Title: The MESS and dualities of cosmological perturbations

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Abstract: We introduce two new effective quantities for the study of comoving curvature perturbations Î¶: the space dependent effective sound speed (SESS) and the momentum dependent effective sound speed (MESS) . We use the SESS and the MESS to derive a new set of equations, not involving explicitly entropy or anisotropies, which can be applied to any system described by an effective stress-energy-momentum tensor (EST), including any multi-fields systems, supergravity and modified gravity theories.

The MESS is the natural quantity to parametrize in a model independent way the effects produced on curvature perturbations by multi-fields systems, particle production and modified gravity theories and could be conveniently used in the analysis of LSS observations, such as the ones from the upcoming EUCLID mission or CMB radiation measurements. It can be also useful to study in a model independent way the production of primordial black holes.

Beside the degeneracy due to different theoretical scenarios producing the same MESS, we show that in absence of entropy or effective anisotropic stress there is an additional degeneracy related to the freedom in the choice of the initial energy scale of inflation, or to the sign of the Hubble parameter. This implies the existence of an infinite family of dual slow-roll parameters histories which can produce the same spectrum of comoving curvature perturbations, implying that in general there is no one-to-one correspondence between the spectrum and higher order correlation functions. Bounce models are examples of the members of this infinite class of dual models.

The combined analysis of data from future CMB and gravitational wave experiments could allow to distinguish between dual models because the primordial tensor perturbations spectra of dual models are in general different.

The MESS and dualities of cosmological perturbations

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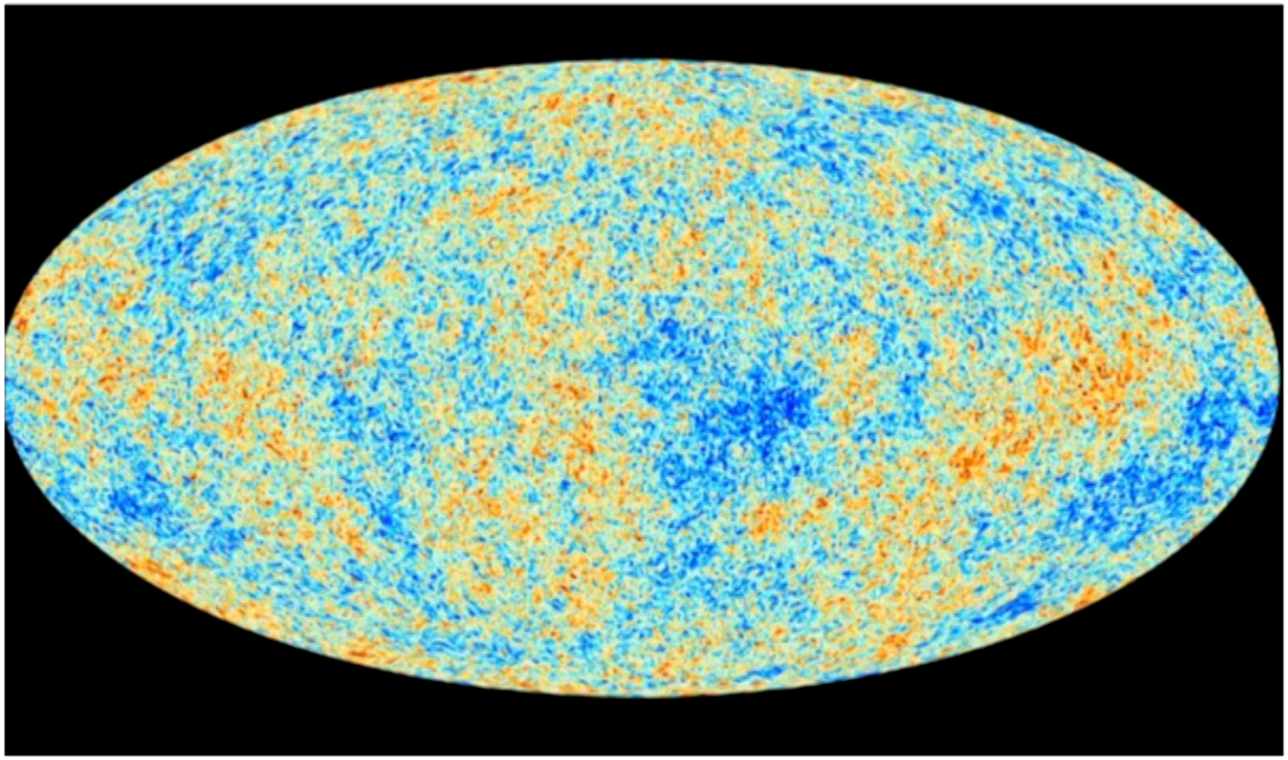
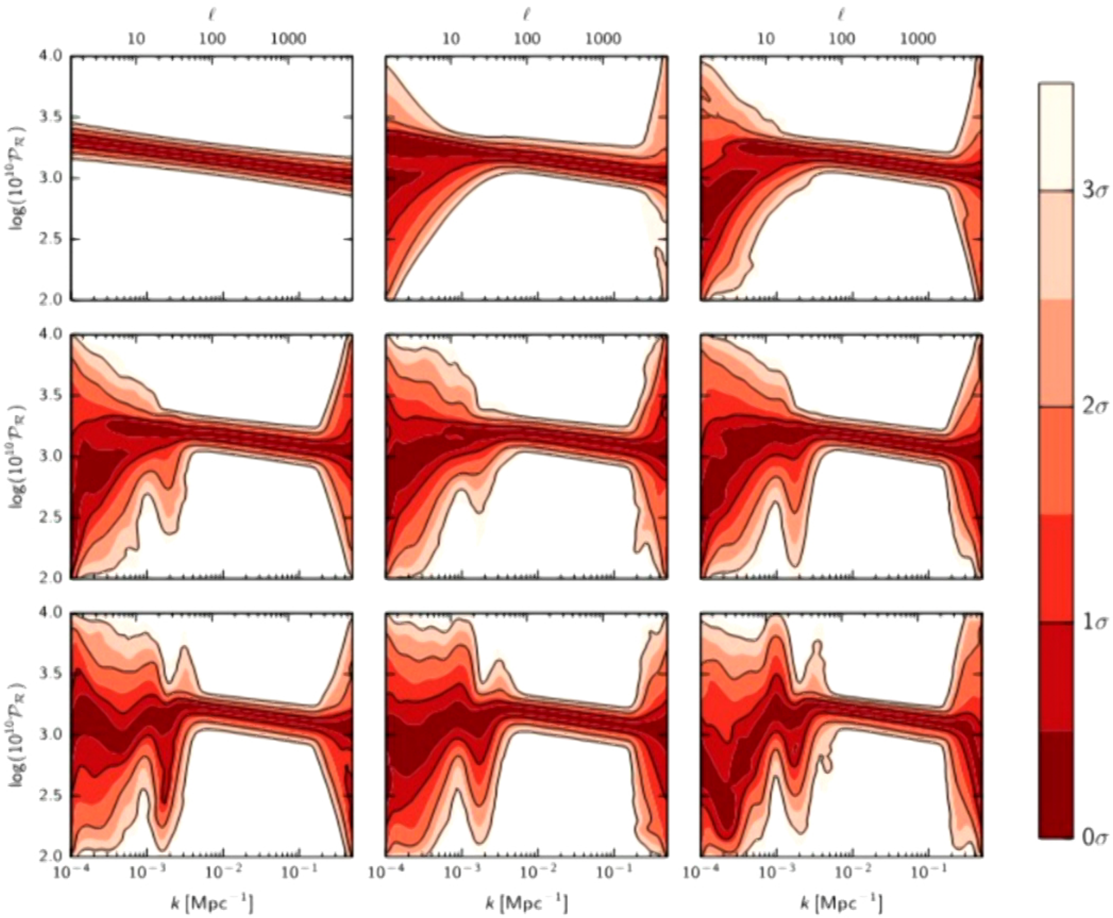
Planck CMB temperature anisotropy map



