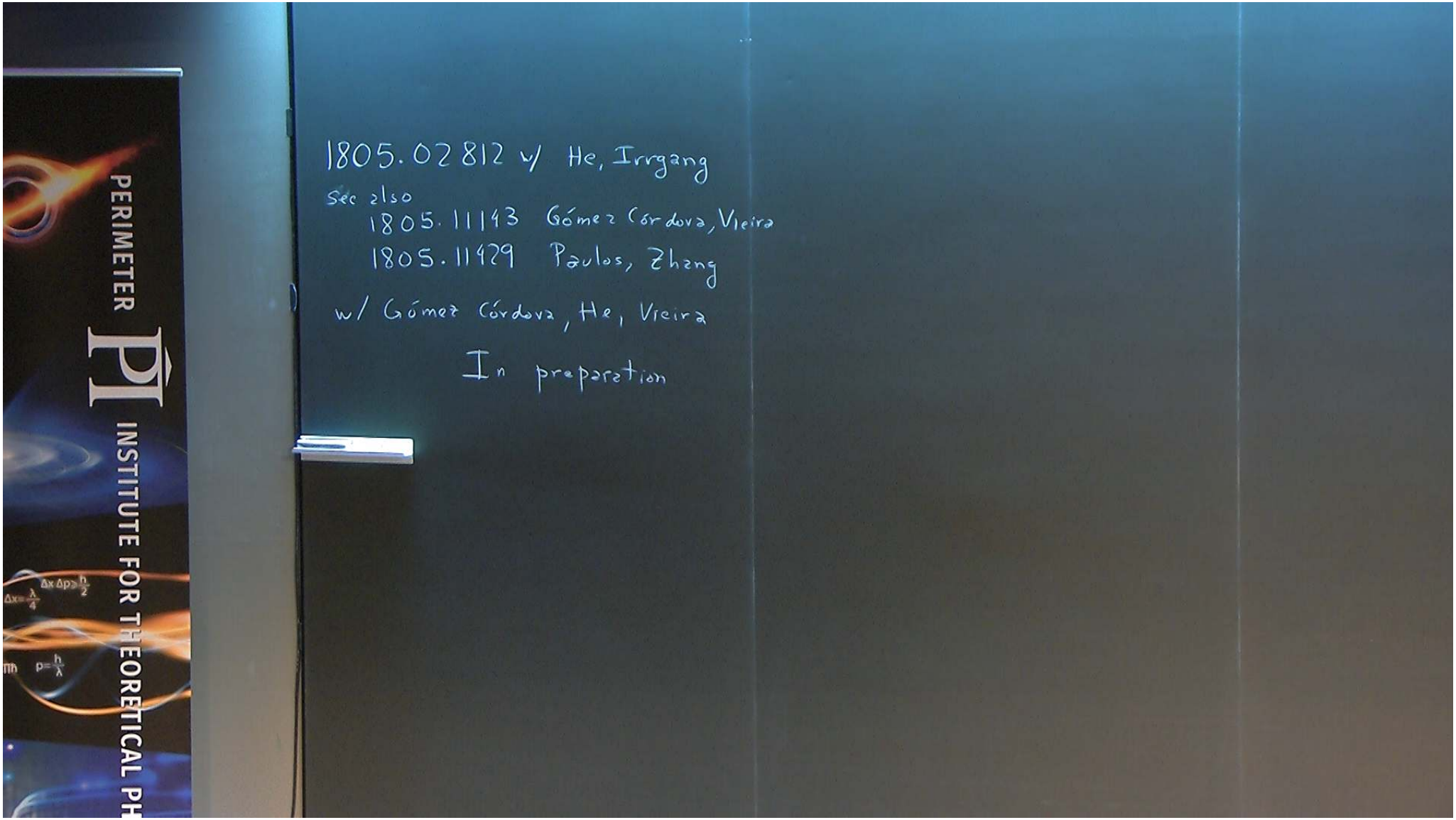
Title: Talk 15 via live stream

Speakers: Martin Kruczenski

Collection: Bootstrap 2019

Date: July 24, 2019 - 10:00 AM

URL: http://pirsa.org/19070040


PERIMETER
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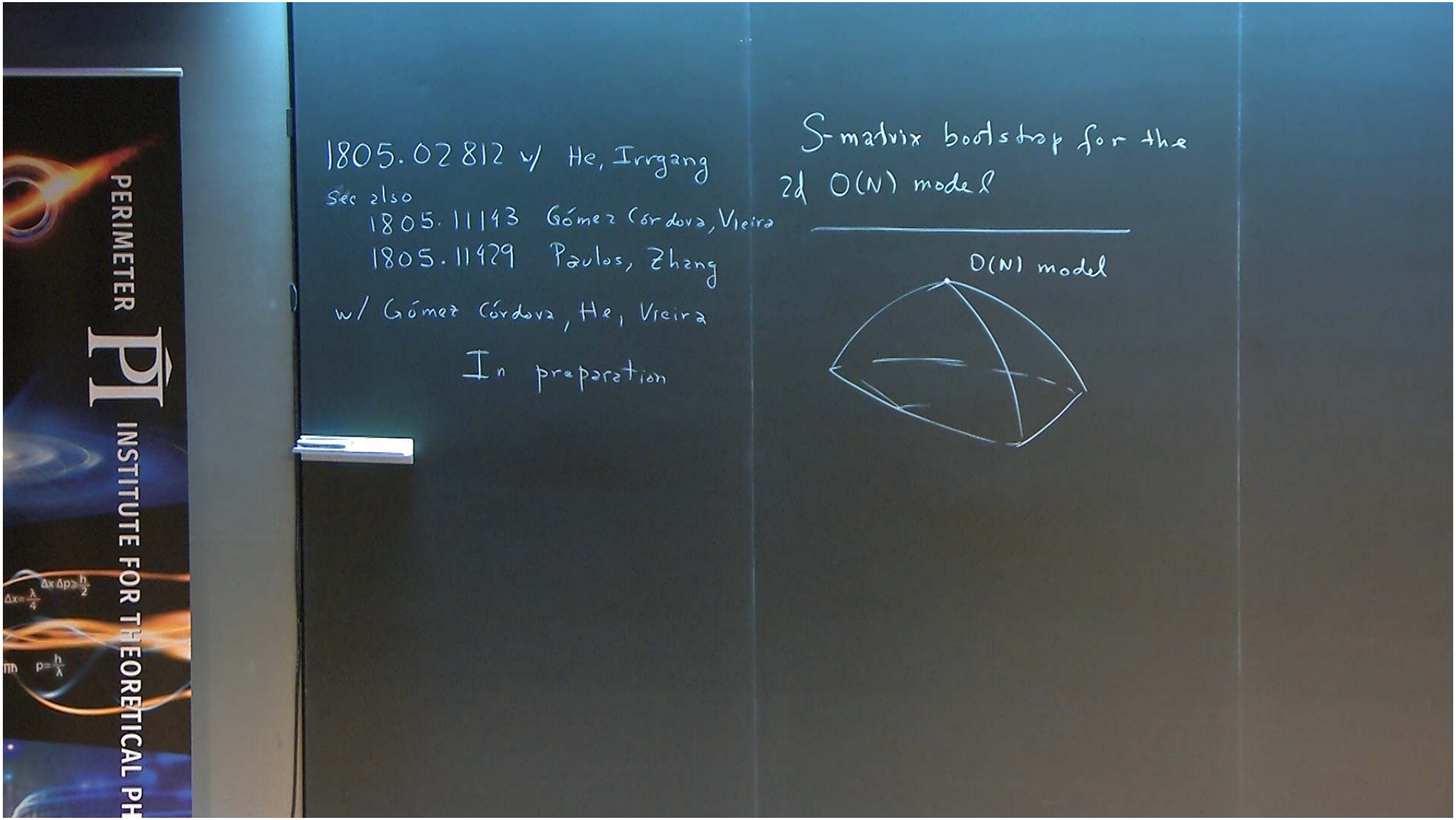
1805.02812 w/ He, Irrgang
See also
1805.11143 Gómez Córdova, Vieira
1805.11429 Paulos, Zhang
w/ Gómez Córdova, He, Vieira
In preparation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In preparation
S-matrix bootstrap for the
2d O(N) model
O(N) model

He, Irrgang

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Paulos, Zheng

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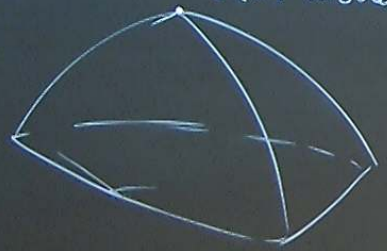
reparation

