

Title: PSI 2019/2020 - Classical Physics - Lecture 8

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YESTERDAY. MAXWELL'S THEORY IN RELATIVISTIC NOTATION

$$\boxed{F_{\mu\nu, \lambda} J^{\lambda} = 0 \quad \Leftrightarrow \quad \partial_{\mu} (*F)^{\mu} = 0} \quad \dots \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

$$\boxed{\frac{dp^{\mu}}{d\tau} = F^{\mu}_{L} = -e F^{\mu\nu} u_{\nu}}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$$F^{0i} = E^i, \quad F^{ij} = \epsilon^{ijk} B_k$$

YESTERDAY. MAXWELL'S THEORY IN RELATIVISTIC NOTATION

$$\boxed{F_{[\mu\nu]\lambda} = 0 \quad \Leftrightarrow \quad \partial_\mu (*F)^{\mu\nu} = 0 \quad \dots \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu}$$

$$\partial_\mu F^{\mu\nu} = -4\pi J^\nu$$

$$J^\nu = (\rho, \mathbf{j})$$

$$\boxed{\frac{dp^\mu}{d\tau} = F^\mu_L = -e F^{\mu\nu} u_\nu}$$

$$F^{\mu\nu} u_\mu$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ 0 & B_z & -B_y & 0 \\ 0 & 0 & B_x & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$F_{0i} = E^i, \quad F_{ij} = \epsilon^{ijk} B_k$$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ 0 & 0 & B_x \\ 0 & 0 & 0 \end{pmatrix}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$$

$\underbrace{\partial_\nu \partial_\mu F^{\mu\nu}}_{\text{STATIONARY}} = -4\pi \underbrace{\partial_\nu J^\nu}_{\text{ANTISTATIONARY}}$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ 0 & 0 & B_x \\ 0 & 0 & 0 \end{pmatrix}$$

$$\vec{E} = \epsilon_0 \nabla \phi$$

$$\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$$

$\uparrow$
 $\text{SYM}(\nu\mu)$ $\text{ANTISYMM}(\nu\mu)$

$$\partial_\nu J^\nu = 0$$

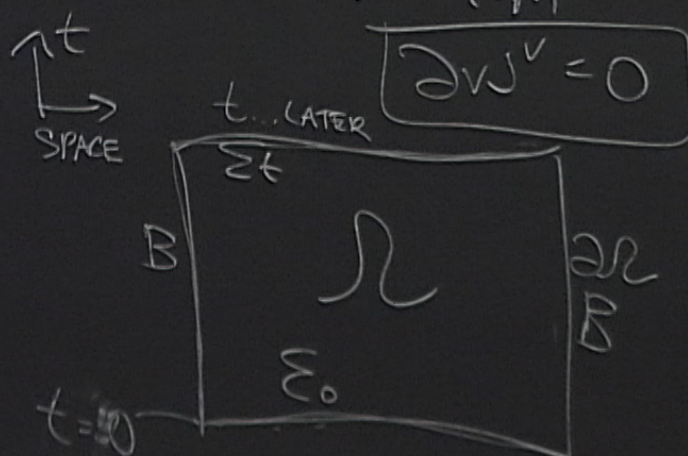
CURRENT
CONSERVATION

$$\begin{pmatrix} E_y & E_z \\ B_z & -B_y \\ 0 & B_x \\ 0 & 0 \end{pmatrix}$$

$$= \epsilon_0 \mu_0 B_x$$

$$\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$$

STATION (v_μ) ANTISTATION (v_μ)



CURRENT
CONSERVATION

$$0 = \int_{\Omega} \partial_{\mu} J^{\mu} d^4x = \int_{\partial\Omega} J^{\mu} d\Sigma_{\mu} = \int_{\Sigma_0} J^{\mu} d\Sigma_{\mu} + \int_B J^{\mu} d\Sigma_{\mu} - \int_{\Sigma_t} J^{\mu} d\Sigma_{\mu}$$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ 0 & 0 & B_x \\ 0 & 0 & 0 \end{pmatrix}$$

$$\epsilon_{ijk} = \epsilon_{ijk} B_k$$

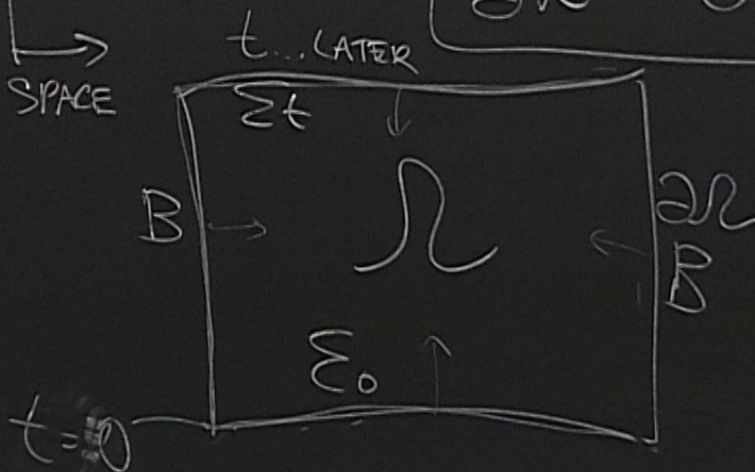
$$\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$$