The CMB leaves room from deviations from a power law spectrum,
Planck 2015 results. XX. Constraints on inflation



What can cause these features?

- Similar features, but on other scales, in the spectrum of primordial curvature perturbations could also cause **PBH** production which have been claimed to be within the LIGO observable range, and could also affect **LSS**.

These features can be due to several different causes such as:

- Multi-fields
- Slow-roll violation in single field
- Modification of gravity
- A combination of the above

Despite their apparent difference do all these phenomena share something?

Yes ..

SESS and MESS

SESS

Space dependent effective sound speed

$$v_s^2(t, x^i) \equiv \frac{\alpha(t, x^i)}{\beta(t, x^i)}$$

Ratio of pressure to density perturbations
In the comoving gauge

In absence of anisotropies, from the perturbed Einstein's equations we get a completely general equation

$$\partial_t \left(\frac{a^3 \epsilon}{v_s^2} \dot{\zeta} \right) - a\epsilon \overset{(3)}{\Delta} \zeta = 0$$

$$\ddot{\zeta} + \frac{\partial_t (Z^2)}{Z^2} \dot{\zeta} - \frac{v_s^2}{a^2} \overset{(3)}{\Delta} \zeta = 0 \quad , \quad Z^2 = \frac{\epsilon a^3}{v_s^2}$$

MESS

Momentum dependent effective sound speed

$$\tilde{v}_k^2(t) \equiv \frac{\alpha_k(t)}{\beta_k(t)}$$

$$\ddot{\zeta}_k + \frac{\partial_t(\tilde{Z}_k^2)}{\tilde{Z}_k^2}\dot{\zeta}_k + \frac{\tilde{v}_k^2}{a^2}k^2\zeta_k = 0 \quad , \quad \tilde{Z}_k^2 = \epsilon a^3/\tilde{v}_k^2$$

$$u_k \equiv \tilde{Z}_k \zeta_k$$

$$\ddot{u}_k + \left(\tilde{v}_k^2 k^2 - \frac{\ddot{\tilde{Z}}_k}{\tilde{Z}_k}\right) u_k = 0$$

Some notation for scalar perturbations : No gauge fixing

$$ds^2 = -(1+2A)dt^2 + 2a\partial_i B dx^i dt + $$
$$+ a^2 \{\delta_{ij}(1+2C) + 2\partial_i\partial_j E\} dx^i dx^j ,$$

TOTAL energy momentum tensor
Includes any matter, multi-fields,
Vector, scalar fields,
Modified gravity

$$T^0{}_0 = -(\rho + \delta\rho) \quad , \quad T^0{}_i = (\rho + P)\partial_i(v + B)$$
$$T^i{}_j = (P + \delta P)\delta^i{}_j + \delta^{ik}\partial_k\partial_j\Pi - \frac{1}{3}\delta^i{}_j \overset{(3)}{\Delta}\Pi .$$

Comoving slices gauge :

$$(T^0{}_i)_c = 0 \longrightarrow \alpha = \delta P_c, \beta = \delta\rho_c, \gamma = A_c, \mu = B_c, \zeta = C_c, \nu = E_c$$

$$ds^2 = -(1+2\gamma)dt^2 + 2a\partial_i\mu\, dx^i dt +$$
$$+ a^2 \{\delta_{ij}(1+2\zeta) + 2\partial_i\partial_j\nu\} dx^i dx^j$$
$$(T^0{}_0)_c = -(\rho + \beta) \quad , \quad (T^i{}_j)_c = (P + \alpha)\delta^i{}_j$$

