Title: The Real Quantum Gravity

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Series: Quantum Foundations

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Abstract: Canonical quantization is not well suited to quantize gravity, while affine quantization is. For those unfamiliar with affine quantization the talk will include a primer. This procedure is then applied to deal with three nonrenormalizable, field theoretical, problems of increasing difficulty, the last one being general relativity itself.

The Real Quantum Gravity

1. A Toy Model

The role of 'free' and 'pseudofree' models

2. An Ultralocal Model

A model with no spatial continuity

3. A Covariant Scalar

Adding spatial continuity to the previous model

4. Affine Quantum Gravity

The mother of all problems!

SOLVED !?! 🙂

NR: = NONRENORMALIZABLE

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NR: A Toy Model

$$A_{g_0} = \int_0^T \{\tfrac{1}{2}[\dot{y}(t)^2 - y(t)^2] - g_0 y(t)^{-4}\}\, dt$$

$$\mathfrak{D}(A_{g_0=0}) \neq \mathfrak{D}(A_{g_0>0})$$

$f \neq pf$ 🙂

Classical to Quantum

$$\langle y'', T \,|\, y', 0\rangle_f = \sum_{n=0,1,2,3,\cdots} h_n(y'')h_n(y')e^{-i(n+1/2)T/\hbar}$$

$$\langle y'', T \,|\, y', 0\rangle_{pf} = 2\,\theta(y''y') \sum_{n=1,3,5,7,\cdots} h_n(y'')h_n(y')e^{-i(n+1/2)T/\hbar}$$

Affine Quantization-1

affine classical position and dilation variables

$$\{q,p\} = 1\,,\;\; q\{q,p\} = q\,,\;\; \{q,qp\} \equiv \{q,d\} = q \lessgtr 0$$

affine quantum position and dilation variables

$$[Q,P] = i\hbar I\,,\;\; Q[Q,P] = i\hbar Q\,,\;\; [Q,QP] = i\hbar Q$$
$$= [Q,(QP+PQ)+(QP-PQ)]/2 = i\hbar Q$$
$$= [Q,(QP+PQ)/2] \equiv [Q,D] = i\hbar Q\,, \quad Q \lessgtr 0$$

the Lie algebra for the affine group

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Affine Quantization-2

canonical coherent states

$$|p,q\rangle = e^{-iqP/\hbar}\, e^{ipQ/\hbar}\, |0\rangle$$

affine coherent states

$$|p,q\rangle = e^{ipQ/\hbar} e^{-i\ln(q)D/\hbar}\, |b\rangle$$

action for Schrödinger's equation

$$A_Q = \int \langle \psi(t)\,|\,[i\hbar(\partial/\partial t) - \mathscr{H}(P,Q)]\,|\,\psi(t)\rangle\, dt$$

$$A'_Q = \int \langle \psi(t)\,|\,[i\hbar(\partial/\partial t) - \mathscr{H}'(D,Q)]\,|\,\psi(t)\rangle\, dt$$

action for enhanced classical equations

$$A_C = \int \langle p(t), q(t)\,|\,[i\hbar(\partial/\partial t) - \mathscr{H}(P,Q)]\,|\,p(t), q(t)\rangle\, dt$$

$$= \int \{p(t)\dot{q}(t) - H(p(t), q(t))\}\, dt$$

$$A'_C = \int \langle p(t), q(t)\,|\,[i\hbar(\partial/\partial t) - \mathscr{H}'(D,Q)]\,|\,p(t), q(t)\rangle\, dt$$

$$= \int \{-q(t)\dot{p}(t) - H'(p(t), q(t))\}\, dt,$$

Both operator pairs lead to similar classical stories and with hbar > 0.

Favored Coordinates-1

Dirac: "Cartesian coordinates should lead to H(p,q)—>H(P,Q)"

canonical quantization

$$H(p,q) = \langle p,q\,|\,\mathscr{H}(P,Q)\,|\,p,q\rangle\ , \qquad (\omega Q + iP)\,|0\rangle = 0$$

$$= \langle 0\,|\,\mathscr{H}(P+p,Q+q)\,|\,0\rangle = \mathscr{H}(p,q) + \mathscr{O}(\hbar;p,q)$$

$$0$$

$$2\hbar[\,\|\,d|p,q\rangle\,\|^2 - |\langle p,q\,|\,d|p,q\rangle|^2] = \omega^{-1}dp^2 + \omega dq^2$$

