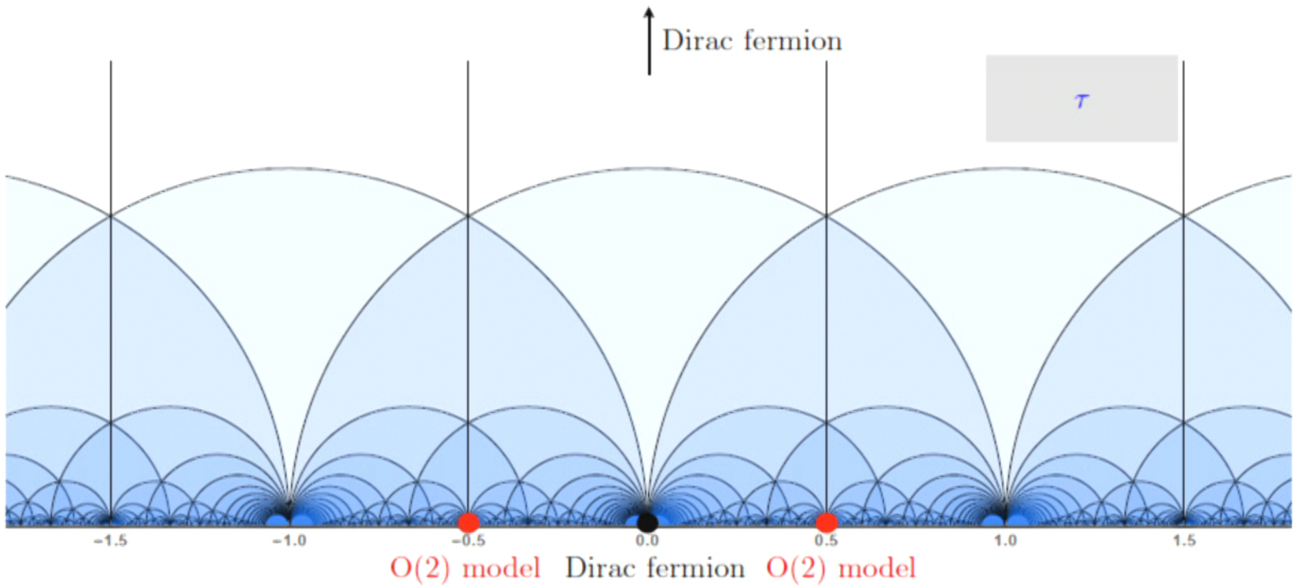
$$F_\partial \underset{\tau'\to\infty}{\sim} -\frac{1}{4}\log\left[\frac{2\,\mathrm{Im}\tau'}{|\tau'|^2}\right] + F^{\mathsf{CFT}}_{p,q} + \mathcal{O}(|\tau'|^{-1})\ .$$

Same singularity in terms of $\tau'$, but different finite piece

Compute $F_\partial$ perturbatively, extrapolate to the gauged cusp, subtract the free-vector contribution in the new cusp

Application: $O(2)$ model and large $N_f$ $QED_3$ from free fermions

# From free Dirac to the $O(2)$ model




Shift in $\tau$ s.t. the Dirac fermion is T-invariant

Perturbation theory around $\tau = \infty$ and $\tau = 0$ and extrapolation to $\tau = 1/2$ to get observables of the $O(2)$ model

## Anomalous dimension of $\bar{\psi}\psi$

$\bar{\psi}\psi$ is a good operator around the Dirac cusp. It will be related to the energy operator at the of the $O(2)$ model

Dimreg ($d = 3 - 2\epsilon$ with fixed codimension). Non-local photon propagator between two points on the boundary

$$\langle A_a(\vec{p},0)A_b(-\vec{p},0)\rangle = 2\pi \frac{\mathrm{Im}\tau}{|\tau|^2}\left[\frac{\delta_{ab}}{|\vec{p}|} + \frac{\mathrm{Re}\tau}{\mathrm{Im}\tau}\epsilon_{abc}\frac{p^c}{\vec{p}^2}\right] .$$

$$\boxed{\gamma_{\bar{\psi}\psi} = -\frac{8}{3\pi}\frac{\mathrm{Im}\tau}{|\tau|^2} + \frac{36\pi^2 - 32}{27\pi^2}\frac{(\mathrm{Im}\tau)^2}{|\tau|^4} - \frac{8}{3}\frac{(\mathrm{Re}\tau)^2}{|\tau|^4} + \mathcal{O}(|\tau|^{-3})}$$