$\uparrow$
 $\Sigma M(\nu\mu)$ ANTISYMM
 $(\nu\mu)$

$\uparrow t$
 $\rightarrow$ SPACE

$$\partial_\nu J^\nu = 0$$

CURRENT
CONSERVATION



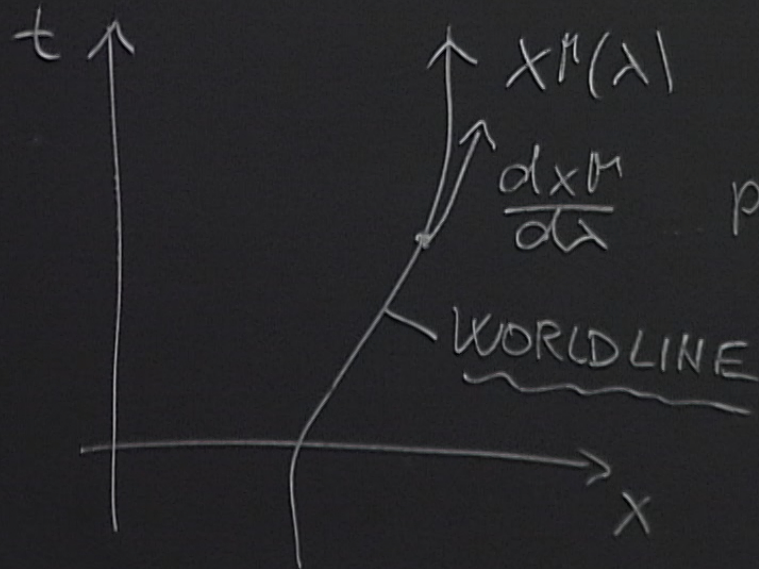
$$0 = \int_{\Omega} \partial_{\mu} J^{\mu} d^4x = \int_{\partial\Omega} J^{\mu} d\Sigma_{\mu} = \int_{\Sigma_0} J^{\mu} d\Sigma_{\mu} + \underbrace{\int_{\Sigma_B} J^{\mu} d\Sigma_{\mu}}_{\substack{\text{NO FLUX} \\ \downarrow \\ \cancel{0}}} - \int_{\Sigma_t} J^{\mu} d\Sigma_{\mu}$$

$$Q = \int_{\Sigma_0} J^{\mu} d\Sigma_{\mu} = \int_{\Sigma_t} J^{\mu} d\Sigma_{\mu}$$

CONSERVED CHARGE

• RELATIVISTIC PARTICLE

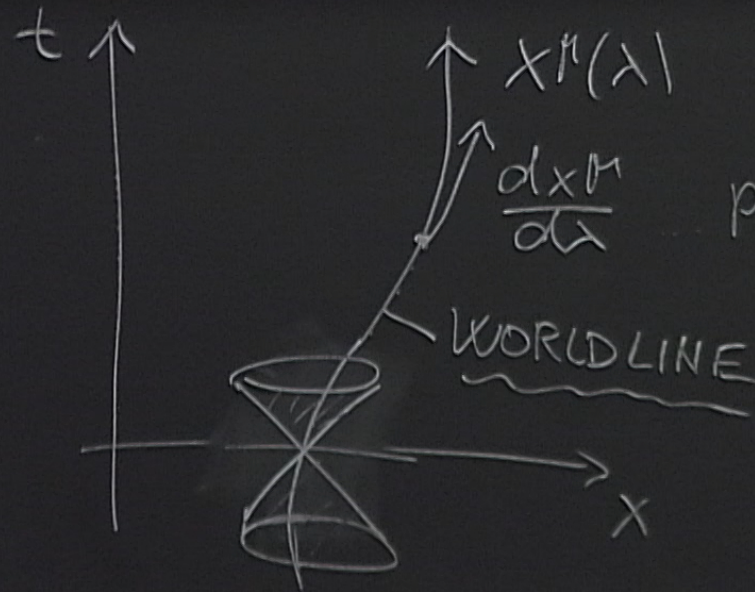
λ . PARAMETER



PARAMETRIC 4-VELOCITY

$$\eta_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} \begin{cases} > 0 & \text{SPACELIKE} \\ = 0 & \dots \text{NULL (LIGHTLIKE)} \dots \text{MASSLESS PARTICLES} \\ < 0 & \dots \text{TIMELIKE} \dots \text{MASSIVE PARTICLES} \end{cases}$$