🙂

affine quantization

$$H'(pq,q) = \langle p,q\,|\,\mathscr{H}'(D,Q)\,|\,p,q\rangle\ , \quad [(Q-1)+iD/b]\,|\,b\rangle = 0$$

$$= \langle b\,|\,\mathscr{H}'(D+pqQ,qQ)\,|\,b\rangle = \mathscr{H}'(pq,q) + \mathscr{O}'(\hbar;p,q)$$

$$-2/b$$

$$2\hbar[\,d|p,q\rangle\,\|^2 - |\langle p,q\,|\,d|p,q\rangle|^2] = b^{-1}q^2dp^2 + bq^{-2}dq^2$$

🙂

NR: Ultralocal Model-1

$$H(\pi,\varphi) = \int \{\tfrac{1}{2}[\pi(x)^2 + m_0^2\varphi(x)^2] + g_0\varphi(x)^4\}\, d^s x\,, \quad s \geq 1$$

$$f \neq pf$$

affine variables

$$\kappa(x) \equiv \pi(x)\varphi(x), \quad \varphi(x) \lessgtr 0$$

$$\{\varphi(x), \kappa(x')\} = \delta^s(x-x')\,\varphi(x), \quad \varphi(x) \lessgtr 0$$

$$H'(\kappa,\varphi) = \int \{\tfrac{1}{2}[\kappa(x)\varphi(x)^{-2}\kappa(x) + m_0^2\varphi(x)^2] + g_0\varphi(x)^4\}\, d^s x$$

Classical to Quantum

$$[\hat{\varphi}(x), \hat{\kappa}(x')] = i\hbar\, \delta^s(x-x')\,\hat{\varphi}(x)\,, \quad \hat{\varphi}(x) \lessgtr 0$$

🙂 🙂

$$\hat{\varphi}(x) = \int B(x,\lambda)^\dagger \lambda B(x,\lambda)\, d\lambda\,, \qquad B(x,\lambda) = A(x,\lambda) + c(\lambda)I\,, \quad A(x,\lambda)\,|0\rangle = 0$$

$$\hat{\kappa}(x) = -\,i\hbar\tfrac{1}{2}\int B(x,\lambda)^\dagger[\lambda(\partial/\partial\lambda) + (\partial/\partial\lambda)\lambda]B(x,\lambda)\; d\lambda$$

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NR: Ultralocal Model-2

$$\hat{\varphi}(x) = \varphi(x) \quad , \quad \hat{\kappa}(x) = -\, i\hbar(1/2)[\varphi(x)\,(\delta/\delta\varphi(x)) + (\delta/\delta\varphi(x))\,\varphi(x)]$$

Schrödinger's equation

$$i\hbar\, \partial\Psi(\varphi, t)/\partial t = \left[\int \{(1/2)[\hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) + m_0^2\varphi(x)^2] + g_0\varphi(x)^4\}\; d^s x\right]\; \Psi(\varphi, t)$$

$$\hat{\kappa}(x)\,\varphi(x)^{-1/2} = 0 \;, \qquad \Psi(\varphi) = e^{-W(\varphi)}\,\Pi_x\,|\,\varphi(x)\,|^{-1/2}$$ **formal**

regularization

$$\varphi(x) \to \varphi_{\mathbf{k}} \equiv \varphi(\mathbf{k}a)\,, \quad a > 0\,,\; \mathbf{k} \in \{0, \pm 1, \pm 2, \pm 3, \cdots\}^s$$

$$\Psi_r(\varphi) = e^{-W_r(\varphi)}\,\Pi_{\mathbf{k}}(ba^s)^{1/2}\,|\,\varphi_{\mathbf{k}}\,|^{-(1-2ba^s)/2}$$ **regularized**

$$\int |\,\Psi_r(\varphi)\,|^2\,\Pi_{\mathbf{k}} d\varphi_{\mathbf{k}} = 1$$

NR: Ultralocal Model-3

$$\Psi_r(\varphi) = \Pi_{\mathbf{k}} e^{-W_r(\varphi_{\mathbf{k}})} (ba^s)^{1/2} \,|\, \varphi_{\mathbf{k}} \,|^{-(1-2ba^s)/2}$$ 🙂

