

Title: Anomalies and the Conformal Bootstrap

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Collection: Bootstrap 2019

Date: July 31, 2019 - 11:30 AM

URL: <http://pirsa.org/19070068>

Anomalies of the Conformal Bootstrap

1904.04833 + work in progress
w/ Ying-Hsuan Lin

ϵ Holographic
Anomaly

Conformal
Bootstrap



Bootstrap

In CFT with global symmetry G
and 't Hooft anomaly α

1. Is there an upper bound
on the lightest G -charged op.?

Bootstrap

In CFT with global symmetry G
and 't Hooft anomaly α

1. Is there an upper bound
on the lightest G -charged op.?
2. How does the bound depend
on the anomaly α ?

G Today: 2d Bosonic CFT

$$G = \mathbb{Z}_2$$

Non-Anomalous

Anomalous

? Bound on
 $\mathbb{Z}_2$ -odd op.

X

✓

◦ Symmetry and Anomaly in 2D

1)
anomalous
D-form
global sym

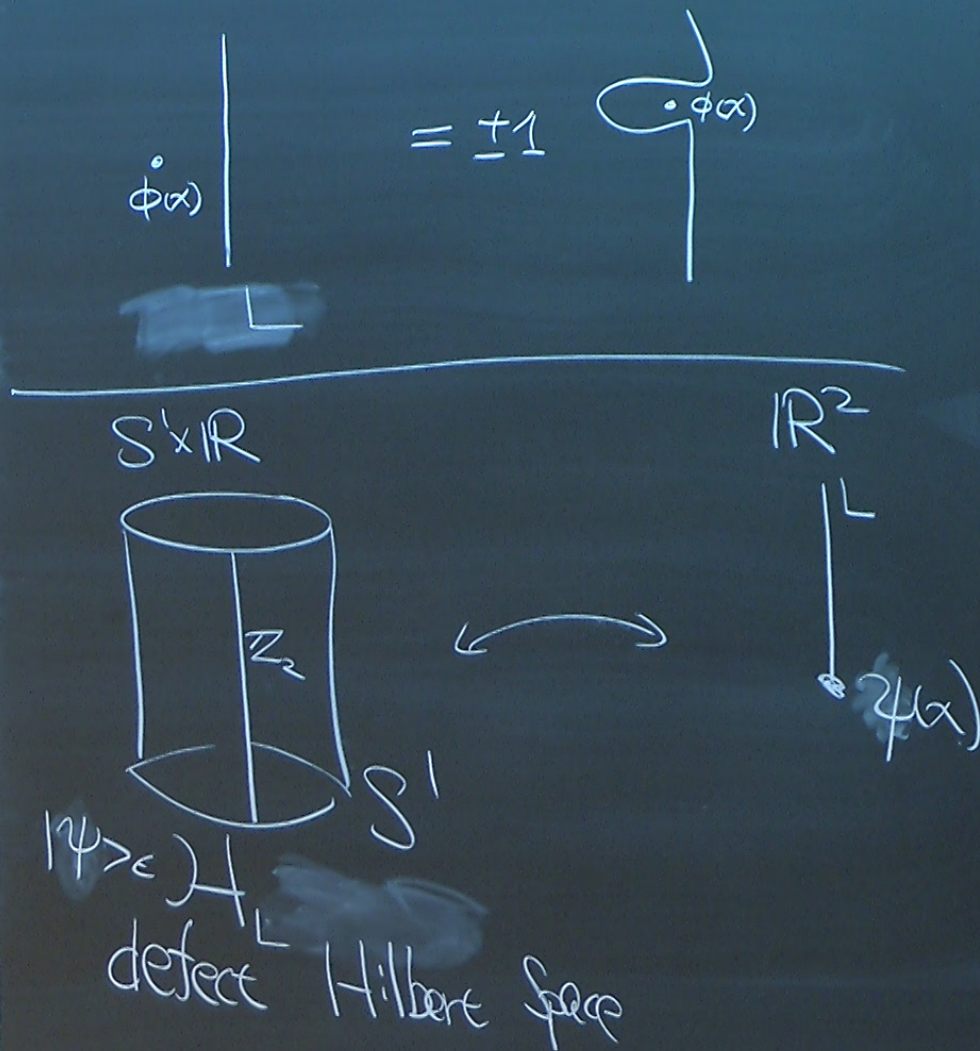
$\Rightarrow$ Codim-1
Topological Defect