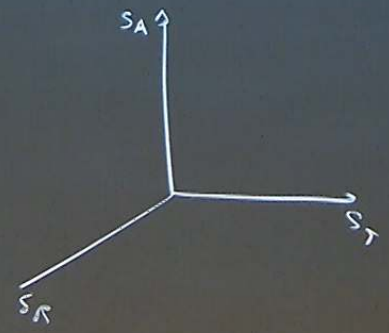
S-matrix bootstrap for the 2d O(N) model

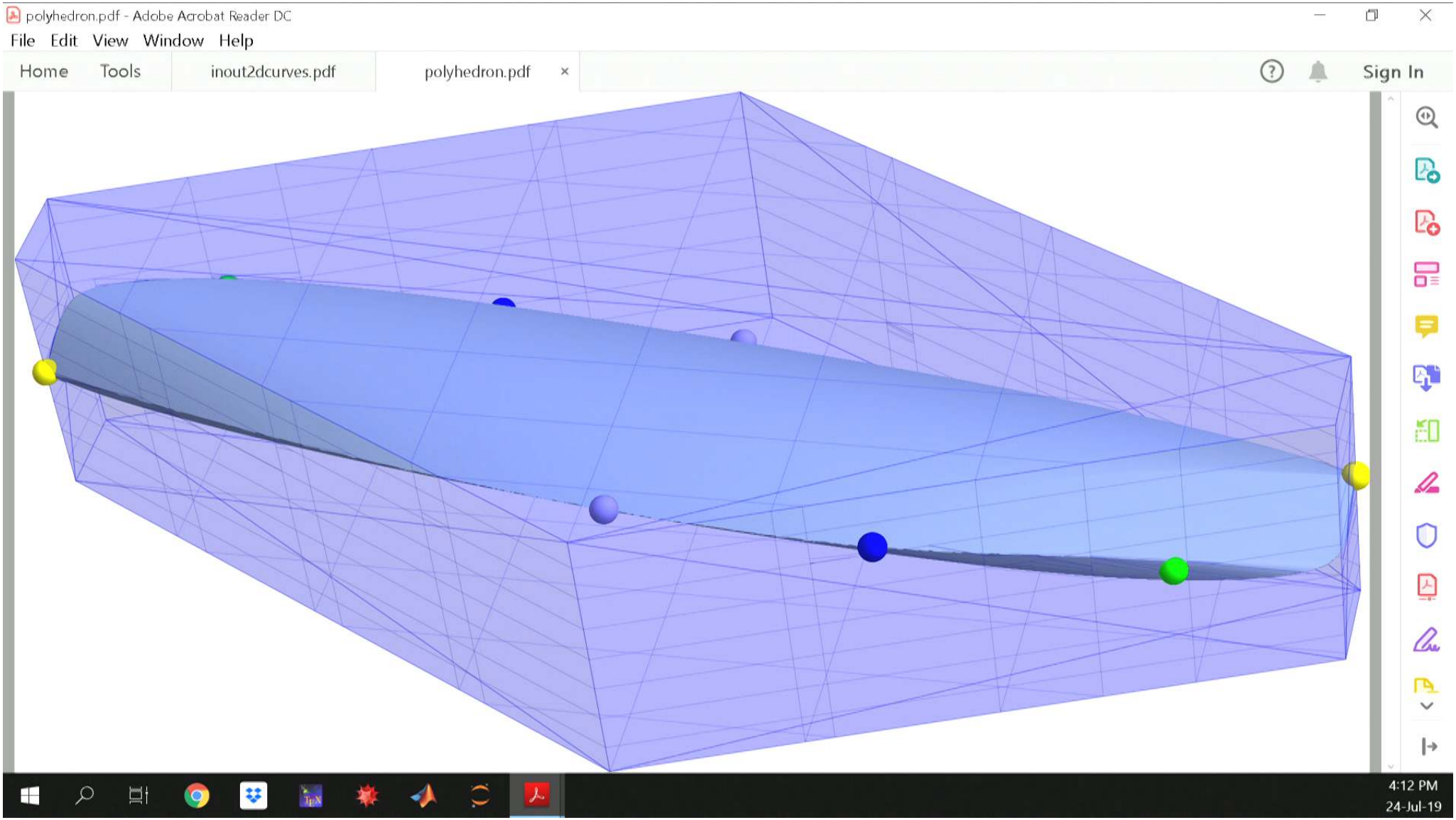
O(N) model



$S_R^{(s)}$, $S_T^{(s)}$, $S_A^{(s)}$

$s=3$

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