Title: Spinning Conformal Bootstrap in 4d

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Collection: Bootstrap 2019

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Spinning Conformal Bootstrap in 4d

based on the paper with
- Petr Kravchuk
- Marco Serone
- Alessandro Vichi

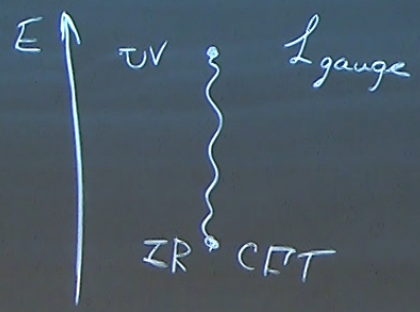
Overview:
1. Motivation
2. Technical setup
3. Rational approximation of conformal blocks
4. Numerical bounds
5. Fake primary effect

$$\langle \psi^{+}(p_1)\, \psi(p_2)\, \psi^{+}(p_3)\, \psi(p_4) \rangle$$

in 4d

d on the paper
uth
avchuk
Serone
andro Vichi

$$\langle \psi^+(p_1)\,\psi(p_2)\,\psi^+(p_3)\,\psi(p_4)\rangle$$



$\langle J^a J^b J^c J^d \rangle \sim ?\times ?\sim 50$

a) global group

b) global anomal. $\langle J^a J^b J^c \rangle$

① QCD

	$SU(N_c)$
ψ_I	$\square$
X_J	$\overline{\square}$

$I, J = 1, \ldots, N_f$

$G = SU(N_f) \times SU(N_f) \times U(1)$

4 fermions ~ 7

$\langle JJJJ \rangle \sim 7$

$\langle TTTT \rangle \sim 25$

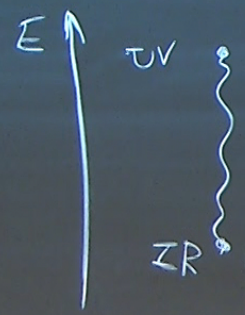
② AQCD

	$SU(N_c)$
ψ_I	adj

$I = 1 \ldots N_f$

$G = SU(N_f)$

$\langle \psi^\dagger(p_1)\ \psi(p_2)\ \psi^\dagger(p_3)\ \psi(p_4) \rangle$



$\mathcal{L}_{gauge}$

① QCD

$SU(3)$ $\epsilon_{ijk}\ \psi^i \psi^j \psi^k$ $g/2$

	$SU(N_c)$
ψ_I	$\square$
X_J	$\overline{\square}$

$I, J = 1, \ldots, N_f$

$G = SU(N_f) \times SU(N_f) \times U(1)$

② AQCD

$\psi^a F^a_{\mu\nu}$ $g/2$

	$SU(N_c)$
ψ_I	adj

$I = 1 \ldots N_f$

$G = SU(N_f)$

$\langle J^a J^b J^c J^d \rangle \sim N_f^4 \sim 50$

a) global group

b) global anomal. $\langle J^a J^b J^c \rangle$

fermions $\sim N_f$

$\langle JJJJ \rangle \sim N_f$

$\langle TTTT \rangle \sim 25$

Spinning Conformal Bootstrap in 4d

$$\langle \psi^\dagger(p_1)\, \psi(p_2)\, \psi^\dagger(p_3)\, \psi(p_4) \rangle$$

$$\mathcal{O}^{(\ell,\bar{\ell})}_{\Delta}(x,s,\bar{s}) \equiv \mathcal{O}^{\dot{\beta}_1 \dots \dot{\beta}_{\bar{\ell}}}_{\alpha_1 \dots \alpha_\ell}(x)\, (s^{\alpha_1} \dots s^{\alpha_\ell})(\bar{s}_{\dot{\beta}_1} \dots \bar{s}_{\dot{\beta}_{\bar{\ell}}})$$

Weyl fermions: $(1,0)$ and $(0,1)$

currents: $(1,1)$

× no parity or time-reversal

global $U(1)$

$$\begin{cases} Q[\psi] = q \\ Q[\psi^\dagger] = -q \end{cases} \quad q \neq 0$$

$q = 0$ $\quad \Psi_M = \begin{pmatrix} \psi \\ \psi^\dagger \end{pmatrix}$

$\langle \psi\psi\psi\psi \rangle$

- view:
- vation
- hnical setup
- tional approximation of conformal blocks
- merical bounds
- he primary effect

$\langle \psi(p_2)\, \psi^+(p_3)\, \psi(p_4) \rangle = \sum_{I=1}^{4} g_I(z,\bar{z})\, T^I$

bootstrap equations

s-t $\quad \langle \psi^+ \psi\, \psi^+ \psi \rangle = \langle \psi^+ \psi\, \psi^+ \psi \rangle$

$\bar{S}_{P,z}$)

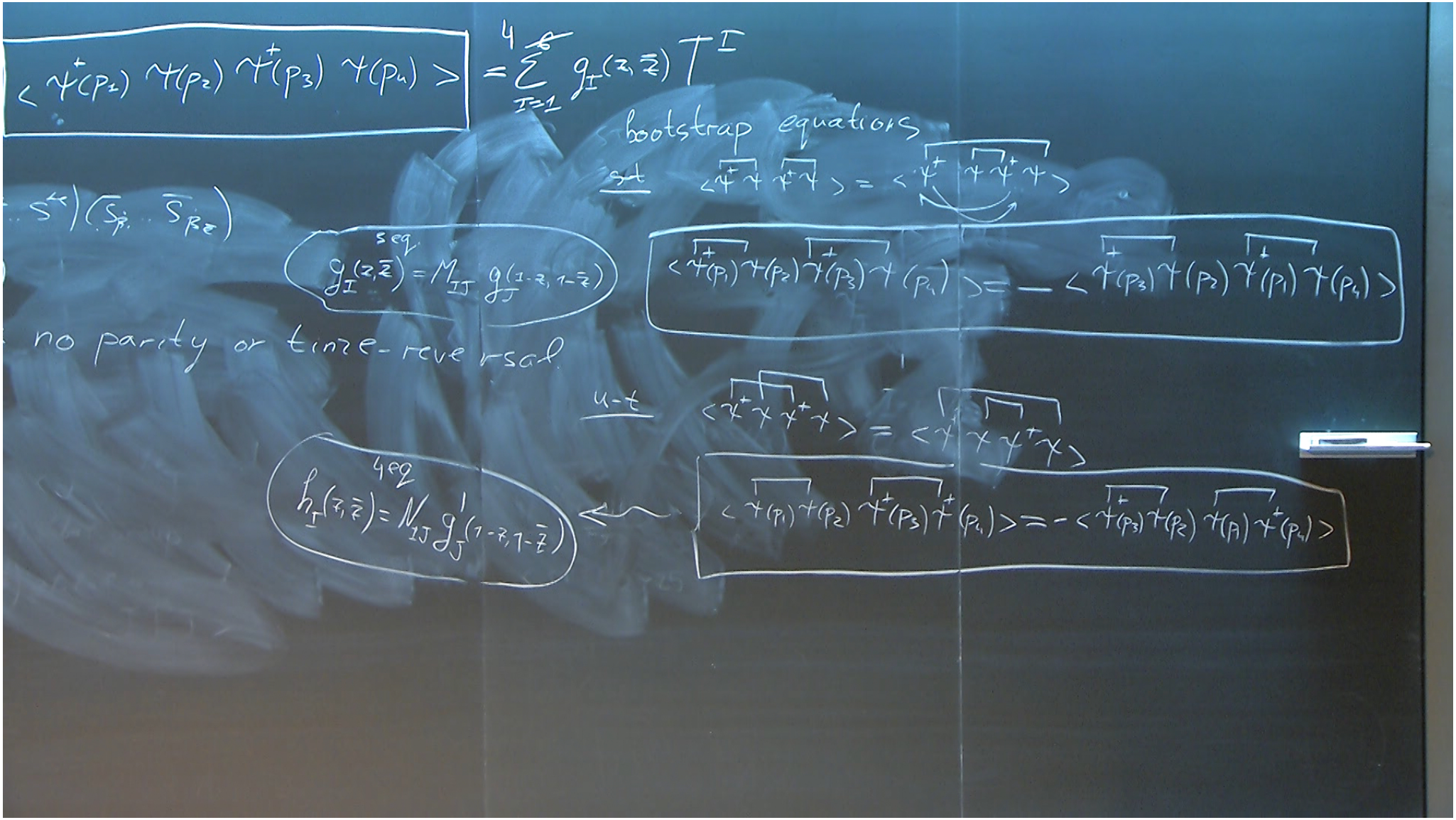
$g_I(z,\bar{z}) = M_{IJ}\, g_J(1-z, 1-\bar{z})$

$\langle \psi^+(p_1) \psi(p_2) \psi^+(p_3) \psi(p_4) \rangle = - \langle \psi^+(p_3) \psi(p_2) \psi^+(p_1) \psi(p_4) \rangle$

rity or time-reversal

u-t $\quad \langle \psi^+ \psi\, \psi^+ \psi \rangle = \langle \psi \psi\, \psi^+ \psi \rangle$

$\langle \psi(p_1) \psi(p_2) \psi^+(p_3) \psi^+(p_4) \rangle = - \langle \psi^+(p_3) \psi(p_2) \psi(p_1) \psi^+(p_4) \rangle$


$\langle \psi^+(p_1)\,\psi(p_2)\,\psi^+(p_3)\,\psi(p_4)\rangle = \sum_{I=1}^{4} g_I(z,\bar z)\,T^I$
bootstrap equations
s-t
$\langle \psi^+\psi\,\psi^+\psi\rangle = \langle \psi^+\,\psi\,\psi^+\psi\rangle$
$g_I(z,\bar z) = M_{IJ}\, g_J(1-z,1-\bar z)$
$\langle \psi^+(p_1)\psi(p_2)\psi^+(p_3)\psi(p_4)\rangle = -\langle \psi^+(p_3)\psi(p_2)\psi^+(p_1)\psi(p_4)\rangle$
no parity or time-reversal
u-t
$\langle \psi^+\psi\,\psi^+\psi\rangle = \langle \psi^+\,\psi\,\psi^+\psi\rangle$
$h_I(z,\bar z) = N_{IJ}\, g_J(1-z,1-\bar z)$
$\langle \psi(p_1)\psi(p_2)\psi^+(p_3)\psi^+(p_4)\rangle = -\langle \psi^+(p_3)\psi(p_2)\psi(p_1)\psi^+(p_4)\rangle$

