Title: PSI 2019/2020 - Classical Physics - Lecture 1

Speakers: David Kubiznak

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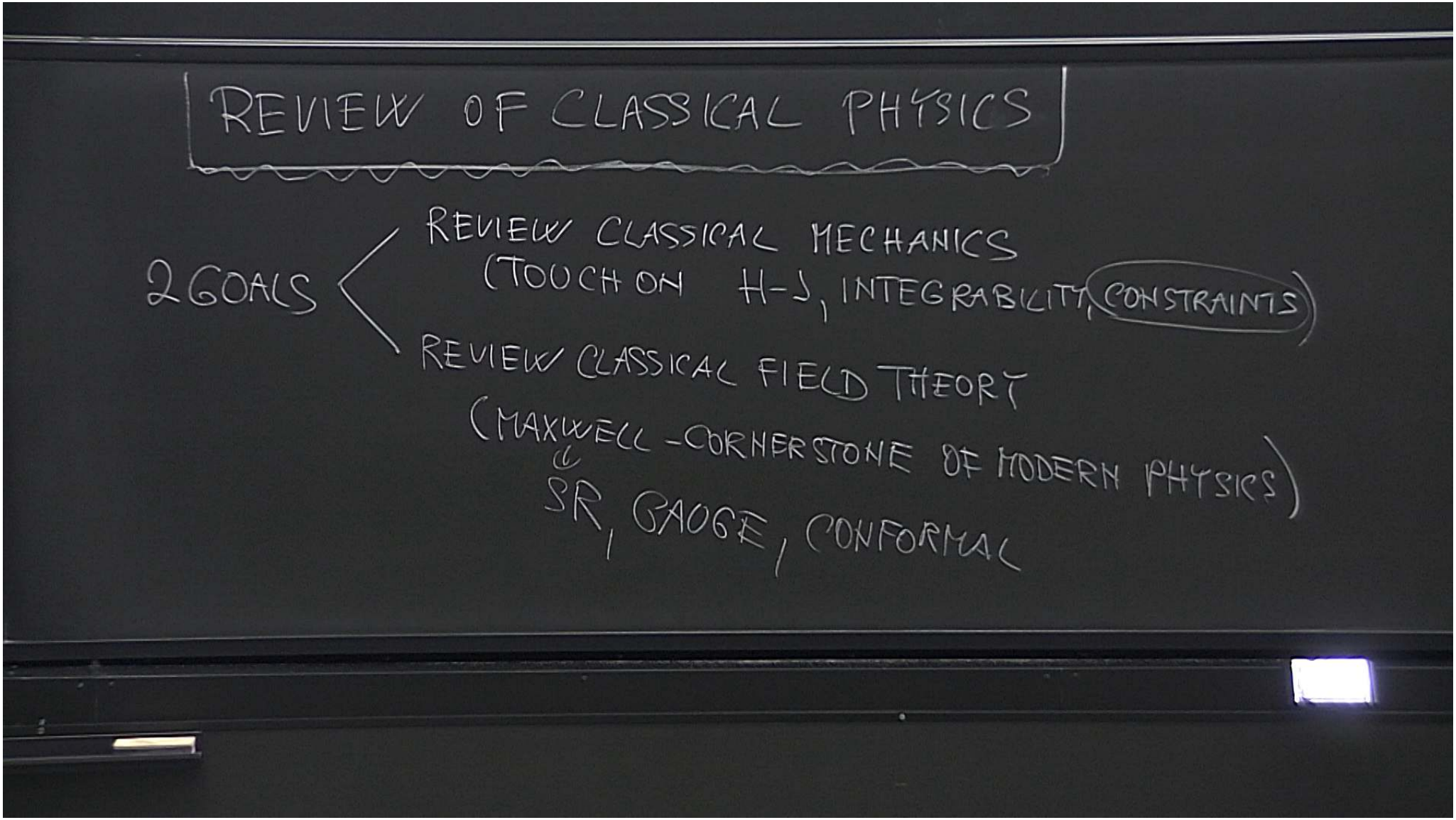
# REVIEW OF CLASSICAL PHYSICS

2 GOALS
- REVIEW CLASSICAL MECHANICS (TOUCH ON H-J, INTEGRABILITY, CONSTRAINTS)
- REVIEW CLASSICAL FIELD THEORY (MAXWELL - CORNERSTONE OF MODERN PHYSICS)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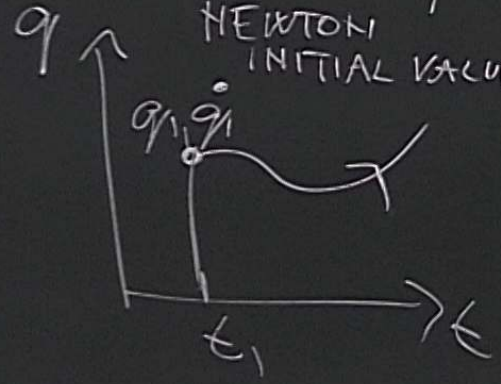
(TOUCH ON H-J, INTEGRABILITY, CONSTRAINTS)
REVIEW CLASSICAL FIELD THEORY
(MAXWELL - CORNERSTONE OF MODERN PHYSICS)
SR, GAUGE, CONFORMAL

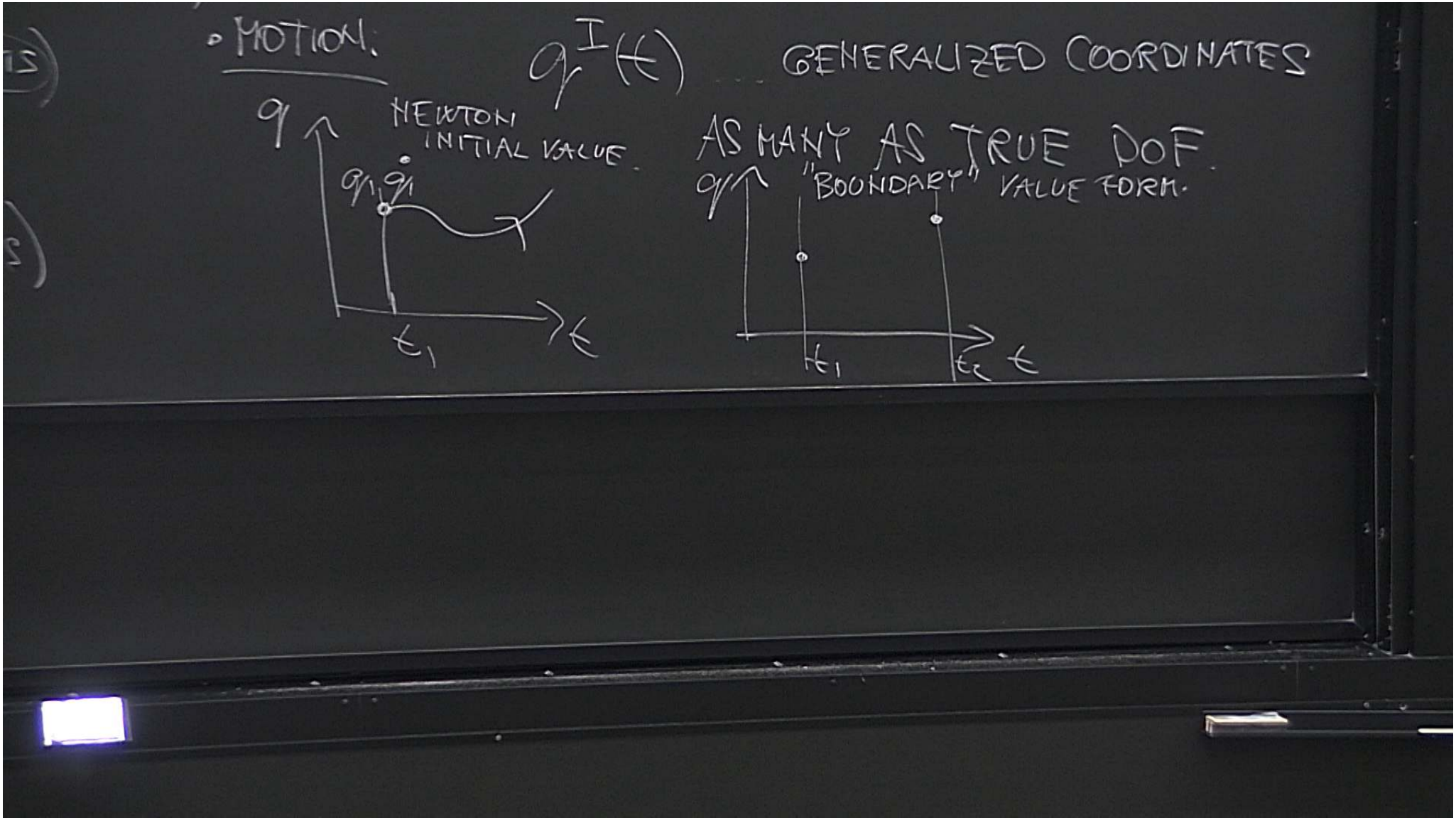
# 1) LAGRANGIAN MECHANICS

## a) HAMILTON'S PRINCIPLE OF LEAST ACTION

- MOTION: $q_i^I(t)$ ... GENERALIZED COORDINATES



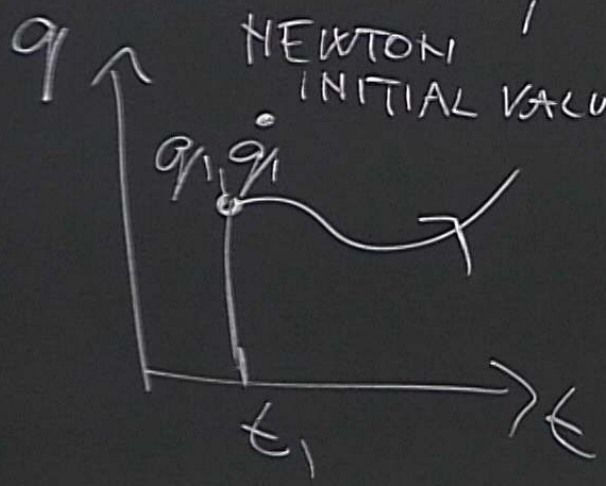

AS MANY AS TRUE DoF.