characteristic functional

$$C(f) = \lim_{a\to 0} \Pi_{\mathbf{k}} \int e^{if_{\mathbf{k}}\varphi_{\mathbf{k}}/\hbar} (ba^s)\, e^{-2W_r(\varphi_{\mathbf{k}})} \,|\, \varphi_{\mathbf{k}} \,|^{-(1-2ba^s)} d\varphi_{\mathbf{k}}$$

$$C(f) = \lim_{a\to 0} \Pi_{\mathbf{k}} \int \{ 1 - (ba^s) \int [1 - e^{if_{\mathbf{k}}\varphi_{\mathbf{k}}/\hbar}]\, e^{-2W_r(\varphi_{\mathbf{k}})} \,|\, \varphi_{\mathbf{k}} \,|^{-(1-2ba^s)} d\varphi_{\mathbf{k}} \}$$

$$C(f) = \exp\{ -b \int d^s x \int [1 - e^{if(x)\lambda/\hbar}]\, e^{-2w(\lambda)} d\lambda / |\, \lambda \,| \}$$ **Poisson** 🙂

pseudofree model: equal-spacing spectrum, NO zero-point energy

NR: Covariant Model-1

$$s \geq 4 \qquad f \neq pf$$

$$H(\pi, \varphi) = \int \{(1/2)[\pi(x)^2 + (\nabla\varphi)(x)^2 + m_0^2\varphi(x)^2] + g_0\varphi(x)^4\}\, d^s x$$

affine variables

$$\kappa(x) \equiv \pi(x)\varphi(x)\,, \quad \varphi(x) \lessgtr 0$$

$$H'(\kappa, \varphi) = \int \{(1/2)[\kappa(x)\varphi(x)^{-2}\kappa(x) + (\nabla\varphi)(x)^2 + m_0^2\varphi(x)^2] + g_0\varphi(x)^4\}\, d^s x$$

Classical to Quantum

$$\hat{\varphi}(x) = \varphi(x) \quad , \qquad \hat{\kappa}(x) = -\, i\hbar(1/2)[\varphi(x)\,(\delta/\delta\varphi(x)) + (\delta/\delta\varphi(x))\,\varphi(x)]$$

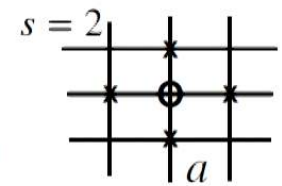
Schrödinger's equation

$$i\hbar\, \partial\Psi(\varphi, t)/\partial t = \left[\int \{(1/2)[\hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) + (\nabla\varphi)(x)^2 + m_0^2\varphi(x)^2] + g_0\varphi(x)^4\}\; d^s x\right]\; \Psi(\varphi, t)$$

NR: Covariant Model-2

$$\hat{\kappa}(x)\,\varphi(x)^{-1/2} = 0\ , \quad \Psi(\varphi) = e^{-Y(\varphi)}\,\Pi_x\,|\,\varphi(x)\,|^{-1/2}$$ **formal**

a different regularization



$$\varphi(x) \to \varphi_{\mathbf{k}} \equiv \varphi(\mathbf{k}a)\,, \quad a > 0\,, \ \mathbf{k} \in \{0, \pm 1, \pm 2, \pm 3, \cdots\}^s$$

$J_{\mathbf{k},\mathbf{l}} \equiv 1/(1+2s)$, $\mathbf{l} = \mathbf{k}$ and 2s nearest spatial neighbors to $\mathbf{k}$; otherwise, $J_{\mathbf{k},\mathbf{l}} \equiv 0$

$$\Psi_r(\varphi) = e^{-Y_r(\varphi)}\,\Pi_{\mathbf{k}}[\Sigma_{\mathbf{l}} J_{\mathbf{k},\mathbf{l}}\,\varphi_{\mathbf{l}}^2]^{-(1-2ba^s)/4}$$ **regularized** 🙂

$$1 = \int |\,\Psi_r(\varphi)\,|^2\,\Pi_{\mathbf{k}} d\varphi_{\mathbf{k}}\ , \quad \varphi_{\mathbf{k}} = \rho\eta_{\mathbf{k}}\ , \quad R = 2ba^s N'$$ **divergence free**

$$= \int e^{-2Y_r(\rho\eta)}\,\delta(1 - \Sigma_{\mathbf{k}}\eta_{\mathbf{k}}^2)\,\Pi_{\mathbf{k}}[\Sigma_{\mathbf{l}} J_{\mathbf{k},\mathbf{l}}\,\eta_{\mathbf{l}}^2]^{-(1-2ba^s)/2}\ d\eta_{\mathbf{k}}]\,\rho^{R-1}\,d\rho$$ 🙂 🙂

