Title: Weyl Anomaly Induced Current and Holography

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Collection: Boundaries and Defects in Quantum Field Theory

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Abstract: We show that when an external magnetic field parallel to the boundary is applied, Weyl anomaly give rises to a new anomalous current transport near the boundary. Similar to the Casimir effect, this transport phenomena has its origin in the effect of the boundary on the quantum fluctuations of the vacuum. The near-boundary current takes universal form for general boundary quantum field theories, which are covariant, gauge invariant, unitary and renormalizable. We verify the universal law of current by studying free QFT and holographic BCFT. Finally, we discuss the so-called "divergence" problem and show that it can be resolved by the law of conservation of charge.

Weyl Anomaly Induced Current and Holography

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collaborated with Prof. Chong-Sun Chu

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Perimeter Institute, 6 August, 2019

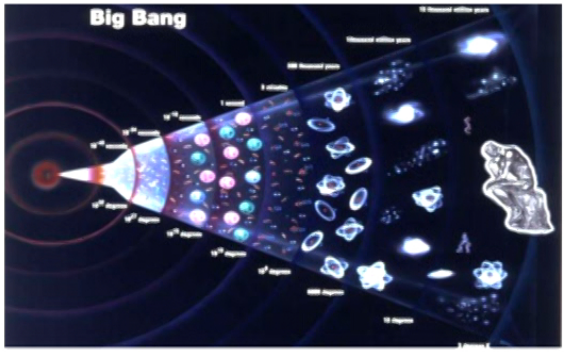
Outline

1. Background
   - Why BCFT/dCFT?
   - Review of Weyl anomaly
2. Main Results
   - Current from Weyl anomaly
   - Test by free BQFT
   - Test by holographic BCFT
3. Comments
   - Finite total current
   - Physical picture

Motivation

Since many physical systems have boundaries, it is interesting to study the boundary effects of quantum systems.

- Casimir effects
- Topological Insulator
- Big Bang of the universe
  It implies that there is a boundary of time.
- Cosmological horizon is also a kind of boundary.



BCFT

- Definitions
  BCFT is a conformal field theory defined on a manifold M with a boundary P, where suitable boundary conditions are imposed.
- Example of free BCFT
  - Conformal free scalar field

