Title: CFT amplitudes

Speakers: Marc Gillioz

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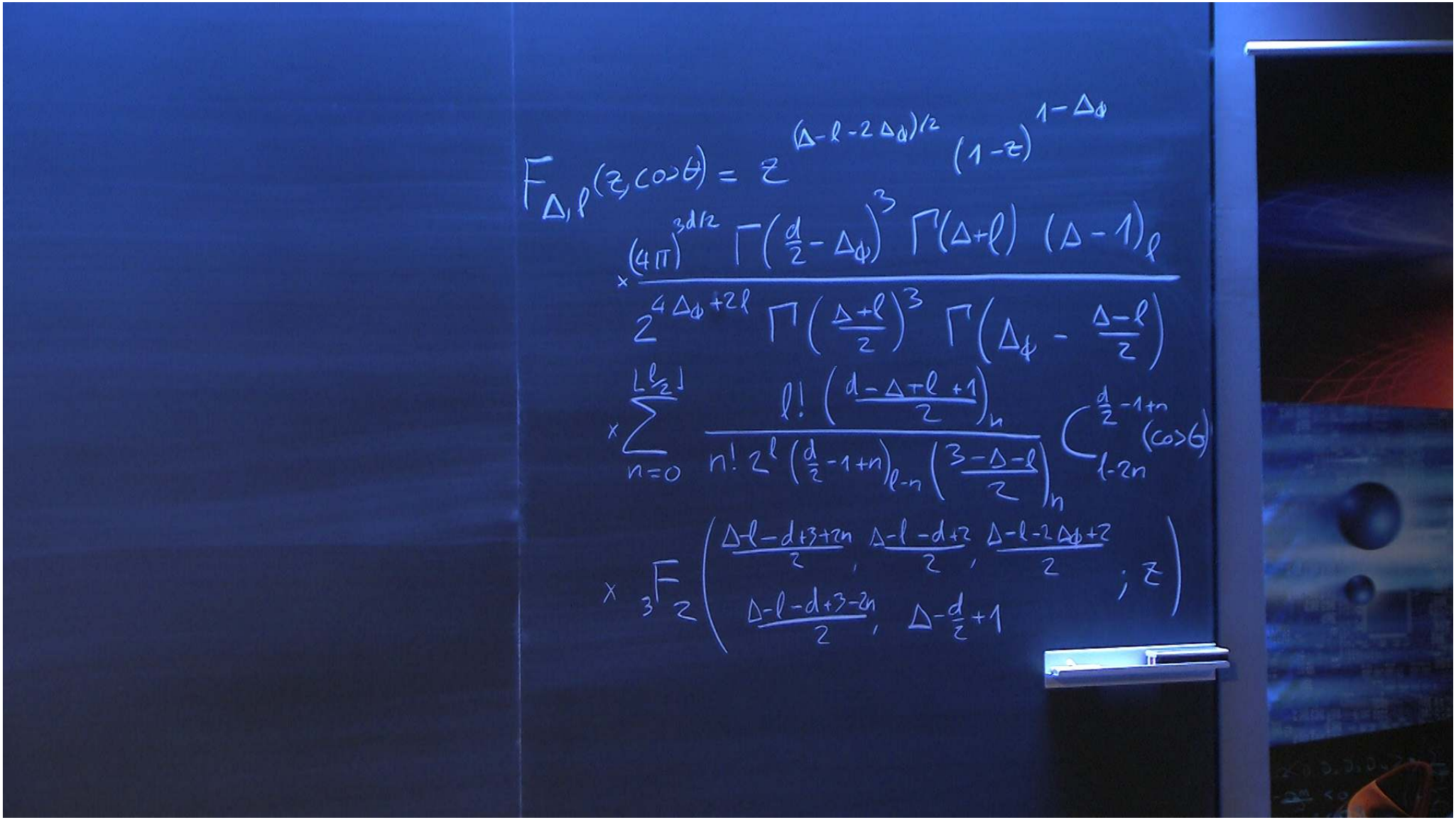
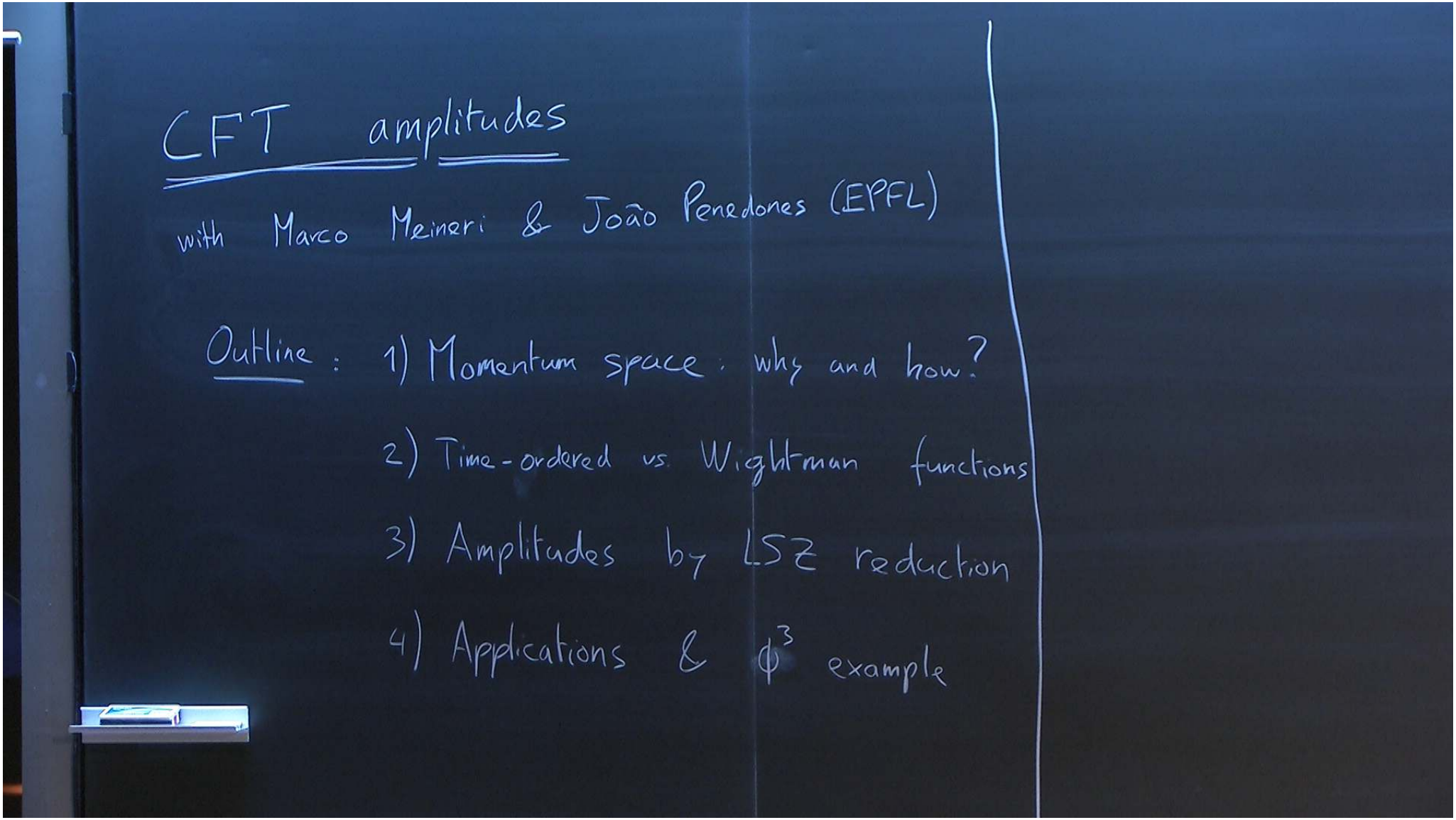
URL: http://pirsa.org/19070046