There are indications that affine quantization of φ_4^4 models is non-trivial 🙂 🙂

NR: Quantum Gravity-1

$$g_{ab}(x)\,dx^a\,dx^b > 0 \;, \quad g_{ac}(x)\,g^{bc}(x) \equiv \delta^b_a \;, \quad \pi^{ac}(x)\,g_{bc}(x) \equiv \pi^a_b(x) \;, \quad g(x) \equiv \det[g_{ab}(x)] > 0$$

$$H'(\pi, g) = \int \{ g^{-1/2}[\pi^a_b \pi^b_a - (1/2)\pi^a_a \pi^b_b] + g^{1/2}\,{}^{(3)}R \}\, d^3x$$

Classical to Quantum

$f \neq pf$

$$[\hat{\pi}^a_b(x), \hat{\pi}^c_d(x')] = i\hbar(1/2)\,\delta^3(x,x')\,[\delta^a_d \hat{\pi}^c_b(x) - \delta^c_b \hat{\pi}^a_d(x)]$$

$$[\hat{g}_{ab}(x), \hat{\pi}^c_d(x')] = i\hbar(1/2)\,\delta^3(x,x')\,[\delta^c_a \hat{g}_{bd}(x) + \delta^c_b \hat{g}_{ad}(x)]$$

$$[\hat{g}_{ab}(x), \hat{g}_{cd}(x')] = 0 \;, \qquad \hat{g}(x) \equiv \det[\hat{g}_{ab}(x)] > 0$$

🙂 🙂

$$\hat{g}_{ab}(x) = \int_+ B^\dagger(x,\gamma)\gamma_{ab} B(x,\gamma) d\gamma \,, \qquad \{\gamma_{ab}\} > 0 \,, \; d\gamma \equiv \Pi_{a \leq b}\, d\gamma_{ab}$$

$$\hat{\pi}^a_b(x) = -\,i\hbar(1/2) \int_+ \{ B(x,\gamma)^\dagger [\gamma_{bc}(\partial/\partial\gamma_{ac}) + (\partial/\partial\gamma_{ac})\gamma_{bc}] B(x,\gamma) \} d\gamma$$

NR: Quantum Gravity-2

$$\hat{g}_{ab}(x) = g_{ab}(x)$$
$$\hat{\pi}^a_b(x) = -\, i\hbar(1/2)[g_{bc}(x)(\delta/\delta g_{ac}(x)) + (\delta/\delta g_{ac}(x))g_{bc}(x)]$$

Schrödinger's equation

$$i\hbar\, \partial\Psi(\{g\},t)/\partial t = \Big[\int [\hat{\pi}^a_b(x) g(x)^{-1/2} \hat{\pi}^b_a(x) - \frac{1}{2}\hat{\pi}^a_a(x) g(x)^{-1/2} \hat{\pi}^b_b(x) + g(x)^{1/2}\, {}^{(3)}\!R(x)]\, d^3x\Big]\ \Psi(\{g\},t)$$ 🙂

$$\hat{\pi}^a_b(x)\, g(x)^{-1/2} = 0\ ,$$

N.B. The field $\pi^{ab}(x)$ is NOT made an operator.

$$\Psi(\{g\}) = Y(\{g\})\, \Pi_x\, g(x)^{-1/2}$$ **formal**

$$\Psi_r(\{g\}) = Y_r(\{g\}) \Pi_{\mathbf{k}}\, [\Sigma_{\mathbf{l}} J_{\mathbf{k},\mathbf{l}}\, g_{\mathbf{l}}]^{-(1-ba^3)/2}$$ **regularized** 🙂