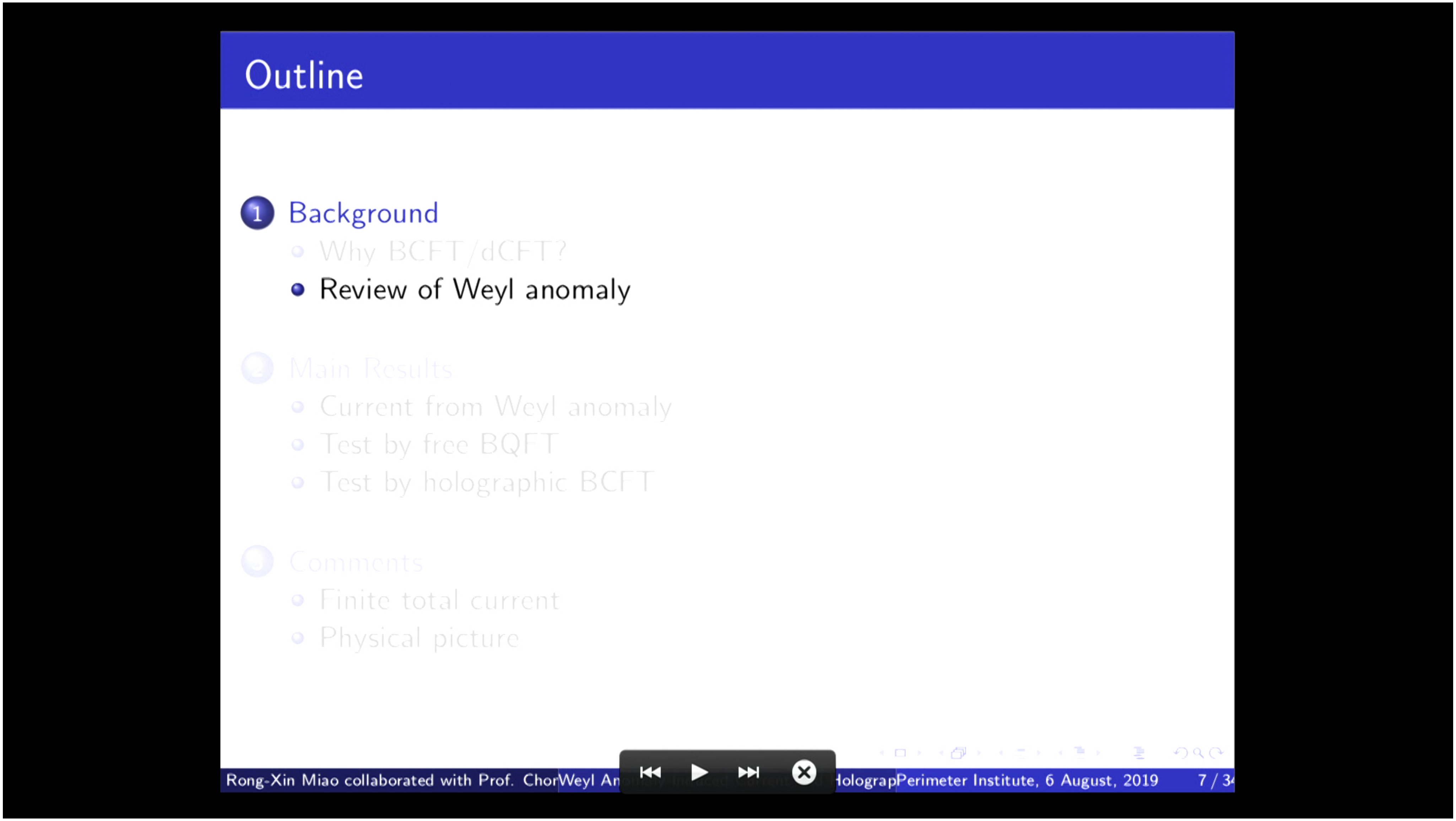
$$I = -\frac{1}{2}\int_M d^d x\sqrt{g}[(\partial\phi)^2 + \xi R\phi^2] - \xi\int_P d^{d-1}y\sqrt{\sigma}K\phi^2 \quad (1)$$

where $\xi = \frac{d-2}{4(d-1)}$, and K is the extrinsic curvature.