$F_{\Delta,\ell}(z,\cos\theta) = z^{(\Delta-\ell-2\Delta_\phi)/2}(1-z)^{1-\Delta_\phi}$
$\times \frac{(4\pi)^{3d/2}\,\Gamma\left(\frac{d}{2}-\Delta_\phi\right)^3\,\Gamma(\Delta+\ell)\,(\Delta-1)_\ell}{2^{4\Delta_\phi+2\ell}\,\Gamma\left(\frac{\Delta+\ell}{2}\right)^3\,\Gamma\left(\Delta_\phi-\frac{\Delta-\ell}{2}\right)}$
$\times \sum_{n=0}^{\lfloor \ell/2 \rfloor} \frac{\ell!\left(\frac{d-\Delta+\ell+1}{2}\right)_n}{n!\,2^\ell\left(\frac{d}{2}-1+n\right)_{\ell-n}\left(\frac{3-\Delta-\ell}{2}\right)_n} C^{\frac{d}{2}-1+n}_{\ell-2n}(\cos\theta)$
$\times {}_3F_2\left(\frac{\Delta-\ell-d+3+2n}{2}, \frac{\Delta-\ell-d+2}{2}, \frac{\Delta-\ell-2\Delta_\phi+2}{2}; z\right)$
$\frac{\Delta-\ell-d+3-2n}{2}, \Delta-\frac{d}{2}+1$


CFT amplitudes
with Marco Meineri & João Penedones (EPFL)
Outline: 1) Momentum space: why and how?
2) Time-ordered vs. Wightman functions
3) Amplitudes by LSZ reduction
4) Applications & ϕ^3 example

1) Why & how?

Why? : - preferred H : D. $\frac{P^0 + K^0/R^2}{2} \xrightarrow{R\to\infty} P^0$

- easy to build conformal blocks

$$G \sim \langle \phi\phi\, \mathcal{O} \rangle \langle \mathcal{O}\, \phi\phi \rangle$$

- GFF are trivial

$$\langle \phi\phi\phi\phi \rangle \sim \delta(p_1+p_2)\,\delta(p_3+p_4) + \text{crossing}$$

$\frac{P^0 + K^0/R^2}{2} \xrightarrow{R\to\infty} P^0$

onformal blocks

$\phi\,\theta\rangle\langle\theta\,\phi\phi\rangle$

$\delta(p_1+p_2)\,\delta(p_3+p_4)$ + crossing

How?

- Wightman $\langle 0|\phi\phi\phi\phi|0\rangle$ $\quad \uparrow \sum_\theta |\theta\rangle\langle\theta|$
  - → OPE
  - → ~~crossing~~
- TO: $\langle 0|T[\phi\phi\phi\phi]|0\rangle$
  - → crossing
  - → ~~OPE~~

$$F_{\Delta,\ell}(z,\cos\theta) = z^{(\Delta-\ell-2\Delta_\phi)/2}(1-z)$$
$$\times \frac{(4\pi)^{3d/2}\,\Gamma\left(\frac{d}{2}-\Delta_\phi\right)^3\,\Gamma(\Delta+\ell)}{2^{4\Delta_\phi+2\ell}\,\Gamma\left(\frac{\Delta+\ell}{2}\right)^3\,\Gamma\left(\Delta_\phi\right.}$$
$$\times \sum_{n=0}^{\lfloor \ell/2 \rfloor} \frac{\ell!\left(\frac{d-\Delta+\ell+1}{2}\right)_n}{n!\,2^\ell\left(\frac{d}{2}-1+n\right)_{\ell-n}\left(\frac{3-\Delta-\ell}{2}\right.}$$
$$\times {}_3F_2\left(\begin{matrix}\frac{\Delta-\ell-d+3+2n}{2}, & \frac{\Delta-\ell-d+2}{2}, & \frac{\Delta-\ell-}{} \\ \frac{\Delta-\ell-d+3-2n}{2}, & \Delta-\frac{d}{2}+1 & \end{matrix}\right.$$

$$\frac{P^0 + K^0/R^2}{2} \xrightarrow{R\to\infty} P^0$$

conformal blocks

$\phi\,\theta\rangle\langle\theta\,\phi\phi\rangle$

$\delta(p_1+p_2)\,\delta(p_3+p_4)$ + crossing

How?

- Wightman $\langle 0|\phi\phi\phi\phi|0\rangle$ $\leftarrow \sum_\theta |\theta\rangle\langle\theta|$
  - → OPE
  - → crossing
- TO: $\langle 0|T[\phi\phi\phi\phi]|0\rangle$
  - → crossing
  - → ~~OPE~~
- Amplitudes

$$\times \frac{(4\pi)^{3d/2}\,\Gamma\left(\frac{d}{2}-\Delta_\phi\right)\,\Gamma(\Delta+\ell)}{2^{4\Delta_\phi+2\ell}\,\Gamma\left(\frac{\Delta+\ell}{2}\right)^3\,\Gamma(\Delta_\phi$$

$$\times \sum_{n=0}^{\lfloor \ell/2 \rfloor} \frac{\ell!\left(\frac{d-\Delta+\ell+1}{2}\right)_n}{n!\,2^\ell\left(\frac{d}{2}-1+n\right)_{\ell-n}\left(\frac{3-\Delta-\ell}{2}\right.}$$

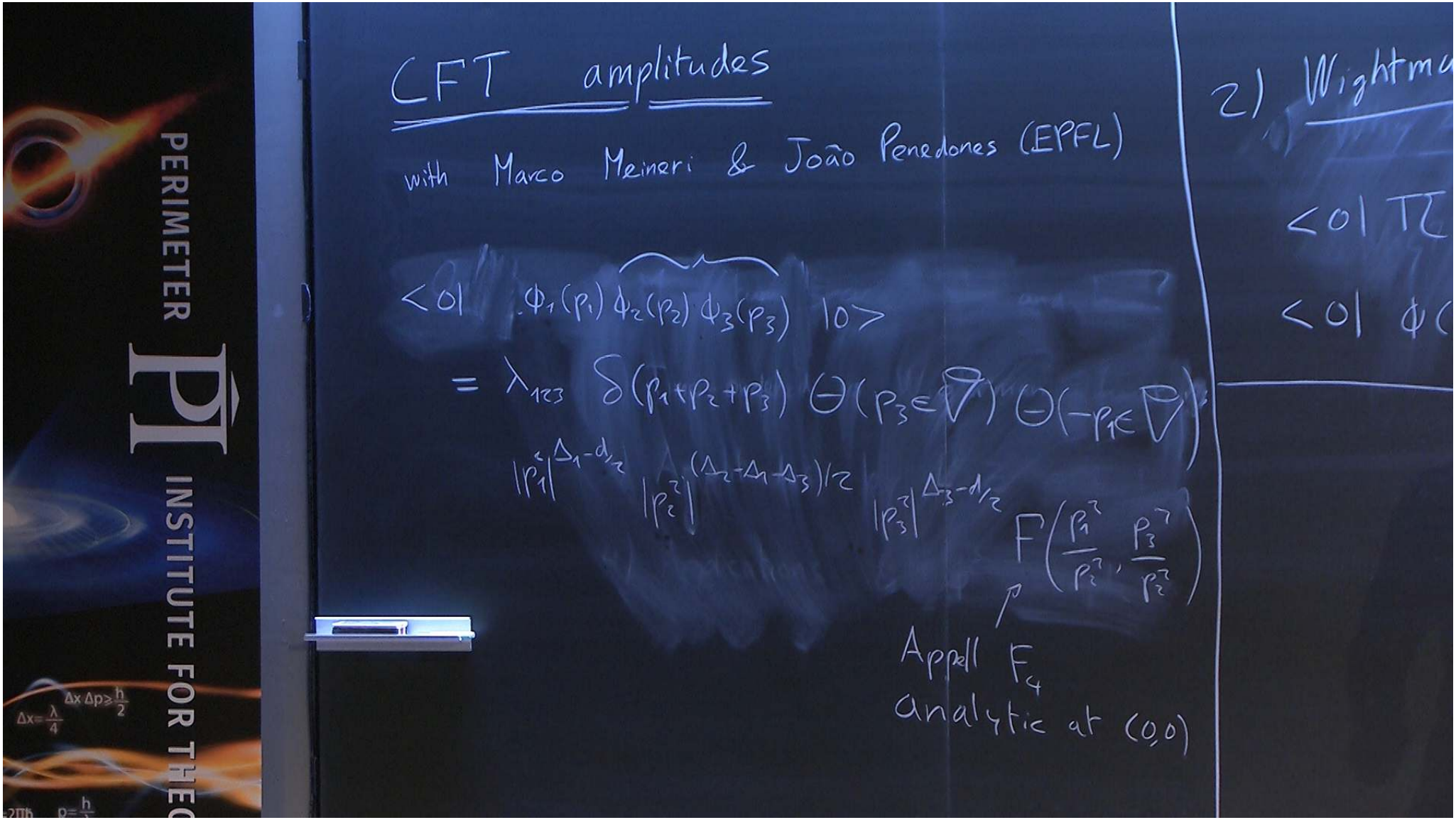
$$\times {}_3F_2\left(\begin{matrix}\frac{\Delta-\ell-d+3+2n}{2}, & \frac{\Delta-\ell-d+2}{2}, & \frac{\Delta-\ell-}{2} \\ \frac{\Delta-\ell-d+3-2n}{2}, & \Delta-\frac{d}{2}+1 & \end{matrix}\right.$$