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NR: Quantum Gravity-3

affine gravity coherent states

$$\eta(x) = \{\eta^a_b(x)\}$$

$$|\pi,\eta\rangle = e^{(i/\hbar)\int \pi^{ab}(x)\,\hat{g}_{ab}(x)\,d^3x}\, e^{-(i/\hbar)\int \eta^a_b(x)\,\hat{\pi}^b_a(x)\,d^3x}\,|\beta\rangle \;\; [\,=|\pi,g\rangle\,]$$

$$\langle\pi,\eta\,|\,\hat{g}_{ab}(x)\,|\,\pi,\eta\rangle = [e^{\eta(x)/2}]^c_a\,\langle\beta\,|\,\hat{g}_{cd}(x)\,|\,\beta\rangle\,[e^{\eta(x)/2}]^d_b \equiv g_{ab}(x)$$ 🙂

$$\langle\pi,\eta\,|\,\hat{\pi}^a_b(x)\,|\,\pi,\eta\rangle = \pi^{ac}(x)\,g_{cb}(x) \equiv \pi^a_b(x)$$ 🙂 🙂

$$\langle\pi'',g''\,|\,\pi',g'\rangle = \exp\Big\{-2\int\beta(x)\,d^3x \times \ln\Big\{\frac{\det\{\frac{1}{2}[g''^{ab}(x)+g'^{ab}(x)]+i\frac{1}{2\hbar}\beta(x)^{-1}[\pi''^{ab}(x)-\pi'^{ab}(x)]\}}{\det[g''^{ab}(x)]^{1/2}\;\det[g'^{ab}(x)]^{1/2}}\Big\}\Big\}$$

ultralocal : NO constraints imposed

Favored Coordinates-2

enhanced classical Hamiltonian density

$$H'(\pi^a_b(x), g_{cd}(x)) = \langle \pi, g \,|\, \mathscr{H}'(\hat{\pi}^a_b(x), \hat{g}_{cd}(x)) \,|\, \pi, g \rangle$$

$$= \langle \beta \,|\, \mathscr{H}'(\hat{\pi}^a_b(x) + \pi^{aj}(x)[e^{\eta(x)/2}]^e_j \hat{g}_{ef}(x)[e^{\eta(x)/2}]^f_b, [e^{\eta(x)/2}]^e_c \hat{g}_{ef}(x)[e^{\eta(x)/2}]^f_d) \,|\, \beta \rangle$$

$$\langle \beta \,|\, [e^{\eta(x)/2}]^e_c \hat{g}_{ef}(x)[e^{\eta(x)/2}]^f_d \,|\, \beta \rangle = [e^{\eta(x)/2}]^e_c \langle \beta \,|\, \hat{g}_{ef}(x) \,|\, \beta \rangle [e^{\eta(x)/2}]^f_d \equiv g_{cd}(x)$$

$$= \mathscr{H}'(\pi^{aj}(x) g_{jb}(x), g_{cd}(x)) + \mathscr{O}'(\hbar; \pi, g)$$

$$= \mathscr{H}'(\pi^a_b(x), g_{cd}(x)) + \mathscr{O}'(\hbar; \pi, g)$$

🙂 🙂

these are favored affine gravity coordinates

Favored Coordinates-3

$$|p,q\rangle = e^{-iqP/\hbar}\, e^{ipQ/\hbar}\, |0\rangle\ , \qquad (\omega Q + iP)\,|0\rangle = 0$$

$$2\hbar[\,\|\,d\,|p,q\rangle\,\|^2 - |\,\langle p,q\,|\,d|\,p,q\rangle\,|^2] = \omega^{-1}dp^2 + \omega dq^2$$

$$|p,q\rangle = e^{ipQ/\hbar} e^{-i\ln(q)D/\hbar}\,|b\rangle\ , \qquad [(Q-1) + iD/b]\,|b\rangle = 0$$

$$2\hbar[\,\|\,d\,|p,q\rangle\,\|^2 - |\,\langle p,q\,|\,d|\,p,q\rangle\,|^2] = b^{-1}q^2dp^2 + bq^{-2}dq^2$$

$$|\pi,g\rangle = e^{(i/\hbar)\int \pi^{ab}(x)\,\hat{g}_{ab}(x)\,d^3x}\, e^{-(i/\hbar)\int \eta^a_b(x)\,\hat{\pi}^b_a(x)\,d^3x}\,|\beta\rangle$$

$$C\hbar[\,\|\,d\,|\pi,g\rangle\,\|^2 - |\,\langle \pi,g\,|\,d|\,\pi,g\rangle\,|^2]$$

$$= \int [(\beta(x)\hbar)^{-1} g_{ab}g_{cd}\, d\pi^{bc}d\pi^{da} + (\beta(x)\hbar)\, g^{ab}g^{cd}\, dg_{bc}dg_{da}]\ d^3x$$