Standard definitions of entropy in the comoving gauge and uniform density gauge

$$\delta P_u = c_w(t)^2 \delta\rho + \delta P^{nad}$$

$c_w^2 = P'/\rho'$ Adiabatic sound speed

$$\delta P_c = c_s(t)^2 \delta\rho_c + \delta P_c^{nad}$$

Comoving curvature pertubation sound speed

$$\alpha(t, x^i) = c_s(t)^2 \beta(t, x^i) + \Gamma(t, x^i)$$

But ….the one in the comoving gauge it is not unique !

$$c_s^2 \to \tilde{c}_s(t)^2 = c_s(t)^2 + \Delta c_s(t)^2,$$

$$\Gamma \to \tilde{\Gamma}(t, x^i) = \Gamma(t, x^i) - \Delta c_s(t)^2 \beta(t, x^i)$$

Comparing it to the SESS we can get the relation between them

$$v_s^2(t, x^i) \equiv \frac{\alpha(t, x^i)}{\beta(t, x^i)}$$

$$\alpha(t, x^i) = c_s(t)^2 \beta(t, x^i) + \Gamma(t, x^i)$$

$$\dot{\zeta} = -\frac{1}{2H\epsilon}\left(\alpha + \frac{2}{3}\overset{(3)}{\Delta}\Pi\right)$$

Relation of SESS with entropy and anisotropy

In presence of anisotropies the definitions of SESS is the same and the relation with entropy is

$$v_s^2 = c_s^2 \left(1 - \frac{\Gamma}{\alpha}\right)^{-1}$$

Using the Einstein's equations $$\dot{\zeta} = -\frac{1}{2H\epsilon}\left(\alpha + \frac{2}{3}\overset{(3)}{\Delta}\Pi\right)$$

We can make explicit the relation with anisotropy

$$v_s^2 = c_s^2 \left[1 + \frac{\Gamma}{2H\epsilon\left(\dot{\zeta} + \frac{1}{3H\epsilon}\overset{(3)}{\Delta}\Pi\right)}\right]^{-1}$$

The most general equation has two source terms related to anisotropy, but no explicit entropy

SESS

$$\dot{\zeta} = -\frac{v_s^2}{a^2 H\epsilon}\overset{(3)}{\Delta}\Psi_B - \frac{1}{3H\epsilon}\overset{(3)}{\Delta}\Pi$$

Without SESS

$$\dot{\zeta} = -\frac{c_s^2}{a^2 H\epsilon}\overset{(3)}{\Delta}\Phi_B - \frac{\Gamma}{2H\epsilon} - \frac{1}{3H\epsilon}\overset{(3)}{\Delta}\Pi$$

$$\ddot{\zeta} + \frac{\partial_t(Z^2)}{Z^2}\dot{\zeta} - \frac{v_s^2}{a^2}\overset{(3)}{\Delta}\zeta + \frac{v_s^2}{\epsilon}\overset{(3)}{\Delta}\Pi + \frac{1}{3Z^2}\partial_t\left(\frac{Z^2}{H\epsilon}\overset{(3)}{\Delta}\Pi\right) = 0.$$

$$\ddot{\zeta} + \frac{\partial_t z^2}{z^2}\dot{\zeta} - \frac{c_s^2}{a^2}\overset{(3)}{\Delta}\zeta + \frac{c_s^2}{\epsilon}\overset{(3)}{\Delta}\Pi + \frac{1}{z^2}\partial_t\left[\frac{a^3}{c_s^2 H}\left(\Gamma + \frac{2}{3}\overset{(3)}{\Delta}\Pi\right)\right] = 0$$

The first and second order equations are obtained using the following important relations, obtained from Manipulating the Einstein's equations in the comoving gauge. The Poisson eq. Is more used in the modified gravity theories literature,

$$\frac{1}{a^2}\overset{(3)}{\Delta}\Psi_B = \frac{1}{2}\beta \qquad \zeta = -\Psi_B + \frac{H^2}{\dot{H}}\left(\Phi_B + H^{-1}\dot{\Psi}_B\right) \qquad \dot{\zeta} = -\frac{1}{2H\epsilon}\left(\alpha + \frac{2}{3}\overset{(3)}{\Delta}\Pi\right)$$

The difference between the uniform density field and the comoving gauge

The uniform density field (aka "unitary") is in general different from the comoving gauge
They coincide for K(X) – inflation, but not for Horndesky theory or multi-fields systems

$$v + B \to v + B - \delta t$$ ► $$\delta t_c = v + B$$

We can now define explicitly gauge invariant quantities:
comoving pressure perturbation α
comoving density perturbation β
comoving curvature perturbation ζ

$$\begin{aligned}
\alpha &= \delta P + \dot{P}\delta t_c \quad , \quad \beta = \delta\rho + \dot{\rho}\delta t_c \,, \\
\gamma &= A + \dot{\delta t}_c \quad , \quad \mu = B - a^{-1}\delta t_c \,, \\
\sigma &= a\dot{E} - B + a^{-1}\delta t_c = a\dot{\nu} - \mu \,, \\
\zeta &= C - H\delta t_c \,.
\end{aligned}$$

Einstein's equations in the comoving gauge

$$\frac{1}{a^2}\overset{(3)}{\Delta}\left[-\zeta + aH\sigma\right] = \frac{\beta}{2}\,,$$

$$\gamma = \frac{\dot{\zeta}}{H}\,,$$

$$-\ddot{\zeta} - 3H\dot{\zeta} + H\dot{\gamma} + (2\dot{H} + 3H^2)\gamma = \frac{\alpha}{2}\,,$$

$$\dot{\sigma} + 2H\sigma - \frac{\gamma + \zeta}{a} = 0\,,$$

How general is this equation?

