Title: PSI 2019/2020 - Lie Groups and Lie Algebras - Lecture 1

Speakers: Tibra Ali

Collection: PSI 2019/2020 - Lie Groups and Lie Algebras (Ali)

Date: August 07, 2019 - 10:45 AM

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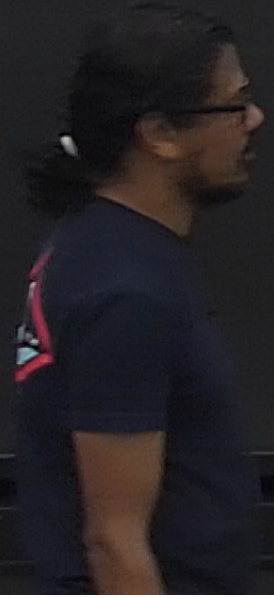
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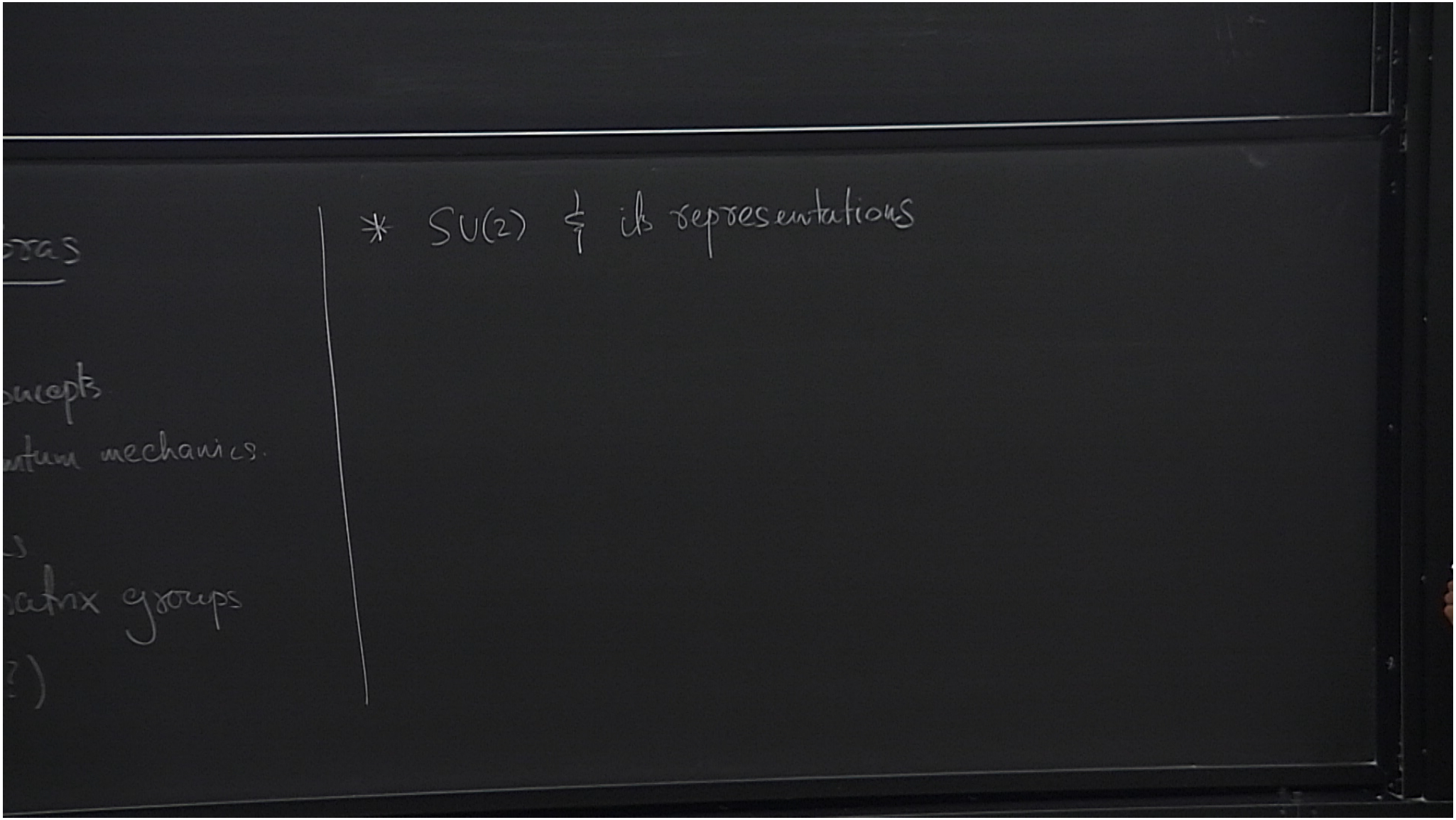
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Tutorial tomorrow at 3:45 – 5:15

Lie Groups and Lie Algebras

* Motivations
* Basic Defn & important concepts
* Group Theory relates to quantum mechanics.
* Lie groups & Lie algebras
* list of some important matrix groups
* Applications to geometry (?)




* SU(2) & its representations

...ras

...ncepts

...ntum mechanics.

...s

...atrix groups

...?)

* SU(2) & its representations
* Young's tableau for SU(N)
* Tensor methods for SU(N)
* Roots & Weights for SU(3)
* Spinor rep of the Lorentz group (Maybe)