Conformal Bootstrap in 4d

$$\langle \psi^+(p_1)\,\psi(p_2)\,\psi^+(p_3)\,\psi(p_4)\rangle = \sum_{I=1}^{4} g_I(z,\bar z)\,T^I$$

bootstrap

$$O_\Delta^{(\ell,\bar\ell)}(x,s,\bar s) \equiv O^{\dot\beta_1\cdots\dot\beta_{\bar\ell}}_{\alpha_1\cdots\alpha_\ell}(x)\,(s^{\alpha_1}\cdots s^{\alpha_\ell})(\bar s_{\dot\beta_1}\cdots\bar s_{\dot\beta_{\bar\ell}})$$

Weyl fermions: $(1,0)$ and $(0,1)$

currents: $(1,1)$

global $U(1)$

$$\begin{cases} Q[\psi]=q \\ Q[\psi^+]=-q \end{cases} \quad q\neq 0$$

$q=0$ $\quad \Psi_M=\begin{pmatrix}\psi\\ \psi^+\end{pmatrix}$

$\langle\psi\psi\psi\psi\rangle$

$$g_I^{\text{seq}}(z,\bar z) = M_{IJ}\, g_J(1-z,1-\bar z)$$

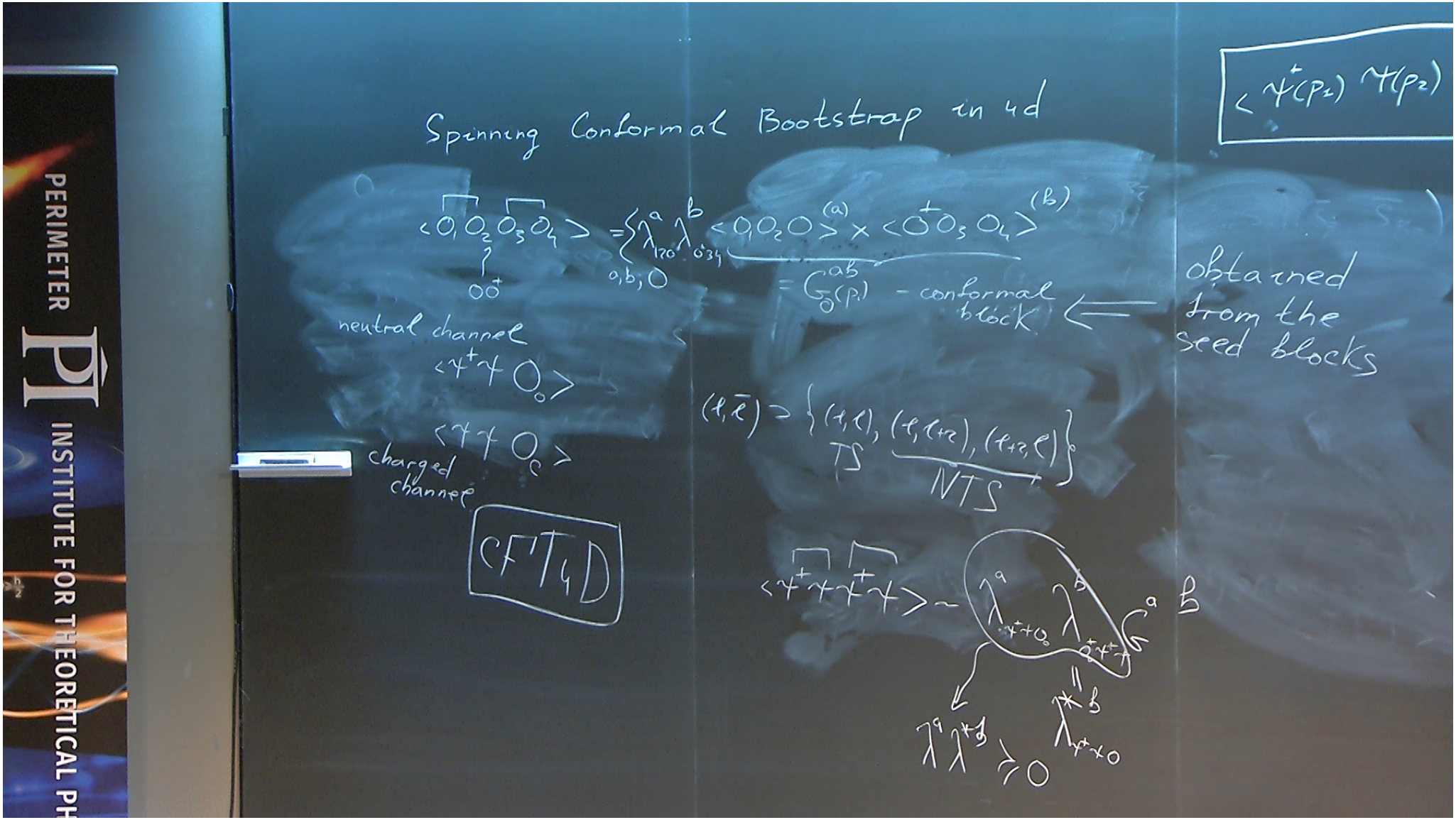
$\times$ no parity or time-reversal

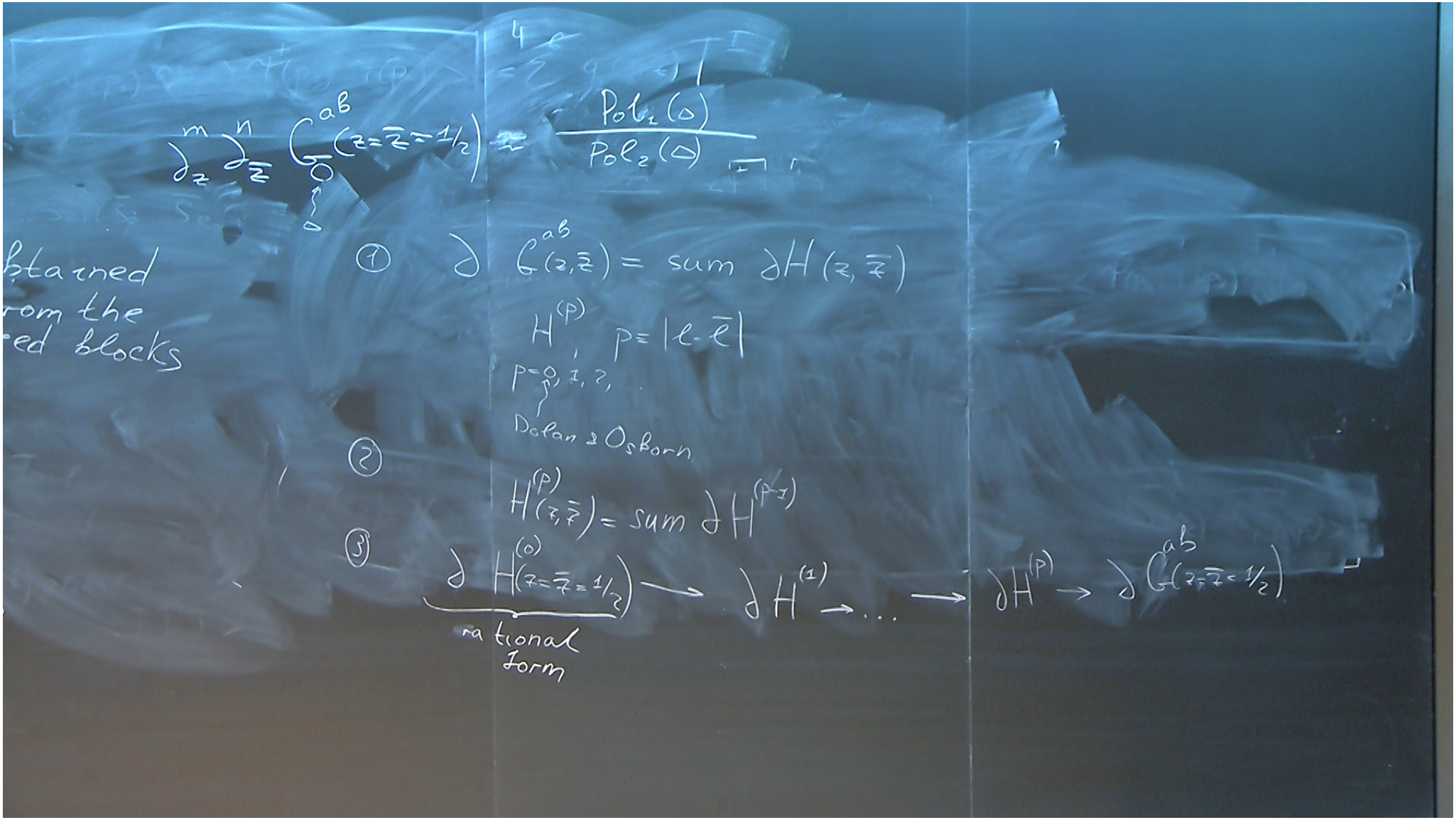
Taylor expand around $z=\bar z=\frac12$

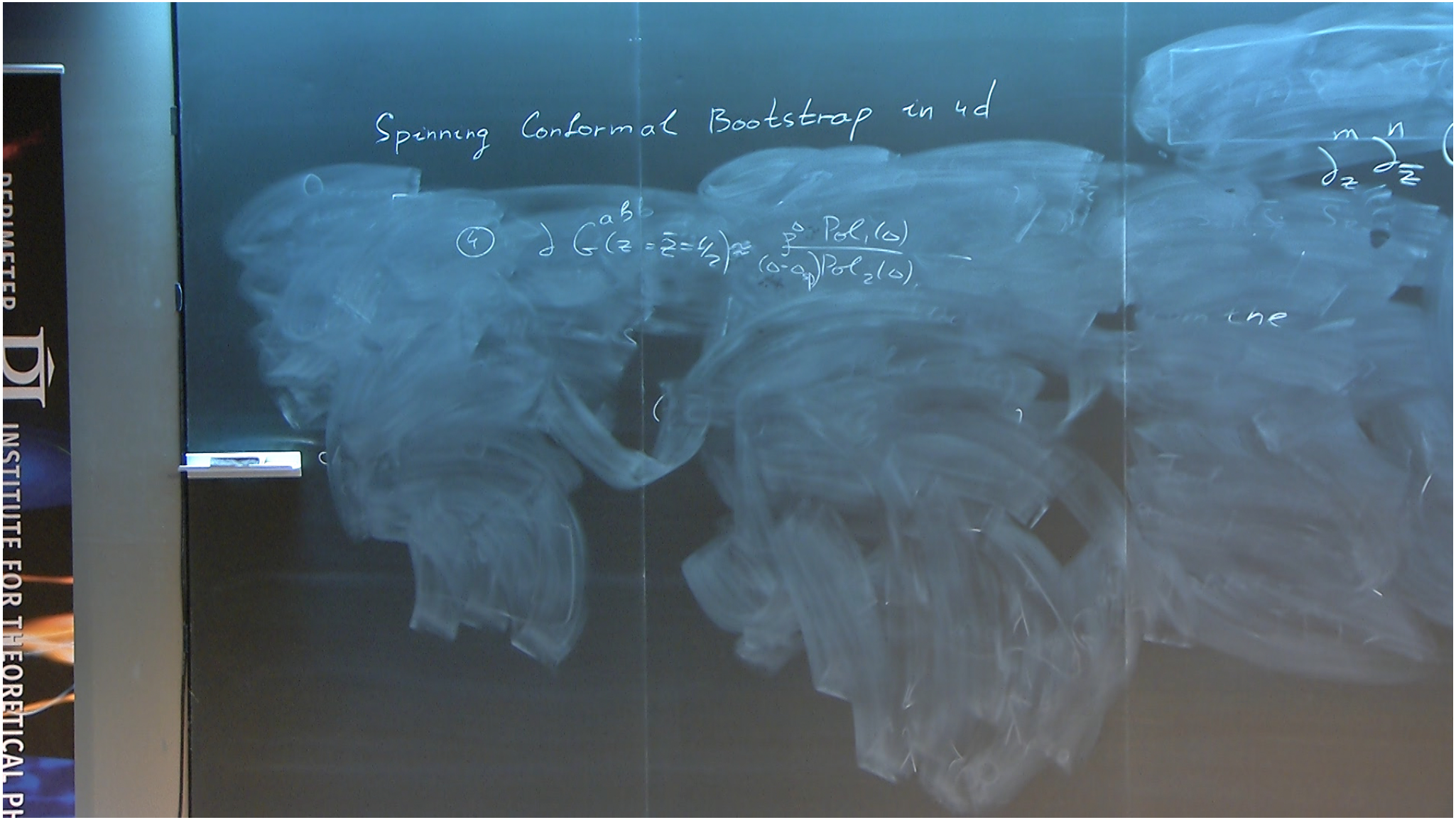
$$h_I^{\text{4eq}}(z,\bar z) = N_{IJ}\, g_J'(1-z,1-\bar z)$$

