Title: Talk 21 via live stream

Speakers: Walter Landry, David Simmons-Duffin

Collection: Bootstrap 2019

Date: July 29, 2019 - 10:00 AM

URL: http://pirsa.org/19070043

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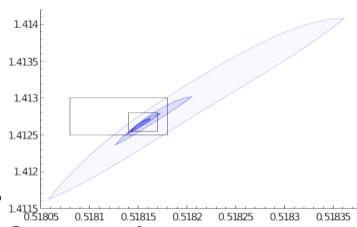
The Bootstrap and Semidefinite Programs

- The conformal bootstrap can be formulated in terms of a semidefinite program.
- Semidefinite programs are generic math problems that occurs in many branchs of science and engineering.
- Existing, off-the shelf solver implementations exist in a variety of environments
 - Matlab, Mathematica, C, Python, ...

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Why SDPB?

 Bootstrap calculations can require extreme numerical precision and computational resources.



- Ising computations ran for weeks.
- SDPB is a solver optimized for bootstrapping.
 - Open-source
 - Arbitrary precision
 - Heavily parallelized

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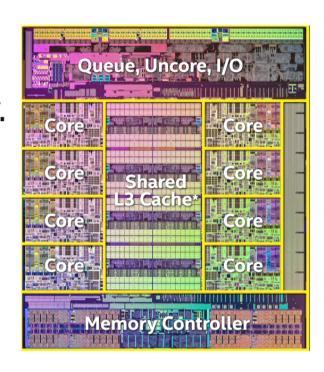
Parallelizing Linear Algebra

- Most of what takes time in SDPB is linear algebra operations on independent blocks of matrices.
- The results of these independent operations are combined into a single, comparatively small, global matrix Q.
- These independent operations can be run on different cores.

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Parallelization with OpenMP

- SDPB was initially parallelized with OpenMP
 - OpenMP is very easy to use, but it relies upon a global view of memory.
 - Works on single nodes up to ~20 cores.
- Global view of memory quickly stops working beyond a single node.



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Parallelism with / MPI



- SDPB has been enhanced to use MPI (Message Passing Interface).
- MPI works by passing messages between cores.
- This works far better than OpenMP on supercomputers.
- It required extensive restructuring of the code.



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Timing Runs

 Part of the restructuring is that we now have to explicitly assign these block computations to specific cores.



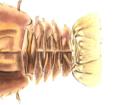
- Doing this well requires measuring how long it takes to run calculations for each block.
- Trying to derive the timings from first principles results in terrible performance.

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Recent Work

- Automatic Timing
- Fake Primal Fix
- Faster Input
- Hot-starting and text checkpoints
- Installations
- Memory Use
- Scalar Blocks
- Spectrum Extraction
- Proposals

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Fake Primal Solution

There is a bug in the original,
 SDPB-OpenMP implementation of the primal error



- Paper says: $primalError = \max\{|\mathbf{p_i}|, |\mathbf{P_i}\mathbf{j}|\}$
- Implementation was: $primalError = max\{|P_ij|\}$
- Usually it makes no difference



SDPB-MPI now reports both
 P and p, but it uses the full
 primalError for deciding when to stop.

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Faster Input

XML : pvm2sdp



- Now fully parallelized
- SDP in Mathematica: sdp2input
 - Directly generates SDPB input files.
 - 16 times faster than SDPB.m on 28 cores
 - Enables some people to work without Mathematica (not all clusters have it).



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Hot-starting is Fully Supported

- Allows you to start a new calculation with an older solution
- Can reduce the number of iterations by a factor of 10.

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Text Checkpoints

- Allows you to add to or modify an existing solution for a new problem.
- Portable across machines
- Not strictly bitwise identical.
 - The last bit can be different.
 - This comes from a limitation in the underlying GMP library.
 - It is unlikely to matter.

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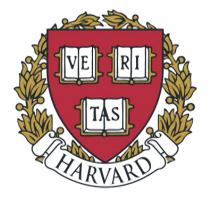
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Easier Installation

- Better autodection of libraries
- No unnecessary dependencies.

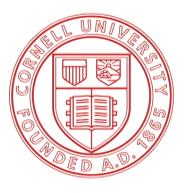
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Installed Everywhere





















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Easiest Option

- For smaller runs on your laptop or desktop
 - Docker (Windows, Mac, Linux) or
 - Singularity (Linux: recommended)
- Download and Run
- Pretty efficient and uses all cores.
 - IAS admins used Singularity for their install on the Helios cluster.
- Instructions in the repository
 https://github.com/davidsd/sdpb/blob/master/docs/Docker.md
 https://github.com/davidsd/sdpb/blob/master/docs/Singularity.md

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Much Better Memory Use

- Memory use is dominated by many cores having their own copy of their contribution to the matrix Q.
- Q is symmetric, so we now explicitly deallocate half of it.
 - The underlying parallel linear algebra library, Elemental, is not accustomed to this, so we have to be a bit careful.

El

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procGranularity

- Added the option procGranularity
- Spreads the local contribution to Q across more cores
- A bit slower, so only use if desperate

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Synchronizing Q

- The local contributions to Q are summed and then distributed to a global Q with the low level routine MPI_Reduce_scatter.
- MPI_Reduce_scatter requires an additional copy of Q on each core.
 - Reimplemented to remove these copies
- With procGranularity, the memory overhead compared to SDPB-OpenMP should now be minimal.

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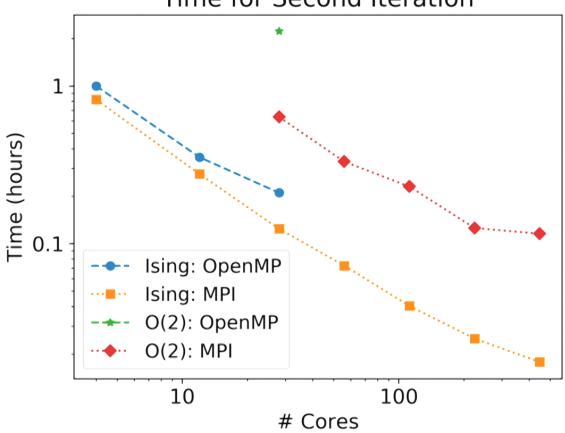
Q Caveats

- It is not as fast for large core counts.
 - Factor of 2-3 for O(2), n_max=18
 with 448 cores at Yale
- However, you would only use large core counts for large problems.
- Previously, you would have a hard time fitting your large problem on the machine at all.
 - O(2), n_max=22 did not fit on Comet

