Title: The Real Quantum Gravity

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Series: Quantum Foundations

Date: July 16, 2019 - 3:30 PM

URL: http://pirsa.org/19070078

Abstract: Canonical quantization is not well suited to quantize gravity, while affine quantization is. For those unfamiliar with affine quantization the talk will include a primer. This procedure is then applied to deal with three nonrenormalizable, field theoretical, problems of increasing difficulty, the last one being general relativity itself.

The Real Quantum Gravity

1. A Toy Model

The role of 'free' and 'pseudofree' models

2. An Ultralocal Model

A model with no spatial continuity

3. A Covariant Scalar

Adding spatial continuity to the previous model

4. Affine Quantum Gravity

The mother of all problems!

SOLVED !?! 🙂

NR: = NONRENORMALIZABLE

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NR: A Toy Model

$$A_{g_0} = \int_0^T \left\{ \frac{1}{2}[\dot{y}(t)^2 - y(t)^2] - g_0 y(t)^{-4} \right\} dt$$

$$\mathfrak{D}(A_{g_0=0}) \neq \mathfrak{D}(A_{g_0>0}) \qquad f \neq pf \quad \text{🙂}$$

Classical to Quantum

$$\langle y'', T | y', 0 \rangle_f = \sum_{n=0,1,2,3,\cdots} h_n(y'') h_n(y') e^{-i(n+1/2)T/\hbar}$$

$$\langle y'', T | y', 0 \rangle_{pf} = 2\, \theta(y'' y') \sum_{n=1,3,5,7,\cdots} h_n(y'') h_n(y') e^{-i(n+1/2)T/\hbar}$$

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Affine Quantization-1

affine classical position and dilation variables

$$\{q,p\} = 1 \,, \quad q\{q,p\} = q \,, \quad \{q,qp\} \equiv \{q,d\} = q \lesseqgtr 0$$

affine quantum position and dilation variables

$$[Q,P] = i\hbar I \,, \quad Q[Q,P] = i\hbar Q \,, \quad [Q,QP] = i\hbar Q$$
$$= [Q,(QP+PQ)+(QP-PQ)]/2 = i\hbar Q$$
$$= [Q,(QP+PQ)/2] \equiv [Q,D] = i\hbar Q \,, \quad Q \lesseqgtr 0$$

the Lie algebra for the affine group

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Affine Quantization-2

canonical coherent states

affine coherent states

$$|p, q\rangle = e^{-iqP/\hbar} \, e^{ipQ/\hbar} \, |0\rangle \qquad\qquad |p, q\rangle = e^{ipQ/\hbar} e^{-i \ln(q)D/\hbar} \, |b\rangle$$

action for Schrödinger's equation

$$A_Q = \int \langle \psi(t) | [i\hbar(\partial/\partial t) - \mathscr{H}(P,Q)] | \psi(t)\rangle \, dt$$

$$A_Q' = \int \langle \psi(t) | [i\hbar(\partial/\partial t) - \mathscr{H}'(D,Q)] | \psi(t)\rangle \, dt$$

action for enhanced classical equations

$$A_C = \int \langle p(t), q(t) | [i\hbar(\partial/\partial t) - \mathscr{H}(P,Q)] | p(t), q(t)\rangle \, dt$$

$$= \int \{ p(t)\dot{q}(t) - H(p(t), q(t)) \} \, dt$$

$$A_C' = \int \langle p(t), q(t) | [i\hbar(\partial/\partial t) - \mathscr{H}'(D,Q)] | p(t), q(t)\rangle \, dt$$

$$= \int \{ -q(t)\dot{p}(t) - H'(p(t), q(t)) \} \, dt,$$

Both operator pairs lead to similar classical stories and with hbar > 0.