• RELATIVISTIC PARTICLE



λ . PARAMETER

PARAMETRIC 4-VELOCITY

$$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$$

ETER

4-VELOCITY

$$\eta_{\mu\nu} \frac{dx^\mu}{dx} \frac{dx^\nu}{dx}$$

> 0 SPACELIKE

$= 0$... NULL (LIGHTLIKE) ... MASSLESS PARTICLES

< 0 ... TIMELIKE ... MASSIVE PARTICLES

FOR MASSIVE PARTICLES CAN INTRODUCE PROPER TIME τ

$$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$

$\eta_{\mu\nu} \frac{dx^\mu}{dx} \frac{dx^\nu}{dx} \begin{cases} > 0 & \text{SPACELIKE} \\ = 0 & \dots \text{NULL (LIGHTLIKE)} \dots \text{MASSLESS PARTICLES} \\ < 0 & \dots \text{TIMELIKE} \dots \text{MASSIVE PARTICLES} \end{cases}$

FOR MASSIVE PARTICLES CAN INTRODUCE PROPER TIME τ

$$\boxed{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1} \quad \Leftrightarrow \quad d\tau = \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu} \quad \dots \text{LORENTZ SCALAR}$$

1-VELOCITY

$$\eta_{\mu\nu} \frac{dx^\mu}{dx} \frac{dx^\nu}{dx}$$

< 0 ... TIMELIKE ... MASSIVE PARTICLES

FOR MASSIVE PARTICLES CAN INTRODUCE PROPER TIME τ

$$\boxed{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1} \quad \Leftrightarrow \quad d\tau = \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu} \quad \dots \text{LORENTZ SCALAR}$$

IN PARTICULAR. $d\tau = \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} dt}$

TIMELIKE ... MASSIVE PARTICLES

PROPER TIME τ

$$\sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu} \quad \dots \text{LORENTZ SCALAR}$$

$$d\tau = dt \sqrt{1 - \frac{v^2}{c^2}}$$

$$x^\mu = (t, x^i)$$

ER

$$\eta_{\mu\nu} \frac{dx^\mu}{dx} \frac{dx^\nu}{dx} \begin{cases} = 0 \dots \text{NULL (LIGHTLIKE)} \dots \text{MASSLESS PARTICLES} \\ < 0 \dots \text{TIMELIKE} \dots \text{MASSIVE PARTICLES} \end{cases}$$

VELOCITY

FOR MASSIVE PARTICLES CAN INTRODUCE PROPER TIME τ

$$\boxed{\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1} \quad \Leftrightarrow \quad d\tau = \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu} \dots \text{LORENTZ SCALAR}$$

IN PARTICULAR, $d\tau = \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} dt = dt \sqrt{1 - \left(\frac{dx^i}{dt}\right)^2} = \frac{dt}{\gamma}$

$$x^\mu = \begin{pmatrix} t \\ x^i \end{pmatrix} \quad \gamma = \frac{1}{\sqrt{1-v^2}}$$

• 4-VELOCITY $\boxed{u^\mu = \frac{dx^\mu}{d\tau}} = \frac{dx^\mu}{dt} \frac{dt}{d\tau} = \gamma(1, \vec{v})$

- 4-VELOCITY $\boxed{u^\mu = \frac{dx^\mu}{d\tau}} = \frac{dx^\mu}{dt} \frac{dt}{d\tau} = \gamma(1, \vec{v})$

- 4-MOMENTUM $\boxed{m(\tau) = \gamma m_0}$ ↖ REST MASS OF A PARTICLE

$$\boxed{p^\mu = \underset{\substack{\uparrow \\ \text{LORENTZ SCALAR}}}{m_0} u^\mu} = \left(\gamma m_0, \underbrace{\gamma m_0}_{\substack{m \\ \vec{p}}} \vec{v} \right) = (\gamma m_0, \vec{p})$$

• EINSTEIN'S FORMULA

$$E = mc^2 = m \quad (\text{TUTORIAL})$$

4-VELOCITY

$$u^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt} \frac{dt}{d\tau} = \gamma(1, \vec{v})$$

• 4-MOMENTUM

$$m(\tau) = \gamma m_0$$

REST MASS OF A PARTICLE

$$p^\mu = \underbrace{m_0}_{\text{LORENTZ SCALAR}} u^\mu = (\gamma m_0, \underbrace{\gamma m_0 \vec{v}}_{\substack{m \\ \vec{p}}} = (\gamma m_0, \vec{p}) = (m, \vec{p})$$

