Title: PSI 2019/2020 - Classical Physics - Lecture 8

Speakers: David Kubiznak

Collection: PSI 2019/2020 - Classical Physics (Kubiznak)

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YESTERDAY. MAXWELL'S THEORY IN RELATIVISTIC NOTATION

$$F_{[\mu\nu,\delta]c} = 0 \quad \Leftrightarrow \quad \partial_\mu (*F)^{\mu\nu} = 0 \qquad \dots \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial_\mu F^{\mu\nu} = -4\pi J^\nu$$

$$\frac{dp^\mu}{d\tau} = F^\mu_{\ L} = e F^{\mu\nu} u_\nu$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$$F^{0i} = E^i, \quad F^{ij} = \varepsilon^{ijk} B_k$$

YESTERDAY. MAXWELL'S THEORY IN RELATIVISTIC NOTATION

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$$J^\nu = (\rho, j)$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

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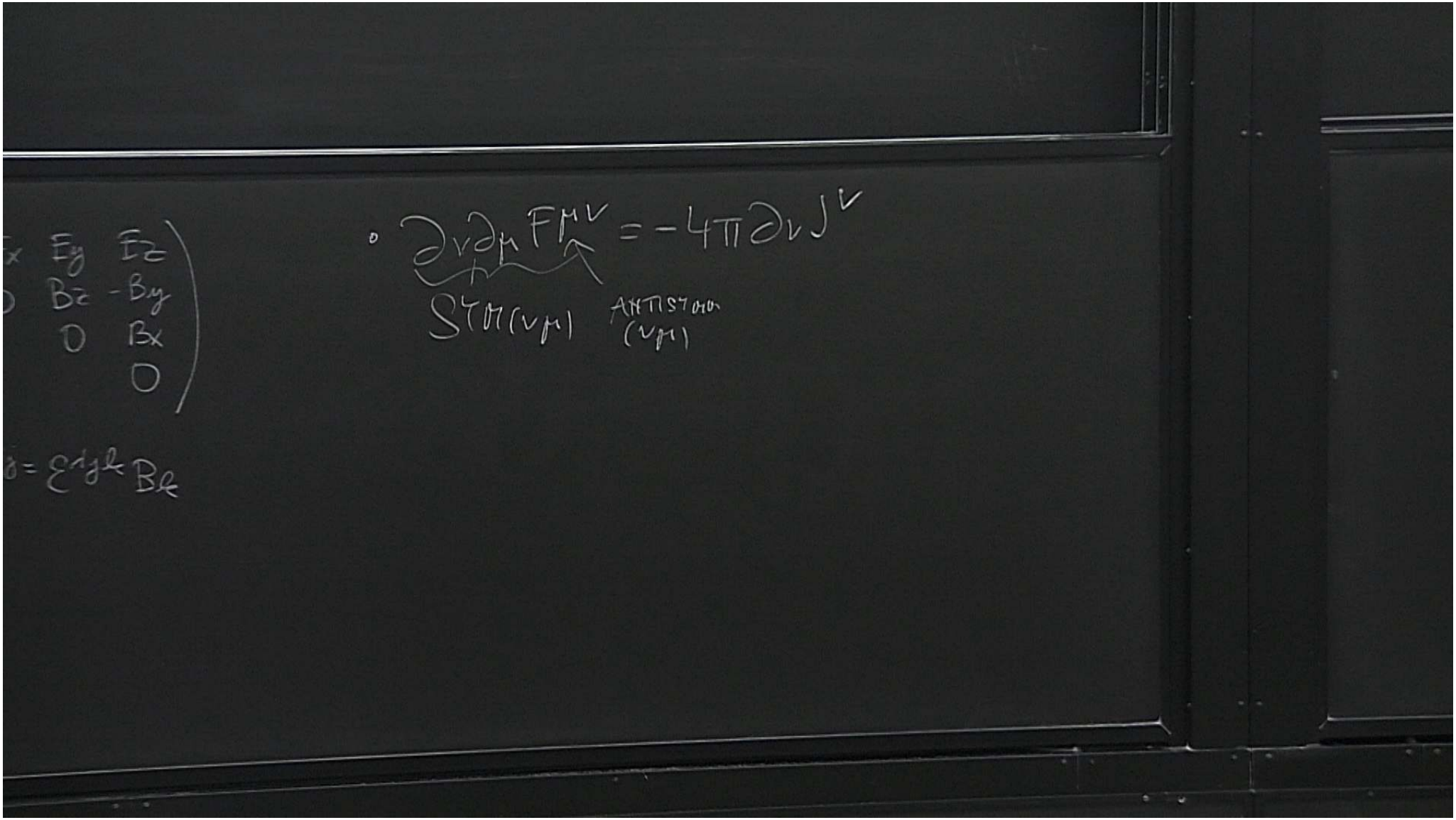
$F^{\mu\nu} \rightarrow u^\mu$


$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ & 0 & B_x \\ & & 0 \end{pmatrix}$
$B = \varepsilon^{ijk} B_k$
$\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$
SYM$(\nu\mu)$
ANTISYM$(\nu\mu)$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ & 0 & B_x \\ & & 0 \end{pmatrix}$$

$\vec{B} = \varepsilon^{ijk} B_k$

- $\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$

SYM($\nu\mu$) ANTISYM ($\nu\mu$)

$$\boxed{\partial_\nu J^\nu = 0}$$

CURRENT CONSERVATION

$$\begin{pmatrix} E_y & E_z \\ B_z & -B_y \\ 0 & B_x \\ & 0 \end{pmatrix}$$

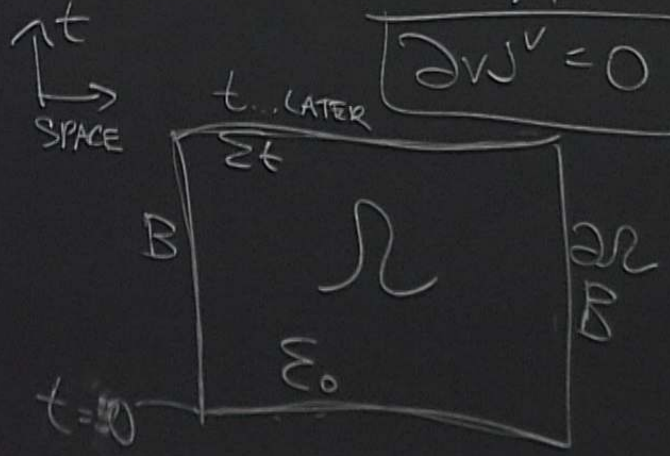
$= \varepsilon_{ijk} B_k$

$\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$

SYMM($\nu\mu$) ANTISYMM($\nu\mu$)

$\partial_\nu J^\nu = 0$

CURRENT CONSERVATION



$$0 = \int_{\Omega} \partial_\nu J^\nu \, d^4x = \int_{\partial\Omega} J^\nu \, d\Sigma_\nu = \int_{\Sigma_0} J^\nu \, d\Sigma_\nu + \int_{B} J^\nu \, d\Sigma_\nu - \int_{\Sigma_t} J^\nu \, d\Sigma_\nu$$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ & 0 & B_x \\ & & 0 \end{pmatrix}$$

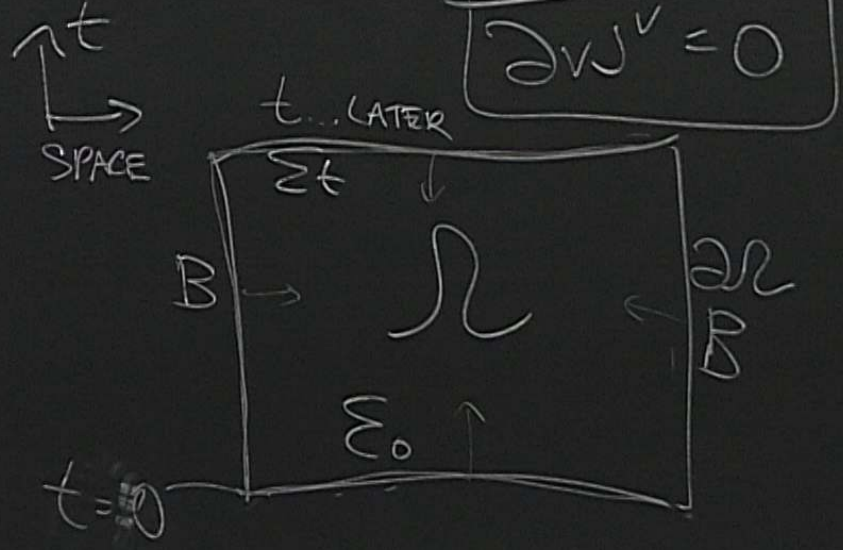
$F_{ij} = \varepsilon_{ijk} B_k$

- $\partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$

SYM($\nu\mu$) ANTISYM ($\nu\mu$)

$\partial_\nu J^\nu = 0$

CURRENT CONSERVATION

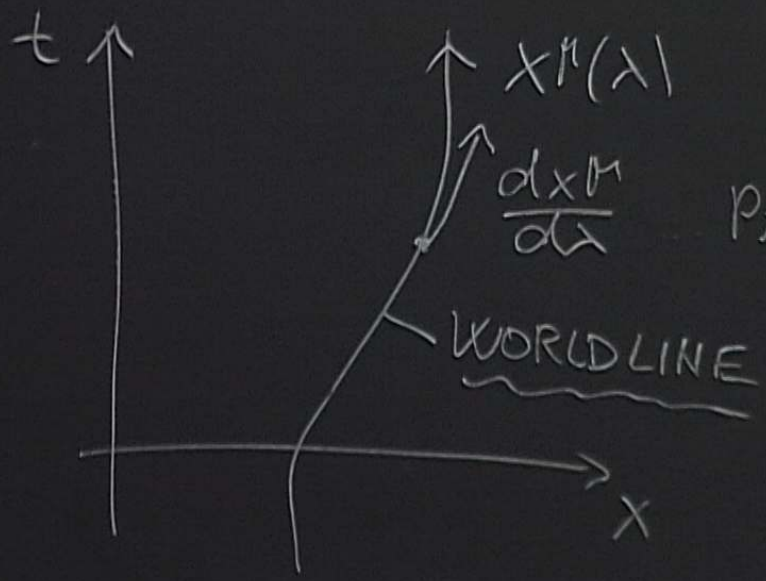


$$0 = \int_{\Omega} \partial_\nu J^\nu d^4x = \int_{\partial\Omega} J^\nu d\Sigma_\nu = \int_{\Sigma_0} J^\nu d\Sigma_\nu + \underbrace{\int_B J^\nu d\Sigma_\nu}_{\text{NO FLUX} \Rightarrow 0} - \int_{\Sigma_t} J^\nu d\Sigma_\nu$$

$$\boxed{Q = \int_{\Sigma_0} J^\nu d\Sigma_\nu = \int_{\Sigma_t} J^\nu d\Sigma_\nu}$$

