Title: Is it possible to be objective in every physical theory?

Speakers: Carlo Maria Scandolo

Series: Quantum Foundations

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Abstract: We investigate the emergence of classicality and objectivity in arbitrary physical theories. First we provide an explicit example of a theory where there are no objective states. Then we characterize classical states of generic theories, and show how classical physics emerges through a decoherence process, which always exists in causal theories as long as there are classical states. We apply these results to the study of the emergence of objectivity, here recast as a multiplayer game. In particular, we prove that the so-called Spectrum Broadcast Structure characterizes all objective states in every causal theory, in the very same way as it does in quantum mechanics. This shows that the structure of objective states is valid across an extremely broad range of physical theories. Finally we show that, unlike objectivity, the emergence of local classical theories is not generic among physical theories, but it becomes possible if a theory satisfies two axioms that rule out holistic behaviour in composite systems.

Is it possible to be objective in every physical theory?

Carlo Maria Scandolo

Department of Mathematics & Statistics, University of Calgary

07/18/2018

Introduction

- Joint work with R. Salazar, J. Korbicz, and P. Horodecki [CMS et al.].

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- Classical theory is objective, different observers agree on what they see.
- We *can't* say the same for quantum theory: measurements are disturbing.

Beyond quantum theory

Why do we observe objective and classical behaviour?

Various proposals

- decoherence [Zeh]

Beyond quantum theory

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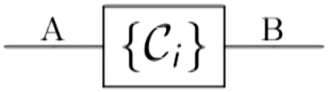
What part of quantum theory is responsible for the emergence of classicality?

- It's useful to look at quantum theory "from the outside".
- We get a new perspective on quantum extensions.

A common framework for all physical theories [Chiribella et al., Hardy]

Test: collection of processes ($\mathcal{C}_i$'s) that can occur in an experiment. i labels the classical outcome.



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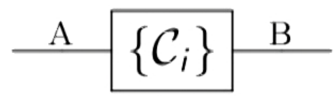

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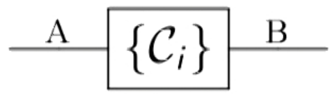

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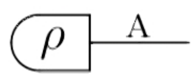

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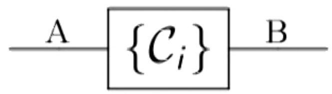

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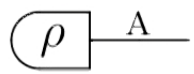
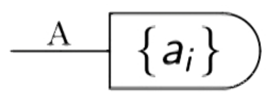
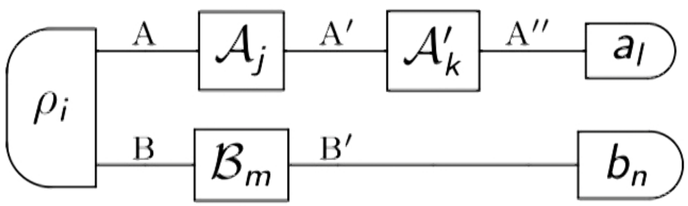


- Some tests destroy the system: measurements, *no* output. They're collections of effects (a_i's).



The probabilistic ingredient

We can build circuits:



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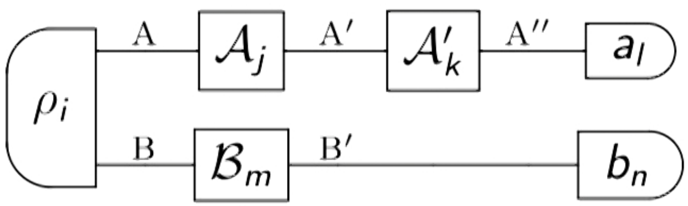
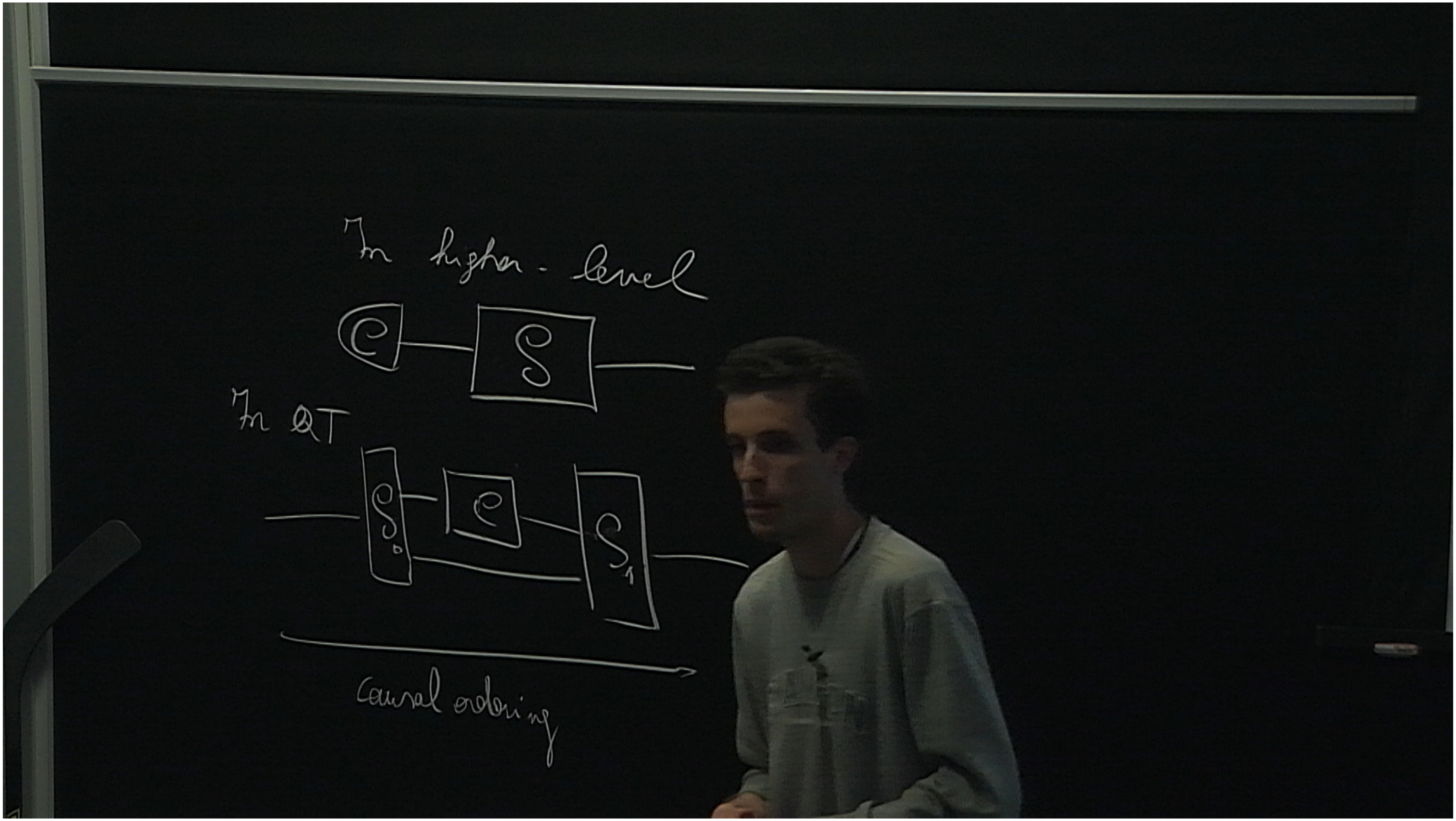


A circuit with no external wires represents a joint probability p_{ijklmn}.


