

Title: BPS Wilson loop in AdS4/CFT3

Speakers: Silvia Penati

Collection: Boundaries and Defects in Quantum Field Theory

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Abstract: I will review recent results concerning a general class of parametric BPS Wilson loops in ABJM theory. In particular, I will present a proposal for their exact quantum expression in terms of a parametric Matrix Model and discuss their role in the exact calculation of physical quantities like the Bremsstrahlung function and in testing the AdS4/CFT3 correspondence.

BPS Wilson loops in AdS₄/CFT₃

Silvia Penati, University of Milano-Bicocca and INFN

August 8, 2019

M. Bianchi, L. Griguolo, M. Leoni, A. Mauri, M. Preti, D. Seminara, J-j. Zhang

1402.4128, 1604.00383, 1705.02322, 1705.10780, 1802.07742, 1808.01397

BPS Wilson Loops

BPS Wilson Loops in supersymmetric gauge theories: gauge invariant non-local operators that preserve some supercharges

The prototype example in 4D $\mathcal{N} = 4$ SYM

$$WL = \text{Tr} P e^{-i \int_{\Gamma} d\tau \mathcal{L}(\tau)}$$

$$\mathcal{L} = \dot{x}^{\mu} A_{\mu} + i |\dot{x}| \theta_I \Phi^I$$

It includes couplings to the six scalars

Maldacena, PRL80 (1998) 4859

Drukker, Gross, Ooguri, PRD60 (1999) 125006; Zarembo, NPB 643

The number of preserved supercharges depends on Γ and θ_I

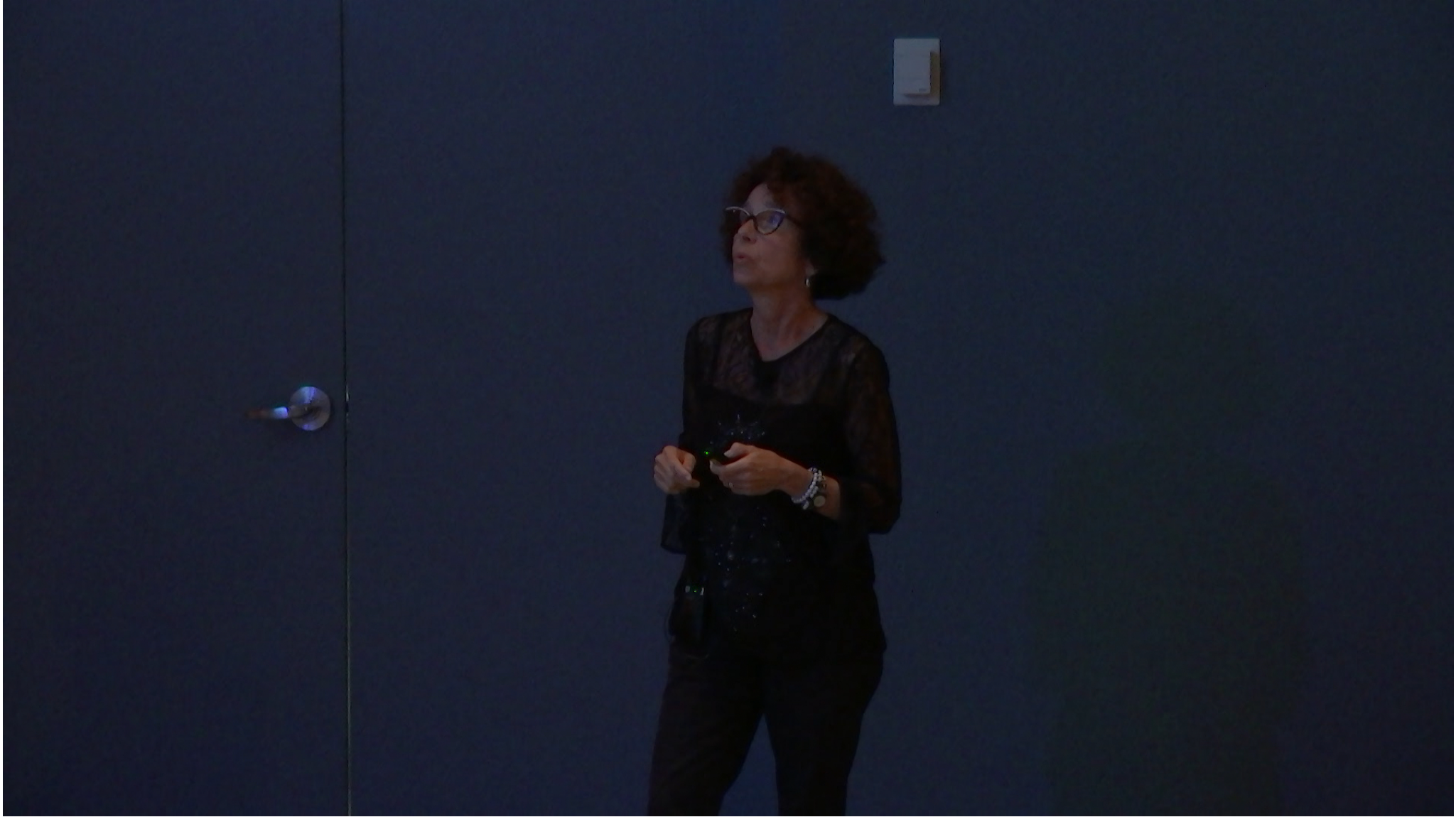
★ They are in general non-protected operators and their expectation values