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Favored Coordinates-1

Dirac: "Cartesian coordinates should lead to H(p,q)—>H(P,Q)"

canonical quantization

$$\underline{0}$$

$$H(p,q) = \langle p,q \,|\, \mathscr{H}(P,Q) \,|\, p,q \rangle \,, \qquad (\omega Q + iP)\,|0\rangle = 0$$

$$= \langle 0 \,|\, \mathscr{H}(P+p, Q+q) \,|\, 0 \rangle = \mathscr{H}(p,q) + \mathscr{O}(\hbar; p,q)$$

$$2\hbar[\,\|\,d|p,q\rangle\,\|^2 - |\langle p,q\,|\,d|p,q\rangle|^2\,] = \omega^{-1}dp^2 + \omega dq^2 \quad \text{🙂}$$

affine quantization

$$\underline{-2/b}$$

$$H'(pq,q) = \langle p,q \,|\, \mathscr{H}'(D,Q) \,|\, p,q \rangle \,, \quad [(Q-1)+iD/b]\,|b\rangle = 0$$

$$= \langle b \,|\, \mathscr{H}'(D + pqQ, qQ) \,|\, b \rangle = \mathscr{H}'(pq,q) + \mathscr{O}'(\hbar; p,q)$$

$$2\hbar[\,d|p,q\rangle\,\|^2 - |\langle p,q\,|\,d|p,q\rangle|^2\,] = b^{-1}q^2 dp^2 + bq^{-2}dq^2 \quad \text{🙂}$$

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NR: Ultralocal Model-1

$$H(\pi, \varphi) = \int \{\tfrac{1}{2}[\pi(x)^2 + m_0^2 \varphi(x)^2] + g_0 \varphi(x)^4\} \, d^s x, \quad s \geq 1$$

affine variables

$f \neq pf$

$$\kappa(x) \equiv \pi(x)\varphi(x), \quad \varphi(x) \lessgtr 0$$

$$\{\varphi(x), \kappa(x')\} = \delta^s(x - x') \, \varphi(x), \quad \varphi(x) \lessgtr 0$$

$$H'(\kappa, \varphi) = \int \{\tfrac{1}{2}[\kappa(x)\varphi(x)^{-2}\kappa(x) + m_0^2 \varphi(x)^2] + g_0 \varphi(x)^4\} \, d^s x$$

Classical to Quantum

$$[\hat{\varphi}(x), \hat{\kappa}(x')] = i\hbar \, \delta^s(x - x') \, \hat{\varphi}(x), \quad \hat{\varphi}(x) \lessgtr 0 \qquad 🙂 \quad 🙂$$

$$\hat{\varphi}(x) = \int B(x, \lambda)^\dagger \lambda B(x, \lambda) \, d\lambda, \qquad B(x, \lambda) = A(x, \lambda) + c(\lambda)I, \quad A(x, \lambda)|0\rangle = 0$$

$$\hat{\kappa}(x) = -i\hbar \tfrac{1}{2} \int B(x, \lambda)^\dagger [\lambda(\partial/\partial\lambda) + (\partial/\partial\lambda)\lambda] B(x, \lambda) \, d\lambda$$

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NR: Ultralocal Model-2

$$\hat{\varphi}(x) = \varphi(x) \;\;,\;\; \hat{\kappa}(x) = -i\hbar(1/2)[\varphi(x)\,(\delta/\delta\varphi(x)) + (\delta/\delta\varphi(x))\,\varphi(x)]$$

Schrödinger's equation

$$i\hbar\,\partial\Psi(\varphi,t)/\partial t = \left[\int \{(1/2)[\hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) + m_0^2\varphi(x)^2] + g_0\varphi(x)^4\}\; d^s x\right]\Psi(\varphi,t)$$

$$\hat{\kappa}(x)\,\varphi(x)^{-1/2} = 0\;,\;\;\;\; \Psi(\varphi) = e^{-W(\varphi)}\,\Pi_x\,|\varphi(x)|^{-1/2} \;\;\; \text{formal}$$