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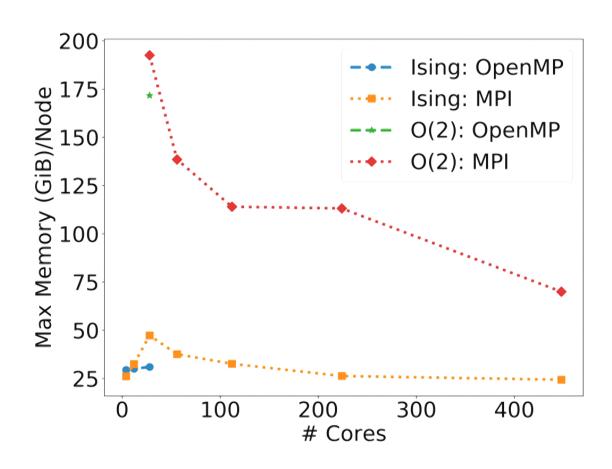
Scaling on Large Machines





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Memory Use



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O(2) Remarks

- The O(2) project has been an excellent driver of progress for SDPB.
- It generated large, concrete benchmarks that people definitely wanted to solve.
- It highlighted bottlenecks when performing a complete bootstrap calculation, motivating improvements to block generation (scalar_blocks) and conversion from Mathematica SDP's to input (sdp2input)

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Scalar Blocks

- Replaces Mathematica block generation
- Written in C++
- 111 times faster on 28 cores
- Available in the socker and simages.

https://gitlab.com/bootstrapcollaboration/scalar_blocks

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Spectrum Extraction

- Python script to extract the spectrum from the SDPB output
- Updated for new output format
- Clarified dependencies and made to work with python 2 or 3



https://gitlab.com/bootstrapcollaboration/spectrum-extraction

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XSEDE Proposal

- XSEDE is an NSF funded clearinghouse for supercomputer time at different centers.
- We wrote a proposal for the O(2) project for 5 million hours on the SDSC Comet cluster.
- Awarded 3 million hours
- Received 1.2 million hours
- Used up 200,000 hours in a few days



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Cannon Cluster Proposal

- Harvard is standing up Cannon, a new cluster with 30,000 cores.
- They are looking for users who can thoroughly exercise the machine.
 - Science would be nice, but is not the driver
- Request for Proposals: Up to 3 days of compute time on the whole cluster.
- We submitted a proposal for ~1 million hours for more O(2) work.

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Better Scaling

- The work so far has pushed the scalability of SDPB from ~20 cores to ~300.
- We have run jobs up to 768 cores.
- The rule of thumb is that each improvement by a factor of 10 takes significant effort.
- The next step will require careful benchmarking on large machines.

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Why Such High Precision?

- I will be looking at a small stress tensor example. It seems non-trivial enough to be useful.
- You might expect to need only to resolve
 - The error threshold: 10^{-40}
 - The duality gap between the primal and dual solutions: 10^{-80}
- In practice, we need much, much higher precision.

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What Breaks?

The first thing that breaks when reducing precision is when solving

$$\begin{pmatrix} S & -B \\ B^T & 0 \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

- S has a block structure made up of symmetric positive-definite matrices.
- We use a Schur complement method, which involves inverting S first.

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S is III-Conditioned

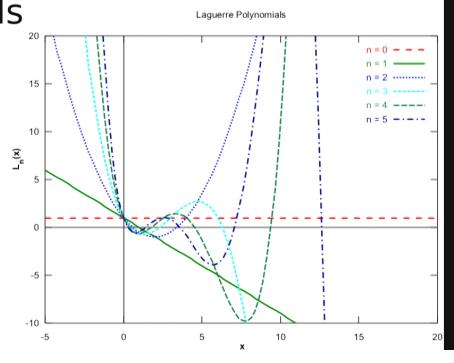
- When precision is low, S is no longer numerically positive.
- This is because S has a very bad condition number: 10^{180}
- This happens immediately, well before we do any real calculations.

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Bad Basis

 By default, we evaluate functions at the roots of Laguerre polynomials.

 Laguerre polynomials mimic exponentials, but the function we are approximating
 is well behaved over the domain.



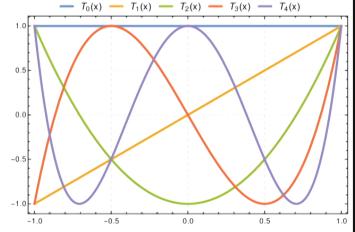
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Chebyshev Polynomials

Chebyshev polynomials are very well

behaved in their domain.

 We tried mapping the Chebyshev roots to the same interval.



 Evaluating functions at these new points yields a dramatic improvement in the condition number of S: 10^4

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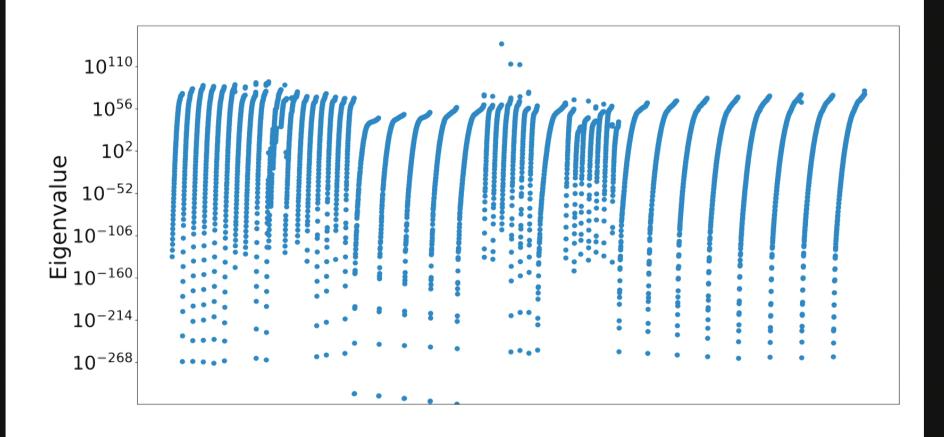
Not the Solution

 Unfortunately, as SDPB converges on a solution,
 S again becomes very, very ill-conditioned:

 10^{400}

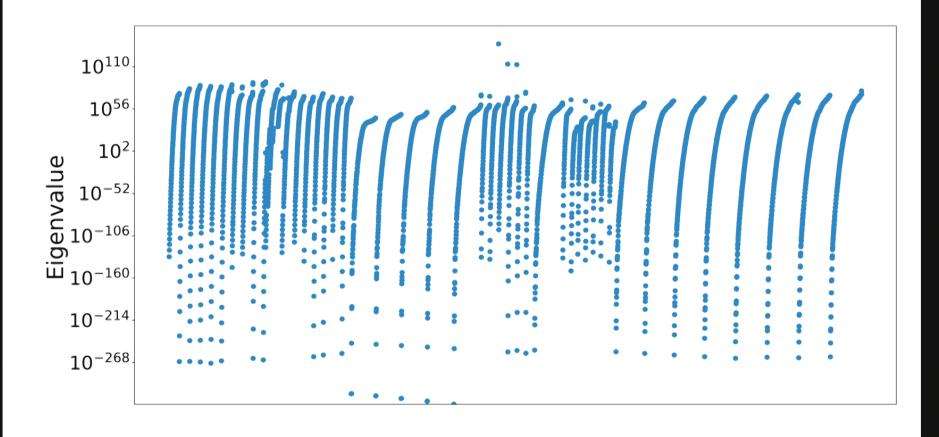
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Eigenvalue Spectrum of S



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Eigenvalue Spectrum of S



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Eigenvector Decomposition

• If we decompose dx and B into eigenvectors of S, it turns out that:

$$dx_i \propto {\lambda_i}^{-1/2}$$
 $B_i \propto {\lambda_i}^{1/3}$

- This implies $r_y = B \cdot dx \propto {\lambda_i}^{-1/6}$
- But r_y lives in a different space and at this point in the calculation, after a dual jump, is essentially zero.

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Small Differences - Big Problems

- The scaling eventually breaks down at small λ_i , but there are still many cancellations over a large range of λ_i .
- It does mean that we can not just ignore small eigenvalues.
- So there is still more to understand.

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New Work

- Gateways
- Cloud
- Job Management

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Gateway

- A web interface to SDPB
 - Simple pointy-clicky
 - Scriptable (https POST)
- You upload input files.
- The Gateway submits these files to a supercomputer.
- You check progress from time to time.
- When finished, you collect the results



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Gateway Users

- This removes the need to understand supercomputers in order to do large calculations.
- Even for those who do understand them, there is no need to get any complex control software running there.
 - For example, Mathematica is not available on XSEDE

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Gateway Implementation

- **XSEDE** is very interested in giving away free time for gateways.
 - I have implemented a gateway with them in the past (seismology simulations)
 - I have also run high traffic web sites with scientific users.
- We would essentially become a mini-funding agency, giving out SDPB time.
- Requires a committee to vet applications

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Scaling to the

- Many different, large providers
- You can make use of enormous compute power.
 - Western Digital burned 8 million core-hours in 8 hours on hard drive simulations.
- Caltech seriously considered using the cloud instead of building their own supercomputer.







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Cloud Details: \$\$\$

- It costs serious money, but sometimes they give away time for free
- Compute is cheap-ish: 2-3 cents/core-hour
 - The Western Digital runs cost \$137,307
- Storage is not cheap: ~2 cents/GB-month
- Transferring data into the cloud is free.
 - Getting data back out is not: ~9 cents/GB
- Good match for SDPB
 - long calculations and small outputs

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Cloud Implementation

- Smaller, 1-node jobs may already work with Docker?
- Larger jobs require more thorough investigation and performance testing.
 - AWS ParallelCluster
 - No Infiniband. Maybe EFA is not terrible?
 - Azure Batch
 - Requires Intel compiler?
 - Not as big a market, so fewer choices

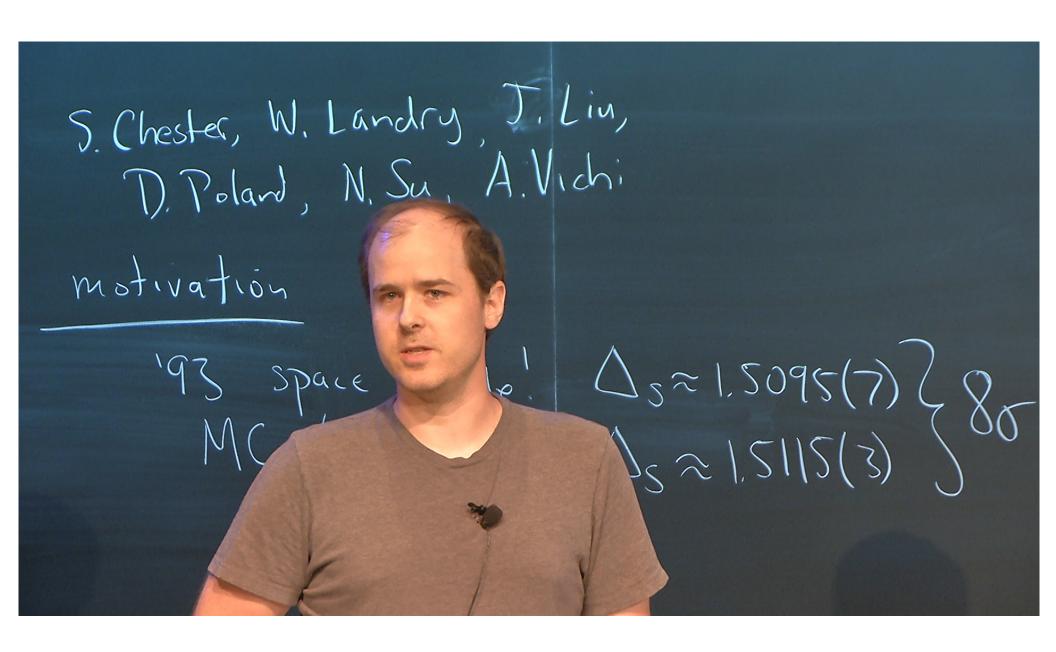
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Managing Computations