$$\langle WL \rangle \sim \int D[A, \hat{A}, C, \bar{C}, \psi, \bar{\psi}] e^{-S} \text{Tr} P \exp \left[-i \int_{\Gamma} d\tau \mathcal{L}(\tau) \right]$$

depend non-trivially on the coupling constant.

- **Weak couplings** $\implies$ Ordinary perturbation theory
- **Strong couplings** $\implies$ Holographic methods: **Dual description** in terms of fundamental strings or M2-branes. The expectation value at strong coupling is given by the exponential of a minimal area surface ending on the WL contour.
- **Finite couplings** $\implies$ Localization techniques (**Matrix Model**)

The matrix model provides an **exact interpolating function** to check the AdS/CFT correspondence



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★ They are related to physical quantities like the **Bremsstrahlung function** and the **Cusp anomalous dimension**. Therefore, they are ultimately related to



INTEGRABILITY IN AdS/CFT

★ Parametric WL (*latitude WL*) are related to correlation functions of the **1D defect CFT** defined on the WL contour.

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INTEGRABILITY IN AdS/CFT

★ Parametric WL (*latitude WL*) are related to correlation functions of the **1D defect CFT** defined on the WL contour.

BPS Wilson loops in $\mathcal{N} = 6$ ABJM theory

Plan of the talk

- “Bosonic” and “fermionic” BPS WL in ABJ(M) theory
 - ▶ Cohomological equivalence & framing
- Latitude Wilson loops
 - ▶ Cohomological equivalence & non-integer framing
 - ▶ Matrix Model for latitude WL
- From framing to Bremsstrahlung functions
- Conclusions and Perspectives

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$\mathcal{N} = 6$ ABJ(M) theory

$U(N_1)_k \times U(N_2)_{-k}$ CS-gauge vectors $A_\mu, \hat{A}_\mu$ minimally coupled to

$SU(4)$ complex scalars $C_I, \bar{C}^I$ and fermions $\psi_I, \bar{\psi}^I$, $I = 1, \dots, 4$

in the (anti)bifundamental representation of the gauge group with non-trivial potential.

$$S = S_{\text{CS}} + S_{\text{mat}} + S_{\text{pot}}^{\text{bos}} + S_{\text{pot}}^{\text{ferm}}$$

$$S_{\text{CS}} = \frac{k}{4\pi i} \int d^3x \varepsilon^{\mu\nu\rho} \left\{ \text{Tr} \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} i A_\mu A_\nu A_\rho \right) - \text{Tr} \left(\hat{A}_\mu \partial_\nu \hat{A}_\rho + \frac{2}{3} i \hat{A}_\mu \hat{A}_\nu \hat{A}_\rho \right) \right\}$$

$$S_{\text{mat}} = \int d^3x \text{Tr} \left[D_\mu C_I D^\mu \bar{C}^I - i \bar{\Psi}^I \gamma^\mu D_\mu \Psi_I \right]$$

$$D_\mu C_I = \partial_\mu C_I + i A_\mu C_I - i C_I \hat{A}_\mu$$

Dual to M-theory on $\text{AdS}_4 \times S^7/Z_k$ or Type IIA on $\text{AdS}_4 \times CP^3$

