Title: Sphere partition functions and Hamiltonian truncation

Speakers: Matthijs Hogervorst

Collection: Bootstrap 2019

Date: August 02, 2019 - 10:00 AM

URL: http://pirsa.org/19080001

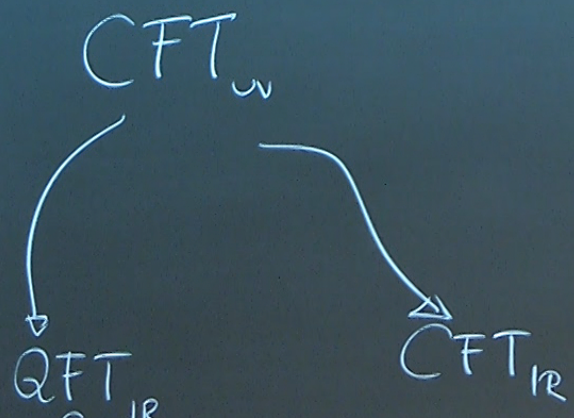
Hamiltonian truncation & Sphere partition functions

1811.00528 + ...

1) Intro to HT
2) QFT on S^d
3) Setup
4) Results
....

$$S = S_{UV} + \lambda \int_M \sqrt{g}\, d^d x\, \mathcal{O}(x)$$

$\Delta \leq d$



$$m_{gap} \sim \mathcal{O}(1) \cdot |\lambda|^{\frac{1}{d-\Delta}} \sim 1/\xi$$

$$E(L) = \sqrt{\frac{\#}{L^2} + m^2} = m + \frac{1}{L^2} \cdots$$

$$L \gg (\text{few}) \cdot |\lambda|^{\#}$$

$$\bar{\lambda} = \lambda L^{d-\Delta} \sim \text{few}$$

CFT_{IR}

$(1) \cdot |\lambda|^{\frac{1}{d-\Delta}} \sim 1/\xi$

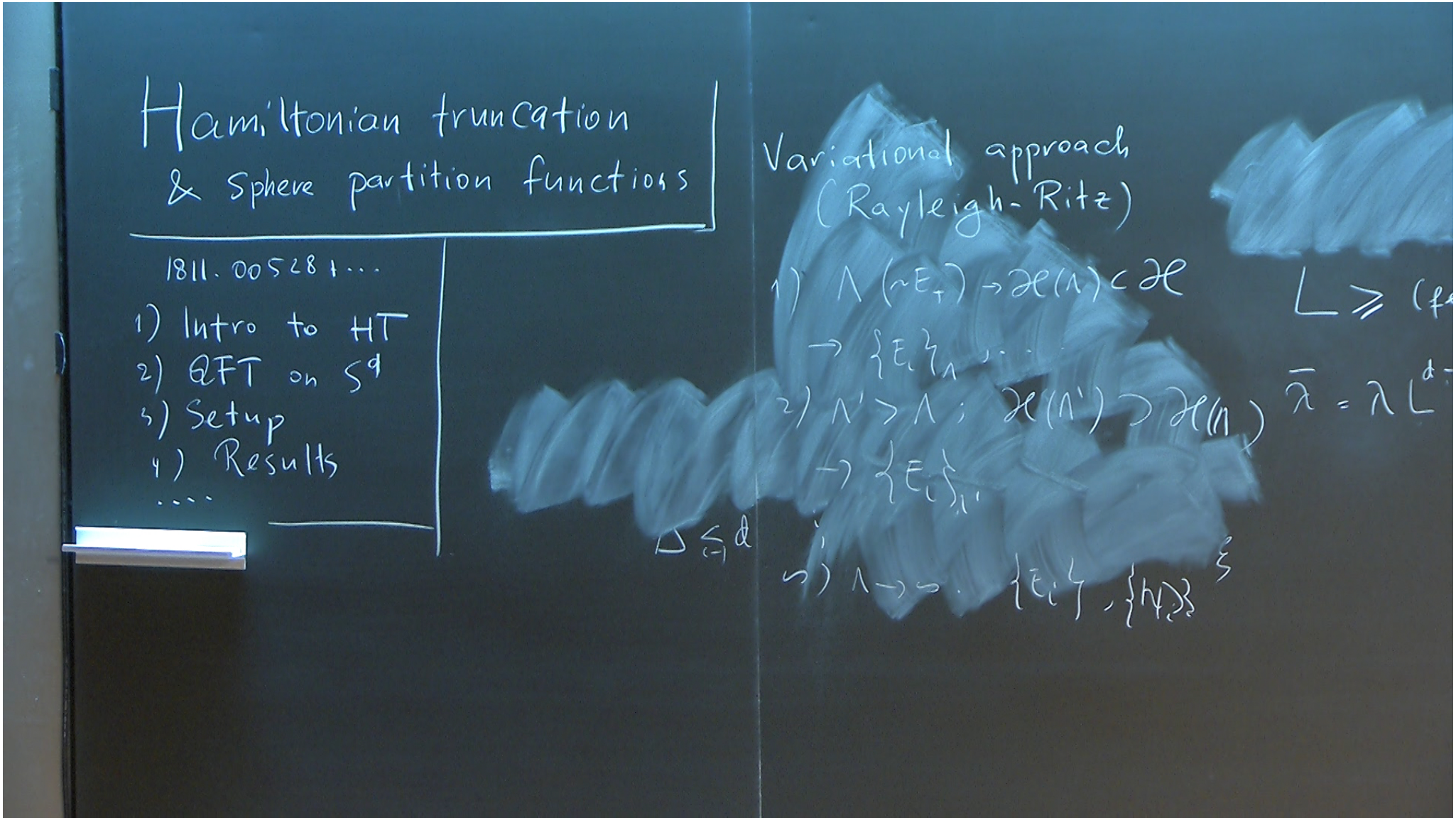
HT: $M = \mathbb{R} \times \Sigma_{d-1}$