In higher-level
$\mathcal{C}$
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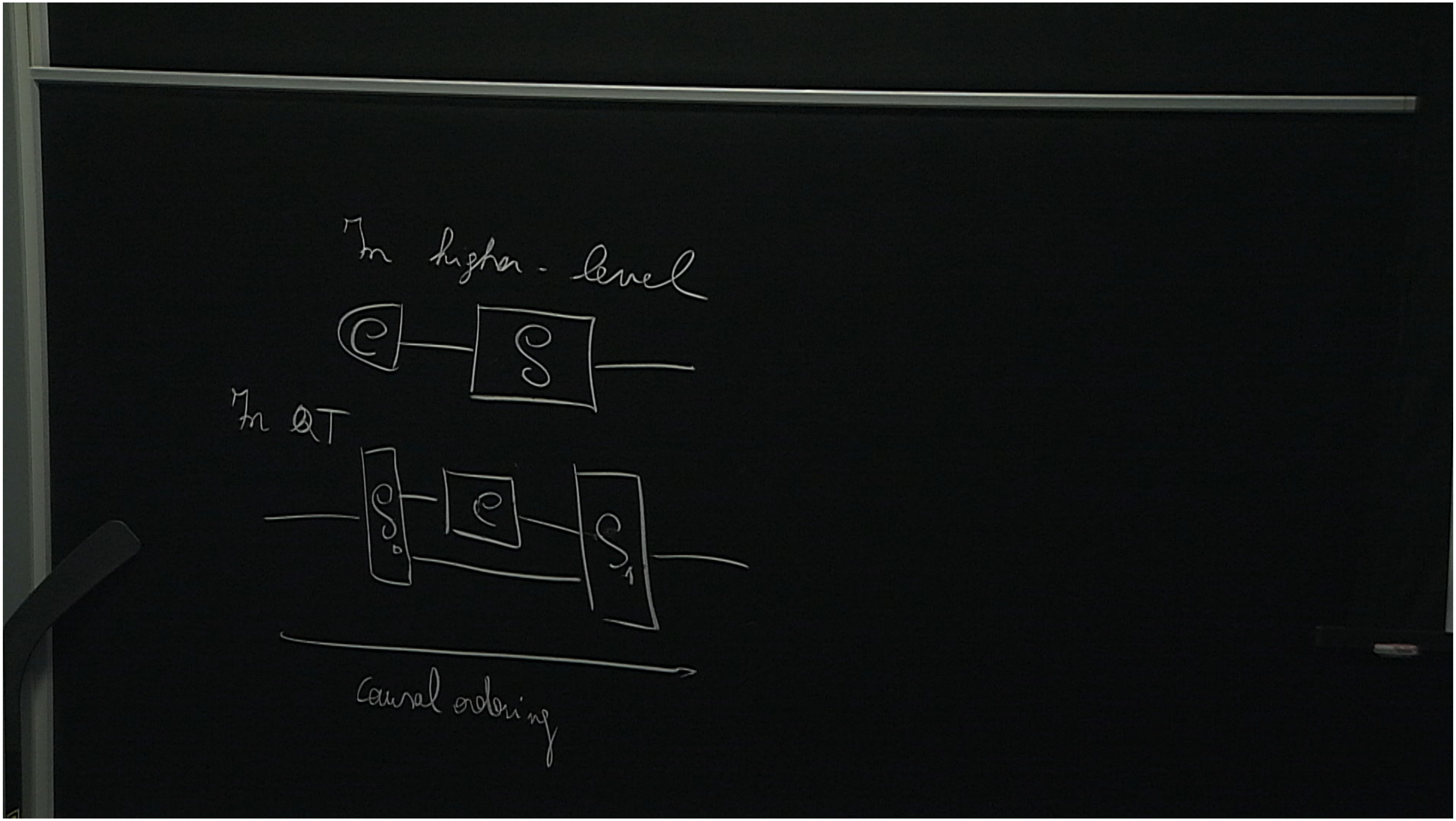
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Purity

Probabilities induce a linear structure on the processes: real (ordered) vector spaces.

Key concept: coarse-graining

Example

Rolling a die, "the outcome is 1" is an atomic event.
"The outcome is odd" is a coarse-graining: we joined together some outcomes (1, 3, 5).

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A transformation is pure if it *can't* be written as a non-trivial sum (coarse-graining) of transformations.

Causality [Chiribella et al.]

In causal theories information *can't* come back from the future.

The probability that a transformation occurs is independent of the choice of tests performed on its output.

Causality forbids signalling.

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A test $\{\mathcal{A}_i\}$ is measure-and-prepare if

$$\mathcal{A}_i = |\rho_i)(a_i|,$$

with $\{a_i\}$ measurement and ρ_i normalised state.

Outline

1. Decoherence

2. Objectivity
   - Local classical theory

Classical theory

Finite classical theory: the theory of probability distributions over a finite set.