- SESS reduces to the standard definition of sound speed for single field K(X) theories
- It is a space dependent quantity which effectively reproduces the effects of the source terms in the EOM which in the standard formulation are associated to entropy perturbations
- Given the generality of the assumptions this formulation is valid for any system for which an energy momentum tensor can be defined, including multi-fields systems or modified gravity theories (MGT)
- It is also valid for MGT, after writing the MGT field equations as Einstein's equations with an appropriate definition of an effective energy momentum tensor

Example: 2 minimally coupled scalar fields

$$L = \sum_n^N X_n + 2V(\Phi_n) \qquad X_n = g^{\mu\nu}\partial_\mu \Phi_n \partial_\nu \Phi_n \qquad \Phi_n(x^\mu) = \phi_n(t) + \delta\phi_n(x^\mu)$$

$$\delta T^0{}_0 = -\dot\phi\dot{\delta\phi} - \dot\psi\dot{\delta\psi} + A(\dot\phi^2 + \dot\psi^2) - V_\phi\delta\phi - V_\psi\delta\psi\,,$$

$$\delta T^i{}_j = \delta^i_j\left[\dot\phi\dot{\delta\phi} + \dot\psi\dot{\delta\psi} - A(\dot\phi^2 + \dot\psi^2) - V_\phi\delta\phi - V_\psi\delta\psi\right]$$

$$\delta T^0{}_i = \partial_i\left(-\frac{\dot\phi\delta\phi + \dot\psi\delta\psi}{a}\right)$$

Comoving gauge

$$\dot\phi\widetilde{\delta\phi} + \dot\psi\widetilde{\delta\psi} = 0 \quad \blacktriangleright \quad \widetilde{\delta\phi} = \delta\phi + \dot\phi\delta t\ , \quad \widetilde{\delta\psi} = \delta\psi + \dot\psi\delta t \quad \blacktriangleright \quad \delta t_c = -\frac{\dot\phi\delta\phi + \dot\psi\delta\psi}{\dot\phi^2 + \dot\psi^2}$$

Comoving field perturbations

Comoving pressure and enegy density

$$U_\phi = \delta\phi - \dot\phi\frac{\dot\phi\delta\phi + \dot\psi\delta\psi}{\dot\phi^2 + \dot\psi^2}\,, \quad U_\psi = \delta\psi - \dot\psi\frac{\dot\phi\delta\phi + \dot\psi\delta\psi}{\dot\phi^2 + \dot\psi^2}$$

$$\alpha = \delta P_c = \dot\phi\dot U_\phi + \dot\psi\dot U_\psi - \gamma(\dot\phi^2 + \dot\psi^2) + $$
$$+ (\ddot\phi + 3H\dot\phi)U_\phi + (\ddot\psi + 3H\dot\psi)U_\psi\,,$$

$$\beta = \delta\rho_c = \dot\phi\dot U_\phi + \dot\psi\dot U_\psi - \gamma(\dot\phi^2 + \dot\psi^2) + $$
$$- (\ddot\phi + 3H\dot\phi)U_\phi - (\ddot\psi + 3H\dot\psi)U_\psi\,.$$

We can substitute the gauge invariant comoving fields in the comoving pressure and density perturbations

$$\beta = -\frac{\dot{\zeta}(\dot{\phi}^2+\dot{\psi}^2)}{H} - \frac{\Theta(\dot{\phi}^2+\dot{\psi}^2)}{2}, \alpha = -\frac{\dot{\zeta}(\dot{\phi}^2+\dot{\psi}^2)}{H}$$

$$\Theta = \left(\frac{\delta\phi}{\dot{\phi}} - \frac{\delta\psi}{\dot{\psi}}\right)\frac{\partial}{\partial t}\left(\frac{\dot{\phi}^2-\dot{\psi}^2}{\dot{\phi}^2+\dot{\psi}^2}\right)$$

Note that this quantity is gauge invariant, as expected

$$\left(\frac{\delta\phi_c}{\dot{\phi}} - \frac{\delta\psi}{\dot{\psi}}\right) = \left(\frac{Q_\phi}{\dot{\phi}} - \frac{Q_\psi}{\dot{\psi}}\right) = \left(\frac{U_\phi}{\dot{\phi}} - \frac{U_\psi}{\dot{\psi}}\right)$$

Assuming a classical trajectory of the form $\psi(\phi)$

► $$\Theta = 4\dot{\phi}\frac{\partial\psi}{\partial\phi}\frac{\partial^2\psi}{\partial\phi^2}\left[\left(\frac{\partial\psi}{\partial\phi}\right)^2+1\right]^{-2}\left(\frac{U_\psi}{\dot{\psi}} - \frac{U_\phi}{\dot{\phi}}\right)$$

The SESS is different from cs only when there is a turn in field space

$$v_s^2 = \left(1 + \frac{H\Theta}{2\dot{\zeta}}\right)^{-1}$$

After substituting SESS we get the "standard" source term

$$\dot{\zeta} = \frac{H}{a^2\dot{H}} \overset{(3)}{\Delta} \Phi_B - \frac{1}{2} H\Theta \,,$$

$$\ddot{\zeta} + \frac{\partial_t(z^2)}{z^2}\dot{\zeta} - \frac{1}{a^2} \overset{(3)}{\Delta} \zeta + \frac{1}{z^2}\partial_t \left(\frac{z^2 H\Theta}{2}\right) = 0$$

Generalization to multi-fields

$$\theta_{ij} = \left(\frac{\delta\phi_i}{\dot{\phi}_i} - \frac{\delta\phi_j}{\dot{\phi}_j}\right) \frac{\partial}{\partial t} \left(\frac{\dot{\phi}_i^{\,2} - \dot{\phi}_j^{\,2}}{\sum_i^n \dot{\phi}_i^{\,2}}\right) \,,\ \Theta = \chi_N \sum_{i>j}^{N} \theta_{ij}$$

Single field KGB : intrinsic entropy

$$L_{KG}(\Phi, X) = K(\Phi, X) + G(\Phi, X)\,\Box\Phi$$

$$\alpha = c_s^2(t)\beta + \Gamma^{int}$$

$$v_{KG}^2 = c_s^2 \left(1 + \frac{\Gamma^{int}}{2\epsilon H\dot{\zeta}}\right)^{-1}$$