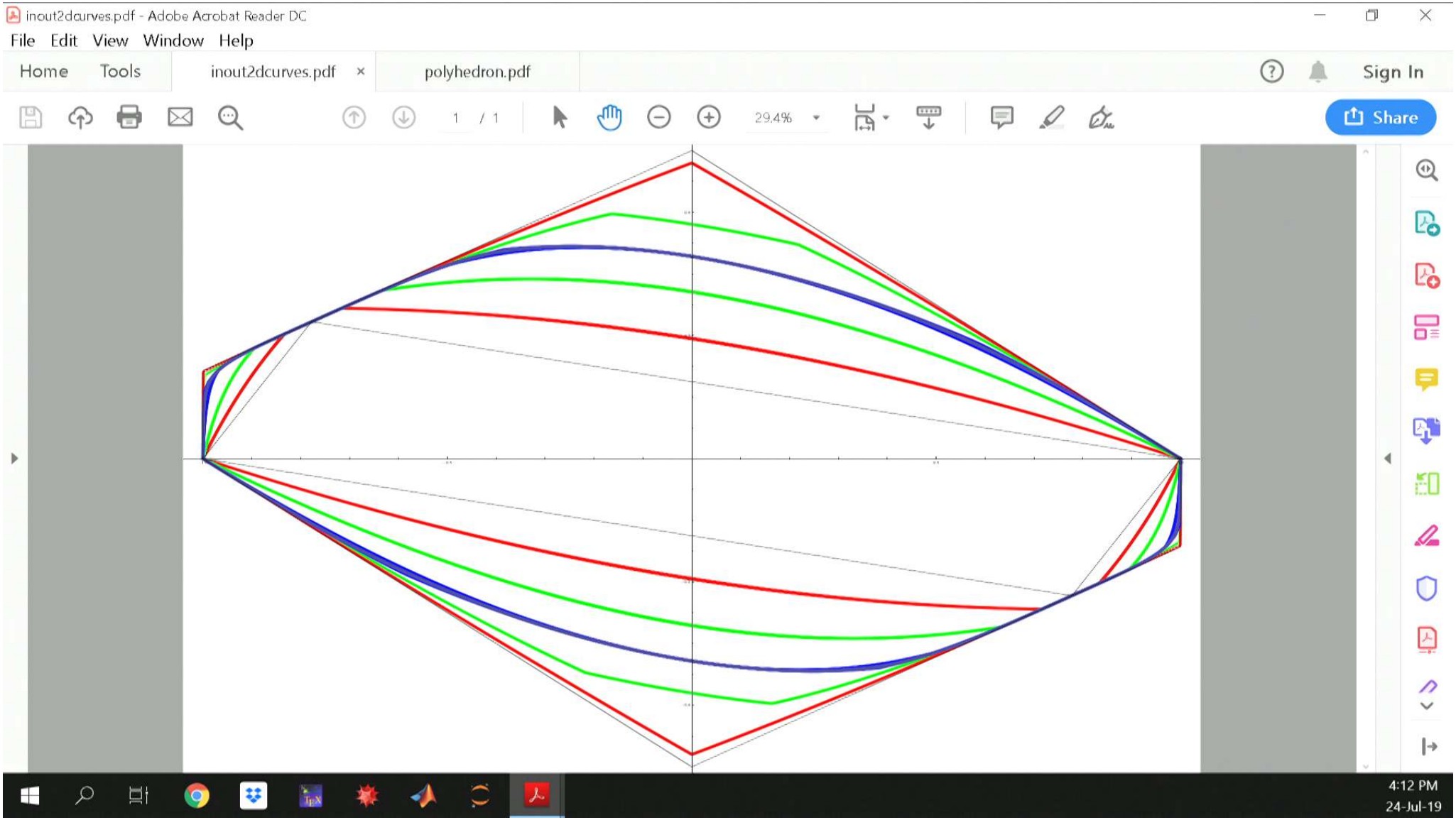
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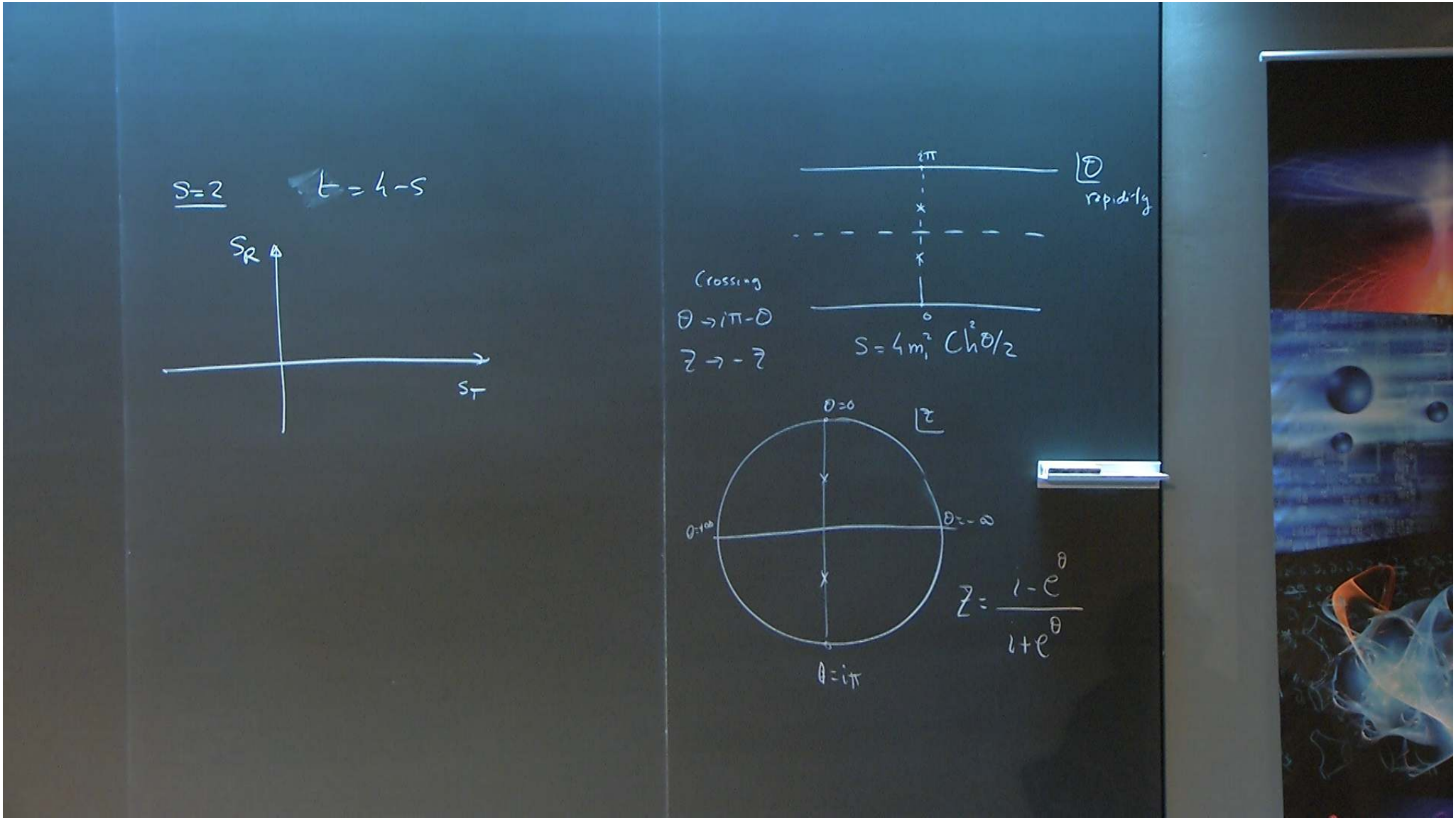
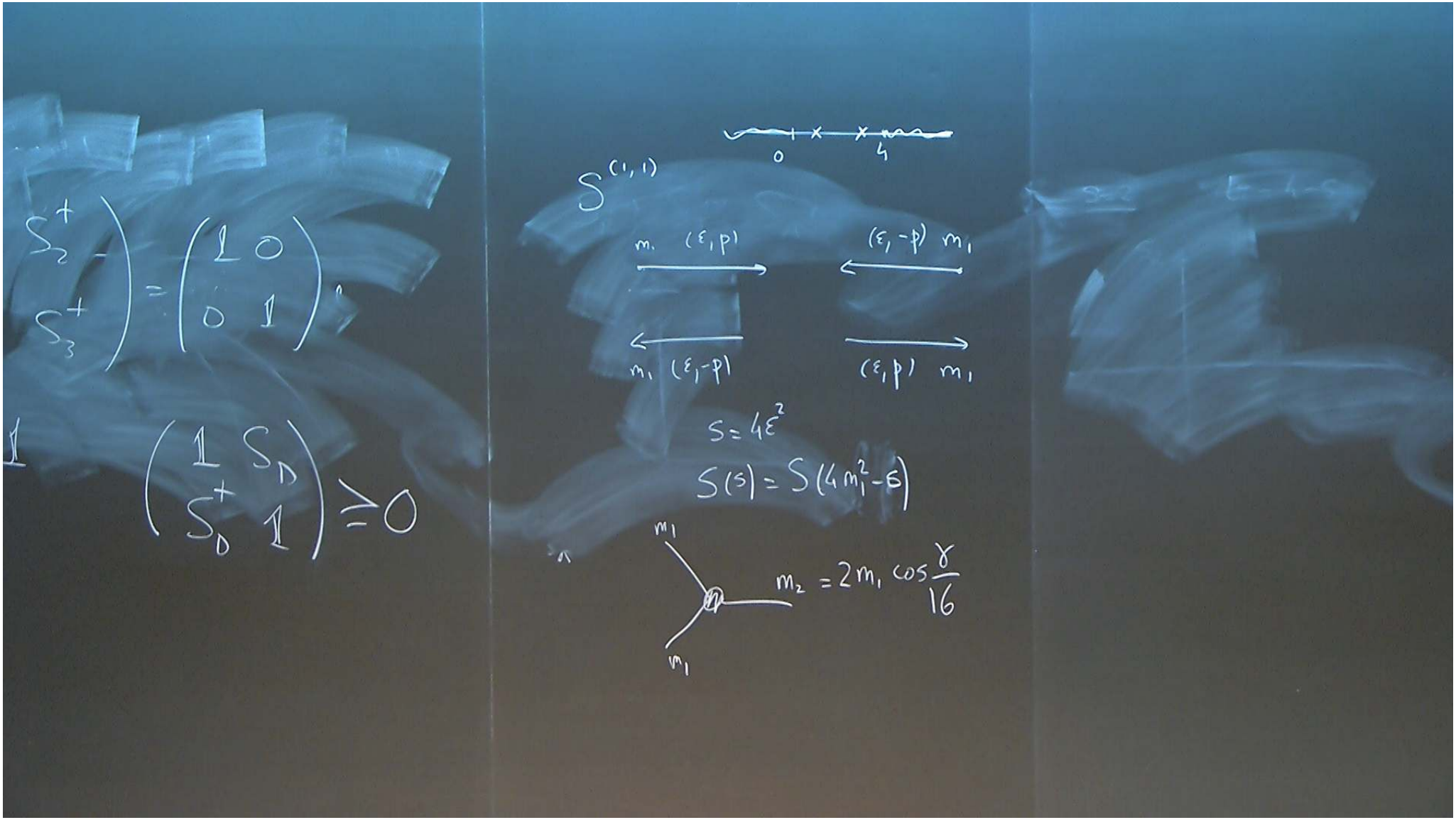
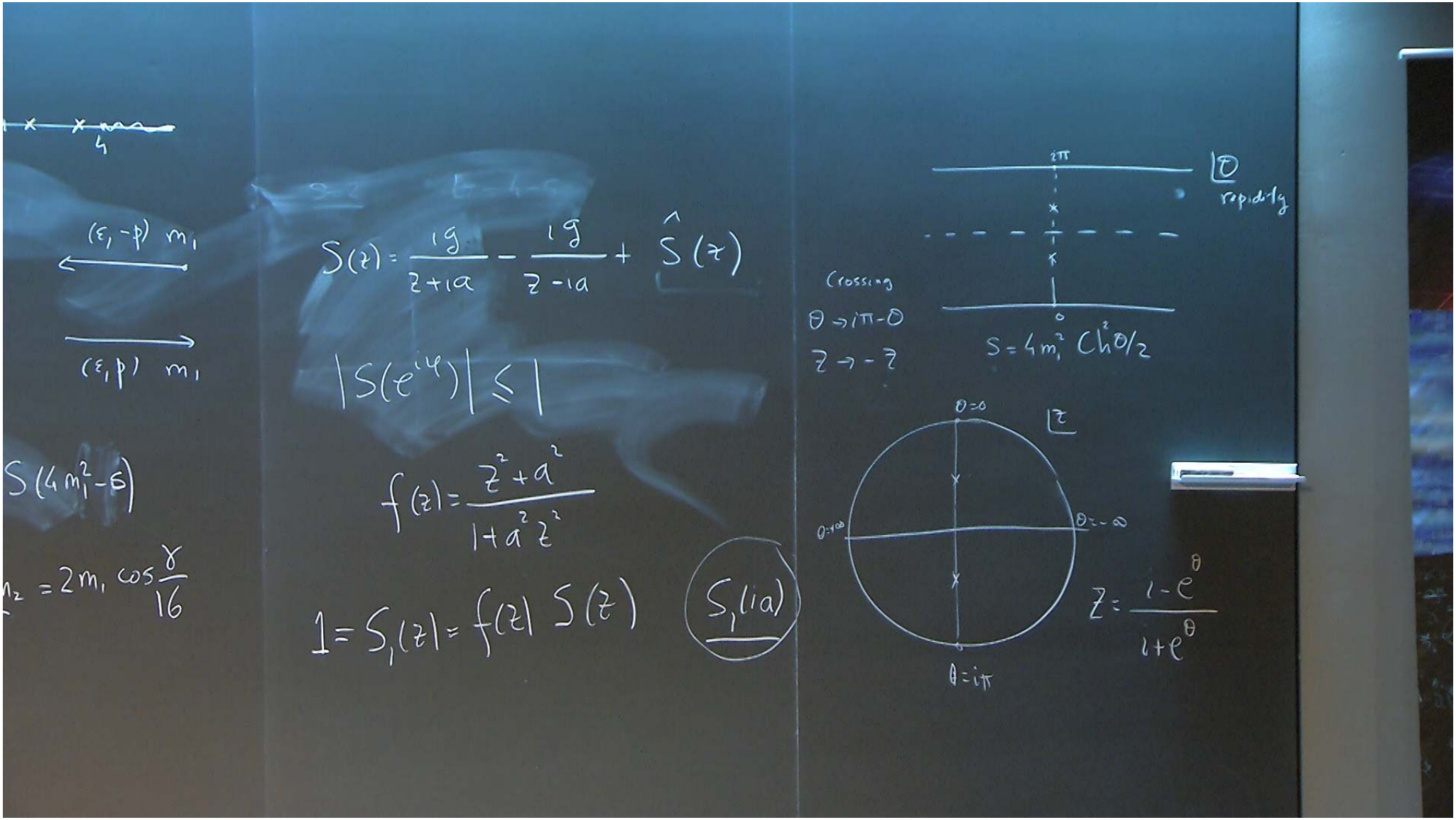