Prototype examples of WLs in ABJ(M)

Bosonic BPS WL:

$$W_B = \text{Tr} P \exp \left[-i \int_{\Gamma} d\tau (\underline{A}_{\mu} \dot{x}^{\mu} - \frac{2\pi i}{k} |\dot{x}| M_J^I C_I \bar{C}^J) \right]$$

$$\hat{W}_B = \text{Tr} P \exp \left[-i \int_{\Gamma} d\tau (\hat{A}_{\mu} \dot{x}^{\mu} - \frac{2\pi i}{k} |\dot{x}| M_J^I \bar{C}^J C_I) \right]$$

For $\Gamma = (0, \cos \tau, \sin \tau)$ and $M = \text{diag}(1, 1, -1, -1) \implies$ **1/6 BPS**

Drukker, Plefka, Young, JHEP 0811 (2008) 019

Chen, Wu, NPB 825 (2010) 38, Rey, Suyama, Yamaguchi, JHEP 0903 (2009)

Exact matrix model result

Kapustin, Willett, Yaakov, JHEP 1003 (2010) 089

Fermionic BPS WL : Holonomy of a $U(N_1|N_2)$ superconnection

$$W_F = \text{Tr} P \exp \left[-i \int_{\Gamma} d\tau \mathcal{L}(\tau) \right]$$

$$\mathcal{L}(\tau) = \begin{pmatrix} A_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| M_J^I C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}} |\dot{x}| \eta_I \bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}} |\dot{x}| \psi_I \bar{\eta}^I & \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| \hat{M}_J^I \bar{C}^J C_I \end{pmatrix}$$

For $\Gamma = (0, \cos \tau, \sin \tau)$ maximal circle

$$M = \text{diag}(1, -1, -1, -1) \text{ and } \eta_I = \delta_{I1}(e^{i\frac{\pi}{2}}, -ie^{-i\frac{\pi}{2}}) \implies \mathbf{1/2 \text{ BPS}}$$

Drukker, Trancanelli, JHEP 02 (2010) 058

Exact matrix model and string dual description

Framing

For the $U(N)_k$ pure Chern–Simons theory ([topological theory](#))

$$S_{CS} = -i \frac{k}{4\pi} \int d^3x \varepsilon^{\mu\nu\rho} \text{Tr} \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} i A_\mu A_\nu A_\rho \right)$$

On a closed path Γ and in fundamental representation

$$\begin{aligned} \langle W_{CS} \rangle &= \langle \text{Tr} P e^{-i \int_\Gamma dx^\mu A_\mu(x)} \rangle \\ &= \sum_{n=0}^{+\infty} \text{Tr} P \int dx_1^{\mu_1} \cdots dx_n^{\mu_n} \langle A_{\mu_1}(x_1) \cdots A_{\mu_n}(x_n) \rangle \end{aligned}$$

- ❶ either by using semiclassical methods in the large k limit

Witten, CMP121 (1989) 351

- ❷ or perturbatively (n -pt correlation functions)

Guadagnini, Martellini, Mintchev, NPB330 (1990) 575

Define $\langle A_{\mu_1}(x_1) A_{\mu_2}(x_2) \rangle$ at coincident points

Using **point-splitting regularization**

$$\Gamma_f : \quad y^\mu(\tau) \rightarrow y^\mu(\tau) + \epsilon n^\mu(\tau)$$



$$\lim_{\epsilon \rightarrow 0} \oint_{\Gamma} dx^\mu \oint_{\Gamma_f} dy^\nu \langle A_\mu(x) A_\nu(y) \rangle = -i\pi \frac{N}{k} \chi(\Gamma, \Gamma_f)$$

$$\chi(\Gamma, \Gamma_f) = \frac{1}{4\pi} \oint_{\Gamma} dx^\mu \oint_{\Gamma_f} dy^\nu \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3} \quad \text{Gauss linking number}$$