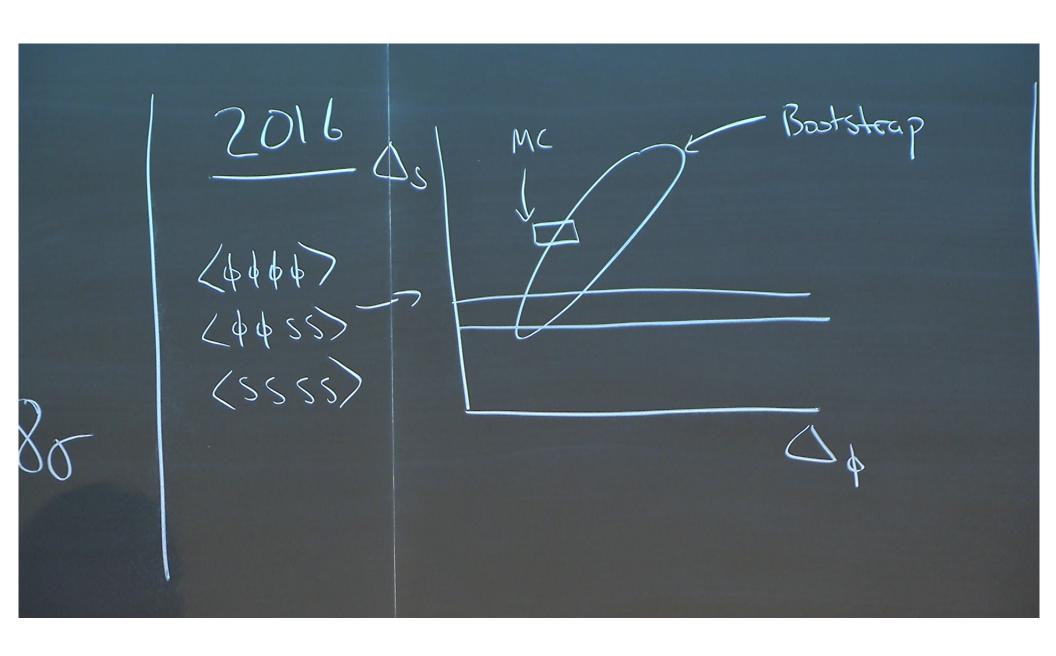
- Assuming that all of these resources are available, we still need a way to manage all of the separate computations that combine into a single result.
- Existing approaches use a variation of Mathematica scripts or Haskell programs.
- We should figure out what is common to all of these approaches and automate that.

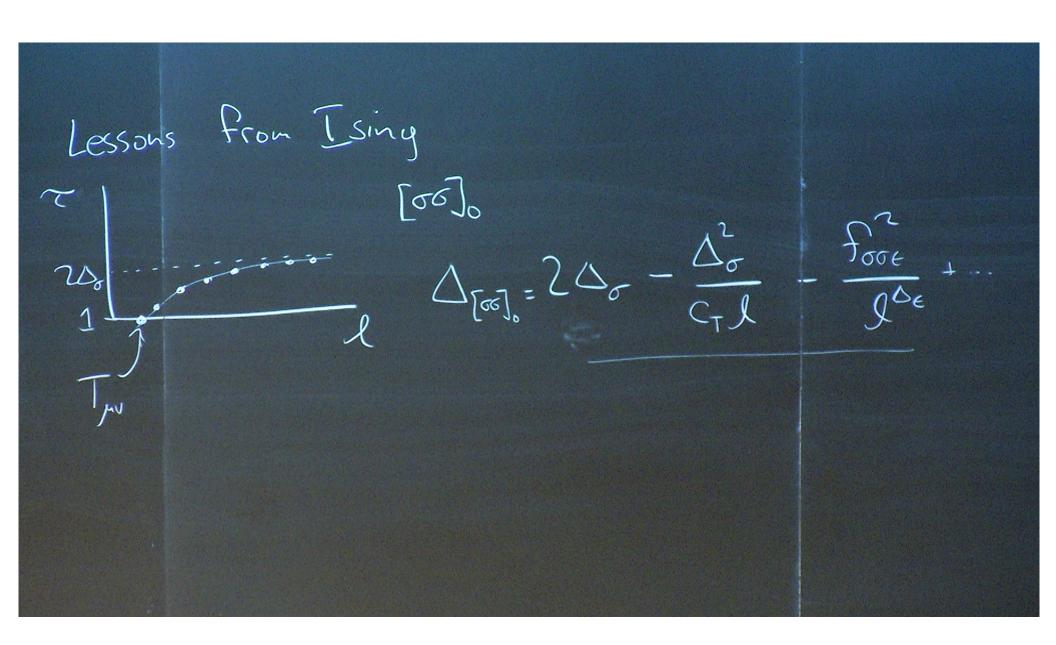
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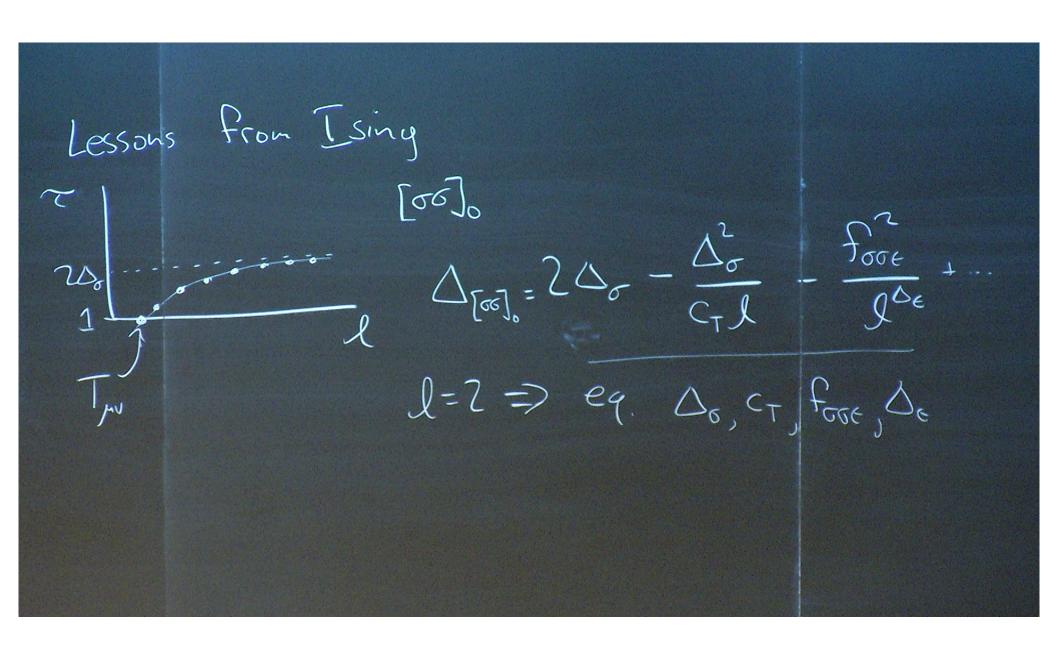
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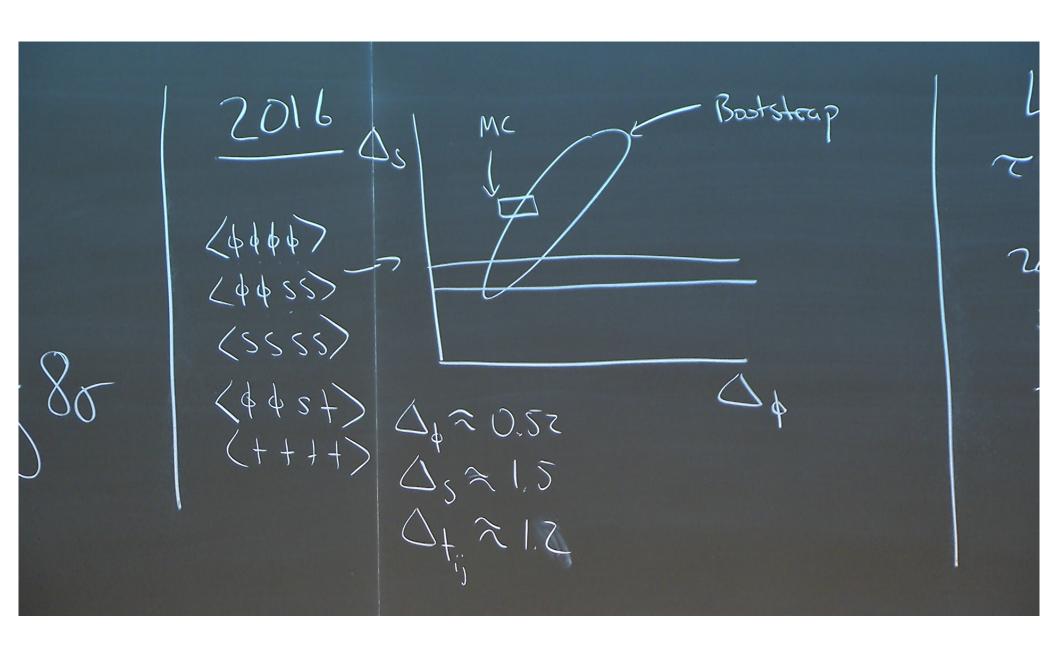