S=2
t = 4 - s
s_R
s_T
iπ
θ
rapidity
Crossing
θ → iπ - θ
z → - z
$s = 4m_1^2 \, Ch^2 \theta/2$
θ=0
z
θ=+∞
θ=-∞
$z = \frac{1 - e^{\theta}}{1 + e^{\theta}}$
θ=iπ

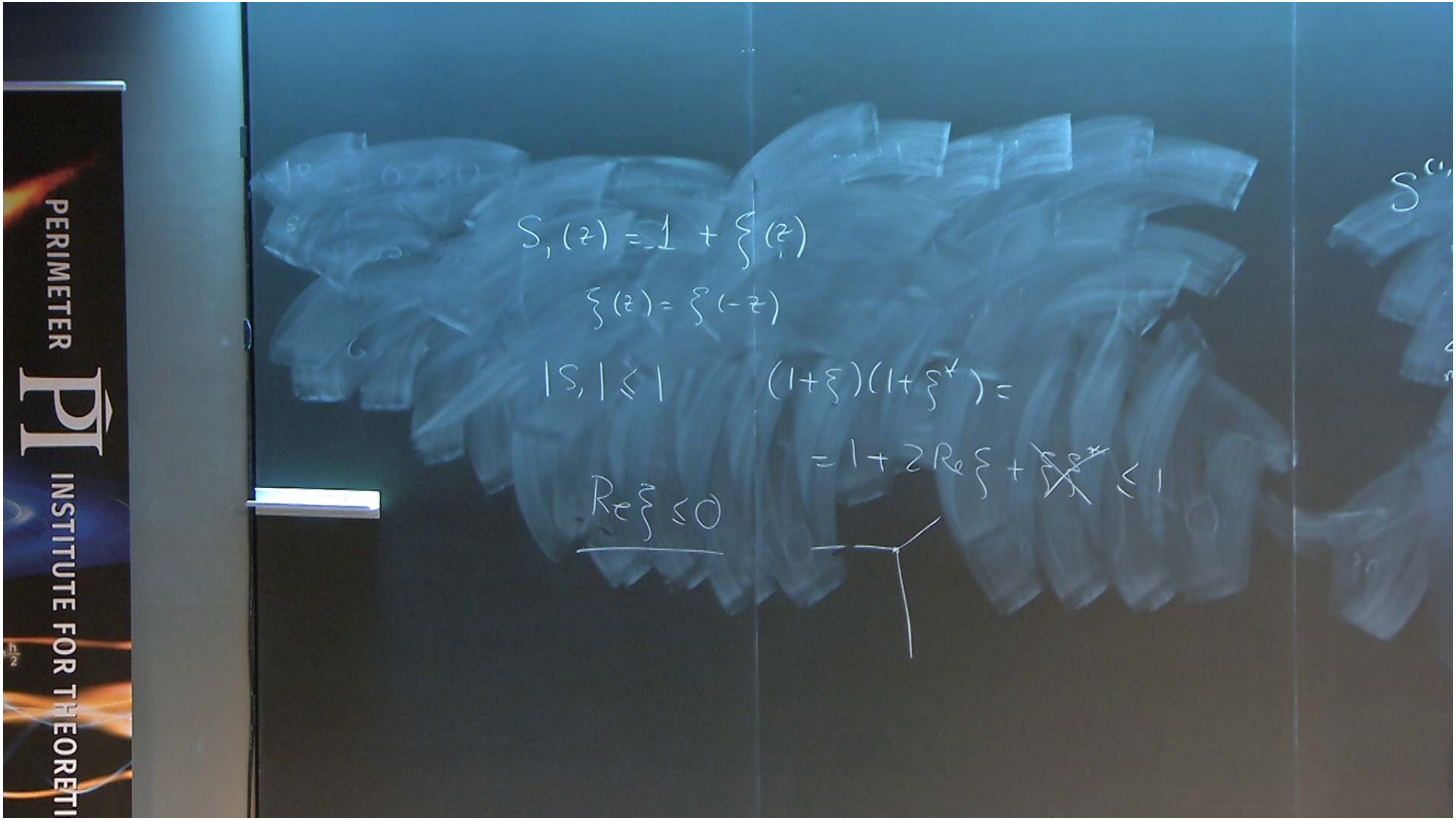
$$S = \overbrace{\begin{pmatrix} S_D & S_1 \\ S_2 & S_3 \end{pmatrix}}^{D} \begin{pmatrix} S_D^+ & S_2^+ \\ S_1^+ & S_3^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S_D S_D^+ + S_1 S_1^+ = 1 \qquad \begin{pmatrix} 1 & S_D \\ S_D^+ & 1 \end{pmatrix} \geq 0$$

$$S_D S_D^+ - 1 \leq 0$$


$S^{(1,1)}$
0
4
m_1 (ε, p)
$(\varepsilon, -p)$ m_1
m_1 $(\varepsilon, -p)$
(ε, p) m_1
$s = 4\varepsilon^2$
$S(s) = S(4m_1^2 - \varepsilon)$
m_1
$m_2 = 2m_1 \cos\frac{\gamma}{16}$
m_1