• MOTION:
$q_r^I(t)$ ... GENERALIZED COORDINATES
$q$
NEWTON
INITIAL VALUE.
$q_1, \dot{q}_1$
$t_1$
$t$
AS MANY AS TRUE DoF.
$q_r$
"BOUNDARY" VALUE FORM.
$t_1$
$t_2$
$t$

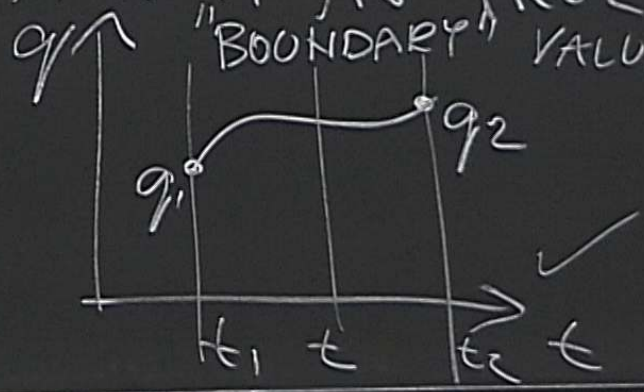
# 1) LAGRANGIAN MECHANICS

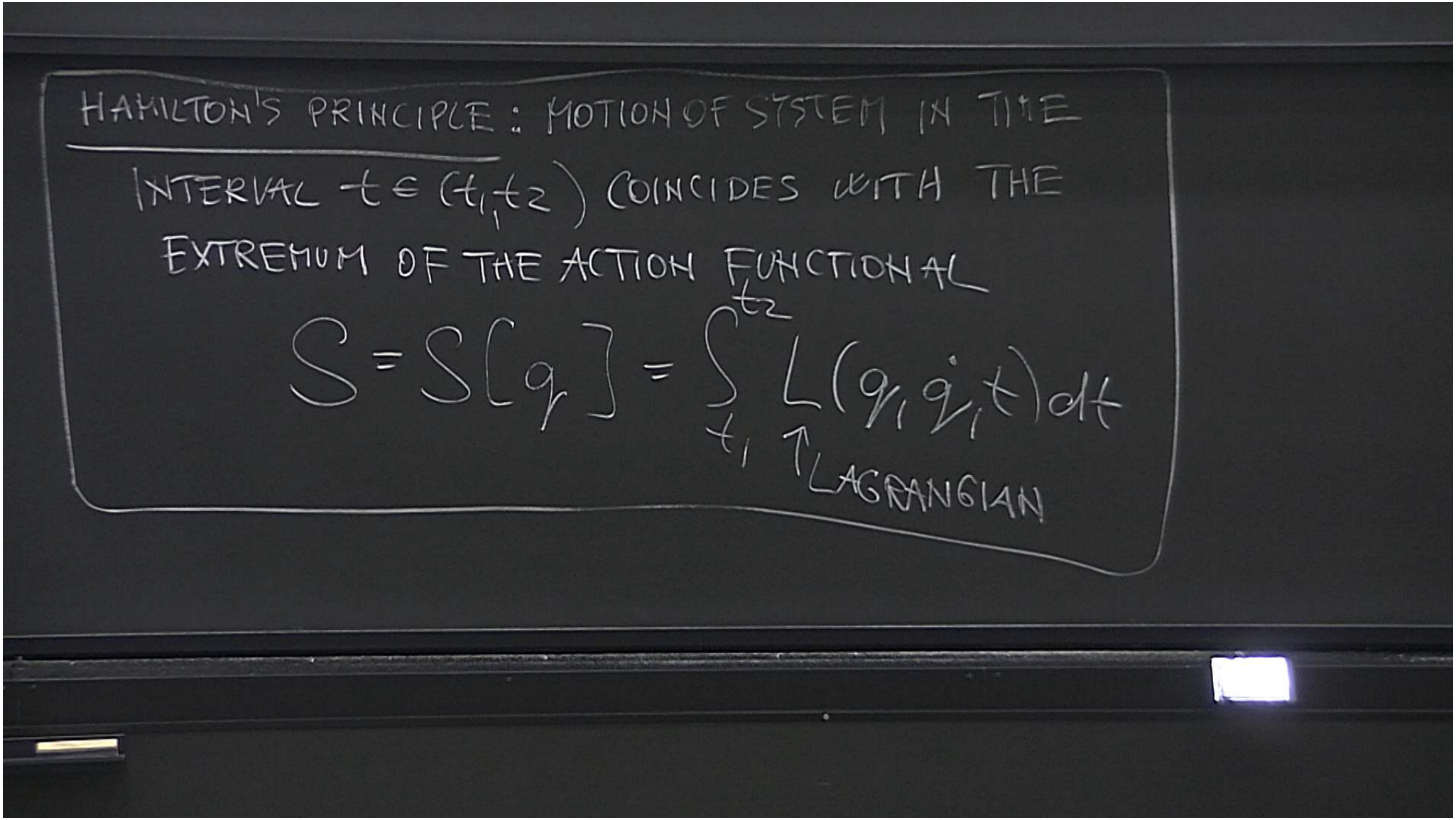
## a) HAMILTON'S PRINCIPLE OF LEAST ACTION

- MOTION: $q_i^I(t)$ ... GENERALIZED COORDINATES

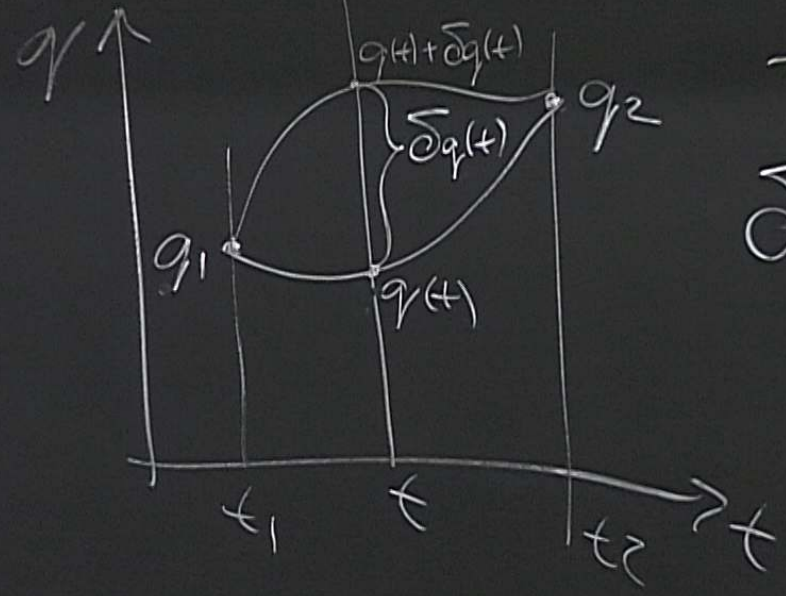



AS MANY AS TRUE DoF.




HAMILTON'S PRINCIPLE: MOTION OF SYSTEM IN TIME
INTERVAL $t \in (t_1, t_2)$ COINCIDES WITH THE
EXTREMUM OF THE ACTION FUNCTIONAL
$S = S[q] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$
↑ LAGRANGIAN

- TO EXTRACT EOM → CONSIDER 'FIXED END POINTS'




TRUE TRAJECTORY EXTREMIZES $S$.