• EINSTEIN'S FORMULA

$$\boxed{E = mc^2 = m} \quad (\text{TUTORIAL})$$

$$p^\mu = (E, \vec{p})$$

$$p^2 = p^\mu p_\mu = \eta_{\mu\nu} p^\mu p^\nu = -E^2 + \vec{p}^2 = -m_0^2$$

$$E = \sqrt{\vec{p}^2 + m_0^2}$$

MASSLESS: $m_0 = 0$

$$\boxed{E = |\vec{p}|}$$

• 4-VELOCITY $\boxed{u^\mu = \frac{dx^\mu}{d\tau}} = \frac{dx^\mu}{dt} \frac{dt}{d\tau} = \gamma (1, \vec{v})$

• 4-MOMENTUM $\boxed{m(\tau) = \gamma m_0}$ ↖ REST MASS OF A PARTICLE

$\boxed{p^\mu = m_0 u^\mu}$ ↑ LORENTZ SCALAR $= (\gamma m_0, \underbrace{\gamma m_0}_{\vec{p}} \vec{v}) = (\gamma m_0, \vec{p}) = (m, \vec{p})$

MASSIVE: $E = m_0 \sqrt{1 + \frac{\vec{p}^2}{m_0^2}}$

$$\boxed{u^\mu = \frac{dx^\mu}{d\tau}} = \frac{dx^\mu}{dt} \frac{dt}{d\tau} = \gamma(1, \vec{v})$$

$$\boxed{m(\tau) = \gamma m_0}$$

REST MASS OF A PARTICLE

$$p^\mu = m_0 u^\mu = (\gamma m_0, \gamma m_0 \vec{v}) = (\gamma m_0, \vec{p})$$

LORENTZ SCALAR

MASSIVE: $E = m_0 \sqrt{1 + \frac{\vec{p}^2}{m_0^2}} \approx m_0 \left(1 + \frac{1}{2} \frac{\vec{p}^2}{m_0^2} + \dots \right) = m_0 + \frac{1}{2} \frac{\vec{p}^2}{m_0} + \dots$

• EINSTEIN'S FORMULA

$$p^\mu = (E, \vec{p})$$

$$p^2 = p^\mu p_\mu = \eta_{\mu\nu} p^\mu p^\nu$$

$$E = \sqrt{\vec{p}^2 + m_0^2}$$

MASSLESS: $m_0 = 0$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ 0 & 0 & B_x \\ 0 & 0 & 0 \end{pmatrix}$$

2ND NEWTON'S LAW.

$$\boxed{\frac{dp^\mu}{d\tau} = F^\mu} \quad 4\text{-FORCE}$$

$$F_{ij} = g_{ij} B_k$$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ 0 & 0 & B_x \\ 0 & 0 & 0 \end{pmatrix}$$

$$F_{ij} = g_{ij} B_k$$

2ND NEWTON'S LAW:

$$\boxed{\frac{dp^\mu}{d\tau} = F^\mu} \quad \text{4-FORCE}$$

NO FORCE .. FREE PARTICLE (GEODESICS) $\boxed{\frac{dp^\mu}{d\tau} = 0}$

$$S = -m_0 c^2 \int d\tau$$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ 0 & 0 & B_x \\ 0 & 0 & 0 \end{pmatrix}$$

$$F_{ij} = \epsilon_{ijk} B_k$$

2ND NEWTON'S LAW:

$$\boxed{\frac{dp^\mu}{d\tau} = F^\mu} \quad \text{4-FORCE}$$

NO FORCE ... FREE PARTICLE (GEODESICS) $\boxed{\frac{dp^\mu}{d\tau} = 0}$

$$S = -m_0 c^2 \int d\tau$$

↑
MAXIMIZING
PROPER TIME

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ 0 & 0 & B_x \\ 0 & 0 & 0 \end{pmatrix}$$

$$F_{ij} = \epsilon_{ijk} B_k$$

2ND NEWTON'S LAW.

$$\boxed{\frac{dp^\mu}{d\tau} = F^\mu} \quad \text{4-FORCE}$$

NO FORCE ... FREE PARTICLE (GEODESICS) $\boxed{\frac{dp^\mu}{d\tau} = 0}$

$$S = -m_0 c^2 \int d\tau = -m_0 c^2 \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} d\lambda$$

↑
MAXIMIZING
PROPER TIME

$$L(x^\mu, \frac{dx^\mu}{d\lambda}, \lambda)$$

$$\left(\frac{1}{m_0} \frac{d\mathbf{p}}{dt} \right) = \left(\frac{1}{m_0} \frac{d\mathbf{p}}{dt} \right) \left(\frac{m_0}{m} \right)$$

d) RELATIVISTIC FIELD THEORIES

$$S[\varphi] = \int$$

↑
FIELD

$$\varphi(t, \vec{x}) = \varphi(x^\mu)$$

$$E = \sqrt{\vec{p}^2 + m_0^2}$$

$$\delta S = 0 = \int \left(\frac{\partial \mathcal{L}}{\partial \psi} \delta \psi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \delta \partial_\mu \psi \right) d^4 x$$