2) Wightman vs TO

$$\langle 0|\, T[\phi(p')\,\phi(p)]\,|0\rangle = \delta^d(p'+p)\;(p^2-i\varepsilon)^{\Delta-d/2}$$

$$\langle 0|\,\phi(p')\,\phi(p)\,|0\rangle = \delta^d(p'+p)\;|p^2|^{\Delta-d/2}\;\underbrace{\theta(p^0)\,\theta(-p^2)}$$

0 unless $p \in$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CFT amplitudes
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2) Wightma
$\langle 0| \Phi_1(p_1)\Phi_2(p_2)\Phi_3(p_3)|0\rangle$
$= \lambda_{123}\,\delta(p_1+p_2+p_3)\,\Theta(p_3\in\bar{V})\,\Theta(-p_1\in\bar{V})$
$|p_1^2|^{\Delta_1-d/2}\,|p_2^2|^{(\Delta_2-\Delta_1-\Delta_3)/2}\,|p_3^2|^{\Delta_3-d/2}\,F\left(\frac{p_1^2}{p_2^2},\frac{p_3^2}{p_2^2}\right)$
Appell F_4
analytic at (0,0)
$\langle 0|$ T
$\langle 0|$ ϕ(

$$\langle 0|\, T[\phi(p')\,\phi(p)]\,|0\rangle = \delta^d(p'+p)\,(p^2 - i\varepsilon)^{\Delta - d/2}$$

$$\langle 0|\, \phi(p')\,\phi(p)\,|0\rangle = \delta^d(p'+p)\,|p^2|^{\Delta - d/2}\,\underbrace{\Theta(p^0)\,\Theta(-p^2)}$$

$$\langle 0|\, T[\phi_1\,\phi_2\,\phi_3]\,|0\rangle = \lambda_{123}\,\delta(p_1+p_2+p_3)\,(p_2^2)^{(\Delta_1+\Delta_2+\Delta_3-2d)/2}$$

$$\times\Big[\, x^{\Delta_1 - d/2}\, y^{\Delta_3 - d/2}\, F^{(1)}(x,y) + x^{\Delta_1 - d/2}\, F^{(2)}(x,y) + y^{\Delta_3 - d/2}\, F^{(3)}(x,y) + F^{(4)}(x,y)\,\Big]$$

$$(x,y) = \left(\frac{p_1^2}{p_2^2}, \frac{p_3^2}{p_2^2}\right)$$

$\in \nabla)$

$\frac{p_3^2}{p_2^2})$

$(0,0)$

How

• W

•

•

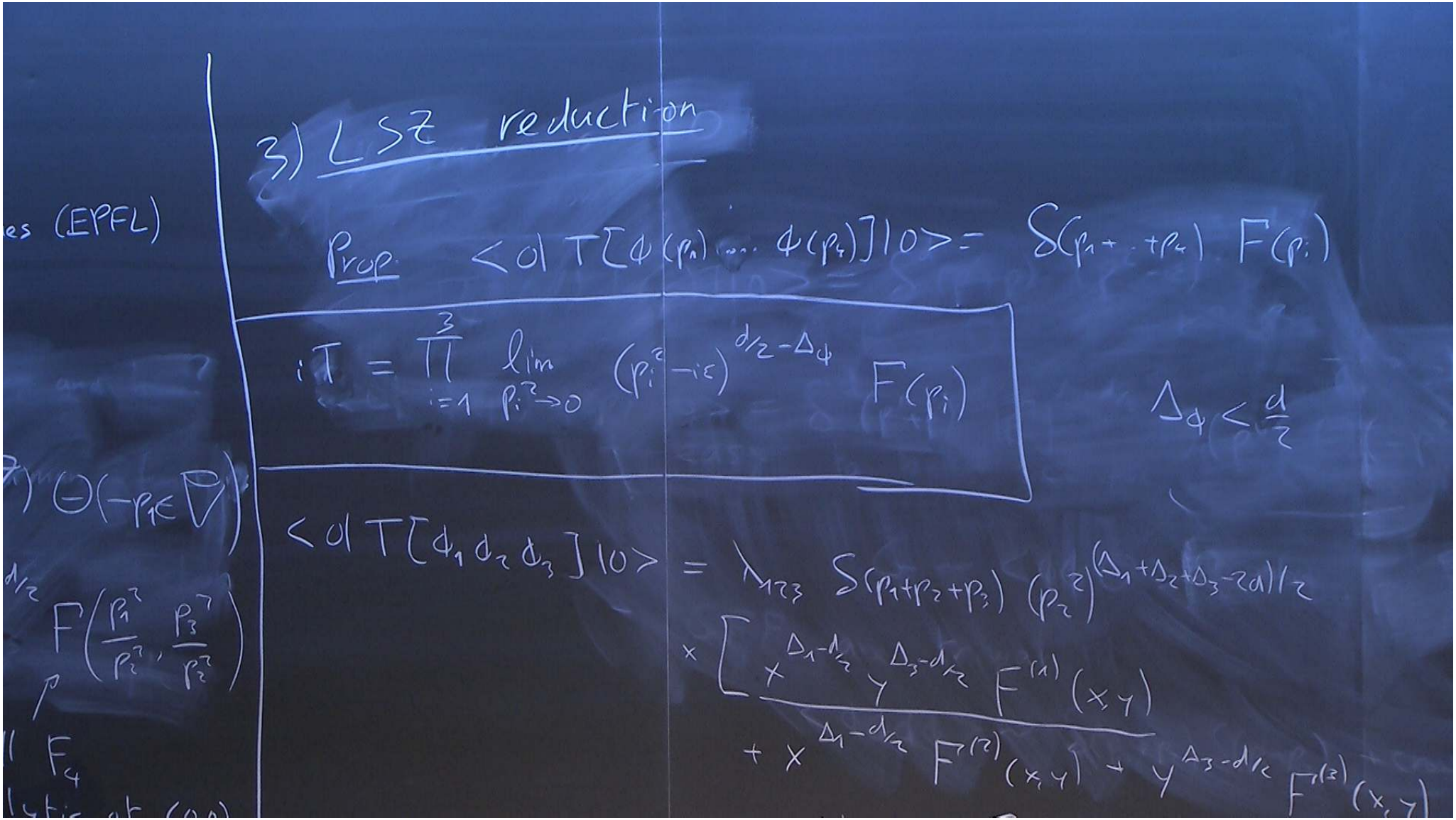
3) LSZ reduction

Prop. $\langle 0|T[\phi(p_1)\dots\phi(p_4)]|0\rangle = \delta(p_1+\dots+p_4)\,F(p_i)$

$$\lim_{p_i^2\to 0}\,(p_i^2-i\varepsilon)^{d/2-\Delta_\phi}\,F(p_i) \qquad \Delta_\phi < \frac{d}{2}$$

$$\langle 0|T[\phi_1\phi_2\phi_3]|0\rangle = \lambda_{123}\,\delta(p_1+p_2+p_3)\,(p_2^2)^{(\Delta_1+\Delta_2+\Delta_3-2d)/2}$$

$$\times\Big[\frac{x^{\Delta_1-d/2}\,y^{\Delta_3-d/2}\,F^{(1)}(x,y)}{}+\dots$$



es (EPFL)