Multi-field Horndeski : nKGB

$$\Gamma_{NKG} = \sum_i^N \Gamma_i^{int} + \chi_N \sum_{i>j}^N \Gamma_{ij}$$

$$v_{NKG}^2 = c_s^2 \left(1 + \frac{\Gamma_{NKG}}{2\epsilon H\dot{\zeta}}\right)^{-1}$$

Applications: features in primordial curvature spectrum motivated by CMB or PBH

Considering the phenomenological ansatz of time independent MESS we get:

$$z^2 = 2a^2\epsilon \qquad \zeta_k'' + \frac{\partial_\eta(z^2)}{z^2}\zeta_k' + \tilde{v}_k^2 k^2 \zeta_k = 0$$

$$u_k \equiv \tilde{Z}_k \zeta_k \qquad u_k'' + \left(\tilde{v}_k^2 k^2 - \frac{z''}{z}\right) u_k = 0$$

Due to the MESS modes freeze after horizon crossing time, around $\eta_k = -\frac{1}{v_k k}$

This super-horizon evolution is the cause of the features in the spectrum

For example for a multi-fields model with standard kinetic term this super-horizon evolution is attributed to entropy perturbations while in the MESS picture it is just due the difference between the freezing time and the horizon crossing time

Effects of a local momentum variation of the MESS: $\tilde{v}_k = 1 + A_c \exp\left[-\left(\frac{k-k_0}{\sigma}\right)^2\right]$