  - Conformally invariant boundary conditions

$$\text{Dirichlet BC}: \phi|_P = 0, \quad (2)$$

$$\text{Robin BC}: (\partial_n + 2\xi K)\phi|_P = 0, \quad (3)$$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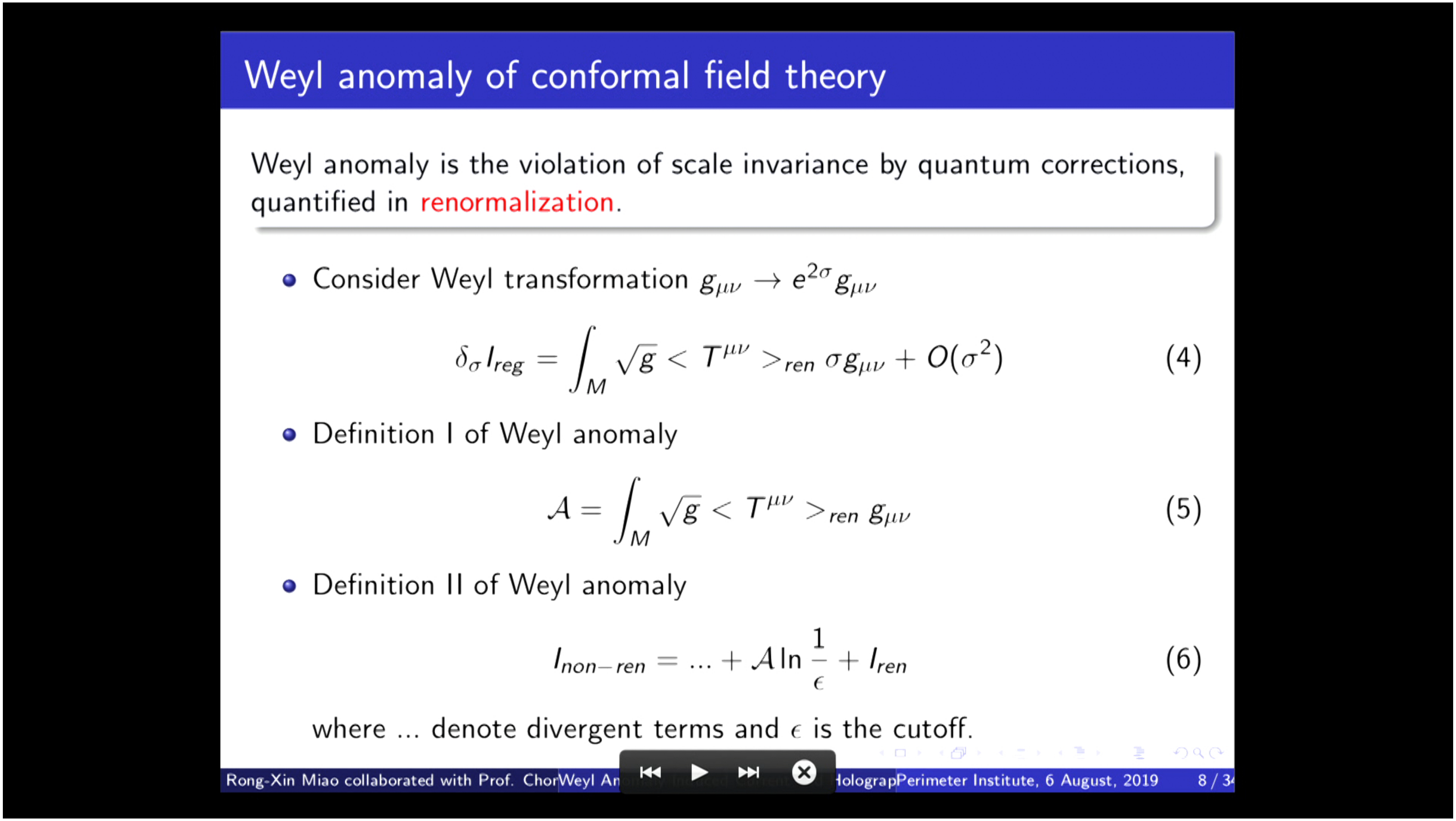

Weyl anomaly of conformal field theory
Weyl anomaly is the violation of scale invariance by quantum corrections, quantified in renormalization.
Consider Weyl transformation $g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$
$\delta_\sigma I_{reg} = \int_M \sqrt{g} < T^{\mu\nu} >_{ren} \sigma g_{\mu\nu} + O(\sigma^2)$ (4)
Definition I of Weyl anomaly
$\mathcal{A} = \int_M \sqrt{g} < T^{\mu\nu} >_{ren} g_{\mu\nu}$ (5)
Definition II of Weyl anomaly
$I_{non-ren} = ... + \mathcal{A} \ln \frac{1}{\epsilon} + I_{ren}$ (6)
where ... denote divergent terms and ϵ is the cutoff.
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Equivalence for two definitions of Weyl anomaly

Key Point: the non-renormalized effective action of CFT is conformally invariant.

Consider Weyl transformation $g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$ and $\epsilon \to e^{\sigma}\epsilon$ for non-renormalized effective action

$$I_{non-ren} = ... + \mathcal{A} \ln \frac{1}{\epsilon} + I_{ren}$$

$$\delta_\sigma I_{non-ren} = \sigma \left(-\mathcal{A} + \int_M \sqrt{g} < T^{\mu\nu} >_{ren} g_{\mu\nu} \right) + O(\sigma^2) = 0 \qquad (7)$$

where the divergent term ... and Weyl anomaly $\mathcal{A}$ are conformally invariant.

Boundary Weyl anomaly

In the presence of boundary, Weyl anomaly of CFT generally pick up a boundary contribution $\langle T^a_a \rangle_P$ in addition to the usual bulk term $\langle T^i_i \rangle_M$, i.e. $\langle T^i_i \rangle = \langle T^i_i \rangle_M + \delta(x_\perp) \langle T^a_a \rangle_P$.