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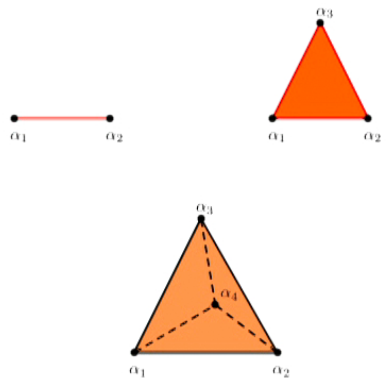

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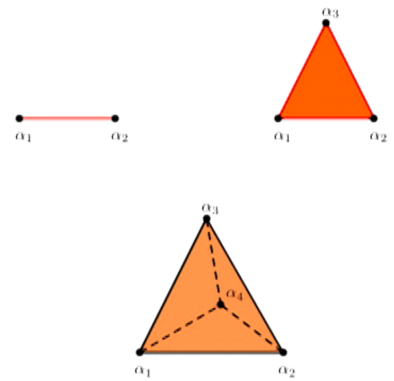

- All pure states are (jointly) perfectly distinguishable.
- The cone of effects is the dual cone.

Identifying classical sub-theories

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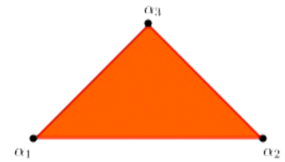
Can we always find them?
No! [CMS et al.]

The theory of restricted trits [CMS et al.]

- Start from a classical trit.



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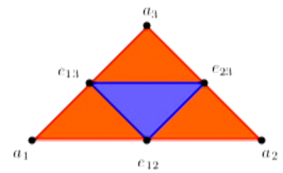

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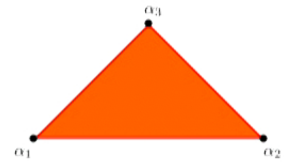

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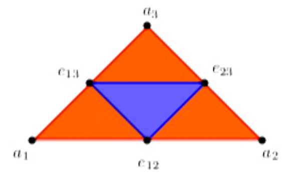

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the pure states are *no longer* perfectly distinguishable!

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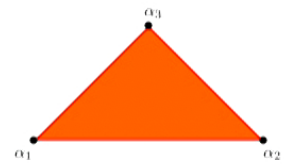

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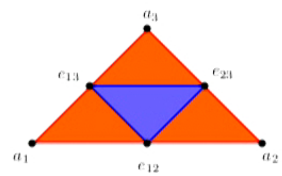

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the pure states are *no longer* perfectly distinguishable!
Not even in the composites!

Describing classical sub-theories

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$$\boldsymbol{\alpha} = \mathrm{Conv}\,\{\alpha_i\}\,.$$

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- Let's restrict the original effects of the theory to $\boldsymbol{\alpha}$.
- Then we take equivalence classes of tomographically indistinguishable effects.

$$e \sim_{\boldsymbol{\alpha}} e' \text{ if } (e|\gamma) = (e'|\gamma)$$

for every $\gamma \in \boldsymbol{\alpha}$.

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The quotient set is generated by the effects $\{a_i\}$ that distinguish the α_i's [CMS et al.].

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Complete decoherence

A channel D_α is a complete decoherence to α if

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Properties of complete decoherence [CMS et al.]

