Title: TBD

Speakers: Leonardo Rastelli

Collection: Boundaries and Defects in Quantum Field Theory

Date: August 06, 2019 - 11:00 AM

URL: http://pirsa.org/19080053

Exploring a Famous Irrelevant Deformation of $\mathcal{N} = 4$ SYM

Leonardo Rastelli

YITP, Stony Brook & CERN

Work in progress with João Caetano and Wolfger Peelaers

Boundaries and Defects in CFT

Perimeter Institute, August 6 2019

Upstream the RG?

In this talk, I will revisit a famous irrelevant deformation of $\mathcal{N} = 4$ SYM,

$$\mathcal{L}_{\mathrm{SYM}} + h\,\mathcal{O}_8 + \ldots$$

$\mathcal{O}_8$ is the leading single-trace *irrelevant* operator $(E = 8)$ that preserves all 16 Q's in $\mathbb{R}^4$.

A maximally SUSY RG flow ending in planar $\mathcal{N} = 4$ SYM generically takes this form in the IR.

Recent remarkable progress in flowing up the RG in $d = 2$:
$T\bar{T}$ deformation appears to be well-defined. The UV theory is *not* a conventional local QFT.

$\mathcal{O}_8$ is in some ways a $d = 4$, $\mathcal{N} = 4$ version of $T\bar{T}$. Analogous story in $\mathcal{N} = 4$ SYM?

To preserve SUSY, need to add top component of a multiplet.

Leading irrelevant deformation is $Q^4\tilde{Q}^4\mathbf{105}$, top component of the 1/2 BPS multiplet in the four-index symmetric traceless irrep of $SO(6)_R$. Unique F-term that preserves full R-symmetry.

Single-trace version:

$$Q^4\tilde{Q}^4\,\mathrm{Tr}\,\phi^{(i}\phi^j\phi^k\phi^{l)} = \mathrm{Tr}\left[F^4 - \frac{1}{4}(F^2)^2 + 2D_m\phi^i D_m\phi^i D_n\phi^j D_n\phi^j - D_m\phi^i D_n\phi^i D_m\phi^j D_n\phi^j + \ldots\right]$$

The field strength piece $F^4 - \frac{1}{4}(F^2)^2 = (F^+)^2(F^-)^2$ is the characteristic structure from expansion of DBI action $\sqrt{\det_4(\eta_{mn} + F_{mn})}$.

Double-trace version:

$$Q^4\tilde{Q}^4\,\mathrm{Tr}\,\phi^{(i}\phi^j\,\mathrm{Tr}\,\phi^k\phi^{l)} = T_{mn}T_{mn} + \mathcal{O}_\tau\mathcal{O}_{\bar\tau} + \ldots$$

A bit like $T\bar{T}$ indeed! But it clearly does *not* have the same "semi-topological" nature.
We will attempt to define the deformation in conformal perturbation theory, not in terms of an evolution equation at finite h.

Most intriguing example of such a flow is the full EFT on N D3 branes, $h \sim (\alpha')^2$.

Long-standing dream to generalize the AdS/CFT duality:
Full D3 brane geometry (asymptotic to flat space) $\leftrightarrow$ full D3 brane effective action?

Intriligator speculated that closed string theory on

$$ds^2 = H^{-1/2} dx_m dx_m + H^{1/2} dx_i dx_i\,, \quad H(r) = \tilde{h} + \frac{R^4}{r^4}\,, \quad R^4 = 4\pi g_s N (\alpha')^4$$

is dual to to

$$\mathcal{L}_{\mathrm{SYM}} + \tilde{h} R^4 \mathcal{O}_8\,.$$

This proposal passes a leading order check. LR van Raamsdonk

An obvious difficulty of this idea is making sense of the field theory side. Full open SFT?
Can we now make progress?

Outline

- Motivations. Universal leading irrelevant deformation.
- The deformation on $S^3 \times \mathbb{R}$. Off-shell classical action.
- All-loop considerations from $SU(2|2) \times SU(2|2)$ symmetry.
- Perturbative analysis.
- Holographic interpretation? Further comments on integrability.
- Outlook

On $S^3 \times \mathbb{R}$

One novelty of our approach is that we will study the deformation on $S^3 \times \mathbb{R}$.

(A clean spectral problem, but the intuition about UV completion being related to full D3 brane geometry is unlikely to apply.)

For $h = 0$, map $\mathbb{R}^4 \to S^3 \times \mathbb{R}$ by Weyl transformation.