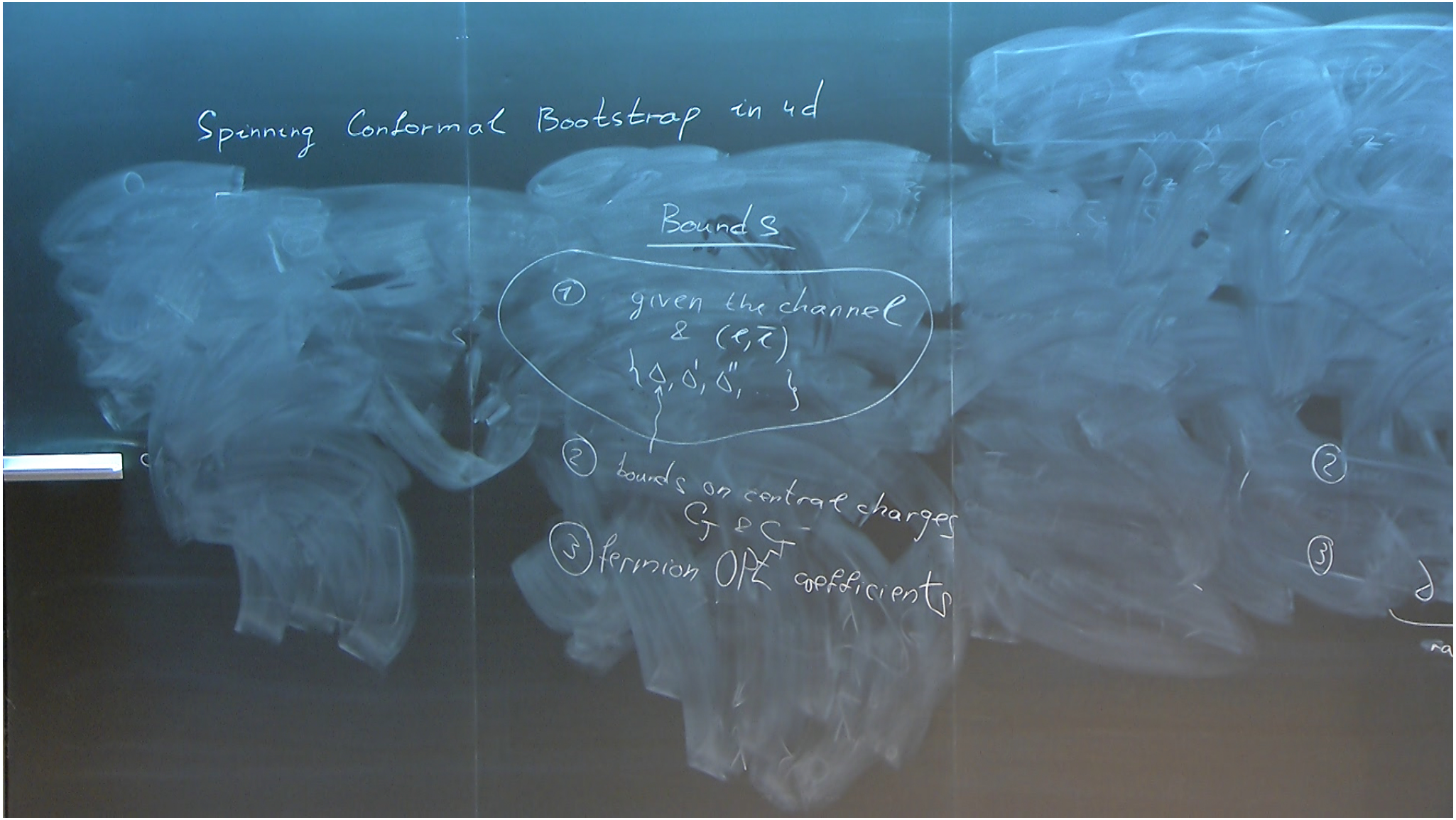
s-t

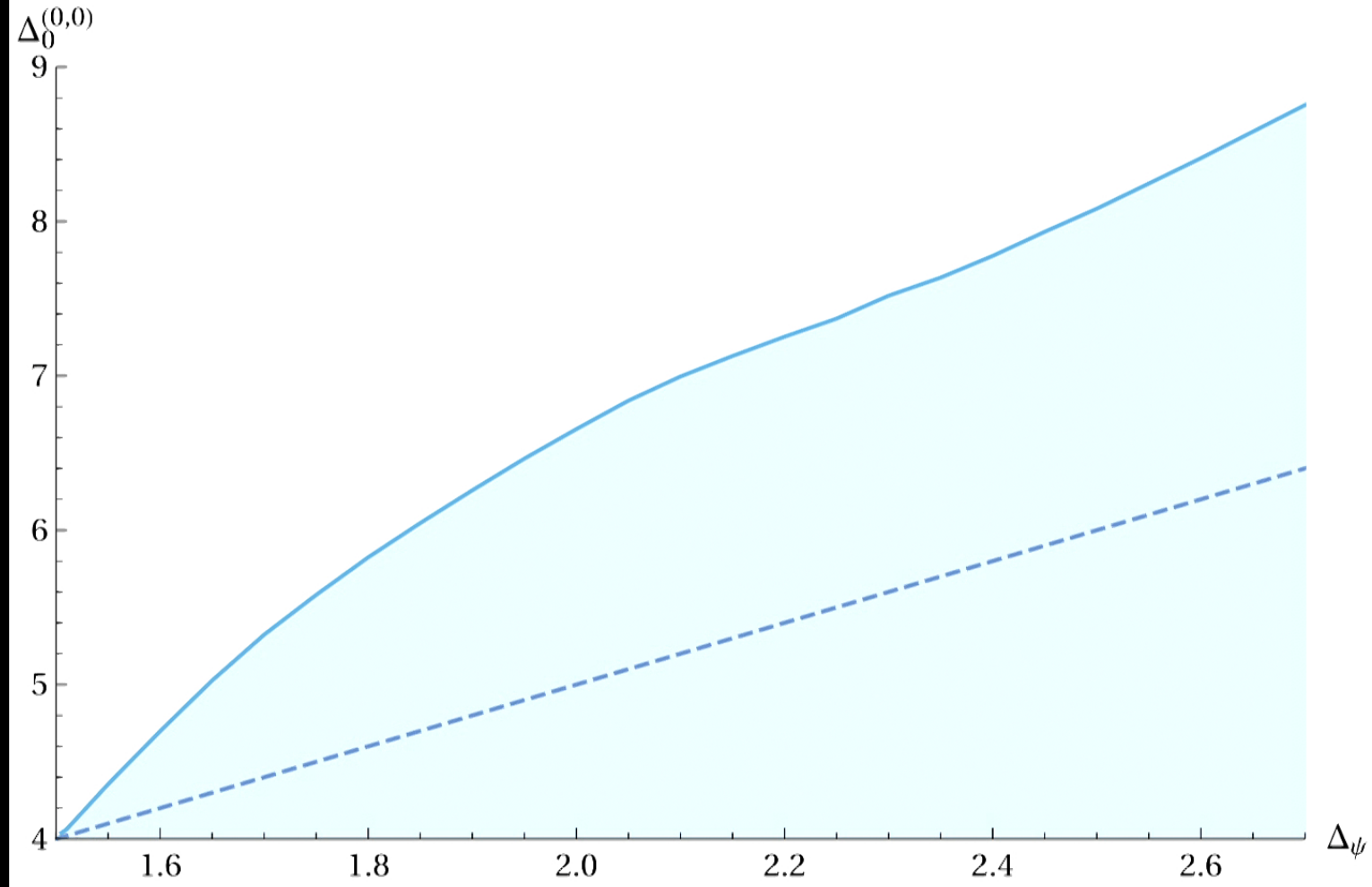
u-t

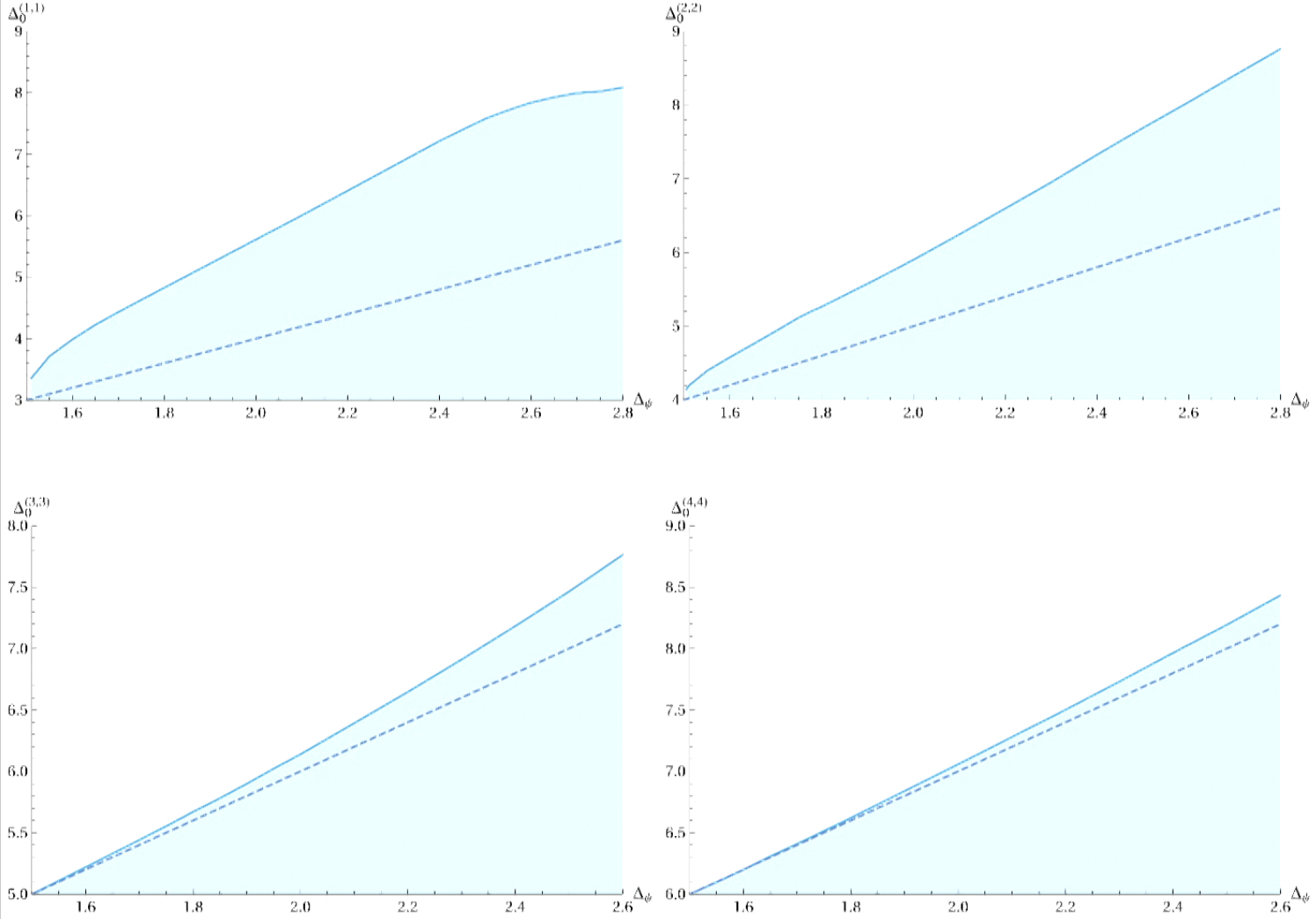

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Spinning Conformal Bootstrap in 4d
neutral channel
charged channel
CFT4D
obtained from the seed blocks
conformal block
TS
NTS

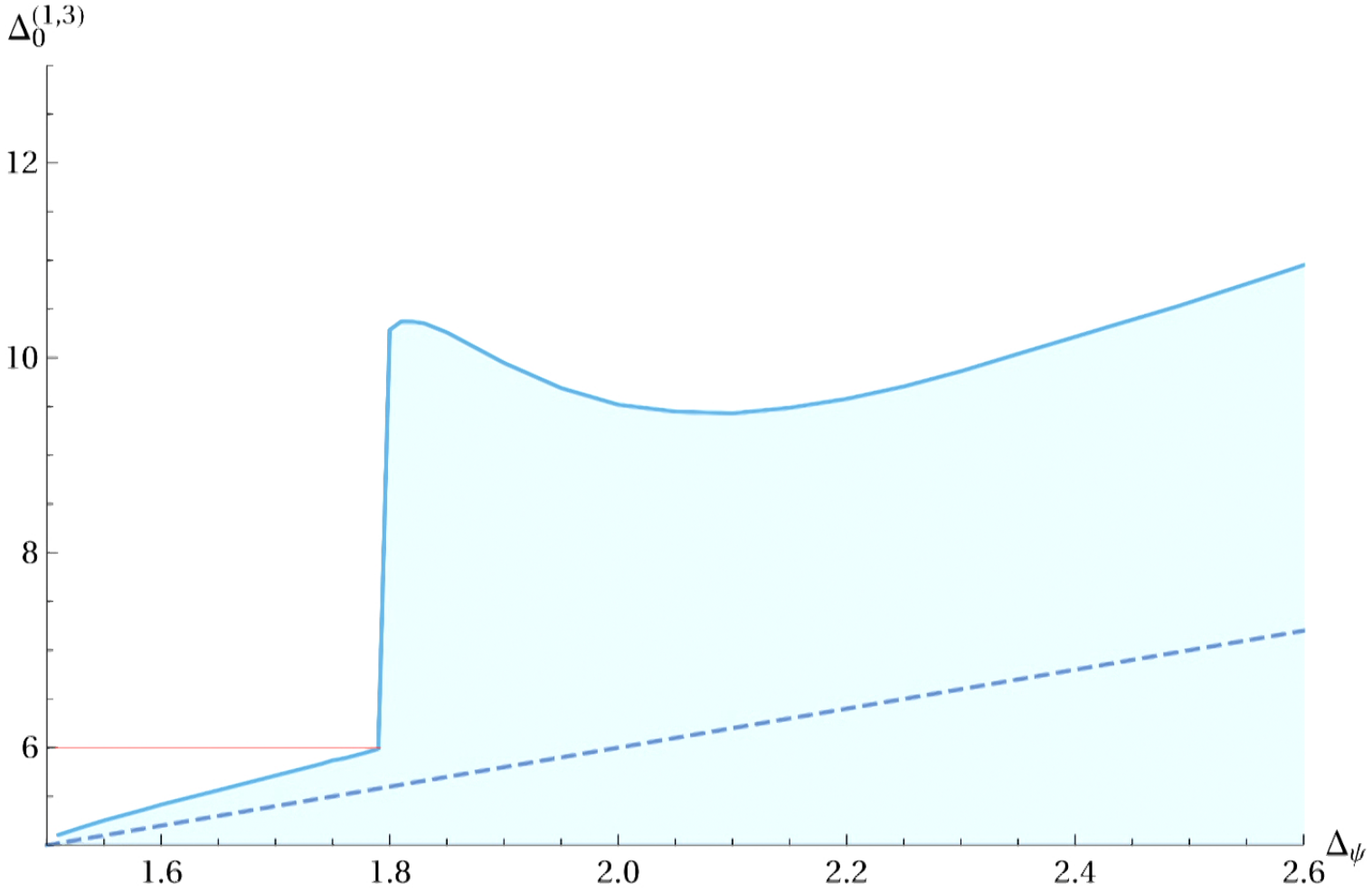

$\partial_z^m \partial_{\bar{z}}^n G^{ab}(z=\bar{z}=1/2) = \frac{Pol_1(0)}{Pol_2(0)}$
obtained from the ed blocks
① $\partial G^{ab}(z,\bar{z}) = sum\ \partial H(z,\bar{z})$
$H^{(p)}$, $p = |l - \bar{l}|$
$p = 0, 1, 2, \ldots$
Dolan & Osborn
② $H^{(p)}(z,\bar{z}) = sum\ \partial H^{(p-1)}$
③ $\partial H^{(0)}(z=\bar{z}=1/2) \rightarrow \partial H^{(1)} \rightarrow \ldots \rightarrow \partial H^{(p)} \rightarrow \partial G^{ab}(z,\bar{z}=1/2)$
rational form