$d^4 x$

$$\delta S = 0 = \int_{\Sigma} \left(\frac{\partial \mathcal{L}}{\partial \psi} \delta \psi + \frac{\partial \mathcal{L}}{\partial \partial_{\mu} \psi} \underbrace{\delta \partial_{\mu} \psi}_{\partial_{\mu} \delta \psi} \right) d^4 x$$

BY PARTS

$$= \int_{\Sigma} \left(\frac{\partial \mathcal{L}}{\partial \psi} - \partial_{\mu} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \right) \delta \psi d^4 x - \int_{\Sigma} d^3 \Sigma_{\mu} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \delta \psi$$

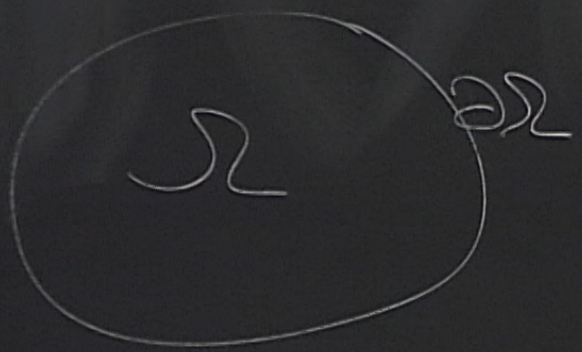
THEORIES

$$\partial_\mu \varphi(x)$$

$$\delta S = 0 = \int_\Omega \left(\frac{\partial \mathcal{L}}{\partial \varphi} \delta \varphi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \underbrace{\delta \partial_\mu \varphi}_{\partial_\mu \delta \varphi} \right) d^4 x$$

BY PARTS

$$= \int_\Omega \left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) \delta \varphi d^4 x - \int_{\partial \Omega} d^3 x$$



ULTR. $d\tau = \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} dt = dt \sqrt{1 - \left(\frac{dx}{dt}\right)^2} = \gamma \quad \gamma = \sqrt{1 - v^2}$

$$\delta S = 0 = \int_{\Sigma} \left(\frac{\partial \mathcal{L}}{\partial \psi} \delta \psi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \delta \partial_\mu \psi \right) d^4 x$$

BY PARTS

$$= \int_{\Sigma} \left(\frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi)} \right) \delta \psi d^4 x + \int_{\partial \Sigma} d^3 \Sigma_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi)} \delta \psi$$

↓
0 FIXED ENDPOINTS

$$\delta \psi / \partial \Sigma = 0$$

$$\delta S = 0 = \int_{\Sigma} \left(\frac{\partial \mathcal{L}}{\partial \psi} \delta \psi + \frac{\partial \mathcal{L}}{\partial \partial_{\mu} \psi} \underbrace{\delta \partial_{\mu} \psi}_{\partial_{\mu} \delta \psi} \right) d^4 x$$

BY PARTS

$$= \int_{\Sigma} \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \psi} - \partial_{\mu} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \right)}_{\mathcal{E}} \delta \psi d^4 x + \int_{\partial \Sigma} d^3 \Sigma_{\mu} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \delta \psi$$

FIXED ENDPOINTS
 $\delta \psi / \partial \Sigma = 0$

(E-L) $\boxed{\frac{\partial \mathcal{L}}{\partial \psi} - \partial_{\mu} \left(\frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \right) = 0}$

$q(t)$

$$q(t)$$

NOETHER LET $\tilde{\delta}$ BE A GLOBAL CONTINUOUS SYMMETRY

THAT IS OFF-SHELL WE FIND $\tilde{\delta} x^\mu = x'^\mu - x^\mu$

$$\tilde{\delta} \varphi(x^\mu) = \varphi(x'^\mu) - \varphi(x^\mu)$$

S.T. $\tilde{\delta} S = 0$

REST MASS OF A PARTICLE

NOETHER LET $\tilde{\delta}$ BE A GLOBAL CONTINUOUS SYMMETRY

THAT IS OFF-SHELL WE FIND $\tilde{\delta} x^\mu = x^{\mu'} - x^\mu$

$$\tilde{\delta} \varphi(x^\mu) = \varphi(x^{\mu'}) - \varphi(x^\mu)$$

S.T. $\tilde{\delta} S = 0 \Rightarrow$

$$J^\mu = T_c^\mu{}_\nu \tilde{\delta} x^\nu + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \tilde{\delta} \varphi$$

IS CONSERVED

$$T_c^\mu{}_\nu = \eta^\mu{}_\nu \mathcal{L} - \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \partial_\nu \varphi$$

$$\partial_\mu J^\mu = 0$$

CANONICAL
ENERGY-MOM. TENSOR

• ENERGY-MOMENTUM TENSOR

ACTION INVARIANT UNDER POINCARÉ TR., SPEC BY TRANSLATIONS.