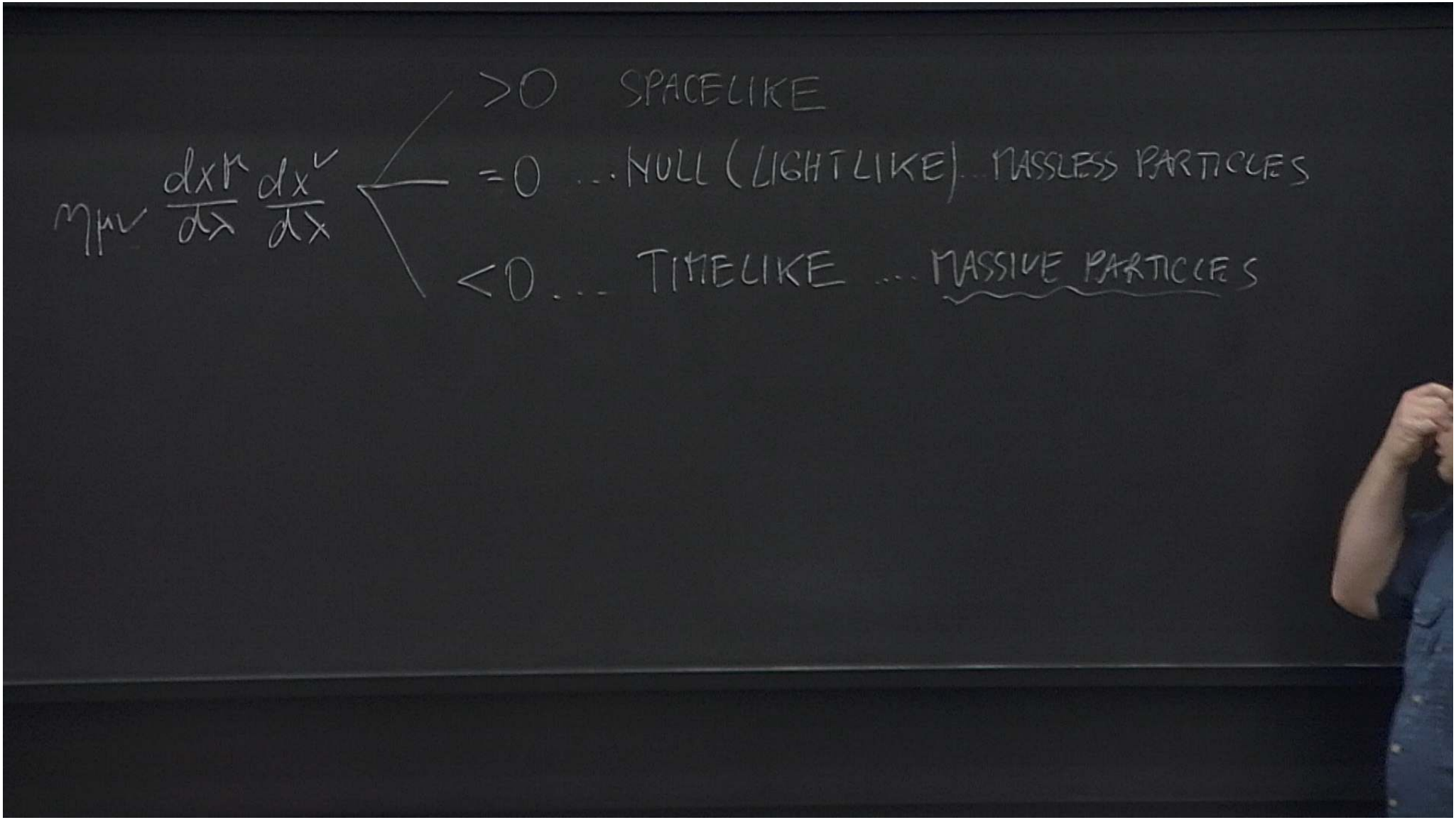
CONSERVED CHARGE

- RELATIVISTIC PARTICLE

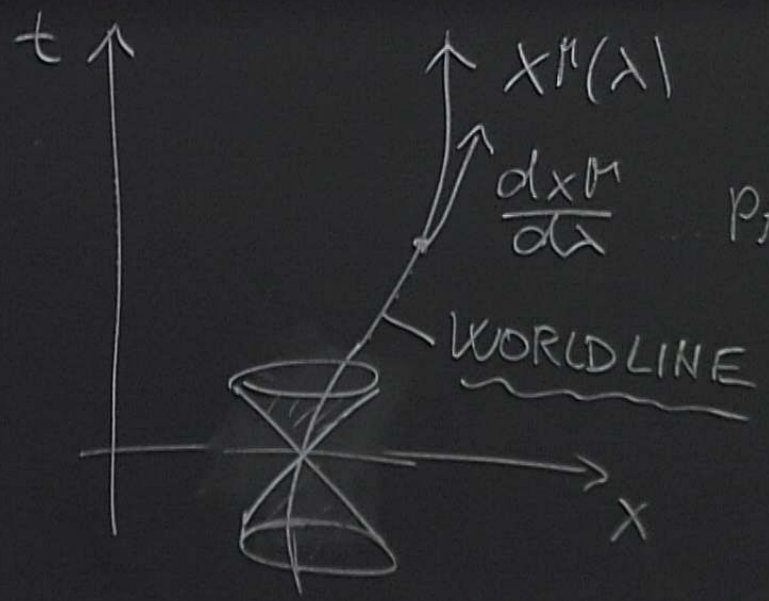


λ . PARAMETER

PARAMETRIC 4-VELOCITY


$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$
>0 SPACELIKE
=0 ...NULL (LIGHTLIKE)...MASSLESS PARTICLES
<0 ... TIMELIKE ... MASSIVE PARTICLES

• RELATIVISTIC PARTICLE



λ . PARAMETER

PARAMETRIC 4-VELOCITY

$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx}{d\lambda}$

ETER

4-VELOCITY

$$\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$$

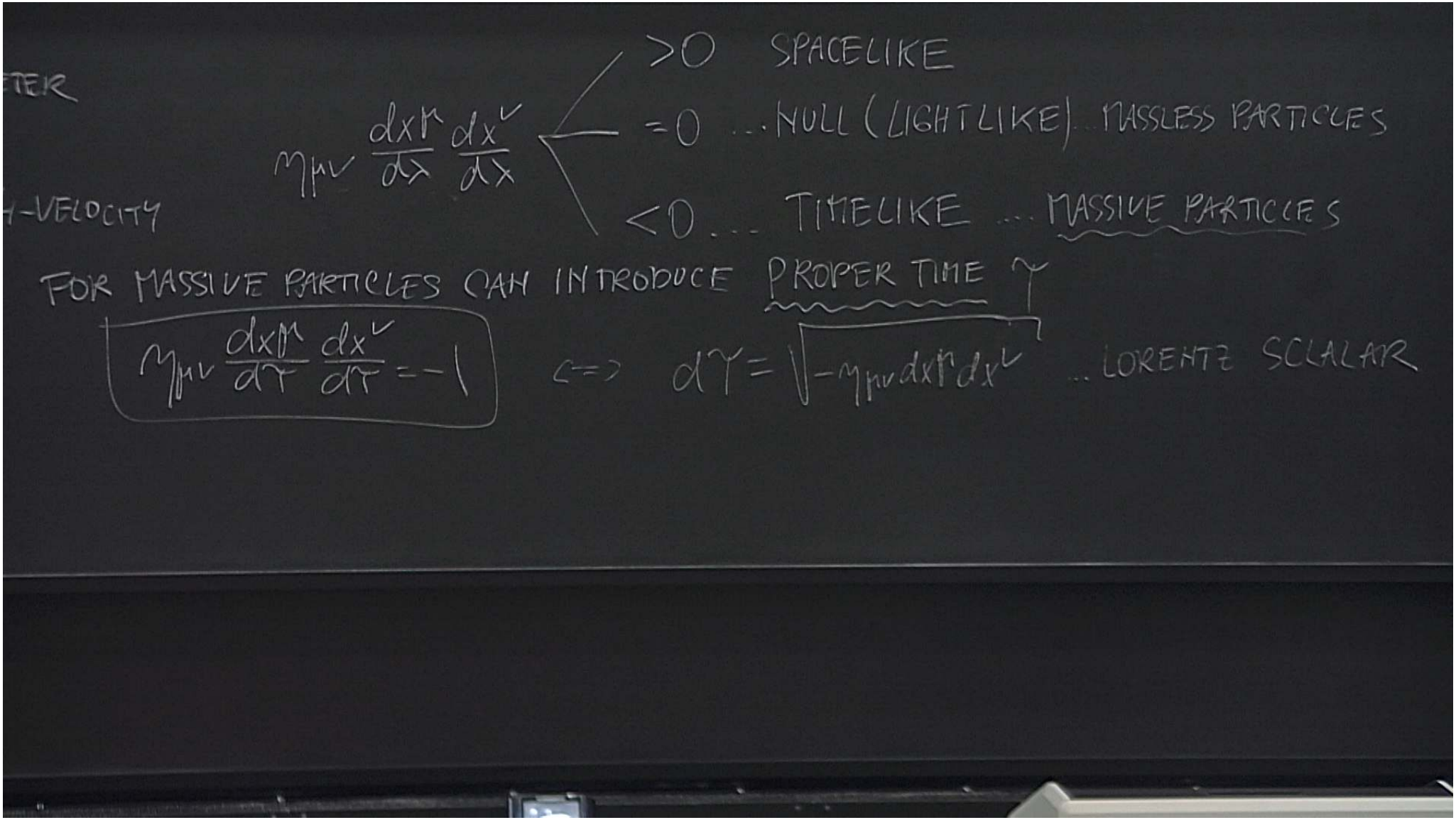
>0 SPACELIKE

$=0$ ... NULL (LIGHTLIKE) .. MASSLESS PA

<0 ... TIMELIKE ... MASSIVE PARTIC

FOR MASSIVE PARTICLES CAN INTRODUCE PROPER TIME τ

$$\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = -1$$


ETER
H-VELOCITY
$\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$
>0 SPACELIKE
=0 ...NULL (LIGHTLIKE) ...MASSLESS PARTICLES
<0 ... TIMELIKE ... MASSIVE PARTICLES
FOR MASSIVE PARTICLES CAN INTRODUCE PROPER TIME τ
$\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}=-1$
$\Leftrightarrow$ $d\tau=\sqrt{-\eta_{\mu\nu}dx^\mu dx^\nu}$ ...LORENTZ SCALAR