regularization

$$\varphi(x) \to \varphi_{\mathbf{k}} \equiv \varphi(\mathbf{k}a)\,,\;\; a > 0\,,\;\; \mathbf{k} \in \{0, \pm 1, \pm 2, \pm 3, \cdots\}^s$$

$$\Psi_r(\varphi) = e^{-W_r(\varphi)}\,\Pi_{\mathbf{k}}(ba^s)^{1/2}\,|\varphi_{\mathbf{k}}|^{-(1-2ba^s)/2} \;\;\; \text{regularized}$$

$$\int |\Psi_r(\varphi)|^2\,\Pi_{\mathbf{k}}d\varphi_{\mathbf{k}} = 1$$

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NR: Ultralocal Model-3

$$\Psi_r(\varphi) = \Pi_{\mathbf{k}} e^{-W_r(\varphi_{\mathbf{k}})} (ba^s)^{1/2} |\varphi_{\mathbf{k}}|^{-(1-2ba^s)/2} \quad \ddot\smile$$

characteristic functional

$$C(f) = \lim_{a\to 0} \Pi_{\mathbf{k}} \int e^{if_{\mathbf{k}}\varphi_{\mathbf{k}}/\hbar} (ba^s) e^{-2W_r(\varphi_{\mathbf{k}})} |\varphi_{\mathbf{k}}|^{-(1-2ba^s)} d\varphi_{\mathbf{k}}$$

$$C(f) = \lim_{a\to 0} \Pi_{\mathbf{k}} \int \{ 1 - (ba^s) \int [1 - e^{if_{\mathbf{k}}\varphi_{\mathbf{k}}/\hbar}] e^{-2W_r(\varphi_{\mathbf{k}})} |\varphi_{\mathbf{k}}|^{-(1-2ba^s)} d\varphi_{\mathbf{k}} \}$$

$$C(f) = \exp\{ -b \int d^s x \int [1 - e^{if(x)\lambda/\hbar}] e^{-2w(\lambda)} d\lambda / |\lambda| \} \quad \textbf{Poisson} \; \ddot\smile$$

pseudofree model: equal-spacing spectrum, NO zero-point energy

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NR: Covariant Model-1

$$s \geq 4 \qquad f \neq pf$$

$$H(\pi, \varphi) = \int \{(1/2)[\pi(x)^2 + (\nabla \varphi)(x)^2 + m_0^2 \varphi(x)^2] + g_0 \varphi(x)^4\} \, d^s x$$

affine variables

$$\kappa(x) \equiv \pi(x)\varphi(x), \quad \varphi(x) \lessgtr 0$$

$$H'(\kappa, \varphi) = \int \{(1/2)[\kappa(x)\varphi(x)^{-2}\kappa(x) + (\nabla \varphi)(x)^2 + m_0^2 \varphi(x)^2] + g_0 \varphi(x)^4\} \, d^s x$$

Classical to Quantum

$$\hat{\varphi}(x) = \varphi(x) \quad , \qquad \hat{\kappa}(x) = -i\hbar(1/2)[\varphi(x)(\delta/\delta\varphi(x)) + (\delta/\delta\varphi(x))\varphi(x)]$$
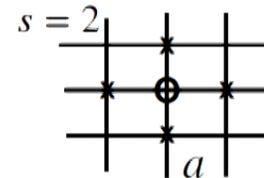
Schrödinger's equation

$$i\hbar \, \partial\Psi(\varphi, t)/\partial t = \left[\int \{(1/2)[\hat{\kappa}(x)\varphi(x)^{-2}\hat{\kappa}(x) + (\nabla \varphi)(x)^2 + m_0^2 \varphi(x)^2] + g_0 \varphi(x)^4\} \, d^s x \right] \Psi(\varphi, t)$$

