Title: SDP / Quantum Lecture Series

Speakers: Jamie Sikora

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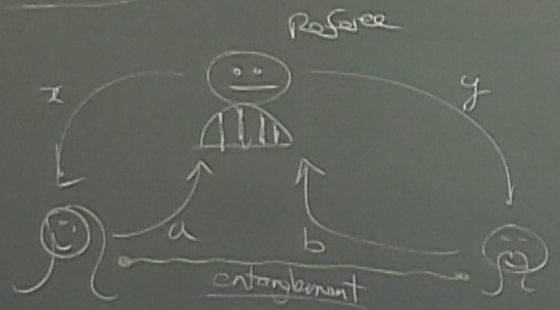
Lecture 5: Nonlocal Games

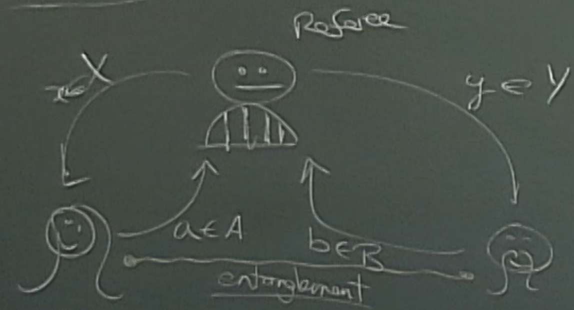


- (x,y) are chosen with probability π_{xy}
- (x,y) are the questions
- (a,b) are the answers
- A & B win if (a,b,x,y) satisfy some predicate

$$V: A\times B\times X\times Y \to \{0,1\}$$

winning

Lecture 5: Nonlocal Games



- (x,y) are chosen with probability π_{xy}
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$$V : A\times B\times X\times Y \to \{0,1\}$$

winning

Quantum Correlation

- Alice and Bob share a q state $|\psi\rangle$
- Alice has a different POVM for every $x\in X$.

  and each has outcomes $a\in A$

  POVM $\{P^x_a : a\in A\}$
- Bob has a different POVM for every $y\in Y$

  and each has outcomes $b\in B$

  POVM $\{Q^y_b : b\in B\}$

$$p(ab|xy) = \langle\psi| P^x_a \otimes Q^y_b |\psi\rangle$$

Quantum Correlation

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  POVM $\{Q_b^y : b \in B\}$

$$p(ab|xy) = \langle\psi| P_a^x \otimes Q_b^y |\psi\rangle$$

The probability they win the game is

$$\sum_{xy} \pi_{xy} \sum_{ab} V(ab|xy)\, p(ab|xy)$$

We define the set Q where $p(ab|xy) \in Q$

when it looks like this

<u>Ex</u>: CHSH game has $A = B = X = Y = \{0,1\}$

$\pi_{00} = \pi_{01} = \pi_{10} = \pi_{11} = 1/4$

$V(ab|xy) = 1 \iff a \oplus b = xy$

$\omega^*(CHSH) = \cos^2(\pi/8) \approx 85\%$

$\omega(CHSH) = 75\%$ (classical)

Ex: CHSH game has $A = B = X = Y = \{0,1\}$

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$\omega^*(CHSH) = \cos^2(\pi/8) \approx 85\%$

$\omega(CHSH) = 75\%$ (classical)

Can we characterize Q using SDPs?

Lecture 5 : Nonlocal Games

NPA Hierarchy

Navascués - Pironio - Acín

$$p(ab|xy) = \langle \psi | P^x_a \otimes Q^y_b | \psi \rangle$$

$$\longrightarrow p(ab|xy) = \langle \psi | E_{xa} E_{yb} | \psi \rangle$$

come from a Hilbert space

commute

Quantum Correlation

- Alice and Bob share a q. sta
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Lecture 5 : Nonlocal Games
NPA Hierarchy
Navascués - Pironio - Acín
$p(ab|xy) = \langle\psi| P_a^x \otimes Q_b^y |\psi\rangle$
$\rightarrow p(ab|xy) = \langle\psi| E_{xa} E_{yb} |\psi\rangle$
commute
a Hilbert space
$E_{xa} = P_a^x \otimes \mathbb{1}$
$E_{yb} = \mathbb{1} \otimes Q_b^y$

Lecture 5 : Nonlocal Games

NPA Hierarchy

Navascués – Pironio – Acín

$$p(ab|xy) = \langle\psi| P^x_a \otimes Q^y_b |\psi\rangle$$

$$(P^x_a \otimes \mathbb{1})(\mathbb{1} \otimes Q^y_b)$$

$$\longrightarrow \; p(ab|xy) = \langle\psi| E_{xa} E_{yb} |\psi\rangle$$

come from a Hilbert space

commute

orthogonal projections

$$E_{xa} = P^x_a \otimes \mathbb{1}$$

$$E_{yb} = \mathbb{1} \otimes Q^y_b$$

$$E_{xa}E_{xa} = E_{xa}$$

$$E_{xa}E_{xa'} = 0$$

E_{xa} self-adjoint

Lecture 5 : Nonlocal Games

NPA Hierarchy

Navascués - Pironio - Acín

$(x,a) \in X \times A$

$p(ab|xy) = \langle \psi | P_a^x \otimes Q_b^y | \psi \rangle$

$(P_a^x \otimes \mathbb{1})(\mathbb{1} \otimes Q_b^y)$

$\longrightarrow p(ab|xy) = \langle \psi | E_{xa} E_{yb} | \psi \rangle$

come from a Hilbert Space

commute

orthogonal projections

but, we don't have to assume this

$$\begin{cases} E_{xa} = P_a^x \otimes \mathbb{1} \\ E_{yb} = \mathbb{1} \otimes Q_b^y \end{cases}$$

$E_{xa} E_{xa} = E_{xa}$

$E_{xa} E_{xa'} = 0$

E_{xa} self-adjoint

$\sum_a E_{xa} = \mathbb{1}_{\mathcal{H}}$

ocal Games

ierarchy

onio - Acín

$P^x_a \otimes Q^y_b |\psi\rangle$

$(P^x_a \otimes I)(I \otimes Q^y_b)$

$\psi | E_{xa} E_{yb} |\psi\rangle$

commute

orthogonal Projections

finite Hilbert Space

$E_{xa}E_{xa} = E_{xa}$

$E_{xa}E_{xa'} = 0$

E_{xa} self-adjoint

$\sum_a E_{xa} = I_{\mathcal{H}}$

$$\Gamma = \text{Gram}\left(\overset{\varepsilon}{|\psi\rangle}, \overset{(x,a)}{E_{xa}|\psi\rangle}, \overset{(y,b)}{E_{yb}|\psi\rangle}\right) \geq 0$$

finite

$x \in X,\ y \in Y$

$a \in A,\ b \in B$

$\Gamma = \text{Gram}(x_1, \ldots, x_n)$

$\Gamma_{ij} = \langle x_i, x_j \rangle$

$X \geq 0 \iff X$ is a Gram matrix

Properties

⓪ It has rows & cols indexed by $\overset{\text{empty string}}{\varepsilon}, (x,a), (y,b)$.