Higher-order contributions exponentiate the one-loop result

$$\langle W_{\text{CS}} \rangle = \underbrace{e^{-i\pi \frac{N}{k} \chi}}_{\text{framing factor}} \rho(\Gamma) \quad \chi \in \mathbb{Z} \quad \text{topological invariants}$$

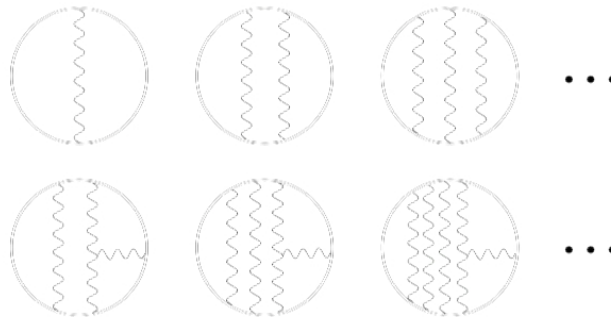
Exponentiation of one-loop framing term relies on the following distinguishing properties

Alvarez, Labastida, NPB395 (1993) 198

- 1 The gauge propagator is one-loop exact

$$\langle A_\mu^a(x) A_\nu^b(y) \rangle = \delta^{ab} \frac{i}{2k} \varepsilon_{\mu\nu\rho} \frac{(x-y)^\rho}{|x-y|^3}$$

- 2 Only diagrams with **collapsible propagators** contribute to framing



- 3 Factorization theorem

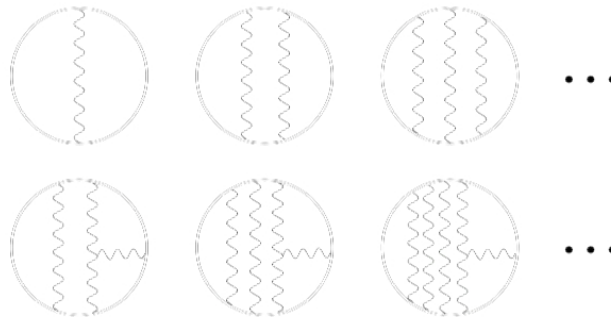
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$\mathcal{N} = 2$ susy CS theory

We are primarily interested in supersymmetric theories for which **localization** can be used.

$$\langle W_{\text{SCS}} \rangle = \langle \text{Tr } P e^{-i \int_{\Gamma} d\tau (\dot{x}^{\mu} A_{\mu}(x) - i |\dot{x}| \sigma)} \rangle$$

Localization always provides the result at framing $\chi(\Gamma, \Gamma_f) = 1$. This follows from requiring consistency between point-splitting regularization and supersymmetry used to localize: **The only point-splitting compatible with susy is the one where the contour and its frame wrap two different Hopf fibers of S^3**

Kapustin, Willett, Yankov, JHEP 1003 (2010) 089

Localization is sensible to framing!

Framing identified as imaginary contributions

$$\langle W_{\text{SCS}} \rangle = e^{-i\pi \frac{N}{k}} \rho(\Gamma)$$



Adding matter $\rightarrow$ ABJ(M) case

$$\mathbf{1/6\text{-BPS WL}} \quad \langle W_B \rangle = \langle \text{Tr} P \exp \left[-i \int_{\Gamma} d\tau (A_{\mu} \dot{x}^{\mu} - \frac{2\pi i}{k} |\dot{x}| M_J^I C_I \bar{C}^J) \right] \rangle$$

Matter contributes to framing. Exponentiation still works, so we can write
($\lambda_i \equiv N_i/k$)

$$\langle W_B \rangle_1 = \underbrace{e^{i\pi \left(\lambda_1 - \frac{\pi^2}{2} \lambda_1 \lambda_2^2 + \mathcal{O}(\lambda^5) \right)}}_{\text{perturbative framing function}} \left(1 - \frac{\pi^2}{6} (\lambda_1^2 - 6\lambda_1 \lambda_2) + \mathcal{O}(\lambda^4) \right)$$