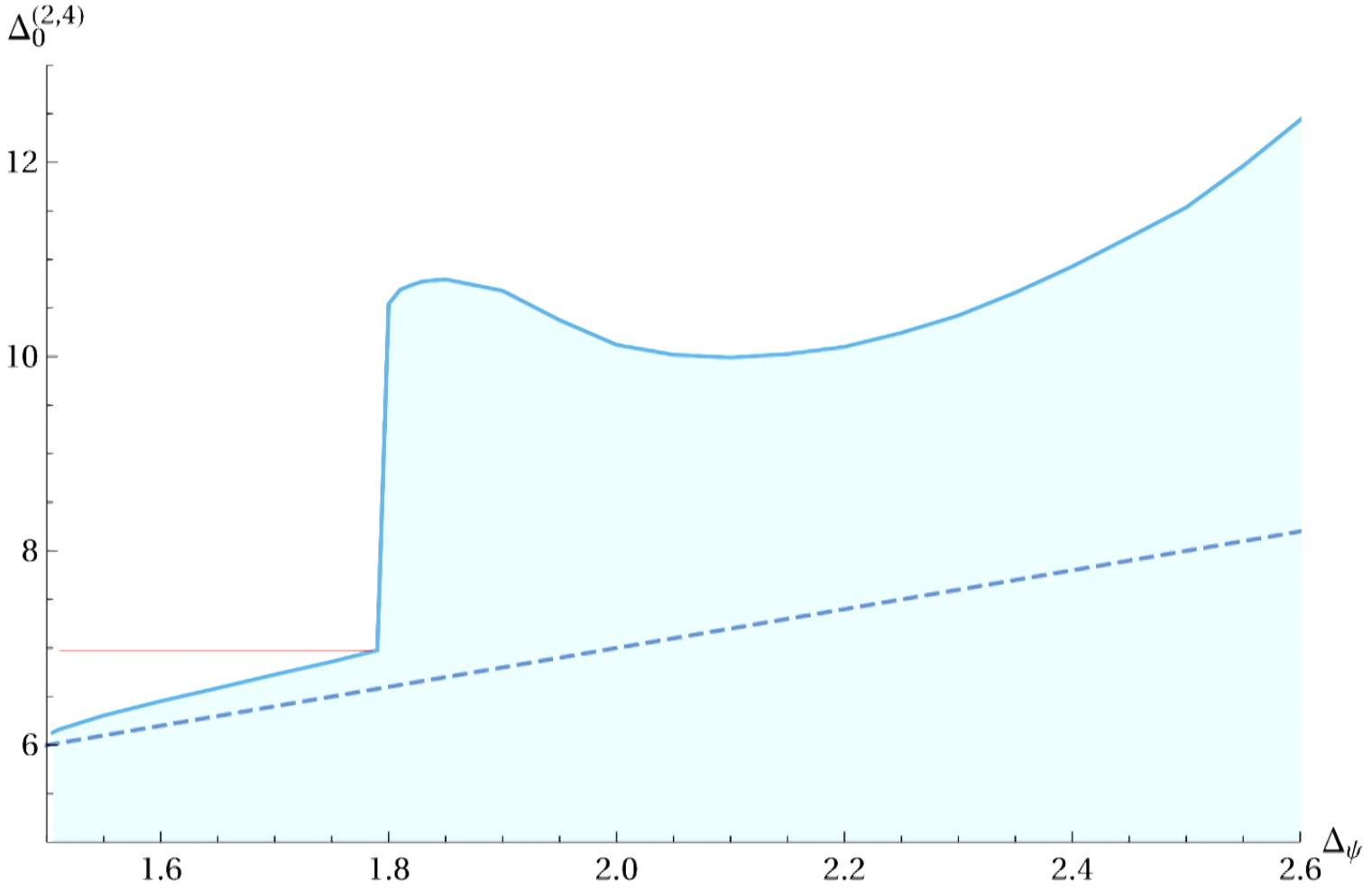

Spinning Conformal Bootstrap in 4d
④
$\partial^m \bar{\partial}^n G^{ab}_{cd}(z=\bar{z}=1/2) \approx \frac{\beta^\Delta \, Pol_1(\Delta)}{(\Delta-\Delta_0) Pol_2(\Delta)}$
the
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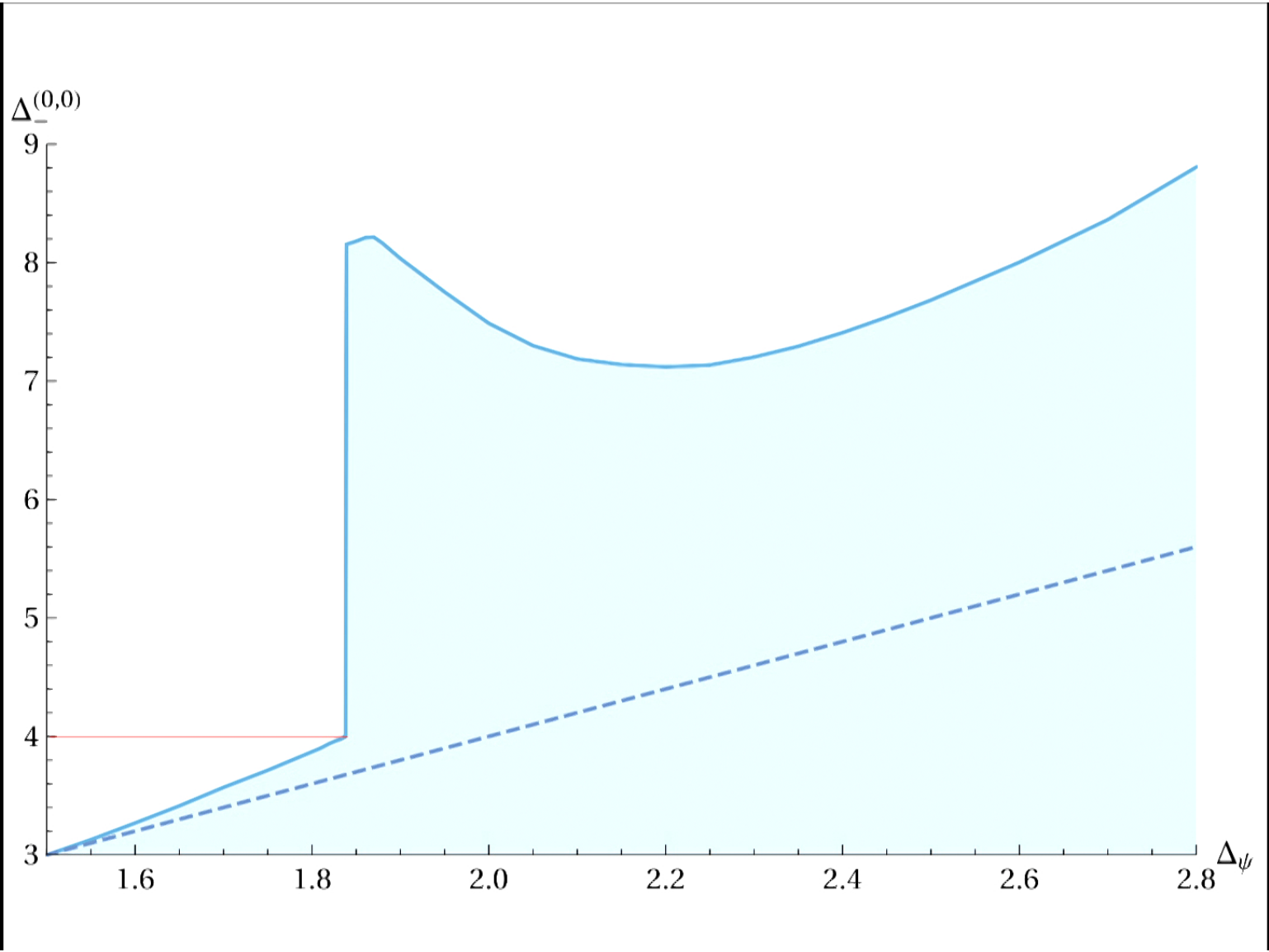

Spinning Conformal Bootstrap in 4d
Bounds
① given the channel & $(z,\bar{z})$
$\{\Delta, \Delta', \Delta'', \ldots\}$
② bounds on central charges C_T & C_J
③ fermion OPE coefficients
②
③