$$\text{e.g. } G=U(1) \Rightarrow \vec{j}_\mu(x), \partial^\mu j_\mu = 0$$

$$\Rightarrow U_\theta = e^{i\theta \int \vec{j}}$$

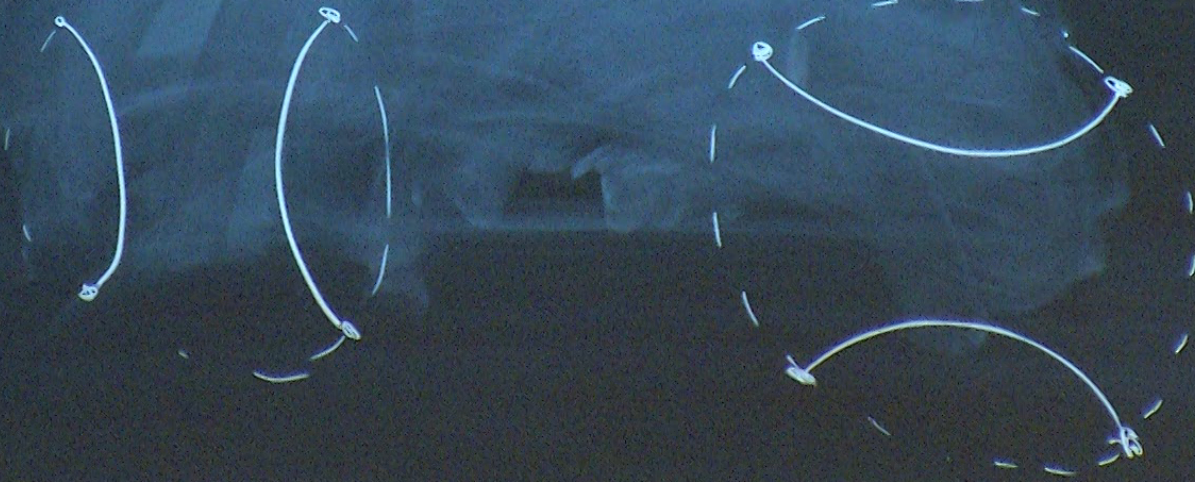
$m-1$
 Logical Defect
 line in 2D)

$$\partial^a j_a = 0$$



w/ Ying-Hsuan Lin

Crossing



1.