$\Downarrow$

$$\text{perturbative framing function} \quad \Phi_B = \lambda_1 - \frac{\pi^2}{2} \lambda_1 \lambda_2^2 + \mathcal{O}(\lambda^5)$$

It agrees with the **localization** result

Kapustin, Willett, Yaakov, JHEP 1003

Drukker, Marino, Putrov, (2011); Klemm, Marino, Schiereck, Soroush, (2013)



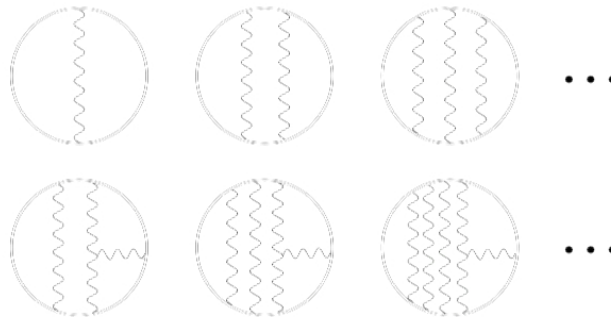
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- 1 The gauge propagator is one-loop exact

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- 2 Only diagrams with **collapsible propagators** contribute to framing



- 3 Factorization theorem

Cohomological equivalence

Classical equivalence

Drukker, Trancanelli (2010)

$$W_F = \frac{N_1 W_B + N_2 \hat{W}_B}{N_1 + N_2} + \mathcal{Q}(\text{something})$$

Localizing the path integral with $\mathcal{Q} \Rightarrow$ cohomological equivalence implemented at quantum level at [framing one](#)

$$\langle W_F \rangle_1 = \frac{N_1 \langle W_B \rangle_1 + N_2 \langle \hat{W}_B \rangle_1}{N_1 + N_2}$$

Up to two loops

$$\langle W_B \rangle_1 \equiv e^{i\pi \frac{N_1}{k}} \langle W_B \rangle_0 \quad , \quad \langle \hat{W}_B \rangle_1 \equiv e^{-i\pi \frac{N_2}{k}} \langle \hat{W}_B \rangle_0$$

$$\langle W_F \rangle_1 \equiv e^{i\pi \frac{N_1 - N_2}{k}} \langle W_F \rangle_0$$

Plan of the talk

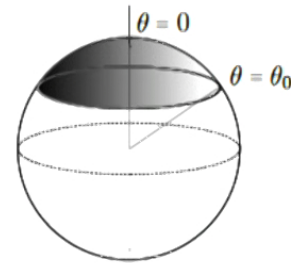
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Latitude Wilson loops

Bosonic & Fermionic Wilson loops depending on a **real parameter**

$$\nu = \sin 2\alpha \sin \theta_0 \quad \nu \in [0, 1]$$

accounting for



- **latitude contours**

$$\Gamma = (\cos \theta_0, \sin \theta_0 \cos \tau, \sin \theta_0 \sin \tau) \quad \theta_0 \in [0, \pi]$$

- **internal angle α** for scalar and fermion couplings $M_I^J(\alpha), \eta_I(\alpha), \bar{\eta}^I(\alpha)$

M.S. Bianchi, L. Griguolo, M. Leoni, SP, D. Seminara (2014)

Bosonic BPS WL

$$W_B(\nu) = \text{Tr} P \exp \left[-i \int_{\Gamma} d\tau (A_{\mu} \dot{x}^{\mu} - \frac{2\pi i}{k} |\dot{x}| M_J^I(\nu) C_I \bar{C}^J) \right]$$

$$M_J^I(\nu) = \begin{pmatrix} -\nu & e^{-i\tau} \sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau} \sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

It preserves 1/12 of susy charges $\mathcal{Q}_1(\nu), \mathcal{Q}_2(\nu) \implies \mathbf{1/12 \text{ BPS}}$

Back to the undeformed case: $W_B(\nu = 1) = W_B$

$$\text{Fermionic BPS WL} \quad W_F(\nu) = \text{Tr} P \exp \left[-i \int_{\Gamma} d\tau \mathcal{L}(\nu, \tau) \right]$$