Fubini-Study metrics

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NR: Quantum Gravity-4

$$|\pi, g\rangle = e^{(i/\hbar)\int \pi^{ab}(x)\,\hat{g}_{ab}(x)\,d^3x}\, e^{-(i/\hbar)\int \eta^a_b(x)\,\hat{\pi}^b_a(x)\,d^3x}\,|\beta\rangle$$

$$\langle \pi, g\,|\,\pi', g'\rangle = \exp\Big\{-2\int \beta(x)\,\ln\Big\{\frac{\det\{\frac{1}{2}[g^{ab}(x)+g'^{ab}(x)]+i\frac{1}{2\hbar}\beta(x)^{-1}[\pi^{ab}(x)-\pi'^{ab}(x)]\}}{\det[g^{ab}(x)]^{1/2}\ \det[g'^{ab}(x)]^{1/2}}\Big\}\,d^3x\Big\}$$

complex polarization

$$C^r_s(x)\,\langle \pi, g\,|\,\Psi\rangle \equiv \Big[-i\hbar g^{rt}(x)\frac{\delta}{\delta\pi^{st}(x)} + \delta^r_s + \beta(x)^{-1} g_{st}(x)\frac{\delta}{\delta g_{rt}(x)}\Big]\,\langle \pi, g\,|\,\Psi\rangle = 0$$

$$A \equiv \frac{1}{2\hbar}\int C^s_r(x)^\dagger C^r_s(x)\,\beta(x)\,d^3x \quad , \quad \lim_{\nu\to\infty} \mathcal{N}_\nu e^{-\nu T A}\,\delta\{\pi-\pi'\}\delta\{g-g'\} = \langle \pi, g\,|\,\pi', g'\rangle$$

an analog to Wiener measure

NR: Quantum Gravity-5

$$\pi'' = \pi^{ab}(\,\cdot\,,T)\ ,\ g'' = g_{ab}(\,\cdot\,,T) \quad ; \quad \pi' = \pi^{ab}(\cdot,0)\ ,\ g' = g_{ab}(\cdot,0) \quad ; \quad T > 0$$

kinematic and physical functional integrals

$$\langle \pi'', g'' | \pi', g' \rangle = \lim_{\nu\to\infty} \mathcal{N}_\nu \int \exp[-(i/\hbar)\int g_{ab}\,\dot{\pi}^{ab}\ d^3x\,dt]$$

$$\times \exp\{-(1/2\nu\hbar)\int[(\beta(x)\hbar)^{-1} g_{ab}g_{cd}\dot{\pi}^{bc}\dot{\pi}^{da} + (\beta(x)\hbar)\,g^{ab}g^{cd}\dot{g}_{bc}\dot{g}^{da}]\ d^3x\,dt\}$$

$$\times \Pi_{x,t}\Pi_{[a\leq b]}\,d\pi^{ab}(x,t)\,dg_{ab}(x,t)$$ 🙂

projection operator IE enforces the constraints

$$\langle \pi'', g'' | \mathbb{E} | \pi', g' \rangle = \lim_{\nu\to\infty} \mathcal{N}'_\nu \int \exp\{-(i/\hbar)\int[g_{ab}\,\dot{\pi}^{ab} + N^a H_a + NH]\,d^3x\,dt\}$$

$$\times \exp\{-(1/2\nu\hbar)\int[(\beta(x)\hbar)^{-1} g_{ab}g_{cd}\dot{\pi}^{bc}\dot{\pi}^{da} + (\beta(x)\hbar)\,g^{ab}g^{cd}\dot{g}_{bc}\dot{g}^{da}]\ d^3x\,dt\}$$

$$\times [\Pi_{x,t}\Pi_{[a\leq b]}\,d\pi^{ab}(x,t)\,dg_{ab}(x,t)\,]\,\mathcal{D}R\{N^a, N\}$$ 🙂 🙂

first and second class constraints

Comparison List

The Canonical Story

$$g(x) = \det[g_{ab}(x)] \geqslant 0$$

complex variables

part Euclidean

discrete metric

$$\{\tilde{E}^a_{\ i}(x)\ ,\ A_b^{\ j}(y)\} = -\,i\delta^a_b\delta^j_i\ \delta^3(x,y)$$

$$\hat{\tilde{E}}^a_{\ i}(x) = -\,i\hbar\ \delta/\delta A_a^{\ i}(x)$$

FAVORED CANONICAL VARIABLES ???