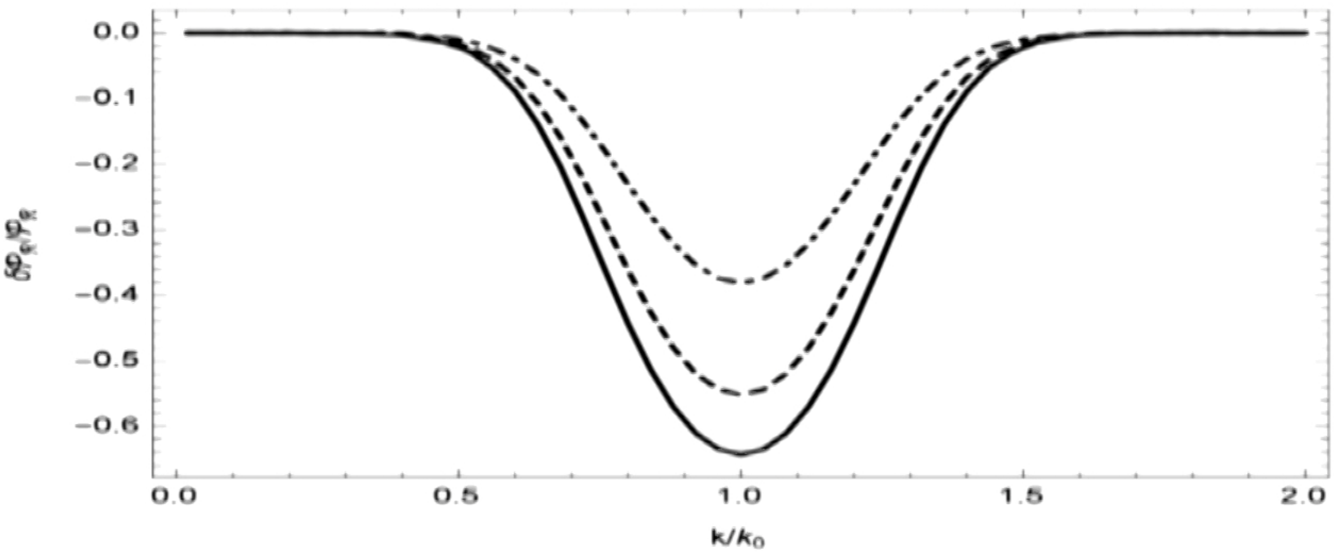
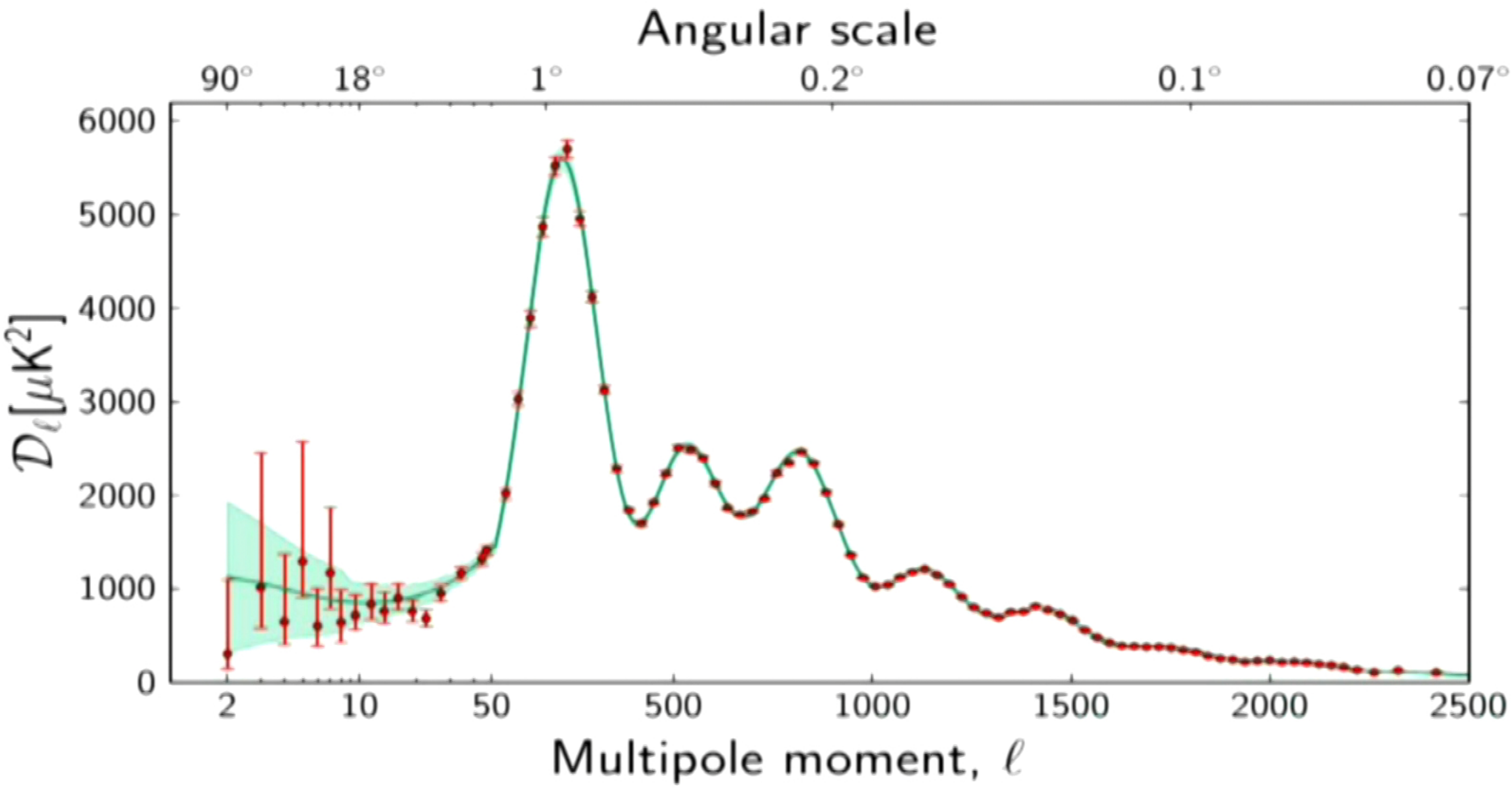
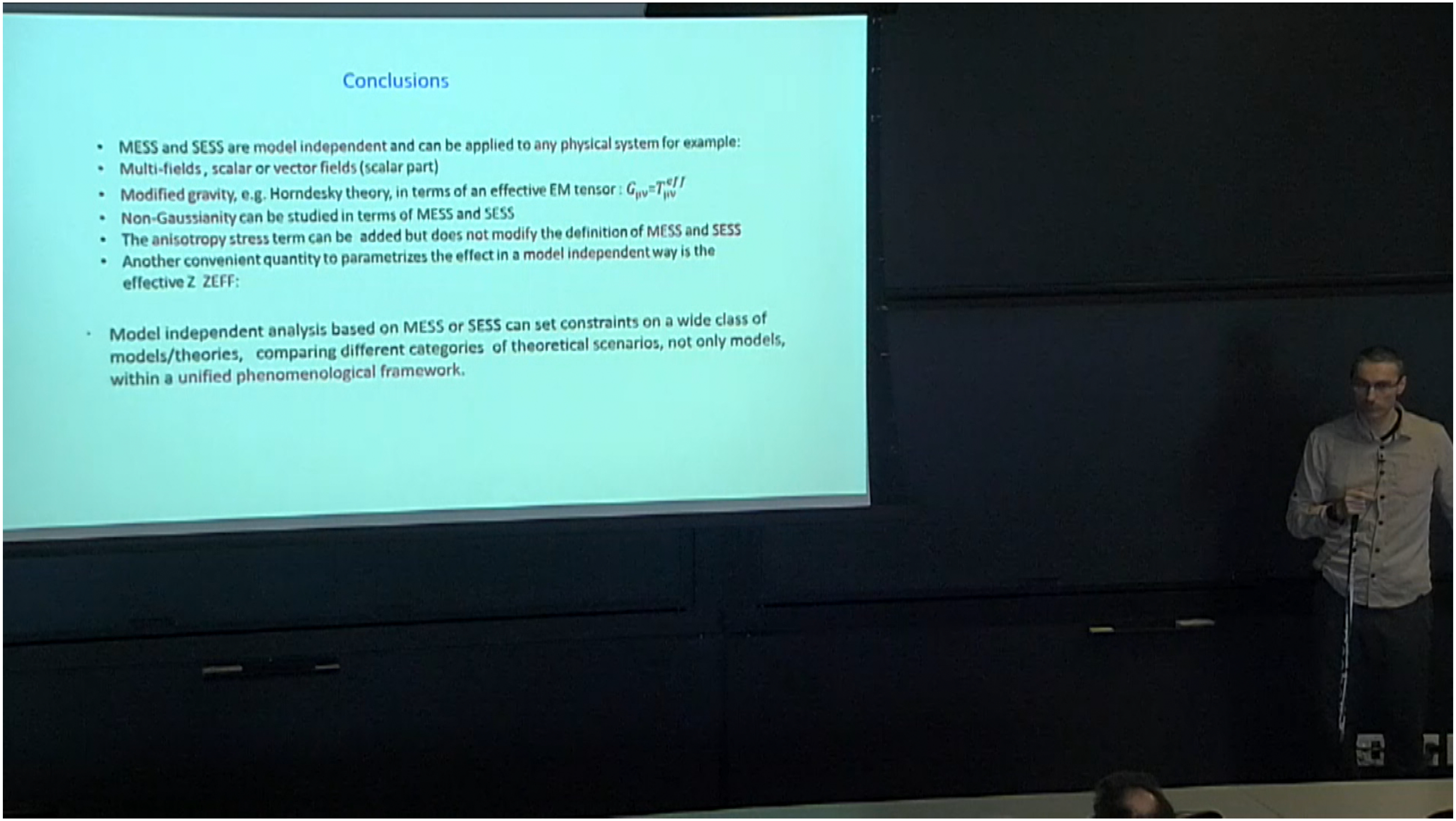


FIG. 1: The relative difference $\Delta\mathcal{P}_\zeta/\mathcal{P}_\zeta$ is plotted as a function of k/k_0. The solid, dashed and dot-dashed lines correspond $\sigma = 2.5 \times 10^{-1} k_0$ and $A_c = 4 \times 10^{-1}$, $A_c = 3 \times 10^{-1}$ and $A_c = 1.7 \times 10^{-1}$ respectively.