$\delta q(t)$... VARIATIONS

TIME IS FIXED:

$$\Rightarrow \boxed{\delta \frac{d}{dt} = \frac{d}{dt} \delta}$$

$$= \delta \int_{t_1}^{t_2} L\,dt = \int_{t_1}^{t_2} \delta L\,dt = \int_{t_1}^{t_2} \left( \frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} \right) dt$$

$$\delta \frac{dq}{dt} = \frac{d}{dt} \delta q$$

PARTS $$= \int_{t_1}^{t_2} \left( \frac{\partial L}{\partial q} - \frac{d}{dt}\left( \frac{\partial L}{\partial \dot{q}} \right) \right) \delta q\,dt + \left[ \frac{\partial L}{\partial \dot{q}} \delta q \right]_{t_1}^{t_2} = 0$$

0... FIXED ENDPOINTS

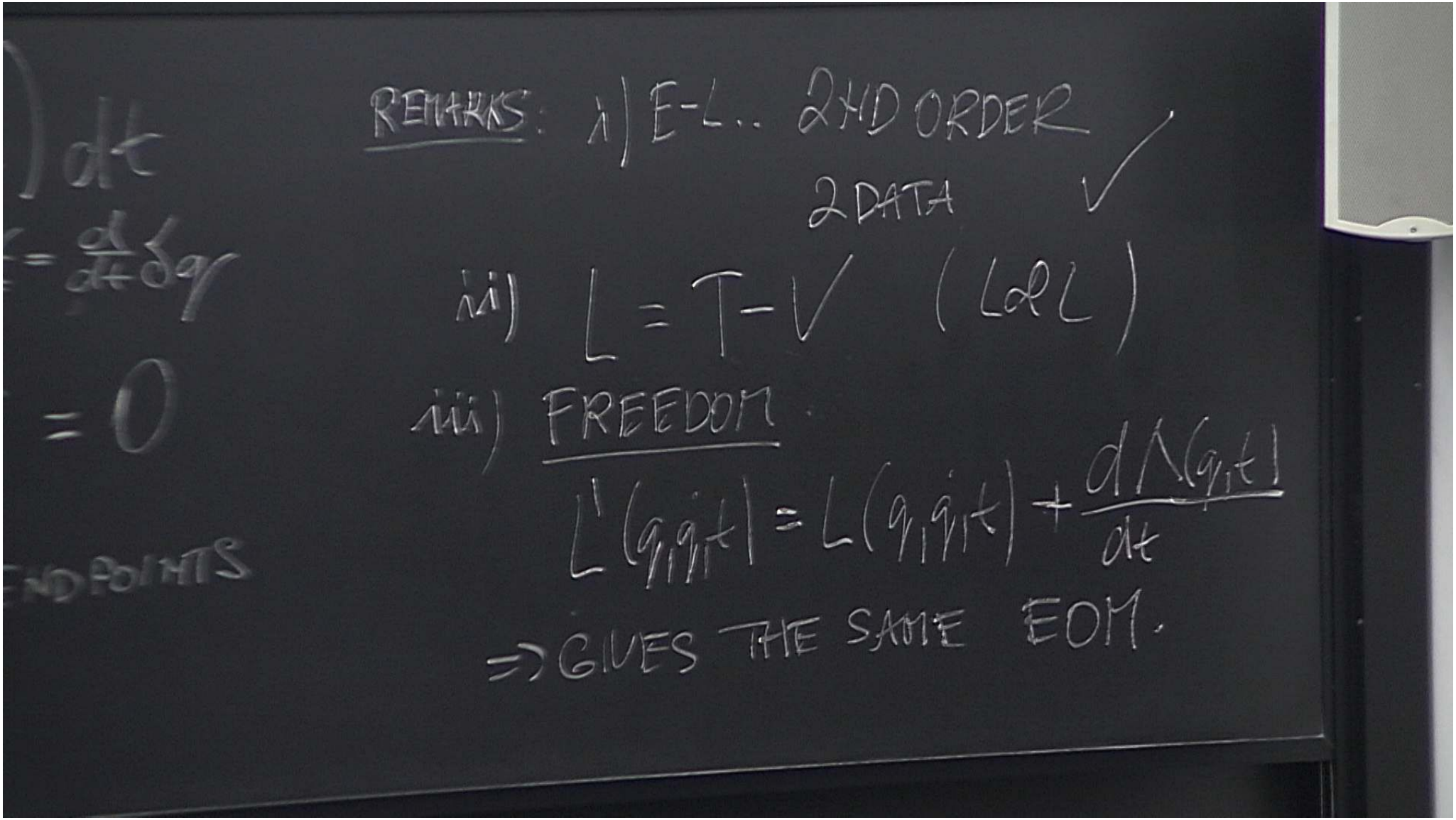
$$\delta S = \delta \int_{t_1}^{t_2} L\,dt = \int_{t_1}^{t_2} \delta L\,dt = \int_{t_1}^{t_2} \left( \frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} \right)$$