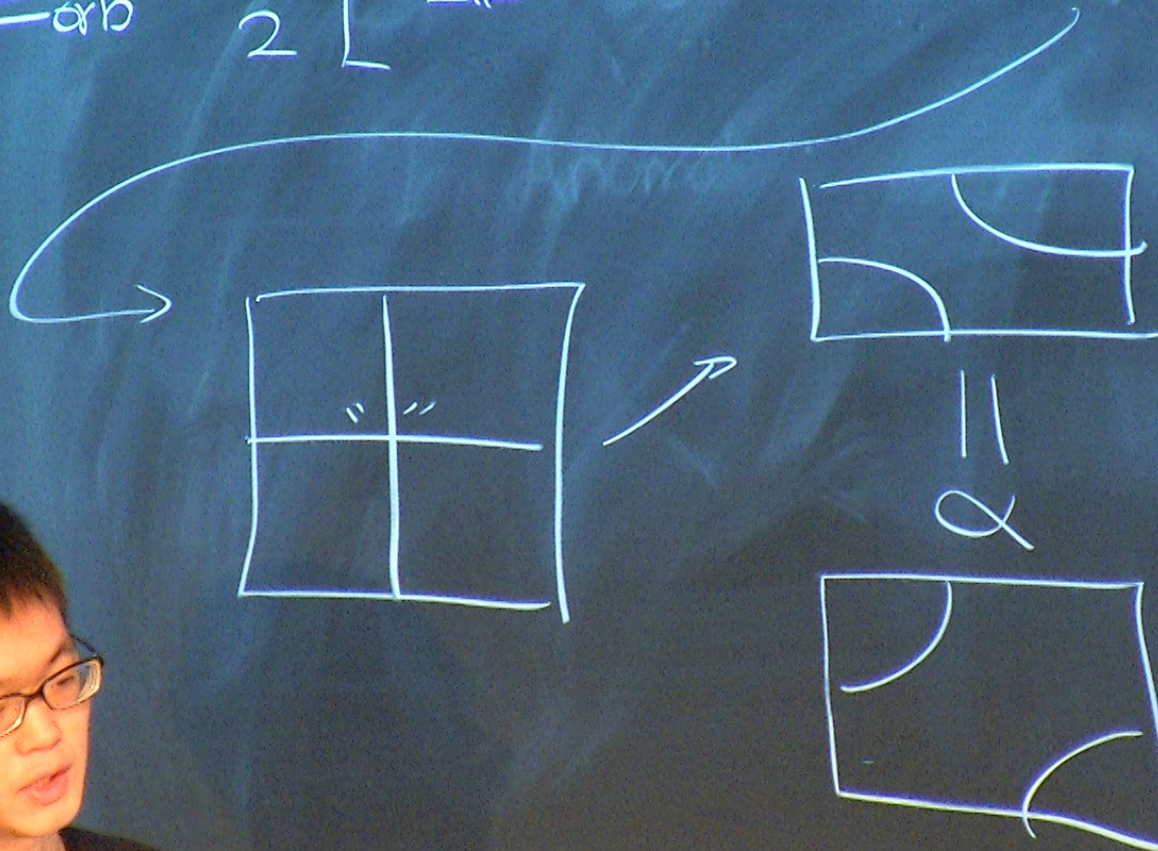
on

2. H

D

progress

$$Z_{\text{orb}} = \frac{1}{2} \left[\begin{array}{|c|} \hline \text{diagram 1} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{diagram 2} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{diagram 3} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{diagram 4} \\ \hline \end{array} \right]$$



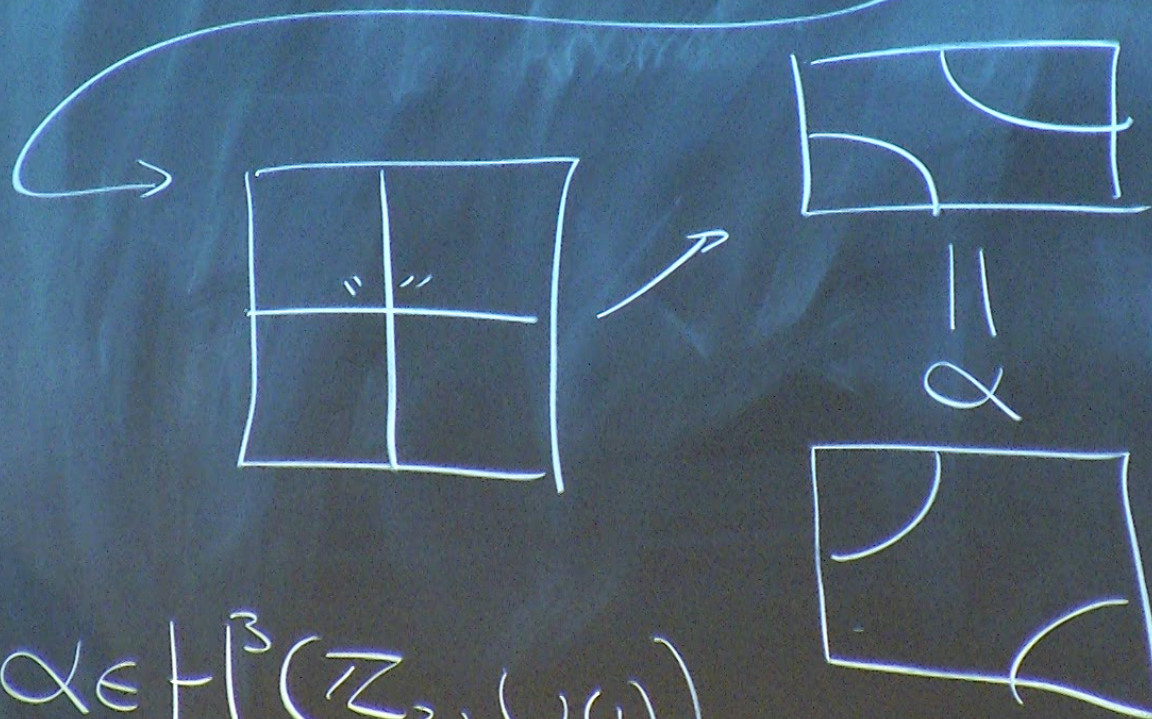
◦ Symmetry

0-f
global

e.g. $G = \dots$

$\Rightarrow$

$$Z_{\text{orb}} = \frac{1}{2} \left[\begin{array}{|c|} \hline \text{---} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} \right]$$