Full $\mathfrak{psu}(2,2|4)$ superalgebra is of course preserved. Fermionic generators

$$\mathcal{Q}^A_\alpha\,, \qquad \mathcal{Q}^{\dagger\alpha}_A\,, \qquad \tilde{\mathcal{Q}}_{A\dot\alpha}\,, \qquad \tilde{\mathcal{Q}}^{\dagger A\dot\alpha}$$

A, B $SU(4)_R$ indices, α, $\dot\alpha$ spinor indices of $SO(4) \cong SU(2)_\alpha \times SU(2)_{\dot\alpha}$ isometry of S^3.

For $h \neq 0$, we can preserve the subgroup of "rigid" (i.e. non-conformal) superisometries

$$\mathfrak{psu}(2|2) \times \mathfrak{psu}(2|2) \ltimes \mathbb{R}^2$$

The preserved bosonic symmetries comprise the isometries

$$SO(4) \times \mathbb{R}_\tau \cong SU(2)_\alpha \times SU(2)_{\dot{\alpha}} \times \mathbb{R}_\tau$$

and the R-symmetries

$$SU(2)_a \times SU(2)_{\dot{a}} \times U(1)_J \subset SU(4)_R$$

The preserved fermionic generators

$$\mathcal{Q}^a_\alpha\,, \qquad \mathcal{Q}^{\dagger\alpha}_a\,, \qquad \tilde{\mathcal{Q}}_{\dot{a}\dot{\alpha}}\,, \qquad \tilde{\mathcal{Q}}^{\dagger\dot{a}\dot{\alpha}}$$

close into *two* copies of $\mathfrak{su}(2|2)$, with *common* central extension $H - J$, where H is the generator of τ translations.

To preserve SUSY on $S^3 \times \mathbb{R}$, the irrelevant deformation $h\mathcal{O}_8$ must be supplemented with curvature corrections $O(1/\ell^k)$, where ℓ is the radius of S^3.

A priori a hard problem, leading to an *infinite* expansion (unlike the case of relevant deformations).

Fortunately, we found an elegant off-shell formalism (following Berkovits, Evans, Pestun).

In 10d notation,

$$\begin{aligned}
\delta_{(\varepsilon,\nu_\ell)} A_M &= \bar{\varepsilon}\Gamma_M \Psi \\
\delta_{(\varepsilon,\nu_\ell)} \Psi &= -\frac{1}{2} F_{MN}\Gamma^{MN}\varepsilon - \frac{1}{2}\Gamma_{\mu i}\phi^i \nabla^\mu \varepsilon - K^\ell \nu_\ell \\
\delta_{(\varepsilon,\nu_\ell)} K_\ell &= -\nu_\ell \Gamma^M D_M \Psi \ .
\end{aligned}$$

(M, N 10d indices; μ, ν $4d$ spacetime indices, $i = 5, 6, 7, 8, 9, 0$ $6d$ R-symmetry indices).

Seven auxiliary fields K^ℓ. These transformations close off-shell, if the auxiliary spinors ν_ℓ satisfy the following constraints

$$\bar{\varepsilon}\Gamma^M \nu_\ell = 0 \ , \qquad \bar{\nu}_\ell \Gamma^M \nu_m = \delta_{\ell m}\bar{\varepsilon}\Gamma^M \varepsilon \ .$$

We can linearly realize 8 of the 16 supersymmetries, the ones with say $J = 1/2$, by setting

$$\nu_{\hat{m}} = \Gamma_{\hat{m}4}\varepsilon_- \qquad \text{and} \qquad \nu_s = \Gamma_{s4}\varepsilon_- \ ,$$

where ε_- has $J = -1/2$.

Remarkably, this $7 = 4 + 3$ split preserves the full isometries of $S^3 \times \mathbb{R}$ and the full $SO(4)_R \times U(1)_J$ R-symmetry, if we declare that $K^{\hat{\mu}}$ transforms as a spatial vector on S^3 and K^s as a vector of $SO(4)_R$.

The remaining 8 supersymmetries are realized on-shell, provided we turn on an imaginary $U(1)_J$ background connection along (Euclidean) time direction τ

$$V = \frac{i}{\ell} d\tau \, , \quad D_\tau \equiv \partial_\tau + \frac{J}{\ell}$$

It is convenient to rename the six scalar fields as

$$Z\,, \qquad \bar{Z}\,, \qquad \phi_{a\dot{a}}\,,$$

with $U(1)_J$ assignments $J(Z) = +1$, $J(\bar{Z}) = -1$, $J(\phi_{a\dot{a}}) = 0$.