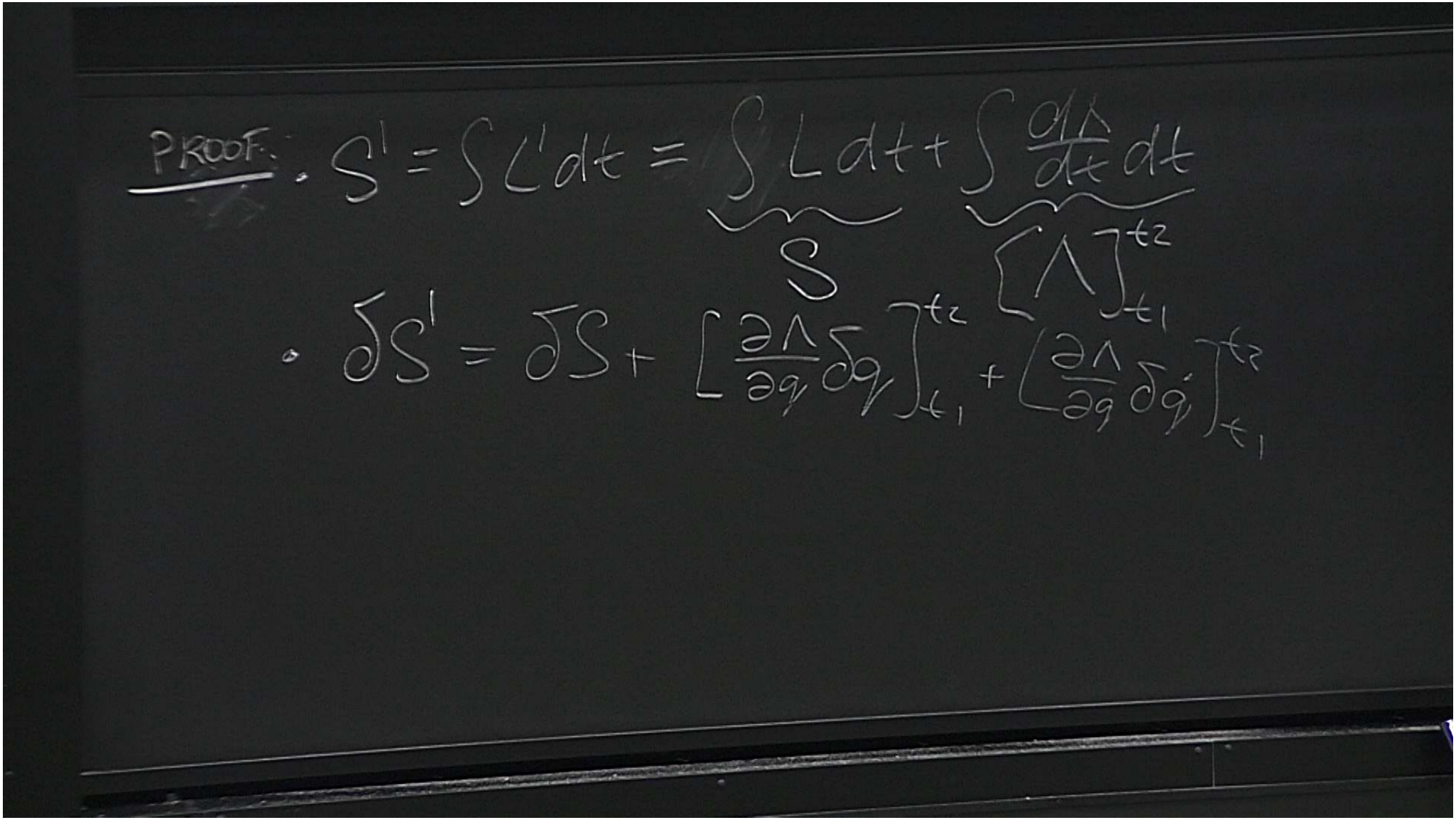
$$\delta \frac{dq}{dt} =$$

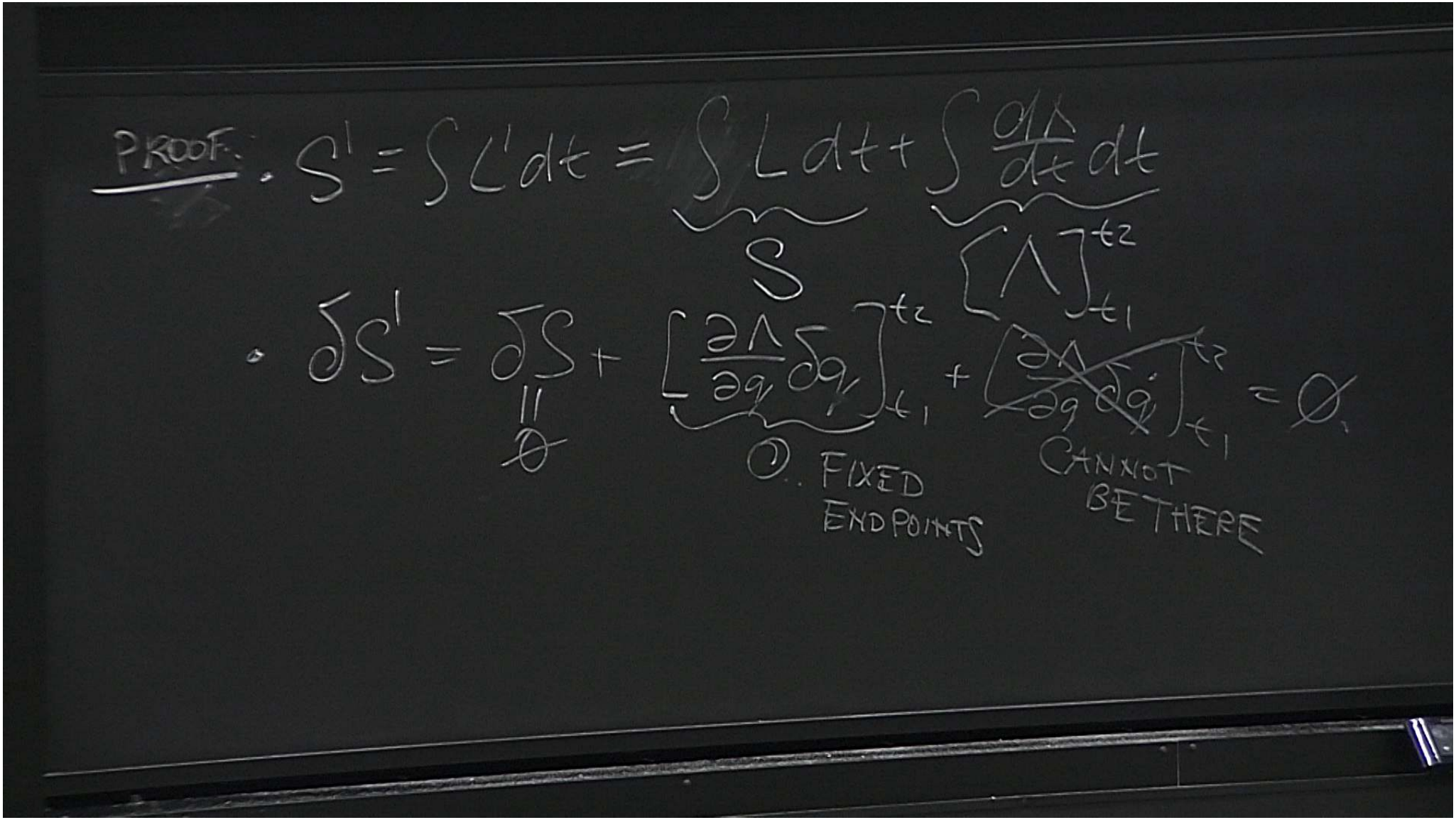
BY PARTS $=$ $\int_{t_1}^{t_2} \left( \frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \right) \delta q\,dt + \left[ \frac{\partial L}{\partial \dot{q}} \delta q \right]_{t_1}^{t_2} =$

$0$ ... $0$ ... FIXED END

EULER-LAGRANGE EQUATIONS $\boxed{\frac{\partial L}{\partial q^I} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^I}\right) = 0}$


REMARKS: i) E-L.. 2ND ORDER
2 DATA
ii) L = T − V (LAGRANGIAN)
iii) FREEDOM
L'(q, q̇, t) = L(q, q̇, t) + dΛ(q, t)/dt
=> GIVES THE SAME EOM.


PROOF:
$S' = \int L' dt = \int L dt + \int \frac{d\Lambda}{dt} dt$
$S$
$[\Lambda]_{t_1}^{t_2}$
$\delta S' = \delta S + \left[\frac{\partial \Lambda}{\partial q_i} \delta q_i\right]_{t_1}^{t_2} + \left[\frac{\partial \Lambda}{\partial \dot{q}} \delta \dot{q}_i\right]_{t_1}^{t_2}$


PROOF:
$S' = \int L' dt = \int L dt + \int \frac{d\Lambda}{dt} dt$
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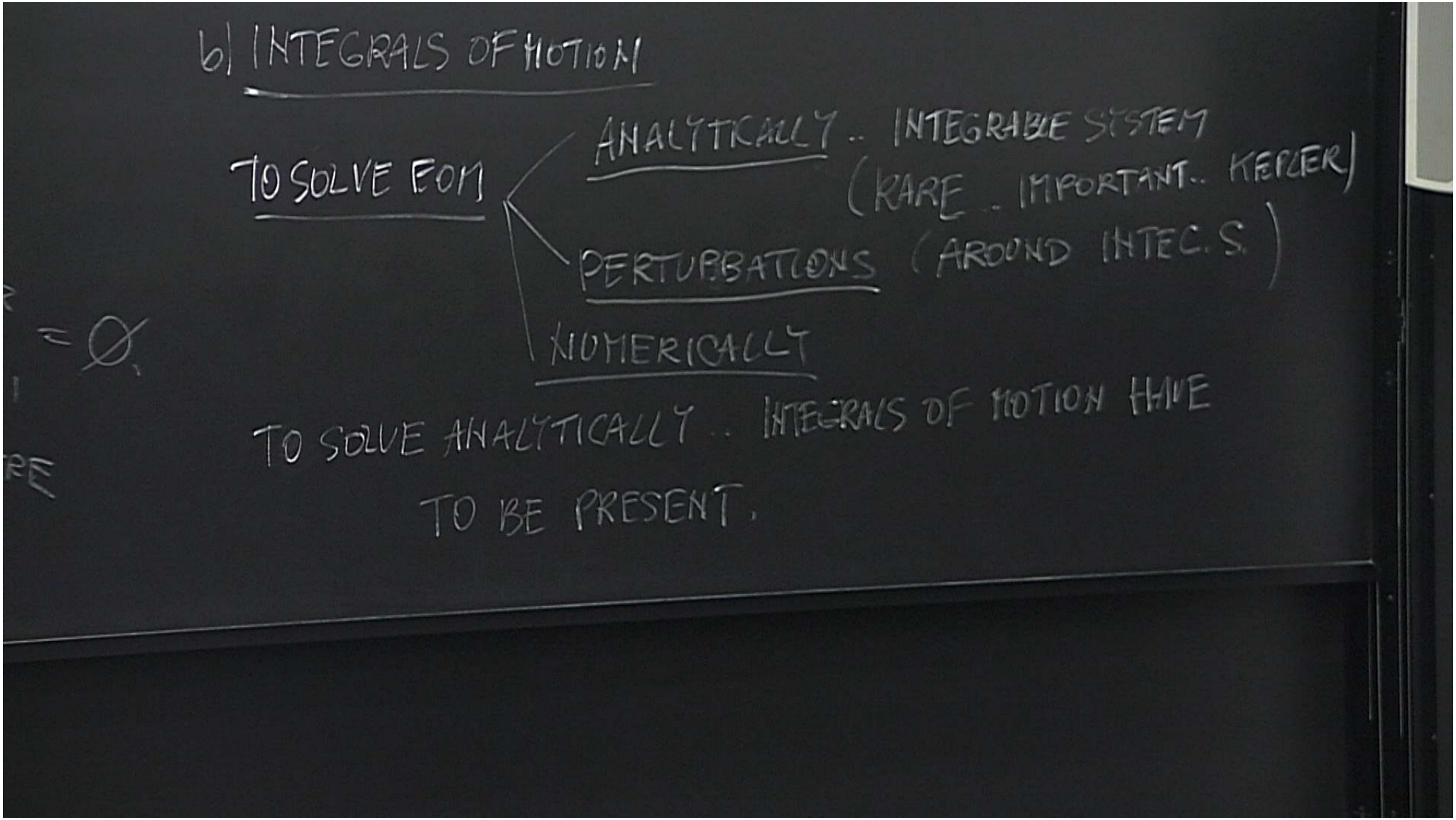
$[\Lambda]_{t_1}^{t_2}$
$\delta S' = \delta S + \left[\frac{\partial \Lambda}{\partial q} \delta q\right]_{t_1}^{t_2} + \left[\frac{\partial \Lambda}{\partial \dot{q}} \delta \dot{q}\right]_{t_1}^{t_2} = 0$
$\delta S = 0$
0. FIXED END POINTS
CANNOT BE THERE


b) INTEGRALS OF MOTION
TO SOLVE EOM
ANALYTICALLY .. INTEGRABLE SYSTEM
(RARE . IMPORTANT .. KEPLER)
PERTURBATIONS (AROUND INTEC. S.)
NUMERICALLY
TO SOLVE ANALYTICALLY . INTEGRALS OF MOTION HAVE
TO BE PRESENT.

DEF: INTEGRAL (CONSTANT) OF MOTION IS A FUNCTION $I = I(q, \dot{q}, t)$ SUCH THAT

$$\frac{dI}{dt} = 0$$ FOR A SOLUTION OF EOM.