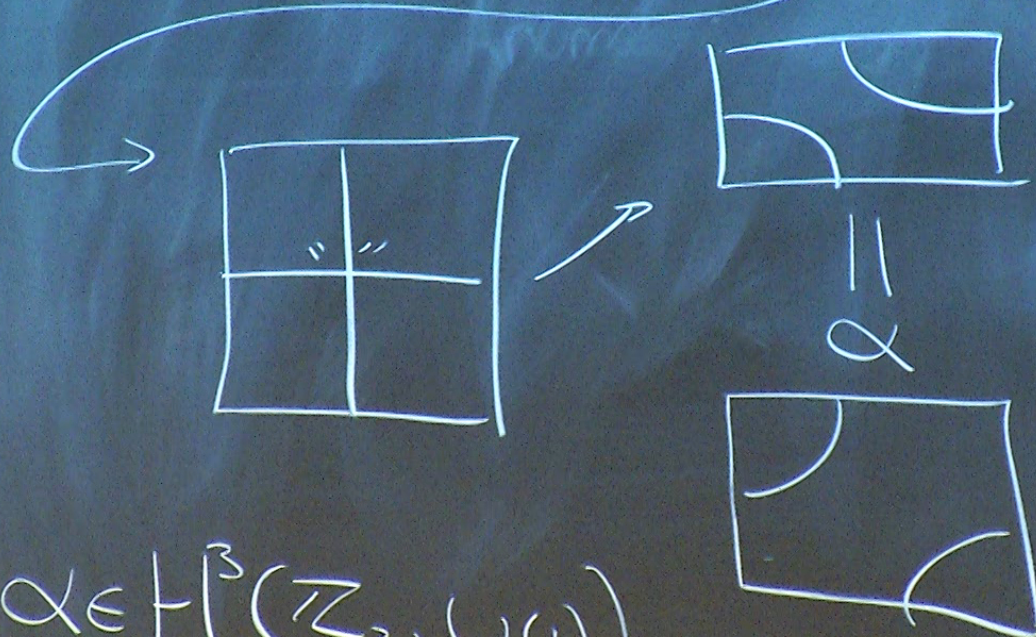
$$\alpha \in H^3(\mathbb{Z}_2, \mathbb{C}(1))$$

0-fc
global

e.g. $G = U$



$$Z_{\text{orb}} = \frac{1}{2} \left[\begin{array}{|c|} \hline \text{---} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{---} \\ \hline \end{array} \right]$$



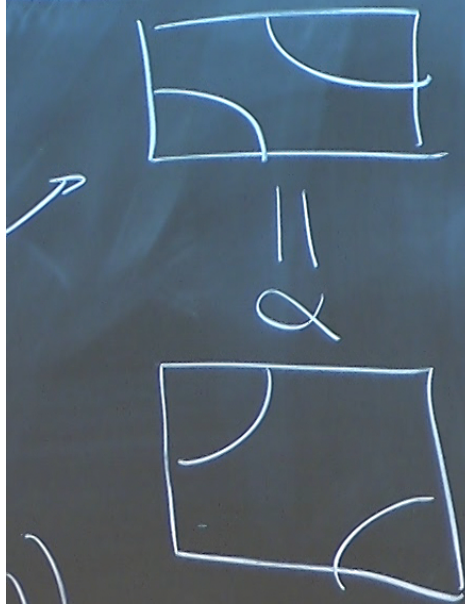
$$\alpha \in H^3(\mathbb{Z}_2, \mathbb{C}(1))$$

$$= \mathbb{Z}_2$$

-anomalies

omaloes

$$\boxed{} + \boxed{} + \boxed{} \Big]$$



o Bootstrapping

* Positivity

$$\boxed{} = \text{Tr}_{\mathcal{H}} \left[e^{h - \frac{c}{24}} e^{\bar{h} - \frac{c}{24}} \right]$$

$$= \sum_{h, \bar{h} \in \mathcal{H}} \frac{n_{h, \bar{h}}}{\text{IV}_0} \chi_h(\tau) \chi_{\bar{h}}(\bar{\tau})$$

$$\boxed{} = \text{Tr}_{\mathcal{H}} \left[\mathbb{Z}_2 e^{h - \frac{c}{24}} e^{\bar{h} - \frac{c}{24}} \right]$$

