Title: Integrability of one-point functions in AdS/dCFT with and without supersymmetry

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Collection: Boundaries and Defects in Quantum Field Theory

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Abstract: We review recent results on the calculation of one-point functions in dCFTs corresponding to N=4 SYM with domain walls, discussing supersymmetric as well as non-supersymmetric cases. In particular, we address the integrability properties of the theories and the status of the comparison to dual string theoretical computations.

Integrability of one-point functions in AdS/dCFT with and without susy

Charlotte Kristjansen

Niels Bohr Institute

Based on:

- M. de Leeuw, C.K., G. Linardopoulos, ArXiv:1802.01258[hep-th], Phys.Lett. B781 (2018) 238
- A. Gimenez-Grau, C.K., M. Volk, M. Wilhelm, Arxiv:1810.11463[hep-th], JHEP 1901 (2019) 007
- M. de Leeuw, C.K., K. Vardinghus, ArXiv: 1906.10714 [hep-th]

Boundaries and Defects in QFT

PI

August 7th, 2019

Plan of the talk

I. The (domain wall) AdS/dCFT set-up and its parameters

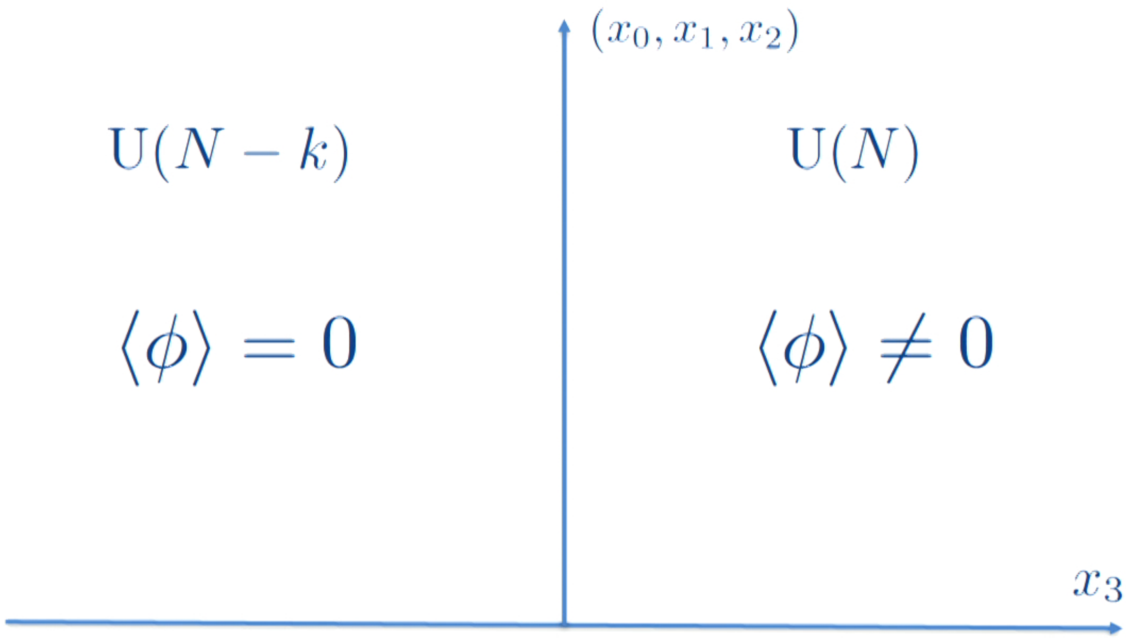
II. One point functions and their integrability properties with and without supersymmetry

III. Comparison between gauge and string theory in a double scaling limit. Positive test of AdS/dCFT with and without susy.