## Boundary Free Energy

$F_\partial$ is completely determined in terms of the current central charges $c_{\hat{I}\hat{I}}, c_{\hat{I}\hat{J}}, c_{\hat{J}\hat{J}}$

The relevant diagrams for NLO were computed by [Klebanov-Giombi-Tarnopolsky],

$$c_{\hat{J}\hat{J}} = \frac{1}{8\pi^2} + \frac{368 - 45\pi^2}{576\pi^3}\frac{\mathrm{Im}\tau}{|\tau|^2} + \mathcal{O}(|\tau|^{-2}) \ ,$$

$$F_\partial = -\frac{1}{4}\log\left[\frac{2\,\mathrm{Im}\tau}{|\tau|^2}\right] + F_{\mathrm{Dirac}} + \frac{\pi}{16}\frac{\mathrm{Im}\tau}{|\tau|^2} + \frac{(368-45\pi^2)(\mathrm{Im}\tau)^2 + (144+45\pi^2)(\mathrm{Re}\tau)^2}{2304|\tau|^4} + \mathcal{O}(|\tau|^{-3})$$

# Extrapolation to $O(2)$

Duality-improved Padé approximant at 2 loops [Beem-Rastelli-Sen-van Rees] manifestly invariant under $\tau \leftrightarrow \mathrm{S}\tau$

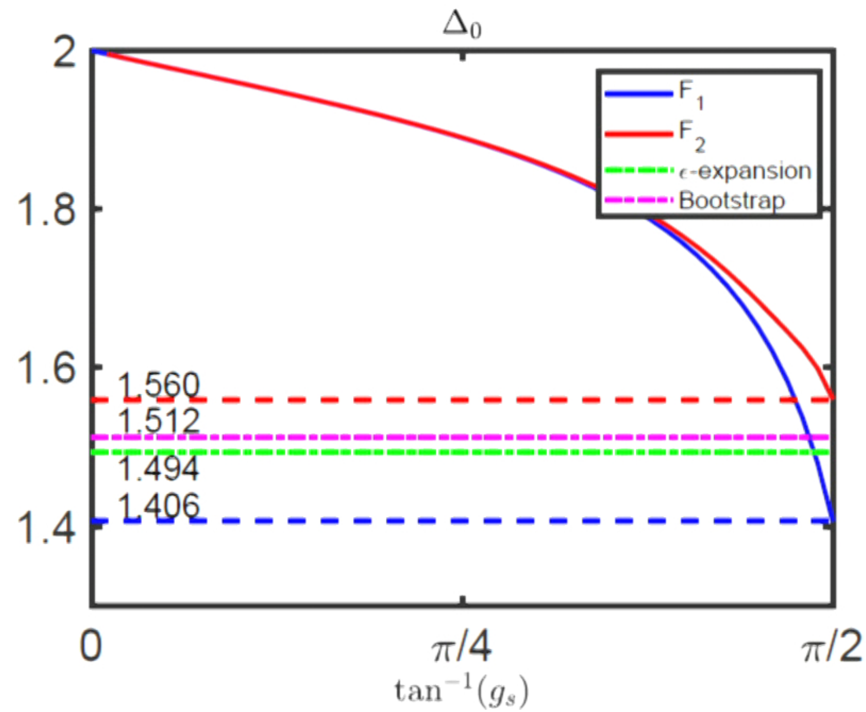
$$F_1(g_s, \theta) = \frac{h_1}{g_s^{-1} + (\mathrm{S} \cdot g_s)^{-1} - h_2} \,,$$

$$F_2(g_s, \theta) = \frac{h_3 \left( g_s^{-1/2} + (\mathrm{S} \cdot g_s)^{-1/2} \right)}{g_s^{-3/2} + (\mathrm{S} \cdot g_s)^{-3/2} + h_4 \left( g_s^{-1/2} + (\mathrm{S} \cdot g_s)^{-1/2} \right)} \,.$$