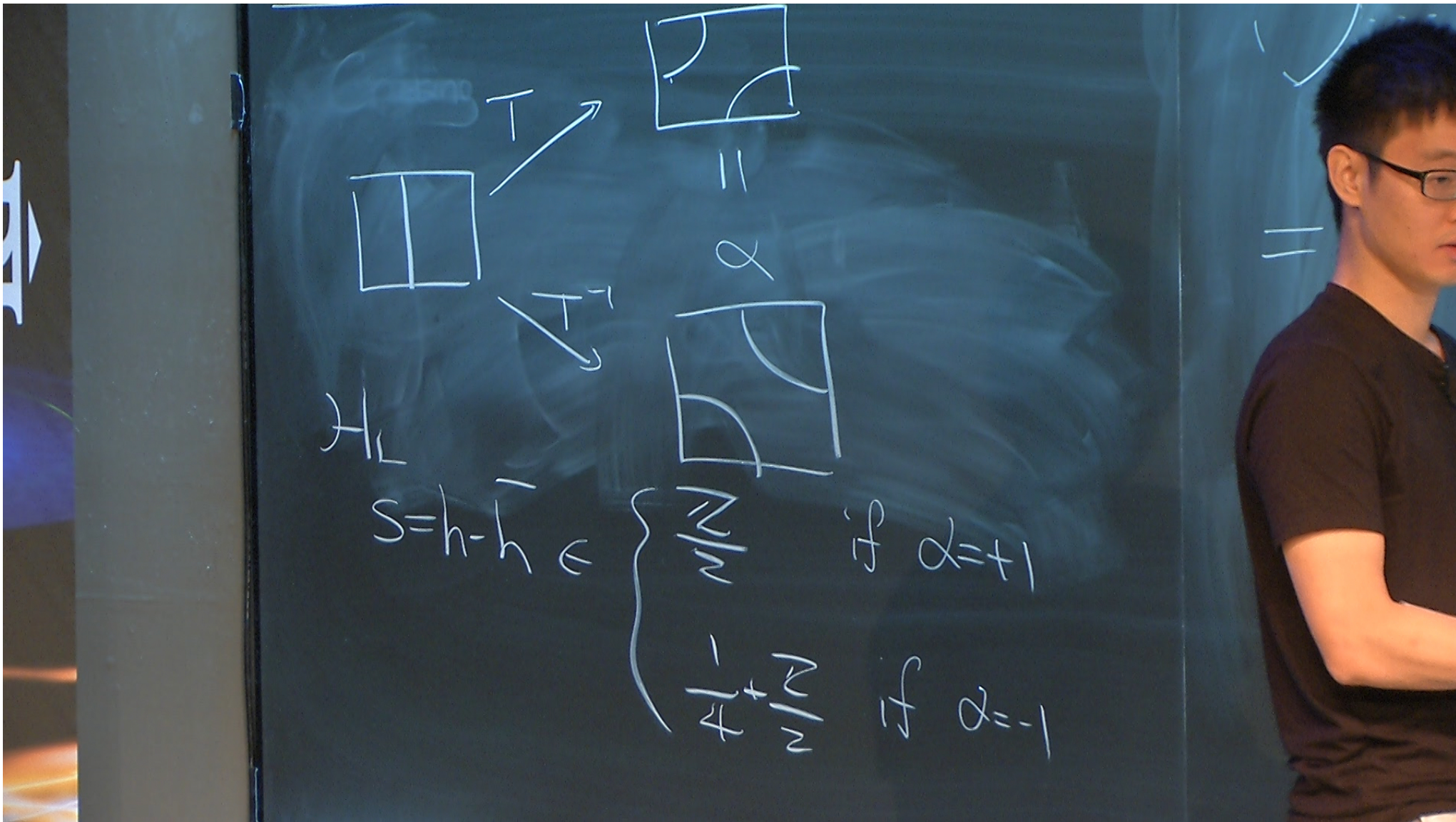
$$= \sum_{h, \bar{h} \in \mathcal{H}} (n_{h, \bar{h}}^+ - n_{h, \bar{h}}^-) \chi_h \chi_{\bar{h}}$$

$$\boxed{\square} = \text{Tr}_{\mathcal{H}_L} \left[q^{h-\frac{c}{24}} \bar{q}^{\bar{h}-\frac{\bar{c}}{24}} \right]$$