It is immediate to set up a superspace formalism, defining the superfield

$$\bar{Z}(\theta_{a\alpha}, \tilde{\theta}^{\dot{a}\dot{\alpha}}) = \bar{Z} - 2i\epsilon^{ab}\epsilon^{\alpha\beta}\Psi_{-a\alpha}\,\theta_{b\beta} - 2i\epsilon_{\dot{a}\dot{b}}\epsilon_{\dot{\alpha}\dot{\beta}}\Psi_{-}^{\dot{a}\dot{\alpha}}\,\tilde{\theta}^{\dot{b}\dot{\beta}} + \ldots$$

On the other hand,

$$\delta_{\varepsilon_-} Z = 0\,.$$

We now have the full classical action

$$S(g_{\mathrm{YM}}, h) = S_{\mathrm{SYM}} + h \int_{S^3\times\mathbb{R}} \sqrt{g}\, d^4x \int d^4\theta d^4\tilde{\theta}\, \mathrm{Tr}\, \bar{Z}(\theta,\tilde{\theta})^4 + \mathrm{h.c.}$$

Invariance under 8 supercharges manifest, and one easily checks (on-shell) invariance under the other 8.

Integrating out the auxiliary fields generate an infinite power expansion in h.

$$
\begin{aligned}
\bar{Z}(\theta_{-I\alpha}, \theta_{-}^{\hat{I}\dot{\alpha}}) = {} & \bar{Z} - 2i\epsilon^{IJ}\epsilon^{\alpha\beta}\Psi_{-I\alpha}\,\theta_{-J\beta} - 2i\epsilon_{\hat{I}\hat{J}}\epsilon_{\dot{\alpha}\dot{\beta}}\Psi_{-}^{\hat{I}\dot{\alpha}}\,\theta_{-}^{\hat{J}\dot{\beta}} \\
& + \frac{1}{2}2i\epsilon^{IJ}\epsilon^{\alpha\beta}\left(+\frac{1}{2}F^{mn}(\sigma_{mn})_{\alpha}{}^{\gamma}\delta_{I}^{K} + \frac{1}{2}[\phi^{a}, \phi^{b}](\sigma_{a-4,b-4})_{I}{}^{K}\delta_{\alpha}^{\gamma} + K^{\hat{m}}(\sigma_{\hat{m}4})_{\alpha}{}^{\gamma}\delta_{I}^{K}\right)\theta_{-K\gamma}\theta_{-J\beta} \\
& + \frac{1}{2}2i\epsilon_{\hat{I}\hat{J}}\epsilon_{\dot{\alpha}\dot{\beta}}\left(\frac{1}{2}F^{mn}(\bar{\sigma}_{mn})^{\dot{\alpha}}{}_{\dot{\gamma}}\delta^{\hat{I}}_{\hat{K}} + \frac{1}{2}[\phi^{a}, \phi^{b}](\bar{\sigma}_{a-4,b-4})^{\hat{I}}{}_{\hat{K}}\delta^{\dot{\alpha}}_{\dot{\gamma}} + K^{\hat{m}}(\bar{\sigma}_{\hat{m}4})^{\dot{\alpha}}{}_{\dot{\gamma}}\delta^{\hat{I}}_{\hat{K}}\right)\theta_{-}^{\hat{K}\dot{\gamma}}\theta_{-}^{\hat{J}\dot{\beta}} \\
& + 2i\epsilon_{\hat{K}\hat{J}}\epsilon_{\dot{\zeta}\dot{\beta}}\left(D_{m}\phi_{a} - \delta_{m}^{4}\left(K^{a} + \frac{1}{\ell}\phi^{a}\right)\right)(\bar{\sigma}^{a-4})^{\hat{K}K}(\bar{\sigma}^{m})^{\dot{\zeta}\zeta}\theta_{-K\zeta}\theta_{-}^{\hat{J}\dot{\beta}} \\
& - \frac{1}{2}2i\epsilon_{\hat{K}\hat{J}}\epsilon_{\dot{\zeta}\dot{\beta}}\Big(\delta_{\hat{L}}^{\hat{K}}\Big((-(\bar{\sigma}^{m})^{\dot{\gamma}\zeta}D_{m}\Psi^{\dot{\alpha}}_{+I} + \frac{4}{2\ell}\Psi^{\dot{\alpha}}_{+I}(\bar{\sigma}_{4})^{\dot{\gamma}\zeta})\epsilon^{IK}(\epsilon_{\dot{\alpha}\dot{\eta}}\delta^{\dot{\zeta}}_{\dot{\gamma}} + \epsilon_{\dot{\gamma}\dot{\eta}}\delta^{\dot{\zeta}}_{\dot{\alpha}}) \\