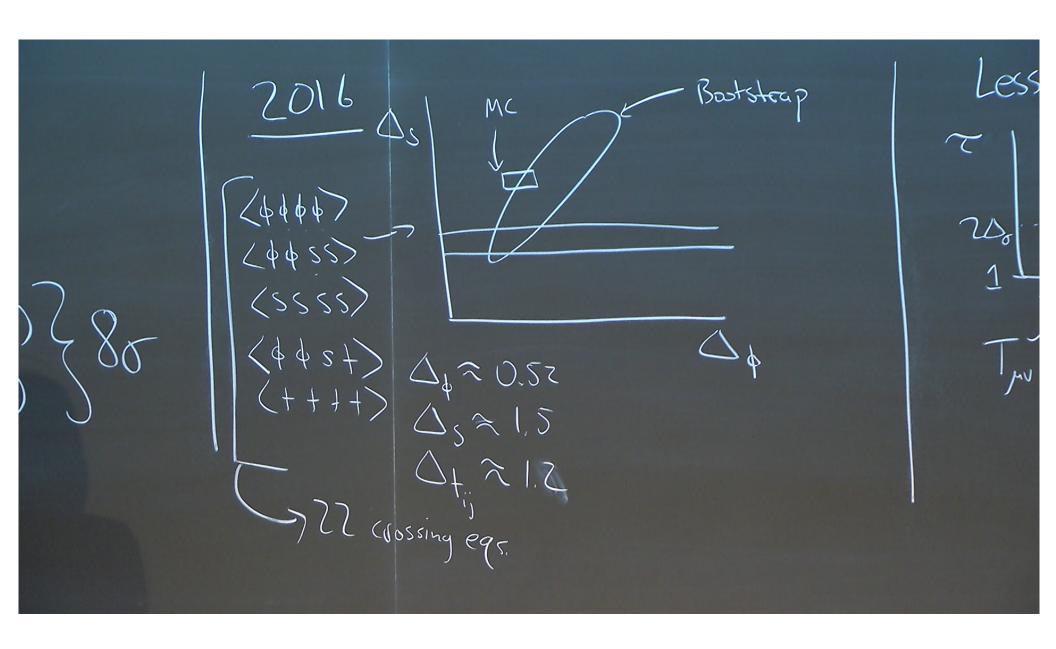
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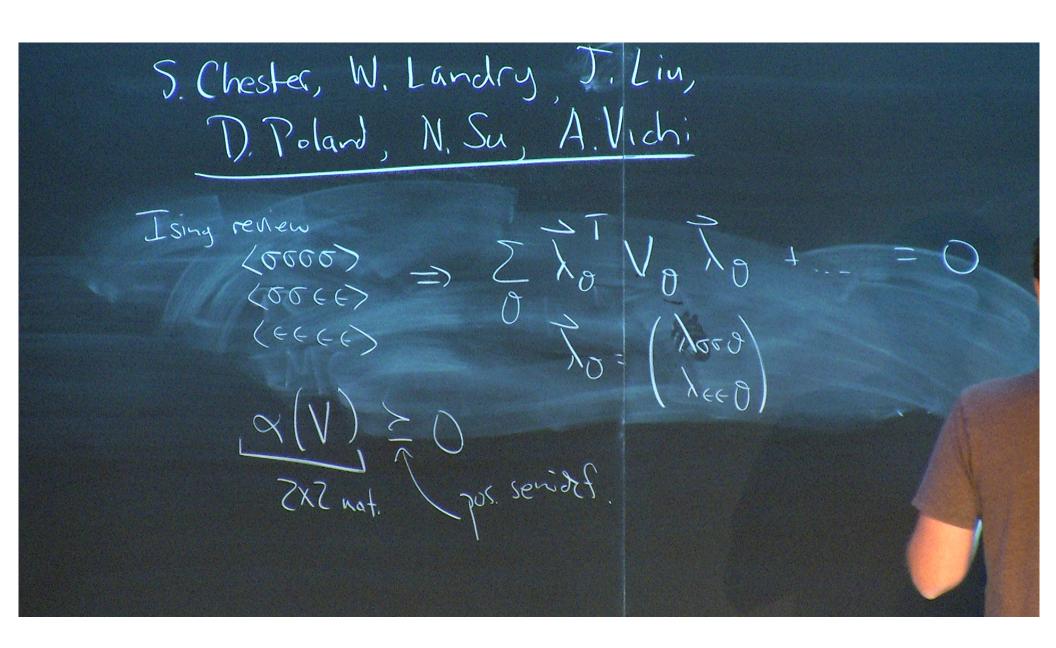