- Bulk Weyl anomaly

$$\langle T^i_i \rangle_M = \frac{c}{16\pi^2} C^{ijkl} C_{ijkl} - \frac{a}{16\pi^2} E_4 + b\, F_{ij} F^{ij}, \quad d = 4, \tag{8}$$

- Boundary Weyl anomaly

$$\langle T^a_a \rangle_P = d_1 \mathrm{Tr} \bar{k}^3 + d_2 C^{ac}{}_{bc} \bar{k}^b{}_a, \quad d = 4, \tag{9}$$

where $\bar{k}_{ab}$ is the traceless part of extrinsic curvature, C_{ijkl} is the Weyl tensor, F_{ij} is the field strength of gauge field.

- a, b, c are the bulk central charges independent of BC.
- d_i are boundary central charges which depend on BC.

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Weyl anomaly of general quantum field theory

Weyl anomaly can be defined as the difference between the trace of renormalized stress tensor and the renormalized trace of stress tensor.

- Definition I of Weyl anomaly

$$\mathcal{A} = \int_M \sqrt{g}\,[< T^{\mu\nu} >_{ren}\, g_{\mu\nu} - < T^{\mu\nu} g_{\mu\nu} >_{ren}] \tag{10}$$

- Definition II of Weyl anomaly

$$I_{non-ren} = ... + \mathcal{A} \ln \frac{1}{\epsilon} + I_{ren} \tag{11}$$

- For unitary, renormalizable and gauge invariant QFT

$$\mathcal{A} = \int_M \sqrt{g}[bF_{\mu\nu}F^{\mu\nu} + O(R^2)] + \int_{\partial M} \sqrt{h}O(Rk), \tag{12}$$

where b is beta function.

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where b is beta function.

Current near the boundary

Renormalized current is divergent near the boundary. However, nothing goes wrong since there is boundary current cancel the bulk "divergence".

- Near-boundary current

$$\langle J_\mu \rangle = \frac{1}{x^3} J_\mu^{(3)} + \frac{1}{x^2} J_\mu^{(2)} + \frac{1}{x} J_\mu^{(1)} + \cdots, \qquad x \sim 0, \tag{13}$$

where x is the proper distance from the boundary, $J_\mu^{(n)}$ depend on only the background geometry, the background gauge field strength.

- Imposing $\nabla_\mu J^\mu = 0$, we get

$$\begin{aligned} &J_\mu^{(3)} = 0, \qquad J_\mu^{(2)} = 0, \\ &J_\mu^{(1)} = \alpha_1 F_{\mu\nu} n^\nu + \alpha_2 \mathcal{D}_\mu k + \alpha_3 \mathcal{D}_\nu k_\mu^\nu + \alpha_4 \star F_{\mu\nu}\, n^\nu \end{aligned} \tag{14}$$

Current from Weyl anomaly

Recall that Weyl anomaly can be obtained as the logarithmic UV divergent term of the effective action.

- Vary the vector and focus on the boundary term

$$(\delta\mathcal{A})_{\partial M} = \delta I_{\text{eff}}\Big|_{\ln 1/\epsilon} = \left(\int_{x\geq\epsilon}\sqrt{g}J^{\mu}\delta A_{\mu}\right)_{\log(1/\epsilon)}, \tag{15}$$

- Variation of Weyl anomaly

$$(\delta\mathcal{A})_{\partial M} = 4b\int_{\partial M}\sqrt{h}F^{b}{}_{n}\,\delta A_{b}. \tag{16}$$

- Variation of effective action

$$\int_{\partial M}\sqrt{h}(\alpha_1 F^{b}{}_{n} + \alpha_2\mathcal{D}^{b}k + \alpha_3\mathcal{D}_{j}k^{jb} + \alpha_4 \star F^{b}{}_{n})\delta A_{b}. \tag{17}$$

- Identifying (16) with (17), we get $\alpha_1 = 4b, \quad \alpha_2 = \alpha_3 = \alpha_4 = 0.$

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Weyl anomaly can be defined as the difference between the trace of renormalized stress tensor and the renormalized trace of stress tensor.

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where b is beta function.

Key result: current from Weyl anomaly

The expectation value of the current take universal form

$$J_a = \frac{4bF_{an}}{x}, \quad x \sim 0, \tag{18}$$

near the boundary.

- The universal law holds for general BQFTs which are covariant, gauge invariant, unitary and renormalizable.
- The current is independent of boundary conditions.
- The magnitude of the induced current is large.

$$J_a = \frac{e^2 c}{\hbar} \frac{4bF_{an}}{x}. \tag{19}$$

- This current comes from the vacuum magnetization near boundary.