& \qquad + (D_{m}\Psi^{\dot{\alpha}}_{+I}(\sigma_{m})_{\alpha\dot{\alpha}}\epsilon^{IK} - i[Z, \Psi_{-I\alpha}]\epsilon^{IK} - [\phi_{b}, \Psi^{\hat{I}}_{+\alpha}]\epsilon_{\hat{I}\hat{M}}(\bar{\sigma}_{b-4})^{\hat{M}K})(\bar{\sigma}^{4})^{\dot{\alpha}\zeta}(\bar{\sigma}_{4})^{\dot{\delta}\alpha}(\epsilon_{\dot{\alpha}\dot{\eta}}\delta^{\dot{\zeta}}_{\dot{\delta}} + \epsilon_{\dot{\delta}\dot{\eta}}\delta^{\dot{\zeta}}_{\dot{\alpha}}) \\
& \qquad + [\Psi^{\hat{I}}_{+\alpha}\epsilon^{\alpha\zeta}\delta^{\dot{\zeta}}_{\dot{\eta}}, (\bar{\sigma}^{a-4})^{\hat{M}K}\phi_{a}](\epsilon_{\hat{I}\hat{L}}\delta^{\hat{K}}_{\hat{M}} + \epsilon_{\hat{M}\hat{L}}\delta^{\hat{K}}_{\hat{I}})\Big)\theta_{-}^{\hat{L}\dot{\eta}}\theta_{-K\zeta}\theta_{-}^{\hat{J}\dot{\beta}} \\
& - \frac{1}{2}2i\epsilon_{\hat{K}\hat{J}}\epsilon_{\dot{\zeta}\dot{\beta}}\Big(\epsilon^{LK}\Big((D_{m}\Psi^{\hat{K}}_{+\alpha}(\bar{\sigma}^{m})^{\dot{\zeta}\theta} - \frac{4}{2\ell}\Psi^{\hat{K}}_{+\alpha}(\bar{\sigma}_{4})^{\dot{\zeta}\theta})(\epsilon^{\alpha\zeta}\delta^{\eta}_{\theta} + \epsilon^{\alpha\eta}\delta^{\zeta}_{\theta}) \\
& \qquad + (-D_{m}\Psi^{\hat{K}}_{+\delta}(\bar{\sigma}_{m})^{\dot{\alpha}\delta} + i[Z, \Psi_{-}^{\hat{K}\dot{\alpha}}] + [(\bar{\sigma}_{b-4})^{\hat{K}I}\phi_{b}, \Psi^{\dot{\alpha}}_{+I}])(\bar{\sigma}^{4})^{\dot{\zeta}\theta}(\sigma_{4})_{\alpha\dot{\alpha}}(\epsilon^{\alpha\zeta}\delta^{\eta}_{\theta} + \epsilon^{\alpha\eta}\delta^{\zeta}_{\theta})\Big) \\
& \qquad + [-\Psi^{\dot{\zeta}}_{+I}\epsilon^{\zeta\eta}, (\bar{\sigma}^{a-4})^{\hat{K}M}\phi_{a}](\epsilon^{IL}\delta^{K}_{M} + \epsilon^{IK}\delta^{L}_{M})\Big)\theta_{-L\eta}\theta_{-K\zeta}\theta_{-}^{\hat{J}\dot{\beta}}
\end{aligned}
$$

To leading order in h, classical Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm SYM} + h\left[\mathcal{O}_8 + \frac{\mathcal{O}_7}{\ell} + \ldots \frac{\mathcal{O}_4}{\ell^4}\right] + O(h^2)$$

where $\mathcal{O}_i$ are components of the $\mathbf{105}$ supermultiplet. For example,

$$\mathcal{O}_4 = \mathrm{STr}(3Z^2\bar{Z}^2 - 6Z\bar{Z}\phi_p\phi_p + \phi_p\phi_p\phi_q\phi_q)$$

is the $SO(4)_R \times U(1)_J$ singlet piece of the superprimary.

At the quantum level, must of add counterterms and fine tune.
The procedure is intrinsically ambiguous, barring additional input.

Spectral problem and spin chains

State/operator map is lost, but it makes perfect sense to ask how the energy spectrum of *states*. Usual spin-chain picture.