with $g_s = g^2$ and

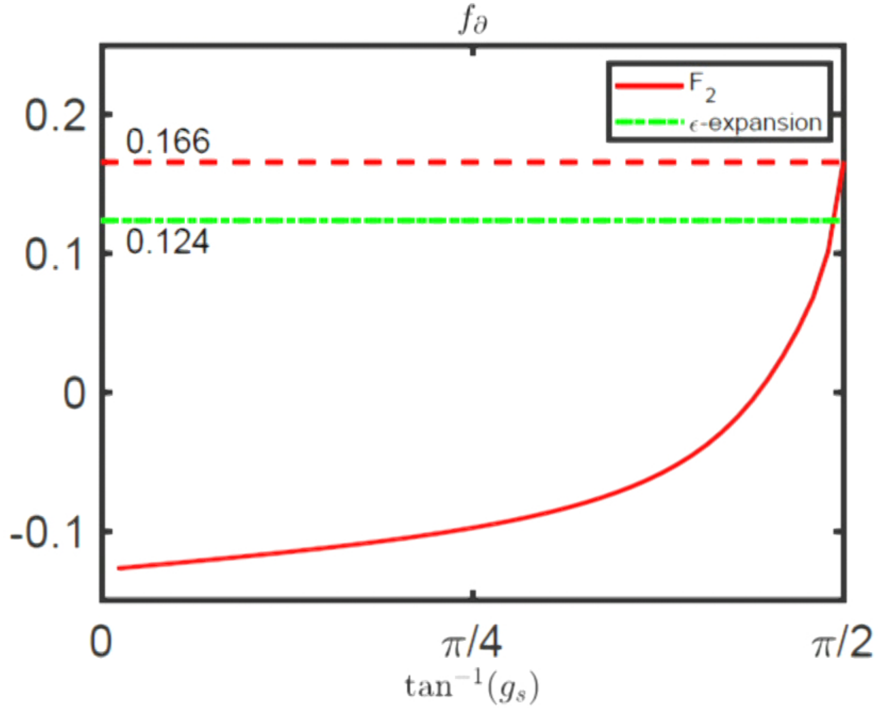
$$\mathrm{S} \cdot g_s = \frac{g_s^2 \theta^2 + 16\pi^4}{\pi^2 g_s} \,.$$

# Energy operator of the $O(2)$ model




Comparison with $\epsilon$-expansion and bootstrap predictions (at $\tan^{-1}(g_s) = \pi/2$) [Kos,Poland,Simmons-Duffin, Vichi][Kleinert,Neu,Schulte-Frohlinde,Chetyrkin,Larin]

# $S^3$ Free energy in $O(2)$




Comparison with $4-\epsilon$ -expansion $\mathcal{O}(\epsilon^5)$ predictions (at $\tan^{-1}(g_s) = \pi/2$)[Fei-Giombi-Klebanov-Tarnopolsky]

# Large $N_f$ QED$_3$

$2N_f$ free Dirac fermions (with same charge) at the boundary and take large $N_f$ with $\lambda = g^2 N_f$ fixed.

Compute $F_\partial$ exactly in the 't Hooft coupling $\lambda$

By Witten's $SL(2,\mathbb{Z})$ action, $\lambda \to \infty$ should correspond to large $N_f$ $QED_3$.

We recover [Klebanov-Pufu-Sachdev-Safdi] and predict $O(N_f^{-1})$

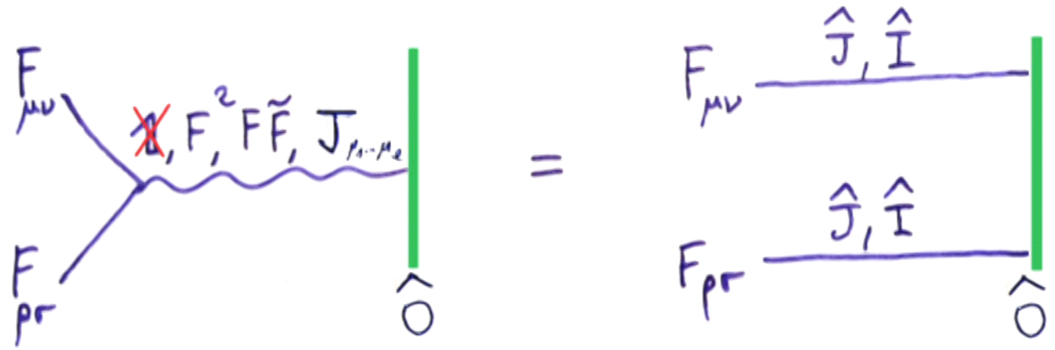
$$F_{\mathrm{QED}_3} = 2N_f F_{\mathrm{Dirac}} + \frac{1}{2}\log\left(\frac{\pi N_f}{4}\right) + \frac{92 - 9\pi^2}{18\pi^2}\frac{1}{N_f} + \mathcal{O}\left(N_f^{-2}\right) \ .$$