Another derivation of current from Weyl anomaly

Consider the Weyl transformation

$$g'_{ij} = e^{2\sigma} g_{ij}, \qquad F'_{ij} = F_{ij} \tag{20}$$

- Variation of effective action

$$\delta_\sigma I_{\text{eff}} = \mathcal{A}\delta\sigma = b \int_M dx^4 \sqrt{g} F^{ij} F_{ij} \delta\sigma(x) + O(R^2, \sigma). \tag{21}$$

- Anomalous action

$$I_{\text{anomalous}} = I(e^{2\sigma} g_{ij}) - I(g_{ij}) = b \int_M dx^4 \sqrt{g'} F'^{ij} F'_{ij} \sigma(x). \tag{22}$$

- Weyl transformation of current

$$J'^{i} = e^{-4\sigma} J^{i} + 4b \nabla'_j (F'^{ij} \sigma). \tag{23}$$

Another derivation of current from Weyl anomaly

Key observation: BCFT in the half space

$$ds^2 = dx^2 + dy_a^2, \quad x \geq 0 \tag{24}$$

is conformally equivalent to CFT in the Poincare patch of AdS

$$ds^2 = \frac{dx^2 + dy_a^2}{x^2}, \quad x \geq 0. \tag{25}$$

- Finite current in AdS

$$\delta I_{\text{ren}} = \int_M dx^4 \sqrt{g} J^i \delta A_i = \int_M dx^4 \frac{J^i}{x^4} \delta A_i, \tag{26}$$

- Current in half space from Weyl transformation law (23)

$$J^i_{\text{BCFT}} = \frac{J^i}{x^4} + 4b\, \nabla'_j (F'^{ij} \ln x) = \frac{4b\; F'^{ix}}{x} + O(\ln x, x^0) \tag{27}$$

Test by free BQFT

The free boundary quantum field theory (BQFT) indeed satisfy the universal law for near-boundary current.

- Free complex scalar

$$I = -\int_M \sqrt{g}[(D^i\phi)^* D_i\phi + E\phi^*\phi] \quad (28)$$

where $D_i = \nabla_i + iA_i$ and E are functions including only the coupling constants with zero or positive mass dimension (renormalizable).

- Boundary conditions

$$\begin{aligned} &\text{Dirichlet BC} : \phi|_{\partial M} = 0, \\ &\text{Robin BC} : (D_n + f)\phi|_{\partial M} = 0 \end{aligned} \quad (29)$$

where f defines a renormalizable theory, for example, $f = 2\lambda_0 k + \lambda_1 m + \ldots$.

Test by free BQFT

The free boundary quantum field theory (BQFT) indeed satisfy the universal law for near-boundary current.

- Renormalized current for both Dirichlet BC and Robin BC (McAvity, Osborn 1991)

$$J_a = -\frac{F_{an}}{24\pi^2 x}, \qquad x \sim 0. \tag{30}$$

- Central charge (beta function)

$$b = \frac{-1}{96\pi^2} \tag{31}$$

- Complex scalars obey the universal law $J_a = \frac{4bF_{an}}{x}$.
- The near-boundary current is indeed independent of BCs.
- The universal law works for not just BCFT but also BQFT.

Geometry Setup of AdS/BCFT

The d dimensional manifold M is extended to a $d+1$ dimensional asymptotically AdS space N so that $\partial N = M \cup Q$, where Q is a d dimensional manifold which satisfies $\partial Q = \partial M = P$.

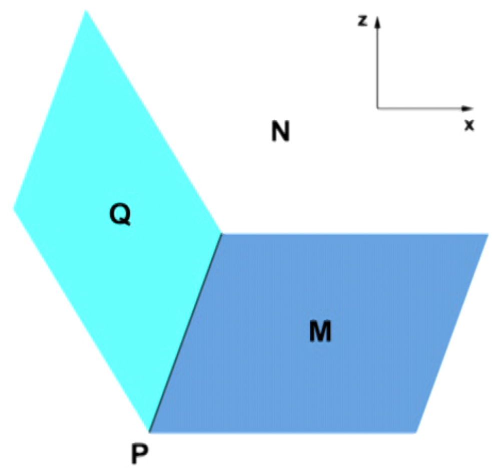

Figure: Geometry of holographic BCFT

Boundary Conditions of AdS/BCFT

A central issue in the construction of the AdS/BCFT is the determination of the location of Q in the bulk.