① $\Gamma((x,a),(y,b))$

The probability they win the g

$$\sum_{xy} \pi_{xy} \sum_{ab} V(ab|xy)\, p(ab$$

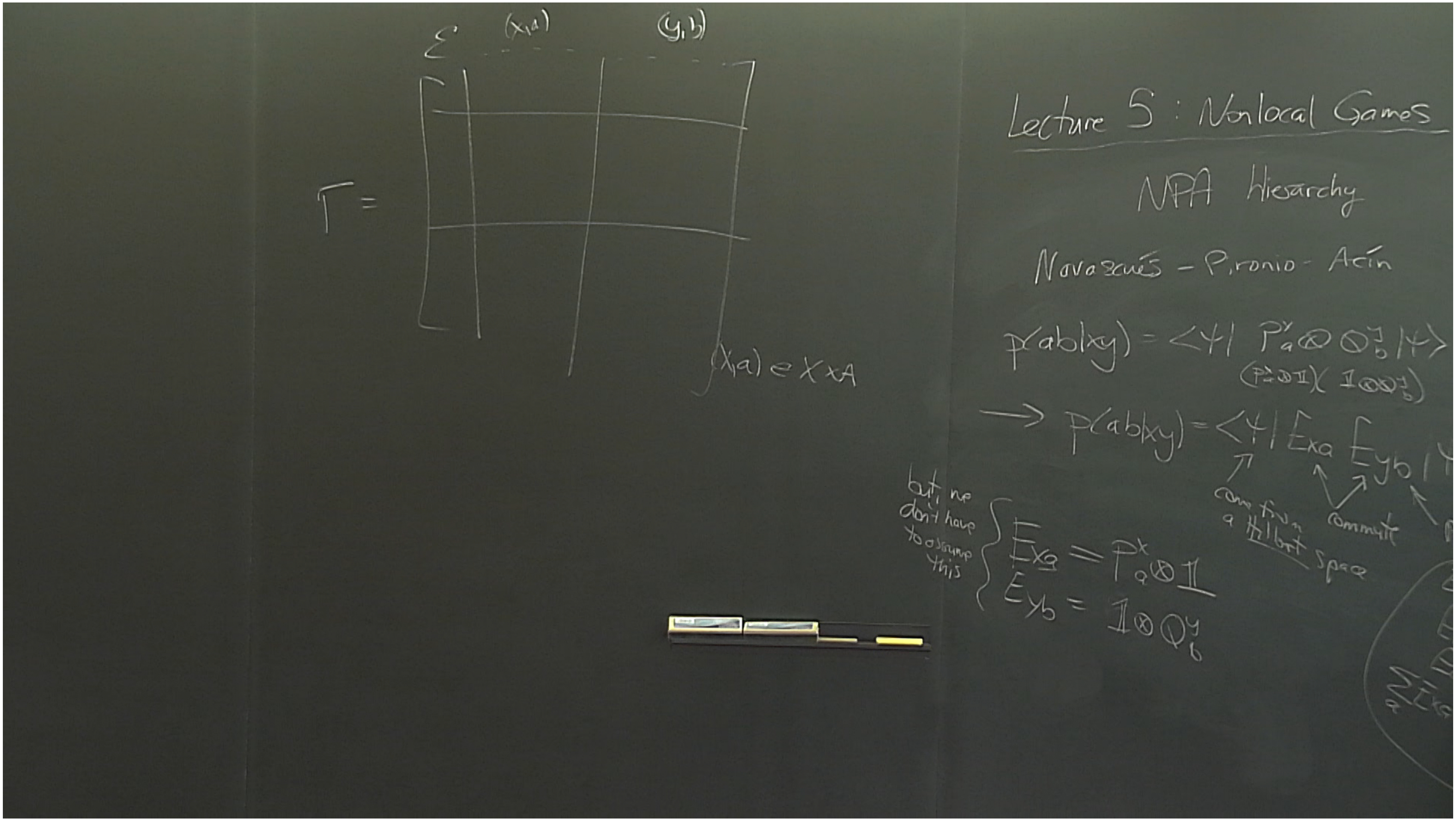
We define the set Q w

when it looks like this

Def'n: The entangled va

$$\omega^* = \sup_{p \in Q} \langle C, p \rangle$$

$C(ab|xy) = \pi_{xy} V(ab|xy)$


Lecture 5 : Nonlocal Games
NPA Hierarchy
Navascués - Pironio - Acín
p(ab|xy) = <ψ| P^x_a ⊗ Q^y_b |ψ>
(P^x_a ⊗ 𝟙)(𝟙 ⊗ Q^y_b)
→ p(ab|xy) = <ψ| E_xa E_yb |ψ>
come from a Hilbert space
commute
but we don't have to assume this
E_xa = P^x_a ⊗ 𝟙
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Γ =
(x,a)
(y,b)
(x,a) ∈ X × A

ocal Games

ierarchy

onio - Acín

$P^x_a \otimes Q^y_b |\psi\rangle$

$(P^x_a \otimes I)(I \otimes Q^y_b)$

$\psi| E_{xa} E_{yb} |\psi\rangle$

commute

orthogonal projections

finite Hilbert space

$E_{xa}E_{xa} = E_{xa}$

$E_{xa}E_{xa'} = 0$

E_{xa} self-adjoint

$\sum_a E_{xa} = I_{\mathcal{H}}$

$\varepsilon \quad (x,a) \quad (y,b)$

$$\Gamma = \text{Gram}\left(|\psi\rangle, E_{xa}|\psi\rangle, E_{yb}|\psi\rangle\right) \succeq 0$$

finite

$x \in X,\ y \in Y$
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empty string

0. It has rows & cols indexed by ε, (x,a), (y,b).
1. $\Gamma((x,a),(y,b)) = \langle\psi|E_{xa}E_{yb}|\psi\rangle = p(ab|xy)$
2. $\Gamma(\varepsilon,\varepsilon) = \langle\psi|\psi\rangle = 1$

The probability they win the g

$$\sum_{xy} \pi_{xy} \sum_{ab} V(ab|xy)\, p(ab$$

We define the set Q

when it looks like this

Def'n: The entangled va

$$\omega^* = \sup_{p \in Q} \langle G, p \rangle$$

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Lecture 5 : Nonlocal Game

NPA Hierarchy

Navascués - Pironio - Acín

$$p(ab|xy) = \langle\psi| P^x_a \otimes Q^y_b |\psi\rangle$$

$$(P^x_a \otimes \mathbb{1})(\mathbb{1} \otimes Q^y_b)$$

$$\longrightarrow p(ab|xy) = \langle\psi| E_{xa} E_{yb} \ldots$$

come from a Hilbert space

commute

but we don't have to assume this

$$\begin{cases} E_{xa} = P^x_a \otimes \mathbb{1} \\ E_{yb} = \mathbb{1} \otimes Q^y_b \end{cases}$$

$$\Gamma = \begin{array}{c|ccc} & \varepsilon & (x,a) & (y,b) \\ \hline \varepsilon & 1 & & \\ (x,a) & & & p(ab|xy) \\ (y,b) & & & \end{array}$$

$(x,a) \in X \times A$

Nonlocal Games

NPA Hierarchy

– Pironio - Acín

$\langle \psi | P_a^x \otimes Q_b^y | \psi \rangle$

$(P_a^x \otimes \mathbb{I})(\mathbb{I} \otimes Q_b^y)$

$p(ab|xy) = \langle \psi | E_{xa} E_{yb} | \psi \rangle$

come from a Hilbert space

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orthogonal projections

$\otimes \mathbb{I}$

Q_b^y

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E_{xa} self-adjoint

$\sum_a E_{xa} = \mathbb{I}_{\mathcal{H}}$

$$\Gamma = \mathrm{Gram}\big(\underset{\varepsilon}{|\psi\rangle}, \underset{(x,a)}{E_{xa}|\psi\rangle}, \underset{(y,b)}{E_{yb}|\psi\rangle}\big) \succeq 0$$

$x \in X,\ y \in Y$

$a \in A,\ b \in B$

finite

$\Gamma = \mathrm{Gram}(x_1, \dots, x_n)$

$\Gamma_{ij} = \langle x_i, x_j \rangle$

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Properties

0. It has rows & cols indexed by $\varepsilon, (x,a), (y,b)$. (empty string)
1. $\Gamma((x,a),(y,b)) = \langle \psi | E_{xa} E_{yb} | \psi \rangle = p(ab|xy)$
2. $\Gamma(\varepsilon, \varepsilon) = \langle \psi | \psi \rangle = 1$
3. $\Gamma((x,a)(x,a)) = \langle \psi | E_{xa} E_{xa} | \psi \rangle = \langle \psi | E_{xa} | \psi \rangle = p(a|x)$
4. $\Gamma((x,a),(x,a')) = \langle \psi | E_{xa} E_{xa'} | \psi \rangle = 0.$ (diff)

The probability they wi[n]

$\sum_{xy} \pi_{xy} \sum_{ab} V(ab|xy)$

We define the set

when it looks like

Def'n: The entang[led]

$\omega^* = \sup_{p \in Q} \langle$

$C(ab|xy) = \pi_{xy} V(ab|xy)$

(x,a) (y,b)

$(E_{xa}|\psi\rangle, E_{yb}|\psi\rangle) \geq 0$

finite

$x \in X,\ y \in Y$

$a \in A,\ b \in B$

Gram matrix

empty slot

replaced by $\mathcal{E}_1\ (x,a),\ (y,b)$.

$\langle\psi|E_{xa}E_{yb}|\psi\rangle = p(ab|xy)$

$\langle\psi|\psi\rangle = 1$

$\langle\psi|E_{xa}E_{xa}|\psi\rangle = \langle\psi|E_{xa}|\psi\rangle = p(a|x)$

$\langle\psi|E_{xa}E_{xa'}|\psi\rangle = 0.$

(3) $P(\mathcal{E}_1(x,a)) = \langle\psi|E_{xa}|\psi\rangle = p(a|x)$

Similarly, the (y,b) conditions also hold

$\tilde{Q}$ be this new set of "quantum" correlations

(Note $Q \subseteq \tilde{Q}$)

Ex: CH

Can

$(x,a) \qquad (y,b)$

$(E_{xa}|\psi\rangle, E_{yb}|\psi\rangle) \geq 0$

finite

$x \in X,\ y \in Y$

$a \in A,\ b \in B$

Gram matrix

empty string

indexed by $\varepsilon, (x,a), (y,b)$.