NOETHER'S THEOREM VI: FOR EVERY GLOBAL CONTINUOUS SYMMETRY OF THE SYSTEM THERE IS A CORRESPONDING INTEGRAL OF MOTION

"GEOMETRIZES PHYSICS"

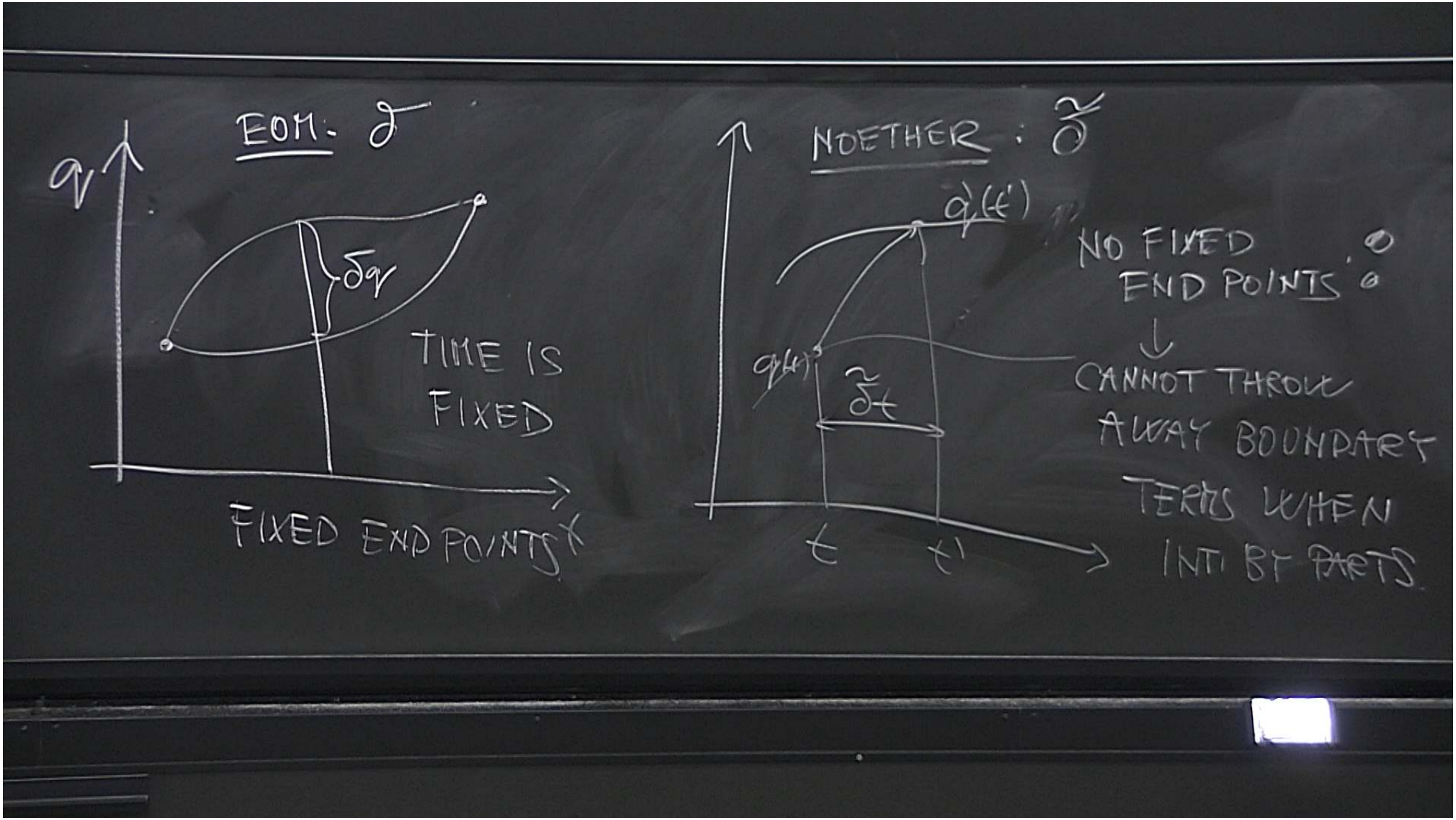
TERMINOLOGY: • ON-SHELL .. EOM ARE SATISFIED

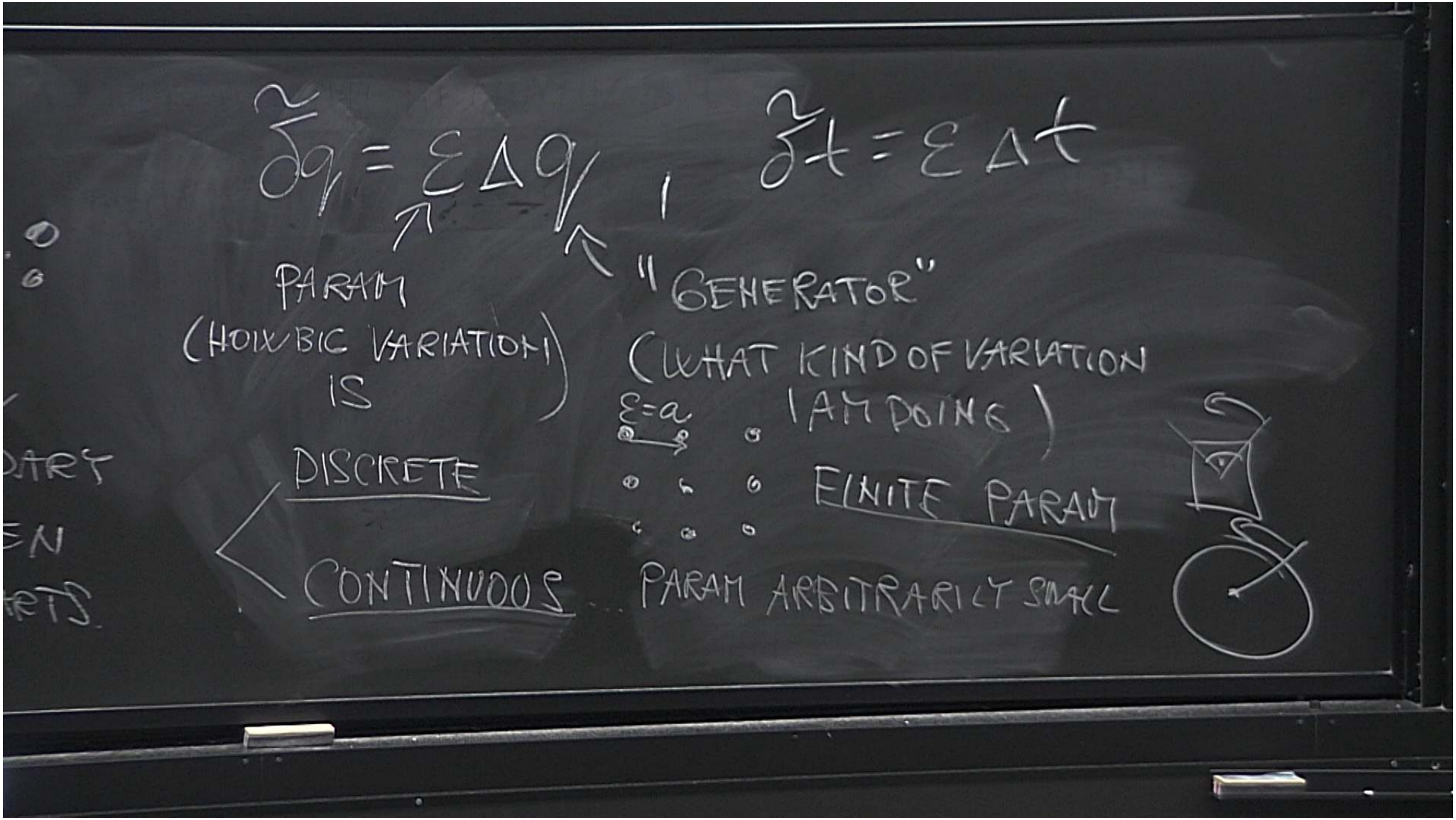
OFF-SHELL .. IRRESPECTIVE OF EOM.