non-Cartesian coordinates yield a FALSE quantum theory

The Affine Story

$$g(x) = \det[g_{ab}(x)] > 0$$

real variables

all Lorentzian

continuous metric* C.R. *lives*

$$[\hat{\pi}^a_b(x), \hat{\pi}^c_d(y)] = i\hbar(1/2)\,\delta^3(x,y)\,[\delta^a_d\hat{\pi}^c_b(x) - \delta^c_b\hat{\pi}^a_d(x)]$$

$$[\hat{g}_{ab}(x), \hat{\pi}^c_d(y)] = i\hbar(1/2)\,\delta^3(x,y)\,[\delta^c_a\hat{g}_{bd}(x) + \delta^c_b\hat{g}_{ad}(x)]$$

$$[\hat{g}_{ab}(x), \hat{g}_{cd}(y)] = 0$$

FAVORED AFFINE VARIABLES !!! 🙂

proper affine coordinates yield a VALID quantum theory 🙂

Today's Message

1. Nonrenormalizable models are NOT continuously connected to their own free model.

2. If Q&P then Q&D, where D=(PQ+QP)/2. Note that [Q,P]=i hbar, and then [Q,D]=i hbar Q, which is the Lie algebra of the affine group.

3. The favored pair q&p to promote are "Cartesian coordinates" which make a flat plane. The favored pair q&pq to promote have an affine form on a Lobachevsky plane of constant negative curvature.

4. Canonical quantization of nonrenormalizable models fails, but affine quantization is successful.

5. Affine quantization of gravity offers self-adjoint momentric and metric fields that respect positivity requirements.

THANK YOU

arXiv:1811.09582

arXiv:1903.11211

Freedman, et al, 1982

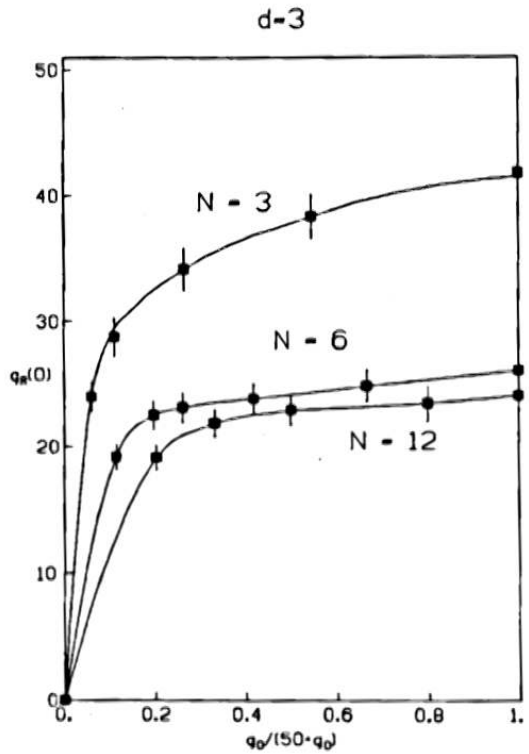


Fig. 3. The zero-momentum coupling constant $g_R(0)$ as a function of the bare coupling constant ratio $g_0/(50+g_0)$ for $(\phi^4)_3$ on lattices of size $N = 3$, 6 and 12.