IV. Summary & Open problems

The defect set-up

$$\mathcal{N} = 4 \quad \text{SYM}$$



Classical Fields (simplest case)

$$\Phi_i^{\rm cl} \neq 0, \quad i = 1,2,3, \qquad \Phi_4^{\rm cl} = \Phi_5^{\rm cl} = \Phi_6^{\rm cl} = 0$$

$A_\mu = 0, \qquad \Psi_A = 0$ Assume only x_3 -dependence and $x_3 > 0$

Classical e.o.m.:

(x_3 is distance to defect)

$$\frac{d^2 \Phi_i^{\rm cl}}{dx_3^2} = \left[\Phi_j^{\rm cl}, \left[\Phi_j^{\rm cl}, \Phi_i^{\rm cl}\right]\right].$$

Solution:

$$\Phi_i^{\rm cl} = \frac{1}{x_3} \begin{pmatrix} (t_i)_{k\times k} & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1,2,3$$

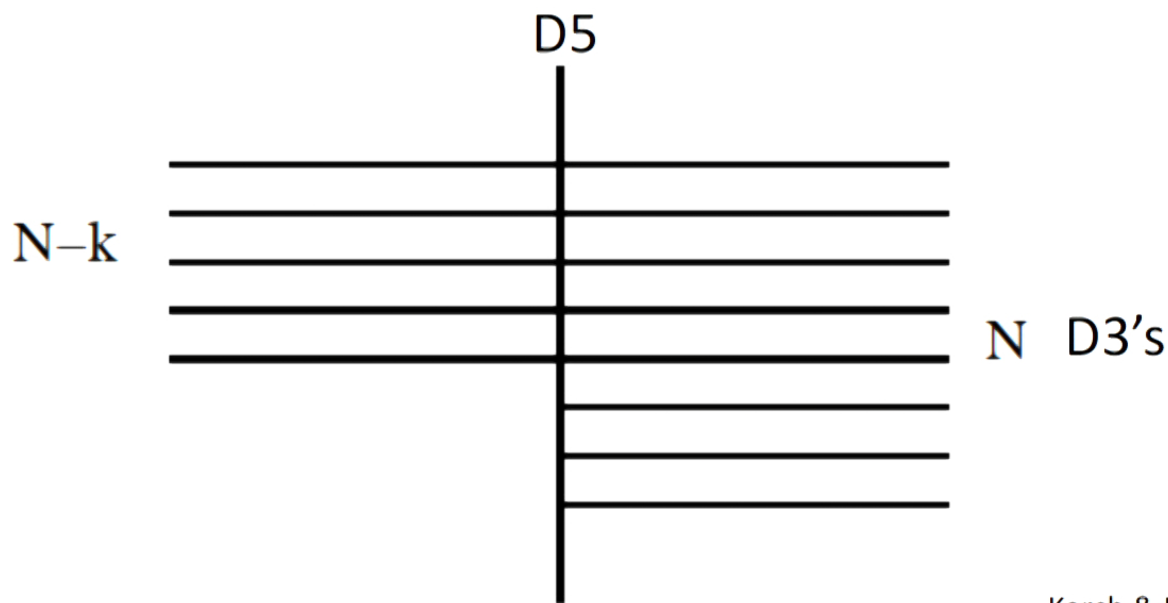
Constable, Myers & Tafjord '99

where t_i, i=1,2,3, constitute a k-dimensional irreducible repr. of SU(2). (Nahm eqns. also fulfilled.)

Set-up ½ BPS (for appropriate choice b.c. for zero-modes, Gaiotto & Witten '08)

AdS/dCFT --- The string theory side

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
$D3$	$\times$	$\times$	$\times$	$\times$						
$D5$	$\times$	$\times$	$\times$		$\times$	$\times$	$\times$			



Karch & Randall '01,

Geometry of D5 brane: $AdS_4 \times S^2$

Background gauge field: k units of magnetic flux on S^2

AdS/dCFT set-ups

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$\mathrm{AdS}_4\times \mathrm{S}^2$	$\mathrm{AdS}_4\times \mathrm{S}^2 \times \mathrm{S}^2$	$\mathrm{AdS}_4\times \mathrm{S}^4$
Flux	k	k_1, k_2	$d_G = \frac{(n+1)(n+2)(n+3)}{6}$
Embedding symmetry	$\mathrm{SO}(3)\times \mathrm{SO}(3)$	$\mathrm{SO}(3) \times \mathrm{SO}(3)$	$\mathrm{SO}(5)$
Gauge Groups	$\mathrm{SU}(N)$, $\mathrm{SU}(N-k)$	$\mathrm{SU}(N)$, $\mathrm{SU}(N-k_1k_2)$	$\mathrm{SU}(N)$, $\mathrm{SU}(N-d_G)$