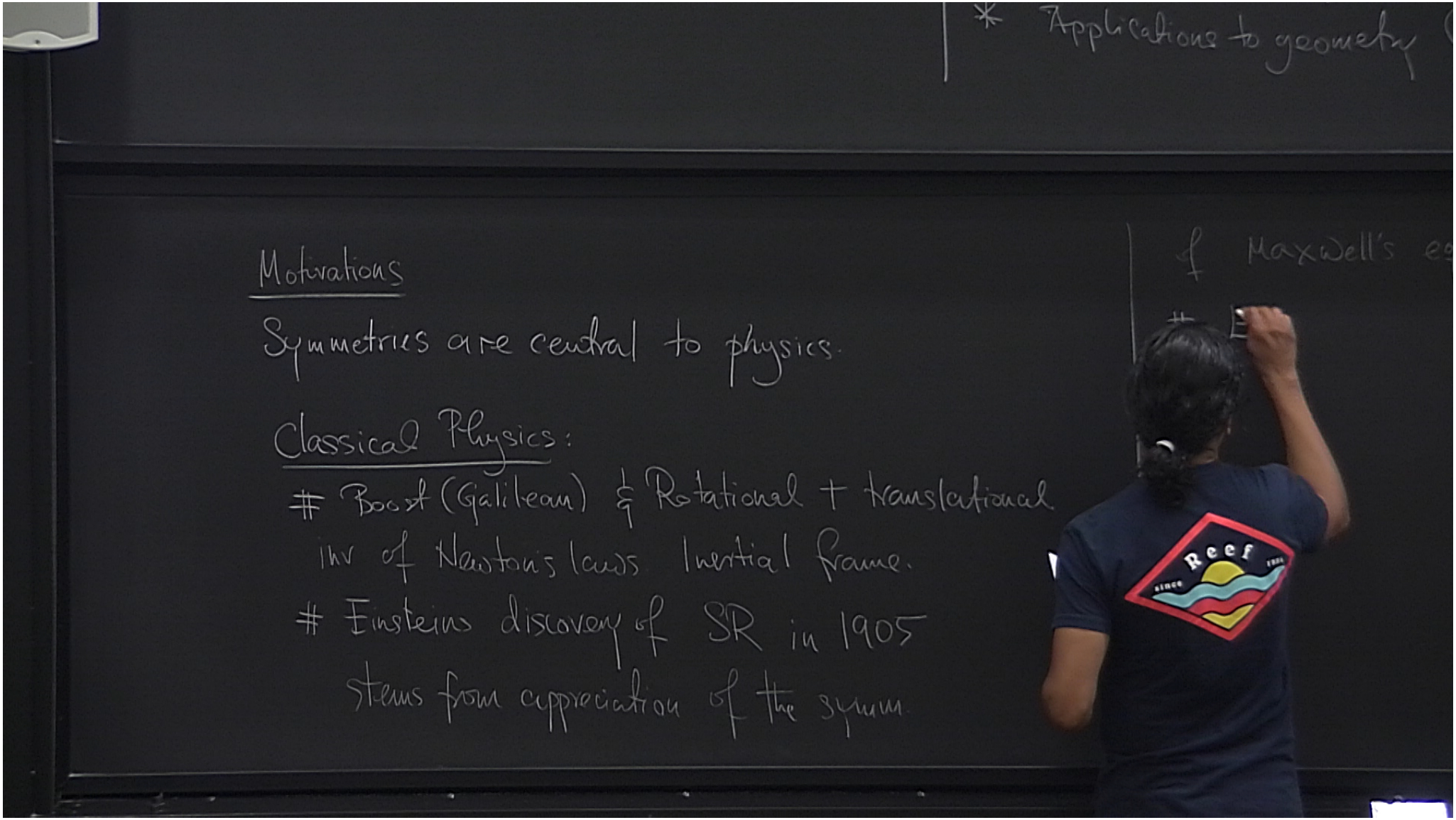
* Applications to geometry

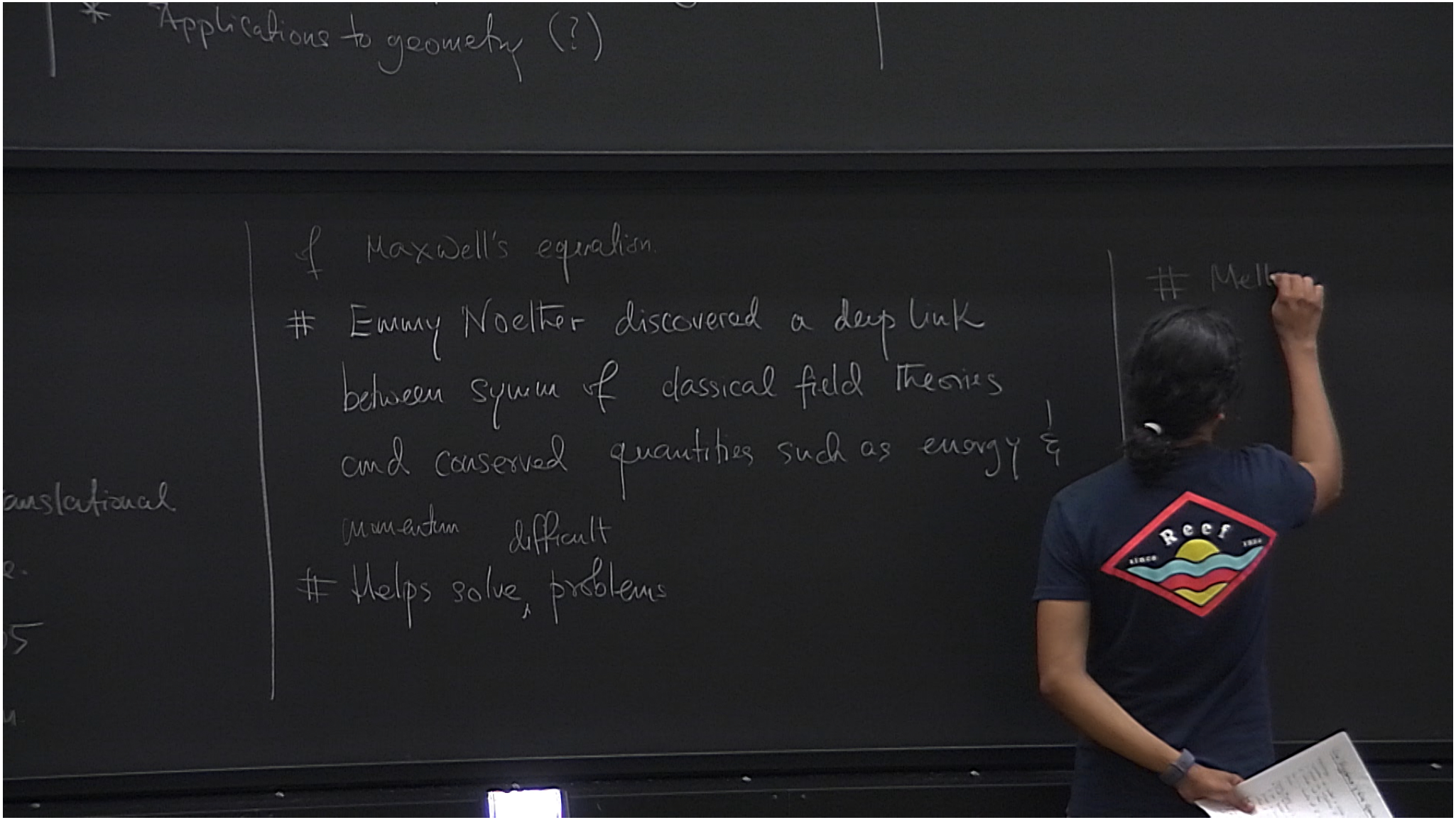
Motivations

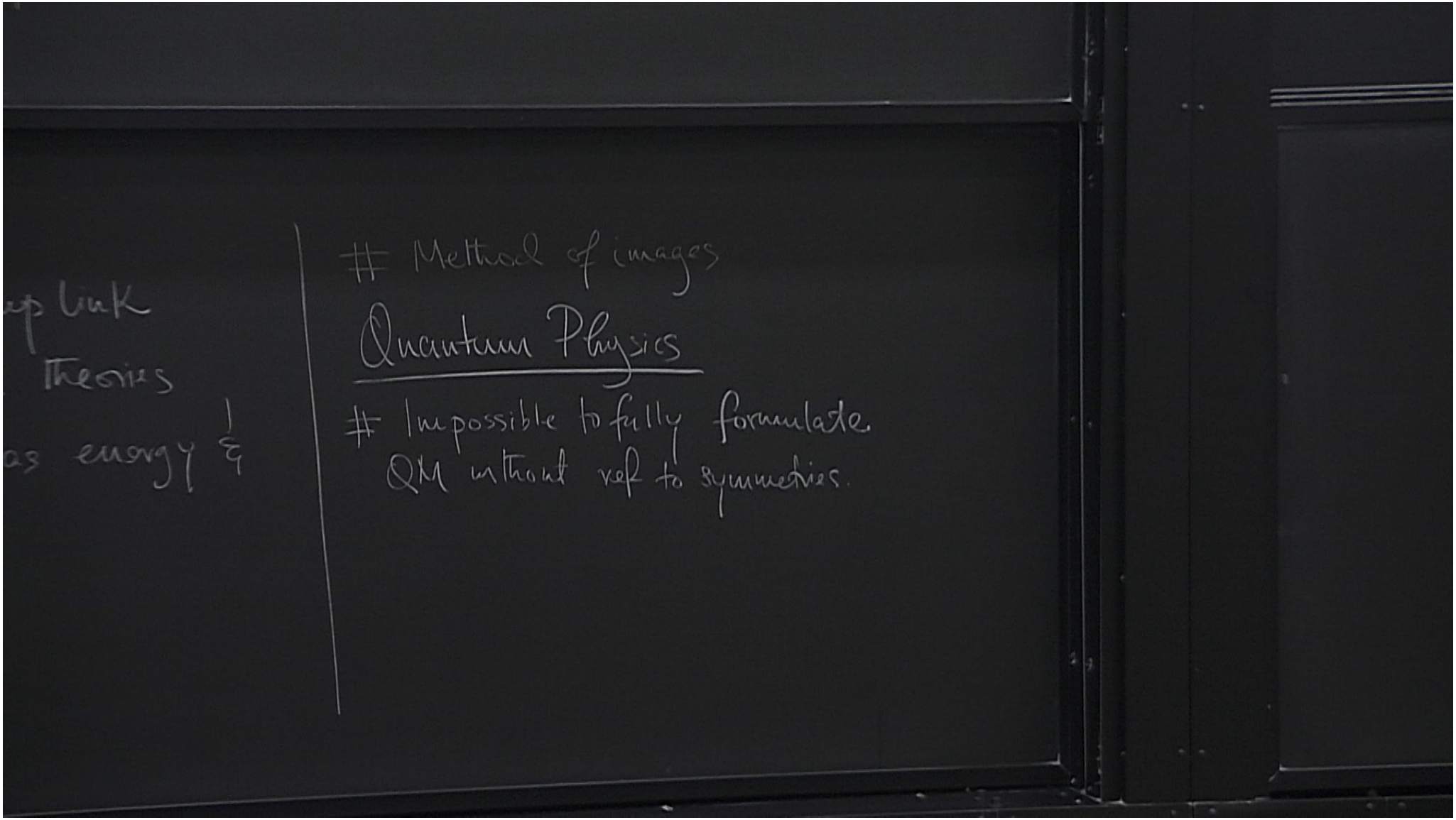
Symmetries are central to physics.

Classical Physics:

Boost (Galilean) & Rotational + translational inv of Newton's laws Inertial frame.


* Applications to geometry
of Maxwell's eq
Motivations
Symmetries are central to physics.
Classical Physics:
Boost (Galilean) & Rotational + translational
inv of Newton's laws Inertial frame.
Einstein's discovery of SR in 1905
stems from appreciation of the symm
Reef


* Applications to geometry (?)
of Maxwell's equation
Emmy Noether discovered a deep link
between symm of classical field theories
and conserved quantities such as energy &
momentum difficult
Helps solve problems
Mell
Reef


ep link
Theories
as energy 1/E
Method of images
Quantum Physics
Impossible to fully formulate
QM without ref to symmetries.