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NR: Covariant Model-2

$$\hat{\kappa}(x)\,\varphi(x)^{-1/2} = 0 \;, \quad \Psi(\varphi) = e^{-Y(\varphi)}\,\Pi_x\,|\,\varphi(x)\,|^{-1/2} \quad \text{formal}$$

a different regularization

$$\varphi(x) \to \varphi_{\mathbf{k}} \equiv \varphi(\mathbf{k}a)\,, \quad a > 0\,, \quad \mathbf{k} \in \{0, \pm 1, \pm 2, \pm 3, \cdots\}^s$$

$$J_{\mathbf{k},\mathbf{l}} \equiv 1/(1 + 2s)\,, \quad \mathbf{l} = \mathbf{k} \text{ and } 2s \text{ nearest spatial neighbors to } \mathbf{k}\,; \text{ otherwise, } J_{\mathbf{k},\mathbf{l}} \equiv 0$$

$$\Psi_r(\varphi) = e^{-Y_r(\varphi)}\,\Pi_{\mathbf{k}}[\Sigma_{\mathbf{l}}J_{\mathbf{k},\mathbf{l}}\,\varphi_{\mathbf{l}}^2]^{-(1-2ba^s)/4} \quad \text{regularized} \;\; 🙂$$

$$1 = \int |\,\Psi_r(\varphi)\,|^2\,\Pi_{\mathbf{k}}d\varphi_{\mathbf{k}}\,, \quad \varphi_{\mathbf{k}} = \rho\eta_{\mathbf{k}}\,, \quad R = 2ba^s N' \quad \text{divergence free}$$

$$= \int e^{-2Y_r(\rho\eta)}\,\delta(1 - \Sigma_{\mathbf{k}}\eta_{\mathbf{k}}^2)\,\Pi_{\mathbf{k}}[\Sigma_{\mathbf{l}}J_{\mathbf{k},\mathbf{l}}\,\eta_{\mathbf{l}}^2]^{-(1-2ba^s)/2}\,d\eta_{\mathbf{k}}]\,\rho^{R-1}\,d\rho \quad 🙂 \; 🙂$$

There are indications that affine quantization of φ_4^4 models is non-trivial 🙂 🙂

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NR: Quantum Gravity-1

$$g_{ab}(x)\,dx^a\,dx^b > 0 \ , \quad g_{ac}(x)\,g^{bc}(x) \equiv \delta_a^b \ , \quad \pi^{ac}(x)\,g_{bc}(x) \equiv \pi_b^a(x) \ , \quad g(x) \equiv \det[g_{ab}(x)] > 0$$

$$H'(\pi, g) = \int \{ g^{-1/2}[\pi_b^a \pi_a^b - (1/2)\pi_a^a \pi_b^b] + g^{1/2}\,{}^{(3)}R \} \, d^3x$$

Classical to Quantum $\qquad f \neq pf$

$$[\hat{\pi}_b^a(x), \hat{\pi}_d^c(x')] = i\hbar(1/2)\,\delta^3(x, x')\,[\delta_d^a \hat{\pi}_b^c(x) - \delta_b^c \hat{\pi}_d^a(x)]$$

$$[\hat{g}_{ab}(x), \hat{\pi}_d^c(x')] = i\hbar(1/2)\,\delta^3(x, x')\,[\delta_a^c \hat{g}_{bd}(x) + \delta_b^c \hat{g}_{ad}(x)]$$

$$[\hat{g}_{ab}(x), \hat{g}_{cd}(x')] = 0 \ , \qquad \hat{g}(x) \equiv \det[\hat{g}_{ab}(x)] > 0 \qquad \text{🙂 🙂}$$

$$\hat{g}_{ab}(x) = \int_+ B^\dagger(x, \gamma)\gamma_{ab}B(x, \gamma)d\gamma \ , \qquad \{\gamma_{ab}\} > 0 \ , \ d\gamma \equiv \Pi_{a \leq b}\, d\gamma_{ab}$$

$$\hat{\pi}_b^a(x) = -i\hbar(1/2)\int_+ \{B(x, \gamma)^\dagger[\gamma_{bc}(\partial/\partial\gamma_{ac}) + (\partial/\partial\gamma_{ac})\gamma_{bc}]B(x, \gamma)\}d\gamma$$