Positivity

$$\frac{1}{2} \left[\boxed{\square} \pm \boxed{\Xi} \right]$$

$$\boxed{\square}$$

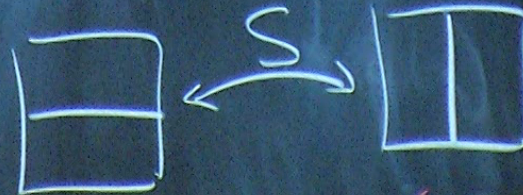
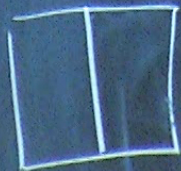
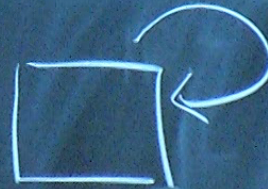


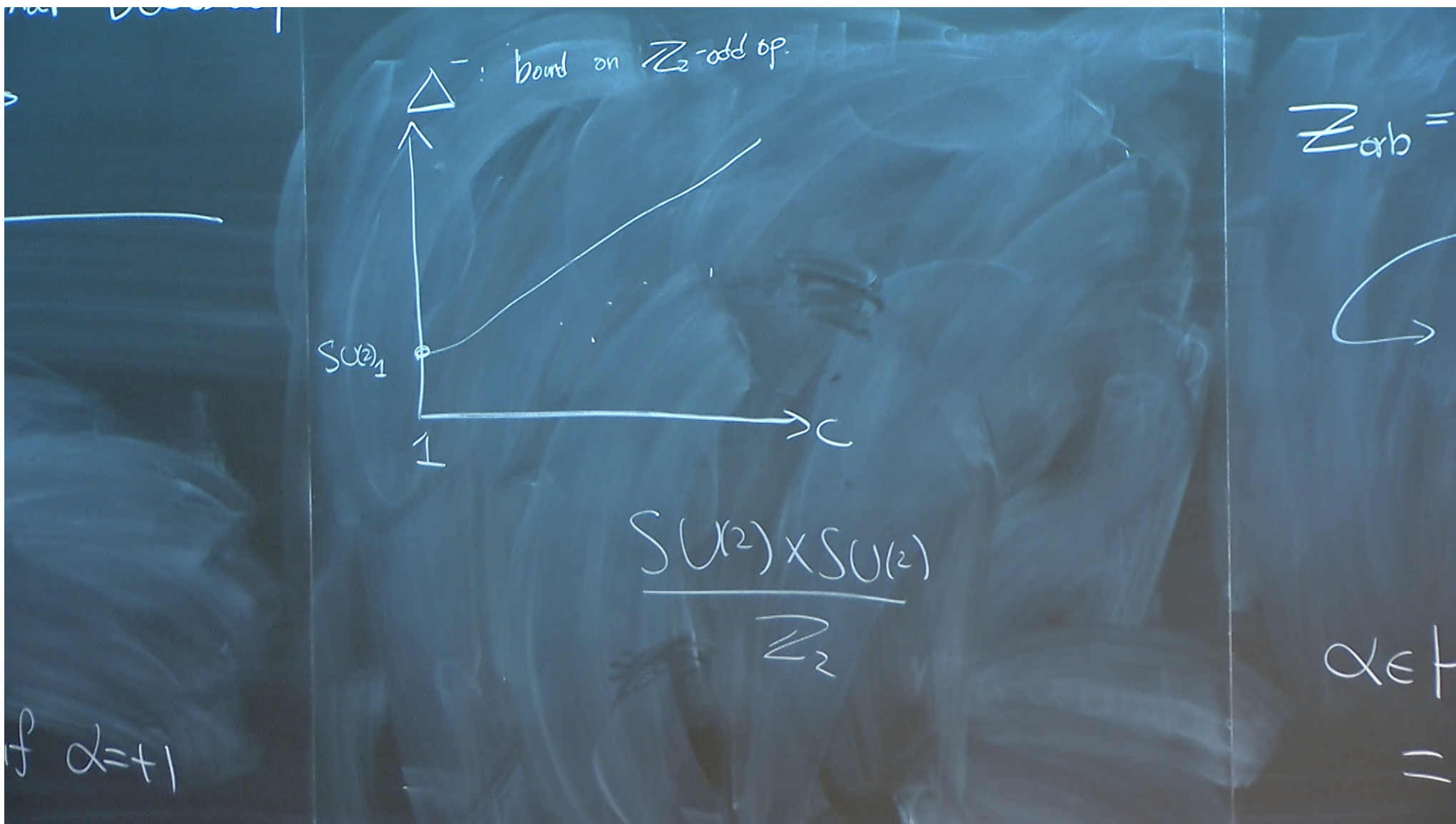
$$\boxed{\square} = \text{Tr}_{\mathcal{H}_L} \left[q^{h-\frac{c}{24}} \bar{q}^{\bar{h}-\frac{c}{24}} \right]$$