$\Theta(-p_1 \in \nabla)$

$d/2$ $F\left(\frac{p_1^2}{p_2^2}, \frac{p_3^2}{p_2^2}\right)$

F_4

lytic at (0,0)

3) LSZ reduction

Prop. $\langle 0 | T[\phi(p_1) \dots \phi(p_4)] | 0 \rangle = \delta(p_1 + \dots + p_4)\, F(p_i)$

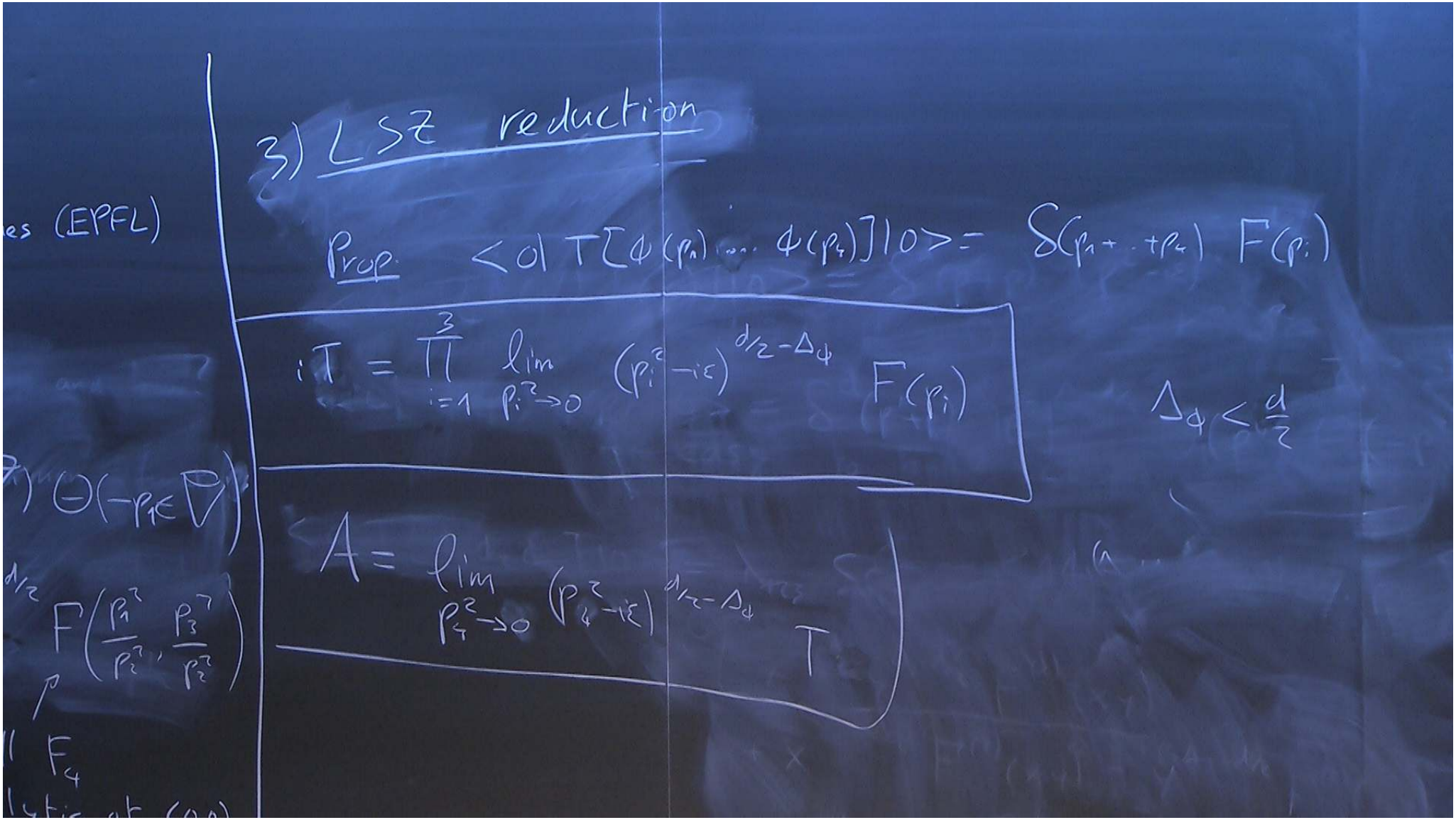
$$\boxed{iT = \prod_{i=1}^{3} \lim_{p_i^2 \to 0} (p_i^2 - i\varepsilon)^{d/2 - \Delta_\phi} F(p_i)}$$