$\eta_{\mu\nu} \frac{dx}{d\tau} \frac{dx}{d\tau}$

4-VELOCITY

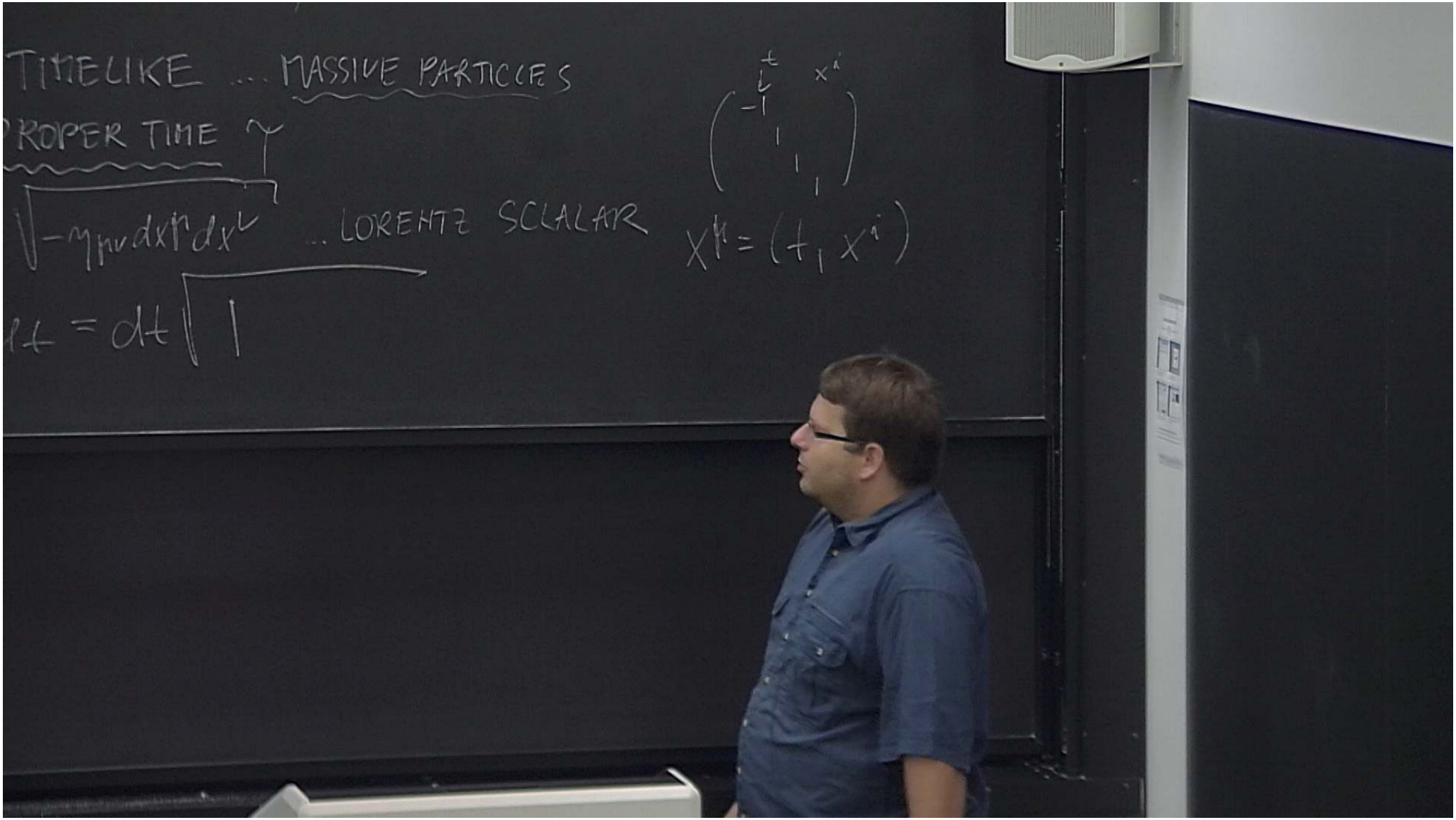
< 0 ... TIMELIKE ... MASSIVE PARTICLES

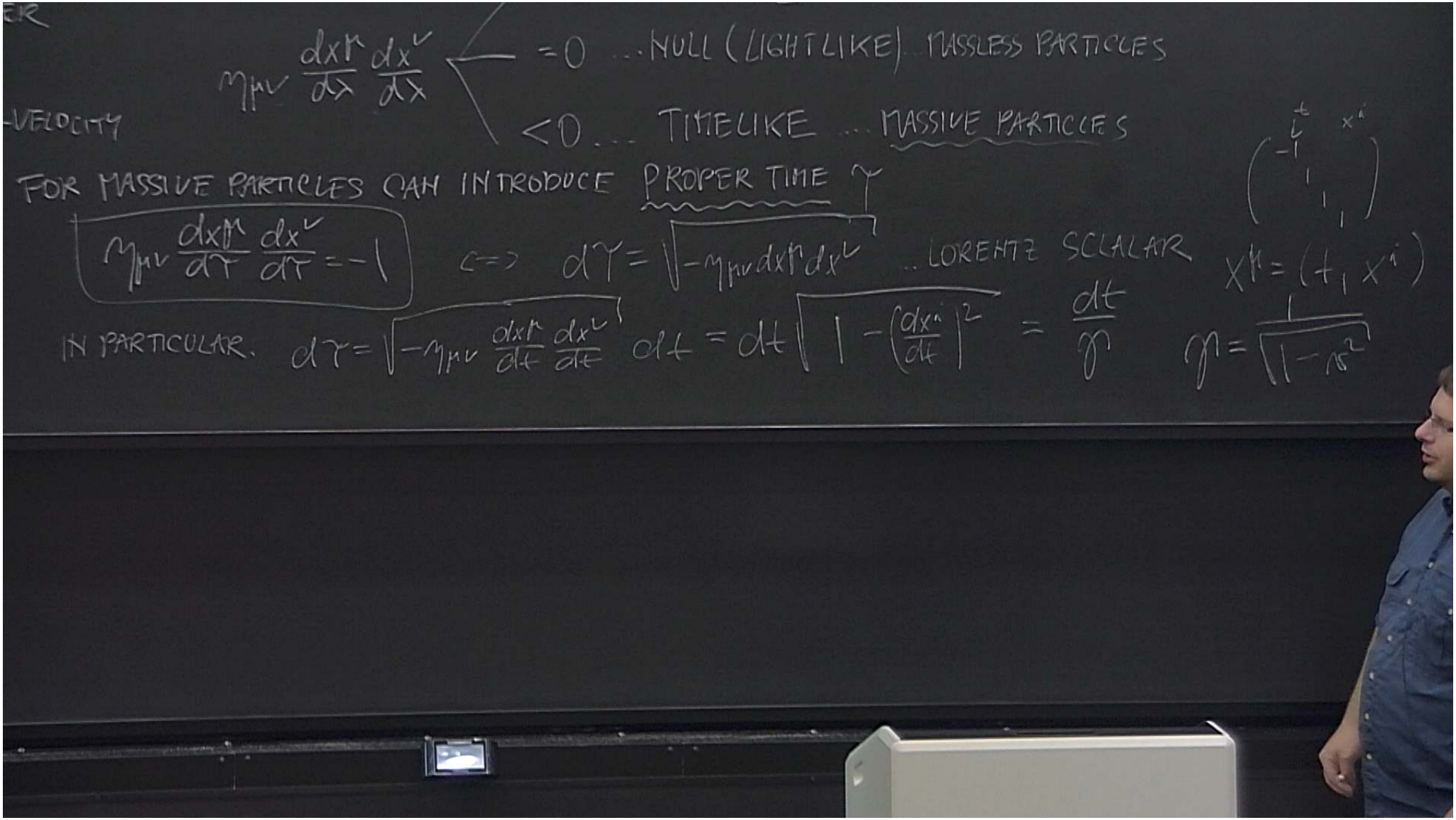
FOR MASSIVE PARTICLES CAN INTRODUCE PROPER TIME τ

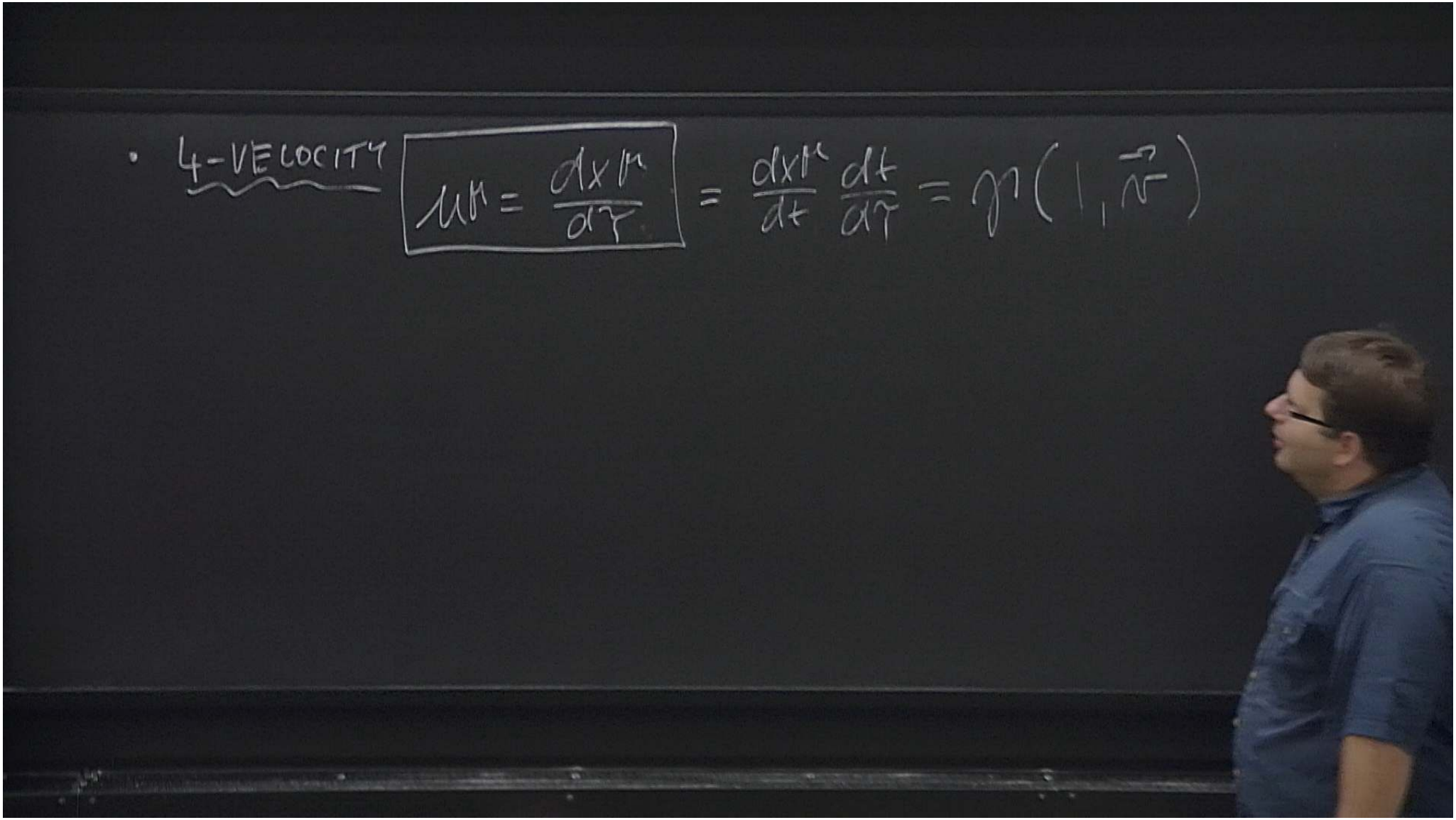
$$\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1 \quad \Leftrightarrow \quad d\tau = \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}$$

.. LORENTZ SCALAR

IN PARTICULAR. $d\tau = \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}}\, dt$


TIMELIKE ... MASSIVE PARTICLES
PROPER TIME
LORENTZ SCLALAR
$X^\mu = (t, x^i)$


ER
$\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$ $=0$ ... NULL (LIGHTLIKE) MASSLESS PARTICLES
-VELOCITY
<0 ... TIMELIKE ... MASSIVE PARTICLES
FOR MASSIVE PARTICLES CAN INTRODUCE PROPER TIME τ
$\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}=-1$ $\Leftrightarrow$ $d\tau=\sqrt{-\eta_{\mu\nu}dx^\mu dx^\nu}$ .. LORENTZ SCALAR
$x^\mu=(t,x^i)$
IN PARTICULAR. $d\tau=\sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt=dt\sqrt{1-\left(\frac{dx^i}{dt}\right)^2}=\frac{dt}{\gamma}$
$\gamma=\frac{1}{\sqrt{1-v^2}}$


• 4-VELOCITY $u^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt}\frac{dt}{d\tau} = \gamma(1, \vec{v})$

- 4-VELOCITY $\boxed{u^\mu = \frac{dx^\mu}{d\tau}} = \frac{dx^\mu}{dt}\frac{dt}{d\tau} = \gamma(1, \vec{v})$

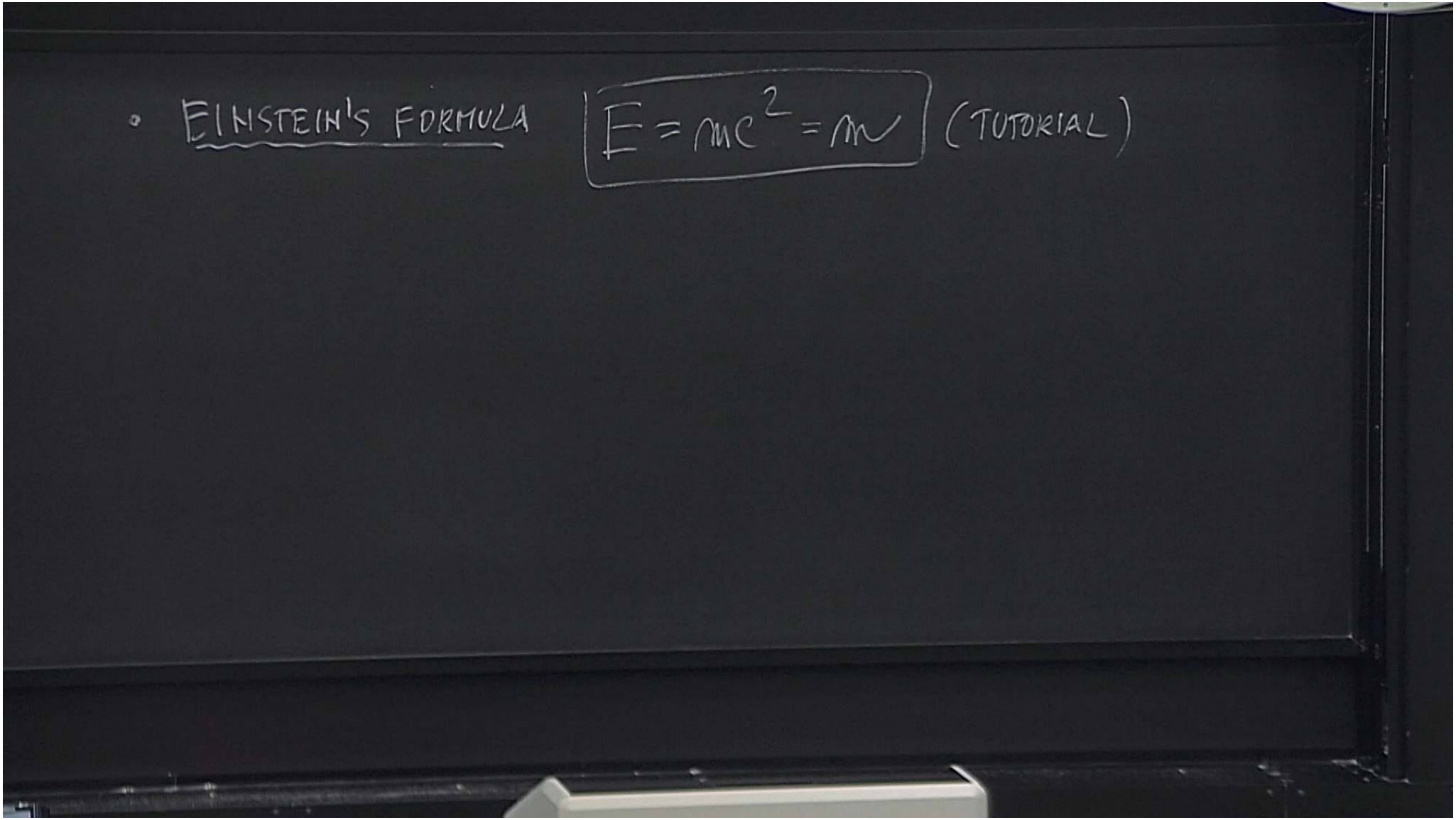
- 4-MOMENTUM $\boxed{m(v) = \gamma m_0}$ ← REST MASS OF A PARTICLE