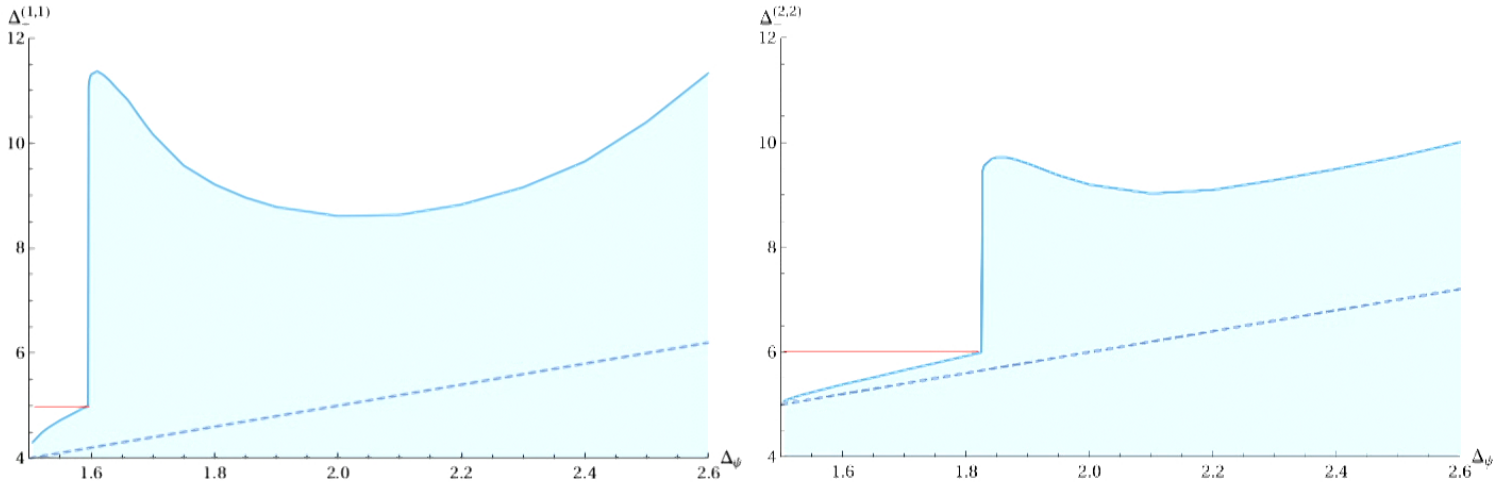

$\Delta_0^{(0,0)}$
9
8
7
6
5
4
1.6
1.8
2.0
2.2
2.4
2.6
Δ_ψ

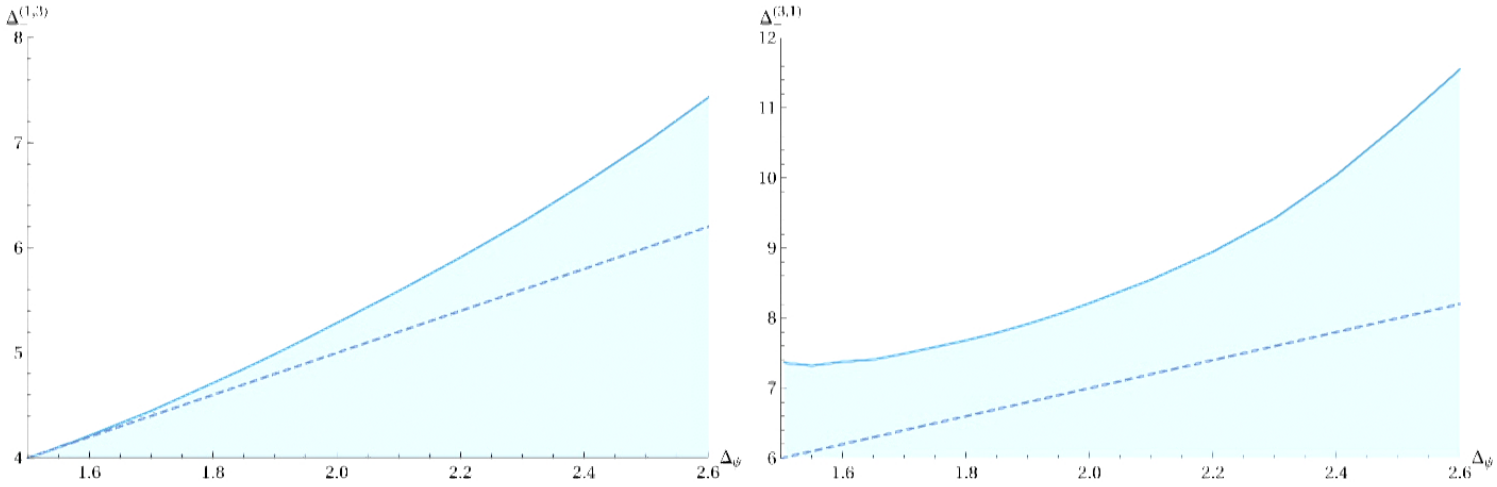

$\Delta_0^{(1,1)}$
$\Delta_0^{(2,2)}$
$\Delta_0^{(3,3)}$
$\Delta_0^{(4,4)}$
Δ_ϕ

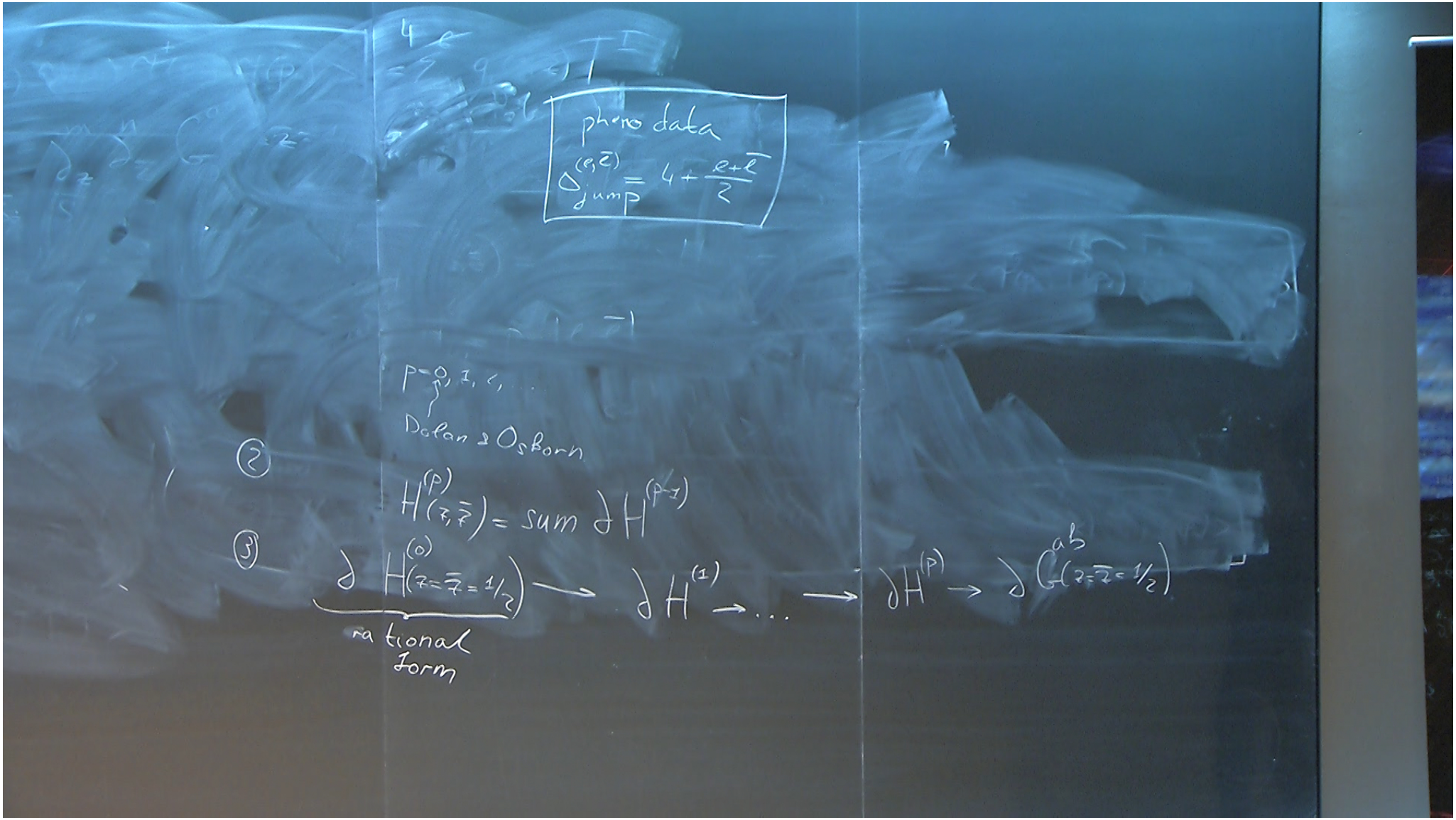

$\Delta_0^{(1,3)}$
12
10
8
6
1.6
1.8
2.0
2.2
2.4
2.6
Δ_ψ

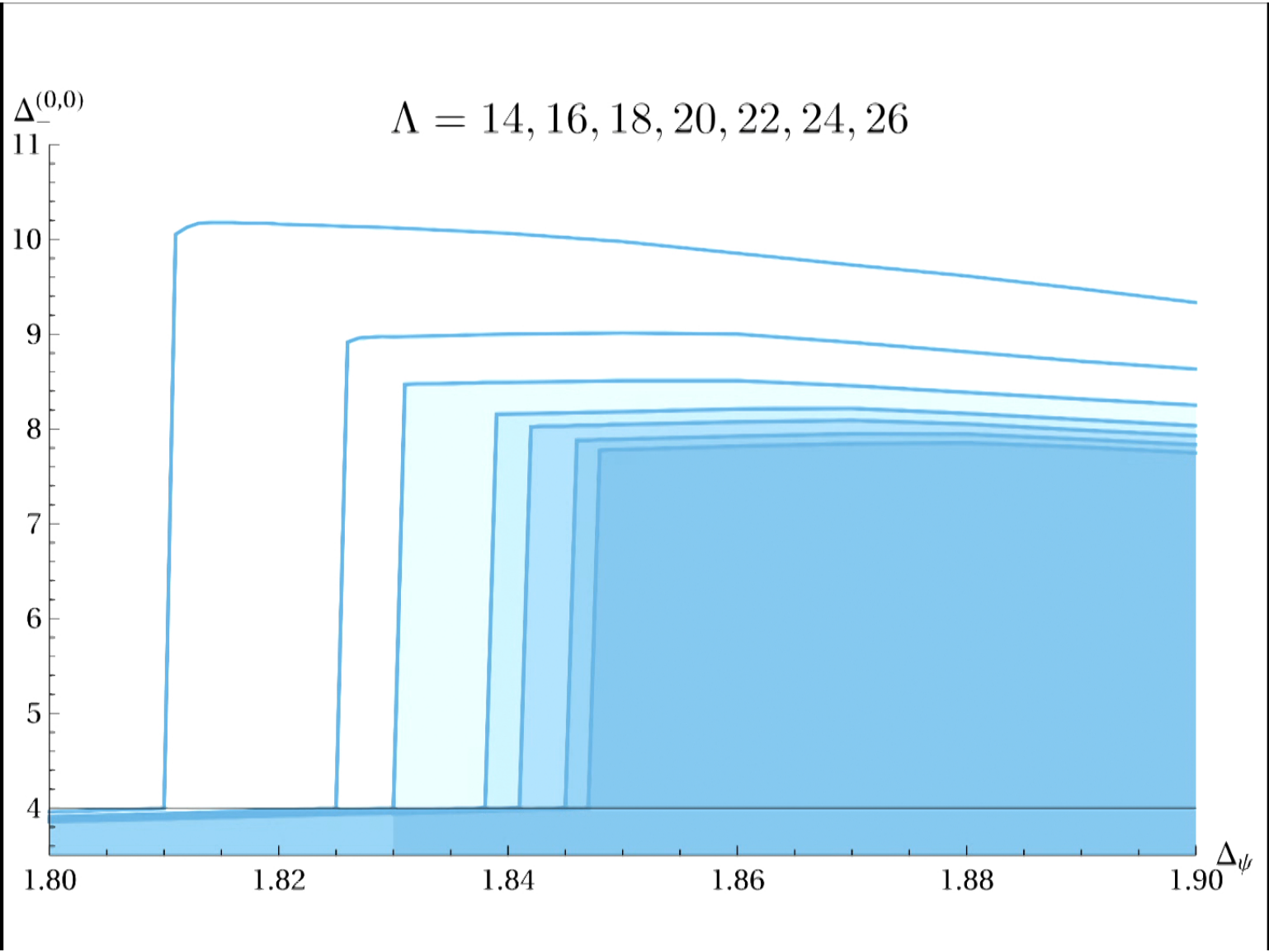

$\Delta_0^{(2,4)}$
12
10
8
6
1.6
1.8
2.0
2.2
2.4
2.6
Δ_ψ