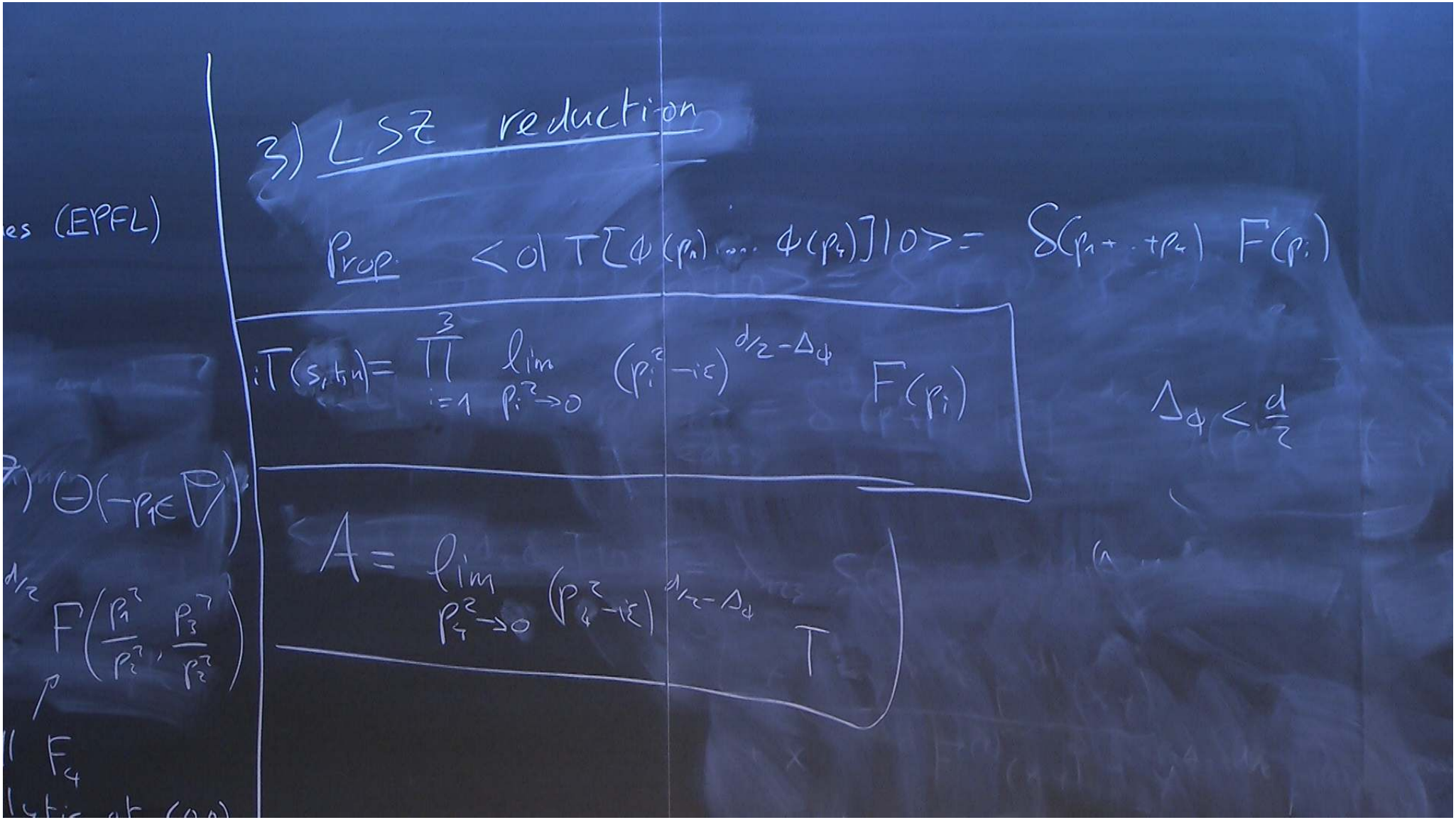
$\Delta_\phi < \frac{d}{2}$

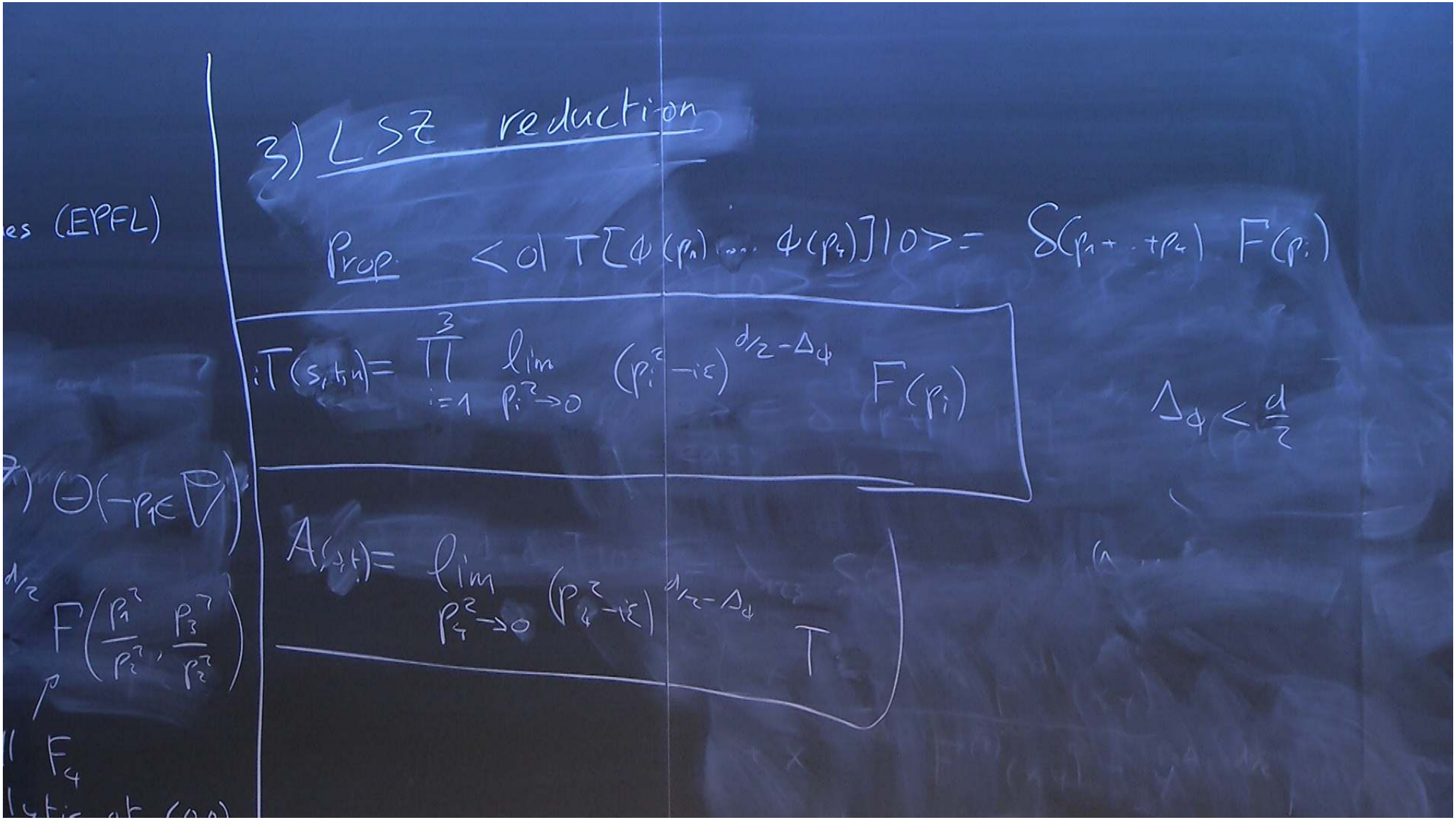
$$\langle 0 | T[\phi_1 \phi_2 \phi_3] | 0 \rangle = \lambda_{123}\, \delta(p_1 + p_2 + p_3)\, (p_2^2)^{(\Delta_1 + \Delta_2 + \Delta_3 - 2d)/2}$$

$$\times \Big[\underline{x^{\Delta_1 - d/2}\, y^{\Delta_3 - d/2}\, F^{(1)}(x,y)}$$

$$+ x^{\Delta_1 - d/2} F^{(2)}(x,y) + y^{\Delta_3 - d/2} F^{(3)}(x,y)$$


3) LSZ reduction
Prop: $\langle 0|T[\phi(p_1)\dots\phi(p_4)]|0\rangle = \delta(p_1+\dots+p_4)\,F(p_i)$
$iT = \prod_{i=1}^{3} \lim_{p_i^2\to 0} (p_i^2 - i\varepsilon)^{d/2-\Delta_\phi}\, F(p_i)$
$\Delta_\phi < \frac{d}{2}$
$A = \lim_{p_4^2\to 0} (p_4^2 - i\varepsilon)^{d/2-\Delta_\phi}\, T$


3) LSZ reduction
Prop: $\langle 0|T[\phi(p_1)\dots\phi(p_4)]|0\rangle = \delta(p_1+\dots+p_4)\,F(p_i)$
$T(s,t,u) = \prod_{i=1}^{3} \lim_{p_i^2\to 0} (p_i^2 - i\varepsilon)^{d/2-\Delta_\phi}\, F(p_i)$
$\Delta_\phi < \frac{d}{2}$
$A = \lim_{p_4^2\to 0} (p_4^2 - i\varepsilon)^{d/2-\Delta_\phi}\, T$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$A(s,t) = \lim_{p_4^2\to 0} (p_4^2 - i\varepsilon)^{d/2-\Delta_\phi}\, T$
es (EPFL)
$\Theta(-p_1\in \bar{V})$
$F\left(\frac{p_1^2}{p_2^2}, \frac{p_3^2}{p_2^2}\right)$
F_4

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$$\lim_{p_1^2 \to 0} \left(p_1^2 - i\varepsilon\right)^{\frac{d}{2}-\Delta_\phi} \int d^d x_1 e^{ip_1 x_1} \langle 0 | T[\phi(x_1) \cdots \phi(x_n)] | 0 \rangle$$

$$\begin{pmatrix} p_1^0 < 0 \\ p_1^2 \geq 0 \end{pmatrix} \Rightarrow \int d^d x_1 e^{ip_1 x_1} \langle 0 | \phi(x_1) T[\phi(x_2) \cdots \phi(x_n)] | 0 \rangle$$