The SO(5) symmetric D3-D7 brane case

For $x_3 > 0$

$$\phi_i^{\rm cl} = \frac{1}{\sqrt{8}x_3} \begin{pmatrix} (G_i)_{d_G \times d_G} & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1,2,3,4,5, \qquad \phi_6^{\rm cl} = 0$$

where $G_{ij} = [G_i, G_j]$ generate a certain d_G-dimensional irreducible representation of $SO(5)$

Construction of G_i: Start from the $4d$ gamma matrices

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij}, \qquad i,j = 1,\ldots,5$$

$$G_i^{(n)} = (\underbrace{\gamma_i \otimes 1 \otimes \ldots \otimes 1}_{n \text{ terms}} + \ldots + 1 \otimes 1 \ldots \otimes \gamma_i)_{sym}$$

$$d_G = \frac{(n+3)!}{3!n!} = \frac{1}{6}(n+1)(n+2)(n+3)$$

Novel features in defect CFTs

Cardy ´84

McAvity & Osborn '95

1. One-point functions

$$\langle \mathcal{O}_{\Delta}^{\text{bulk}}(x) \rangle = \frac{C}{|x_3|^{\Delta}}$$

Normalization given by: $\lim_{x_3 \to \infty} \langle \mathcal{O}_{\Delta}^{\text{bulk}}(y+x)\mathcal{O}_{\Delta'}^{\text{bulk}}(z+x) \rangle = \frac{\delta_{\Delta\Delta'}}{|y-z|^{2\Delta}}$

2. Two-point functions between op's with different conf. dims.

3. Mixed correlators involving bulk and defect fields

One-point functions --- Motivation

- Possibly the simplest observables beyond the spectrum
- Integrable cases, closed determinant formulas
- Give a positive test of AdS/CFT in a situation where conformal symmetry is partially broken and susy symmetry is partially or fully broken.
- Interesting connections to statistical physics: Matrix product states, Quantum quenches
- Provide input for the boundary conformal bootstrap program 1+2=3

Main questions to be addressed

- Is the one-point function problem integrable ?

  Can we match one-point functions from gauge theory and string theory ?

One-point functions
and their integrability properties

One-point functions in gauge theory

Scalar operators can have non-zero 1-pt fcts already at tree-level

Wish: A Systematic approach to the computation of 1-pt functions of *conformal* scalar operators using the tools of integrability

Consider single trace operators built from the scalar fields

$$\phi_i, \quad i = 1, \ldots, 6$$

$$\mathrm{Tr}(\phi_{i_1}\phi_{i_2}\ldots\phi_{i_L}) \sim |\phi_{i_1}\phi_{i_2}\ldots\phi_{i_L}\rangle$$

Conformal scalar operators=Eigenstates of integrable SO(6) spin chain

Minahan & Zarembo ´02

Eigenstates of length L: $|u_i, v_j^+, v_k^-\rangle_L$
characterized by three sets of rapidities $\{u_i\}_{i=1}^{M}, \ \{v_j^+\}_{i=1}^{N^+}, \ \{v_j^-\}_{i=1}^{N^-}$

One-point functions at tree level

$$\langle \mathcal{O}_\Delta(x) \rangle = (\mathrm{Tr}(\phi_{i_1} \ldots \phi_{i_\Delta}) + \ldots) \,|_{\phi_{i_a} \to \phi_{i_a}^{\mathrm{cl}}} \equiv \frac{C(\{u_i, v_j^+, v_l^-\})}{x_3^\Delta}$$

Matrix Product State associated with the defect:
(D3-D5 case for simplicity)

deLeeuw, C.K. & Zarembo '15,

$$|\mathrm{MPS_k}\rangle = \sum_{\vec{i}} \mathrm{tr}[t_{i_i} \ldots t_{i_L}] |\phi_{i_1} \ldots \phi_{i_L}\rangle,$$