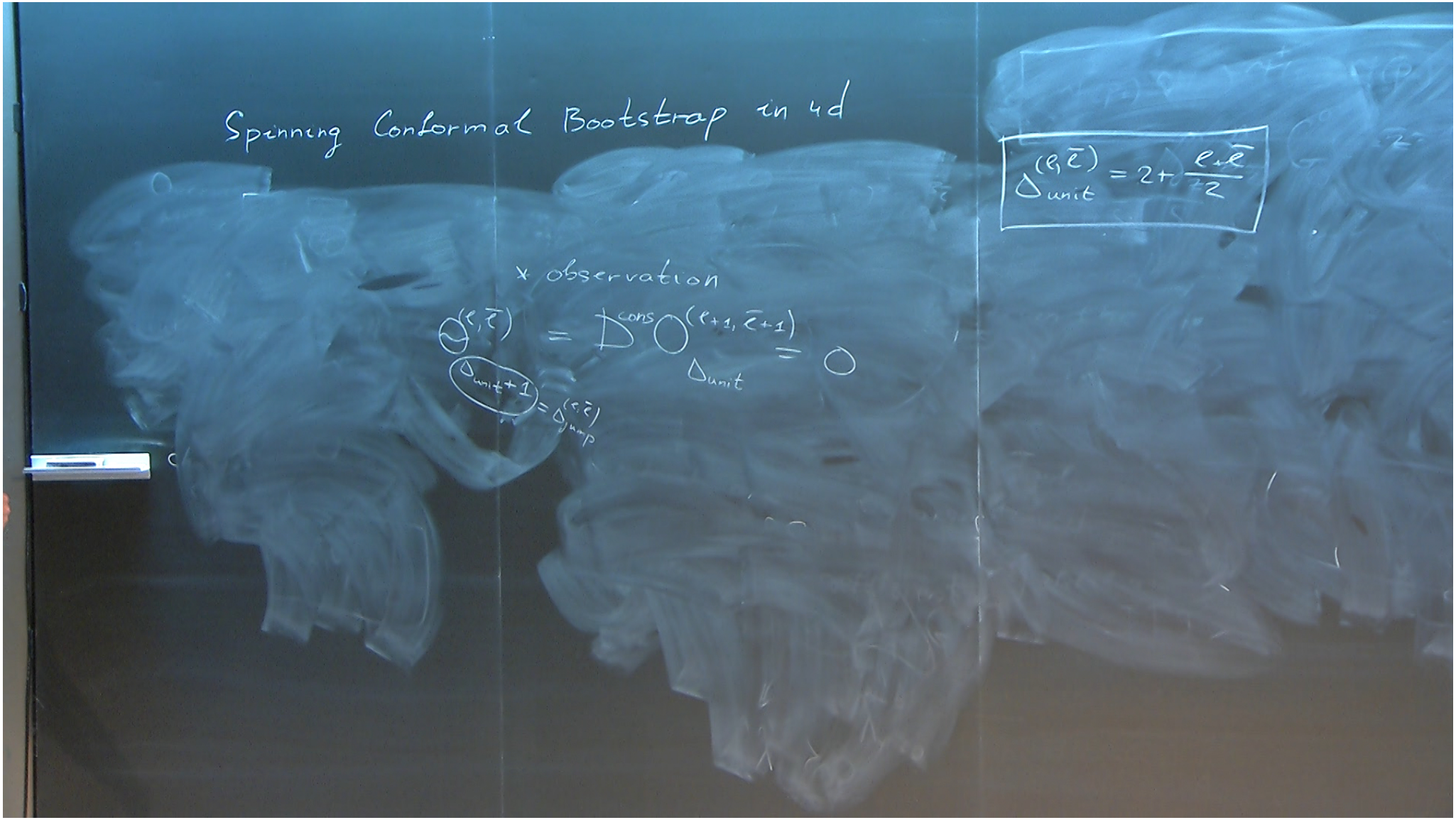

$\Delta_-^{(0,0)}$
9
8
7
6
5
4
3
1.6
1.8
2.0
2.2
2.4
2.6
2.8
Δ_ψ

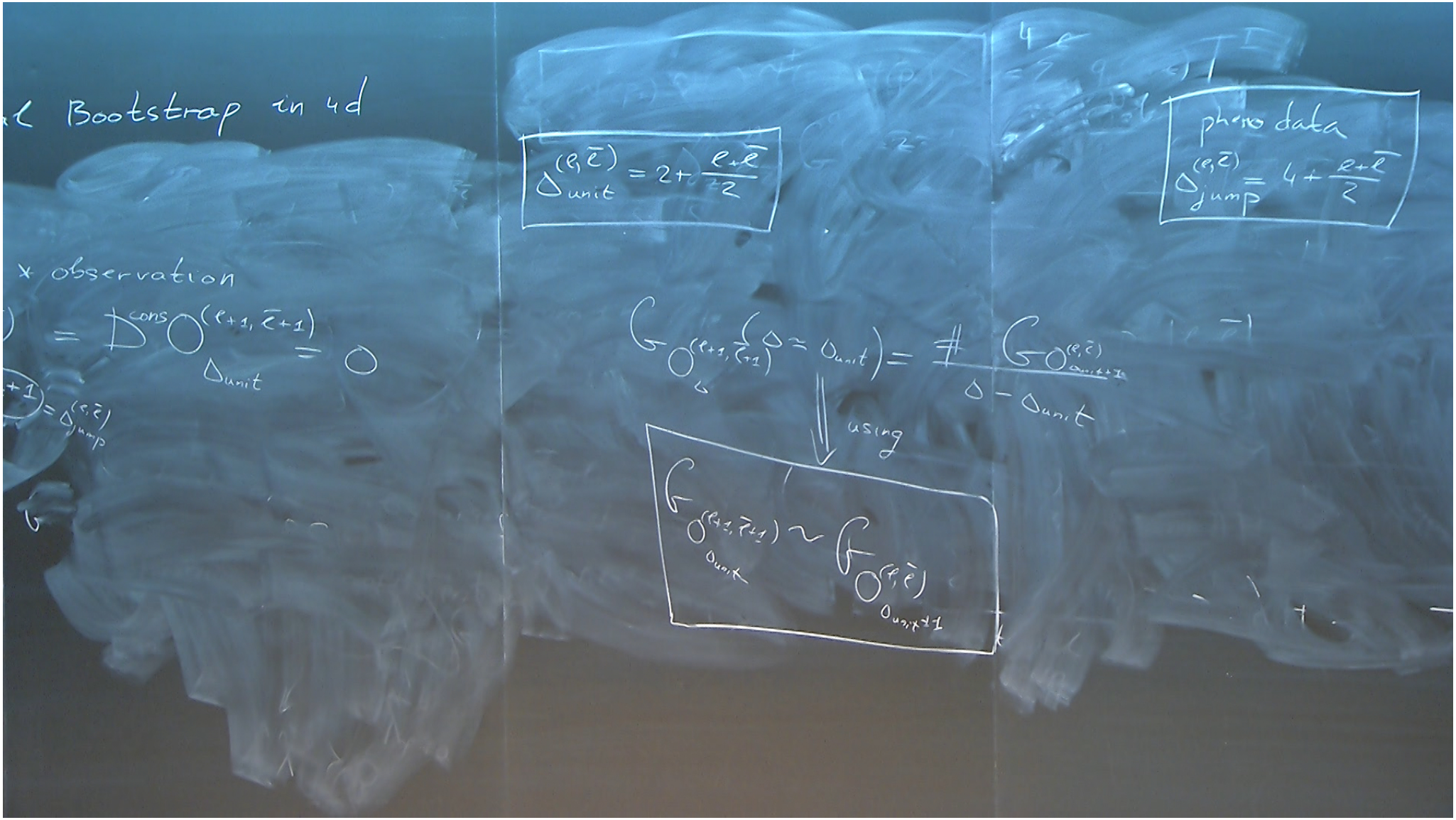

$\Delta_-^{(1,1)}$
12
10
8
6
4
1.6
1.8
2.0
2.2
2.4
2.6
Δ_ψ
$\Delta_-^{(2,2)}$
12
10
8
6
4
1.6
1.8
2.0
2.2
2.4
2.6
Δ_ψ

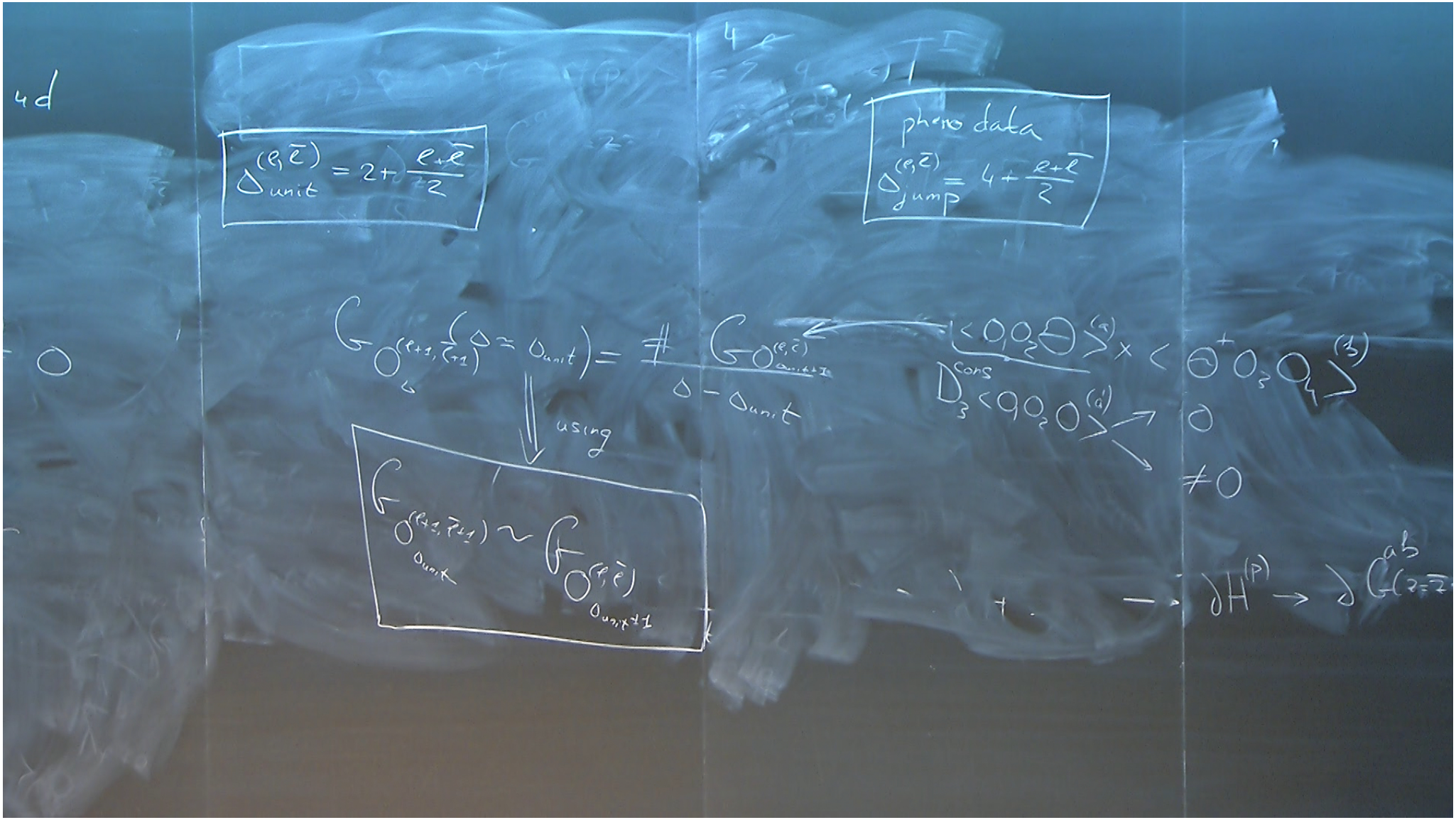

$\Delta_-^{(1,3)}$
8
7
6
5
4
1.6
1.8
2.0
2.2
2.4
2.6
Δ_ϕ
$\Delta_-^{(3,1)}$
12
11
10
9
8
7
6
1.6
1.8
2.0
2.2
2.4
2.6
Δ_ϕ