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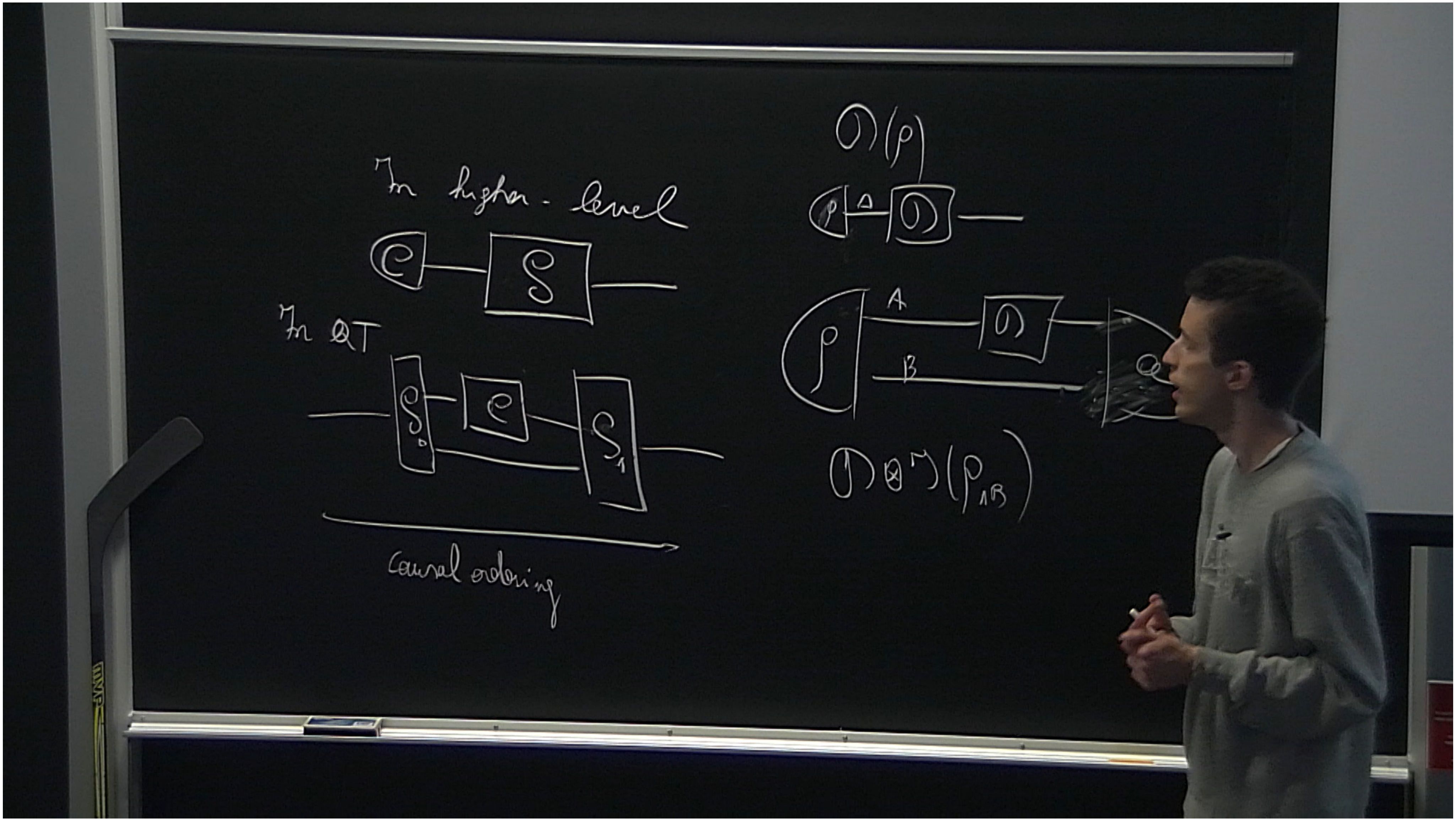
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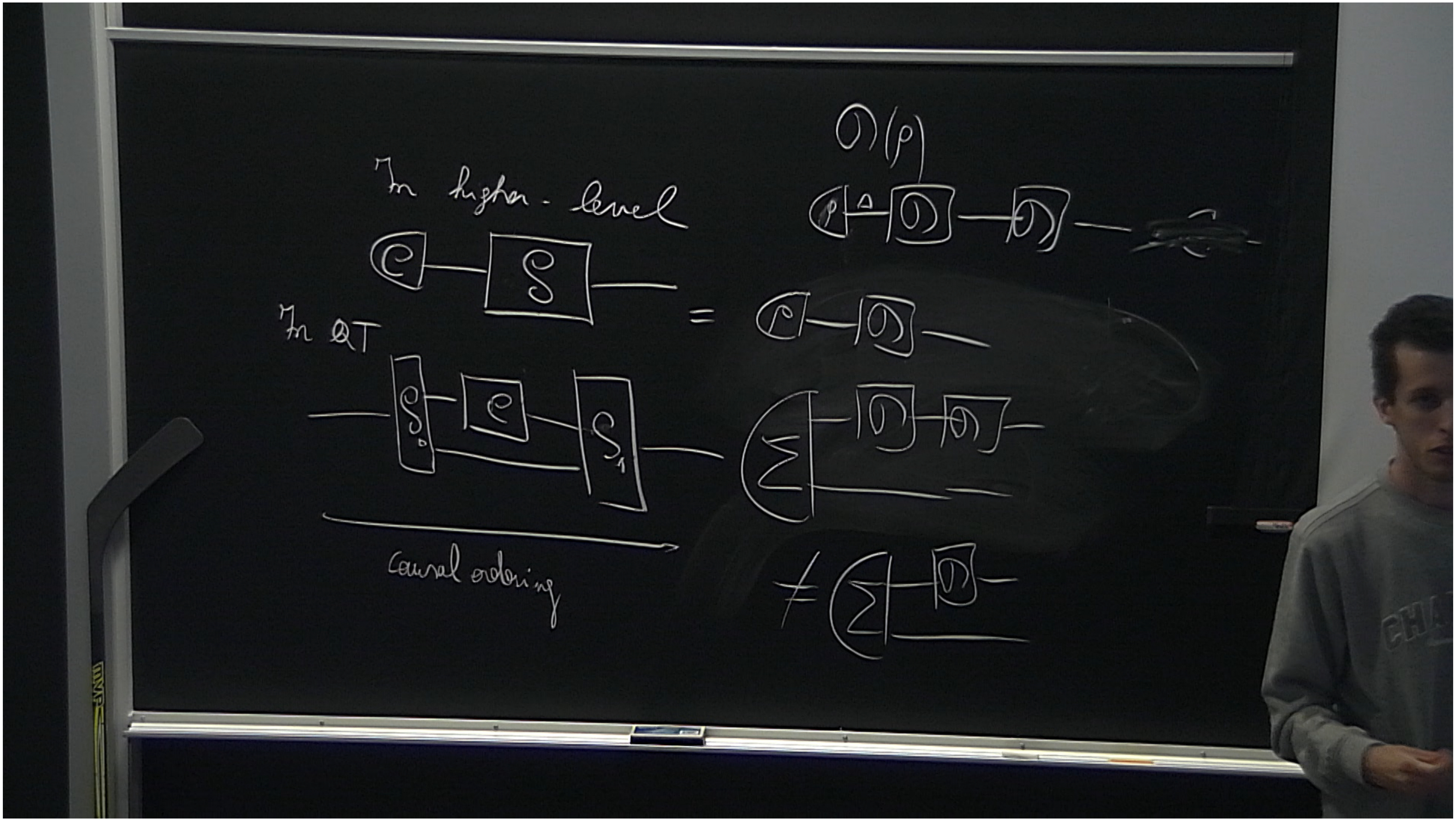
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An asymmetric behaviour