$\langle\psi|E_{xa}E_{yb}|\psi\rangle = p(ab|xy)$

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③ $P(\varepsilon, (x,a)) = \langle\psi|E_{xa}|\psi\rangle = p(a|x)$

Similarly, the (y,b) conditions also hold

$\tilde{Q}$ be this new set of "quantum" correlations (☺)

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$p \in \tilde{Q} \Rightarrow \Gamma$ with those properties exist

Ex: CH

Can

(x,a) (y,b)

$(E_{xa}|\psi\rangle, E_{yb}|\psi\rangle) \geq 0$

finite

$x \in X,\ y \in Y$

$a \in A,\ b \in B$

Gram matrix

empty slot

indexed by $\varepsilon, (x,a), (y,b)$,

$\langle\psi|E_{xa}E_{yb}|\psi\rangle = p(ab|xy)$

$\rangle = 1$

$\langle\psi|E_{xa}E_{xa}|\psi\rangle = \langle\psi|E_{xa}|\psi\rangle = p(a|x)$

$\langle\psi|E_{xa}E_{xa'}|\psi\rangle = 0.$

③ $P(\varepsilon,(x,a)) = \langle\psi|E_{xa}|\psi\rangle = p(a|x)$

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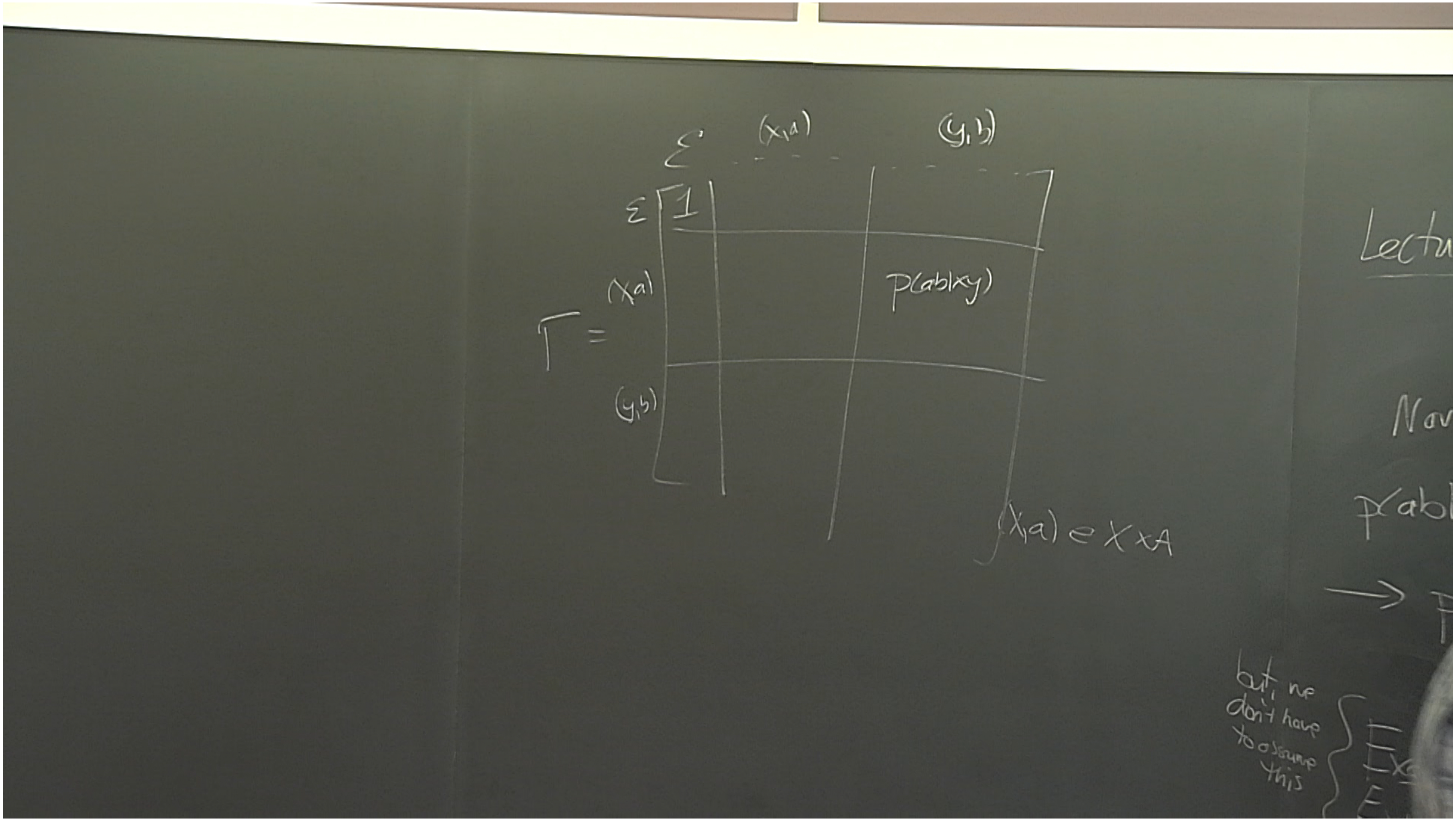
But, Γ may exist while $p(ab|xy) = \Gamma((x,a),(y,b))$

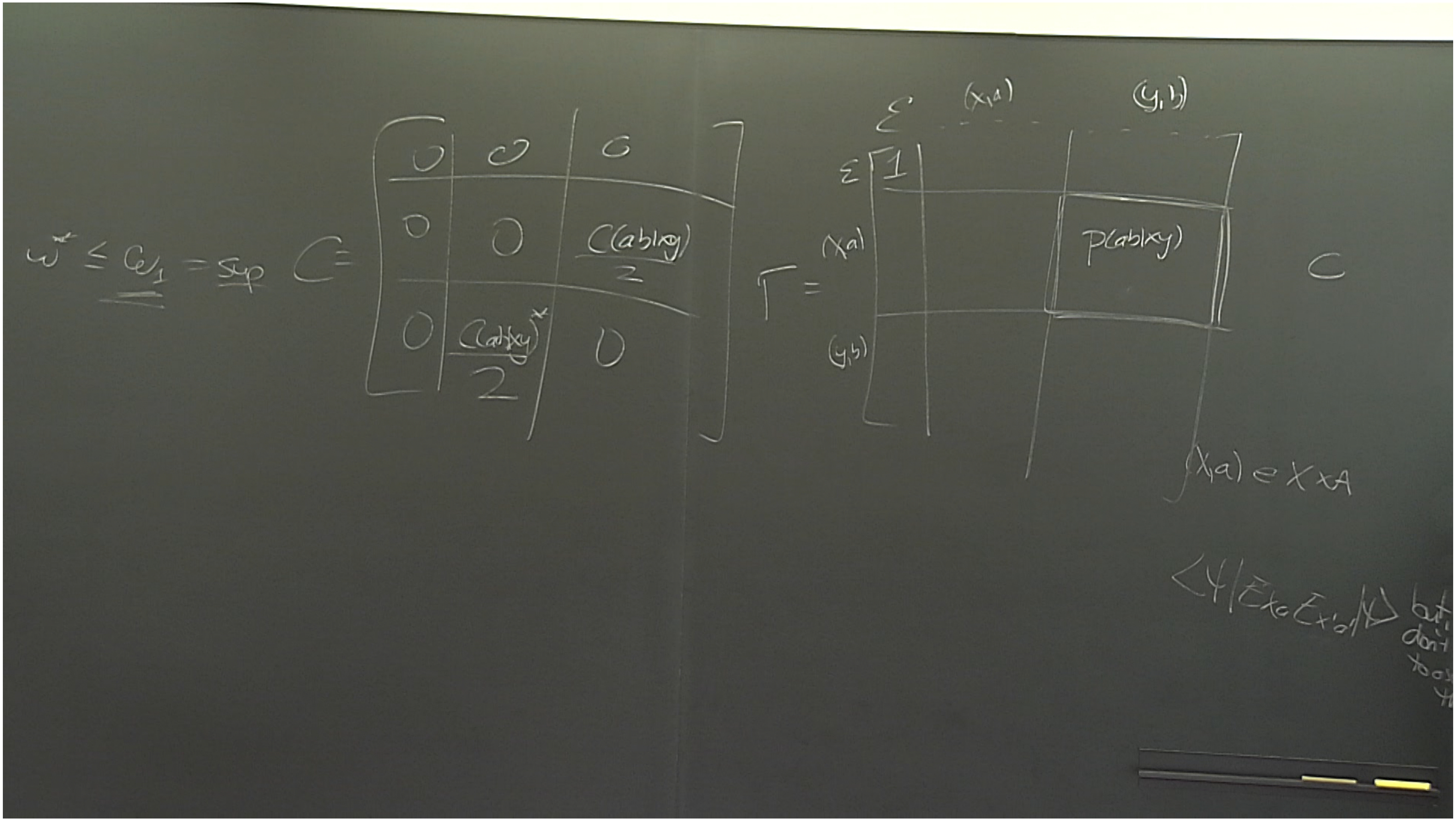
may not be quantum

Ex: CH

Can



(x,a)
(y,b)
Γ =
1
p(ab|xy)
(x,a) ∈ X×A
but, we don't have to assume this


C =
T =
p(ab|xy)
(x,a)
(y,b)
(x,a) ∈ X × A

$|\Psi\rangle)\big) \geq 0$

finite

③ $P(\xi_1(x,a)) = \langle\Psi|E_{xa}|\Psi\rangle = p(a|x)$

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may not be quantum

$\langle\Psi|E_{xa}E_{x'a'}E_{yb}E_{y'b'}E_{x''a''}\cdots|\Psi\rangle$

ts share

$(x,a),\ (y,b),$

$\rangle = p(ab|xy)$

$\langle\Psi|E_{xa}|\Psi\rangle = p(a|x)$

$=0.$

Alphabets

$\Sigma_A = X \times A$, $\Sigma_B = Y \times B$, $\Sigma = \Sigma_A \sqcup \Sigma_B$.