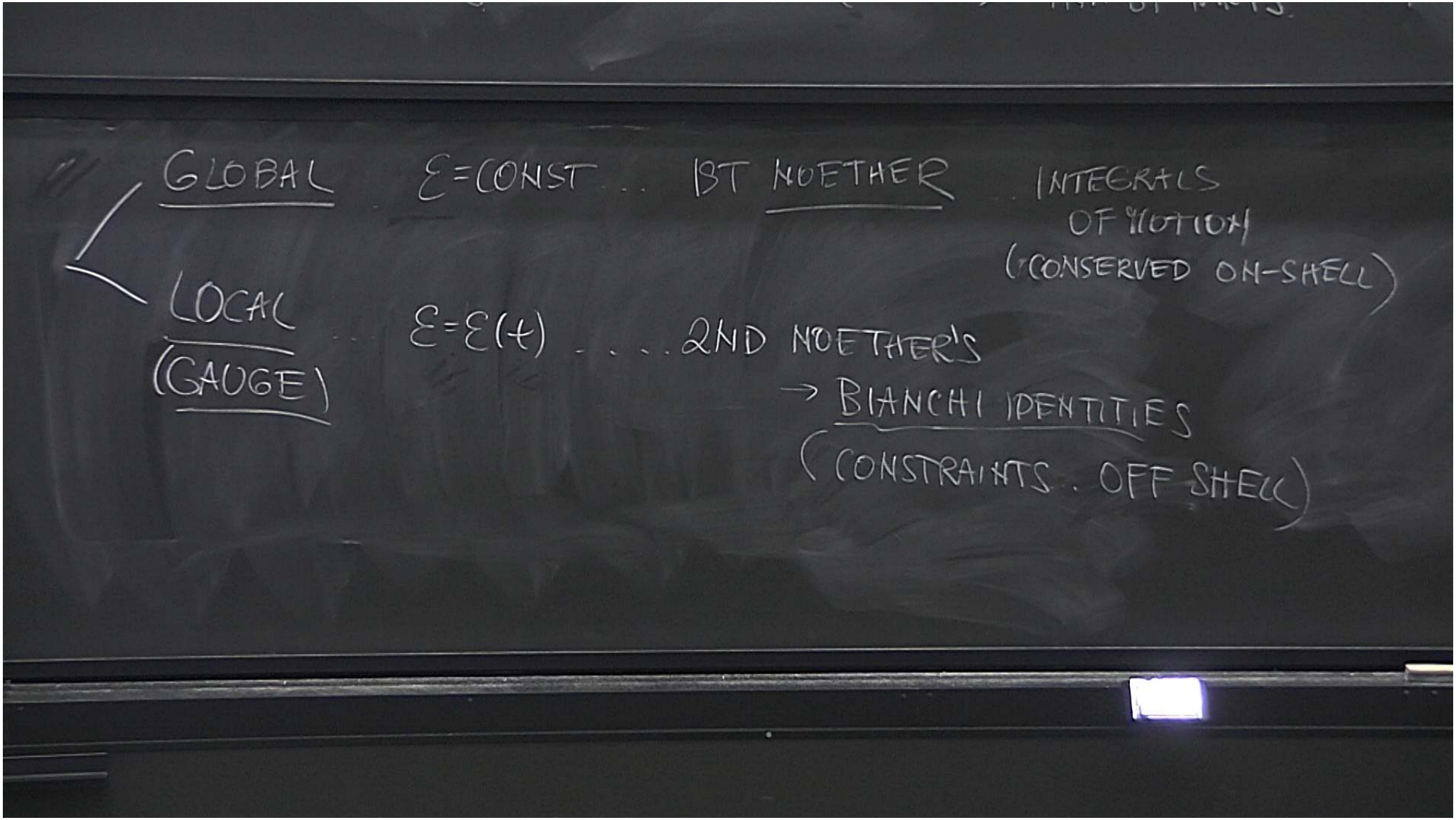
$\frac{dH}{dt} = 0$ FOR A SOLUTION OF EOM. TERMINOLOGY:

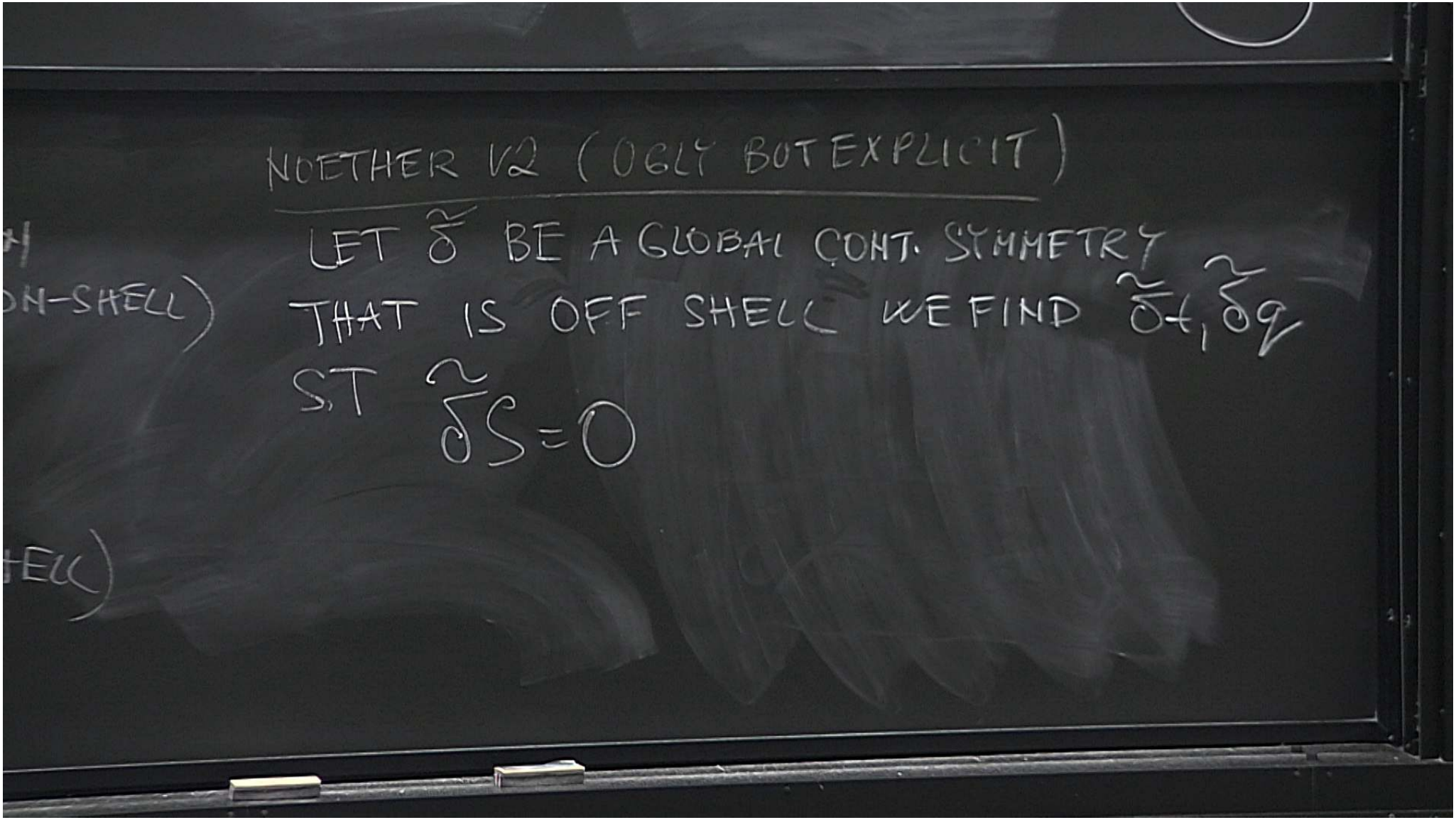
- TYPES OF SYMMETRIES: CONSIDER A TRANSFORMATION

$$t \to t' = t + \tilde{\delta} t, \quad q \to q'(t') = q(t) + \tilde{\delta} q(t)$$


EOM: $\delta$
$q$
$\delta q$
TIME IS FIXED
FIXED END POINTS
$x$
NOETHER : $\tilde{\delta}$
$q'(t')$
$q(t)$
$\tilde{\delta} t$
$t$
$t'$
NO FIXED END POINTS
↓
CANNOT THROW AWAY BOUNDARY TERMS WHEN INT BY PARTS


$\tilde{\delta q_i} = \varepsilon \Delta q_i$ , $\tilde{\delta t} = \varepsilon \Delta t$
PARAM
(HOW BIG VARIATION)
IS
"GENERATOR"
(WHAT KIND OF VARIATION
I AM DOING)
$\varepsilon = a$
DISCRETE
FINITE PARAM
CONTINUOUS
PARAM ARBITRARILY SMALL


GLOBAL
ε=CONST
1ST NOETHER
INTEGRALS OF MOTION (CONSERVED ON-SHELL)
LOCAL (GAUGE)
ε=ε(t)
2ND NOETHER'S
→ BIANCHI IDENTITIES (CONSTRAINTS. OFF SHELL)


NOETHER V2 (OGLY BUT EXPLICIT)
LET $\tilde{\delta}$ BE A GLOBAL CONT. SYMMETRY
THAT IS OFF SHELL WE FIND $\tilde{\delta}t, \tilde{\delta}q$
S.T $\tilde{\delta}S=0$

NOETHER V2 (OGLY BUT EXPLICIT)

LET $\tilde{\delta}$ BE A GLOBAL CONT. SYMMETRY THAT IS OFF SHELL WE FIND $\tilde{\delta}t, \tilde{\delta}q$ S.T

$$\tilde{\delta}S = 0 \Rightarrow I = \frac{\partial L}{\partial \dot{q}}\tilde{\delta}q + \left(L - \dot{q}\frac{\partial L}{\partial \dot{q}}\right)\tilde{\delta}t$$