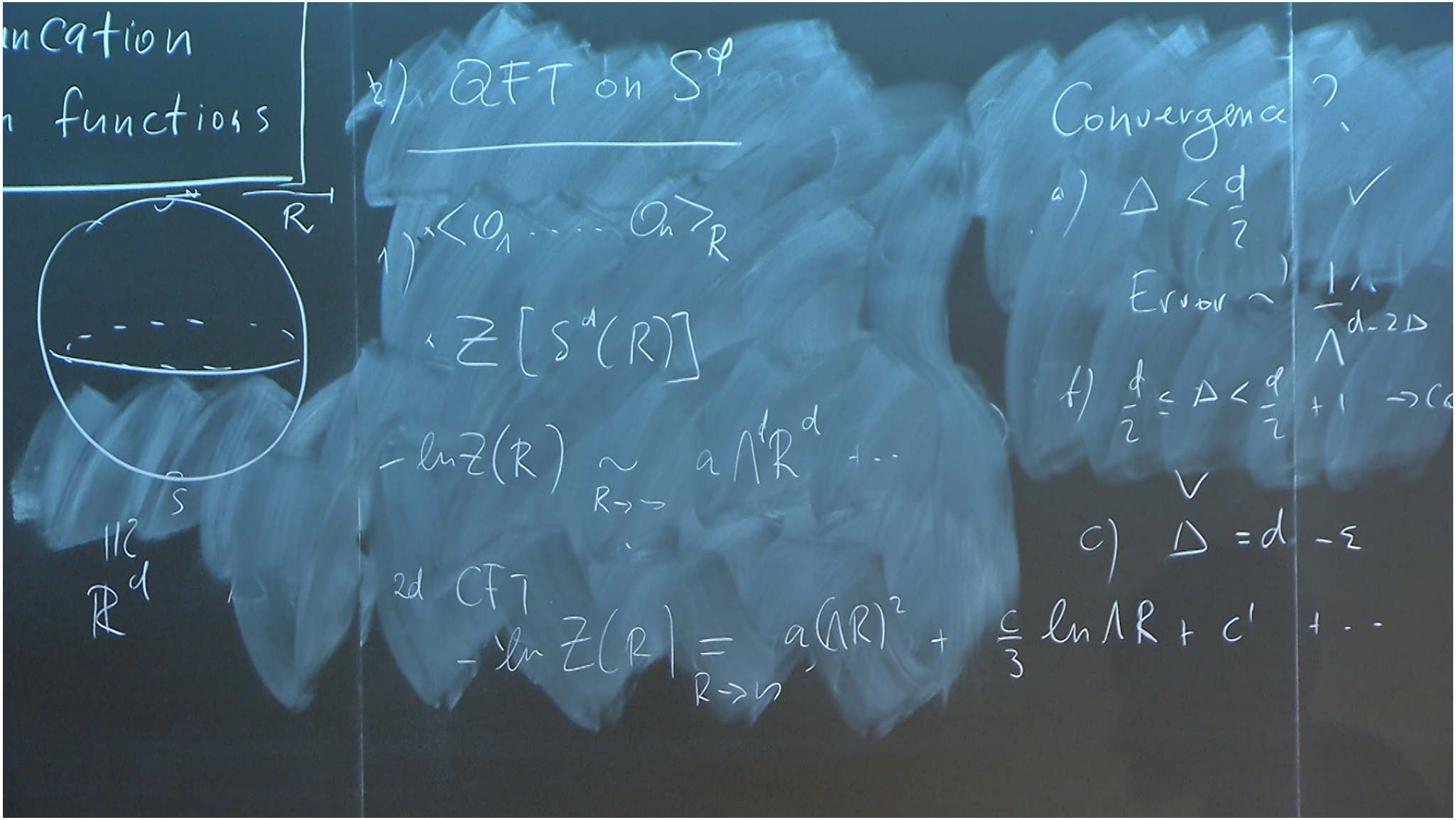
$\uparrow$ t

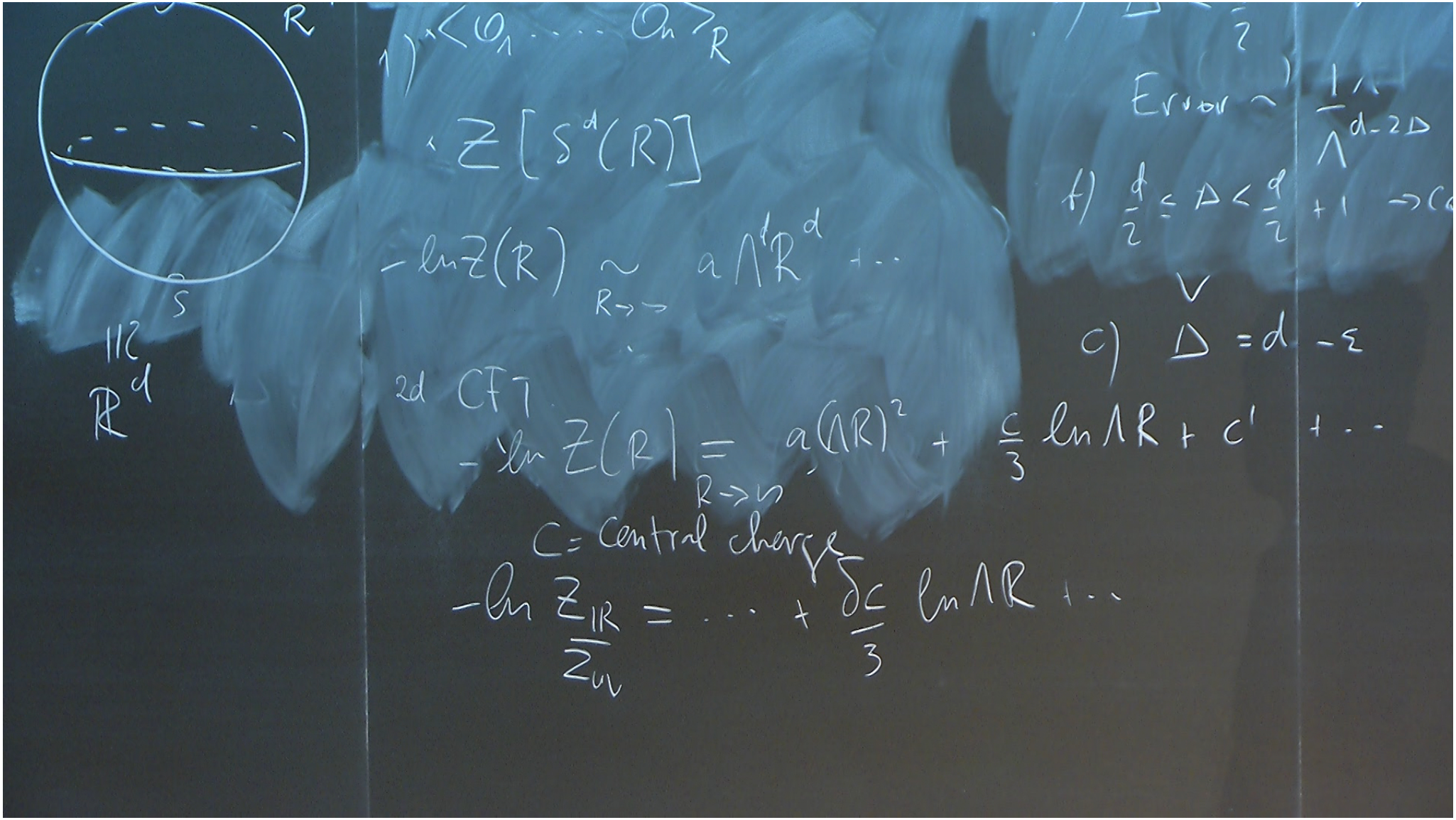
- Hilbert space $\mathcal{H}$.
- Hamiltonian

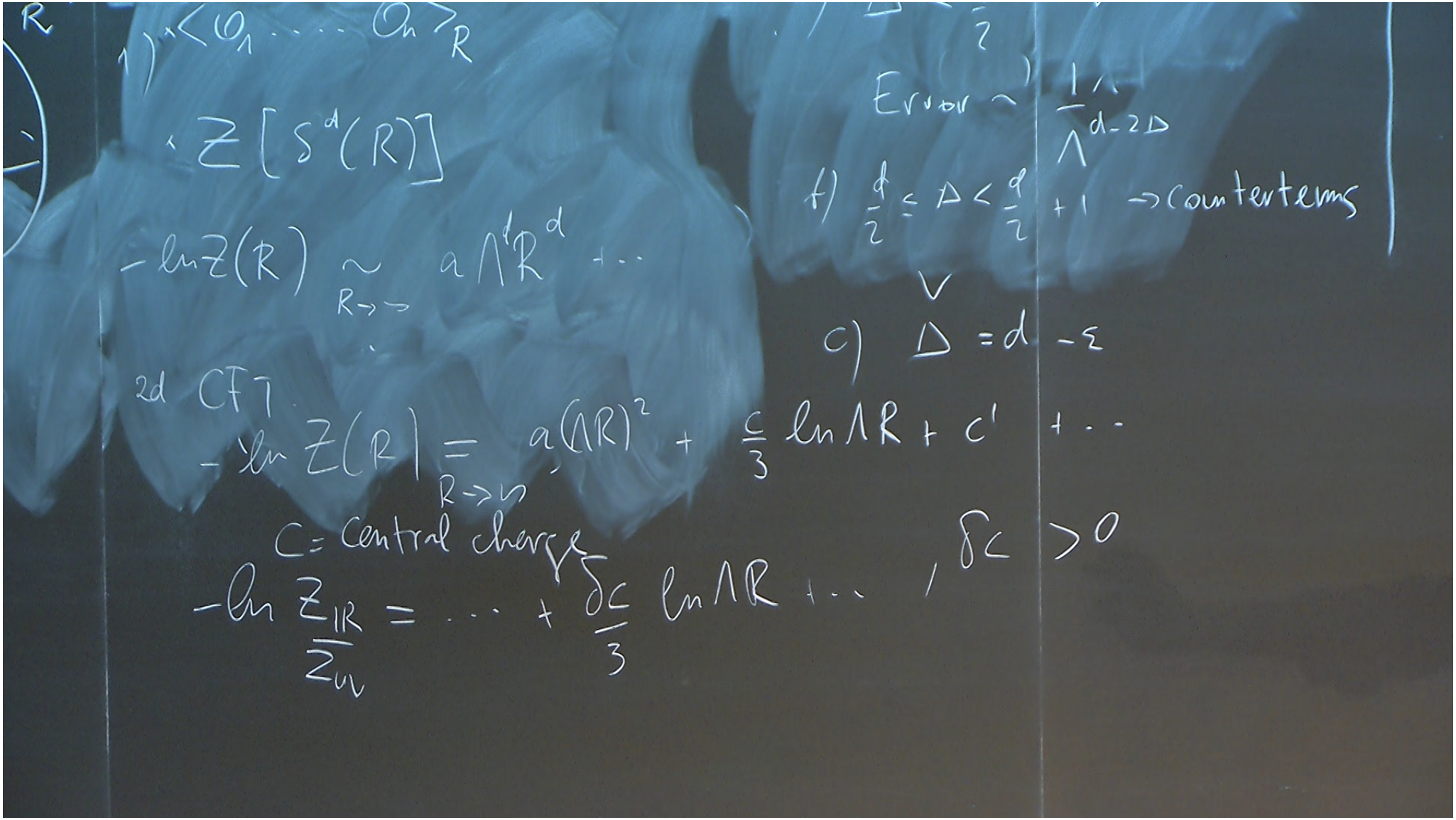
$$H = H_0 + V, \quad V = \lambda \int d^{d-1}x\, \mathcal{O}(x)$$