$(\varepsilon_1, -p)$ m_1
(ε_1, p) m_1
$S(4m_1^2 - 6)$
$m_2 = 2m_1 \cos\frac{\gamma}{16}$
$S(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} + \hat{S}(z)$
$|S(e^{i\varphi})| \leq 1$
$f(z) = \frac{z^2+a^2}{1+a^2z^2}$
$1 = S_1(z) = f(z)\, S(z)$
$S_1(ia)$
Crossing
$\theta \to i\pi - \theta$
$z \to -z$
2π
rapidity
$S = 4m_1^2 \cosh^2\theta/2$
$\theta = 0$
z
$\theta = +\infty$
$\theta = -\infty$
$\theta = i\pi$
$z = \frac{1-e^{\theta}}{1+e^{\theta}}$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$S_1(z) = 1 + \xi(z)$
$\xi(z) = \xi(-z)$
$|S_1| \leq 1$
$(1+\xi)(1+\xi^*) =$
$= 1 + 2\mathrm{Re}\,\xi + \xi\xi^* \leq 1$
$\mathrm{Re}\,\xi \leq 0$

$+\xi)(1+\xi^*) =$

$= 1 + 2\mathrm{Re}\,\xi + \cancel{\xi\xi^*} \leq 1$

$$\mathrm{Re}\,S(0) = \mathrm{Re}\,\frac{1}{2\pi i}\oint \frac{S(z)}{z}\,dz$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{Re}\,S\,d\varphi \leq \frac{1}{2\pi}\int_0^{2\pi} |S|\,d\varphi \leq 1$$

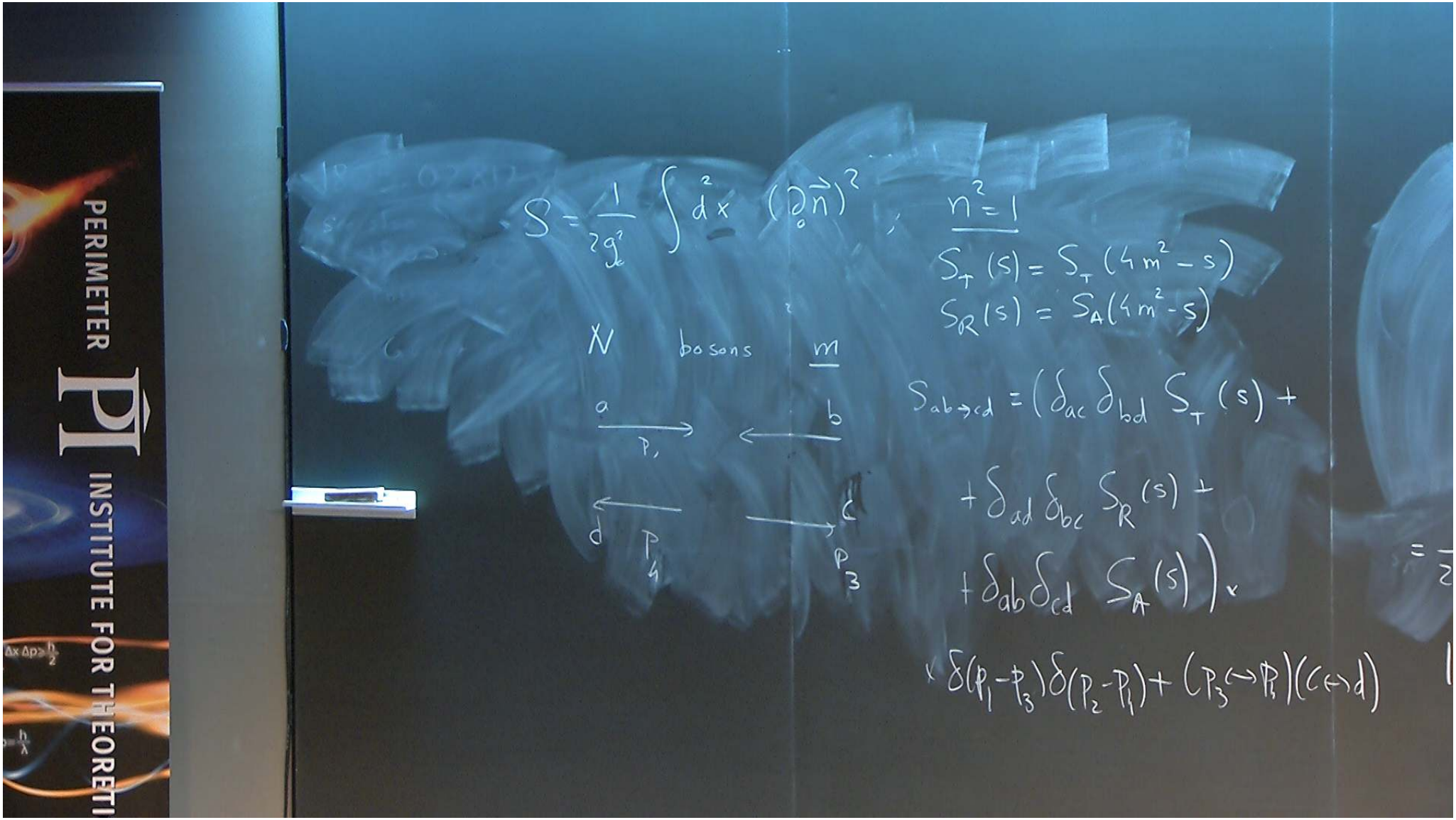
$|S| = 1 \quad \mathrm{Im}\,S = 0 \quad \mathrm{Re}\,S \geq 0$

$$S(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} +$$

$$|S(e^{i\varphi})| \leq 1$$

$$f(z) = \frac{z^2+a^2}{1+a^2z^2}$$

$$1 = S_1(z) = f(z)\,S(z)$$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$S = \frac{1}{2g^2}\int d^2x\ (\partial_\mu \vec{n})^2$
$\vec{n}^2 = 1$
$S_T(s) = S_T(4m^2 - s)$
$S_R(s) = S_A(4m^2 - s)$
N bosons m
$S_{ab\to cd} = (\delta_{ac}\delta_{bd} S_T(s) +$
$+ \delta_{ad}\delta_{bc} S_R(s) +$
$+ \delta_{ab}\delta_{cd} S_A(s))\times$
$\times \delta(p_1 - p_3)\delta(p_2 - p_4) + (p_3 \leftrightarrow p_4)(c \leftrightarrow d)$

$n^2 = 1$

$S_T(s) = S_T(4m^2 - s)$

$S_R(s) = S_A(4m^2 - s)$

$S_{ab\to cd} = (\delta_{ac}\delta_{bd} S_T(s) +$

$+ \delta_{ad}\delta_{bc} S_R(s) +$

$+ \delta_{ab}\delta_{cd} S_A(s))\times$

$\times \delta(p_1 - p_3)\delta(p_2 - p_4) + (p_3 \leftrightarrow p_4)(c \leftrightarrow d)$

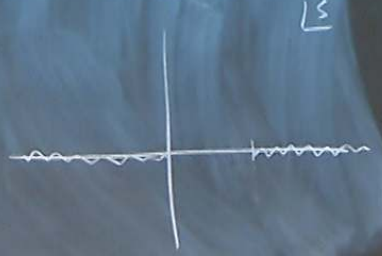
$S_I = N S_A + S_T + S_R$

$S_\pm = S_T \pm S_R$

$|S_I(s>4)| \leq 1$

$|S_+| \leq 1$

$|S_-| \leq 1$



$S(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} + \hat{S}$

$|S(e^{iq})| \leq 1$

$f(z) = \frac{z^2 + a^2}{1 + a^2 z^2}$

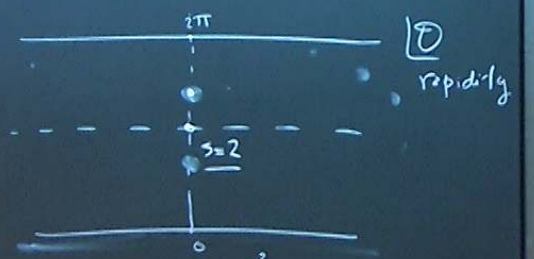
$1 = S_1(z) = f(z)\, S(z)$

$$(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} + \hat{S}(z)$$

$$|S(e^{i\varphi})| \leq 1$$

$$f(z) = \frac{z^2+a^2}{1+a^2z^2}$$

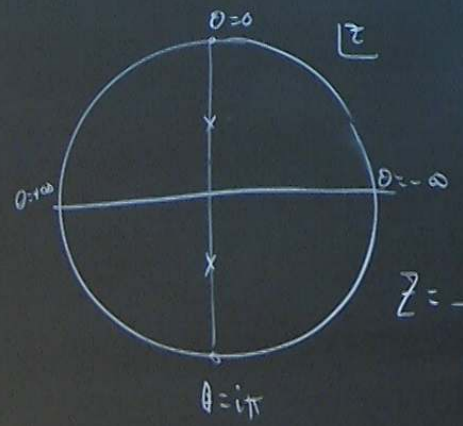
$$S_1(z) = f(z)\, S(z) \qquad \underline{S_1(ia)}$$