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NR: Quantum Gravity-2

$$\hat{g}_{ab}(x) = g_{ab}(x)$$
$$\hat{\pi}_b^a(x) = -i\hbar(1/2)[g_{bc}(x)(\delta/\delta g_{ac}(x)) + (\delta/\delta g_{ac}(x))g_{bc}(x)]$$

Schrödinger's equation

$$i\hbar\,\partial\Psi(\{g\},t)/\partial t = \left[\int[\hat{\pi}_b^a(x)g(x)^{-1/2}\hat{\pi}_a^b(x) - \frac{1}{2}\hat{\pi}_a^a(x)g(x)^{-1/2}\hat{\pi}_b^b(x)\right.$$
$$\left. + g(x)^{1/2\,(3)}R(x)]\,d^3x\right]\Psi(\{g\},t) \quad 🙂$$

$$\hat{\pi}_b^a(x)\,g(x)^{-1/2} = 0\ ,$$

N.B. The field $\pi^{ab}(x)$ is NOT made an operator.

$$\Psi(\{g\}) = Y(\{g\})\,\Pi_x\,g(x)^{-1/2} \qquad \textbf{formal}$$
$$\Psi_r(\{g\}) = Y_r(\{g\})\Pi_{\mathbf{k}}\,[\Sigma_{\mathbf{l}}J_{\mathbf{k},\mathbf{l}}\,g_{\mathbf{l}}]^{-(1-ba^3)/2} \qquad \textbf{regularized} \quad 🙂$$

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NR: Quantum Gravity-3

affine gravity coherent states $\eta(x) = \{\eta_b^a(x)\}$

$$|\pi, \eta\rangle = e^{(i/\hbar) \int \pi^{ab}(x)\, \hat{g}_{ab}(x)\, d^3x}\, e^{-(i/\hbar) \int \eta_b^a(x)\, \hat{\pi}_a^b(x)\, d^3x}\, |\beta\rangle \quad [= |\pi, g\rangle]$$

$$\langle \pi, \eta |\, \hat{g}_{ab}(x)\, |\pi, \eta\rangle = [e^{\eta(x)/2}]_a^c\, \langle \beta |\, \hat{g}_{cd}(x)\, |\beta\rangle\, [e^{\eta(x)/2}]_b^d \equiv g_{ab}(x) \quad \text{🙂}$$

$$\langle \pi, \eta |\, \hat{\pi}_b^a(x)\, |\pi, \eta\rangle = \pi^{ac}(x)\, g_{cb}(x) \equiv \pi_b^a(x) \quad \text{🙂} \quad \text{🙂}$$

$$\langle \pi'', g'' | \pi', g' \rangle = \exp\Big\{ -2 \int \beta(x)\, d^3x \\
\times \ln\Big\{ \frac{\det\{\frac{1}{2}[g''^{ab}(x) + g'^{ab}(x)] + i\frac{1}{2\hbar}\beta(x)^{-1}[\pi''^{ab}(x) - \pi'^{ab}(x)]\}}{\det[g''^{ab}(x)]^{1/2}\, \det[g'^{ab}(x)]^{1/2}} \Big\} \Big\}$$

ultralocal : NO constraints imposed

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Favored Coordinates-2

enhanced classical Hamiltonian density

$$H'(\pi_b^a(x), g_{cd}(x)) = \langle \pi, g \,|\, \mathcal{H}'(\hat{\pi}_b^a(x), \hat{g}_{cd}(x)) \,|\, \pi, g \rangle$$

$$= \langle \beta \,|\, \mathcal{H}'(\hat{\pi}_b^a(x) + \pi^{aj}(x)[e^{\eta(x)/2}]_j^e \hat{g}_{ef}(x)[e^{\eta(x)/2}]_b^f, \, [e^{\eta(x)/2}]_c^e \hat{g}_{ef}(x)[e^{\eta(x)/2}]_d^f) \,|\, \beta \rangle$$