Given α, we have $D_\alpha \alpha_i = \alpha_i$, but in general $a_i D_\alpha \neq a_i$, and we have only $a_i D_\alpha \sim_\alpha a_i$.

Unique decoherence?

The complete decoherence to α may be non unique in two ways:

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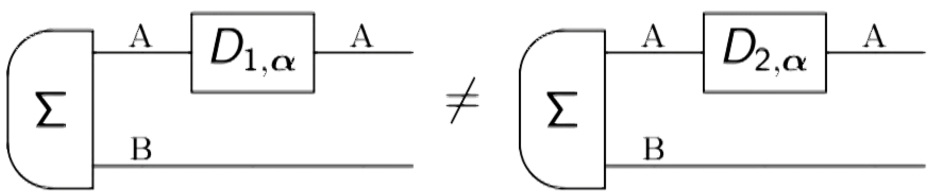

for some Σ.

Measurement-induced decoherence

- But above all, does at least one decoherence exist?
- Yes!
- Consider the measure-and-prepare test $\{|\alpha_i)\,(a_i|\}_{i=1}^{d}$.
- It exists by Causality.

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$$\widehat{D}_{\boldsymbol{\alpha}} = \sum_{i=1}^{d} |\alpha_i)\,(a_i|$$

It's a complete decoherence to $\boldsymbol{\alpha}$.

In this case $\widehat{D}_{\boldsymbol{\alpha}}^2 = \widehat{D}_{\boldsymbol{\alpha}}$ and $a_i \widehat{D}_{\boldsymbol{\alpha}} = a_i$.