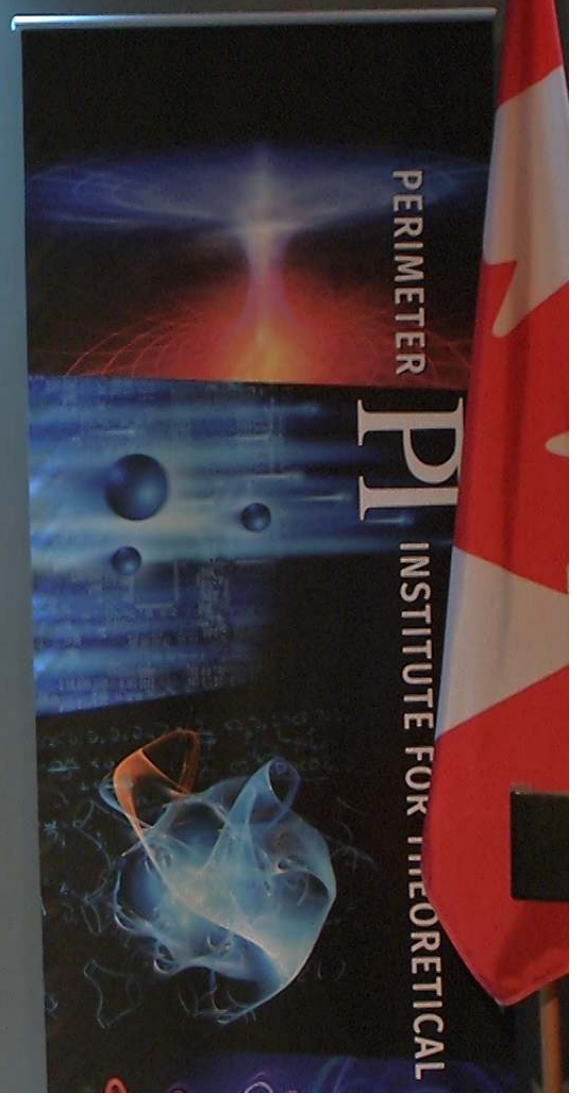
Crossing

$\theta \to i\pi - \theta$

$z \to -z$



$$z = \frac{1-e^{\theta}}{1+e^{\theta}}$$



$n^2 = 1$

$S_+(s) = S_T(4m^2 - s)$

$S_R(s) = S_A(4m^2 - s)$

$S_{ab\to cd} = (\delta_{ac}\delta_{bd} S_T(s) +$

$+ \delta_{ad}\delta_{bc} S_R(s) +$

$+ \delta_{ab}\delta_{cd} S_A(s))\times$

$\times \delta(p_1 - p_3)\delta(p_2 - p_4) + (p_3 \leftrightarrow p_4)(c \leftrightarrow d)$