Σ^k: strings of length k

$\Sigma^{\leq k}$: strings of length $\leq k$

Σ^*: all strings

Define the relation:

- $s\sigma t \sim s\sigma\sigma t$, $\forall s \in \Sigma^*, t \in \Sigma^*, \sigma \in \Sigma$

Alphabets

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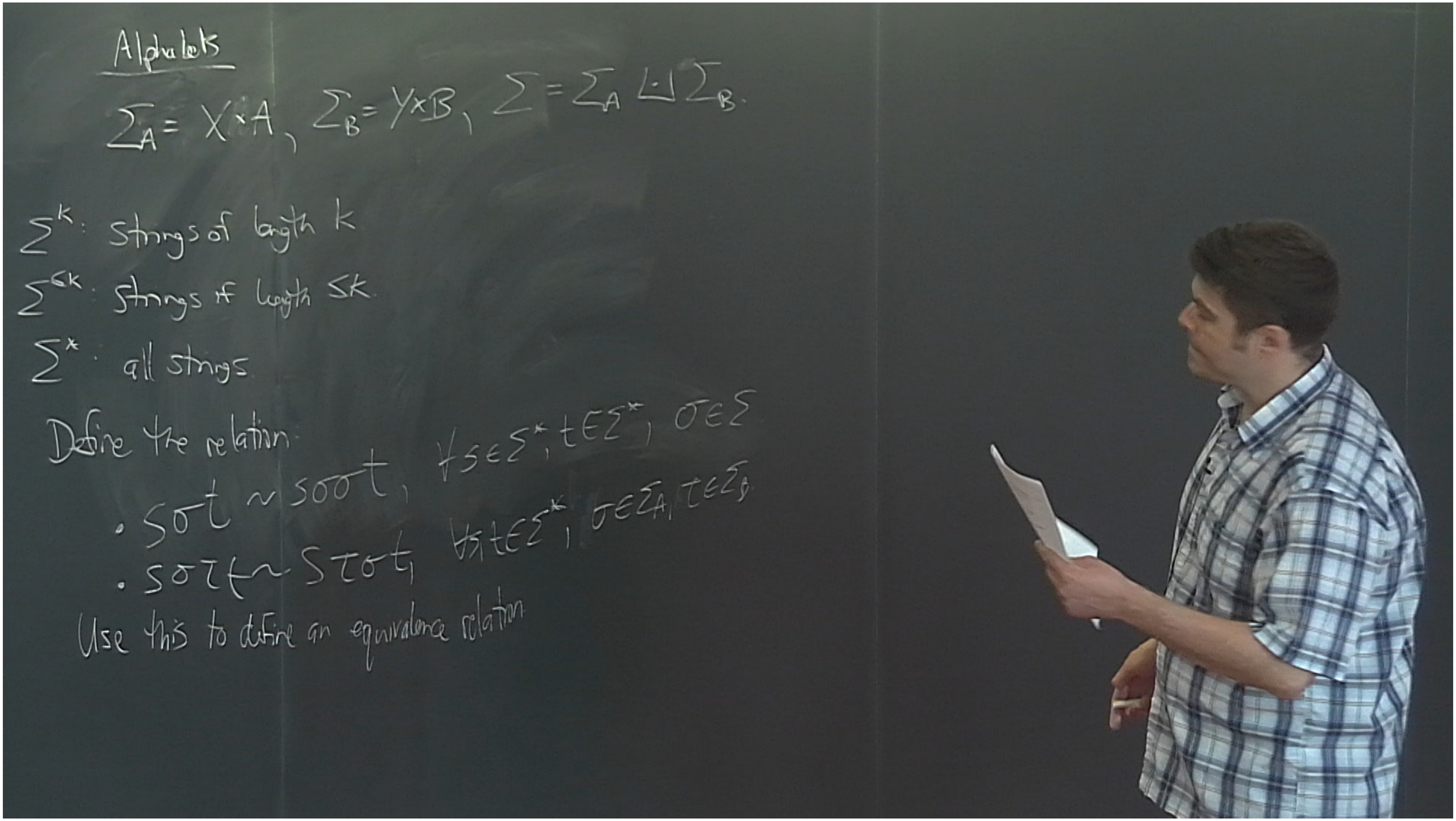
Define the relation:

- $s\sigma t \sim s\sigma\sigma t$, $\forall s \in \Sigma^*, t \in \Sigma^*, \sigma \in \Sigma$
- $s\sigma\tau t \sim s\tau\sigma t$, $\forall s, t \in \Sigma^*, \sigma \in \Sigma_A, \tau \in \Sigma_B$


Alphabets
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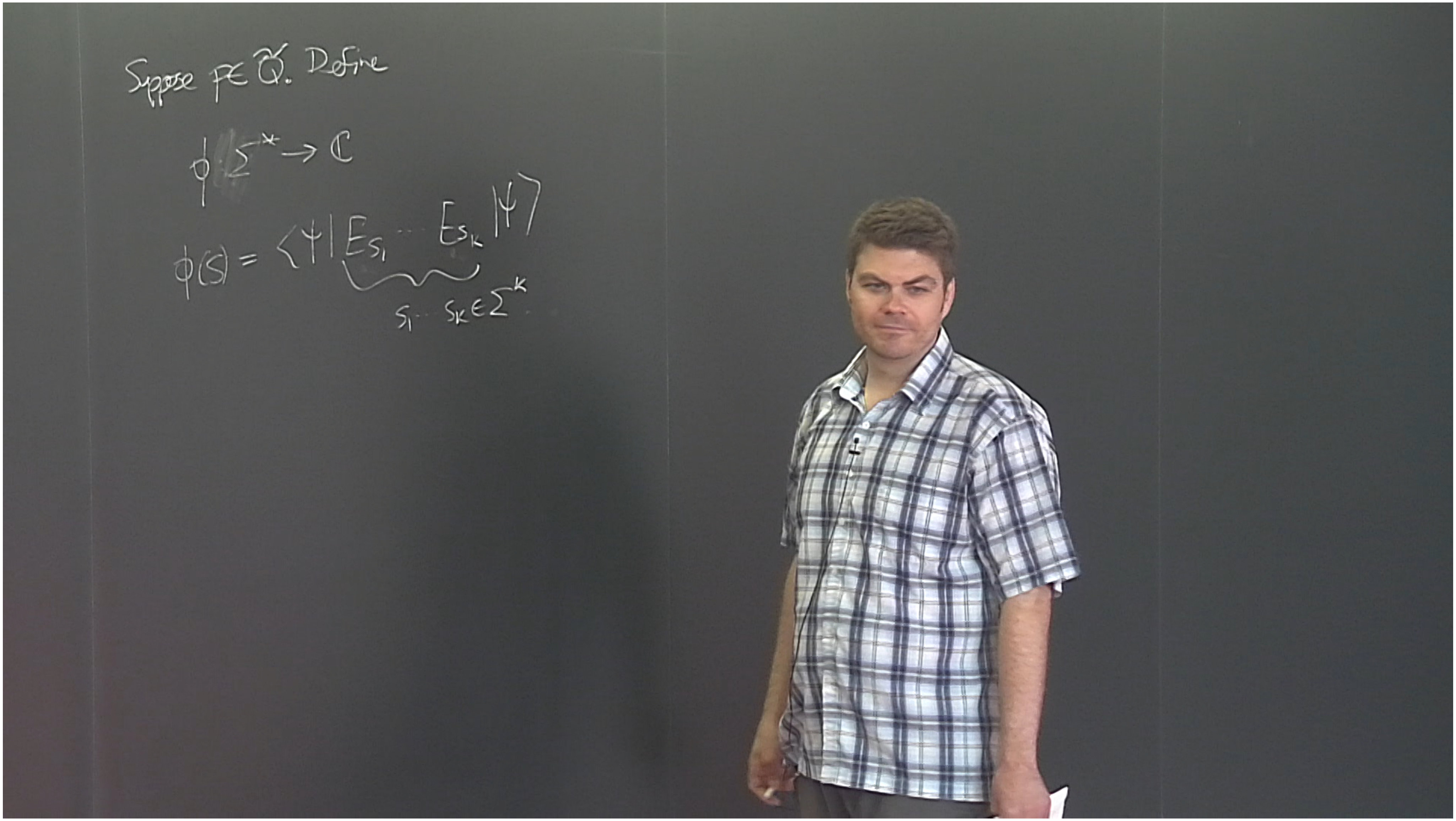
• $s\sigma\tau t \sim s\tau\sigma t$, $\forall s, t \in \Sigma^*, \sigma \in \Sigma_A, \tau \in \Sigma_B$
Use this to define an equivalence relation