The scale k0 could have different origins: turning point in multi-fields modes, particle production, modification of gravity, etc.

CMB anisotropy spectrum : there exists some anomalies which could be explained by MESS




Conclusions
MESS and SESS are model independent and can be applied to any physical system for example:
Multi-fields , scalar or vector fields (scalar part)
Modified gravity, e.g. Horndesky theory, in terms of an effective EM tensor : $G_{\mu\nu}=T^{eff}_{\mu\nu}$
Non-Gaussianity can be studied in terms of MESS and SESS
The anisotropy stress term can be added but does not modify the definition of MESS and SESS
Another convenient quantity to parametrizes the effect in a model independent way is the effective Z ZEFF:
Model independent analysis based on MESS or SESS can set constraints on a wide class of models/theories, comparing different categories of theoretical scenarios, not only models, within a unified phenomenological framework.

One spectrum to rule them all?

$$\mathcal{R}_c''(k) + 2\frac{z'}{z}\mathcal{R}_c'(k) + c_s^2 k^2 \mathcal{R}_c(k) = 0,$$

$$h_k'' + 2\frac{z_\gamma'}{z_\gamma}h_k' + c_\gamma^2 k^2 h_k = 0\,,$$

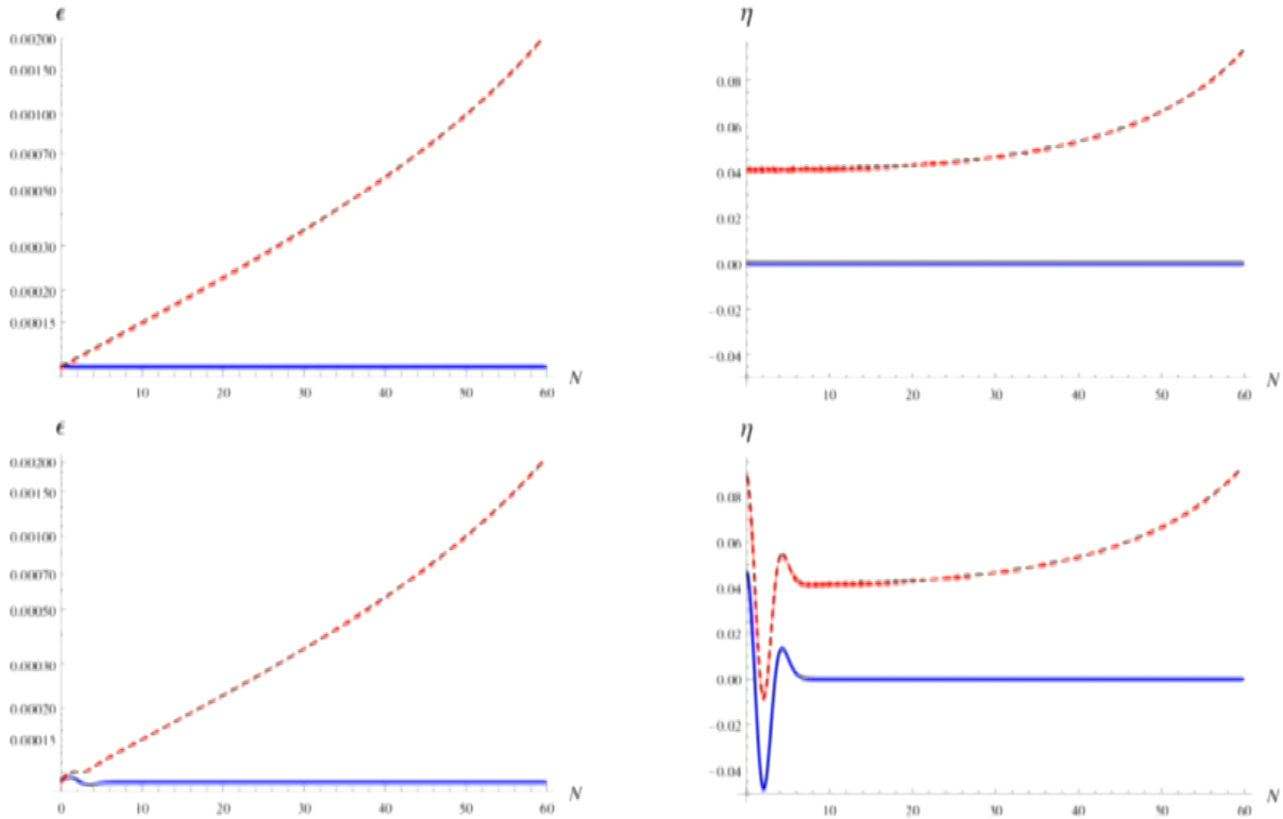
$$z = \frac{a\sqrt{2\epsilon}}{c_s} = \frac{1}{c_s}\sqrt{2\left(a^2 - \frac{a^3\ddot{a}}{\dot{a}^2}\right)}.$$

Freedom to choose the initial condition
condition for a(t) for a given z(t) !!