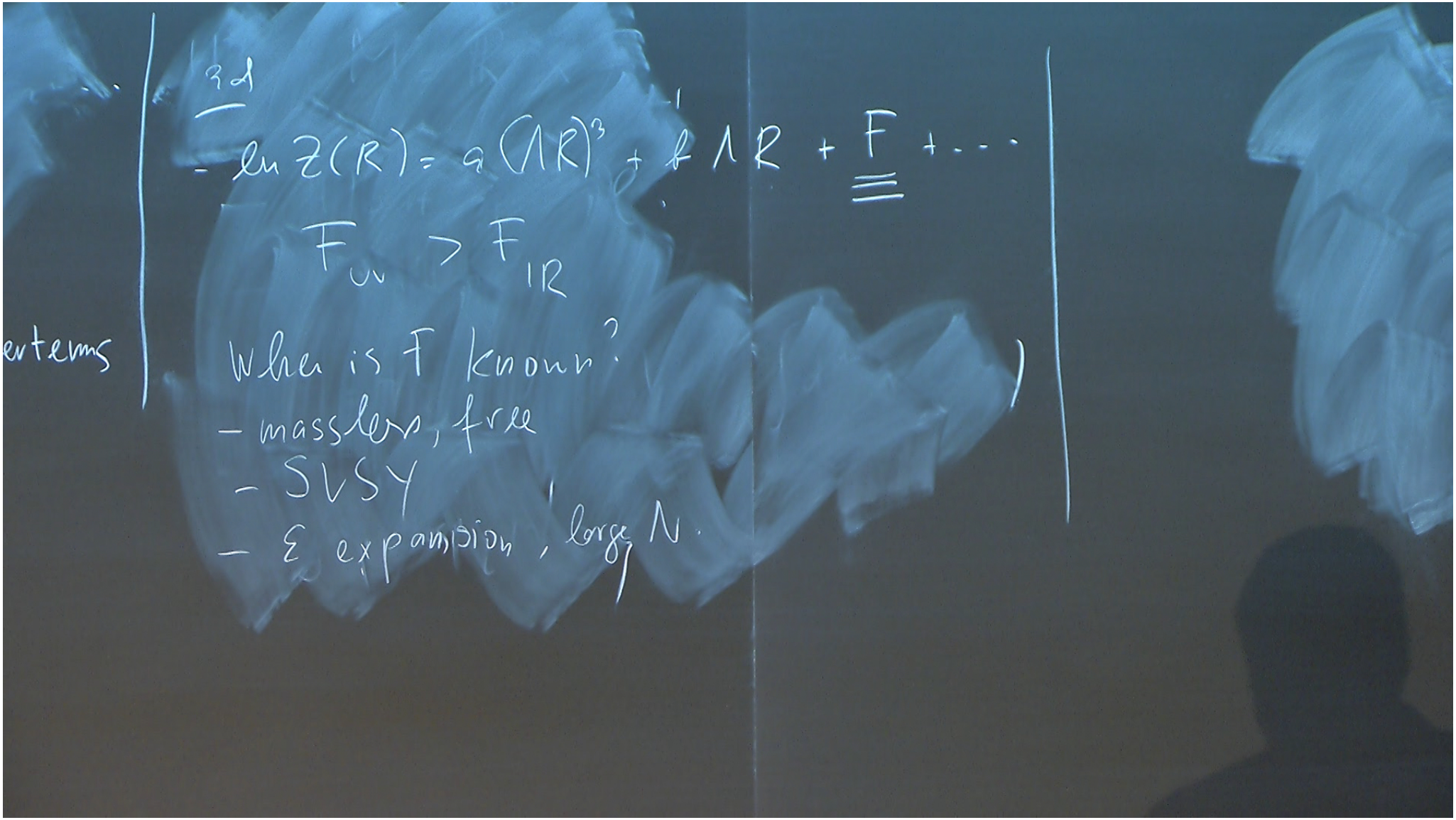
- energies $\{E_i\}$
- eigenstates $\{|\psi_i\rangle\}$


Hamiltonian truncation
& Sphere partition functions
1811.00528 + ...
1) Intro to HT
2) QFT on S^d
3) Setup
4) Results
....
Variational approach
(Rayleigh-Ritz)
1) $\Lambda(\sim E_+) \to \mathcal{H}(\Lambda) \subset \mathcal{H}$
$\to \{E_i\}_\Lambda$ ...
2) $\Lambda' > \Lambda$; $\mathcal{H}(\Lambda') \supset \mathcal{H}(\Lambda)$
$\to \{E_i\}_{\Lambda'}$
$L \geq$ (f
$\bar{\lambda} = \lambda L^d$
$\Delta \leq$ d
3) $\Lambda \to \infty$. $\{E_i\}$. $\{H_{ij}\}$ ξ


nCation
functions
R
S
$\mathbb{R}^d$
2) QFT on S^d
1) $\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle_R$
$Z[S^d(R)]$
$-\ln Z(R) \underset{R\to\infty}{\sim} a \Lambda^d R^d + \dots$
2d CFT
$-\ln Z(R) \underset{R\to\infty}{=} a(\Lambda R)^2 + \frac{c}{3}\ln \Lambda R + c' + \dots$
Convergence ?
a) $\Delta < \frac{d}{2}$ ✓
Error $\sim \frac{1}{\Lambda^{d-2\Delta}}$
f) $\frac{d}{2} \le \Delta < \frac{d}{2} + 1 \to$ (c
c) $\Delta = d - \varepsilon$


R
S
$\mathbb{R}^d$
$-\ln Z(R) \underset{R\to\infty}{\sim} a\Lambda^d R^d + \dots$
2d CFT
$-\ln Z(R) = a(\Lambda R)^2 + \frac{c}{3}\ln\Lambda R + c' + \dots$
c = central charge
$-\ln \frac{Z_{IR}}{Z_{UV}} = \dots + \frac{\delta c}{3}\ln\Lambda R + \dots$
Error
$\frac{1}{\Lambda^{d-2\Delta}}$
f) $\frac{d}{2} \le \Delta < \frac{d}{2}+1$
c) $\Delta = d - \varepsilon$


R
1) $\langle O_1 \dots O_n \rangle_R$
$Z[S^d(R)]$
$-\ln Z(R) \underset{R\to\infty}{\sim} a \Lambda^d R^d + \dots$
2d CFT
$-\ln Z(R) \underset{R\to\infty}{=} a(\Lambda R)^2 + \frac{c}{3} \ln \Lambda R + c' + \dots$
c = central charge
$-\ln \frac{Z_{IR}}{Z_{UV}} = \dots + \frac{\delta c}{3} \ln \Lambda R + \dots$, $\delta c > 0$
Error ~ $\frac{1}{\Lambda^{d-2\Delta}}$
f) $\frac{d}{2} \le \Delta < \frac{d}{2} + 1$ → counterterms
c) $\Delta = d - \varepsilon$


3d
$-\ln Z(R) = a(\Lambda R)^3 + b\Lambda R + F + \dots$
$F_{UV} > F_{IR}$
erterms
When is F known?
- massless, free
- SUSY
- ε expansion, large N.