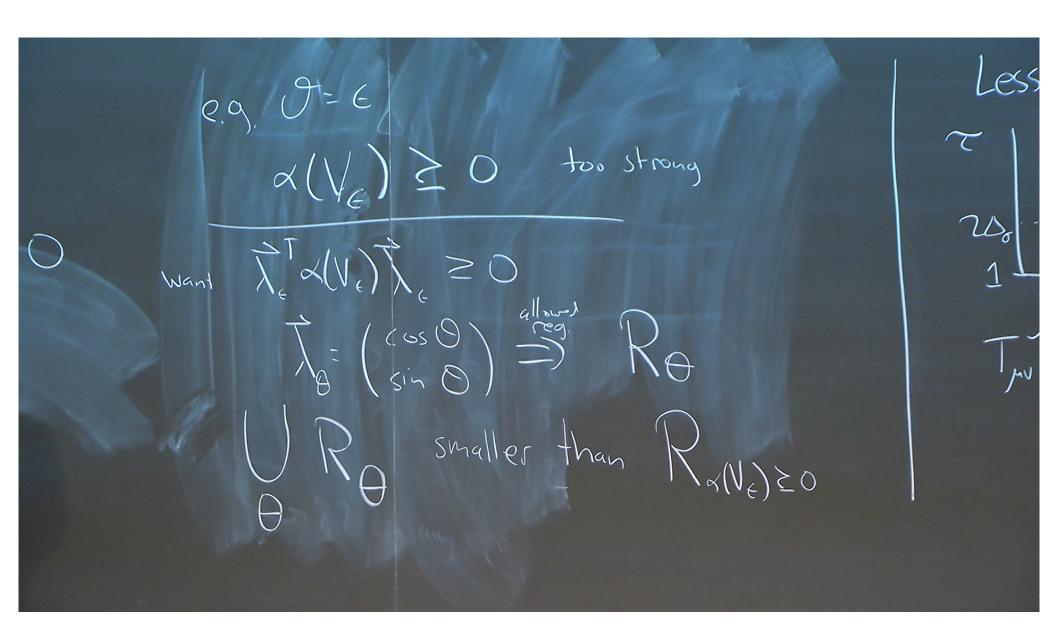


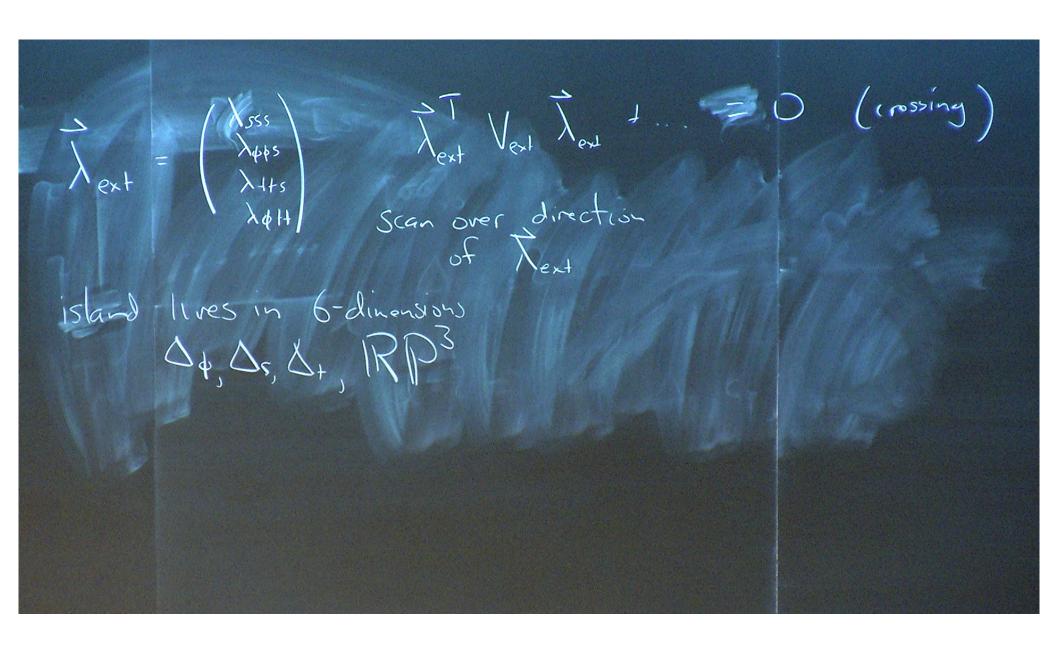




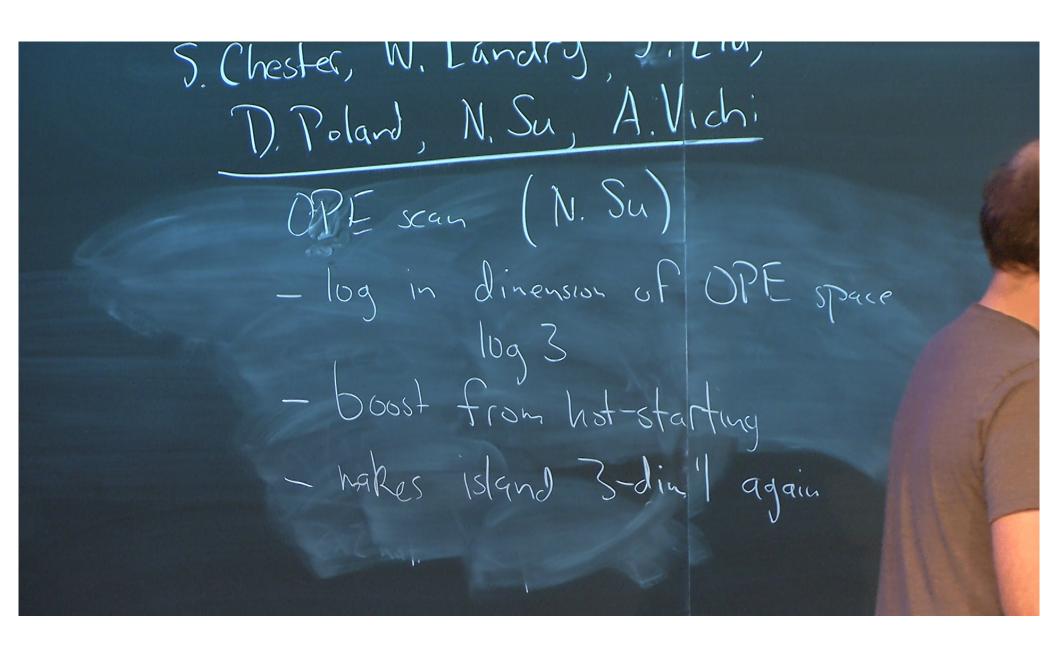


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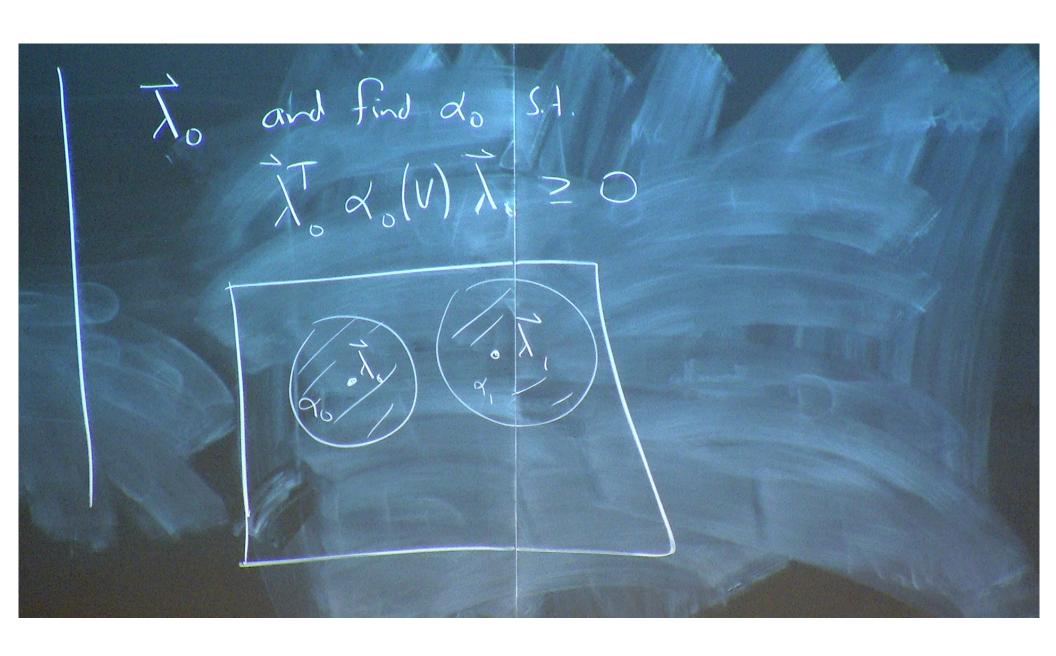