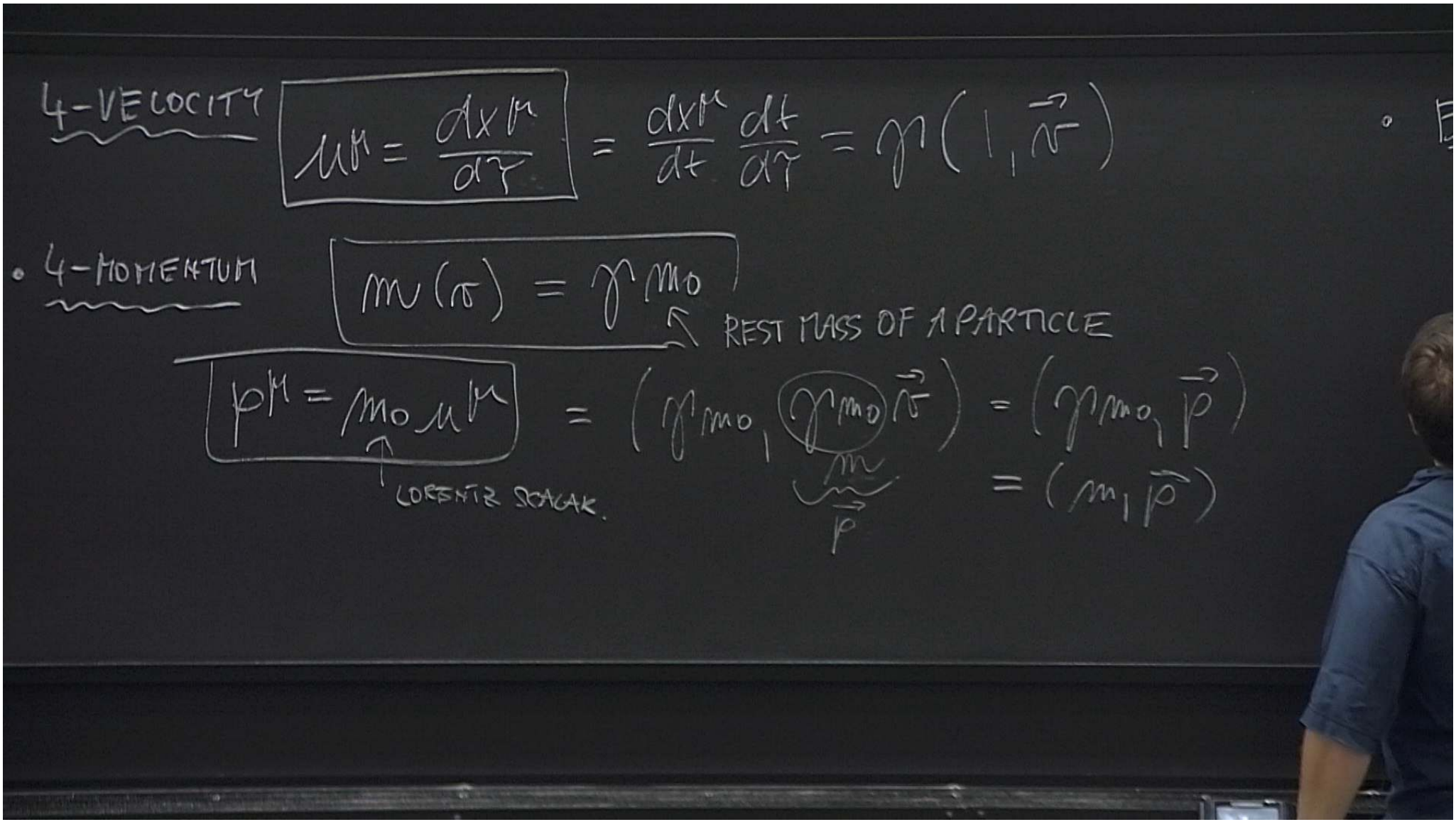
$\boxed{p^\mu = m_0 u^\mu} = (\gamma m_0, \gamma m_0 \vec{v}) = (\gamma m_0, \vec{p})$

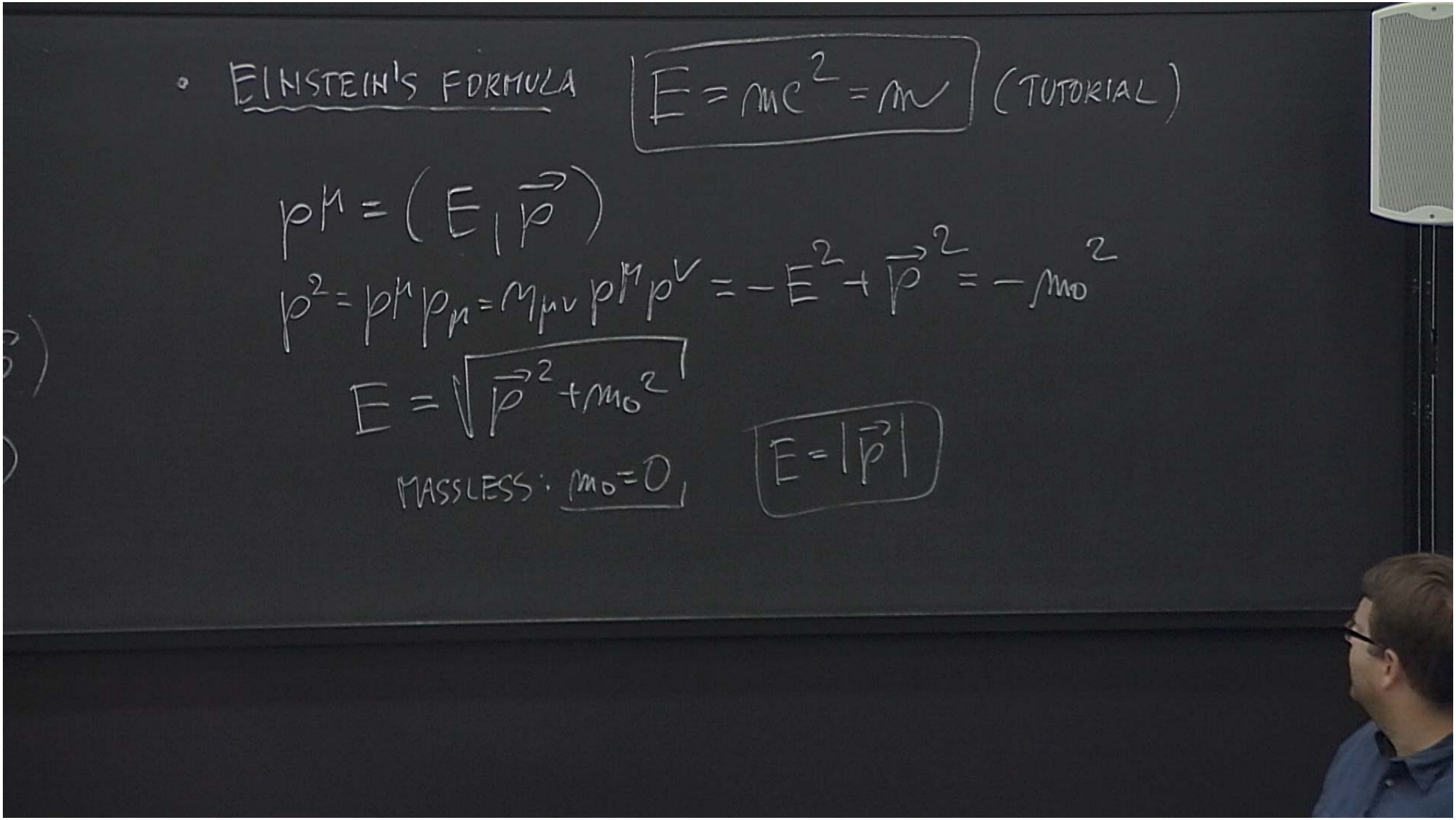
↑ LORENTZ SCALAR.

m

$\vec{p}$


• EINSTEIN'S FORMULA $E = mc^2 = m\nu$ (TUTORIAL)


4-VELOCITY
$u^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt}\frac{dt}{d\tau} = \gamma(1, \vec{v})$
4-MOMENTUM
$m(v) = \gamma m_0$
REST MASS OF A PARTICLE
$p^\mu = m_0 u^\mu = (\gamma m_0, \gamma m_0 \vec{v}) = (\gamma m_0, \vec{p})$
$= (m, \vec{p})$
LORENTZ SCALAR.


• EINSTEIN'S FORMULA $E = mc^2 = m$ (TUTORIAL)
$p^\mu = (E, \vec{p})$
$p^2 = p^\mu p_\mu = \eta_{\mu\nu} p^\mu p^\nu = -E^2 + \vec{p}^2 = -m_0^2$
$E = \sqrt{\vec{p}^2 + m_0^2}$
MASSLESS: $m_0 = 0$, $E = |\vec{p}|$

- 4-VELOCITY $\boxed{u^\mu = \frac{dx^\mu}{d\tau}} = \frac{dx^\mu}{dt}\frac{dt}{d\tau} = \gamma(1, \vec{v})$

- 4-MOMENTUM $\boxed{m(v) = \gamma m_0}$ REST MASS OF A PARTICLE

$$\boxed{p^\mu = m_0 u^\mu} = (\gamma m_0, \underbrace{\gamma m_0}_{m} \vec{v}) = (\gamma m_0, \vec{p})$$

LORENTZ SCALAR.

$$= (m, \vec{p})$$

MASSIVE: $E = m_0 \sqrt{1 + \frac{\vec{p}^2}{m_0^2}}$

$$u^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt}\frac{dt}{d\tau} = \gamma(1, \vec{v})$$

$$m(v) = \gamma m_0$$

↖ REST MASS OF A PARTICLE

$$p^\mu = m_0 u^\mu = (\gamma m_0, \gamma m_0 \vec{v}) = (\gamma m_0, \vec{p})$$

m

$$= (m, \vec{p})$$

↑ LORENTZ SCALAR.

MASSIVE: $E = m_0\sqrt{1 + \frac{\vec{p}^2}{m_0^2}} \approx m_0\left(1 + \frac{1}{2}\frac{\vec{p}^2}{m_0^2} + \dots\right) = m_0 + \frac{1}{2}\frac{\vec{p}^2}{m_0} + \dots$

• EINSTEIN'S FORMULA

$$p^\mu = (E, \vec{p})$$

$$p^2 = p^\mu p_\mu = \eta_{\mu\nu} p^\mu$$

$$E = \sqrt{\vec{p}^2 + m_0}$$

MASSLESS: $m_0 =$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ & 0 & B_x \\ & & 0 \end{pmatrix}$$

$$F^{ij} = \varepsilon^{ijk} B_k$$

2ND NEWTON'S LAW:

$$\boxed{\frac{dp^\mu}{d\tau} = F^\mu} \quad \ldots \text{ 4-FORCE}$$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ & 0 & B_x \\ & & 0 \end{pmatrix}$$

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2ND NEWTON'S LAW:

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NO FORCE .. FREE PARTICLE (GEODESICS) $\frac{dp^\mu}{d\tau} = 0$

$$S = -m_0 c^2 \int d\tau$$

$$\begin{pmatrix} E_x & E_y & E_z \\ 0 & B_z & -B_y \\ & 0 & B_x \\ & & 0 \end{pmatrix}$$

$F^{ij} = \varepsilon^{ijk} B_k$

2ND NEWTON'S LAW:

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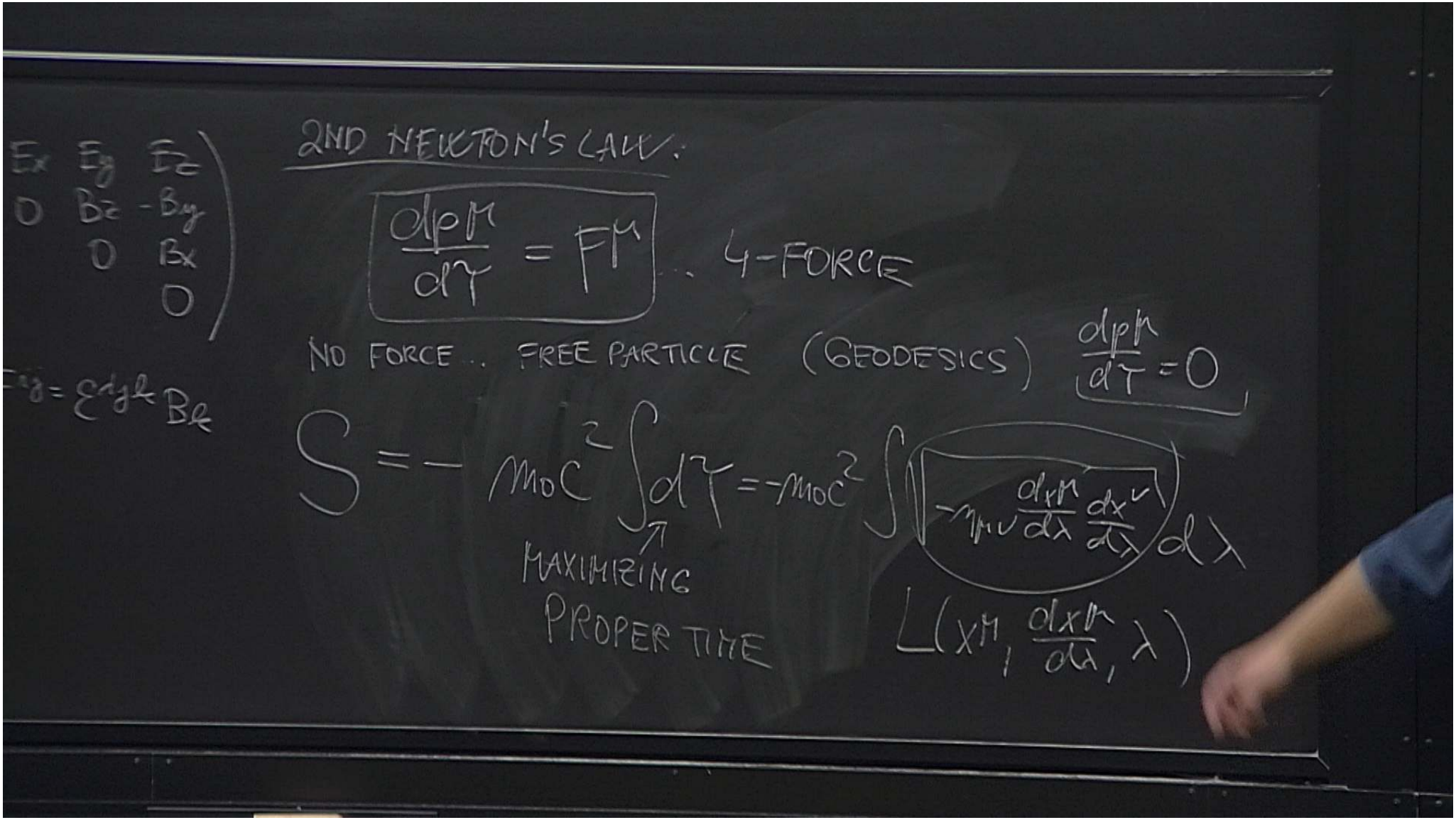
$$S = -m_0 c^2 \int d\tau$$

↑ MAXIMIZING PROPER TIME

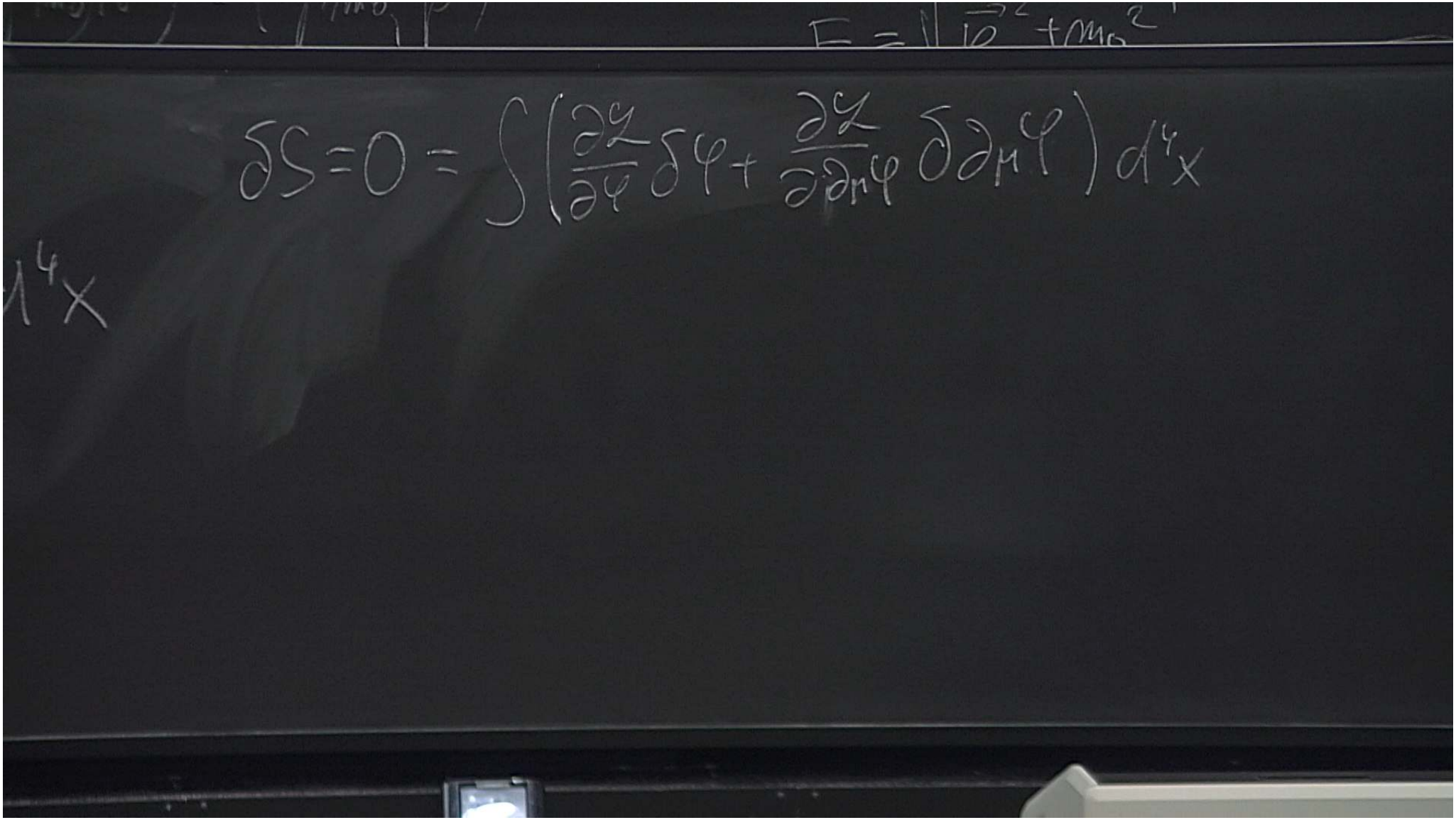

2ND NEWTON'S LAW:
$\frac{dp^\mu}{d\tau} = F^\mu$ . . 4-FORCE
NO FORCE . . FREE PARTICLE (GEODESICS) $\frac{dp^\mu}{d\tau} = 0$
$S = -m_0 c^2 \int d\tau = -m_0 c^2 \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, d\lambda$
MAXIMIZING
PROPER TIME
$L(x^\mu, \frac{dx^\mu}{d\lambda}, \lambda)$

d) RELATIVISTIC FIELD THEORIES

$$S[\varphi] = \int$$

φ: FIELD

$$\varphi(t, \vec{x}) = \varphi(x^\mu)$$


$E = \sqrt{\vec{p}^2 + m_0^2}$
$\delta S = 0 = \int \left(\frac{\partial \mathcal{L}}{\partial \varphi} \delta\varphi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \delta \partial_\mu \varphi \right) d^4x$
d^4x

$$\delta S = 0 = \int_{\Omega} \left(\frac{\partial \mathcal{L}}{\partial \varphi} \delta\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \underbrace{\delta(\partial_\mu \varphi)}_{\partial_\mu \delta\varphi} \right) d^4x$$

BY PARTS

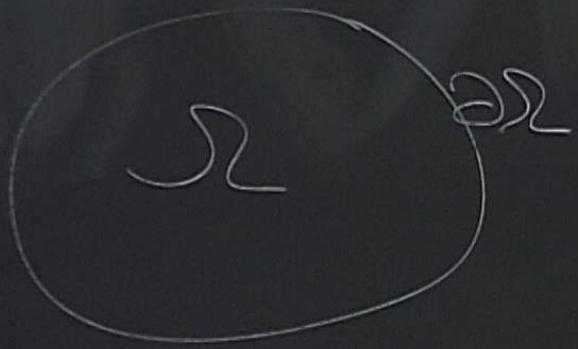
$$= \int_{\Omega} \left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) \delta\varphi \, d^4x - \int_{\partial\Omega} d^3\Sigma_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \delta\varphi$$

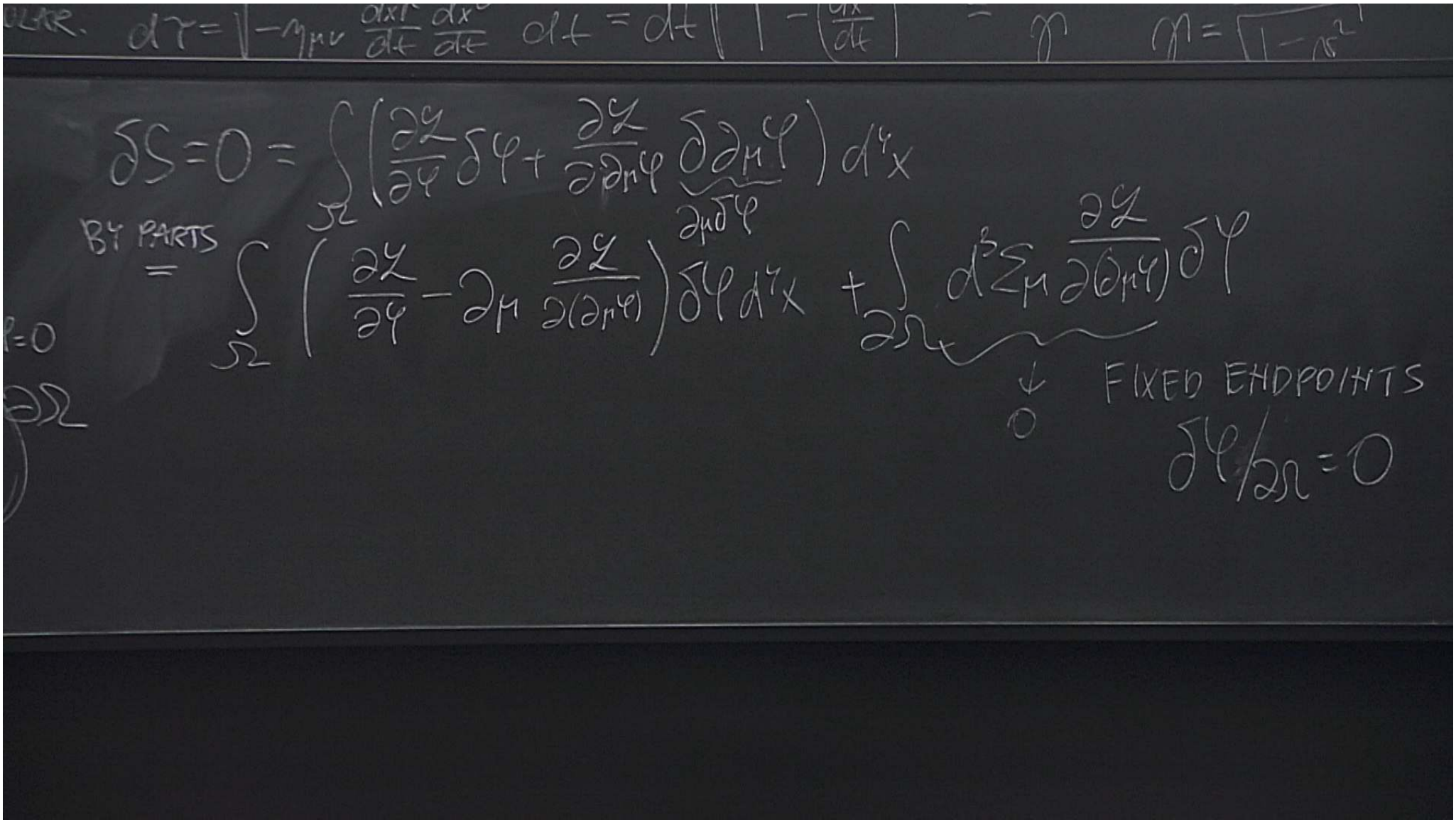
THEORIES

$$\delta S = 0 = \int_\Omega \left(\frac{\partial \mathcal{L}}{\partial \varphi} \delta\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \underbrace{\delta(\partial_\mu \varphi)}_{\partial_\mu \delta\varphi} \right) d^4x$$

$$\overset{\text{BY PARTS}}{=} \int_\Omega \left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) \delta\varphi \, d^4x - \int_{\partial\Omega} d^3$$