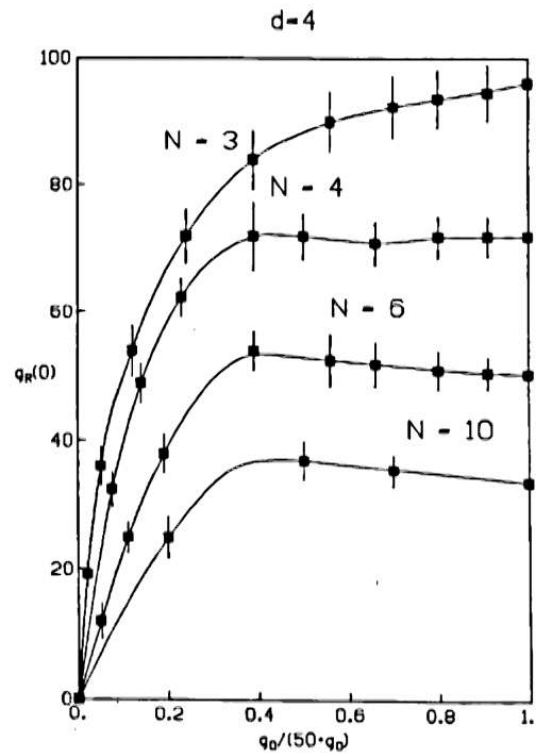
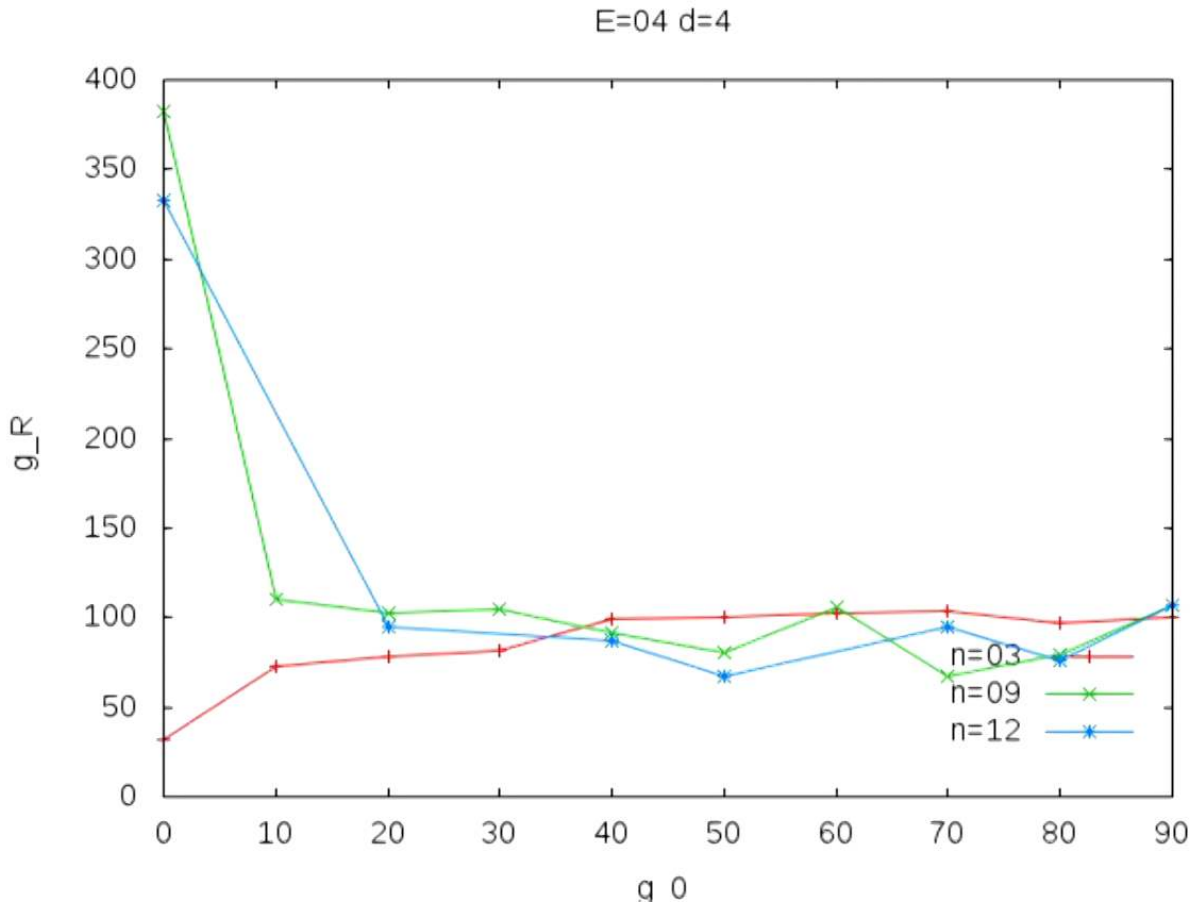


Fig. 1. The zero-momentum renormalized coupling constant $g_R(0)$ as a function of the bare coupling constant ratio $g_0/(50+g_0)$ for $(\phi^4)_4$ on lattices of sizes $N = 3$, 4, 6 and 10.

Phi^4_4 With Counter Term