Non-perturbative test (in $g$) of the this construction

# A Bootstrap perspective

Considered a continuous family of interacting b.c. for Maxwell theory. Some universal features:

- EoM & Bianchi $\Rightarrow$ boundary conserved currents $\hat{J}_a, \hat{I}_a$
- Bulk is free $\Rightarrow$ relations between $\langle \hat{I}\hat{I}\ldots\rangle, \langle \hat{I}\hat{J}\ldots\rangle, \langle \hat{J}\hat{J}\ldots\rangle$. Constraint on bdry theories!



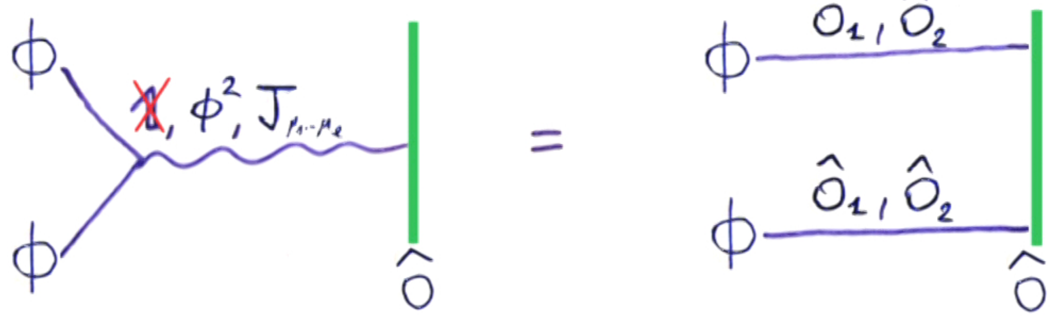

$$\boxed{\rightsquigarrow \lambda_{\hat{I}\hat{I}\hat{O}} = A(\hat{O})\lambda_{\hat{I}\hat{J}\hat{O}}, \ \lambda_{\hat{J}\hat{J}\hat{O}} = B(\hat{O})\lambda_{\hat{I}\hat{J}\hat{O}}}$$

- Boundary bootstrap of mixed correlators of $\hat{J}_a, \hat{I}_a$ subjected to these relations.

## Similar problem (WIP)

Space of conformal b.c. for a free scalar $\phi$ in $d$-dimensions. Some universal features:

- EoM $\Rightarrow$ protected operators $\widehat{O}_1 = \phi|_\partial$, $\widehat{O}_2 = \partial_\perp \phi|_\partial$
- Bulk is free $\Rightarrow$ relations between correlators $\langle \widehat{O}_1 \widehat{O}_1 \ldots \rangle, \langle \widehat{O}_1 \widehat{O}_2 \ldots \rangle, \langle \widehat{O}_2 \widehat{O}_2 \ldots \rangle$. Constraint on bdry theories!




$$\boxed{\rightsquigarrow \lambda_{11\widehat{O}} \lambda_{22\widehat{O}} = \frac{(\widehat{\Delta} - 2)(\cos(\pi \widehat{\Delta}) + 1)}{(\widehat{\Delta} - 1)(\cos(\pi \widehat{\Delta}) - 1)} \lambda^2_{12\widehat{O}}, \quad (d = 4)}$$

- Numerical bootstrap of mixed correlators of $\widehat{O}_1$, $\widehat{O}_2$ subjected to these relations.

## Conclusions and Directions

- Explored the space of conformal b.c. a free 4d $U(1)$ gauge theory
- In the absence of phase transition we can approach the data of an infinite family of 3d abelian gauge theories
- 3d dualities + perturbation theory + improved Padé resummation $\rightarrow$ new computational tool

Some directions

- More observables of the $O(2)$ model/higher loops/resummation
- Bootstrap perspective: space of conformal b.c. for the free vector? Space of conformal b.c. for a free scalar? (WIP)
- Away from free theory in the bulk?

Thanks!