$$\langle \beta \,|\, [e^{\eta(x)/2}]_c^e \hat{g}_{ef}(x)[e^{\eta(x)/2}]_d^f \,|\, \beta \rangle = [e^{\eta(x)/2}]_c^e \langle \beta \,|\, \hat{g}_{ef}(x) \,|\, \beta \rangle [e^{\eta(x)/2}]_d^f \equiv g_{cd}(x)$$

$$= \mathcal{H}'(\pi^{aj}(x) g_{jb}(x), g_{cd}(x)) + \mathcal{O}'(\hbar; \pi, g)$$

$$= \mathcal{H}'(\pi_b^a(x), g_{cd}(x)) + \mathcal{O}'(\hbar; \pi, g) \qquad 🙂 \quad 🙂$$

these are favored affine gravity coordinates

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Favored Coordinates-3

$$|p, q\rangle = e^{-iqP/\hbar} e^{ipQ/\hbar} |0\rangle , \qquad (\omega Q + iP)|0\rangle = 0$$

$$2\hbar[\| d|p, q\rangle \|^2 - |\langle p, q | d|p, q\rangle|^2] = \omega^{-1} dp^2 + \omega dq^2$$

$$|p, q\rangle = e^{ipQ/\hbar} e^{-i\ln(q)D/\hbar} |b\rangle , \qquad [(Q-1) + iD/b]|b\rangle = 0$$

$$2\hbar[\| d|p, q\rangle \|^2 - |\langle p, q | d|p, q\rangle|^2] = b^{-1} q^2 dp^2 + bq^{-2} dq^2$$

$$|\pi, g\rangle = e^{(i/\hbar) \int \pi^{ab}(x) \, \hat{g}_{ab}(x) \, d^3x} \, e^{-(i/\hbar) \int \eta_b^a(x) \, \hat{\pi}_a^b(x) \, d^3x} |\beta\rangle$$

$$C\hbar[\| d|\pi, g\rangle \|^2 - |\langle \pi, g | d|\pi, g\rangle|^2]$$

$$= \int [(\beta(x)\hbar)^{-1} g_{ab} g_{cd} \, d\pi^{bc} d\pi^{da} + (\beta(x)\hbar) \, g^{ab} g^{cd} \, dg_{bc} dg_{da}] \, d^3x$$

Fubini-Study metrics

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NR: Quantum Gravity-4

$$|\pi, g\rangle = e^{(i/\hbar) \int \pi^{ab}(x)\, \hat{g}_{ab}(x)\, d^3x}\, e^{-(i/\hbar) \int \eta_b^a(x)\, \hat{\pi}_a^b(x)\, d^3x}\, |\beta\rangle$$

$$\langle \pi, g \,|\, \pi', g'\rangle = \exp\left\{ -2 \int \beta(x) \ln\left\{ \frac{\det\{\frac{1}{2}[g^{ab}(x) + g'^{ab}(x)] + i\frac{1}{2\hbar}\beta(x)^{-1}[\pi^{ab}(x) - \pi'^{ab}(x)]\}}{\det[g^{ab}(x)]^{1/2}\ \det[g'^{ab}(x)]^{1/2}} \right\} d^3x \right\}$$

complex polarization

$$C_s^r(x)\,\langle \pi, g \,|\, \Psi\rangle \equiv \left[-i\hbar g^{rt}(x)\frac{\delta}{\delta \pi^{st}(x)} + \delta_s^r + \beta(x)^{-1} g_{st}(x)\frac{\delta}{\delta g_{rt}(x)} \right]\langle \pi, g \,|\, \Psi\rangle = 0$$

$$A \equiv \frac{1}{2\hbar}\int C_r^s(x)^\dagger C_s^r(x)\,\beta(x)\, d^3x \quad , \quad \lim_{\nu \to \infty} \mathcal{N}_\nu e^{-\nu\, TA}\, \delta\{\pi - \pi'\}\delta\{g - g'\} = \langle \pi, g \,|\, \pi', g'\rangle$$