Object to calculate:

$$C_k\left(\{u_i, v_j^+, v_l^-\}\right) = \frac{\langle \mathrm{MPS}_k \left| \{u_i, v_j^+, v_l^-\} \right\rangle_L}{\left\langle \{u_i, v_j^+, v_l^-\} \middle| \{u_i, v_j^+, v_l^-\} \right\rangle^{\frac{1}{2}}}$$

NB: Parameters:

$L = \Delta$,

M, N_+, N_- number of Bethe roots/fields of various types,

$(N_- = 0: \ \mathrm{SU}(3)$ sector, $N_- = N_+ = 0: \ \mathrm{SU}(2)$ sector$)$

k representation label

Solution D3-D5 case

Selection rules:

de Leeuw, C.K & Linardopoulos, 18.

- Momentum of Bethe state equal to zero
- M and $L + N_+ + N_-$ even
- The rapidities come in pairs, i.e. $\{u_i\}, \{v_j^+\}, \{v_l^-\} = \{-u_i\}, \{-v_j^+\}, \{-v_l^-\}$
  Follows from the fact that $Q_{2n+1}|\mathrm{MPS}_k\rangle = 0$ for all $n \in N_0$

Result for C_k:

- Exact formula valid for any, L, M, N^+, N^- and k
- Expressed in terms of objects well known from integrability
  - Baxter polynomials $Q(u) = \prod_{i=1}^{M}(u - u_i)$, $Q_+(v)$, $Q_-(v)$
  - Determinant of Gaudin matrix, G
    $\langle\{u_i, v_j^+, v_l^-\}|\{u_i, v_j^+, v_l^-\}\rangle = \det G = \det G_+ \det G_-$
  - Transfer matrix in higher reps (or some projection thereof).

Solution D3-D5 case

Result for C_k:

de Leeuw, C.K & Linardopoulos, '18.

- Exact formula valid for any, L, M, N^+, N^- and k

$$C_k^{SO(6)} = \sqrt{\frac{Q(0)Q(\frac{i}{2})Q(\frac{ik}{2})Q(\frac{ik}{2})}{\bar{Q}_+(0)\bar{Q}_+(\frac{i}{2})\bar{Q}_-(0)\bar{Q}_-(\frac{i}{2})}} \cdot \mathbb{T}_{k-1}(0) \cdot \sqrt{\frac{\det G_+}{\det G_-}}$$

$$\mathbb{T}_n(x) = \sum_{a=-\frac{n}{2}}^{\frac{n}{2}} (x+ia)^L \frac{Q_+(x+ia)Q_-(x+ia)}{Q(x+i(a+\frac{1}{2}))Q(x+i(a-\frac{1}{2}))}.$$

- Can be proved analytically for the $SU(2)$ subsector ($N_+ = N_- = 0$). Here $\mathbb{T}_n$ is the transfer matrix in the $n+1$ dimensional rep.

  Buhl-Mortensen, de Leeuw, C.K & Zarembo, 1'5.

- Has been checked numerically up to $L = 16, k = 6$ for $SU(3)$ ($N_- = 0$). (Involves summing 10^{12} terms.)

  de Leeuw, C.K & Mori, '16.

- Has been checked numerically up to $L = 8, k = 6$ for $SO(6)$

  de Leeuw, C.K & Linardopoulos, '18.

SO(5) symmetric D3-D7 brane set-up

Selection rules:

de Leeuw, C.K & Linardopoulos, 17.

- Momentum of Bethe state equal to zero
- $(L, M, N_+, N_-) = (L, M, M/2, M/2)$
- The rapidities come in pairs, i.e. $\{u_i\}, \{v_j^+\}, \{v_l^-\} = \{-u_i\}, \{-v_j^+\}, \{-v_l^-\}$
  Follows from the fact that $Q_{2p+1}|\mathrm{MPS}_n\rangle = 0$ for all $p \in N_0$

Result for C_n:

- Trivialises for $SU(2)$ sub-sector, $N_- = N_+ = 0$. ($\Longrightarrow$ $M = 0$)
- Trivialises for $SU(3)$ sub-sector, $N_- = 0$. ($\Longrightarrow$ $N_+ = M = 0$)
- No closed expression found for the full $SO(6)$ sector (yet)

Should we expect a closed expression?