Suppose $\rho \in \tilde{Q}$. Define
$\phi: \Sigma^* \to \mathbb{C}$
$\phi(s) = \langle \psi | \underbrace{E_{s_1} \cdots E_{s_k}}_{s_1 \cdots s_k \in \Sigma^*} | \psi \rangle$

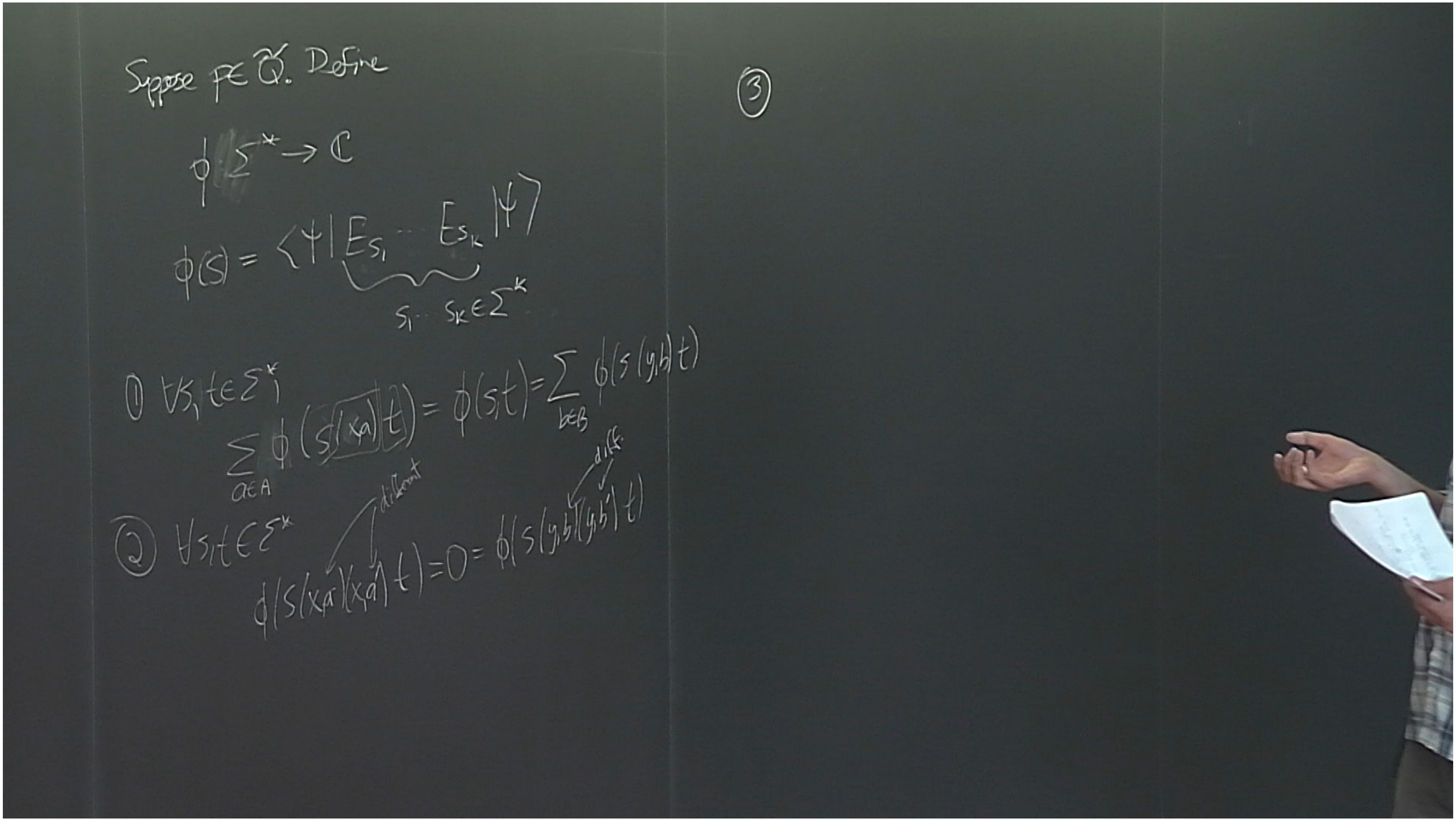
Suppose $p \in \tilde{Q}$. Define

$$\phi : \Sigma^* \to \mathbb{C}$$

$$\phi(s) = \langle \psi | \underbrace{E_{s_1} \cdots E_{s_k}}_{s_1 \cdots s_k \in \Sigma^*} | \psi \rangle$$

① $\forall s, t \in \Sigma^*$,

$$\sum_{a \in A} \phi\left(s \boxed{(x, a)} t\right) = \phi(s, t) = \sum_{b \in B} \phi(s\,(y, b)\,t)$$


Suppose $\rho \in \tilde{Q}_0$. Define
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$\phi(s) = \langle \psi | \underbrace{E_{s_1} \cdots E_{s_k}}_{s_1 \cdots s_k \in \Sigma^*} | \psi \rangle$
① $\forall s, t \in \Sigma^*$;
$\sum_{a \in A} \phi(s(x,a)t) = \phi(s,t) = \sum_{b \in B} \phi(s(y,b)t)$
different
diff.
② $\forall s, t \in \Sigma^*$
$\phi(s(x,a)(x,a')t) = 0 = \phi(s(y,b)(y,b')t)$
③

$E_{s_1} \cdots E_{s_k} |\psi\rangle$

$s_1 \cdots s_k \in \Sigma^k$

$(t)) = \phi(s,t) = \sum_{b \in B} \phi(s(y,b)t)$

diff.

$t) = 0 = \phi(s(y,b)(y,b')t)$

③ $\forall s, t \in \Sigma^*$

$$\phi(s) = \phi(t) \text{ when } s \sim t.$$

If ϕ satisfies ①, ②, ③

we call it admissible

C

$(x,a) \in X \times A$

Lecture 5: Nonlocal Games

Def'n: We say that $\Gamma_k \in \mathrm{Pos}\left(\mathbb{C}^{\Sigma^{\leq k}}\right)$

is k-th order admissible if there exists

$\phi : \Sigma^{\leq 2k} \to \mathbb{C}$ such that

$$\Gamma(s,t) = \phi(\underset{\leq k}{s^R}\ \underset{\leq k}{t})$$

$(\text{Jamie})^R = \text{eimaJ}$

Def'n: Q_k is the set of $p(a,b|x,y)$ such that

$\exists\, \Gamma_k$ k-th order admissible es

$\Gamma_k \in \mathrm{Pos}\left(\mathbb{C}^{\Sigma^{\leq k}}\right)$

le if there exists

n that

t)

(ab|xy) such that

missible

$\alpha_k = \sup t$

Γ_k satisfies 1, 2, 3

$\Gamma_k \geq t\,\mathbb{I}$

$\Gamma_k\left[(x,a),(y,b)\right] = p(ab|xy)$

$\alpha_k \geq 0 \iff p(ab|xy) \in Q_k$

3 $P(\xi_1(x,a)) = \langle\psi|E_{xa}|\psi\rangle = p($

Similarly, the (y,b) conditions a

$\tilde{Q}$ be this new set of "qu

(Note $Q \subseteq \tilde{Q}$)

$p \in \tilde{Q} \Rightarrow \Gamma$ with those properties

But, Γ may exist while $p(ab|xy) = \Gamma($

may not be

$\langle\psi|\underline{E_{xa}}\,\underline{E_{x'a'}}\,\underline{E_{yb}}\,\underline{E_{y'b'}}\,\underline{E_{x''a''}}$

tisfies ①,②,③

$\in \mathrm{II}$

$),(y,b)] = p(ab|xy)$

$b|xy) \in Q_K$

③ $P(E_1(x,a)) = \langle\psi|E_{xa}|\psi\rangle = p(a|x)$

Similarly, the (y,b) conditions also hold

$\tilde{Q}$ be this new set of "quantum" correlations (∗)

(Note $Q \subseteq \tilde{Q}$)