ep link
Theories
as energy 1/E

Method of images

Quantum Physics

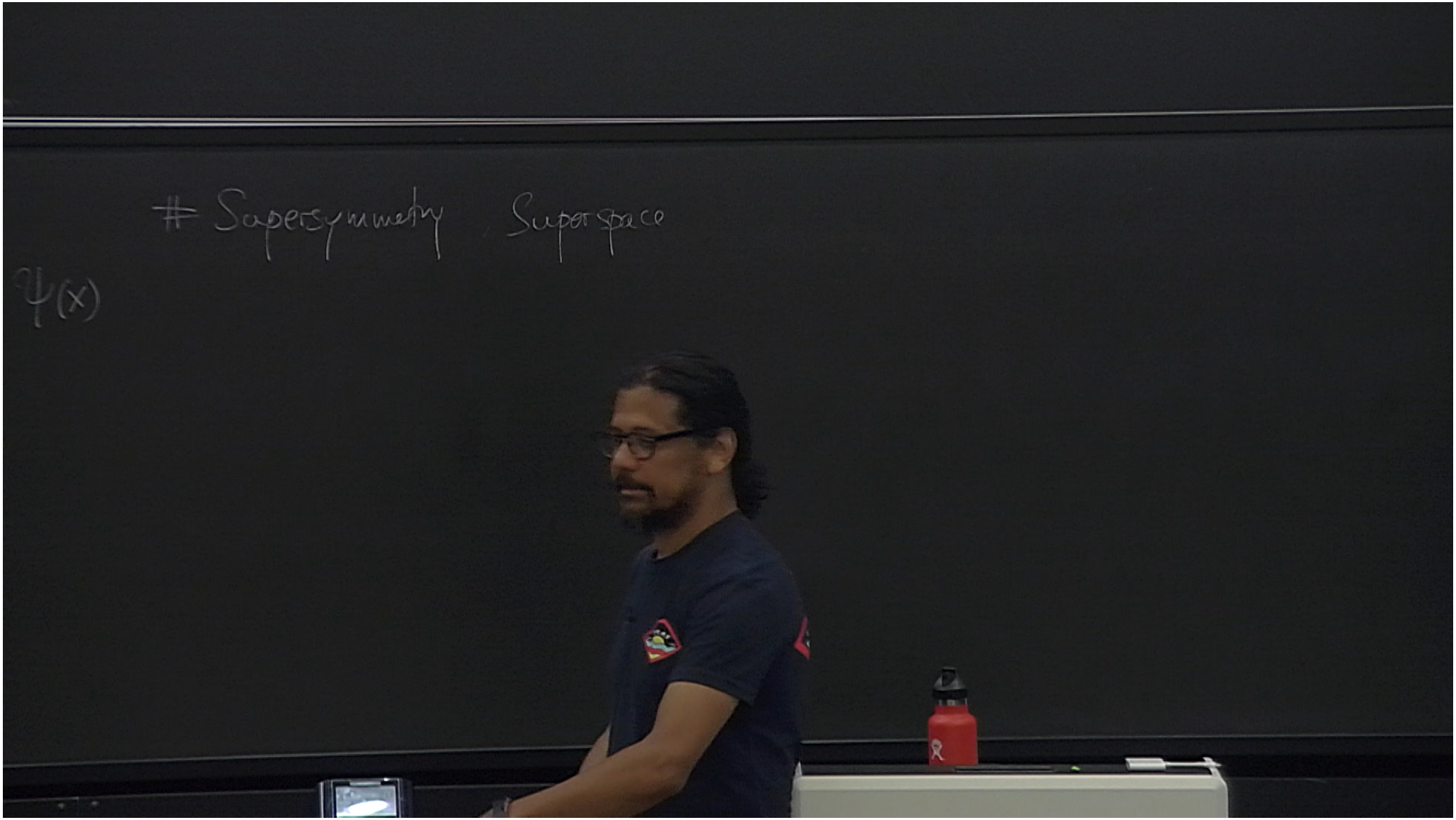
Impossible to fully formulate QM without ref to symmetries.
→ Theorem by Wigner

Classifications of particles according to rep of the Poincaré group

In quantum field theories we also have

'internal symmetries' (continuous) that lead to conserved charges according to Noether's theorem. $e^{i\theta}\,\psi(x)$

Ex: The electric charge can be thought of as the conserved charge associated to a global phase symmetry