IS AN INTEGRAL OF MOTION

2 EXAMPLES.

i) TIME TRANSLATION SYMMETRY ... $L \neq L(t)$

$$\tilde{\delta} t = \varepsilon, \quad \tilde{\delta} q = 0 \Rightarrow \tilde{\delta} S = 0$$ (PLEASE CHECK)

$$\boxed{I = L - \dot{q}^I \frac{\partial L}{\partial \dot{q}^I}}$$ ... GENERALIZED ENERGY

ER'S THEOREM VI. FOR EVERY GLOBAL CONTINUOUS SYMMETRY

PROOF: $\frac{dI}{dt} = \frac{\partial L}{\partial q}\dot{q} + \left(\frac{\partial L}{\partial \dot{q}}\ddot{q} + \frac{\partial L}{\partial t} - \frac{d}{dt}\left(\dot{q}\frac{\partial L}{\partial \dot{q}}\right)\right)$

$\frac{dL}{dt}$

$-\ddot{q}\frac{\partial L}{\partial \dot{q}} - \dot{q}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$

$= \left(\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right)\dot{q}$

ER'S THEOREM V1. FOR EVERY GLOBAL CONTINUOUS SYMMETRY

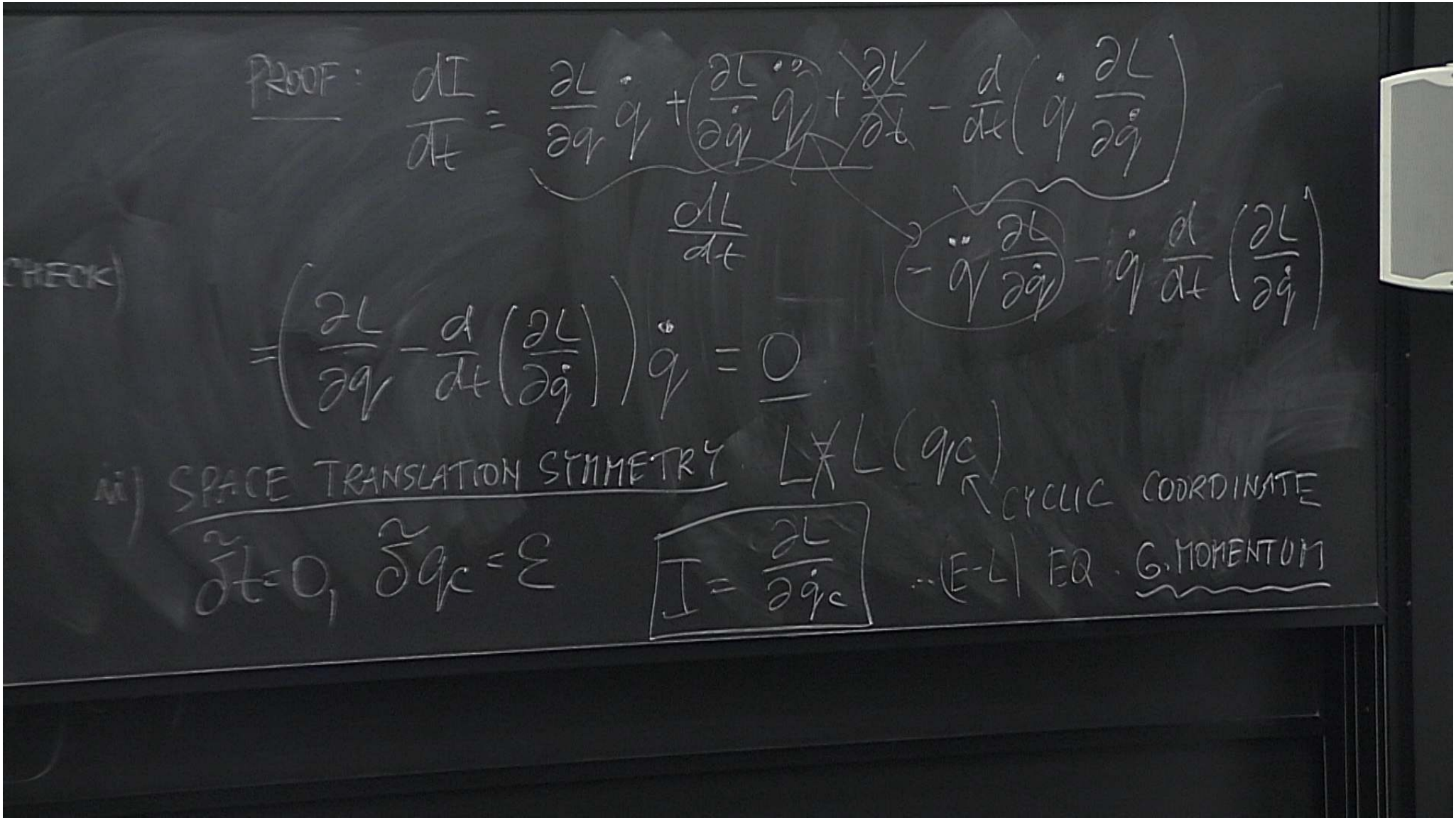
PROOF: $\frac{dI}{dt} = \frac{\partial L}{\partial q}\dot{q} + \left(\frac{\partial L}{\partial \dot{q}}\ddot{q} + \frac{\partial L}{\partial t} - \frac{d}{dt}\left(\dot{q}\frac{\partial L}{\partial \dot{q}}\right)\right)$