$x_1^0 > x_i^0$

$$\sum_{\mathcal{O}} |\mathcal{O}\rangle\langle\mathcal{O}|$$

$$= \frac{1}{2i \sin \pi(\frac{d}{2} - \Delta_\phi)} \langle \phi(p_1) | T[\phi(x_2)\phi(x_3)\phi(x_4)] | 0 \rangle$$

3) LSZ reduction

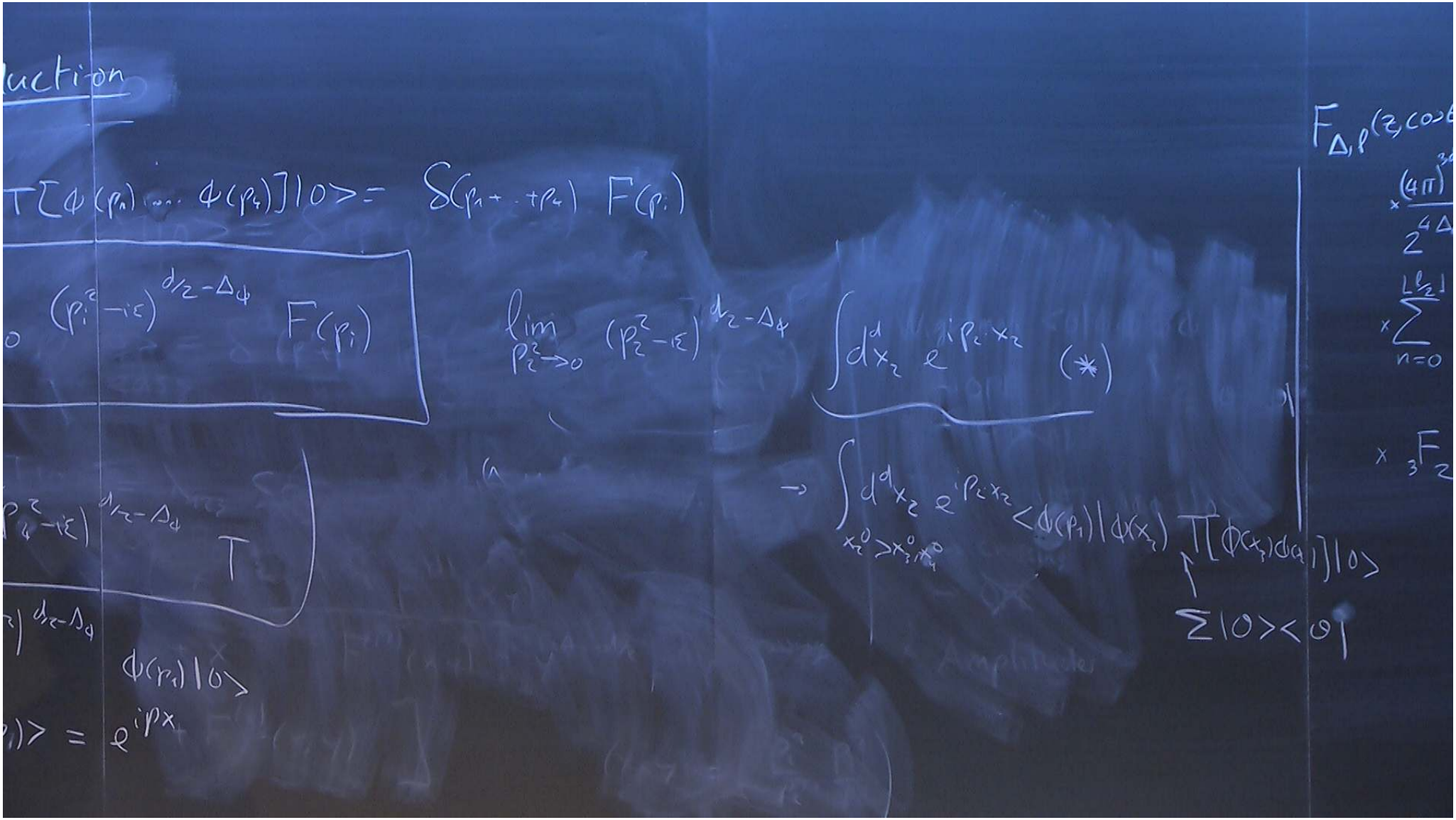
Prop $\langle 0 | T[\phi(p_1) \cdots \phi(p_n)] | 0$

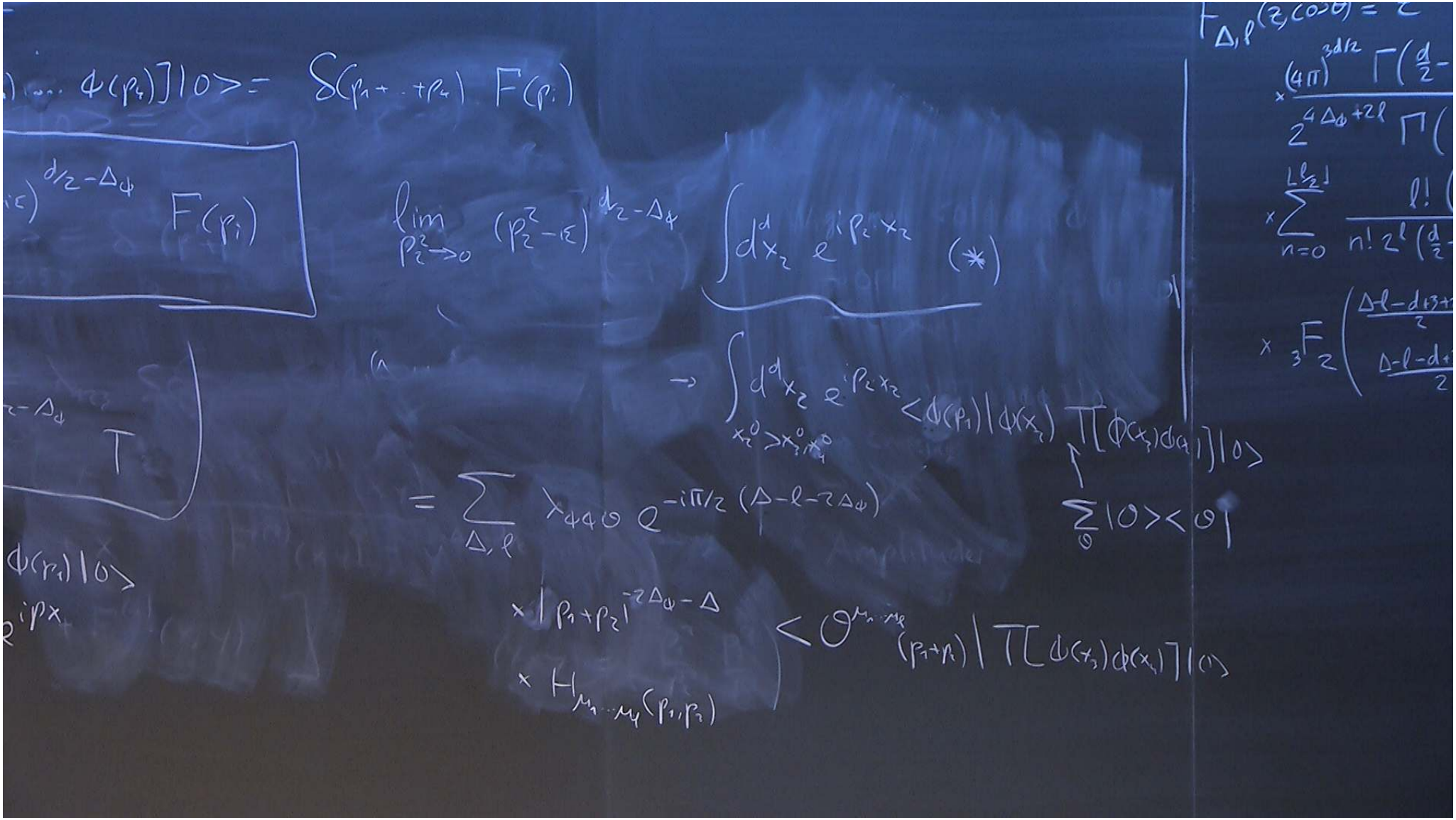
$$iT(s,t,u) = \prod_{i=1}^{3} \lim_{p_i^2 \to 0} \left(p_i^2 - i\varepsilon\right)^{d/2-\Delta_\phi} \quad FC$$