$$\tilde{\delta} X^\mu = \epsilon^\mu, \quad \tilde{\delta} \psi = 0 \quad \dots \text{SYMMETRY.}$$

• ENERGY-MOMENTUM TENSOR

ACTION INVARIANT UNDER POINCARÉ TR., SPEC BY TRANSLATIONS.

$$\delta \chi = 0, \quad \delta \psi = 0 \quad \text{.. SYMMETRY.}$$

$$J^\mu = T_\alpha^\mu \chi^\alpha \rightarrow \boxed{J^\mu(\chi) = T_\alpha^\mu \chi^\alpha}$$

4 CONSERVED
CURRENT.

• ENERGY-MOMENTUM TENSOR

ACTION INVARIANT UNDER POINCARÉ TR., SPEC BY TRANSLATIONS

$$\delta \tilde{\chi}^M = \epsilon^M, \quad \delta \tilde{\psi} = 0 \quad \dots \text{SYMMETRY.}$$

$$J^M = T_C^M \sim \epsilon^\nu$$

$$J^M_\nu = T_C^M{}_\nu$$

4 CONSERVED
CURRENT.

$$\partial_\mu J^M_\nu = 0$$

$J^M_{(0)} \dots$ ENERGY CONS.

$J^M_{(i)} \dots$ MOMENTUM CONS.

• ENERGY-MOMENTUM TENSOR

ACTION INVARIANT UNDER POINCARÉ TR., SPEC BY TRANSLATIONS

$$\tilde{\delta} X^\mu = \epsilon^\mu, \quad \tilde{\delta} \psi = 0 \quad \dots \text{SYMMETRY.}$$

$$J^\mu = T^\mu_\nu \epsilon^\nu$$

$$\boxed{J^\mu(\nu) = T^\mu_\nu \epsilon^\nu}$$

4 CONSERVED CURRENT.

$$\partial_\mu J^\mu(\nu) = 0$$

$J^\mu_{(0)} \dots$ ENERGY CONS.

$J^\mu_{(i)} \dots$ MOMENTUM CONS.