Decoherence and purity

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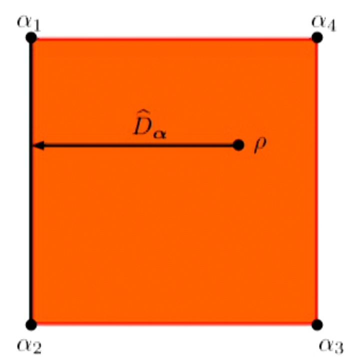
- In other words, a mixed state *can't* be decohered to a pure state.
- Is it true in general?
- No! Counterexample of the square bit.



Section 2

Objectivity

Quantum objectivity

Objective states

A state is objective if many observers can determine it without disturbance, and agree on their findings [Ollivier et al.].

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Theorem [Horodecki et al.]

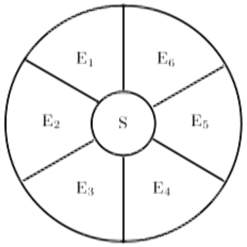
Objective quantum states are of the form (SBS)

$$\rho_{\mathrm{SE_1\ldots E}_n} = \sum_j p_j \left|j\right\rangle \left\langle j\right|_{\mathrm{S}} \otimes \rho_{j,\mathrm{E_1}} \otimes \ldots \otimes \rho_{j,\mathrm{E}_n}$$

with $\rho_{j,\mathrm{E}_l}\rho_{k,\mathrm{E}_l} = 0$ for $j \neq k$.

Beyond quantum: a multiplayer game [CMS et al.]

- Setting of quantum Darwinism: a target system, several environments, one for each observer.



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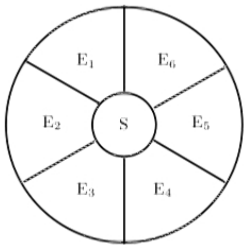

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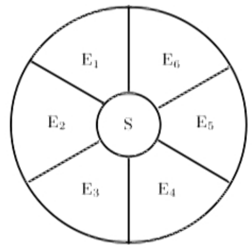

- An observer on the system acts as a referee who measures on some classical set $\boldsymbol{\alpha}$.
- The players win if they find out the state of S without disturbing the *joint* state.

A test $\{\mathcal{A}_i\}$ is non-disturbing on ρ if $\sum_i \mathcal{A}_i \rho = \rho$.

Sharply repeatable measurements [Perinotti, CMS et al.]

Sharply repeatable test (SRTs): a test $\{P_i\}$ with $P_i P_j = \delta_{ij} P_i$.

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In quantum theory

von Neumann measurements and $\{\mathcal{M}_j\}$ with

$$\mathcal{M}_j(\rho) = \mathrm{tr}(P_j \rho)\, \sigma_j,$$

σ_j's with orthogonal support, P_j's projectors on their support.

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If $\{\rho_i\}$ are perfectly distinguished by $\{e_i\}$, $\{|\rho_i)(e_i|\}$ is a (measure-and-prepare) SRT.

General objective states [CMS et al.]

The players *can't* cooperate.

Strong independence [Horodecki et al.]

The only correlation between the players is the common information about the system.

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SRTs exist in all causal theories!

Local classical theory

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- When is this possible?
- In a bipartite system AB, we have a classical set $\boldsymbol{\alpha}$ for A, and a classical set $\boldsymbol{\beta}$ for B.

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$$\boldsymbol{\alpha}\boldsymbol{\beta} = \mathrm{Conv}\left\{\alpha_i \otimes \beta_j\right\}$$

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$$\boldsymbol{\alpha\beta} = \mathrm{Conv}\left\{\alpha_i \otimes \beta_j\right\}$$

- In this case we also want $D_{\boldsymbol{\alpha\beta}} = D_{\boldsymbol{\alpha}} \otimes D_{\boldsymbol{\beta}}$ (cf. [Richens et al.]).

Local classical theory

- In the objectivity game, we had localised classical observers.
- When is this possible?
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These are non-trivial requirements!

Two locality axioms

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Axiom

The product of two pure states is pure.

Axiom (Information locality [Hardy])

The product of two pure maximal sets is still a pure maximal set.

Holistic behaviour without the two axioms

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The theory is extremely holistic.

2. If only Information Locality fails, we can construct the classical set for AB partially out of α and β.

We need some extra states to make it maximal; there is still some holism left.

Local decoherence

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- However, if we take the measurement-induced decoherence, we have

$$\widehat{D}_{\alpha\beta} = \widehat{D}_{\alpha} \otimes \widehat{D}_{\beta}.$$

This suggests that the measurement-induced decoherence is the most physically-motivated one.