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$$\mathcal{L}(\nu, \tau) = \begin{pmatrix} A_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| M_J^I(\nu) C_I \bar{C}^J & -i \sqrt{\frac{2\pi}{k}} |\dot{x}| \eta_I(\nu) \bar{\psi}^I \\ -i \sqrt{\frac{2\pi}{k}} |\dot{x}| \psi_I \bar{\eta}^I(\nu) & \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{k} |\dot{x}| M_J^I(\nu) \bar{C}^J C_I \end{pmatrix}$$

$$M_I{}^J = \begin{pmatrix} -\nu & e^{-i\tau}\sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau}\sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \eta_I^\alpha = \frac{e^{\frac{i\nu\tau}{2}}}{\sqrt{2}} \begin{pmatrix} \sqrt{1+\nu} \\ -\sqrt{1-\nu}e^{i\tau} \\ 0 \\ 0 \end{pmatrix}_I (1, -ie^{-i\tau})^\alpha$$

It preserves 1/6 of susy charges $Q_{1,2,3,4}(\nu) \implies$ **1/6 BPS**

Back to the undeformed case: $W_F(\nu = 1) = W_F$

Cohomological equivalence

Classical equivalence

$$W_F(\nu) = \frac{N_1 e^{-\frac{i\pi\nu}{2}} W_B(\nu) - N_2 e^{\frac{i\pi\nu}{2}} \hat{W}_B(\nu)}{N_1 e^{-\frac{i\pi\nu}{2}} - N_2 e^{\frac{i\pi\nu}{2}}} + Q(\nu)(\text{something})$$

Localizing the path integral with $Q(\nu) \Rightarrow$ cohomological equivalence implemented at quantum level at **framing ν**

$$\langle W_F(\nu) \rangle_\nu = \frac{N_1 e^{-\frac{i\pi\nu}{2}} \langle W_B(\nu) \rangle_\nu - N_2 e^{\frac{i\pi\nu}{2}} \langle \hat{W}_B(\nu) \rangle_\nu}{N_1 e^{-\frac{i\pi\nu}{2}} - N_2 e^{\frac{i\pi\nu}{2}}}$$

where up to two loops we have defined (and checked)

$$\langle W_B(\nu) \rangle_\nu \equiv e^{i\pi \frac{N_1}{k} \nu} \langle W_B(\nu) \rangle_0, \quad \langle \hat{W}_B(\nu) \rangle_\nu \equiv e^{-i\pi \frac{N_2}{k} \nu} \langle \hat{W}_B(\nu) \rangle_0$$

$$\langle W_F(\nu) \rangle_\nu \equiv e^{i\pi \frac{N_1 - N_2}{k} \nu} \langle W_F(\nu) \rangle_0$$

New finding: **Non-integer framing !!**

Navigation icons: back, forward, search, etc.

The Matrix Model

In order to respect cohomological equivalence at quantum level, we would like to perform localization with $\mathcal{Q}(\nu) \implies$ **Matrix Model will depend on ν**

But we are not able yet to implement localization, since $\mathcal{Q}(\nu)$ is not chiral.

Alternatively, we guess the structure of the MM and perform consistency checks

$$\langle W_B(\nu) \rangle_1 = \left\langle \frac{1}{N_1} \sum_{a=1}^{N_1} e^{2\pi \lambda_a} \right\rangle$$

$$Z = \int \prod_{a=1}^{N_1} d\lambda_a e^{i\pi k \lambda_a^2} \prod_{b=1}^{N_2} d\mu_b e^{-i\pi k \mu_b^2} \\ \times \frac{\prod_{a < b}^{N_1} \sinh \pi(\lambda_a - \lambda_b) \prod_{a < b}^{N_2} \sinh \pi(\mu_a - \mu_b)}{\prod_{a=1}^{N_1} \prod_{b=1}^{N_2} \cosh \pi(\lambda_a - \mu_b)}$$

The Matrix Model

In order to respect cohomological equivalence at quantum level, we would like to perform localization with $Q(\nu) \Rightarrow$ **Matrix Model will depend on ν**

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Alternatively, we guess the structure of the MM and perform consistency checks

$$\langle W_B(\nu) \rangle_\nu = \left\langle \frac{1}{N_1} \sum_{a=1}^{N_1} e^{2\pi \sqrt{\nu} \lambda_a} \right\rangle$$