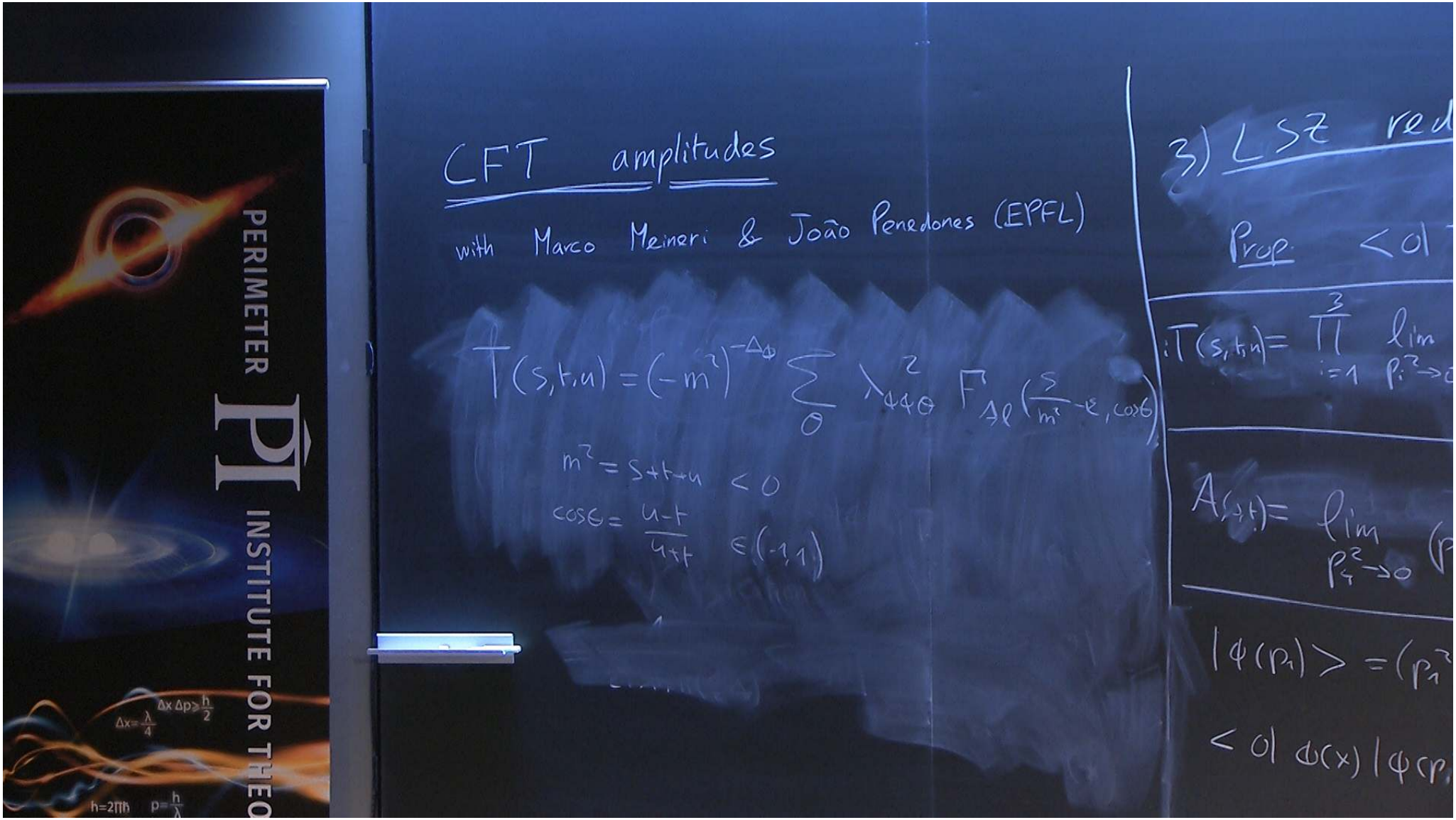
$$A(s,t) = \lim_{p_4^2 \to 0} \left(p_4^2 - i\varepsilon\right)^{d/2-\Delta_\phi} T$$

$$|\phi(p_1)\rangle = (p_1^2)^{d/2-\Delta_\phi} \phi(p_1)|0\rangle$$

$$\langle 0 | \phi(x) | \phi(p_1) \rangle = e^{ipx}$$


uction
T[φ(p1) ... φ(p4)]|0> = δ(p1 + ... + p4) F(pi)
(pi² − iε)^(d/2 − Δφ) F(pi)
lim p2² → 0 (p2² − iε)^(d/2 − Δφ) ∫ d^d x2 e^(i p2 · x2) (*)
→ ∫ d^d x2 e^(i p2 · x2) <φ(p1)|φ(x2) T[φ(x3)φ(x4)]|0>
x2^0 > x3^0, x4^0
Σ|0><0|
φ(p1)|0>
= e^(ipx)
F_{Δ,ℓ}(z, cos
(4π)^3
2^4Δ
Σ_{n=0}^{⌊ℓ/2⌋}
3F2