SO(3) x SO(3) symmetric D3-D7 brane set-up

de Leeuw, C.K & Vardinghus, 19.

Selection rules:

- Momentum of Bethe state equal to zero
- L and M even for SU(2) sector

NB: $Q_3|MPS\rangle \neq 0$. Hence no pairing of roots

Result for C_n:

- No closed expression even for the $SU(2)$ sector

Relation to quantum quenches

Prepare quantum system in eigenstate $|n\rangle$ of one Hamiltonian $\mathcal{H}_0$.
Have the eigenstate evolve under a different Hamiltonian $\mathcal{H}_0 + \mathcal{H}_1$

or

Set out quantum system in initial state $|\Psi_0\rangle$
which is not an eigenstate of its Hamiltonian $\mathcal{H}_0$

Study time evolution of local observable after a quantum quench:

$$\langle \mathcal{O}(t) \rangle = \langle \Psi_0 | e^{i\mathcal{H}_0 t} \mathcal{O} e^{-i\mathcal{H}_0 t} | \Psi_0 \rangle$$
$$= \sum_{n,m} \langle \Psi_0 | n \rangle \langle m | \Psi_0 \rangle \langle n | \mathcal{O} | m \rangle e^{-i(E_m - E_n)t}$$

Role of MPS: $\langle \mathrm{MPS} \, | \, \{ u_i, v_j^+, v_l^- \} \rangle \sim \langle \mathrm{Initial} \, | n \rangle \equiv \langle \Psi_0 | n \rangle$

Assume $\mathcal{H}_0$ Hamiltonian of an integrable spin chain

Proposed criterion for integrability of MPS: $\hat{Q}_{2m+1} |\mathrm{MPS}\rangle = 0, \quad m \geq 1$ Piroli, Pozsgay Vernier '17

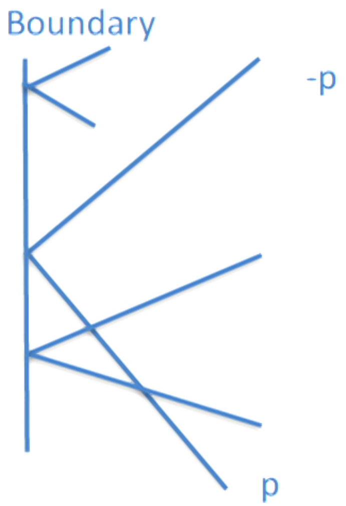
NB: Imply pairing of roots for Bethe states in order to have non-vanishing overlap with MPS

Motivation

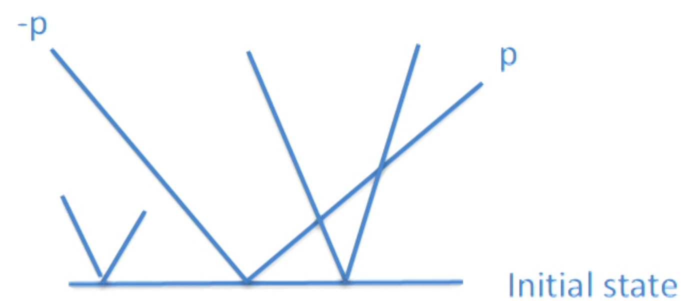
(i) Fullfilled for all cases where closed overlap formula is known

(ii) Discrete version of integrable boundary state condition Piroli, Pozsgay Vernier '17

Ghoshal, Zamolodchikov '93



Wick rotation



Pure reflection
BYB for reflection matrix

Entangled (p,-p) pairs $Q_{2m+1}|\Psi_0\rangle = 0$
BYB for initial state

"Rationalization"

Reflection matrix which fulfills BYB of SO(6) spin chain and has the appropriate symmetries can be found for the two cases with $Q_{2m+1}|\Psi_0\rangle = 0$

D3-D5 brane case:

Piroli, Pozsgay Vernier '19

C.K, Pozsgay Vardinghus, Wilhelm, to appear

$$K_{ij}(u) = g(u)S_iS_j + \tilde{g}(u)S_jS_i + f(u)\delta_{ij}, \qquad [S_i, S_j] = i\epsilon_{ijk}S_k$$
$$K_{iI}(u) = K_{Ii}(u) = 0,$$
$$K_{IJ}(u) = h(u)\delta_{IJ},$$

$$i, j \in \{1, 2, 3\}, \quad I, J \in \{4, 5, 6\}$$

$$g(u) = 2(u^2 - 1), \quad \tilde{g}(u) = -2u(u+1),$$
$$f(u) = u\,(u^2 + u + C), \quad h(u) = -u\,(u^2 + u - C)$$

New element: Reflection matrix carries internal degrees of freedom. Reflects the auxiliary space of the MPS

D3-D7 set-up with SO(5) symmetry: Same idea works

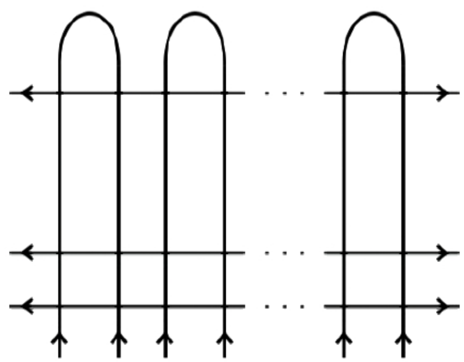
Derivation of the overlap formula

Rough idea: (has so far only been implemented in SU(2) sub-sector for D3-D5 case).

$$\langle \text{MPS}| \sim \langle K(u^*)|^{\otimes L/2} \sim \quad \bigcap \quad \bigcap \quad \ldots \quad \bigcap$$

$$\langle \text{MPS}|\{u_i\}\rangle \sim \langle K(u^*)K(u^*)\ldots K(u^*)|B(u_1)\ldots B(u_M)|0\rangle$$

$$\sim \text{Partition function of vertex model with certain boundary conditions}$$



Korepin '82, Izgerzin '87
Tsuchiya '98
Pozsgay '13, Brockmann et al '14
Foda and Zarembo '15

For SU(2)-subsector: 6-vertex model: has been implemented

For SU(3)-subsector: 15 vertex model
For SO(6): 96 vertex model needed

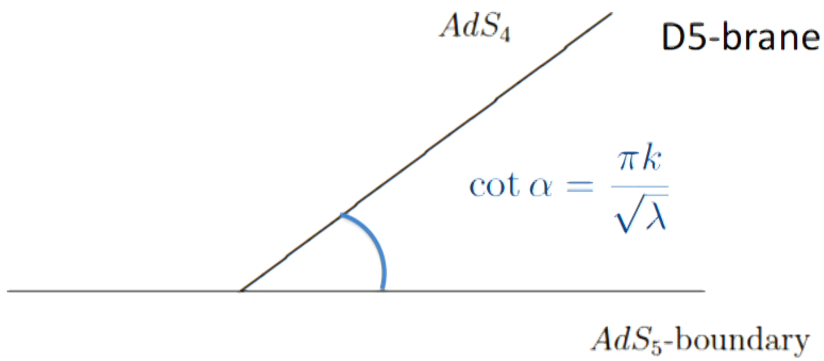
Integrability of MPS

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
Flux	k	k_1, k_2	$d_G = \frac{(n+1)(n+2)(n+3)}{6}$
$\|MPS\rangle$	Integrable	Non-integrable	Integrable
One-point functions	Closed expression found tree level and one-loop	—	— ?

Comparison between gauge and string theory

The double scaling limit

D3-D5 probe brane system suggests a new double scaling limit Nagasaki & Yamaguchi '12,



$$\lambda \to \infty, \; k \to \infty, \; \frac{\lambda}{k^2} \text{ finite} \qquad (N \to \infty)$$

One can compare perturbative gauge theory to semi-classical string theory (or sugra).