The spectrum of the planar theory is calculated by a deformation $H(g^2, h)$ of the spin-chain Hamiltonian $H(g^2)$ of $\mathcal{N} = 4$ SYM.

Much of the familiar story goes through.
Our deformation is hermitian and preserve "parity" (in the spin chain sense).

For $h \neq 0$, magnons are *not* Goldstones, but their dispersion relation and their S-matrix is still constrainted by Beisert's triply centrally extended $\mathfrak{su}(2|2)$.

$$\{\mathcal{S}^{\alpha}{}_{a}, \mathcal{Q}^{b}{}_{\beta}\} = \delta^{a}_{b}\mathcal{L}^{\alpha}{}_{\beta} + \delta^{\alpha}_{\beta}\mathcal{R}^{b}{}_{a} + \frac{1}{2}\delta^{a}_{b}\delta^{\alpha}_{\beta}\,(H - J)\ .$$

$$\{\mathcal{Q}^{a}_{\alpha}\,, \mathcal{Q}^{b}_{\beta}\} = \epsilon^{ab}\epsilon_{\alpha\beta}\,\mathcal{P}\,, \quad \{\mathcal{S}^{\alpha}_{a}\,, \mathcal{S}^{b}_{\beta}\} = \epsilon_{ab}\epsilon^{\alpha\beta}\,\mathcal{K}\,,$$

By the *same* argument, magnon dispersion relation

$$H - J = \sqrt{1 + 8a^2 \sin^2 \frac{p}{2}}\,,$$

where a^2 is *a priori* a function of $g^2 \sim \lambda$ and h.

The $SU(2|2)$ symmetry also completely fixes the matrix structure of $2 \to 2$ scattering.

The only freedom in the $2 \to 2$ S-matrix is thus in the dressing phase.

Of course, still non-trivial to ask whether $n \to n$ scattering factorizes.
To try and answer this question we turn to explicit calculations.

The $SU(2|2) \ltimes U(1)$ sector

Impractical to do perturbation theory using the complicated action we derived.
But symmetry-based methods are very powerful.

We restrict to the subsector with elementary fields

$$\phi_a \equiv \phi_{a\mathrm{i}} , \qquad \psi_\alpha \equiv \psi_{\alpha\mathrm{i}} \quad , \qquad Z .$$

where $a = 1, 2$ and $\alpha = \pm$. This subsector is analogous to the $SU(2|3)$ sector of $\mathcal{N} = 4$ SYM (with $Z \to \phi_3$), but we have the *smaller* symmetry $SU(2|2) \ltimes U(1)$.

We have a double expansion in g^2 and h, but from the abstract symmetry viewpoint they appear on the same footing. A term $O(g^{2i}h^j)$ corresponds to $2i + j$ "loop" order.

Following Beisert, we use symmetry to constrain the action of the generators of $SU(2|2) \ltimes U(1)$ acting on spin-chain states.
As usual, the symbols

$$\left\{ {A_1 \ldots A_n \atop B_1 \ldots B_m} \right\}$$

represent the tensor structures. At a given "loop" order k the generators should take the form

$$J_k \sim \left\{ {A_1 \ldots A_n \atop B_1 \ldots B_m} \right\}, \text{ with } n + m = k + 2$$

By imposing closure of the algebra (and parity) we find that at one-loop

$$\begin{aligned} H_2 \quad &= d_1 (\left\{ {ab \atop ab} \right\} + \left\{ {a\beta \atop a\beta} \right\} + \left\{ {\alpha b \atop \alpha b} \right\} + \left\{ {\alpha\beta \atop \alpha\beta} \right\} - \left\{ {ab \atop ba} \right\} - \left\{ {a\beta \atop \beta a} \right\} - \left\{ {\alpha b \atop b\alpha} \right\} + \left\{ {\alpha\beta \atop \beta\alpha} \right\} \\ &+ \left\{ {Z\beta \atop Z\beta} \right\} + \left\{ {\beta Z \atop \beta Z} \right\} + \left\{ {Zb \atop Zb} \right\} + \left\{ {bZ \atop bZ} \right\} - \left\{ {Z\beta \atop \beta Z} \right\} - \left\{ {Zb \atop bZ} \right\} - \left\{ {bZ \atop Zb} \right\} - \left\{ {\beta Z \atop Z\beta} \right\}) \end{aligned}$$

There is only one overall constant, and this is in fact the familiar one-loop result for the $SU(2|3)$ chain. Automatic symmetry enhancement to this lowest order.