Conclusions

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Promising research area for both the "Darwinist" community and the GPT one.

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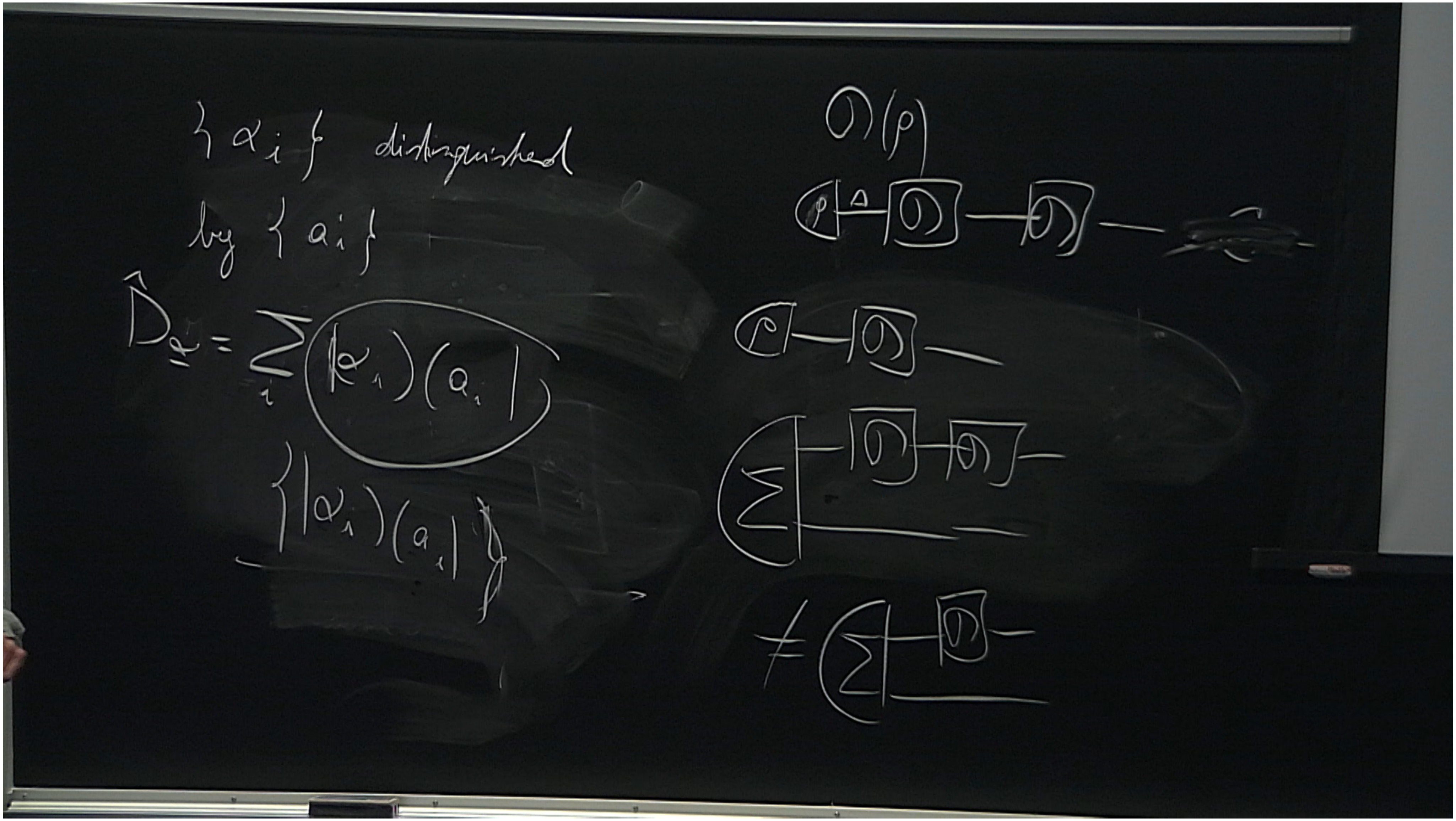
{α_i} distinguished
by {a_i}
D̂_α = Σ_i |α_i)(a_i|
{|α_i)(a_i|}
D(ρ)