Similar idea works for the two D3-D7 set-ups C.K., Semenoff & Young '12,

The double scaling parameter

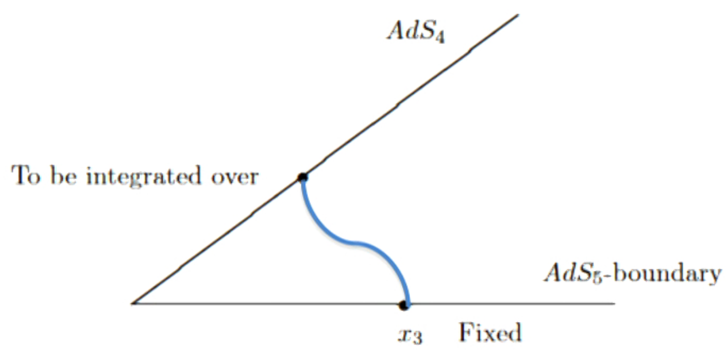
	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$\mathrm{AdS}_4 \times \mathrm{S}^2$	$\mathrm{AdS}_4 \times \mathrm{S}^2 \times \mathrm{S}^2$	$\mathrm{AdS}_4 \times \mathrm{S}^4$
Flux/Instanton number	k	k_1, k_2	$d_G = \frac{(n+1)(n+2)(n+3)}{6}$
Double scaling parameter	$\frac{\lambda}{k^2}$	$\frac{\lambda}{k_1^2+k_2^2}$	$\frac{\lambda}{n^2}$

One-point functions of chiral primaries --- GKPW method

- Only one chiral primary with the appropriate symmetry for each (even) conf.dim. Δ
- Find the variation $\delta S_E = \delta(S_{DBI} + S_{WZ})$
- Expand fluctuations in terms of spherical harmonics on S^5

$$\delta S_E(X,\Omega) = \sum_{\Delta}\sum_{I} s_{\Delta I}(X)\, Y_{\Delta I}(\Omega)$$

- Pick $Y_{\Delta I}(\Omega)$ the wanted unique chiral primary
- Replace $s_{\Delta I}(X)$ with a bulk-to-boundary propagator reaching from a point z on the brane to x_3 at the boundary. Integrate over z.



Nagasaki & Yamaguchi '12,

Results in d.s.l. for chiral primary of length L

Match at leading order in d.s.l. both for D3-D5 and D3-D7

Nagasaki & Yamaguchi '12, C.K. , Semenoff, Young '12,

Next to leading order predictions

D3-D5 set-up:

Nagasaki & Yamaguchi '12,

$$\frac{\langle Y_L(\lambda)\rangle}{\langle Y_L(0)\rangle}\bigg|_{sugra} = 1 + \frac{\lambda}{4\pi^2 k^2}\frac{L(L+1)}{4(L-1)} + \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2 k^2}\right)^2\right)$$

D3-D7 set-up with SO(3) x SO(3) symmetry

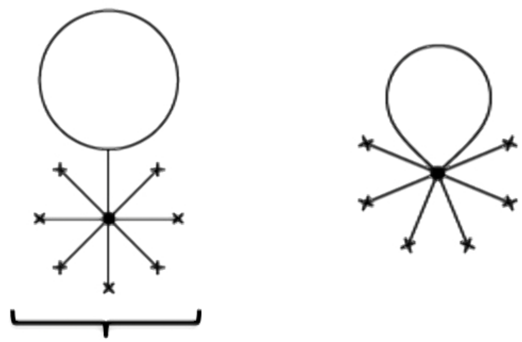
Grau, C.K, Volk & Wilhelm, '18

$$\frac{\langle Y_L(\lambda)\rangle}{\langle Y_L(0)\rangle}\bigg|_{sugra} = 1 + \frac{\lambda}{4\pi^2(k_1^2+k_2^2)}\frac{1}{[k_1^2+k_2^2]^3\,(L-1)\sin(L+2)\phi}\Big[$$
$$+ 4Lk_1k_2\left[(k_1)^4+(k_2)^4+(k_1k_2)^2(L+1)\right]\cos L\phi$$
$$+ \left[(k_2)^2-(k_1)^2\right]\left[4(k_1k_2)^2(L^2+L-1)+((k_1)^4+(k_2)^4)(L^2+3L-2)\right]\sin L\phi\Big]$$
$$+ \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2(k_1^2+k_2^2)}\right)^2\right), \qquad \phi = \arctan\left(\frac{k_1}{k_2}\right)$$