Positivity

$$\frac{1}{2} \left[\boxed{\square} \pm \boxed{\equiv} \right]$$

S Crossing





2d CFT with $U(1)$

holo $U(1) \Rightarrow$ boson
anomalies

2d $c=1$ compact boson

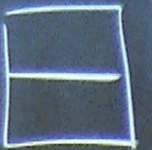
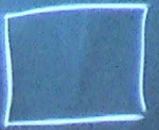
anomaly $U(1)_{\text{momentum}}$ $U(1)_{\text{winding}}$

Δ^-

KK mode $\frac{1}{2R^2}$

winding $\frac{R^2}{2}$

* Pos



2d CFT with $U(1)$

holo $U(1) \Rightarrow$ boson

anomalies

2d $c=1$ compact boson

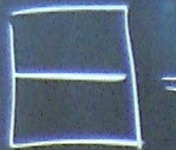
anomaly $U(1)_{\text{momentum}}$ $U(1)_{\text{winding}}$ $U(1)_{\text{diag}}$

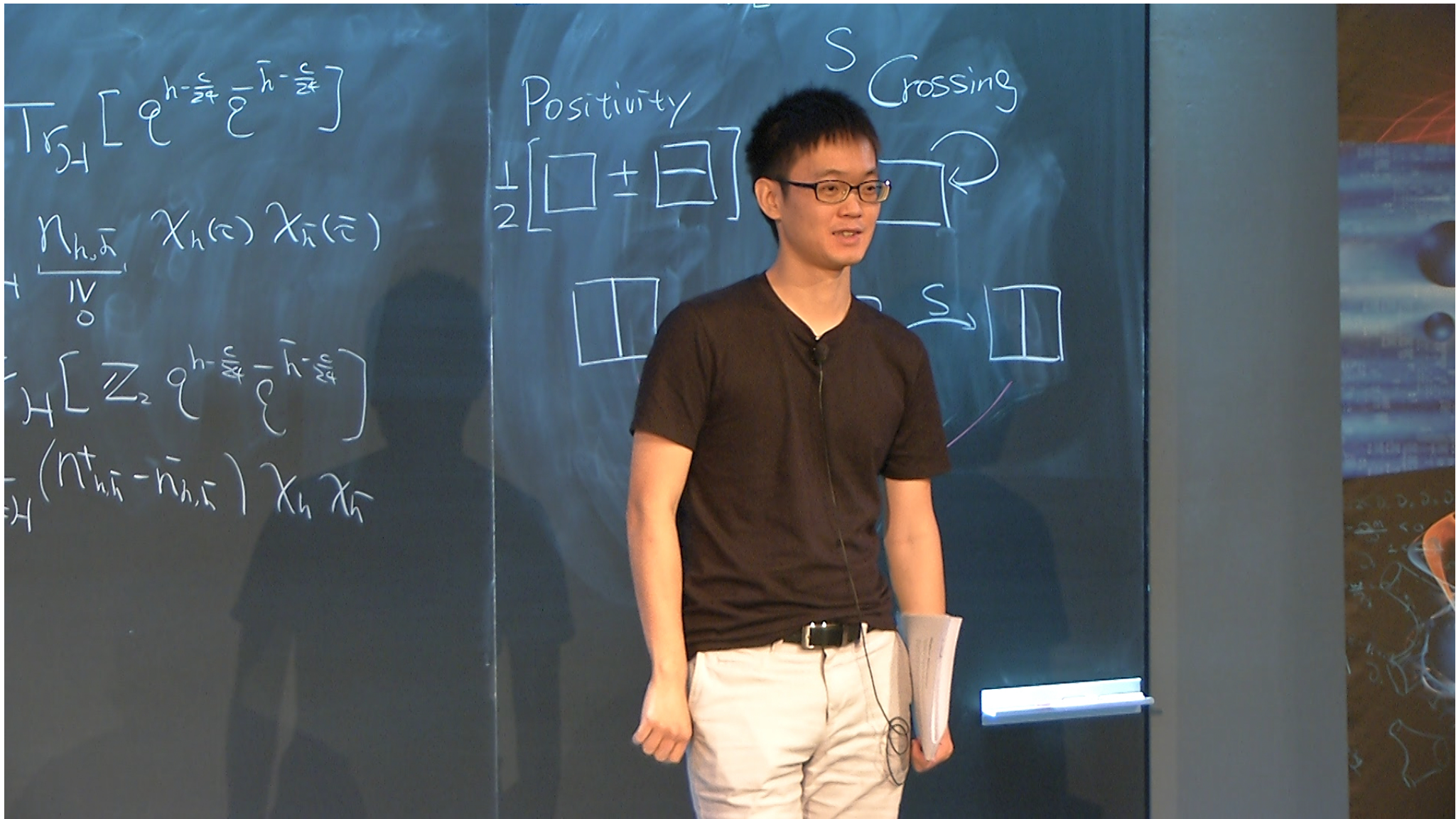
Δ^-

KK mode $\frac{1}{2R^2}$