$$\boxed{\partial_\mu T^\mu_\nu = 0}$$

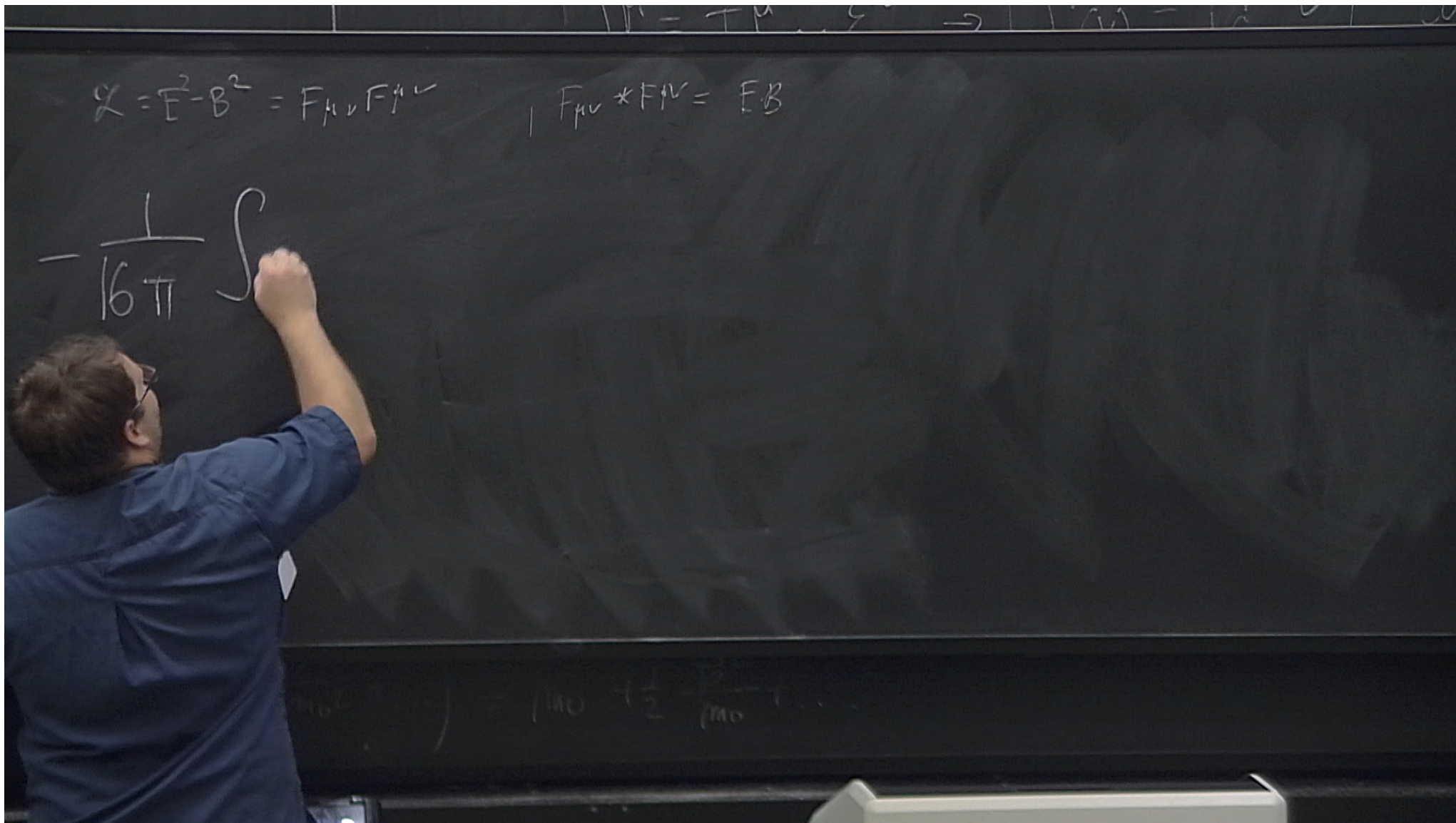
CONSERVATION OF E.M. TENSOR,

$$J^\mu = T^\mu_{\nu} \dot{x}^\nu + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi)} \delta \psi \quad \left. \begin{array}{l} \text{IS CONSERVED} \\ \partial_\mu J^\mu = 0 \end{array} \right\}$$

ACTION FOR MAXWELL:

$$S[A_\mu, \dot{y}^\nu]$$

$\uparrow$ FIELD $\uparrow$ ELECTRON'S TRAJECTORY



$$\text{ST. } \delta S = 0 \Rightarrow$$

$$\frac{\partial \mathcal{L}}{\partial \psi} = 0$$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

ACTION FOR MAXWELL

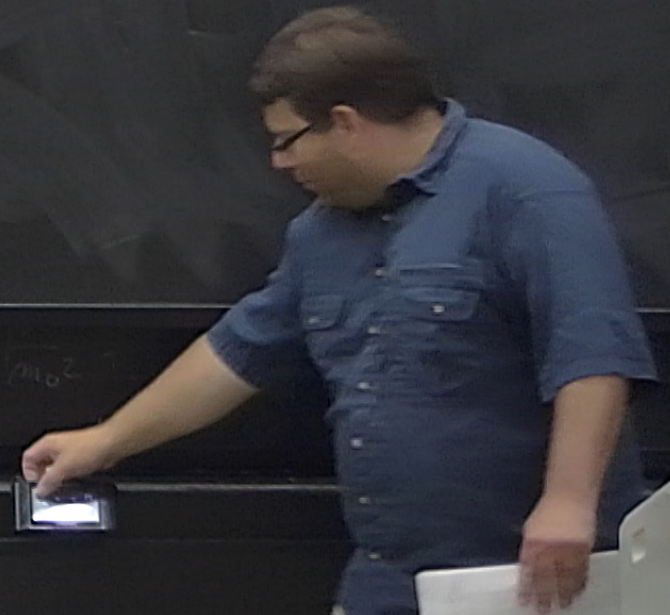
$$\mathcal{L} = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{2} (E^2 - B^2)$$

$$F_{\mu\nu} * F^{\mu\nu} = EB$$

$$S[A_\mu, y^\nu] = \underbrace{-m \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}}}_{\text{ELECTRON}} - \underbrace{\frac{1}{16\pi} \int d^4x F_{\mu\nu} F^{\mu\nu}}_{\text{FIELD}}$$

$\uparrow$ FIELD
 $\uparrow$ ELECTRON'S TRAJECTORY
 $\downarrow$ ELECTRON

PASSIVE: $E = m_0 \sqrt{1 + \frac{p^2}{m_0^2}} \approx m_0 \left(1 + \frac{1}{2} \frac{p^2}{m_0^2} \right)$



ACTION FOR MAXWELL

$$\mathcal{L} = E^2 - B^2 = F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} * F^{\mu\nu} = EB$$

$$S[A_\mu, y^\nu] = \underbrace{-m \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}}}_{\text{ELECTRON}} - \underbrace{\frac{1}{16\pi} \int d^4x F_{\mu\nu} F^{\mu\nu}}_{\text{FIELD}}$$