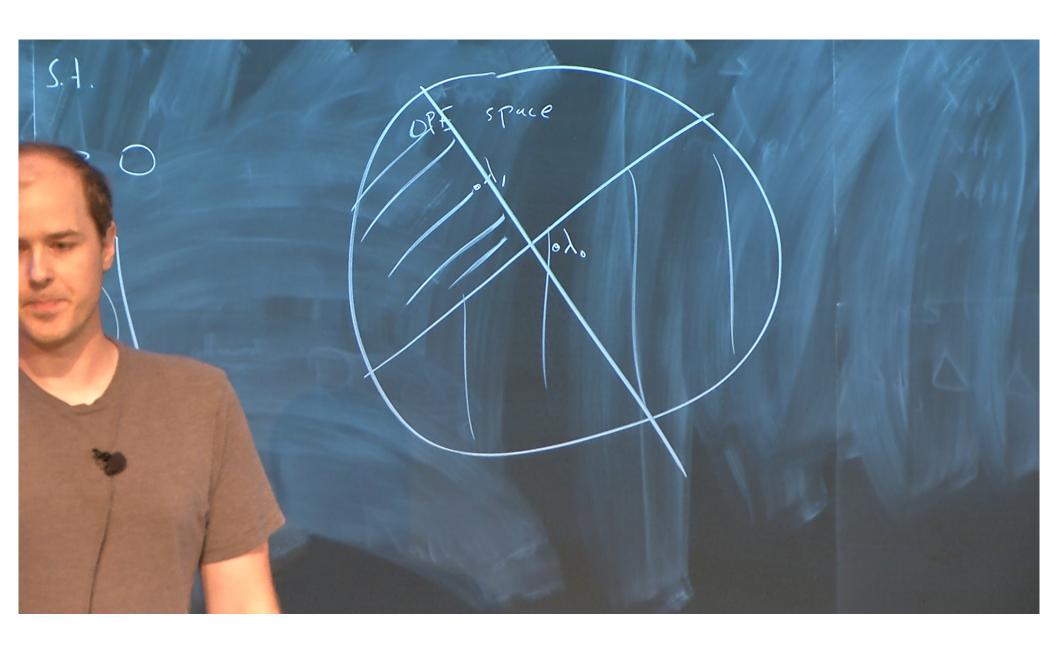
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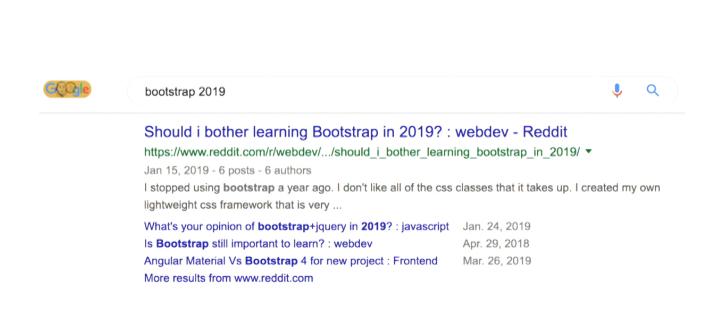


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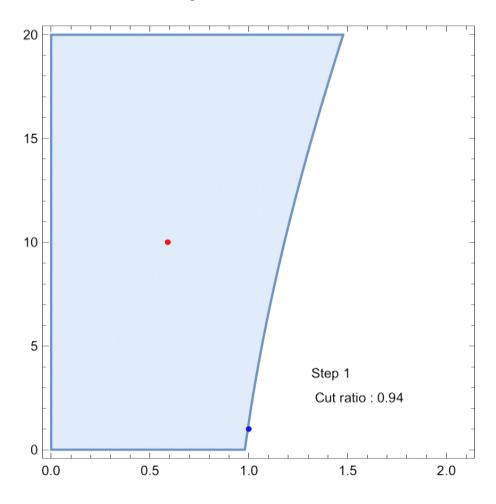