$p \in \tilde{Q} \Rightarrow \Gamma$ with those properties exist

But, Γ may exist while $p(ab|xy) = \Gamma[(x,a),(y,b)]$

may not be quantum

$\langle\psi|E_{xa}E_{xa'}E_{yb}E_{y'b}|\psi\rangle$ $\quad |\psi\rangle$

Al

Σ_A

Σ^k: str

$\Sigma^{\leq k}$: str

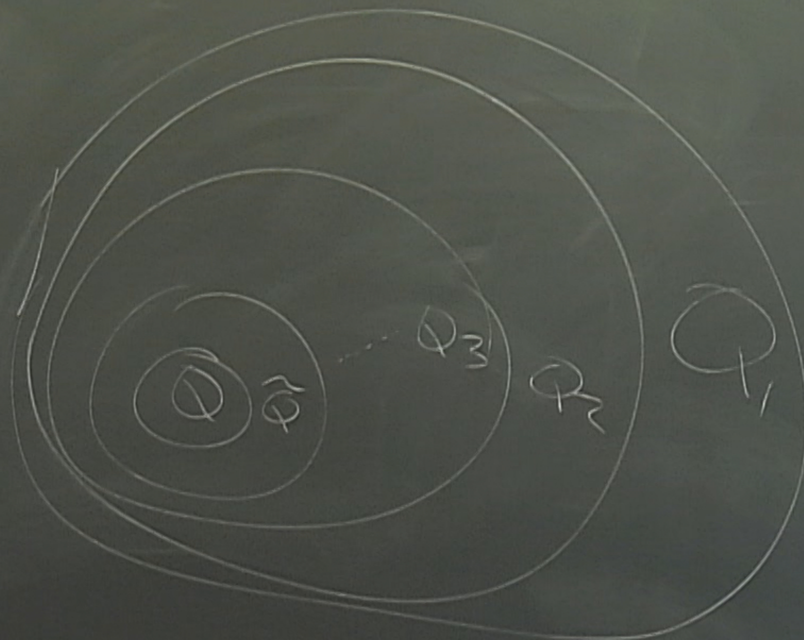
Σ^*: all

Define th

• S

• S

Use this



Lecture 5 : Nonlocal

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$\phi : \Sigma^{\leq 2k} \to \mathbb{C}$ s

$\Gamma(s,t) = \phi$

$(\text{Jamie})^R = \text{eima}$ J

Def'n: Q_k is the set of

$\exists \Gamma_k$. k-th order

②, ③

$(ab|xy)$

k

Fun Fact: $\tilde{Q} \subseteq \bigcap_{k=1}^{\infty} Q_k$

Do we have equality?

Let $p(ab|xy) \in \bigcap_{k=1}^{\infty} Q_k$

$\Rightarrow \exists\, \Gamma_k \geq 0$ kth order admissible matrices

$\Gamma_k((x,a),(y,b)) = p(ab|xy)$ $\quad \forall k \in \mathbb{N}$

$$\tilde{\Gamma}_k = \begin{bmatrix} \Gamma_k & 0 \\ 0 & 0 \end{bmatrix} \in \mathrm{Pos}(\mathbb{C}^{\Sigma^*})$$

Alph

$\Sigma_A =$

Σ^k: string

$\Sigma^{\leq k}$: string

Σ^*: all s

Define the

- $S\sigma$
- $S\sigma$

Use this

$\{\tilde{\Gamma}_k : k \in \mathbb{N}\}$ is a sequence

$\exists$ a convergent subsequence $\{\tilde{\Gamma}_k : k \in \mathcal{I}\}$

(by a compactness argument)

ios

$\Sigma^*)$

$\{\tilde{\Gamma}_k : k \in \mathbb{N}\}$ is a sequence

$\exists$ a convergent subsequence $\{\tilde{\Gamma}_k : k \in \mathcal{I}\}$

(by a compactness argument)

Tychonoff's Thm

ios

$\Sigma^*)$

$\{\tilde{\Gamma}_k : k\in\mathbb{N}\}$ is a sequence

$\exists$ a convergent subsequence $\{\tilde{\Gamma}_k : k \in \mathcal{I}\}$

(by a compactness argument)

Tychonoff's Thm

Banach-Alaoglu Thm

sup t

Γ_k satisfies ①,②,③

$\Gamma_k \geq t \mathbb{I}$

$\Gamma_k\left[(x,a),(y,b)\right] = p(ab|xy)$

$\Longleftrightarrow p(ab|xy) \in Q_k$

Fun Fact: $\tilde{Q} \subseteq \bigcap_{k=1}^{\infty} Q_k$

Do we have equality?

Let $p(ab|xy) \in \bigcap_{k=1}^{\infty} Q_k$

$\Rightarrow \exists \Gamma_k \geq 0$ kth order admissible matrices

$\Gamma_k((x,a),(y,b)) = p(ab|xy) \quad \forall k \in \mathbb{N}$

$$\tilde{\Gamma}_k = \begin{bmatrix} \Gamma_k & 0 \\ 0 & 0 \end{bmatrix} \in \mathrm{Pos}\left(\mathbb{C}^{\Sigma^*}\right)$$

$\{\tilde{\Gamma}_k : k \in \mathbb{N}\}$ i

$\exists$ a convergent s

(by a compa

Tychonoff's Th

Banach-Alaoglu Th

$\{\tilde{\Gamma}_k \cdot k \in \mathbb{N}\}$ is a sequence

$\exists$ a convergent subsequence $\{\tilde{\Gamma}_k \cdot k \in \mathcal{I}\}$

(by a compactness argument) Converge pointwise.

Tychonoff's Thm

Banach-Alaoglu Thm

Let Γ_∞ be the limit

$\{\tilde{\Gamma}_k : k \in \mathbb{N}\}$ is a sequence

$\exists$ a convergent subsequence $\{\tilde{\Gamma}_k : k \in \mathcal{I}\}$

(by a compactness argument) Converge pointwise.

Tychonoff's Thm

Banach-Alaoglu Thm

Let Γ_∞ be the limit

$\exists\, \phi$ admissible

$\Gamma(s,t) = \phi(s^R t)$

Lecture 5 : Nonlocal Games

We need a Hilbert space $V = \mathbb{C}^{S^*}$

$$\langle e_s, e_t \rangle = \Gamma_\infty(s,t)$$

$\alpha_k = \sup t$

Γ_k satisfies ①,②

$\Gamma_k \geq t \mathbb{I}$

$\Gamma_k[(x,a),(y,b)] = p(a$

$\alpha_k \geq 0 \iff p(ab|xy) \in Q_k$

Lecture 5 : Nonlocal Games

We need a Hilbert space $V = \mathbb{C}^{S^*}$

$\langle e_s, e_t \rangle = \Gamma_\infty(s,t)$

$$\begin{array}{c c} & \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \\ s \rightarrow & \end{array}$$

extend linearly.