$+ \int \Lambda R + F + \ldots$

$\equiv$

large N.

a) CFT under control

$CFT_{UV} \to$ empty CFT

?

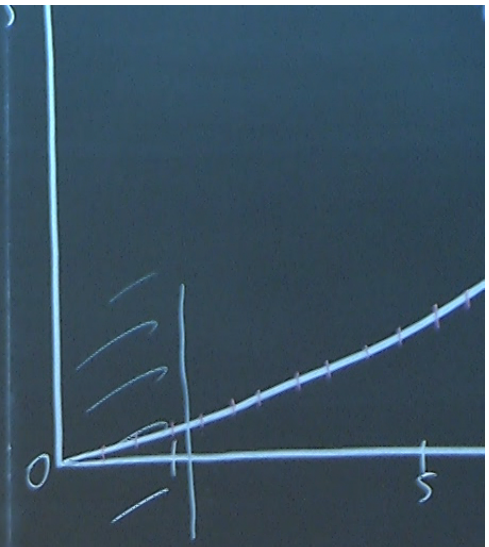
$F=0 \to \delta F = F_{UV}$

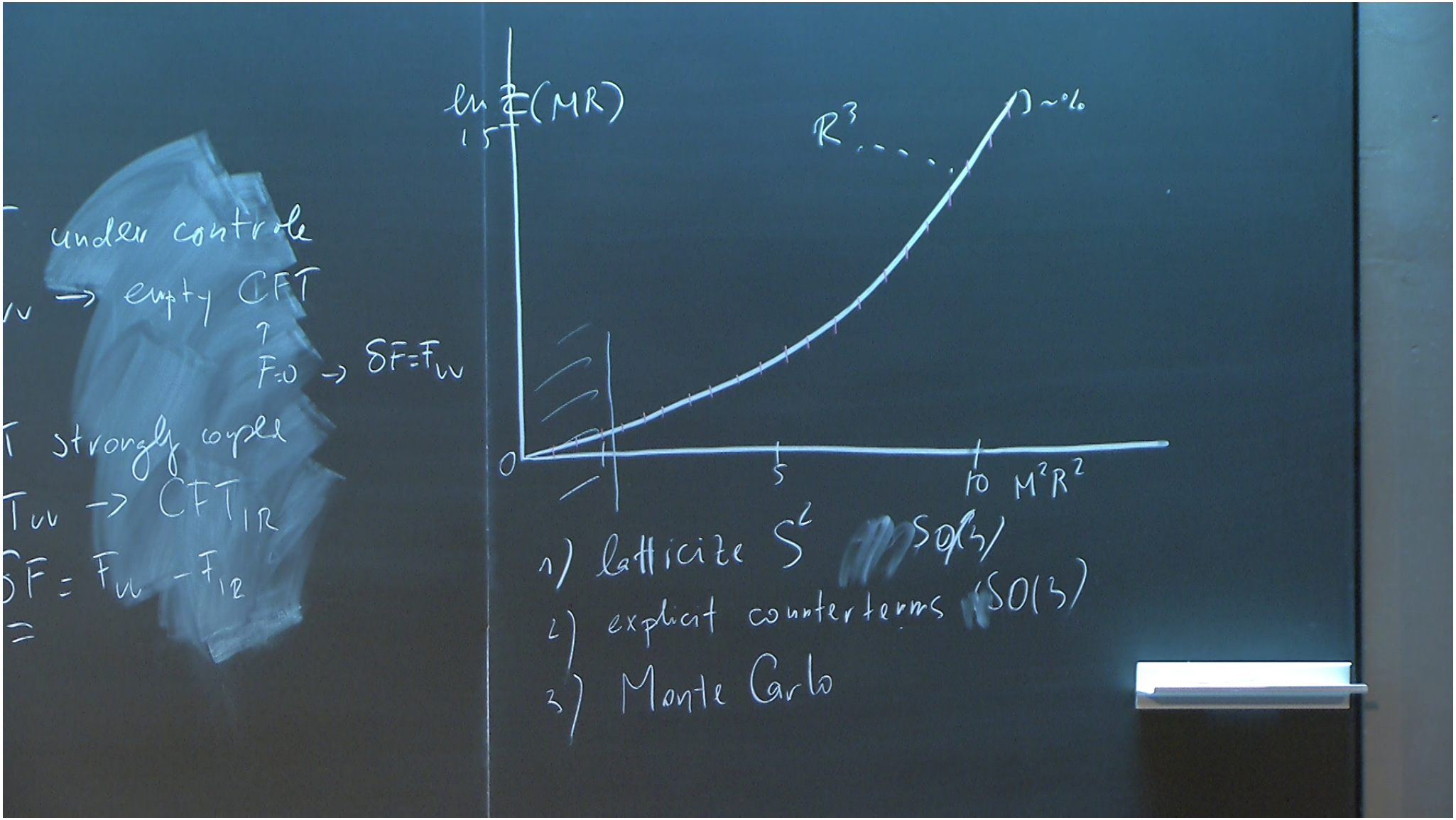
b) CFT strongly coupled

$CFT_{UV} \to CFT_{IR}$

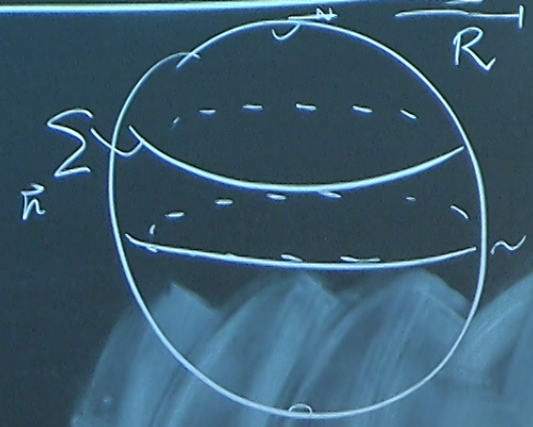
$\delta F = F_{UV} - F_{IR}$

$=$




under control
→ empty CFT
F=0 → δF=F_UV
strongly coupled
T_UV → CFT_IR
δF = F_UV − F_IR
ln Z(MR)
1.5
R³
0
5
10
M²R²
1) lattice S² SO(3)
2) explicit counterterms SO(3)
3) Monte Carlo

rtition functions



$\mathbb{R}^d$

3) HT for S^3?