- Gravitational action

$$I = \int_N \sqrt{G}(R - 2\Lambda) + 2\int_Q \sqrt{h}(K - T), \tag{32}$$

where T is the holographic dual of BC, since it affects the boundary central charges as the BC does.

- Variation of the action on Q

$$\delta I|_Q = -\int_Q \sqrt{\gamma}\left(K^{\alpha\beta} - (K - T)h^{\alpha\beta}\right)\delta h_{\alpha\beta}. \tag{33}$$

- Takayanagi's proposal: Neumann BC

$$K_{\alpha\beta} - (K - T)h_{\alpha\beta}|_Q = 0 \tag{34}$$

- Our proposal: Dirichlet BC

$$\delta h_{\alpha\beta}|_Q = 0 \tag{35}$$

Holographic BCFT

To derive the holographic current of order $O(F)$, it is sufficient to focus on probe limit for bulk metric and embedding function of bulk boundary.

- Action

$$I = \int_N \sqrt{G}(R - 2\Lambda - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}) + 2\int_Q \sqrt{\gamma}(K - 3\tanh\rho). \quad (36)$$

- Probe limit

$$ds^2 = \frac{dz^2 + dx^2 + \delta_{ab}dy^a dy^b}{z^2}, \quad (37)$$

$$x = -\sinh\rho\ z, \quad (38)$$

- BCs for gauge field

$$\begin{aligned} &\text{Absolute BC}: \mathcal{F}_{n\mu}|_Q = 0, \\ &\text{Relative BC}: {}^*\mathcal{F}_{n\mu}|_Q = 0. \end{aligned} \quad (39)$$

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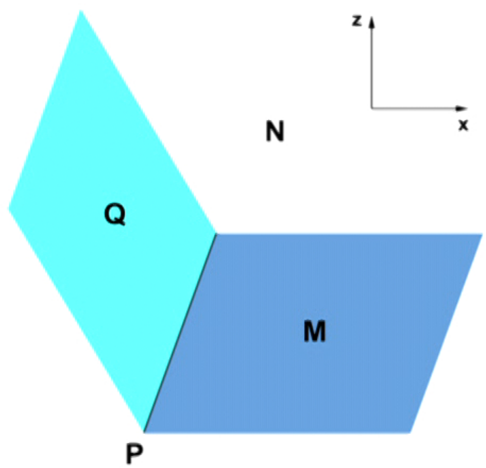

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Holographic BCFT

For plane boundary, $\mathcal{A}_\mu$ depends on only the coordinates z and x. For simplicity, we choose the gauge $\mathcal{A}_z = \mathcal{A}_x = 0$.

- Ansatz for gauge field

$$\mathcal{A}_a = A_a^{(0)} + A_a^{(1)} x f(\frac{z}{x}), \tag{40}$$

where $f(0) = 1$, $A_a^{(i)}$ are constants and $A_a^{(1)} = F_{xa}$.

- Maxwell equations

$$s(s^2+1)f''(s) - f'(s) = 0, \tag{41}$$

- Solutions

$$f(s) = 1 - \alpha + \alpha\sqrt{1+s^2} \tag{42}$$

- Imposing BCs

$$\alpha_A = 1, \qquad \alpha_R = \frac{1}{1+\coth\rho} \tag{43}$$

Holographic BCFT

In general, there are non-trivial contributions on Q to the holographic current.

- Holographic current

$$
\begin{aligned}
< J^a > &= \lim_{z\to 0} \frac{\delta I}{\sqrt{g}\delta A_a} = \lim_{z\to 0} \frac{\sqrt{G}}{\sqrt{g}} [\mathcal{F}^{za}|_M + \mathcal{F}^{na}|_Q] \\
&= -\frac{F_{an}}{x} + O(1) \qquad (44)
\end{aligned}
$$

- Holographic Weyl anomaly

$$
\mathcal{A} = \int_M \sqrt{g}[bF_{ij}F^{ij} + O(R^2)], \quad b = -\frac{1}{4}. \qquad (45)
$$

- Holographic BCFT satisfy the universal law for anomaly-induced current, independent of BCs.