Recipe to construct dual models:

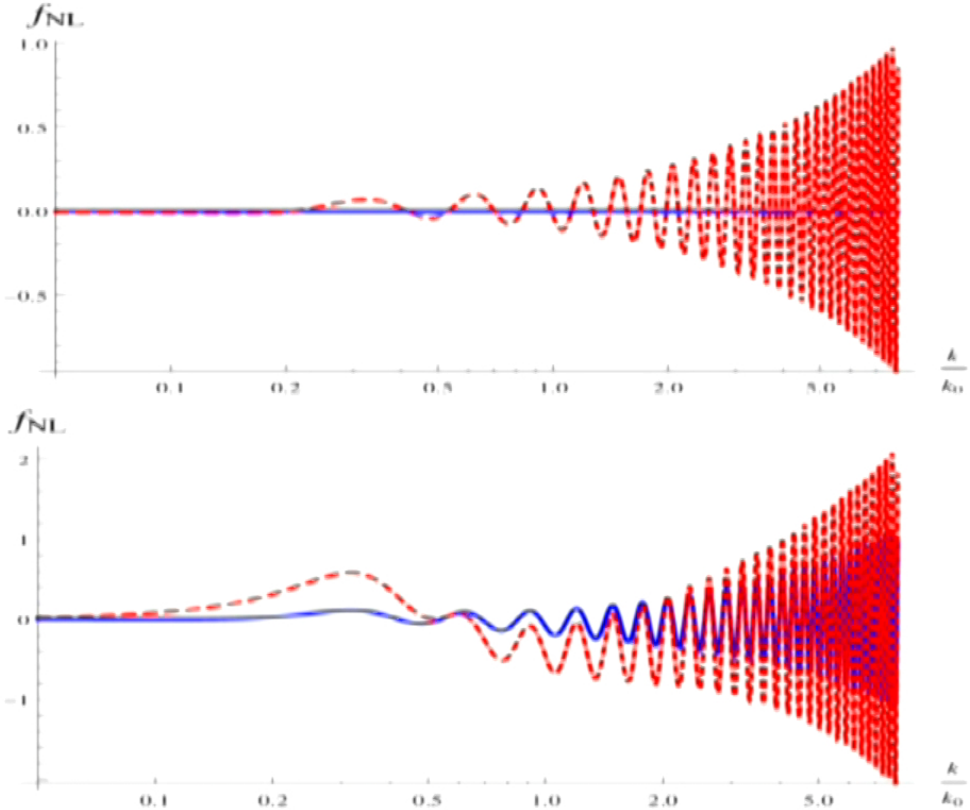
- Fix z0(t), c(t)
- Solve z(t)=z0(t) with different initial H, i.e. different initial derivative a'
- The new a(t) will by construction give the same z(t) but different slow roll parameters
- The spectrua will be the same
- Higher order correlation functions for scalar perturbations will be different
- Gravitational waves spectra will be different

Examples of dual models, with and without features

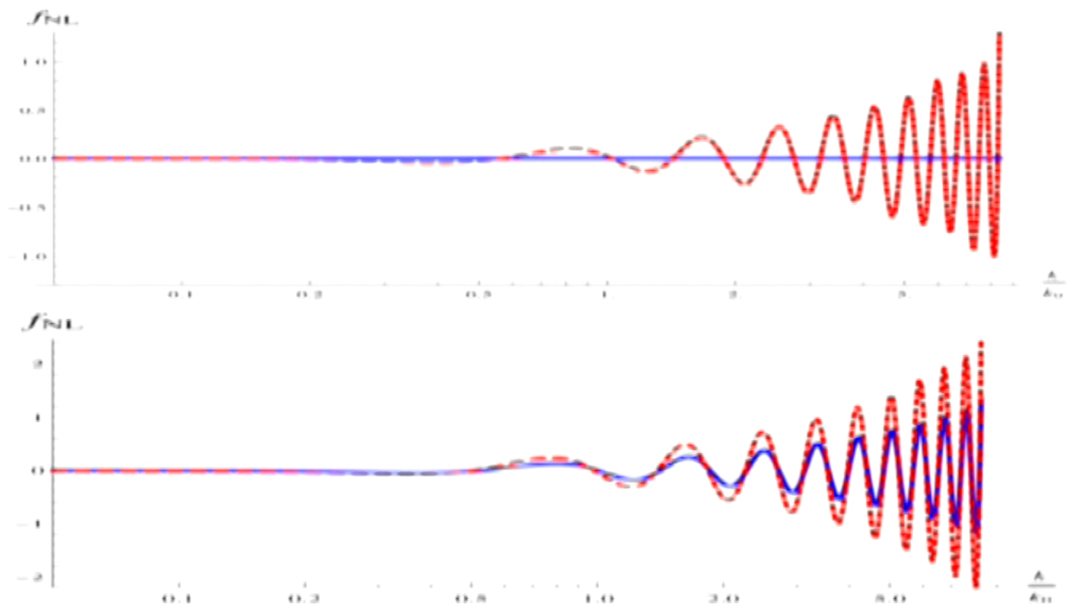


Equilateral configuration

$$a_{\rm ref}(t) = \left(1 + \epsilon_{\rm c} H_{{\rm ref},i} t\right)^{1/\epsilon_{\rm c}} \left[1 + \lambda e^{-(\frac{t-t_0}{\sigma})^2}\right],$$



Squeezed configuration



Violation of "general consistency condition"(JCAP 1504 (2015), Palma), not the squeezed limit Maldacena 's

$$f_{NL} \simeq \frac{5}{12}\frac{k_1k_2k_3}{k_1^3+k_2^3+k_3^3}\left[\frac{d^2}{d\ln k^2}\frac{\Delta P_{\mathcal{R}_c}}{P^0_{\mathcal{R}_c}}(k)\right]_i$$

Conclusions

Any (not just scale invariant) spectrum of comoving curvature perturbation can be obtained

- with an infinite class of background histories, including contracting Universes
- different theoretical scenarios with the same MESS such as multi-fields, modified gravity, or their combination
- Further degeneracy due to combination of MESS and background evolution degeneracy
- Higher order correlation functions and gravitational waves can reduce the degeneracy
- MESS is a useful model independent quantity to span the full space of theoretical scenarios