$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 + \frac{m^2}{2}\varphi^2 + \sum \frac{\lambda_n}{n!}\varphi^n$$

$$ds^2 = \left(\frac{R}{\cosh\tau}\right)^2 \left[d\tau^2 + d\Omega_{d-1}(n)\right]$$

$$[a, a^\dagger) = 1$$

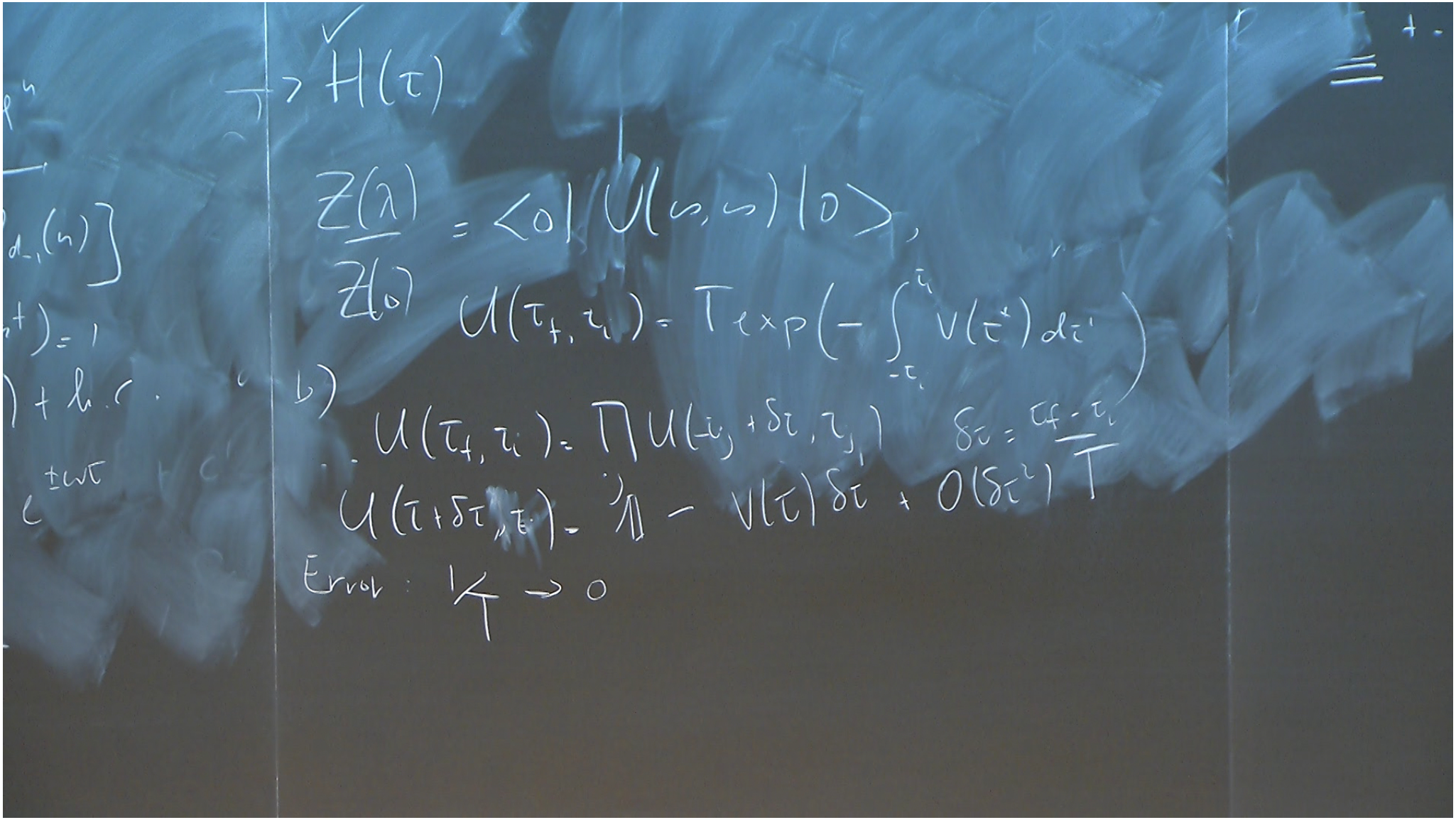
$$\varphi(\tau, \vec{n}) = \sum_{\text{spins } s} a_s \, Y_s(\vec{n}) \, K_s(\tau) + h.c.$$