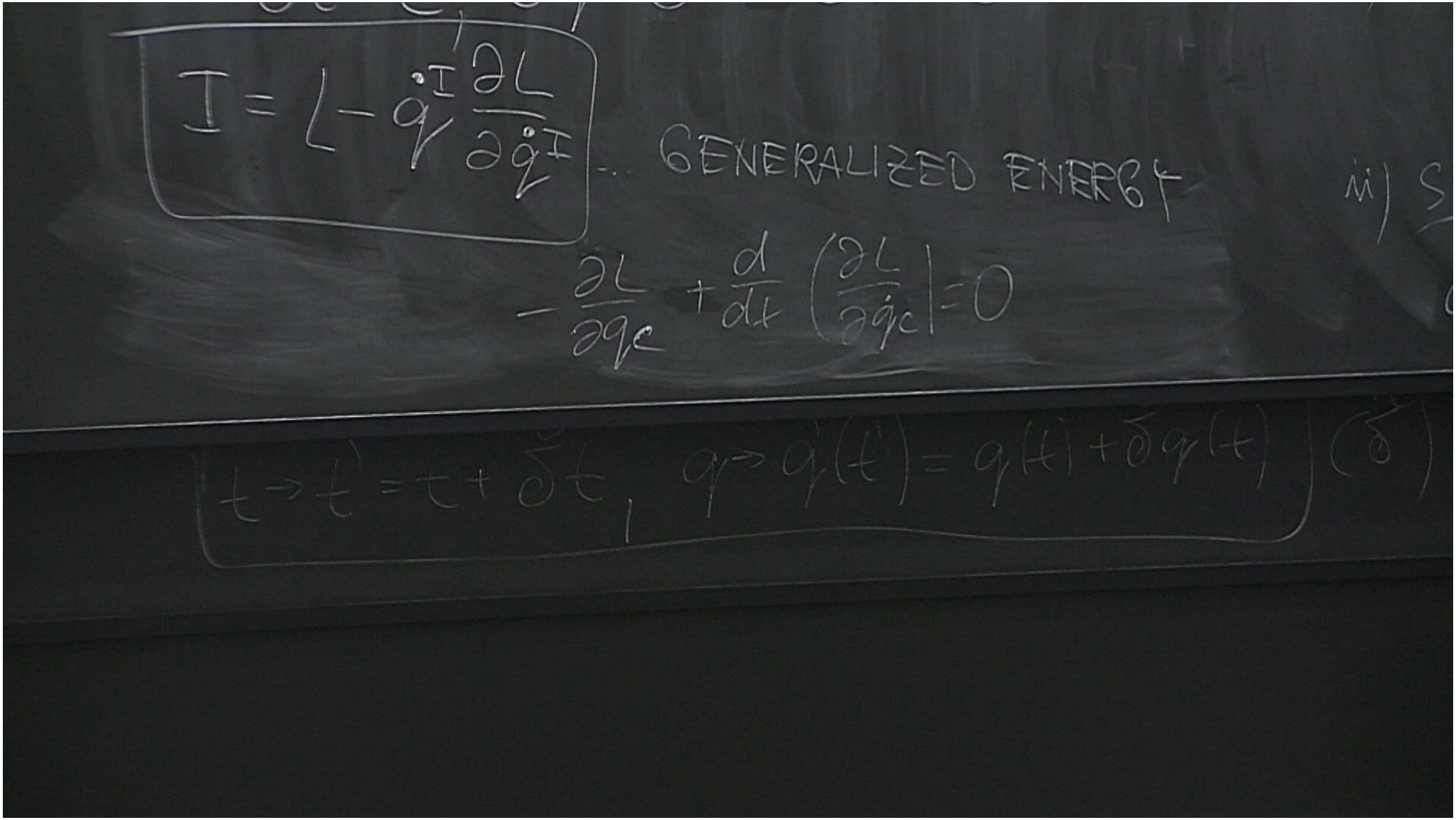
$\frac{dL}{dt}$

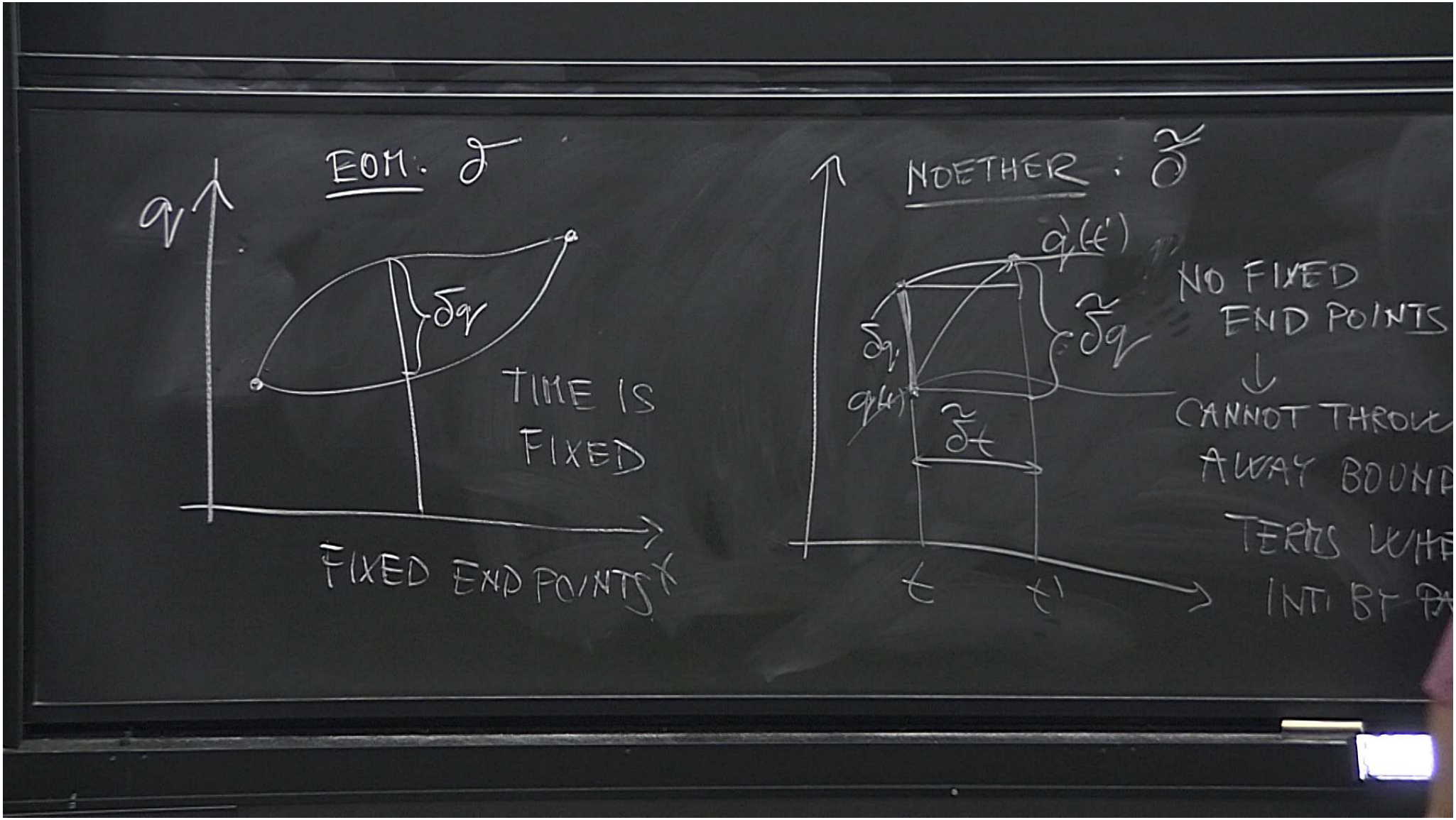
$- \ddot{q}\frac{\partial L}{\partial \dot{q}} - \dot{q}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$

ECK)

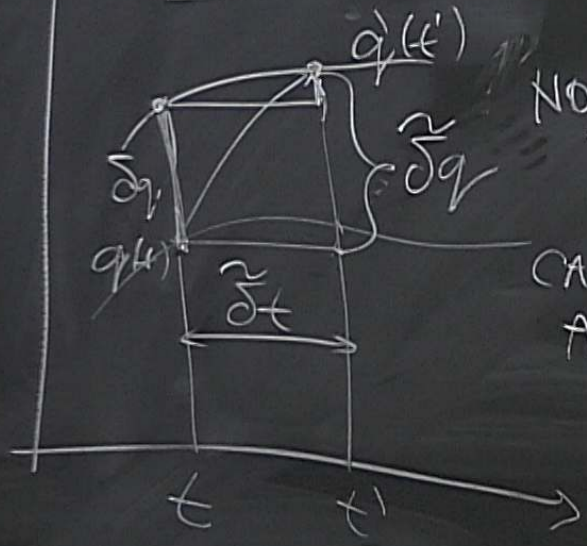
$= \left(\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right)\dot{q} = 0$


PROOF: $\frac{dI}{dt} = \frac{\partial L}{\partial q}\dot{q} + \left(\frac{\partial L}{\partial \dot{q}}\ddot{q} + \frac{\partial L}{\partial t}\right) - \frac{d}{dt}\left(\dot{q}\frac{\partial L}{\partial \dot{q}}\right)$
$\frac{dL}{dt}$
$-\ddot{q}\frac{\partial L}{\partial \dot{q}} - \dot{q}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$
CHECK
$= \left(\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right)\dot{q} = 0$
ii) SPACE TRANSLATION SYMMETRY $L \nexists L(q_c)$
CYCLIC COORDINATE
$\delta t = 0, \ \delta q_c = \varepsilon$
$I = \frac{\partial L}{\partial \dot{q}_c}$
...(E-L) EQ. G. MOMENTUM


$I = L - \dot{q}_I \frac{\partial L}{\partial \dot{q}_I}$
GENERALIZED ENERGY
$-\frac{\partial L}{\partial q_c} + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_c}\right) = 0$
$t \to t' = t + \delta t, \quad q \to q'(t) = q(t) + \delta q(t)$


EOM: $\delta$
$q_i$
$\delta q$
TIME IS FIXED
FIXED END POINTS
$t$
NOETHER : $\tilde{\delta}$
$q_i(t')$
$\tilde{\delta} q$
$\delta q_i$
$q_i(t)$
$\tilde{\delta} t$
NO FIXED END POINTS
CANNOT THROW
AWAY BOUND
TERMS WHE
INTO BY PA
$t$
$t'$

NOETHER : $\tilde{\delta}$




NO FIXED END POINTS

↓

CANNOT THROW AWAY BOUNDARY TERMS WHEN INT BY PARTS

PROOF OF V2

$$\tilde{\delta}q = \delta q + \frac{dq}{dt}\cdot\tilde{\delta}t$$

$f(q, \dot{q}, t)$

$$\boxed{\tilde{\delta} = \delta + \tilde{\delta}t\frac{d}{dt}}$$

FIXED END POINTS $t$ $t'$ → INT BY PARTS

$$\tilde{\delta}S \overset{\text{ASS}}{=} 0 = \int \left( \tilde{\delta}L\,dt + L\,\tilde{\delta}dt \right) \qquad \tilde{\delta}dt = dt' - dt = \frac{d\tilde{\delta}t}{dt}dt$$

$$= \int \left( dt\,\delta L + \underbrace{\tilde{\delta}t \frac{dL}{dt}dt}_{\frac{d}{dt}(L\tilde{\delta}t)dt} - L\,\frac{d\tilde{\delta}t}{dt}dt \right)$$

$$= \int \left[ \delta L + \frac{d}{dt}(L\tilde{\delta}t) \right] dt = \int \frac{d}{dt}\left( \underbrace{L\tilde{\delta}t + \frac{\partial L}{\partial \dot{q}_i}\delta q_i}_{I} \right) + \overbrace{(E\text{-}L)}^{0 \text{ ON-SHELL}} \text{TERM}$$

TERMS WHEN
⇒ INT. BY PARTS.

$dt = dt' - dt = \frac{d\tilde{\delta}t}{dt} dt$

$\frac{\partial L}{\partial \dot{q}} \delta q$ + (E-L) TERM

0 ON-SHELL

NOETHER V2 (UGLY BUT EXPLICIT)

LET $\tilde{\delta}$ BE A GLOBAL CONT. SYMMETRY
THAT IS OFF SHELL WE FIND $\tilde{\delta}t, \tilde{\delta}q$
S.T $\tilde{\delta}S = 0 \Rightarrow I = \frac{\partial L}{\partial \dot{q}} \tilde{\delta}q + \left(L - \dot{q}\frac{\partial L}{\partial \dot{q}}\right)\tilde{\delta}t$

IS AN INTEGRAL OF MOTION