$\partial_\mu \varphi, x^\mu) \, d^4x$




$d\tau = \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt = dt\sqrt{1-\left(\frac{dx}{dt}\right)^2} = \gamma \quad \gamma = \frac{1}{\sqrt{1-v^2}}$
$\delta S = 0 = \int_\Omega \left(\frac{\partial \mathcal{L}}{\partial \varphi}\delta\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi)}\frac{\delta\partial_\mu\varphi}{\partial_\mu\delta\varphi}\right) d^4x$
BY PARTS
$= \int_\Omega \left(\frac{\partial \mathcal{L}}{\partial\varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi)}\right)\delta\varphi\, d^4x + \int_{\partial\Omega} d^3\Sigma_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi)}\delta\varphi$
$\varphi = 0$
$\partial\Omega$
$\downarrow$
0
FIXED ENDPOINTS
$\delta\varphi|_{\partial\Omega} = 0$

$$\delta S = 0 = \int_\Omega \left(\frac{\partial \mathcal{L}}{\partial \varphi}\delta\varphi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \underbrace{\delta \partial_\mu \varphi}_{\partial_\mu \delta\varphi} \right) d^4x$$

BY PARTS

$$= \int_\Omega \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right)}_{\emptyset} \delta\varphi \, d^4x + \int_{\partial\Omega} \underbrace{d^3\Sigma_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)}}_{\downarrow \, 0} \delta\varphi$$

FIXED ENDPOINTS

$$\delta\varphi / \partial\Omega = 0$$

$$(E\text{-}L) \quad \boxed{\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) = 0}$$

$\varphi = 0$

$\partial\Omega$

$q(\underline{t})$

< 0 TIMELIKE MASSIVE PARTICLES

PARTICLES CAN INTRODUCE PROPER TIME τ

$dx^\mu \, dx^\nu$

LORENTZ SCALAR

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$q(t_1)$

NOETHER LET $\tilde{\delta}$ BE A GLOBAL CONTINUOUS SYMMETRY

THAT IS OFF-SHELL WE FIND $\tilde{\delta} x^\mu = x'^\mu - x^\mu$

$\tilde{\delta}\varphi(x^\mu) = \varphi'(x'^\mu) - \varphi(x^\mu)$

S.T. $\tilde{\delta} S = 0$

REST MASS OF A PARTICLE

NOETHER LET $\tilde{\delta}$ BE A GLOBAL CONTINUOUS SYMMETRY

THAT IS OFF-SHELL WE FIND $\tilde{\delta} x^\mu = x^{\mu\prime} - x^\mu$

$\tilde{\delta}\varphi(x^\mu) = \varphi'(x'^\mu) - \varphi(x^\mu)$

S.T. $\tilde{\delta} S = 0 \Rightarrow$

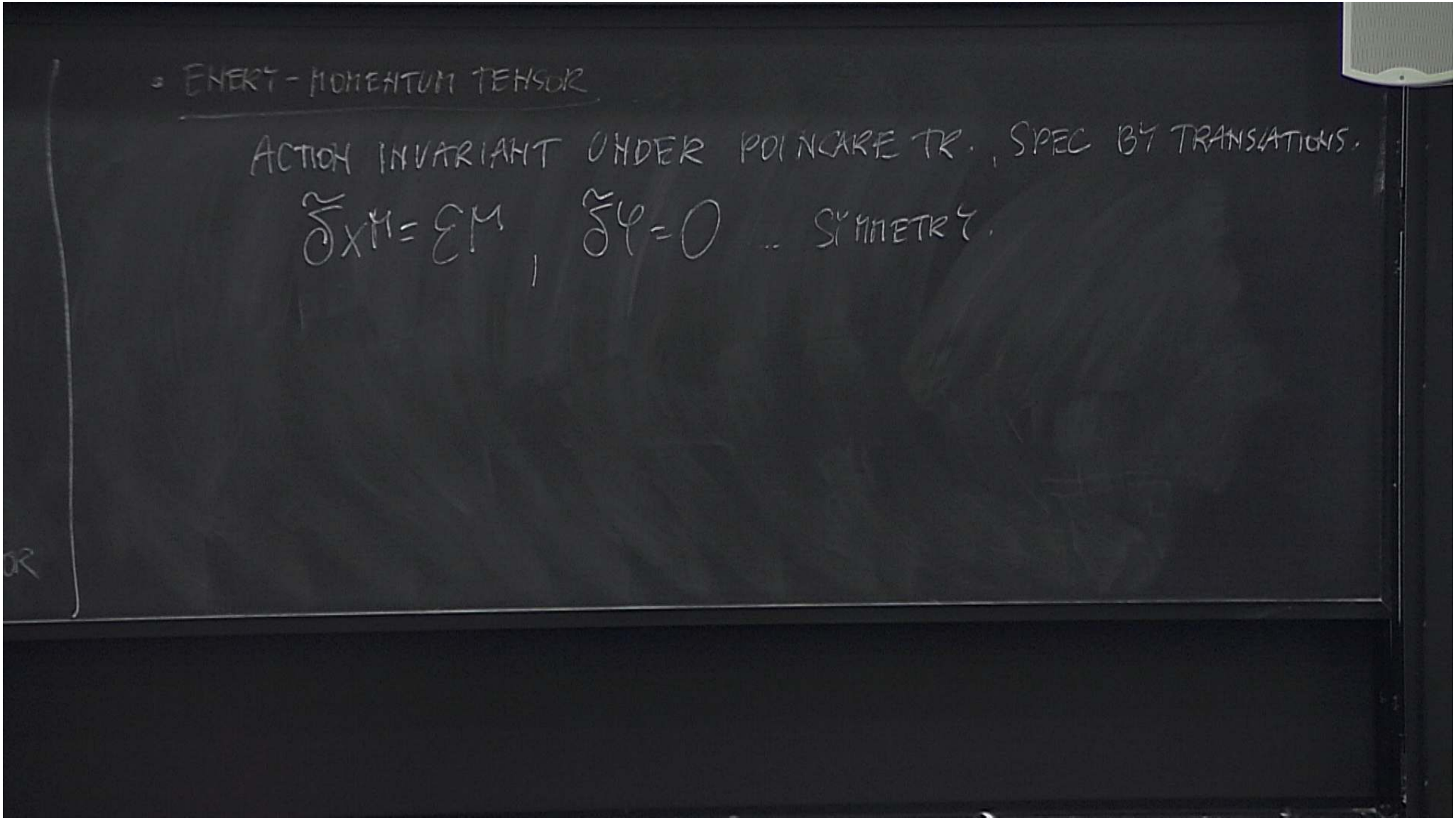
$$J^\mu = T^\mu_{c\,\nu}\tilde{\delta}x^\nu + \frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi)}\tilde{\delta}\varphi$$

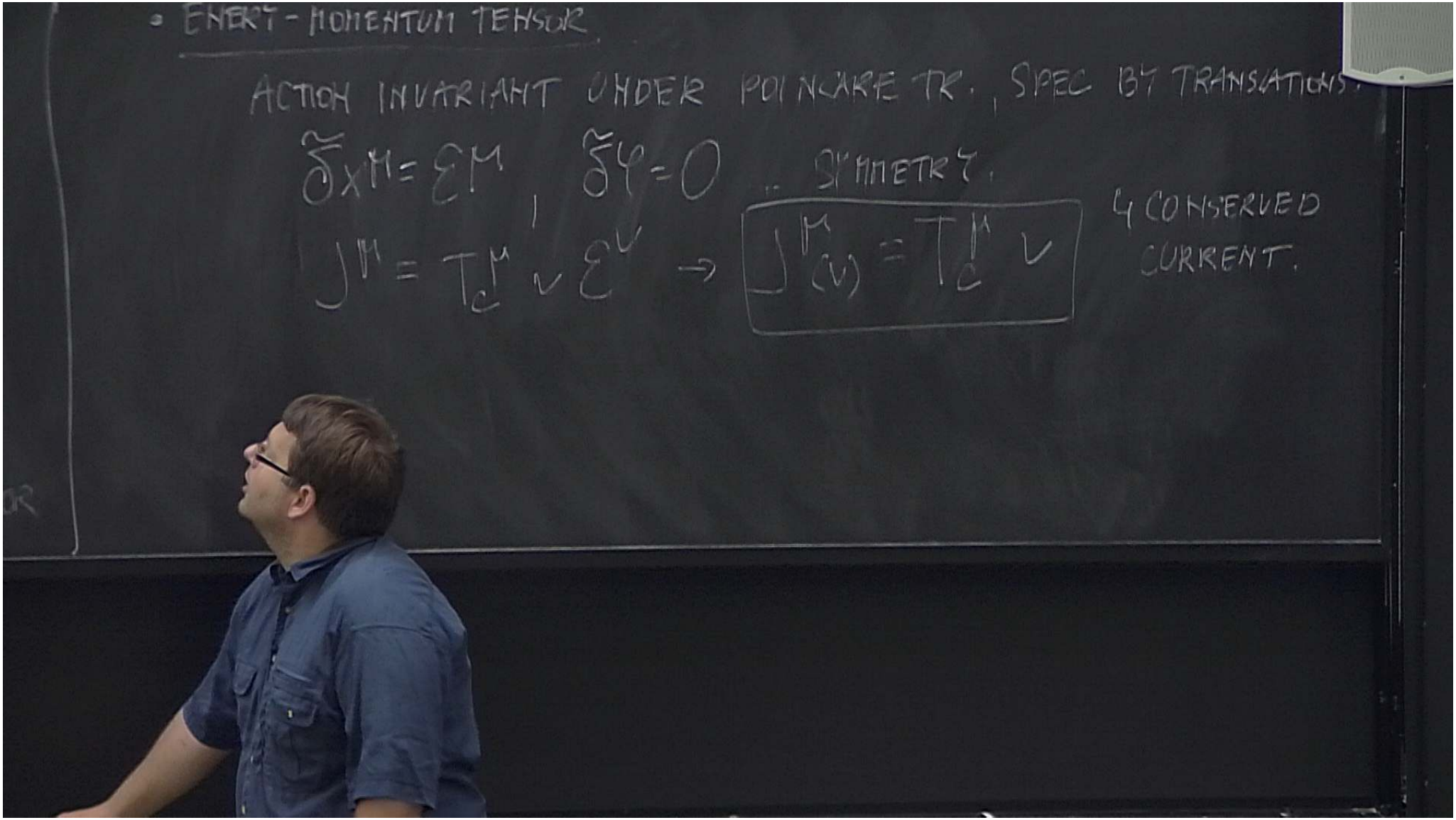
$$T^\mu_{c\,\nu} = g^\mu{}_\nu \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi)}\partial_\nu\varphi$$

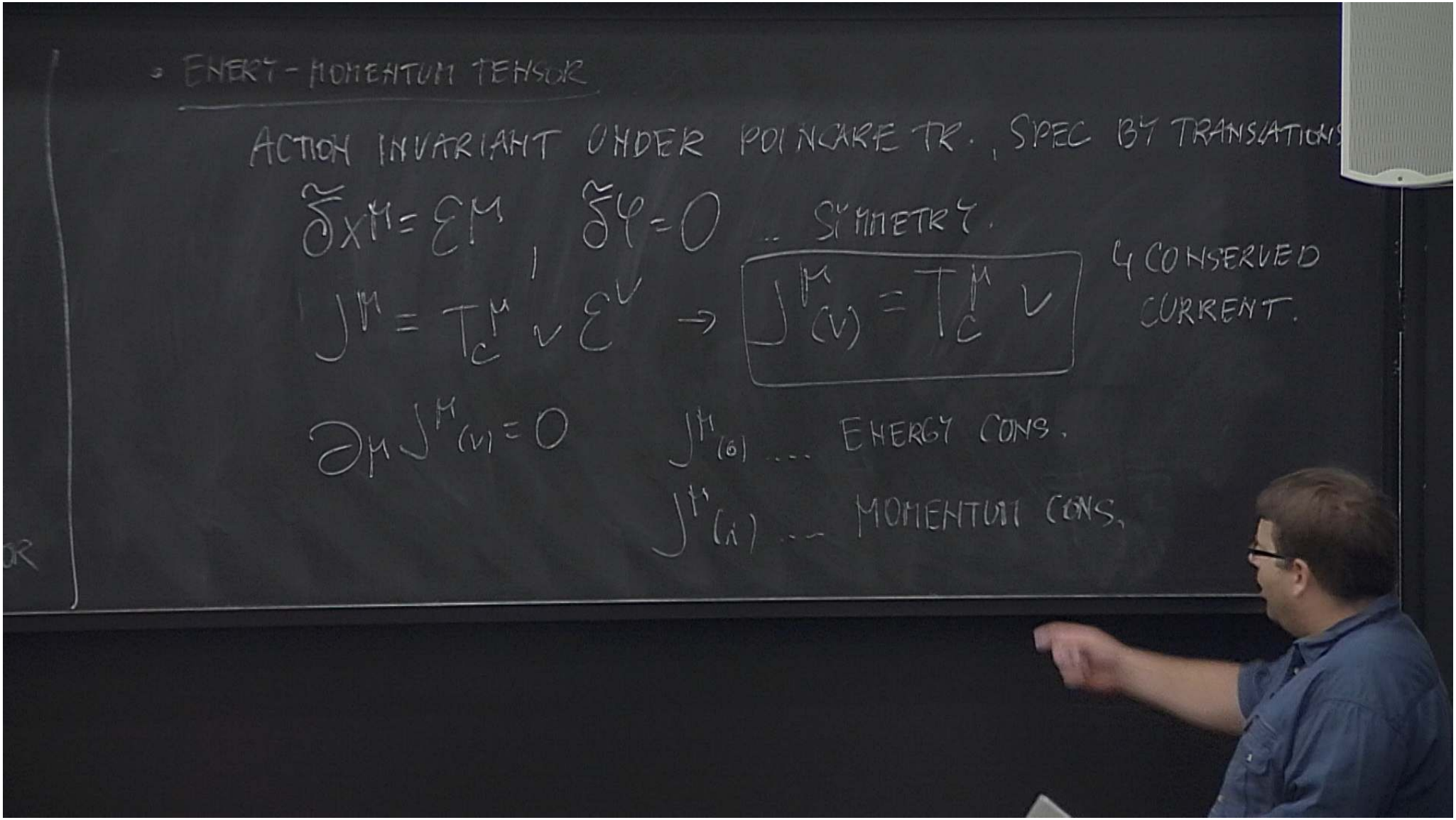
IS CONSERVED

$$\partial_\mu J^\mu = 0$$

CANONICAL ENERGY-MOM. TENSOR


• ENERT - MOMENTUM TENSOR
ACTION INVARIANT UNDER POINCARE TR., SPEC BY TRANSLATIONS.
$\tilde{\delta} x^\mu = \varepsilon^\mu$, $\tilde{\delta}\varphi = 0$ .. SYMMETRY.