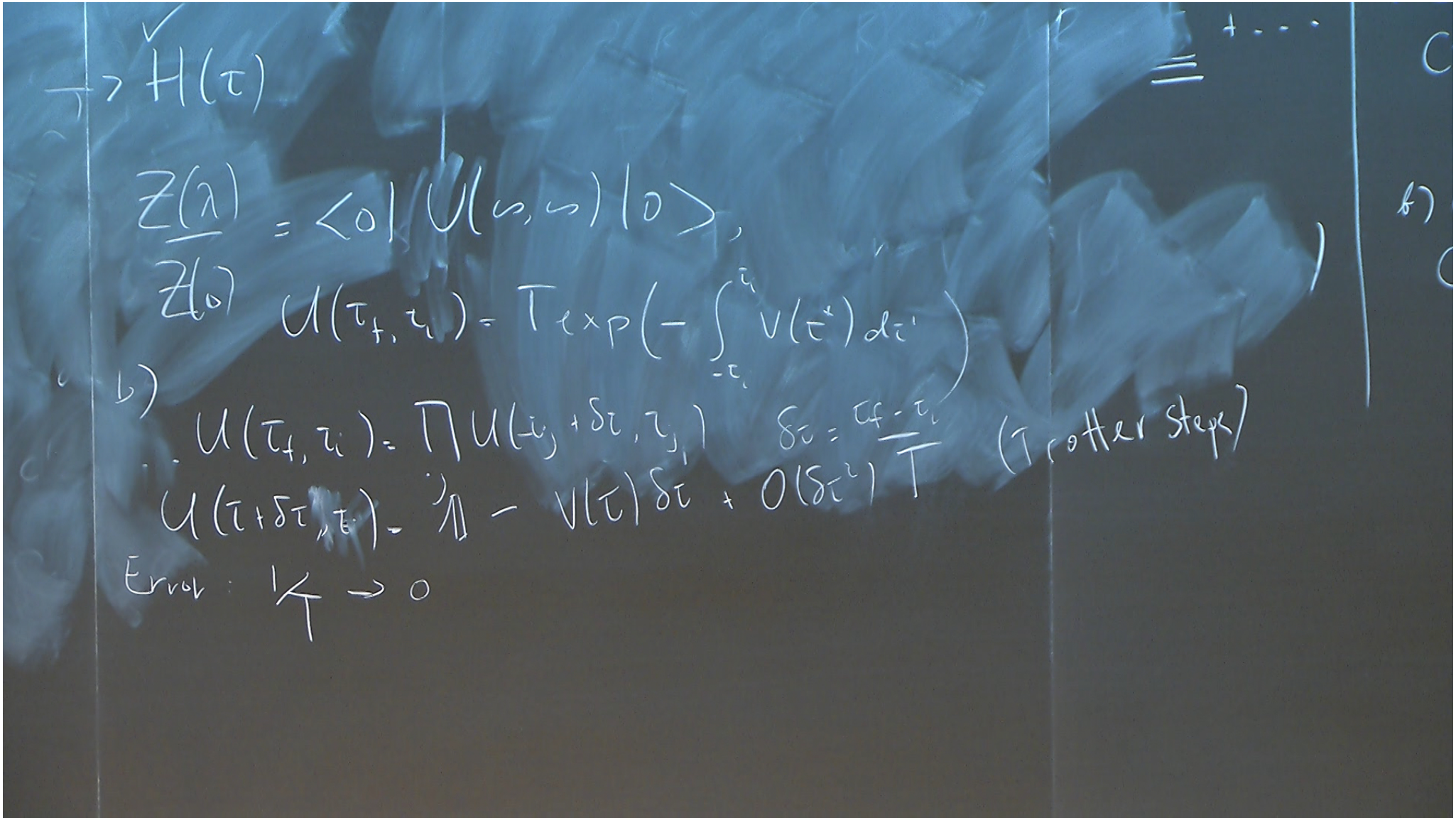
$e^{\pm i\omega\tau}$

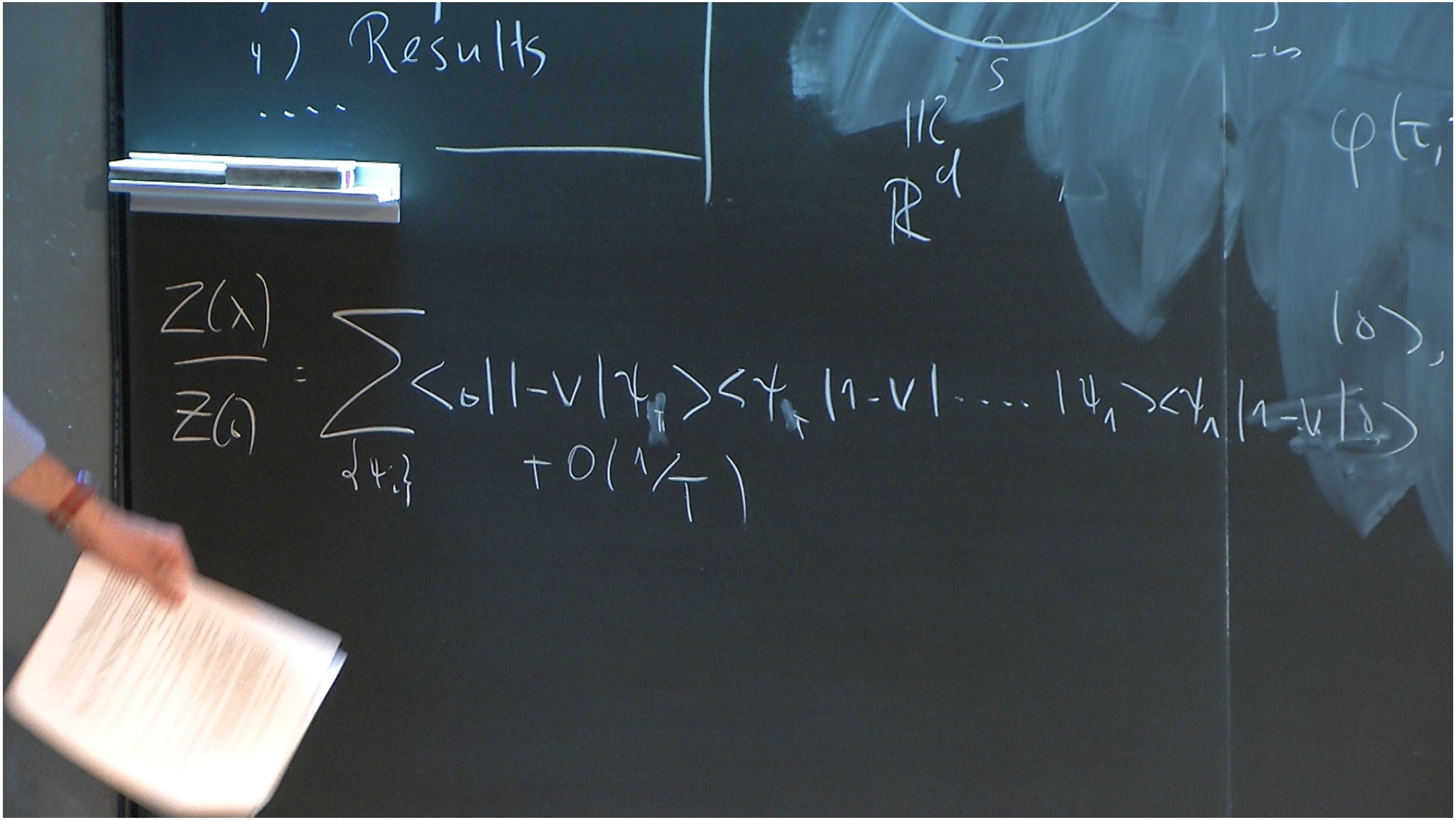
$|0\rangle$

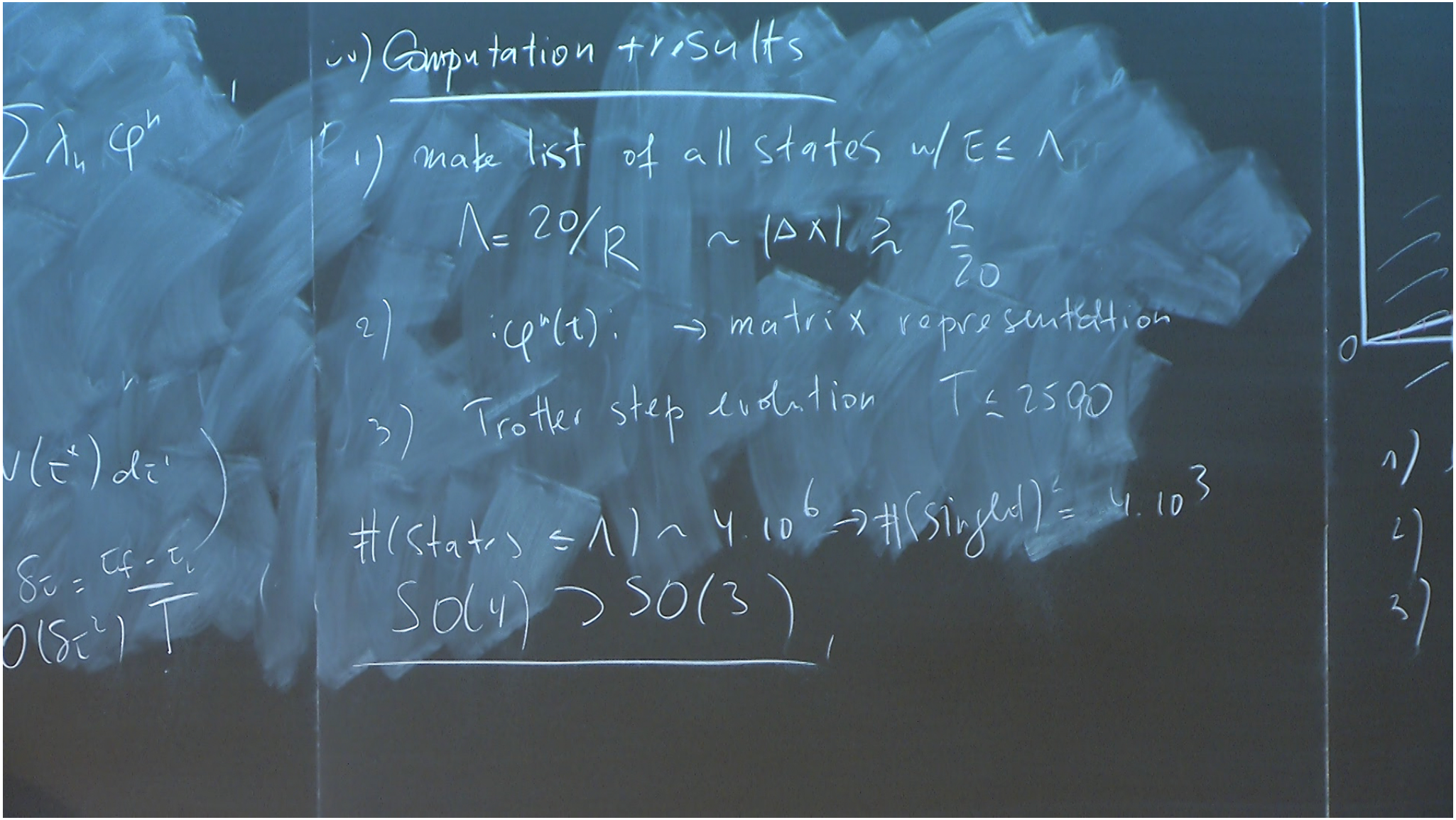
R

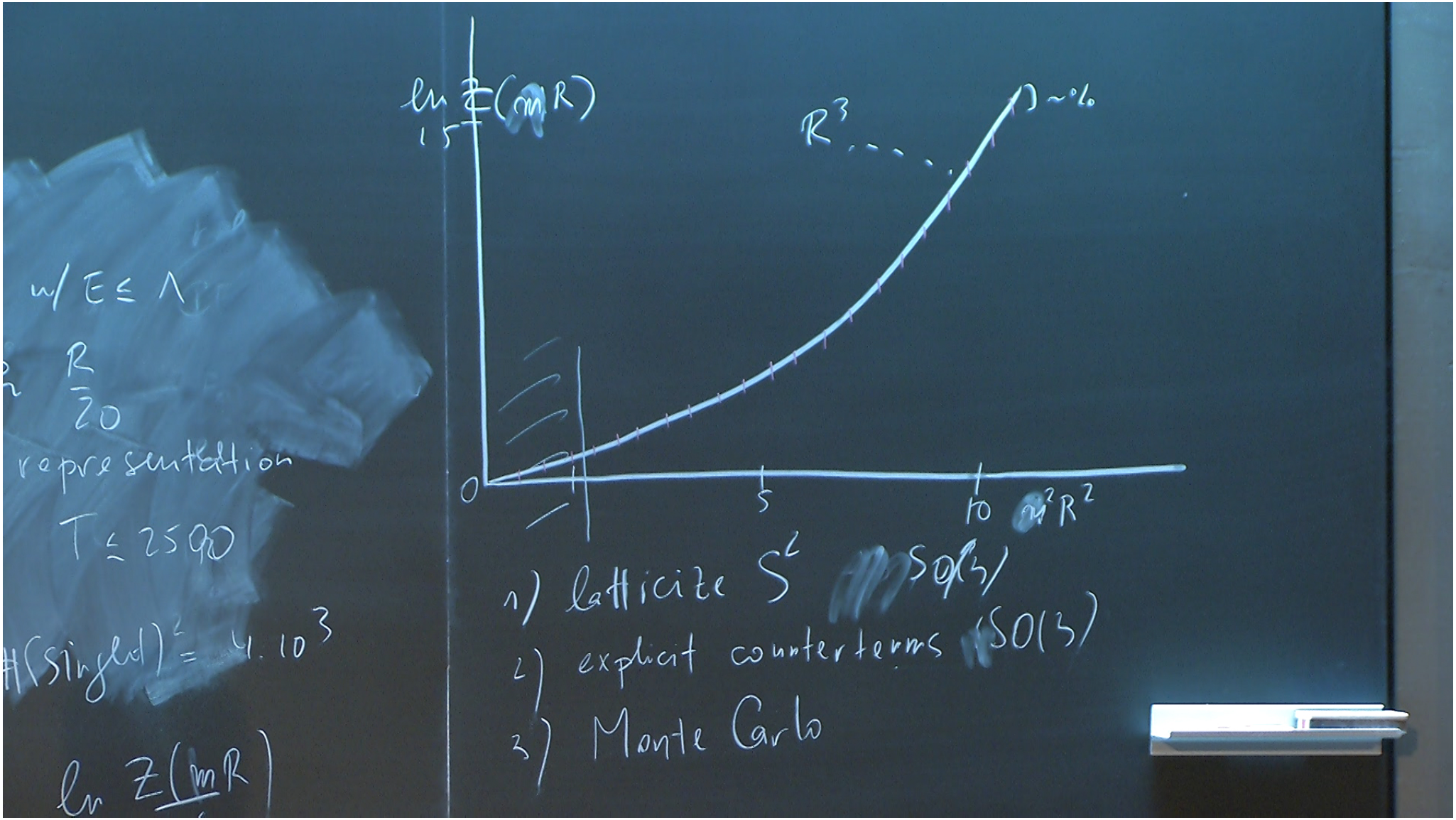
3


$\to H(\tau)$
$Z(\lambda) = \langle 0 | U(\infty,-\infty) | 0 \rangle$,
$Z(0)$
$U(\tau_f,\tau_i) = T\exp\left(-\int_{-\tau_i}^{\tau_f} V(\tau')\,d\tau'\right)$
b)
$U(\tau_f,\tau_i) = \prod U(-\tau_j+\delta\tau,\tau_j)$ $\quad \delta\tau = \frac{\tau_f-\tau_i}{T}$
$U(\tau+\delta\tau,\tau) = \mathbb{1} - V(\tau)\delta\tau + O(\delta\tau^2)$
Error: $\sim \frac{1}{T} \to 0$

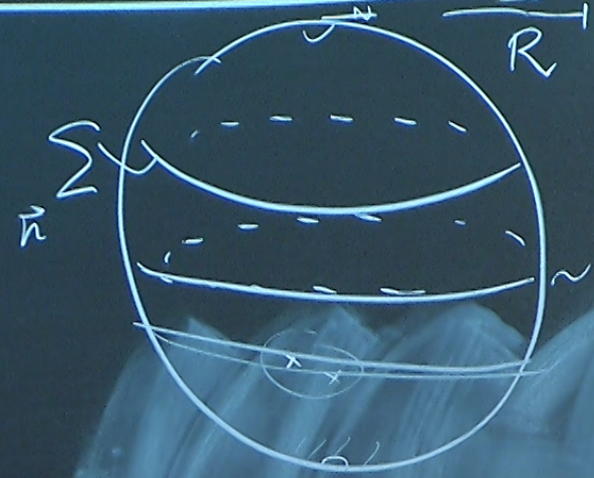

$\hat{H}(\tau)$
$Z(\lambda) = \langle 0 | U(\infty,-\infty) | 0 \rangle$,
$Z(0)$
$U(\tau_f,\tau_i) = T\exp\left(-\int_{-\tau_i}^{\tau_f} V(\tau')\,d\tau'\right)$
b)