↑ ELECTRON'S TRAJECTORY
↑ FIELD

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

LORENTZ FORCE

MASSIVE: $E = m_0 \sqrt{1 + \frac{\vec{p}^2}{m_0^2}} = (m, \vec{p})$

$$\approx m_0 \left(1 + \frac{1}{2} \frac{\vec{p}^2}{m_0^2} + \dots \right) = m_0 + \frac{1}{2} \frac{\vec{p}^2}{m_0}$$

$$\mathcal{L} = \mathbf{E}^2 - \mathbf{B}^2 = F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} * F^{\mu\nu} = \mathbf{E} \cdot \mathbf{B}$$

$$-\frac{1}{16\pi} \int d^4x F_{\mu\nu} F^{\mu\nu} - e \int \frac{d\gamma^\mu}{d\lambda} A_\mu d\lambda$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

INTERACTION TERM

$$\delta A_\mu \rightarrow \text{MAXWELL}$$

$$\delta \gamma^\mu \rightarrow \text{L.F.L.}$$

$$-\psi(x^\mu)$$

IS CONSERVED

ACTION INVARIANT UNDER POINCARÉ TR. SPEC BY TRANSLATIONS.

$$\delta x^\mu = \epsilon^\mu, \quad \delta \psi = 0$$

SYMMETRY

$$J^\mu = T^\mu_\nu \epsilon^\nu \rightarrow \boxed{J^\mu_\nu = T^\mu_\nu}$$

4 CONSERVED
CURRENT.

ACTION FOR MAXWELL

$$\mathcal{L} = E^2 - B^2 = F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} * F^{\mu\nu} = EB$$

$$S[A_\mu, y^\nu] = -m \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}} - \frac{1}{16\pi} \int d^4x F_{\mu\nu} F^{\mu\nu} - e \int \frac{dy^\mu}{d\lambda} A_\mu d\lambda$$

$\uparrow$ FIELD $\uparrow$ ELECTRON'S TRAJECTORY
 $\downarrow$ ELECTRON

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

INTERACTION TERM

$\delta A_\mu \rightarrow$ MAXWELL

$\delta y^\mu \rightarrow$ L.F.L.

THAT IS OFF-SHELL WE FIND $\tilde{\delta} x^\mu = x'^\mu - x^\mu$
 $\tilde{\delta} \varphi(x^\mu) = \varphi(x'^\mu) - \varphi(x^\mu)$

$$\tilde{\delta} S = 0 \Rightarrow$$

$$J^\mu = T_c^\mu \tilde{\delta} x^\nu + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \tilde{\delta} \varphi$$

IS CONSERVED

$$\partial_\mu J^\mu = 0$$

ACTION INVARIANT UNDER POIN

$$\tilde{\delta} x^\mu = \epsilon^\mu, \quad \tilde{\delta} \varphi = 0$$

$$J^\mu = T_c^\mu \nu \epsilon^\nu \rightarrow \int J^\mu$$

$$\partial_\mu J^\mu = 0$$

INTERACTION TERM

$\delta A^M \rightarrow$ MAXWELL

$\delta \psi^M \rightarrow$ L.F.L.

$$(\partial^2 - m^2)\psi = 0$$

$$\tilde{\partial}_\mu \chi^M = \epsilon^M, \quad \tilde{\partial}_\mu \psi = 0 \quad \dots \text{SYMMETRY.}$$

$$J^M = T_C^M \nu \epsilon^\nu$$

$\rightarrow$

$$J^M(\nu) = T_C^M \nu$$

4 CONSERVED
CURRENT.

$$J^M(\nu) = 0$$

$$J^M(\nu)$$

ENERGY CONS.

INTERACTION TERM

$\delta A^M \rightarrow$ MAXWELL

$\delta \psi^M \rightarrow$ L.F.L.

$$\boxed{(\partial^2 - m^2) \psi^* = 0} \quad \boxed{\text{SCALAR EL DYN}}$$

$\tilde{\partial}_M \psi^M = \epsilon^M$, $\tilde{\partial}_M \psi = 0$... SYMMETRY.

$J^M = T_C^M \psi \epsilon^\nu$

$\rightarrow$

$$\boxed{J^M(\psi) = T_C^M \psi}$$

4 CONSERVED CURRENT.

$\psi^M(\psi) = 0$

$\psi^M(\psi)$

ENERGY CONS.

INTERACTION TERM

$\delta A^{\mu} \rightarrow$ MAXWELL

$\delta \psi^{\mu} \rightarrow$ L.F.L.

$$(\partial^2 - m^2) \psi^* = 0$$

SCALAR
ELDYN

DIRAC ... $\frac{1}{2}$ SPIN

$\tilde{\partial} X^{\mu} = \epsilon^{\mu}$, $\tilde{\partial} \psi = 0$... SYMMETRY.

$$J^{\mu} = T^{\mu}_{\nu} \psi \epsilon^{\nu}$$

$\rightarrow$

$$J^{\mu}(\psi) = T^{\mu}_{\nu} \psi$$

4 COM
CUR

$$\psi^{\mu}(\psi) = 0$$

$$\psi^{\mu}(\psi)$$

ENERGY CONS.