$\phi(p_4)]|0\rangle = \delta(p_1 + \cdots + p_4)\, F(p_i)$
$\lim_{p_2^2 \to 0} (p_2^2 - i\varepsilon)^{d/2-\Delta_\phi}$
$\int d^d x_2\, e^{i p_2 \cdot x_2}$ (*)
$\to \int_{x_2^0 > x_3^0, x_4^0} d^d x_2\, e^{i p_2 \cdot x_2} \langle \phi(p_1)|\phi(x_2)\, T[\phi(x_3)\phi(x_4)]|0\rangle$
$= \sum_{\Delta,\ell} \lambda_{\phi\phi\mathcal{O}}\, e^{-i\pi/2(\Delta-\ell-2\Delta_\phi)}$
$\times |p_1+p_2|^{-2\Delta_\phi-\Delta}$
$\times H_{\mu_3\cdots\mu_4}(p_1,p_2)$
$\sum_{\mathcal{O}} |\mathcal{O}\rangle\langle\mathcal{O}|$
$\langle \mathcal{O}^{\mu_3\cdots\mu_\ell}(p_1+p_2)|\, T[\phi(x_3)\phi(x_4)]|0\rangle$
$F_{\Delta,\ell}(z,\cos\theta) =$
$\frac{(4\pi)^{3d/2}\,\Gamma(\frac{d}{2}-$
$2^{4\Delta_\phi+2\ell}\,\Gamma($
$\sum_{n=0}^{\lfloor \ell/2 \rfloor}$ $\frac{\ell!\,(}{n!\,2^\ell(\frac{d}{2}-}$
${}_3F_2\left(\frac{\Delta-\ell-d+3+}{2}\right.$
$\left.\frac{\Delta-\ell-d+}{2}\right)$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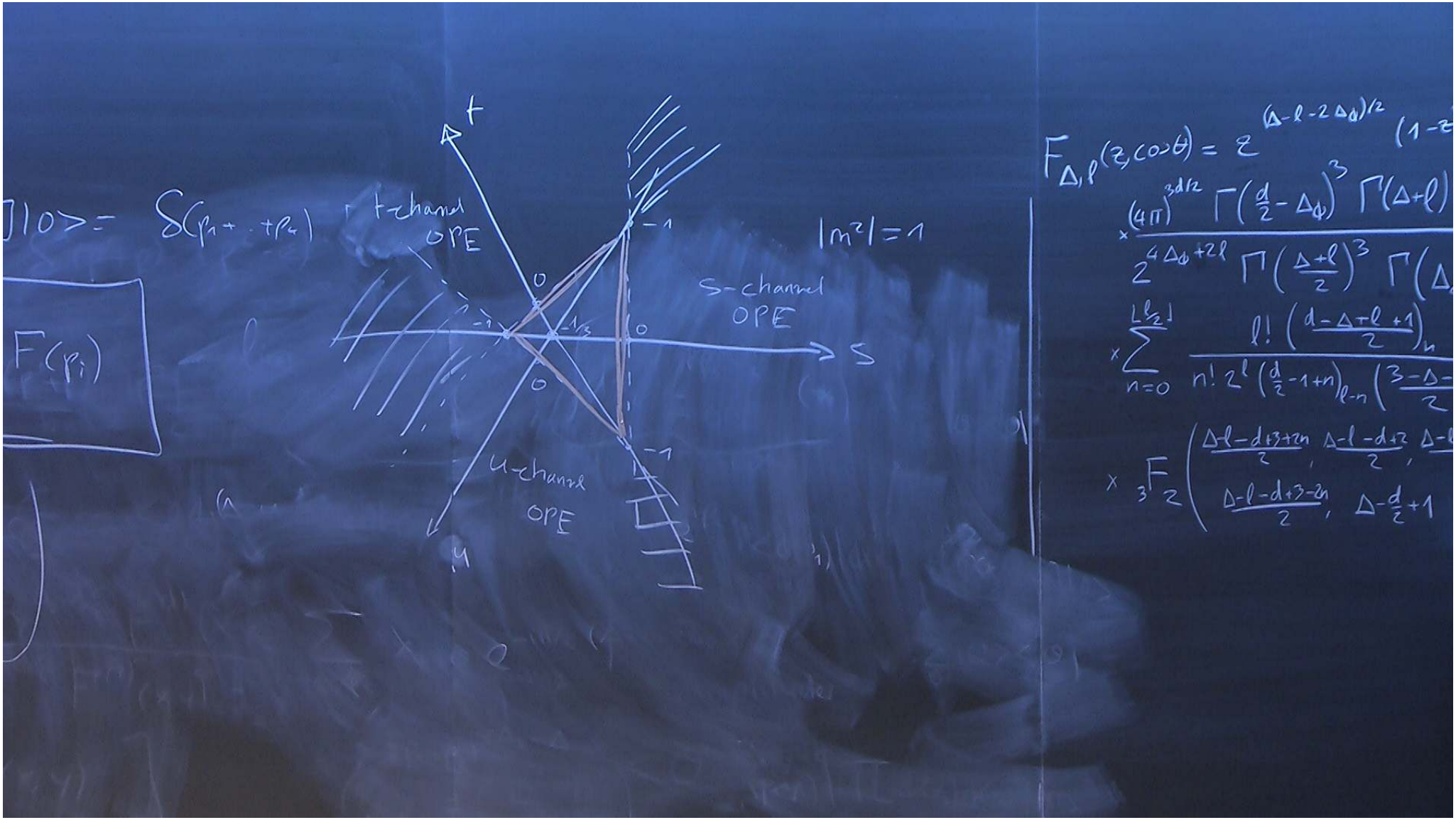
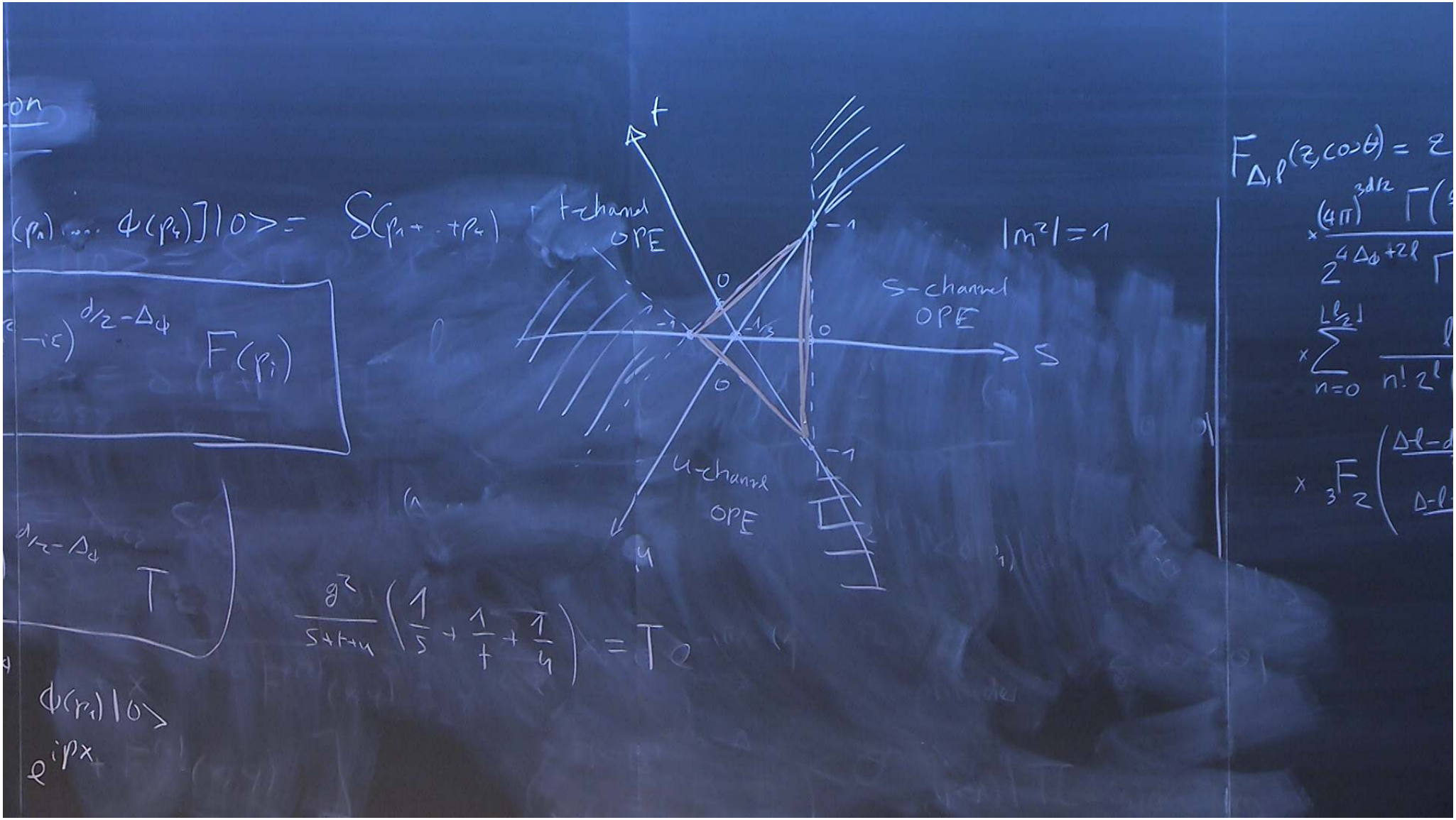
3) LSZ red
Prop:


t
t-channel
OPE
s-channel
OPE
u-channel
OPE
s
u
|m²|=1
F(pᵢ)


t-channel OPE
s-channel OPE
u-channel OPE
$|m^2| = 1$
$F(p_i)$
$\frac{g^2}{s+t+u}\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right) = T$
$\phi(p_1)|0\rangle$
e^{ipx}