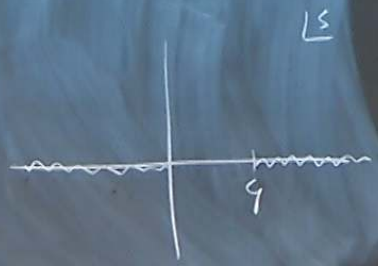
$S_I = N S_A + S_T + S_R$

$S_\pm = S_T \pm S_R$

$|S_I(s>4)| \leq 1$

$|S_+| \leq 1$

$|S_-| \leq 1$

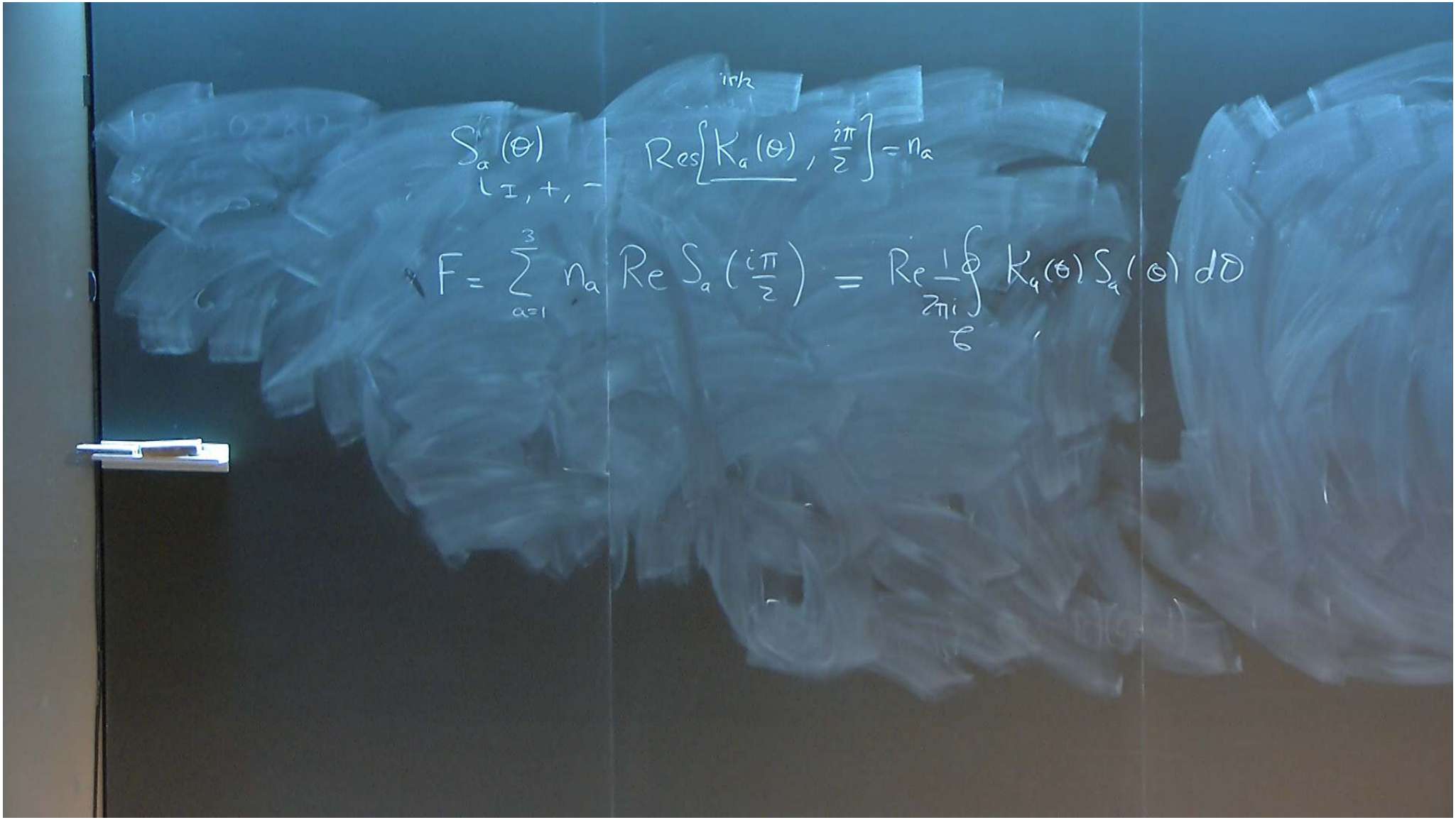


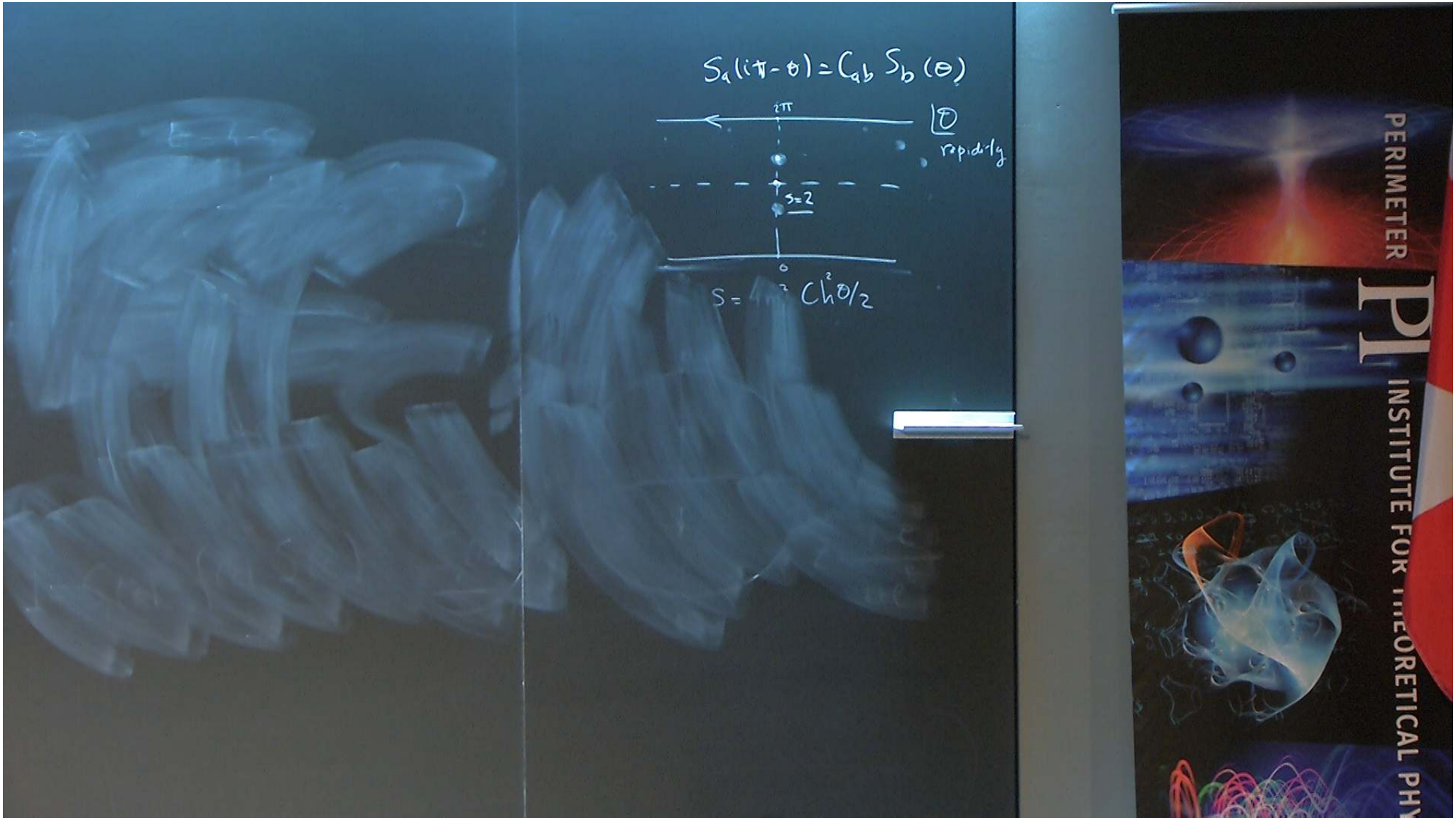
$S(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} + S$

$|S(e^{iq})| \leq 1$

$f(z) = \frac{z^2 + a^2}{1 + a^2 z^2}$

$1 = S_1(z) = f(z)\, S(z)$


$S_a(\theta)$
$I, +, -$
$\mathrm{Res}\left[K_a(\theta), \frac{i\pi}{2}\right] = n_a$
$F = \sum_{a=1}^{3} n_a \, \mathrm{Re}\, S_a\left(\frac{i\pi}{2}\right) = \mathrm{Re}\, \frac{1}{2\pi i} \oint_{\mathcal{C}} K_a(\theta) S_a(\theta) \, d\theta$


$S_a(i\pi - \theta) = C_{ab} S_b(\theta)$
2π
θ
rapidity
$s=2$
0
$S = 4 \cdot Ch^2\theta/2$
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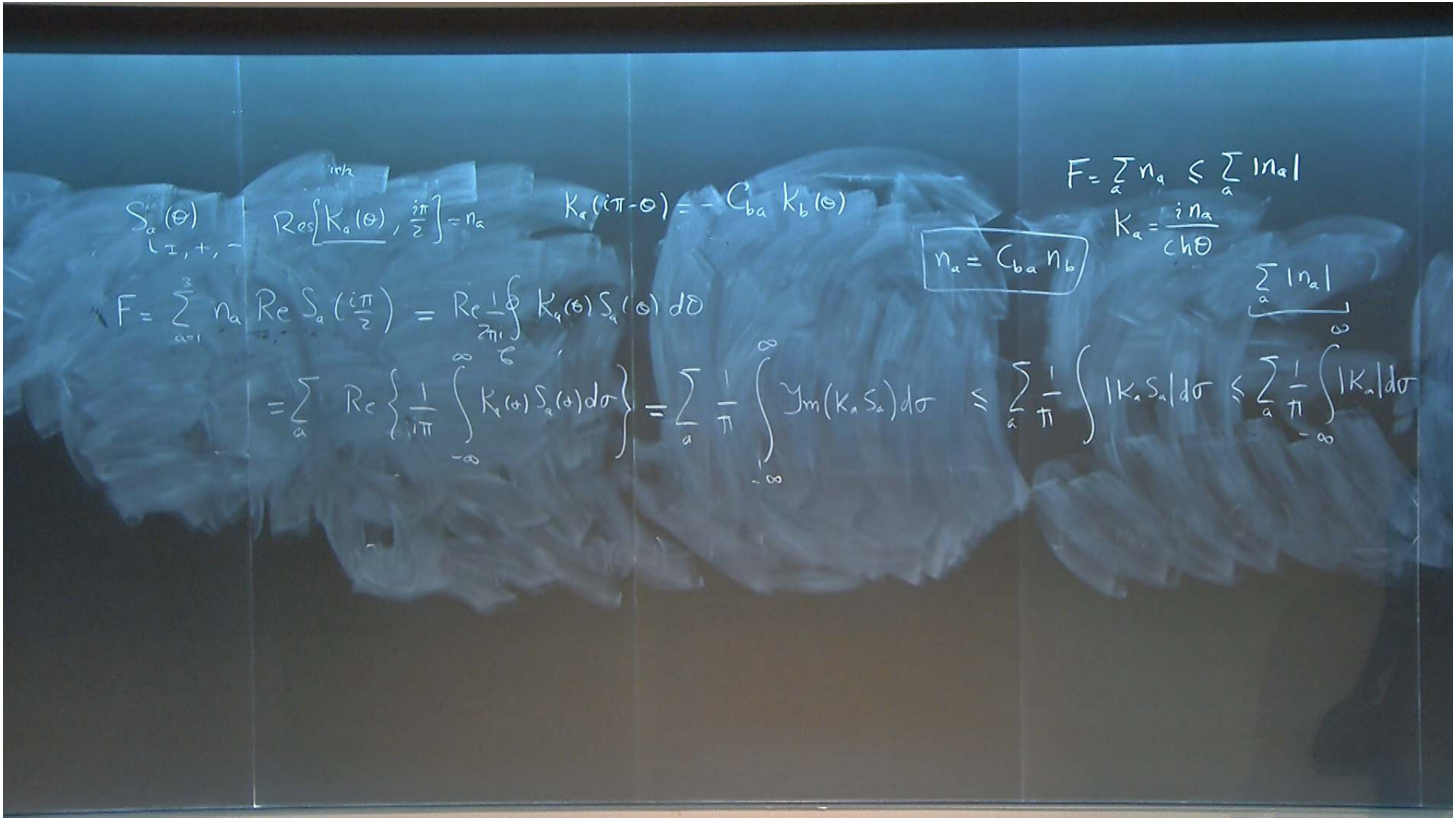