Finite Total Current

There are boundary current, which exactly cancel the apparent "divergence" in the bulk current and make finite total current.

- Gauge invariance

$$\begin{aligned}\delta_\alpha I &= \int_M \sqrt{g} J^i \delta A_i + \int_{\partial M} \sqrt{h} j^b \delta a_b = 0 \\ &= -\int_M \sqrt{g} \nabla_i J^i \alpha - \int_{\partial M} \sqrt{h} (D_b j^b - J_n) \alpha \qquad (46)\end{aligned}$$

- Conservation laws

$$\text{Bulk}: \ \nabla_i J^i = 0 \ \Rightarrow \ J_n = 4bD_a F^a{}_n \ln x + O(1) \qquad (47)$$

$$\text{Boundery}: \ D_a j^a = J_n \ \Rightarrow \ j_a = 4bF_{an} \ln \epsilon. \qquad (48)$$

- Finite total current

$$J_a = \frac{4bF_{an}}{x} + \delta(x; \partial M) 4bF_{an} \ln \epsilon + O(1). \qquad (49)$$

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Physical Picture

When there is a boundary, the contribution from source points at $x < 0$ are missing. This leads to a net amount of charge moving to $-y$ direction.

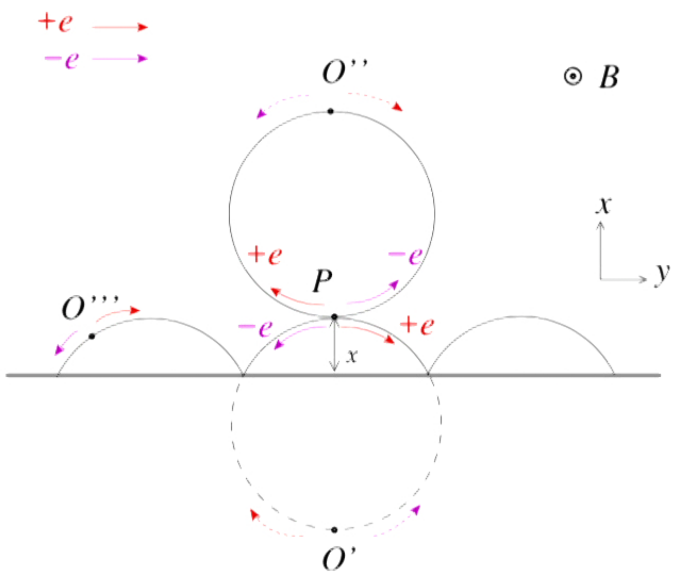

Figure: Induced current from virtual pair creation in presence of boundary.

Vacuum magnetization

The current is due to vacuum magnetization near boundary. One need energy to produce but no energy to maintain this magnetization-current.

- The current is carried by virtual particles instead of real particles. Since Lorentz force does not do work, magnetic field **B** cannot produce real particles.
- Although there is current, there is no energy flux. Virtual particle pairs move in the opposite direction.
- The current does not transfer energy. As a result, there is no dissipation in material with resistance for our vacuum current.

Some generalizations

- Current in higher dimensions

$$J_i = b\frac{F_{xi}}{x^{d-3}} + \dots \tag{50}$$

where b is central charge, which depends on BCs.

- Current for n-forms

$$J_{i_1\dots i_n} = b_n\frac{H_{x\ i_1\dots i_n}}{x^{d-2n-1}} + \dots \tag{51}$$

- Stress tensor near boundary (Deutsch and Candelas 1979)

$$T_{ij} = c\frac{\bar{k}_{ij}}{x^{d-1}} + \dots \tag{52}$$

c is the norm of displacement operator (Chu and Miao 2018, 2019) (Herzog, Huang and Jensen 2018).

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Summary and Outlook

Summary:

- Weyl anomaly induce a current, when external magnetic field parallel to the boundary is applied.
- Near the boundary, the current take universal form for covariant, gauge invariant, unitary and renormalizable QFT.
- The universal law is independent of boundary conditions, temperature and the states of QFT.
- The current is due to vacuum magnetization.

Outlook:

- Generalizations to defect QFT.
- Experimental measurement.

Thank you!


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