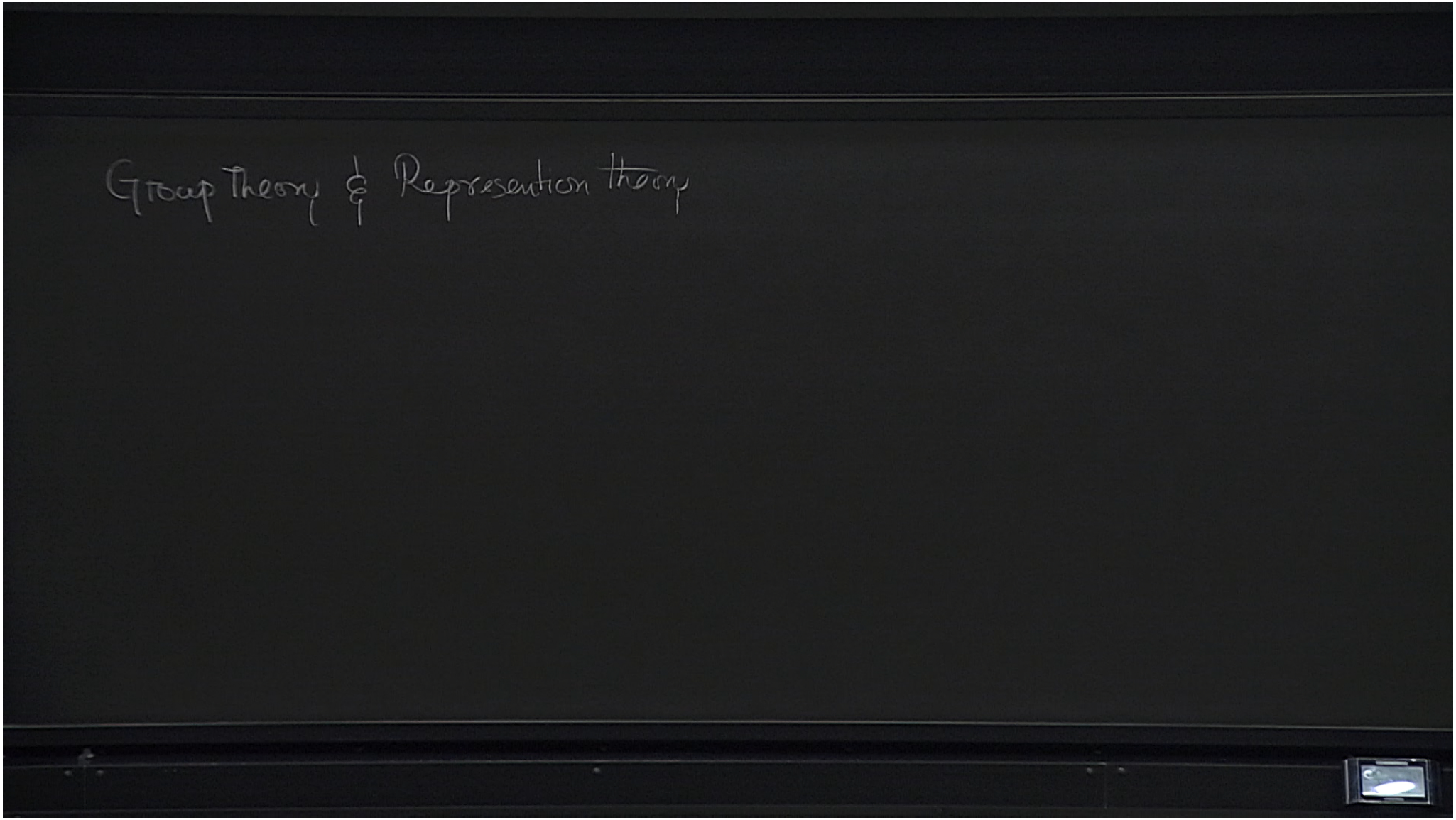

Supersymmetry
Superspace
$\Psi(x)$

Supersymmetry Superspace

$\psi(x)$

One parameter strongly continuous

Unitary trans $\longleftrightarrow$ Observable

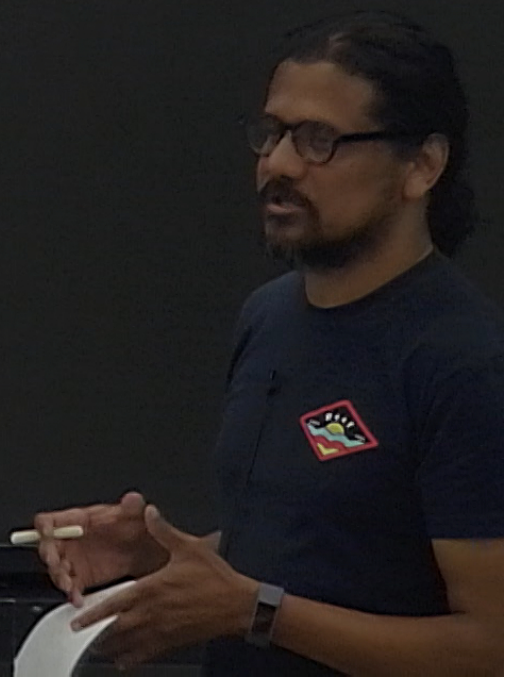

Group Theory & Representation theory

Group Theory & Representation theory

In Physics — states $\in$ Vector Space

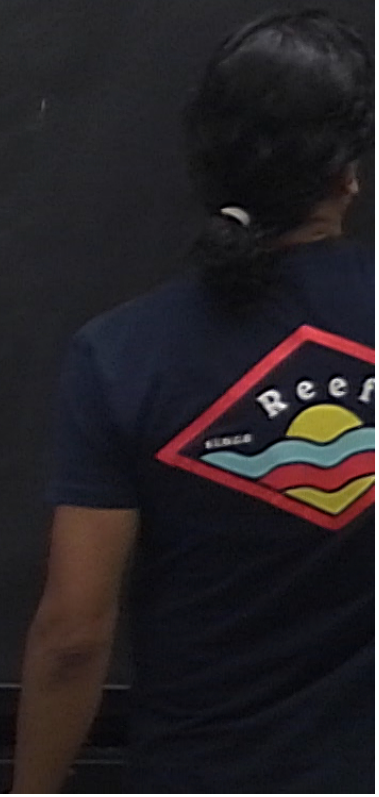
Action of group — linear operator on vectors
(finite or infinite dim)

Representations of the group.



Fundamental Concepts

Group: A group is a set G equipped with a binary operation, called the gp multiplication, $\cdot : G \times G \to G$, such that the following hold:



1. Closure: If $a \& b \in G$ then $a \cdot b \in G$

2. Associative: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$, $\forall a, b, c \in G$

3 Identity element: $\exists$ an element, e, st $a \cdot e = a$ $\forall a \in G$

4. Inverses: $\forall a \in G \; \exists$ an element $a^{-1} \in G$ known as the inverse of a st $a \cdot a^{-1} = e$

hold

$$a\,a^{-1} = e = a^{-1}a$$

$\cdot : G \times G \to G$, such that the following hold:

4. Inverses: $\forall a \in G \; \exists a^{-1}$ s.t. $a \cdot a^{-1} = e$

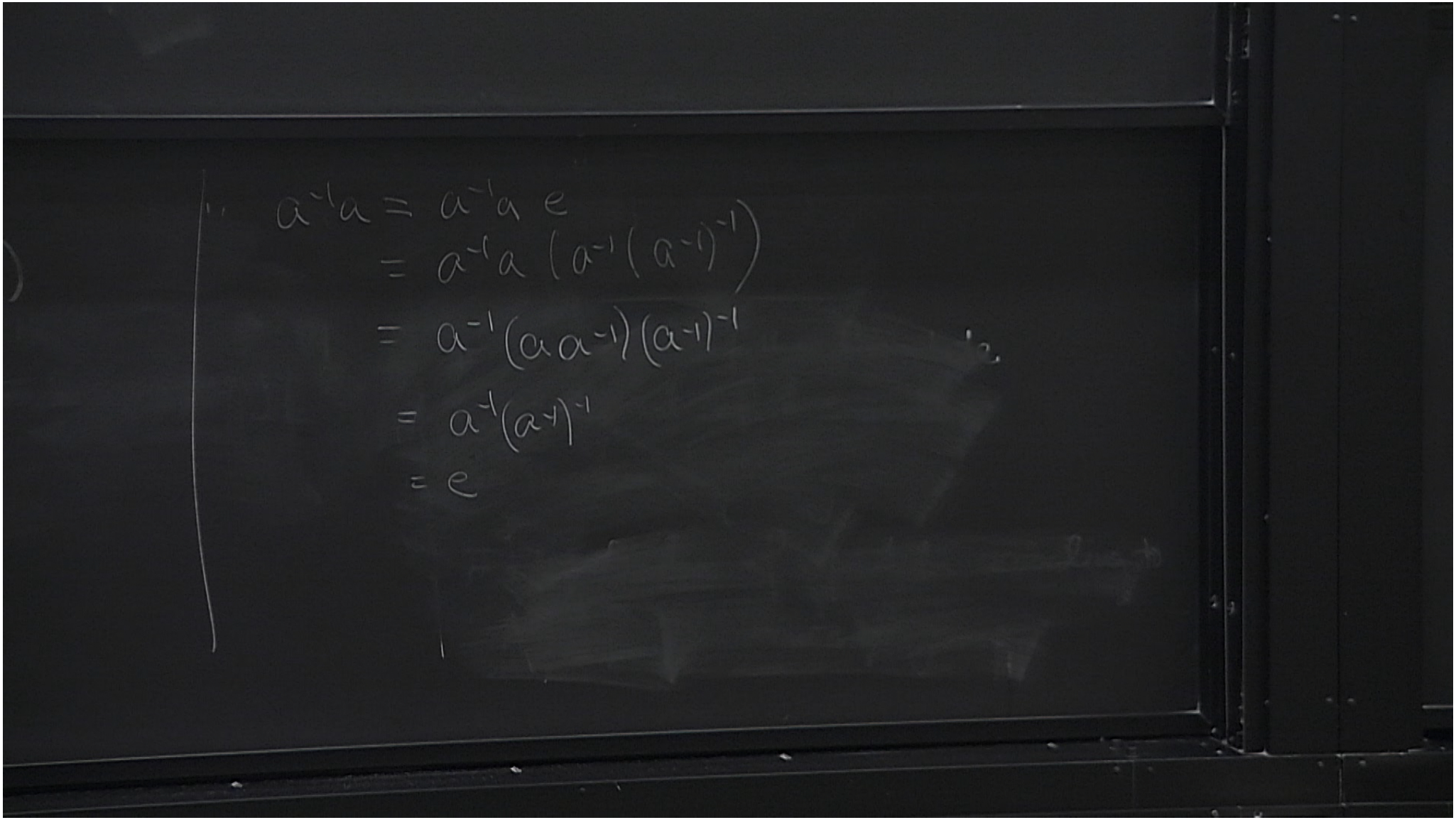
$a^{-1} / \; aa^{-1} = e \; / a$

$$a^{-1}a = (a^{-1}e)a = (a^{-1}(aa^{-1}))a = (a^{-1}a)(a^{-1}a)$$

$$b = a^{-1}a = e$$

$$b = bb$$

$$e = bb^{-1} = bbb^{-1} = b$$


$a^{-1}a = a^{-1}ae$
$= a^{-1}a(a^{-1}(a^{-1})^{-1})$
$= a^{-1}(aa^{-1})(a^{-1})^{-1}$
$= a^{-1}(a^{-1})^{-1}$
$= e$

e is also the 'left' identity.

The G can be finite or infinite (countably or uncountably).

If G is finite then the number of elements of G is called the order of the group: $O(G)$.

$\otimes$ mult is not necessarily commutative, $ab \neq ba$

Defn Abelian & non-Abelian groups

If all the elements of G commute then G is Abelian

If $\exists$ at least two elements $a, b \in G$ st $ab \neq ba$

then G is non-Abelian

If all the elements of G commute then G is Abelian

if ∃ at least two elements $a, b \in G$ s.t. $ab \neq ba$

then G is non-Abelian

Subgroups

G, $H \subset G$ s.t. H forms a gp under the same multiplication of G.

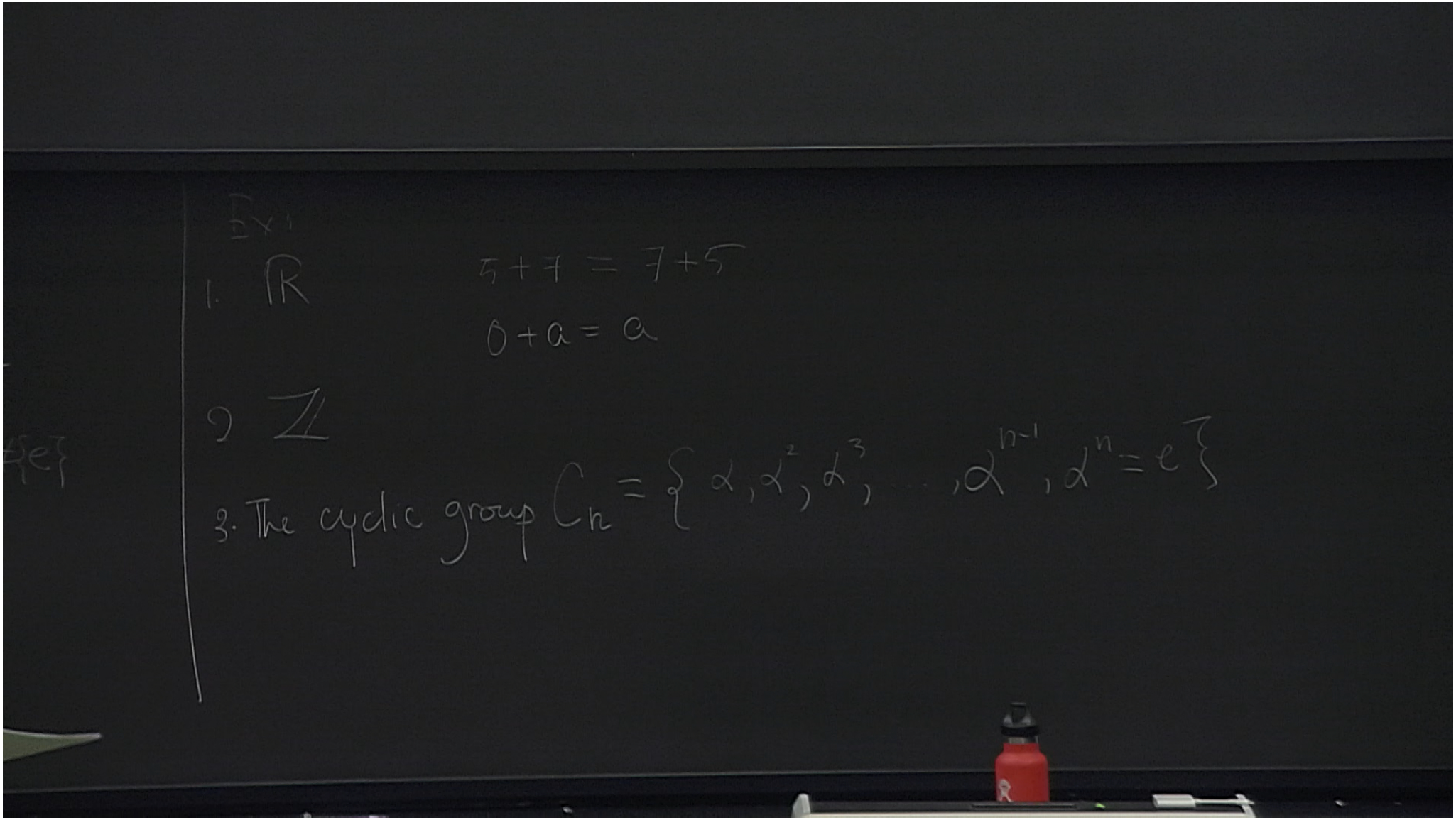
Then H is a subgroup.