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Checks

- These are the simplest expressions that for $\nu = 1$ go back to the well known MM's of [Kapustin, Willett, Yaakov \(2010\) 089](#); [Klemm, Marino, Schiereck, Soroush \(2013\)](#)
- Strong consistency check: Partition function
 - ▶ For ABJM theory ($N_1 = N_2$) Z does not depend on ν
 - ▶ For ABJ theory ($N_1 \neq N_2$) $|Z|$ does not depend on ν
- Its weak coupling expansion matches a genuine three-loop calculation done at [framing \$\nu\$ \(1802.07742\)](#)
- Its leading behavior at strong coupling in the large N limit reproduces the holographic prediction

$$\langle W_F(\nu) \rangle \sim e^{\pi\nu\sqrt{2N/k}}$$

[Correa, Aguilera-Damia, Silva, JHEP 06 \(2014\)](#)

The Matrix Model

In order to respect cohomological equivalence at quantum level, we would like to perform localization with $Q(\nu) \Rightarrow$ **Matrix Model will depend on ν**

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Checks

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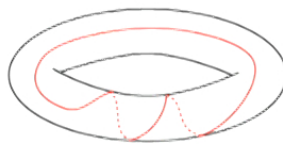
- Recent result for $\left. \frac{\langle W_F(1) \rangle}{\langle W_F(\nu) \rangle} \right|_{1\text{-loop}}$ at strong coupling matches our prediction

Medina-Rincon, 1907.02984

- For ABJM it satisfies the remarkable identity

$$\partial_\nu \log \left(\langle W_B(\nu) \rangle_\nu + \langle \hat{W}_B(\nu) \rangle_\nu \right) = 0$$

- It resembles the Matrix Model computing (P, Q) torus knot invariants in ordinary $U(N_1|N_2)$ Chern–Simons theory



but $P/Q = \nu$ not coprime integers

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Bremsstrahlung function

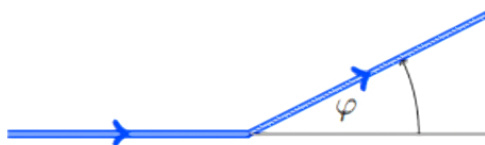
We now prove that latitude WLs have physical meaning, being related to the **Bremsstrahlung functions**. In particular, we disclose the physical meaning of the **framing function**.

Energy lost by a slowly moving massive quark

$$\Delta E = 2\pi B \int dt \dot{v}^2 \quad |\dot{v}| \ll 1$$

In CFT

$$\langle W^\angle \rangle_\varphi \sim e^{-\Gamma_{cusp}(\varphi) \log \frac{L}{\epsilon}} \quad \Gamma_{cusp}(\varphi) \underset{\varphi \ll 1}{\sim} -B \varphi^2$$



Correa, Henn, Maldacena, Sever, JHEP 06 (2012)

Bremsstrahlung functions in ABJM ($N_1 = N_2$)

$$\langle W_F^\angle(\theta) \rangle_\varphi \sim e^{-\Gamma_{cusp}^{1/2}(\varphi, \theta) \log \frac{L}{\epsilon}} \quad \Gamma_{cusp}^{1/2}(\varphi, \theta) \underset{\varphi, \theta \ll 1}{\sim} B_{1/2}(\theta^2 - \varphi^2)$$

$$\langle W_B^\angle(\theta) \rangle_\varphi \sim e^{-\Gamma_{cusp}^{1/6}(\varphi, \theta) \log \frac{L}{\epsilon}} \quad \Gamma_{cusp}^{1/6}(\varphi, \theta) \underset{\varphi, \theta \ll 1}{\sim} B_{1/6}^\theta \theta^2 - B_{1/6}^\varphi \varphi^2$$

They are functions of the coupling constant N/k

How to relate B to quantities exactly computable via localization?