But, there might be $v \in V$ s.t. $\langle v, v \rangle = 0$

$\alpha_k = \sup t$

Γ_k satisfies ①,②

$\Gamma_k \geq t\, \mathbb{I}$

$\Gamma_k[(x,a),(y,b)] = p(a$

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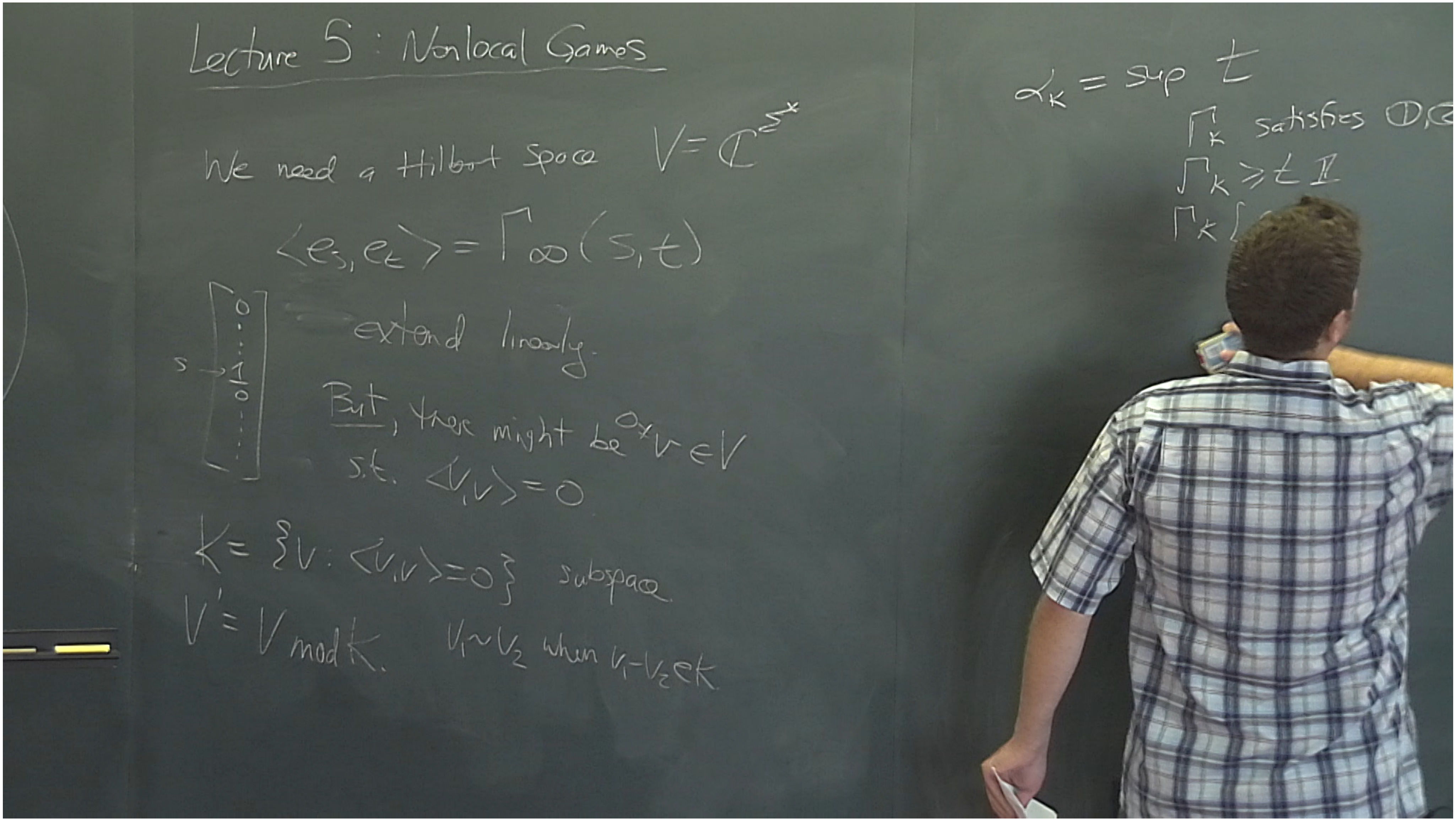
Lecture 5 : Nonlocal Games
We need a Hilbert space $V = \mathbb{C}^{S^*}$
$\langle e_s, e_t \rangle = \Gamma_\infty(s,t)$
extend linearly.
But, there might be $0 \neq v \in V$
s.t. $\langle v, v \rangle = 0$
$K = \{ v : \langle v,v \rangle = 0 \}$ subspace
$V' = V \bmod K.$ $v_1 \sim v_2$ when $v_1 - v_2 \in K$
$\alpha_K = \sup t$
Γ_K satisfies ①,②
$\Gamma_K \geq t\, \mathbb{I}$

Games

space $V = \mathbb{C}^{\Sigma^*}$

$\infty(s,t)$

nearly.

ght be $0 \neq v \in V$

$= 0$

subspace

when $v_1 - v_2 \in K$

Take its Cauchy Completion to get H

Fun Fact: $\tilde{Q} \subseteq$

Do we hav

let $p(ab|xy) \in \bigcap_{k=1}^{\infty} Q_k$

$\Rightarrow \exists \Gamma_k \geq 0$ k.t.

$\Gamma_k((x,a),(y,b)) =$

$$\tilde{\Gamma}_k = \begin{bmatrix} \Gamma_k & \\ & 0 \end{bmatrix}$$

Games

space $V = \mathbb{C}^{\Sigma^*}$

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noonly.

ght be $0 \neq v \in V$

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subspace

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Take its Cauchy Completion to get $\mathcal{H}$

Define operators E_{xa} $((x,a) \in X \times A)$
E_{yb} $((y,b) \in Y \times B)$

$E_{xa} e_s = e_{(xa)s}$

Fun Fact: $\tilde{Q} \subseteq$

Do we hav

let $p(ab|xy) \in \bigcap_{k=1}^{\infty} Q_k$

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$\Gamma_k((xa),(y,b)) =$

$\tilde{\Gamma}_k = \begin{bmatrix} \Gamma_k & \\ & 0 \end{bmatrix}$

Games

space $V = \mathbb{C}^{\Sigma^*}$

$\infty(s,t)$

nearly.

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$= 0$

subspace

when $v_1 - v_2 \in K$

Take its Cauchy Completion to get $\mathcal{H}$

Define operators E_{xa} $((x,a) \in \mathcal{X} \times \mathcal{A})$

E_{yb} $((y,b) \in \mathcal{Y} \times \mathcal{B})$

$E_{xa} e_s = e_{(x,a)s} \quad \forall s \in \Sigma^*$

$E_{yb} e_s = e_{(y,b)s} \quad \forall s \in \Sigma^*$

& extend linearly

$|\psi\rangle = e_\varepsilon$

Fun Fact: $\tilde{Q} \subseteq$

Do we hav

let $p(ab|xy) \in \bigcap_{k=1}^{\infty} Q_k$

$\Rightarrow \exists \Gamma_k \geq 0$ k+

$\Gamma_k((x,a),(y,b)) =$

$$\tilde{\Gamma}_k = \begin{bmatrix} \Gamma_k & \\ & 0 \end{bmatrix}$$

Take its Cauchy Completion to get $\mathcal{H}$

Lemma:

$= \mathbb{C}^{\Sigma^*}$

$)$

$\in V$

Define operators E_{xa} $\quad ((x,a) \in X \times A)$

E_{yb} $\quad ((y,b) \in Y \times B)$

$$E_{xa} e_s = e_{(x,a)s} \quad \forall s \in \Sigma^*$$

$$E_{yb} e_s = e_{(y,b)s} \quad \forall s \in \Sigma^*$$

& extend linearly

$$|\psi\rangle = e_\varepsilon \quad (= |0\rangle_{\mathcal{H}})$$

ompletion to get H

E_{xa} $((x,a) \in X \times A)$

E_{yb} $((y,b) \in Y \times B)$

s $\forall s \in \Sigma^*$

$\forall s \in \Sigma^*$

$\rangle_H)$

<u>Lemma</u>: E_{xa} is self-adjoint

<u>Proof</u>: $\langle e_s, E_{xa} e_t \rangle = \langle e_s, e_{(x,a)t} \rangle = \Gamma_\infty(s\,(x,a)\,t) = \phi(s^R\,(x,a)\,t)$

$s,t \in \Sigma^*$

$\langle E_{xa} e_s, e_t \rangle = \langle e_{(x,a)s}, e_t \rangle = \Gamma_\infty((x,a)\,s,t)$

$= \phi(s^R(x,a)\,t).$

$\square$

ompletion to get $\mathcal{H}$

$E_{xa} \quad ((x,a) \in X \times A)$

$E_{yb} \quad ((y,b) \in Y \times B)$

$\forall s \in \Sigma^*$

$\forall s \in \Sigma^*$

$\rangle_{\mathcal{H}})$

Lemma: E_{xa} is self-adjoint

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$s,t \in \Sigma^*$

$\langle E_{xa} e_s, e_t \rangle = \langle e_{(x,a)s}, e_t \rangle = \Gamma_\infty((x,a)\,s,t)$

$= \phi(s^R (x,a)\,t).$

$\square$

Lemma: $E_{xa} E_{xa} = E_{xa}$

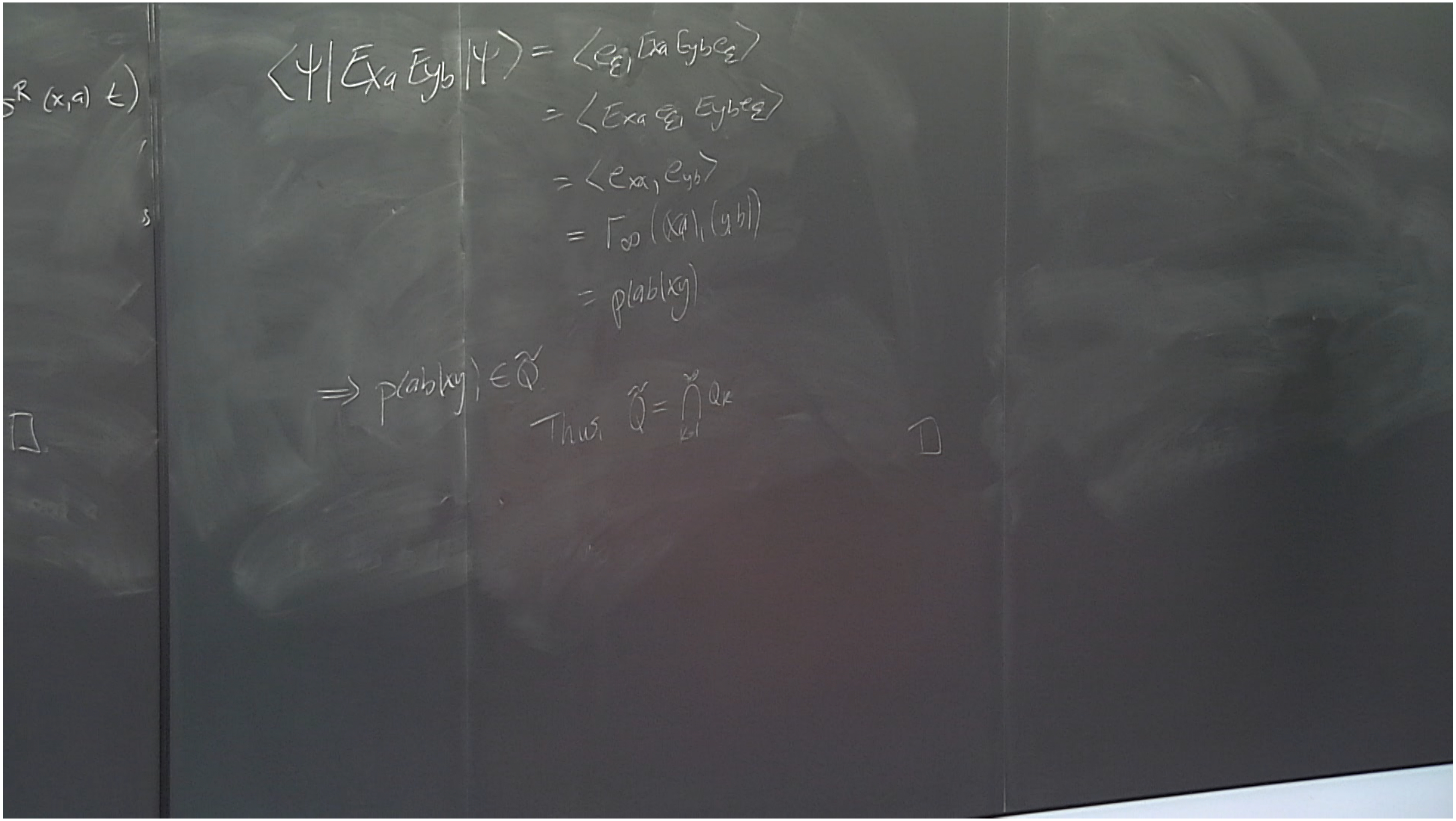
Lemma: $E_{xa} E_{xa'} = 0 \quad a \neq a'$