The one-loop computation in QFT

Two planar diagrams (no corrections to the eigenstate)

Scalars, gauge fields,
fermions, ghosts



$= 0$ for D3-D5 (susy)

$\neq 0$ for D3-D7 (no susy)

Challenges

- Diagonalization of complicated mass matrix needed (all scalars have vevs for SO(3) x SO(3) case and mix with each other and the gauge field)
- Propagators are in (auxiliary) AdS_4 space
- Careful regularization needed (dimensional regularization in combination with dimensional reduction)

Comparison to string theory

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
Flux/Instanton number	k	k_1, k_2	$d_G = \frac{(n+1)(n+2)(n+3)}{6}$
Double scaling parameter	$\frac{\lambda}{k^2}$	$\frac{\lambda}{k_1^2+k_2^2}$	$\frac{\lambda}{n^2}$
Match with string theory to two leading orders	Yes	Yes	Work in progress

Results in d.s.l. for chiral primary of length L

Match at leading order in d.s.l. both for D3-D5 and D3-D7 — Nagasaki & Yamaguchi '12, C.K. , Semenoff, Young '12,

Next to leading order predictions

D3-D5 set-up:

Nagasaki & Yamaguchi '12,

$$\left.\frac{\langle Y_L(\lambda)\rangle}{\langle Y_L(0)\rangle}\right|_{sugra} = 1 + \frac{\lambda}{4\pi^2 k^2}\frac{L(L+1)}{4(L-1)} + \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2 k^2}\right)^2\right)$$

D3-D7 set-up with SO(3) x SO(3) symmetry

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$$\left.\frac{\langle Y_L(\lambda)\rangle}{\langle Y_L(0)\rangle}\right|_{sugra} = 1 + \frac{\lambda}{4\pi^2(k_1^2+k_2^2)}\frac{1}{\left[k_1^2+k_2^2\right]^3(L-1)\sin(L+2)\phi}\Big[$$
$$+ 4Lk_1k_2\left[(k_1)^4+(k_2)^4+(k_1k_2)^2(L+1)\right]\cos L\phi$$
$$+ \left[(k_2)^2-(k_1)^2\right]\left[4(k_1k_2)^2(L^2+L-1)+((k_1)^4+(k_2)^4)(L^2+3L-2)\right]\sin L\phi\Big]$$
$$+ \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2(k_1^2+k_2^2)}\right)^2\right), \qquad \phi = \arctan\left(\frac{k_1}{k_2}\right)$$

Comparison to string theory

	D3-D5	D3-D7	D3-D7
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Brane geometry	$AdS_4\times S^2$	$AdS_4\times S^2\times S^2$	$AdS_4\times S^4$
Flux/Instanton number	k	k_1, k_2	$d_G=\frac{(n+1)(n+2)(n+3)}{6}$
Double scaling parameter	$\frac{\lambda}{k^2}$	$\frac{\lambda}{k_1^2+k_2^2}$	$\frac{\lambda}{n^2}$
Match with string theory to two leading orders	Yes	Yes	Work in progress

Summary and open problems

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
$\vert MPS\rangle$	Integrable	Non-integrable	Integrable
One-point functions	Closed expression exists tree level and one-loop	—	— ?
Match with string theory	Yes	Yes	work in progress

Future directions

- Comparisons with string theory for one-point functions of non-protected operators---f.inst. spinning strings
- Comparison of two-point functions between gauge and string theory
- Derivation of overlap formula in full SO(6) sector (and for D3-D7)
- Moving on to higher loop orders. Suggestion for asymptotic formula exists --- need to understand wrapping
- MPS and determinant formulas also appear for 3-point functions involving giant gravitons. Jiang, Komatsu & Vescovi '19
- Boundary conformal bootstrapping using one and two-point functions as input

Thank you