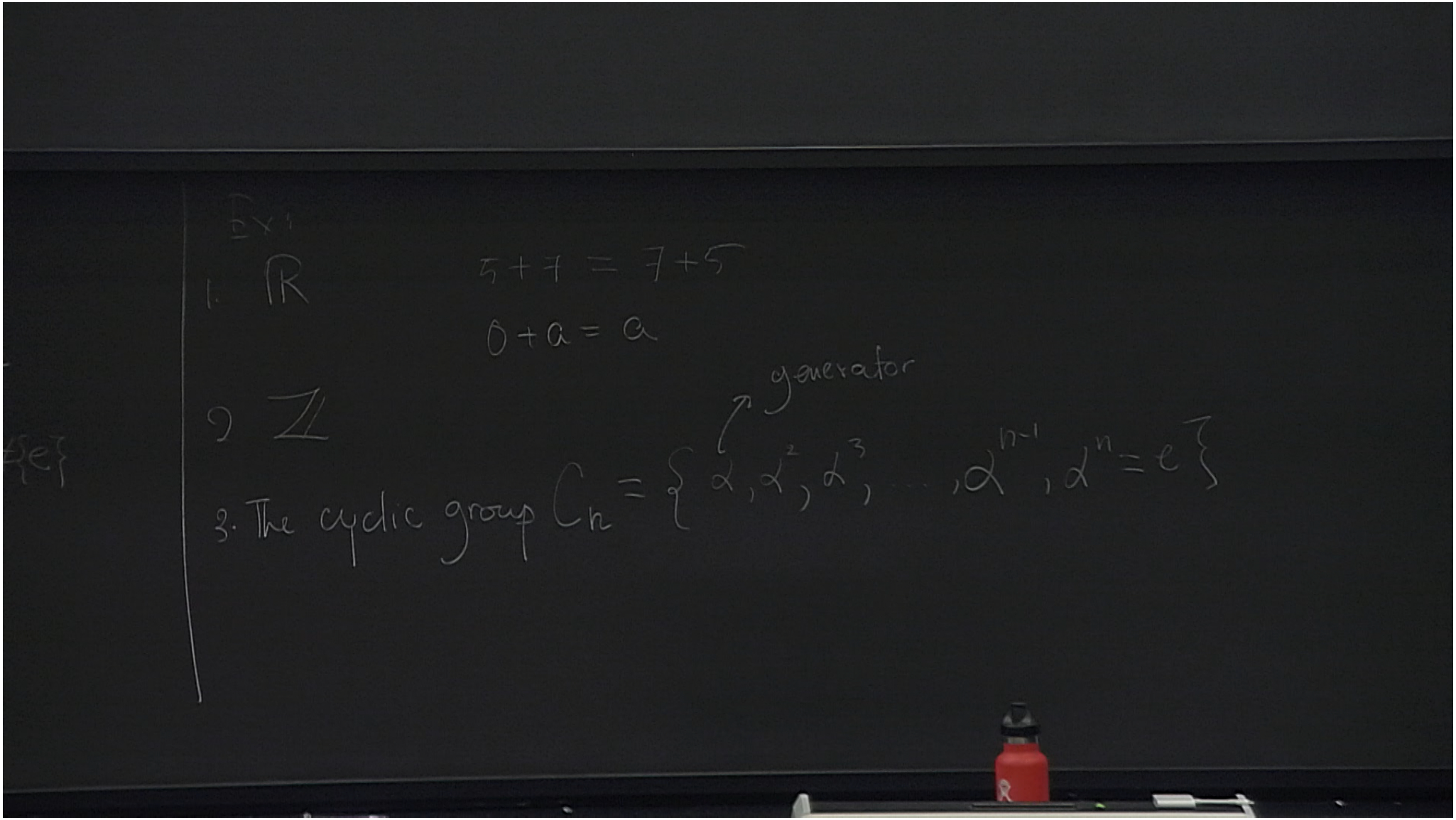
symmetry

Def'n Proper & Improper Subgroups

Both $\{e\}$ & G are trivially subgroups. Such gps are known to be Improper Subgroups If $H \subset G$ but $H \neq \{e\}$

$H \neq G$ then we say H is a proper subgroup


Ex:
1. $\mathbb{R}$
$5+7=7+5$
$0+a=a$
2. $\mathbb{Z}$
3. The cyclic group $C_n = \{a, a^2, a^3, \ldots, a^{n-1}, a^n = e\}$


Ex:
1. $\mathbb{R}$
$5+7=7+5$
$0+a=a$
2. $\mathbb{Z}$
generator
3. The cyclic group $C_n = \{a, a^2, a^3, \ldots, a^{n-1}, a^n = e\}$

4. $\mathbb{Z}_4 = \mathbb{Z}/4\mathbb{Z} = \{0, 1, 2, 3\}$

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$e = 0$
$a = 1$
$b = 2$
$c = 3$

·	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

Group multiplication table.