$S_a(\theta)$
$(I, +, -)$
$\mathrm{Res}\left[K_a(\theta), \frac{i\pi}{2}\right] = n_a$
$K_a(i\pi - \theta) = -C_{ba} K_b(\theta)$
$n_a = C_{ba} n_b$
$F = \sum_a n_a \leq \sum_a |n_a|$
$K_a = \frac{i n_a}{ch\theta}$
$F = \sum_{a=1}^{3} n_a \mathrm{Re}\, S_a\left(\frac{i\pi}{2}\right) = \mathrm{Re}\frac{1}{2\pi i}\oint_C K_a(\theta) S_a(\theta)\, d\theta$
$= \sum_a \mathrm{Re}\left\{\frac{1}{i\pi}\int_{-\infty}^{\infty} K_a(\sigma) S_a(\sigma)\, d\sigma\right\} = \sum_a \frac{1}{\pi}\int_{-\infty}^{\infty} \mathrm{Im}(K_a S_a)\, d\sigma \leq \sum_a \frac{1}{\pi}\int |K_a S_a|\, d\sigma \leq \sum_a \frac{1}{\pi}\int_{-\infty}^{\infty} |K_a|\, d\sigma$
$\sum_a |n_a|$

$$\frac{i\pi}{2}\Big] = n_a \qquad K_a(i\pi-\theta) = -C_{ba}\,K_b(\theta)$$

$$F = \sum_a n_a \le \sum_a |n_a| \qquad K_a = \frac{i n_a}{ch\theta}$$

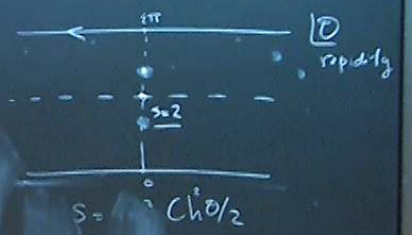
$$\boxed{n_a = C_{ba}\,n_b}$$

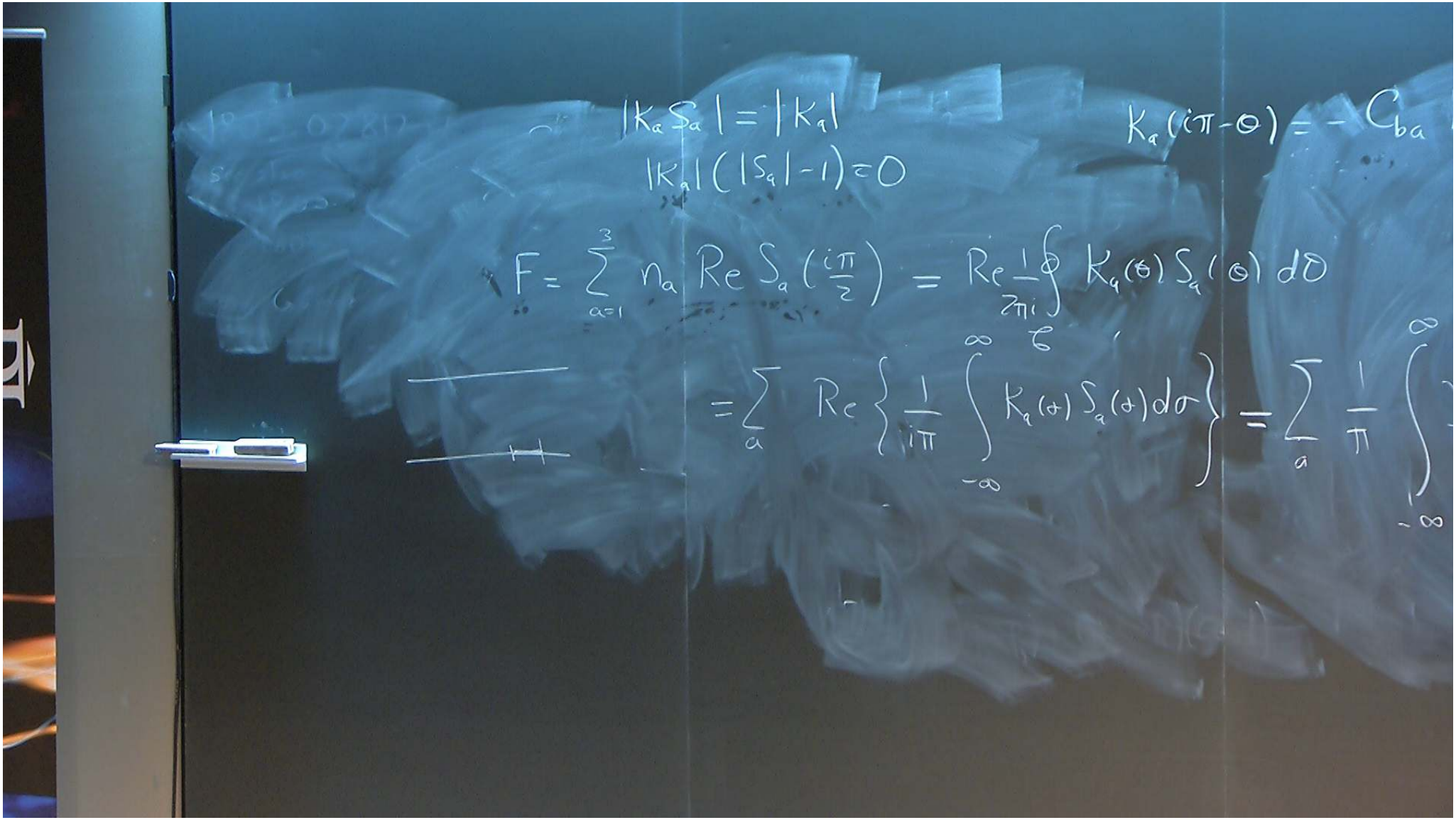
$$= \mathrm{Re}\,\frac{1}{2\pi i}\oint_C K_a(\theta)\,S_a(\theta)\,d\theta$$

$$\left\{\frac{1}{i\pi}\int_{-\infty}^{\infty} K_a(\sigma)\,S_a(\sigma)\,d\sigma\right\} = \sum_a \frac{1}{\pi}\int_{-\infty}^{\infty} \mathrm{Im}(K_a S_a)\,d\sigma \le \sum_a \frac{1}{\pi}\int_{-\infty}^{\infty} |K_a S_a|\,d\sigma \le \sum_a \frac{1}{\pi}\int_{-\infty}^{\infty} |K_a|\,d\sigma$$

$$= \sum_a |n_a|$$

$$S_a(i\pi-\theta) = C_{ab}\,S_b(\theta)$$




$|K_a S_a| = |K_a|$
$|K_a|(|S_a| - 1) = 0$
$K_a(i\pi - \theta) = -C_{ba}$
$F = \sum_{a=1}^{3} n_a \, \mathrm{Re}\, S_a\left(\frac{i\pi}{2}\right) = \mathrm{Re}\frac{1}{2\pi i}\oint_{\mathcal{C}} K_a(\theta) S_a(\theta)\, d\theta$
$= \sum_a \mathrm{Re}\left\{\frac{1}{i\pi}\int_{-\infty}^{\infty} K_a(\sigma) S_a(\sigma)\, d\sigma\right\} = \sum_a \frac{1}{\pi}\int_{-\infty}^{\infty}$

$= |K_a|$

$|S_a| - 1) = 0$

$K_a(i\pi - \theta) = - C_{ba} K_b(\theta)$

$n_a = C_{ba} n_b$

$F = \sum_a n_a \leq \sum_a |n_a|$

$K_a = \frac{i n_a}{ch\theta}$

$\sum_a |n$

$S_a\left(\frac{i\pi}{2}\right) = Re \frac{1}{2\pi i} \oint_C K_a(\theta) S_a(\theta) d\theta$

$\sum_a Re\left\{ \frac{1}{i\pi} \int_{-\infty}^{\infty} K_a(\sigma) S_a(\sigma) d\sigma \right\} = \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} Ym(K_a S_a) d\sigma \leq \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} |K_a S_a| d\sigma \leq \sum_a \frac{1}{\pi}$

$Re(K_a S_a) = 0$

$Ym(K_a S_a) \geq 0$

$S_a = \frac{i \bar{K}_a}{|K_a|}$

$$K_a = \frac{i n_a}{ch\theta}$$

$$K_a = \frac{i\, k_a(\theta)}{ch\theta}$$

$$k_a\left(\frac{i\pi}{2}\right) = n_a$$

$$k_a(\theta) = k_a(\theta + b)$$

$$S_a(\theta) = S_a(\theta + b)$$

$$\mathrm{Re}\, S_a = \alpha\, V_a$$

$$\max \alpha$$

$$n_a V_a = 1$$

$$\min \sum_a |n_a|$$

$$K_a = \frac{i n_a}{\mathrm{ch}\theta}$$

$$K_a = \frac{i\, k_a(\theta)}{\mathrm{ch}\theta}$$

$$k_a\left(\frac{i\pi}{2}\right) = n_a$$

$$k_a(\theta) = k_a(\theta + b)$$