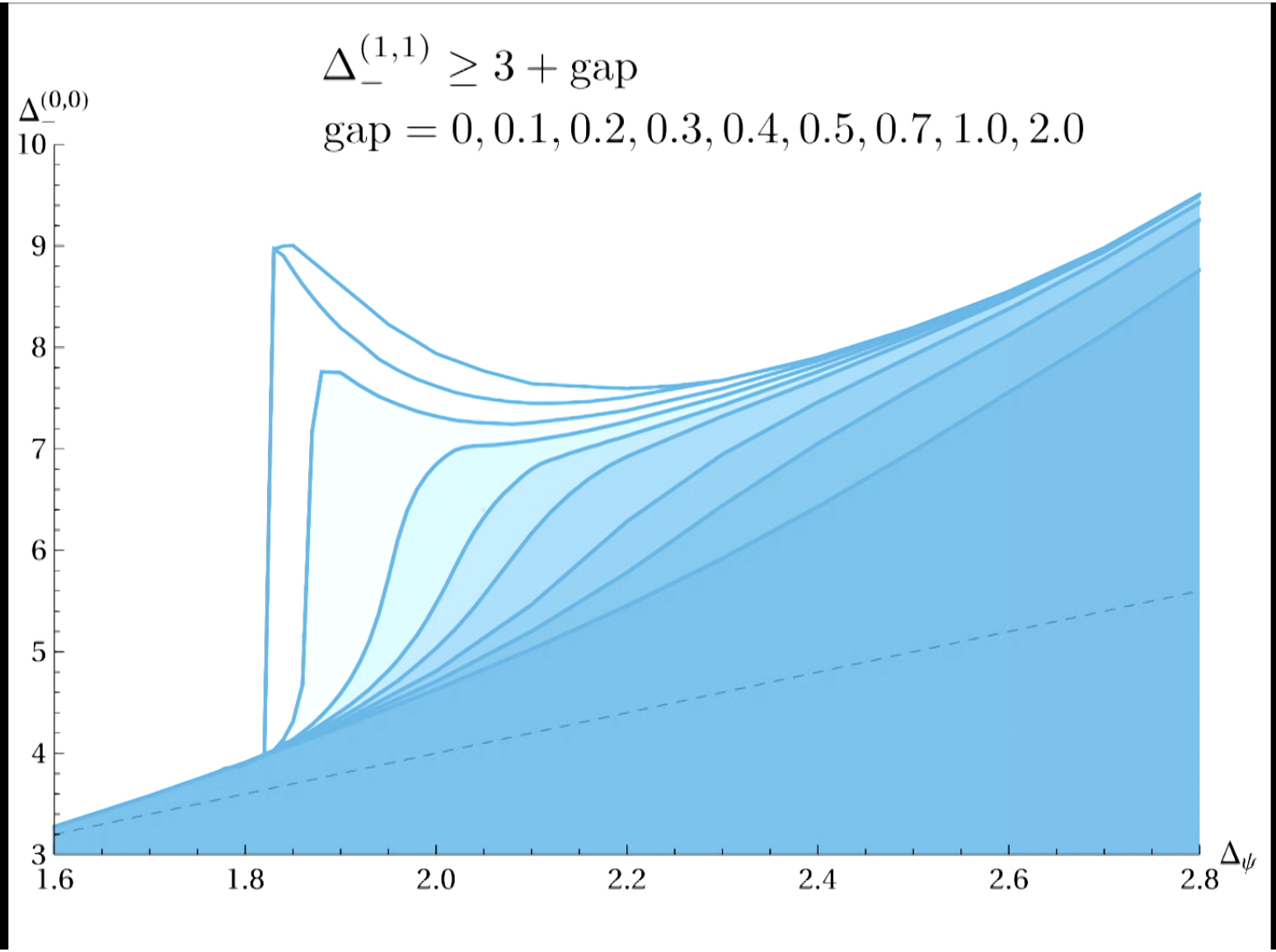

pheno data
$\Delta^{(e,\bar{e})}_{jump} = 4 + \frac{e+\bar{e}}{2}$
$p = 0, 1, 2, \ldots$
Dolan & Osborn
(2) $H^{(p)}_{(z,\bar{z})} = \text{sum}\, \partial H^{(p-1)}$
(3) $\partial H^{(0)}_{(z=\bar{z}=1/2)} \to \partial H^{(1)} \to \ldots \to \partial H^{(p)} \to \partial G^{ab}_{(z=\bar{z}=1/2)}$
rational form

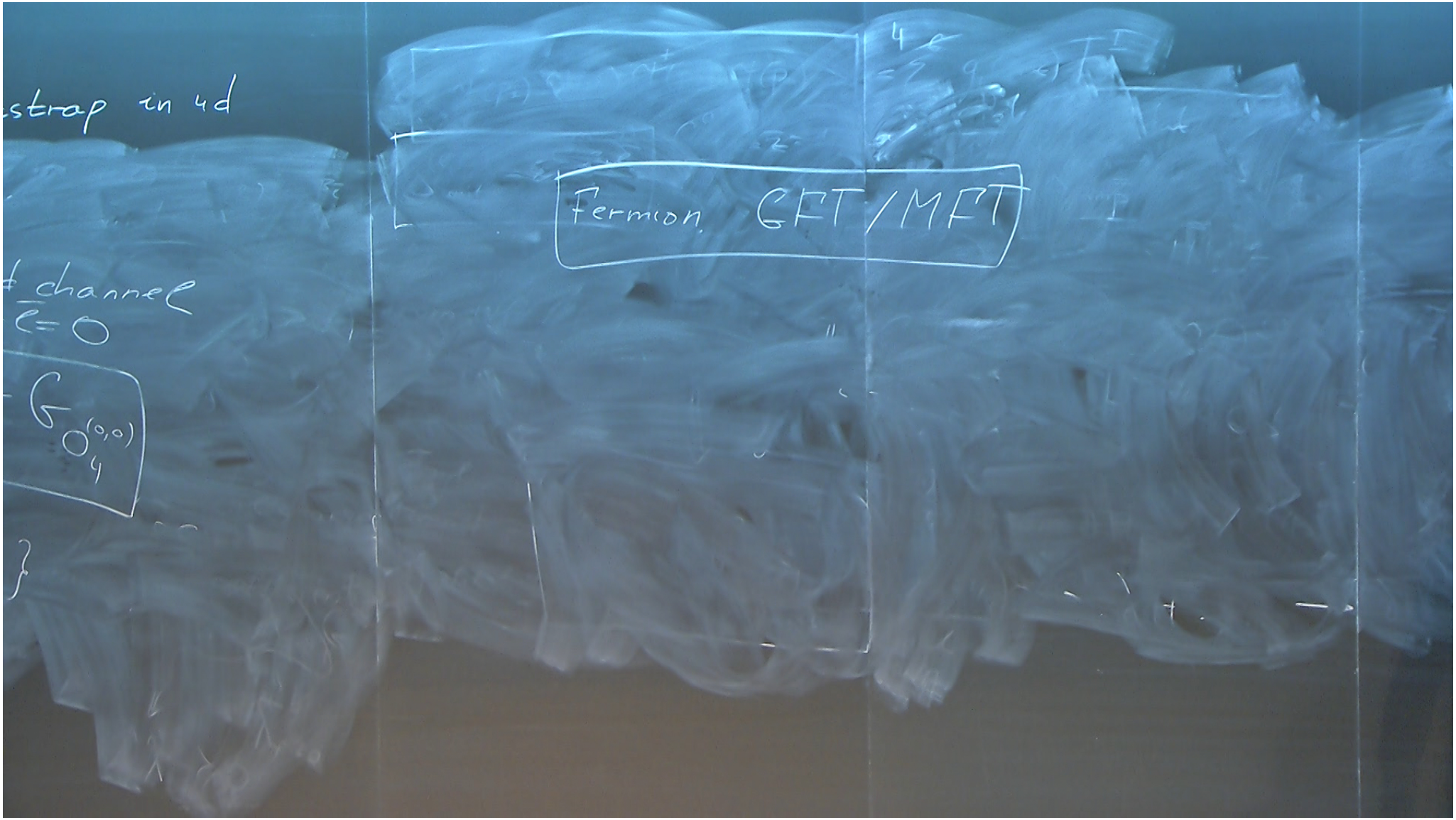

$\Lambda = 14, 16, 18, 20, 22, 24, 26$
$\Delta_-^{(0,0)}$
11
10
9
8
7
6
5
4
1.80
1.82
1.84
1.86
1.88
1.90
Δ_ψ


Spinning Conformal Bootstrap in 4d
$\Delta^{(\ell,\bar{\ell})}_{\text{unit}} = 2 + \frac{\ell+\bar{\ell}}{2}$
* observation
$\mathcal{O}^{(\ell,\bar{\ell})} = D^{\text{cons}} \mathcal{O}^{(\ell+1,\bar{\ell}+1)}_{\Delta_{\text{unit}}} = 0$
$\Delta_{\text{unit}+1} = \Delta^{(\ell,\bar{\ell})}_{\text{jump}}$


Bootstrap in 4d
$\Delta_{unit}^{(\ell,\bar{\ell})} = 2 + \frac{\ell+\bar{\ell}}{2}$
$\Delta_{jump}^{(\ell,\bar{\ell})} = 4 + \frac{\ell+\bar{\ell}}{2}$
phase data
observation
$= D^{cons} O^{(\ell+1,\bar{\ell}+1)}_{\Delta_{unit}} = 0$
$= \Delta_{jump}^{(\ell,\bar{\ell})}$
$G_{O^{(\ell+1,\bar{\ell}+1)}}(\Delta \simeq \Delta_{unit}) = \# \frac{G_{O^{(\ell,\bar{\ell})}}}{\Delta - \Delta_{unit}}$
using
$G_{O^{(\ell+1,\bar{\ell}+1)}_{\Delta_{unit}}} \sim G_{O^{(\ell,\bar{\ell})}_{\Delta_{unit}+1}}$


4d
$\Delta^{(p,\bar{p})}_{unit} = 2 + \frac{p+\bar{p}}{2}$
phase data
$\Delta^{(p,\bar{p})}_{jump} = 4 + \frac{p+\bar{p}}{2}$
$G\mathcal{O}^{(p+1,\bar{p}+1)}(\Delta \simeq \Delta_{unit}) = \# \; G\mathcal{O}^{(p,\bar{p})}_{\Delta_{unit}+1}$
$\Delta - \Delta_{unit}$
$\langle 0|\mathcal{O}_2\Theta\rangle^{(a)} \times \langle\Theta^+\mathcal{O}_3\mathcal{O}_4\rangle^{(3)}$
$D^{conf}_3 \langle \mathcal{O}_2\mathcal{O}_3\rangle^{(a)}$
$= 0$
$\neq 0$
using
$G\mathcal{O}^{(p+1,\bar{p}+1)}_{\Delta_{unit}} \sim G\mathcal{O}^{(p,\bar{p})}_{\Delta_{unit}+1}$
$\partial H^{(p)} \to \partial G^{ab}_{(\gamma,\bar{\gamma}}$


$\Delta_-^{(1,1)} \geq 3 + \text{gap}$
$\text{gap} = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1.0, 2.0$
$\Delta_-^{(0,0)}$
10
9
8
7
6
5
4
3
1.6
1.8
2.0
2.2
2.4
2.6
2.8
Δ_ψ


strap in 4d
Fermion GFT/MFT
channel
(0,0)