an analog to Wiener measure

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NR: Quantum Gravity-5

$$\pi'' = \pi^{ab}(\,\cdot\,,T)\,,\ g'' = g_{ab}(\,\cdot\,,T) \quad;\quad \pi' = \pi^{ab}(\cdot,0)\,,\ g' = g_{ab}(\cdot,0) \quad;\quad T > 0$$

kinematic and physical functional integrals

$$\langle \pi'', g'' \,|\, \pi', g' \rangle = \lim_{\nu \to \infty} \mathcal{N}_\nu \int \exp\big[-(i/\hbar) \int g_{ab}\,\dot\pi^{ab}\,d^3x\,dt \big]$$

$$\times \exp\big\{ -(1/2\nu\hbar) \int [(\beta(x)\hbar)^{-1} g_{ab}g_{cd}\,\dot\pi^{bc}\dot\pi^{da} + (\beta(x)\hbar)\, g^{ab}g^{cd}\dot g_{bc}\dot g^{da}]\,d^3x\,dt \big\}$$

$$\times \Pi_{x,t}\Pi_{[a \le b]}\,d\pi^{ab}(x,t)\,dg_{ab}(x,t) \qquad \text{🙂}$$

projection operator IE enforces the constraints

$$\langle \pi'', g'' \,|\, \mathbb{E} \,|\, \pi', g' \rangle = \lim_{\nu \to \infty} \mathcal{N}'_\nu \int \exp\big\{ -(i/\hbar) \int [g_{ab}\,\dot\pi^{ab} + N^a H_a + N H]\,d^3x\,dt \big\}$$

$$\times \exp\big\{ -(1/2\nu\hbar) \int [(\beta(x)\hbar)^{-1} g_{ab}g_{cd}\,\dot\pi^{bc}\dot\pi^{da} + (\beta(x)\hbar)\, g^{ab}g^{cd}\dot g_{bc}\dot g^{da}]\,d^3x\,dt \big\}$$

$$\times [\Pi_{x,t}\Pi_{[a \le b]}\,d\pi^{ab}(x,t)\,dg_{ab}(x,t)\,]\,\mathscr{D}R\{N^a, N\} \qquad \text{🙂}\ \ \text{🙂}$$

first and second class constraints

Comparison List

The Canonical Story

$$g(x) = \det[g_{ab}(x)] \geqslant 0$$

complex variables

part Euclidean

discrete metric

$$\{\tilde{E}^a_i(x), A_b^{\,j}(y)\} = -i\delta^a_b\delta^j_i\,\delta^3(x,y)$$

$$\hat{\tilde{E}}^a_i(x) = -i\hbar\,\delta/\delta A_a^{\,i}(x)$$

**FAVORED CANONICAL
VARIABLES ???**

**non-Cartesian
coordinates yield a
FALSE quantum theory**

The Affine Story

$$g(x) = \det[g_{ab}(x)] > 0$$

real variables

all Lorentzian

continuous metric*   *C.R. lives*

$$[\hat{\pi}^a_b(x), \hat{\pi}^c_d(y)] = i\hbar(1/2)\,\delta^3(x,y)\,[\delta^a_d\hat{\pi}^c_b(x) - \delta^c_b\hat{\pi}^a_d(x)]$$

$$[\hat{g}_{ab}(x), \hat{\pi}^c_d(y)] = i\hbar(1/2)\,\delta^3(x,y)\,[\delta^c_a\hat{g}_{bd}(x) + \delta^c_b\hat{g}_{ad}(x)]$$

$$[\hat{g}_{ab}(x), \hat{g}_{cd}(y)] = 0$$

FAVORED AFFINE   🙂
VARIABLES !!!