$U(\tau_f,\tau_i) = \prod U(\tau_j+\delta\tau,\tau_j)$ $\delta\tau = \frac{\tau_f-\tau_i}{T}$ (T other steps)
$U(\tau+\delta\tau,\tau) = \mathbb{1} - V(\tau)\delta\tau + O(\delta\tau^2)$
Error: $1/T \to 0$
$\equiv$ + . . .
C
b)


4) Results
....
S
$\mathbb{R}^d$
$\varphi(\tau,$
$|\delta\rangle,$
$\frac{Z(\lambda)}{Z(0)} = \sum_{\{\psi_i\}} \langle 0|1-V|\psi_n\rangle\langle\psi_n|1-V|\ldots|\psi_1\rangle\langle\psi_1|1-V|\delta\rangle$
$+O(1/T)$


iv) Computation results
1) make list of all states w/ E ≤ Λ
Λ = 20/R ~ |Δx| ≥ R/20
2) :φ^n(t): → matrix representation
3) Trotter step evolution T ≤ 2590
#(states ≤ Λ) ~ 4.10^6 → #(singlet) = 4.10^3
SO(4) ⊃ SO(3)


ln Z(mR)
15
R³
0
5
10
m²R²
w/ E ∈ Λ
R
20
representation
T ≤ 2590
#(singlet) ≤ 4.10³
ln Z(mR)
1) latticize S² SO(3)
2) explicit counterterms SO(3)
3) Monte Carlo

tition functions



3) HT for S^3?