At two loops, calculation more involved but still feasible.

Closure of the $SU(2|2) \ltimes U(1)$ algebra and hermiticity fix H_4 up to 10 real parameters, of which 7 are in fact redundant (can be eliminated by similarity or correspond to structures that vanish on cyclic states).

Finally, 2 parameters are fixed by enforcing the dispersion relation. (Equivalently, they could be fixed by working with the triply centrally extended $SU(2|2)$).

The final parameter is the overall normalization of the coupling,

The deformation makes no difference even at two loops!
To this order, again automatic symmetry enhancement to $SU(2|3)$.
Clearly, no test of integrability yet.

Three loop calculation in progress...

We can however ask more structural questions. Restricting for now to the $SU(2|2) \ltimes U(1)$ subsector, does there even *exist* an integrable long-range spin-chain which is *different* from the $\mathcal{N} = 4$ case?

Beisert, Fievét, de Leeuw, Loebbert performed a general analysis of integrable long range XXZ chains. They found a large class of models.

$$\exp(ip(u_k)L) = \prod_{j \neq k} \exp(-2i\theta(u_k, u_j)) \frac{\sinh \hbar(u_k - u_j + i)}{\sinh \hbar(u_k - u_j - i)}$$

$$\theta(u_k, u_j) = \sum_{s>r=2}^{\infty} \beta_{r,s}(q_r(u_k)q_s(u_j) - q_s(u_k)q_r(u_j)) + \sum_{r=2}^{\infty} \eta_r(q_r(u_k) - q_r(u_j))$$

We should repeat the analysis for the $SU(2|2) \times U(1)$ chain!

Note that the XXZ chain can be viewed as a closed of subsector of the $SU(2|2) \ltimes U(1)$ chain. Since the only departure from $\mathcal{N} = 4$ can be in the dressing phase, we must take $\hbar \to 0$. Still, at least for XXZ, ample freedom is left.

In the standard $\mathcal{N}=4$ story, symmetry enhancement $SU(2|2) \to SU(2|3)$ from Bethe roots at infinity (a state with additional roots at infinity still satisfies the Bethe ansatz and has the same energy).

For us, dressing phase will break the symmetry (at sufficiently high loop order), so adding Bethe roots at infinity does *not* in general give a new solution.

From BFdLL analysis, expect that scattering that of two roots at $u=\infty$ is trivial (the $q_i \to 0$). This makes a prediction: the "old" BPS states of $\mathcal{N}=4$ must remain protected for $h \neq 0$. This appears to be compatible with an analysis of the superoconformal index.

Holographic interpretation?

A natural setting are the "bubbling" geometries of Lin Lunin Maldacena (LLM). Generally, $SU(2|2) \times SU(2|2)$ symmetry and one can impose additional $U(1)_J$.

LLM geometries describe the backreaction of "additional" D3 branes (giant gravitons) in global $AdS_5 \times S^5$. The extreme UV asymptotics are always $AdS_5 \times S^5$.

An LLM geometry is fully specified by prescribing $\pm 1/2$ boundary conditions on a two-dimensional plane: bicoloring of the plane.

Dually, it corresponds to considering $\mathcal{N} = 4$ in a non-trivial half-BPS *state*, with $E \sim N^2$. In a sense, an $S^3 \times \mathbb{R}$ version of a Coulomb branch flow. de Mello Koch Huang Tribelhorn have studied this setup with intriguing results.

(a) (b)

(a) is the LLM picture for $AdS_5 \times S^5$

(b) is the simplest non-trivial LLM geometry with $U(1)_J$ isometry.
Denoting the outer circle as R_1 and the inner circle as R_2, we can identify

$$\frac{h}{\ell^4} \sim \frac{R_2^2}{R_1^2}$$

Sadly, Chervonyi and Lunin's analysis appears to indicate that the classical sigma model for (b) is not integrable. The massless geodesic equation (corresponding to pointlike strings) is not separable.

Side note: for full asymptotically flat D3 brane geometry, Stepanchuk and Tseytlin found the string sigma model to be not integrable (though massless geodesics are fine).

Outlook

- 3-loop analysis of $SU(2|2) \ltimes U(1)$ chain in progress.
- $SU(2|2)$ integrable long-range chain? Which RG flow would it describe on field theory side?
- Interesting to explore relation with LLM, regardless of integrability.
- Other geometries: $\mathbb{R}^4$, S^4.
- Double-trace version.