**proper affine
coordinates yield a**   🙂
VALID quantum theory

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Today's Message

1. Nonrenormalizable models are <u>NOT</u> continuously connected to their own free model.

2. If Q&P then Q&D, where D=(PQ+QP)/2. Note that [Q,P]=i hbar, and then [Q,D]=i hbar Q, which is the <u>Lie algebra of the affine group.</u>

3. The favored pair q&p to promote are "Cartesian coordinates" which make a <u>flat plane.</u> The favored pair q&pq to promote have an affine form on a Lobachevsky plane of <u>constant negative curvature.</u>

4. Canonical quantization of nonrenormalizable models fails, but affine quantization is <u>successful.</u>

5. Affine quantization of gravity offers self-adjoint momentric and metric fields that <u>respect positivity requirements.</u>

THANK YOU

arXiv:1811.09582

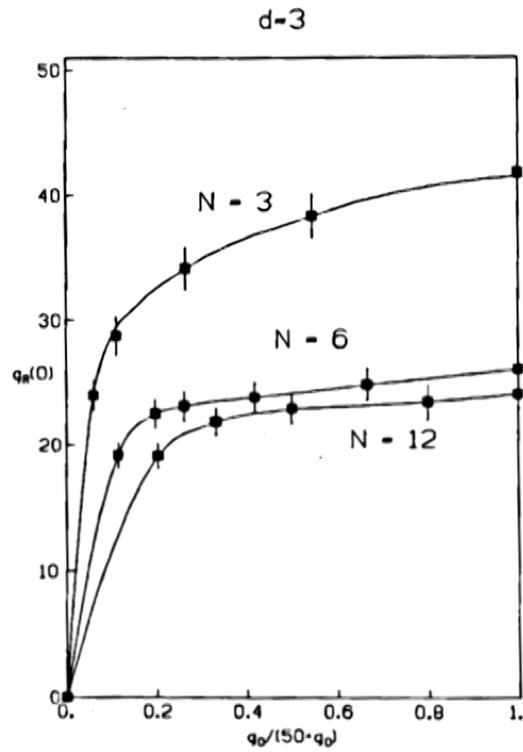**arXiv:1903.11211**

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Freedman, et al, 1982

Fig. 3. The zero-momentum coupling constant $g_R(0)$ as a function of the bare coupling constant ratio $g_0/(50 + g_0)$ for $(\phi^4)_3$ on lattices of size $N = 3$, 6 and 12.
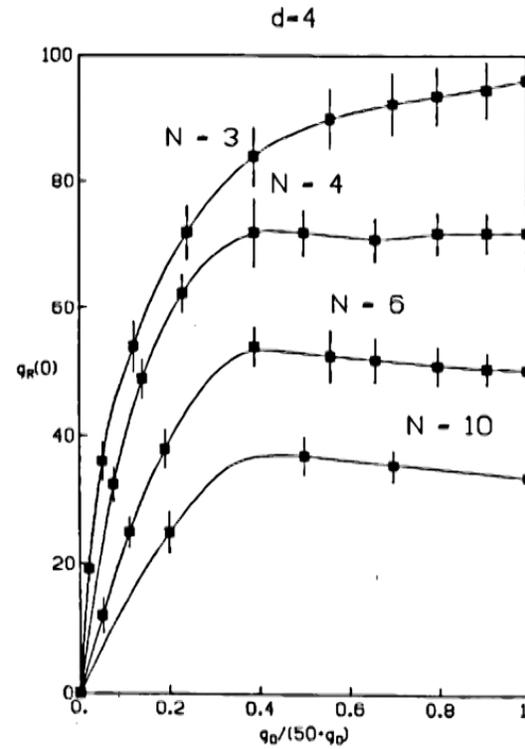
Fig. 1. The zero-momentum renormalized coupling constant $g_R(0)$ as a function of the bare coupling constant ratio $g_0/(50 + g_0)$ for $(\phi^4)_4$ on lattices of sizes $N = 3$, 4, 6 and 10.

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Phi^4_4 With Counter Term

E=04 d=4