First proposal

$$B_{1/6}^\varphi = \frac{1}{4\pi^2} \partial_m \log |\langle W_B^m \rangle| \Big|_{m=1}$$

Lewkowycz, Maldacena, JHEP 05 (2014)

Our proposal

$$B_{1/2} = \frac{1}{4\pi^2} \partial_\nu \log |\langle W_F(\nu) \rangle| \Big|_{\nu=1}, \quad B_{1/6}^\theta = \frac{1}{4\pi^2} \partial_\nu \log |\langle W_B(\nu) \rangle| \Big|_{\nu=1}$$

Tests:

- Perturbative checks up to three loops for $B_{1/2}$, up to four loops for $B_{1/6}^\theta$ by a direct computation of Γ_{cusp}

M. Bianchi, Griguolo, Leoni, SP, Seminara (2014); Griguolo, Marmiroli, Martelloni, Seminara (2014); M. Bianchi, Mauri (2017)

- $B_{1/2}$ matches the string prediction at next-to-leading order

Forini, Giangreco Puletti, Ohlsson Sax (2013); Correa, Aguilera-Damia, Silva 1412.4084

- Exact proof using two-point correlation functions in 1d defect CFT

$B_{1/6}^\theta$: Correa, Aguilera-Damia, Silva 1405.1396

$B_{1/2}$: L. Bianchi, Griguolo, Preti, Seminara (2017)

Relation with framing

M. Bianchi, Griguolo, Leoni, SP, Seminara (2014)
L. Bianchi, Preti, Vescovi (2018)

We write

$$\langle W_B(\nu) \rangle_\nu = e^{i\Phi_B(\nu)} |\langle W_B(\nu) \rangle_\nu|$$

where the phase $\Phi_B(\nu)$ includes all (but not only) framing effects.

Using cohomological equivalence and $\partial_\nu \log \left(\langle W_B(\nu) \rangle_\nu + \langle \hat{W}_B(\nu) \rangle_\nu \right) \Big|_{\nu=1} = 0$ we find

$$B_{1/2} = \frac{1}{8\pi} \tan \Phi_B(1) \quad B_{1/6}^\theta = \frac{1}{2} B_{1/6}^\varphi = \frac{1}{4\pi^2} \tan \Phi_B(1) \partial_\nu \Phi_B(\nu) \Big|_{\nu=1}$$

The undeformed $\Phi_B(1)$ is the **framing function** of 1/6 BPS Wilson loop.

New physical interpretation of framing for non-topological models

Conclusions & Perspectives

1) Matrix Model

- ▶ We have found the Matrix Model for latitude WLs in ABJ(M). This is a new exact, interpolating function that allows to test $\text{AdS}_4/\text{CFT}_3$ in the large N limit. Localization procedure?

2) Bremsstrahlung functions

- ▶ We have provided an exact prescription for computing the Bremsstrahlung functions in terms of latitude WL. Exploiting **integrability** the exact B could be also computed by a system of TBA equations, as done in $\mathcal{N} = 4$ SYM Drukker (2012); Correa-Maldacena-Sever (2012)

The matching would be crucial for checking the famous

interpolating function $h(\lambda)$ of ABJM

Gromov, Sizov (2014)

3) Relations with the 1D defect CFT

(Work in progress)

Derivatives of latitude WL respect to the parameter produce correlation functions of scalar local operators of the 1D defect CFT

$$\partial_\nu \log \langle W_B(\nu) \rangle \Big|_{\nu=1} = -\frac{8\pi^2}{k^2} \int_0^{2\pi} d\tau_1 \int_0^{\tau_1} d\tau_2 \langle\langle \chi(\tau_1) \chi(\tau_2) \rangle\rangle_{\text{WL}}$$

where $\chi(\tau) = m_J^I(\tau) C_I(\tau) \bar{C}^J(\tau)$

$$\text{and} \quad \langle\langle \chi(\tau_1) \chi(\tau_2) \rangle\rangle_{\text{WL}} \equiv \frac{\langle \text{Tr} P \chi(\tau_1) \chi(\tau_2) e^{-i \oint d\tau \mathcal{L}_B(\tau)} \rangle}{\langle W_B \rangle}$$

Interesting relations:

- ▶ Contact terms in the 1D correlation functions induce framing independent imaginary terms in the WL. Checked at three loops.
- ▶ Cohomological equivalence at quantum level leads to non-trivial identities among correlation functions of 1D CFT. Implications for 1D bootstrap?
- ▶ More generally, what is the effect of framing on the 1D CFT? What is its meaning in 1D?