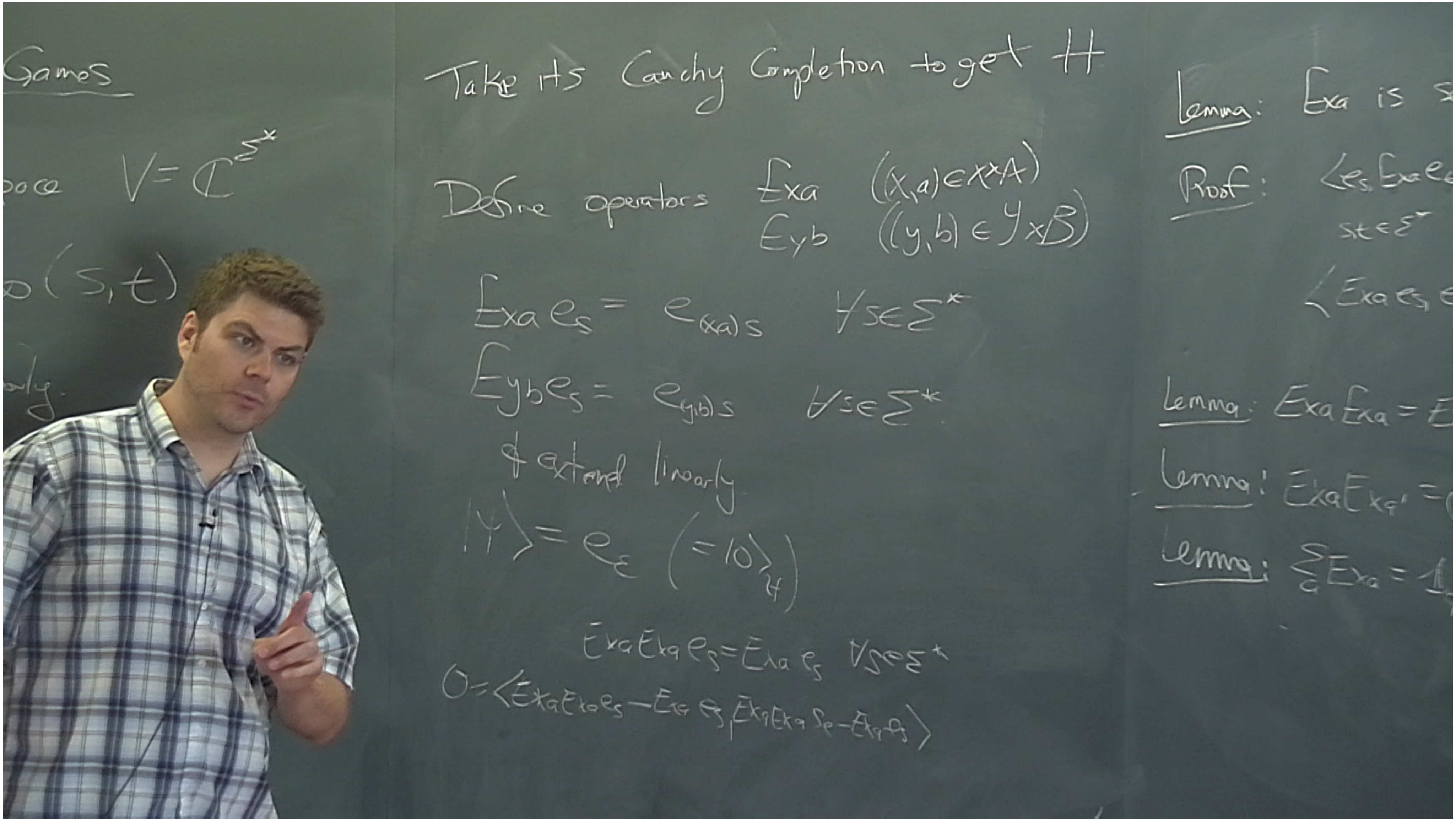
Lemma: $\sum_a E_{xa} = \mathbb{1}_{\mathcal{H}}$

$^{R}(x,a)\ t)$

$$\langle \psi | E_{xa} E_{yb} | \psi \rangle = \langle e_{\varepsilon}, E_{xa} E_{yb} e_{\varepsilon} \rangle$$
$$= \langle E_{xa} e_{\varepsilon}, E_{yb} e_{\varepsilon} \rangle$$
$$= \langle e_{xa}, e_{yb} \rangle$$
$$= \Gamma_{\infty}((x,a),(y,b))$$
$$= p(ab|xy)$$

$\square$


$^R (x,a)\ t)$
$\langle \psi | E_{xa} E_{yb} | \psi \rangle = \langle e_\varepsilon, E_{xa} E_{yb} e_\varepsilon \rangle$
$= \langle E_{xa} e_\varepsilon, E_{yb} e_\varepsilon \rangle$
$= \langle e_{xa}, e_{yb} \rangle$
$= \Gamma_\infty((xa),(yb))$
$= p(ab|xy)$
$\Rightarrow p(ab|xy) \in \tilde{Q}$
Thus $\tilde{Q} = \bigcap_{k=1}^{\infty} Q_k$


Games
Take its Cauchy Completion to get H
Define operators Exa ((x,a)∈X×A)
Eyb ((y,b)∈Y×B)
Exa es = e(x,a)s ∀s∈Σ*
Eyb es = e(y,b)s ∀s∈Σ*
& extend linearly
|ψ⟩ = eε (= |0⟩ψ)
Exa Exa es = Exa es ∀s∈Σ*
Lemma: Exa is
Proof:
s,t∈Σ*
Lemma: Exa Exa = E
Lemma: Exa Exa' =
Lemma: Σa Exa = 𝟙

Games

space $V = \mathbb{C}^{\Sigma^*}$

(s,t)

$v \in V$

space

$v_1 - v_2 \in K$

Take its Cauchy Completion to get H

Define operators E_{xa} $((x,a) \in X \times A)$

E_{yb} $((y,b) \in Y \times B)$

$E_{xa} e_s = e_{(xa)s}$ $\forall s \in \Sigma^*$

$E_{yb} e_s = e_{(yb)s}$ $\forall s \in \Sigma^*$

& extend linearly

$|\psi\rangle = e_\varepsilon$ $\left(= |0\rangle_\psi\right)$

$E_{xa} E_{xa} e_s = E_{xa} e_s$ $\forall s \in \Sigma^*$

$0 = \langle \underbrace{E_{xa} E_{xa} e_s - E_{xa} e_s}_{v}, \underbrace{E_{xa} E_{xa} e_s - E_{xa} e_s}_{v} \rangle$

Lemma: E_{xa} is s

Proof: $\langle e_s, E_{xa} e_t$

$s,t \in \Sigma^*$

$\langle E_{xa} e_s, e$

Lemma: $E_{xa} E_{xa} = E$

Lemma: $E_{xa} E_{xa'} = ($

Lemma: $\sum_a E_{xa} = \mathbb{1}$