winding $\frac{R^2}{2}$

$\min\left[\frac{R^2}{2}, \frac{1}{2R^2}\right]$





Higher dim generalization

$d > 2$ G : unitary, discrete, bosonic

No! Wang-Wen-Witten

$\Rightarrow$ TQFT w/ G and α

$U(1)_{\text{diag}}$

✓

$$\min\left[\frac{R^2}{2}, \frac{1}{2R^2}\right]$$

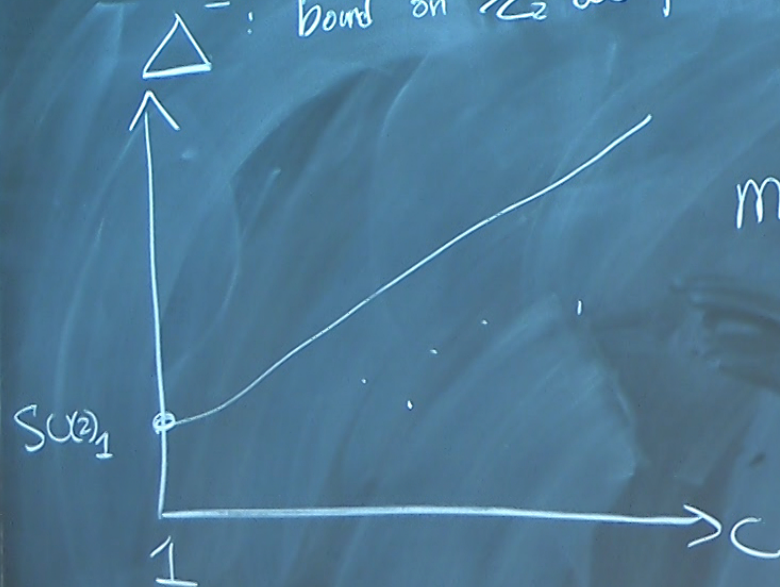
* $G = U(1)$ 4d
 $SU(N)$ $N > 2$ pert anomaly

* $G = SU(2)$ 4d



Bootstrap

Δ^- : bound on $\mathbb{Z}_2$ -odd op.



$$\min[\Delta^-, \Delta^{\text{disorder}}]$$

$\mathbb{Z}_2$ -odd
Order

$\mathcal{H}_L$
disorder

$$\frac{SU(2) \times SU(2)}{\mathbb{Z}_2}$$

anomaly

$$\Delta^-$$

2d CF

holo
anomaly

2d C

U(

K

$$\alpha = +1$$