$\cdot = + \bmod 4$

The multiplicative group mod 5 $(\mathbb{Z}/5\mathbb{Z})^\times$

$[1] = \ldots, -4, 1, 6, 11, 16 \ldots$

$[2] = \ldots, -3, 2, 7, 12, 17 \ldots$

$[3] = \ldots, -2, 3, 8, 13, 18 \ldots$

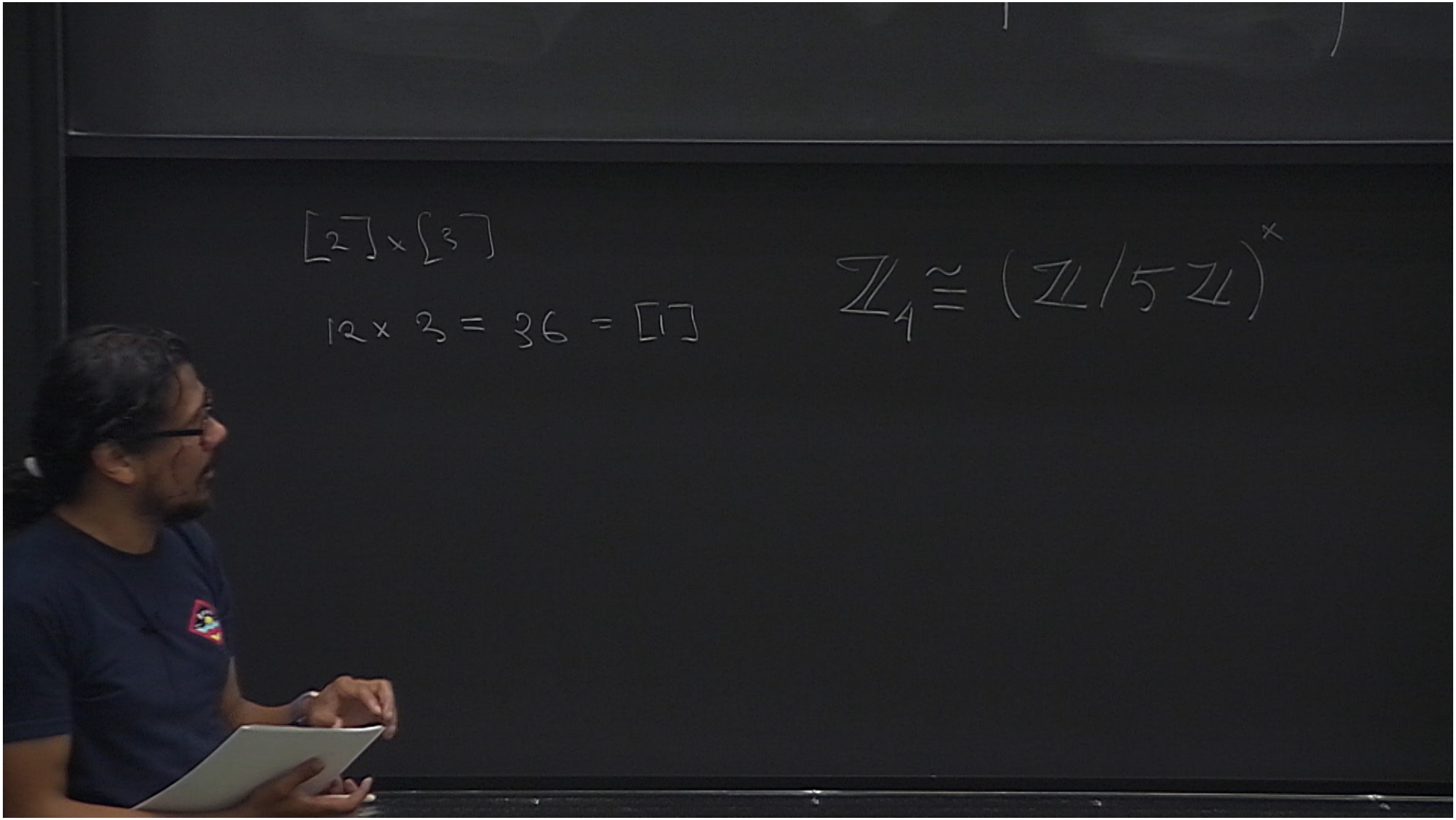
$[4] = \ldots, -1, 4, 9, 14, 19 \ldots$

$[2] \times [3] = 6 = [1]$

$\times$	$[1]$	$[2]$	$[4]$	$[3]$
$[1]$	$[1]$	$[2]$	$[4]$	$[3]$
$[2]$	$[2]$	$[4]$	$[3]$	$[1]$
$[4]$	$[4]$	$[3]$	$[1]$	$[2]$
$[3]$	$[3]$	$[1]$	$[2]$	$[4]$

$e = [1]$ $a = [2]$

$b = [4]$, $c = [3]$


[2] × [3]
12 × 3 = 36 = [1]
$\mathbb{Z}_4 \cong (\mathbb{Z}/5\mathbb{Z})^\times$

$[2] \times [3]$

$12 \times 3 = 36 = [1]$

$\mathbb{Z}_4 \cong (\mathbb{Z}/5\mathbb{Z})^\times$

This is an example of <u>group homomorphism</u>.

A group homomorphism f is a map between

$G_1 \& G_2$, $f : G_1 \to G_2$



st $\forall$ elements $a, b \in G_1$

we have $f(ab) = f(a) f(b)$

mult of G_1 mult of G_2