a) Trotter errors $\frac{1}{T}$

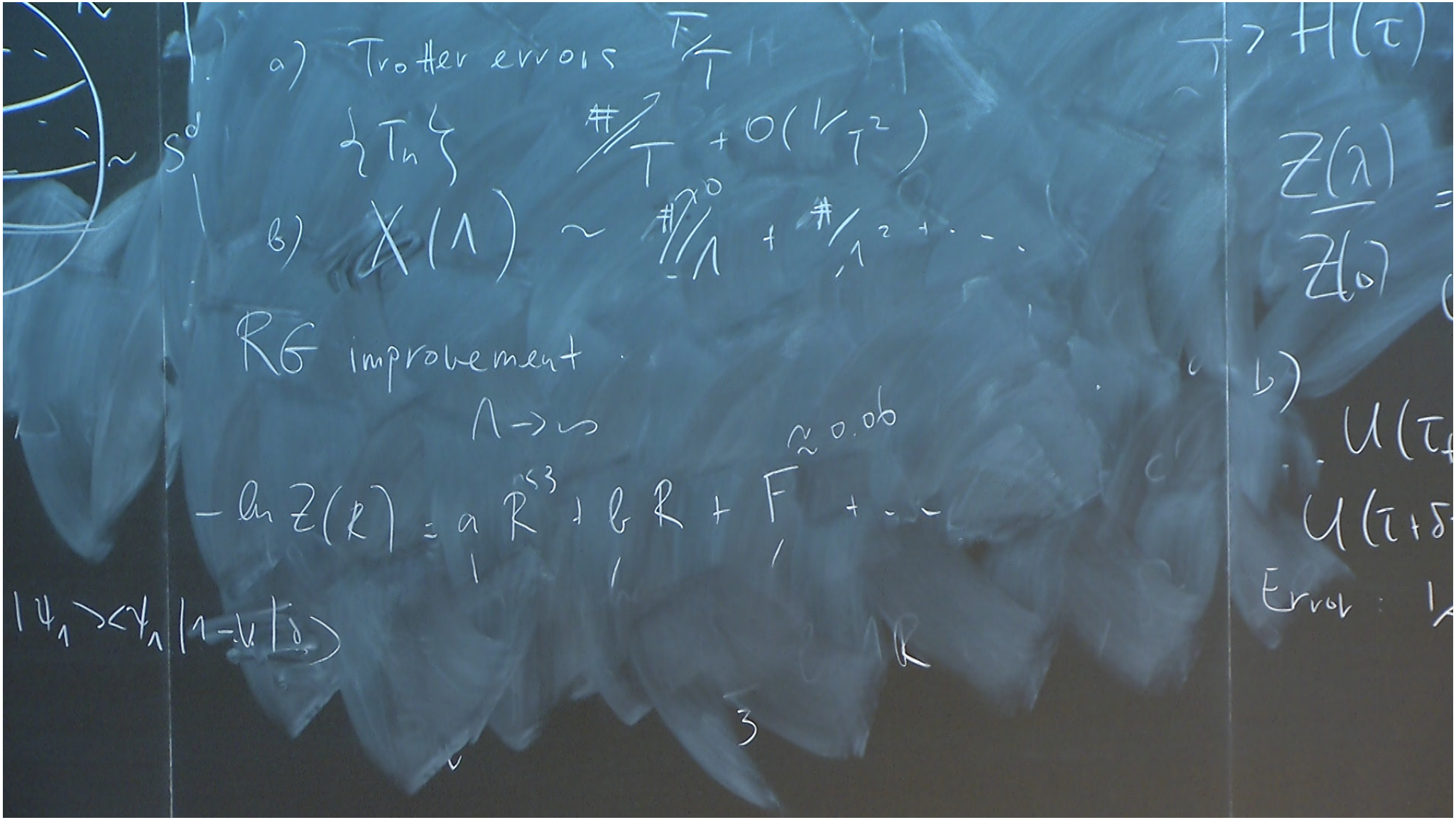
$\{T_n\}$ $\frac{\#}{T} + O(\frac{1}{T^2})$

b) $\chi(\Lambda) \sim \frac{\#}{\Lambda^0} + \frac{\#}{\Lambda^2} + \dots$

RG improvement

$\Lambda \to \infty$

$\langle s_t | 1 - V | \dots | 1 - V | s_0 \rangle \langle s_0 | 1 - V | s_t \rangle$


a) Trotter errors
b)
RG improvement
Error

$\delta\mathcal{L} = g\,\varphi^3$

Landau-Ginsburg → Lee-Yang CFT

$2 \le d < 6$

$\Delta_\varphi[g] = \frac{3}{2} = \frac{d}{2}$ in $d = 3$

$Z[\Lambda] \propto \Lambda$ as $\Lambda \to \infty$

$H_+ \to H_+ + \#\, g^2 \ln \Lambda R + \dots$

> 0

c) Comput

1) make

$\Lambda =$

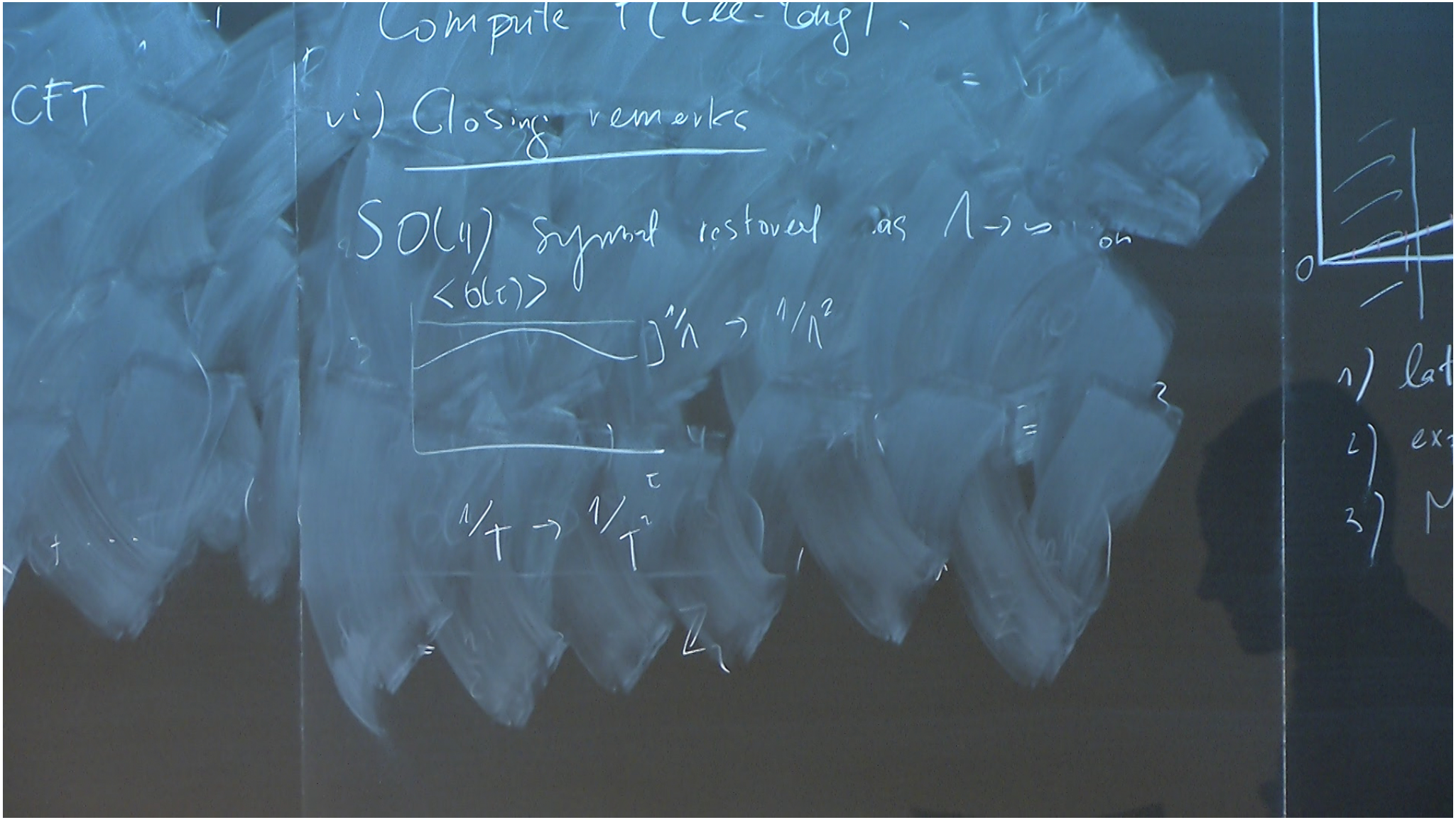
2) $:\varphi$

3) Trot

#(state

SO(4)

$\delta\mathcal{L} = \frac{m^2}{2}$


Compute
CFT
vi) Closing remarks
SO(4) symmetry restored as
<σ(τ)>
1/T → 1/T²
τ
1/Λ → 1/Λ²
0
1)
2)
3)