• ENERGY-MOMENTUM TENSOR
ACTION INVARIANT UNDER POINCARÉ TR., SPEC BY TRANSLATIONS:
$\tilde{\delta}x^\mu = \varepsilon^\mu$, $\tilde{\delta}\varphi = 0$ .. SYMMETRY.
$J^\mu = T^\mu_{\ \nu}\,\varepsilon^\nu \rightarrow \boxed{J^\mu_{(\nu)} = T^\mu_{\ \nu}}$
4 CONSERVED CURRENT.


• ENERGY - MOMENTUM TENSOR
ACTION INVARIANT UNDER POINCARE TR., SPEC BY TRANSLATIONS
$\tilde{\delta}x^\mu = \varepsilon^\mu$, $\tilde{\delta}\varphi = 0$ .. SYMMETRY.
$J^\mu = T^\mu_{\ \nu}\varepsilon^\nu \rightarrow \boxed{J^\mu_{(\nu)} = T^\mu_{\ \nu}}$
4 CONSERVED CURRENT.
$\partial_\mu J^\mu_{(\nu)} = 0$
$J^\mu_{(0)}$ ... ENERGY CONS.
$J^\mu_{(i)}$ ... MOMENTUM CONS.

• ENERGY - MOMENTUM TENSOR

ACTION INVARIANT UNDER POINCARE TR., SPEC BY TRANSLATIONS

$\tilde{\delta} x^\mu = \varepsilon^\mu$, $\tilde{\delta}\varphi = 0$ .. SYMMETRY.

$J^\mu = T^\mu_{c\,\nu}\,\varepsilon^\nu \rightarrow \boxed{J^\mu_{(\nu)} = T^\mu_{c\,\nu}}$ 4 CONSERVED CURRENT.

$\partial_\mu J^\mu_{(\nu)} = 0$ $J^\mu_{(0)}$ ... ENERGY CONS.

$\boxed{\partial_\mu T^\mu_{c\,\nu} = 0}$ CONSERVATION OF E.M TENSOR. $J^\mu_{(i)}$ ... MOMENTUM CONS.

$J^\mu = T^\mu_\nu \tilde{\delta x}^\nu + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \tilde{\delta\varphi}$ IS CONSERVED

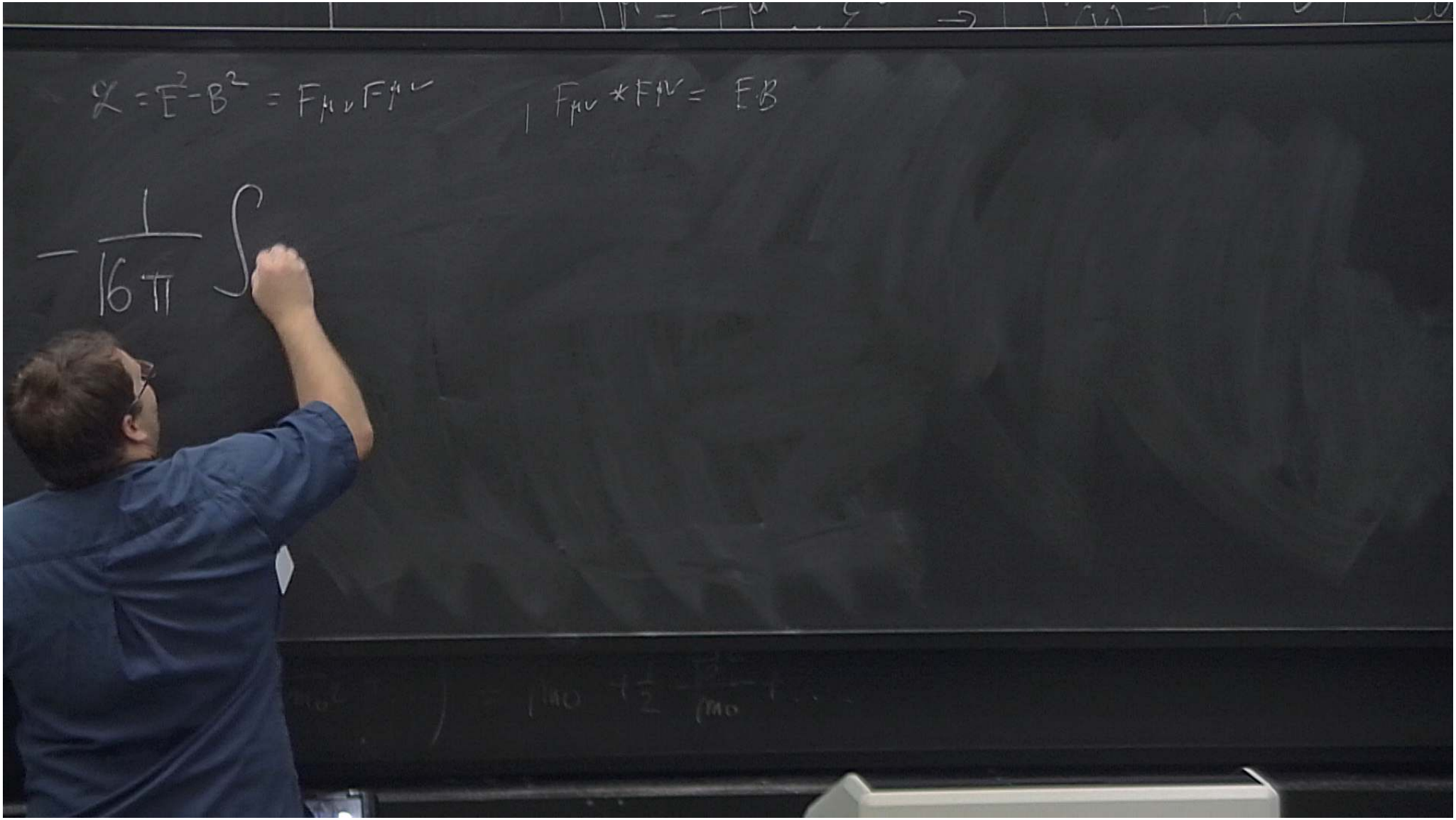
$\partial_\mu J^\mu = 0$

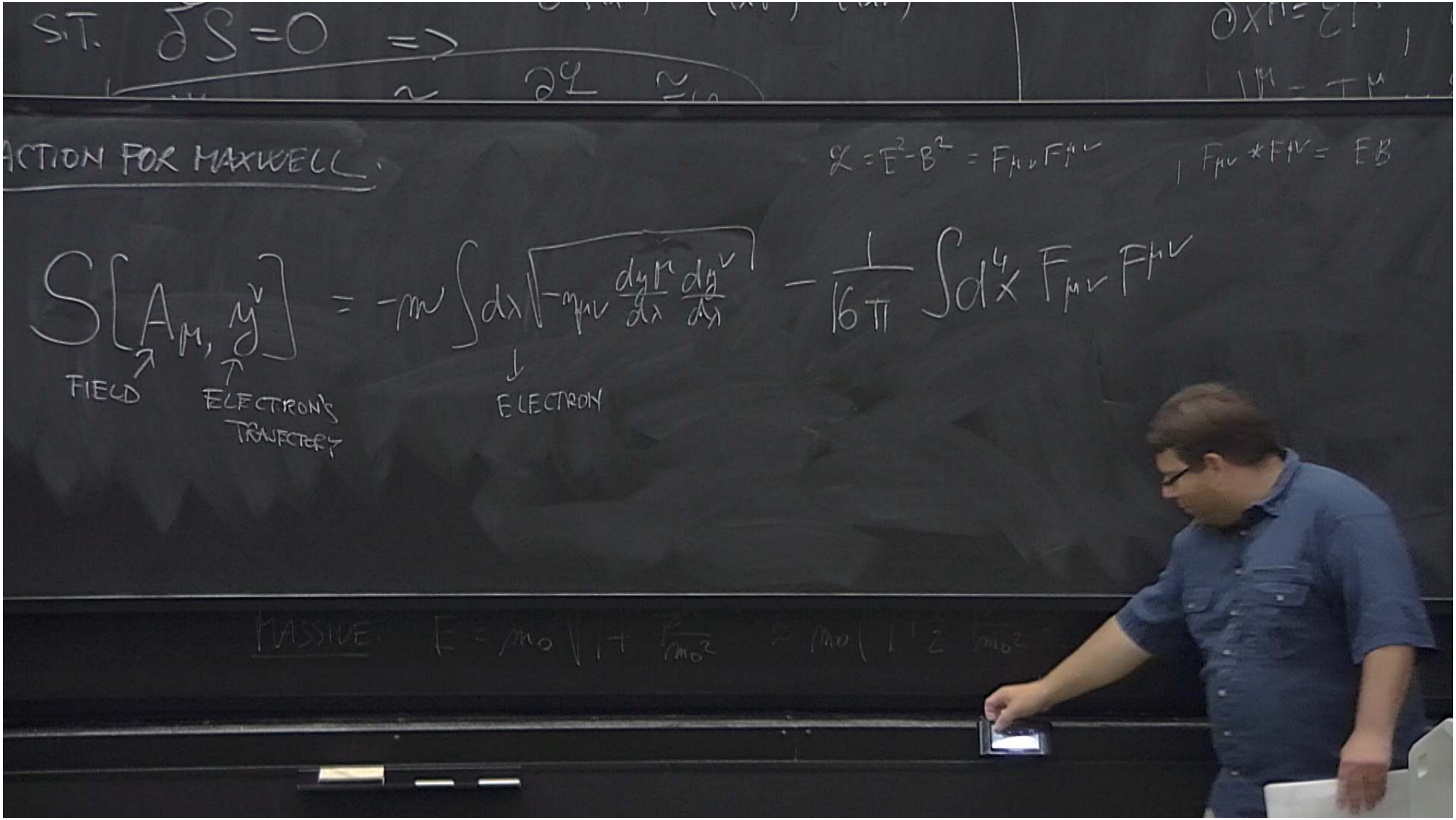
ACTION FOR MAXWELL:

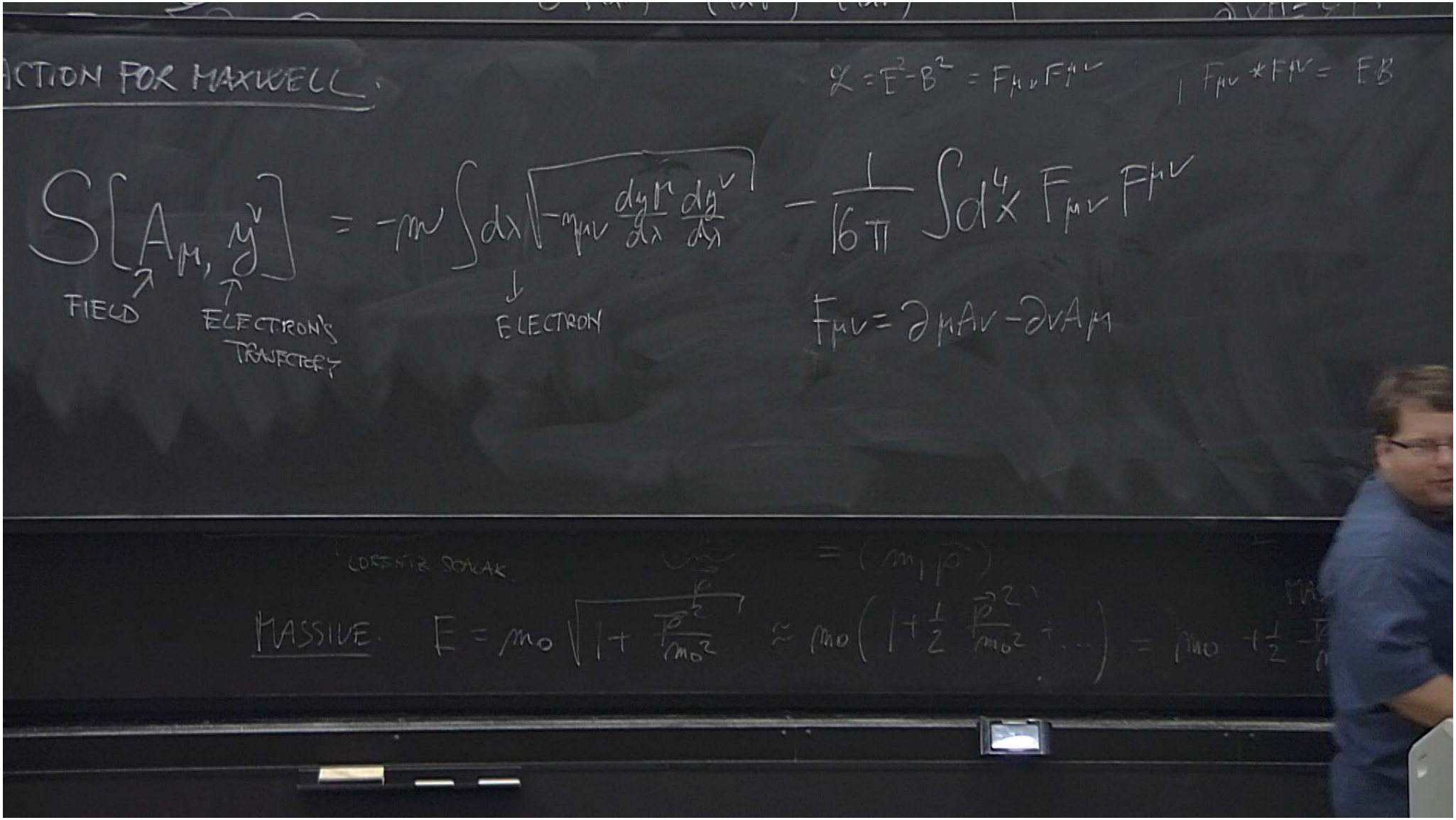
$S[A_\mu, y^\nu]$

FIELD

ELECTRON'S TRAJECTORY


$\mathcal{L} = E^2 - B^2 = F_{\mu\nu} F^{\mu\nu}$
$F_{\mu\nu} * F^{\mu\nu} = E \cdot B$
$-\frac{1}{16\pi} \int$


S.T. $\delta S = 0 \Rightarrow$
ACTION FOR MAXWELL:
$\mathcal{L} = E^2 - B^2 = F_{\mu\nu} F^{\mu\nu}$
$F_{\mu\nu} * F^{\mu\nu} = EB$
$S[A_\mu, y^\nu] = -m \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}} - \frac{1}{16\pi} \int d^4x \, F_{\mu\nu} F^{\mu\nu}$
FIELD
ELECTRON'S TRAJECTORY
ELECTRON
PASSIVE


ACTION FOR MAXWELL:
$\mathcal{L} = E^2 - B^2 = F_{\mu\nu}F^{\mu\nu}$
$F_{\mu\nu} * F^{\mu\nu} = EB$
$S[A_\mu, y^\nu] = -m \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dy^\mu}{d\lambda}\frac{dy^\nu}{d\lambda}} - \frac{1}{16\pi}\int d^4x\, F_{\mu\nu}F^{\mu\nu}$
FIELD
ELECTRON'S TRAJECTORY
ELECTRON
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
LORENTZ SCALAR
$= (m, \vec{p})$
MASSIVE: $E = m_0\sqrt{1 + \frac{\vec{p}^2}{m_0^2}} \approx m_0\left(1 + \frac{1}{2}\frac{\vec{p}^2}{m_0^2} + \dots\right) = m_0 + \frac{1}{2}$

$\mathcal{L} = E^2 - B^2 = F_{\mu\nu} F^{\mu\nu}$, $F_{\mu\nu} * F^{\mu\nu} = EB$

$$-\frac{1}{16\pi} \int d^4x \, F_{\mu\nu} F^{\mu\nu} - e \int \frac{dy^\mu}{d\lambda} A_\mu \, d\lambda$$

INTERACTION TERM

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\delta A^\mu \rightarrow$ MAXWELL

$\delta y^\mu \rightarrow$ L.F.L.

$\varphi(x^\mu)$

IS CONSERVED

ACTION INVARIANT UNDER POINCARE TR., SPEC BY TRANSLATIONS.

$\delta x^\mu = \varepsilon^\mu$, $\delta\varphi = 0$ .. SYMMETRY

$J^\mu = T^\mu_{\ \nu} \varepsilon^\nu \rightarrow$ $J^\mu_{(\nu)} = T$

4 CONSERVED CURRENT.

ACTION FOR MAXWELL:

$\mathcal{L} = E^2 - B^2 = F_{\mu\nu} F^{\mu\nu}$ | $F_{\mu\nu} * F^{\mu\nu} = EB$

$$S[A_\mu, y^\nu] = -m \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}} - \frac{1}{16\pi} \int d^4x \, F_{\mu\nu} F^{\mu\nu} - e \int \frac{dy^\mu}{d\lambda} A_\mu \, d\lambda$$

FIELD — ELECTRON'S TRAJECTORY — ELECTRON — INTERACTION TERM

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\delta A^\mu \rightarrow$ MAXWELL

$\delta y^\mu \rightarrow$ L.F.L

THAT IS OFF-SHELL WE FIND $\tilde{\delta} x^\mu = x'^\mu - x^\mu$

$\tilde{\delta} \varphi(x^\mu) = \varphi'(x'^\mu) - \varphi(x^\mu)$

$$\tilde{\delta} S = 0 \Rightarrow$$

$$J^\mu = T^\mu_{\ c\,\nu} \tilde{\delta} x^\nu + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \tilde{\delta} \varphi$$

IS CONSERVED

$\partial_\mu J^\mu = 0$

ACTION INVARIANT UNDER

$\tilde{\delta} x^\mu = \varepsilon^\mu$, $\tilde{\delta} \varphi = 0$

$J^\mu = T^\mu_{\ c\,\nu} \varepsilon^\nu \rightarrow$

$\partial_\mu J^\mu_{(\nu)} = 0$ $\quad J^\mu_{(\alpha)}$ ...

INTERACTION TERM

$\delta A^\mu \rightarrow$ MAXWELL

$\delta \psi^\alpha \rightarrow$ L.F.L.

$$(\partial^2 - m^2)\varphi = 0$$

$\tilde{\delta} x^\mu = \varepsilon^\mu$, $\tilde{\delta}\varphi = 0$ .. SYMMETRY.

$J^\mu = T^\mu_{c\,\nu}\,\varepsilon^\nu \rightarrow$ $\boxed{J^\mu_{(\nu)} = T^\mu_{c\,\nu}}$ 4 CONSERVED CURRENT.

$\partial_\mu J^\mu_{(\nu)} = 0$ .. ENERGY CONS.

INTERACTION TERM

$\delta A^\mu \rightarrow$ MAXWELL

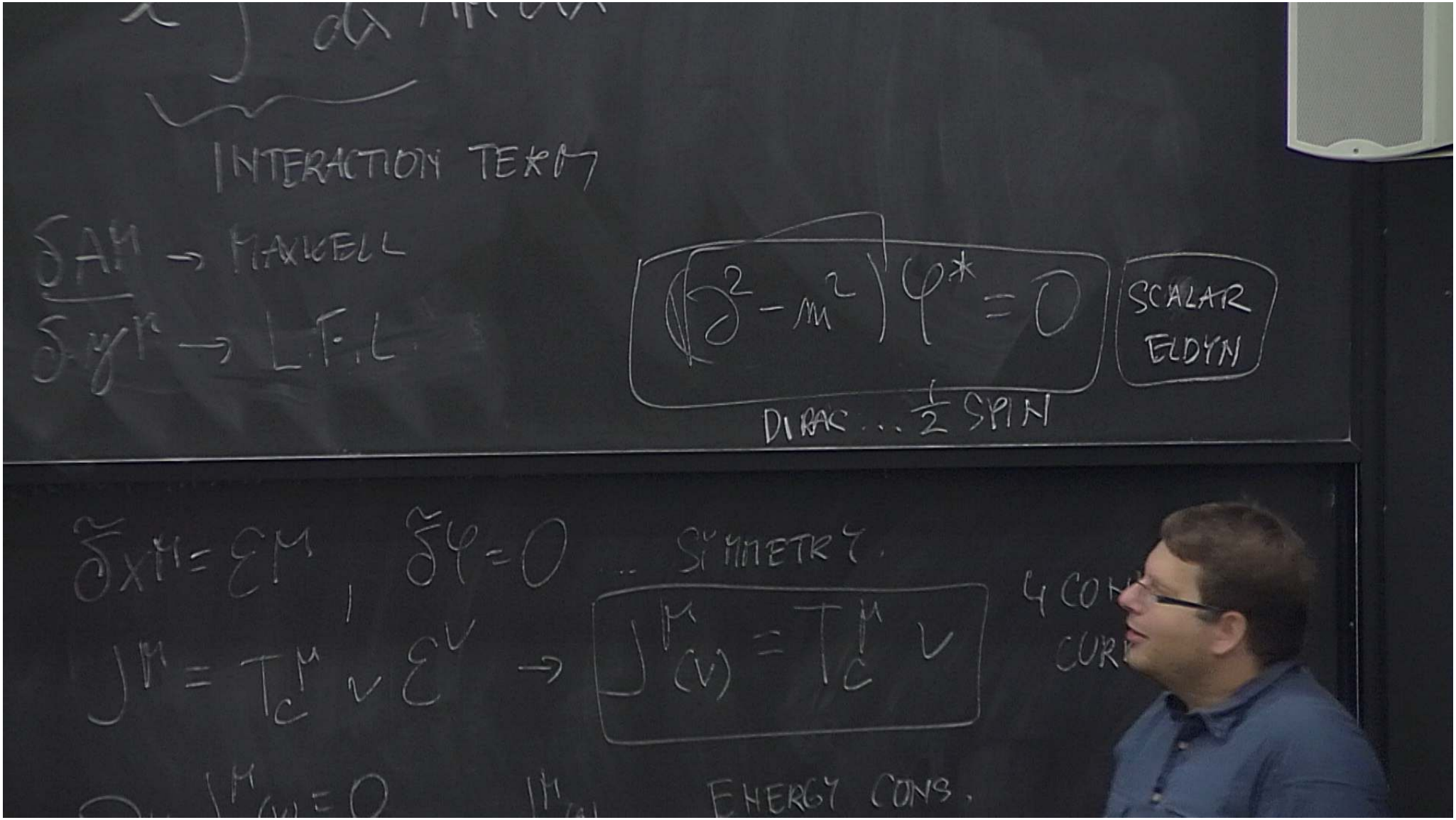
$\delta \psi^* \rightarrow$ L.F.L.

$$(\partial^2 - m^2)\varphi^* = 0$$ SCALAR EL DYN

$\tilde{\delta} x^\mu = \varepsilon^\mu$, $\tilde{\delta}\varphi = 0$ .. SYMMETRY.

$J^\mu = T^\mu_{c\,\nu}\,\varepsilon^\nu \rightarrow$ $J^\mu_{(\nu)} = T^\mu_{c\,\nu}$ 4 CONSERVED CURRENT.

$\partial_\mu J^\mu_{(\nu)} = 0$ $\quad \partial_\mu$ ENERGY CONS.


INTERACTION TERM
$\delta A^\mu \rightarrow$ MAXWELL
$\delta \psi^\alpha \rightarrow$ L.F.L.
$(\partial^2 - m^2)\varphi^* = 0$
SCALAR ELDYN
DIRAC ... $\frac{1}{2}$ SPIN
$\tilde{\delta} x^\mu = \varepsilon^\mu$, $\tilde{\delta}\varphi = 0$ .. SYMMETRY.
$J^\mu = T^\mu_{c\,\nu}\varepsilon^\nu \rightarrow J^\mu_{(\nu)} = T^\mu_{c\,\nu}$
4 CON
CUR
$\partial_\mu J^\mu_{(\nu)} = 0$
ENERGY CONS.