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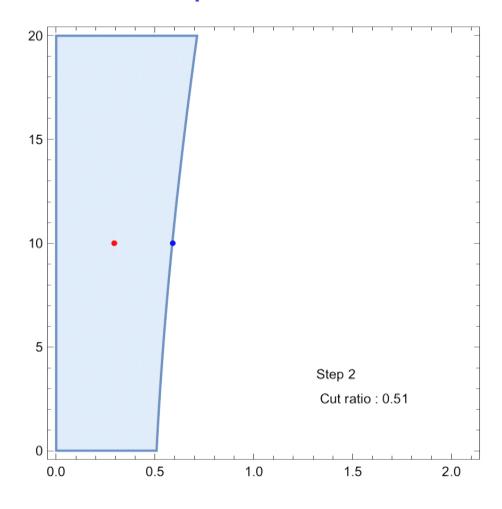
When I tried to google this workshop



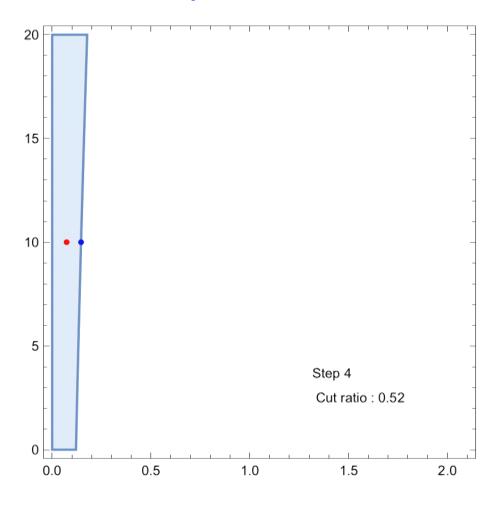
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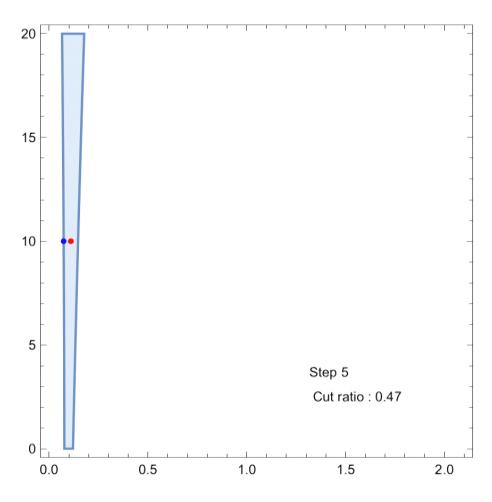
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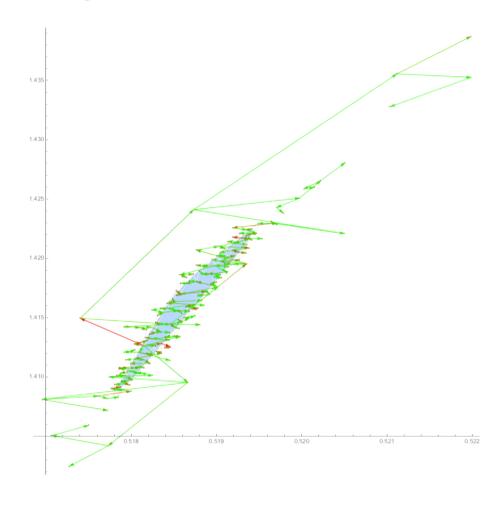


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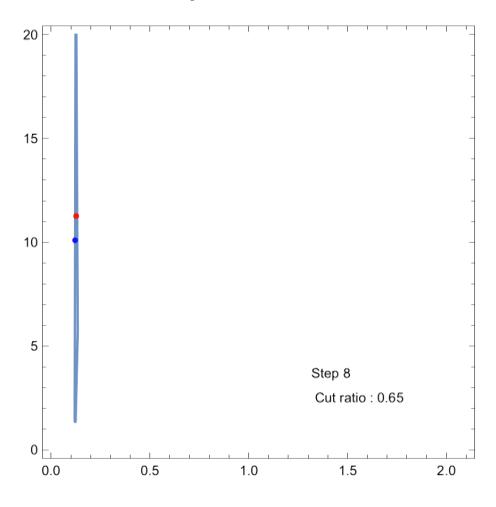


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Hot-starting effectiveness figure: Ning

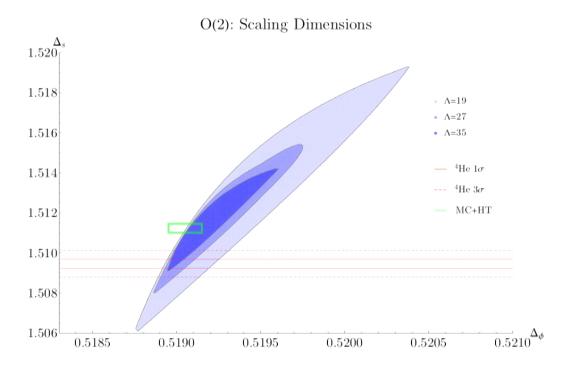


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$\operatorname{Old} O(2)$ results Kos, Poland, DSD, Vichi '16

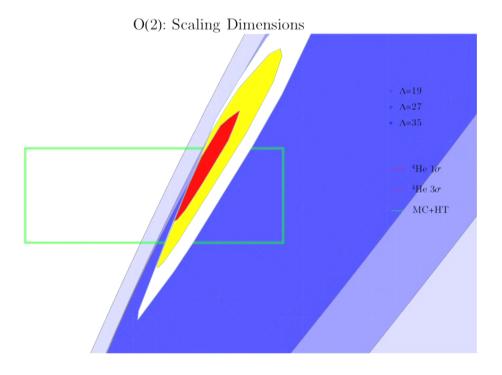


Four-point functions of s, ϕ_i . 7 crossing equations with up to 2×2 matrices. Functionals with size up to 1197.

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New results (preliminary) Chester, Landry, Liu,

Poland, DSD, Su, Vichi '19



Nature is wrong! (jk — our results appear consistent with experimental data).

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