

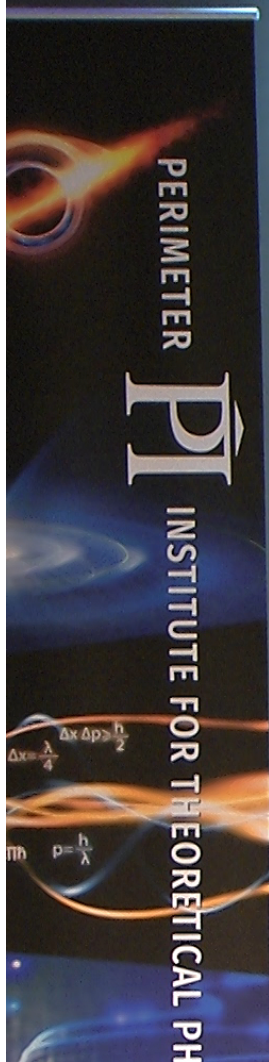
Title: Talk 15 via live stream

Speakers: Martin Kruczenski

Collection: Bootstrap 2019

Date: July 24, 2019 - 10:00 AM

URL: <http://pirsa.org/19070040>



1805.02812 w/ He, Irrgang

See also

1805.11143 Gómez Córdoba, Vieira

1805.11429 Paulos, Zheng

w/ Gómez Córdoba, He, Vieira

In preparation

1805.02812 w/ He, Irrgang

See also

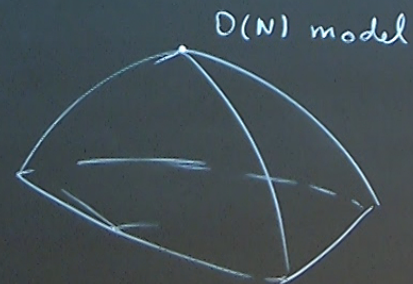
1805.11143 Gómez Córdova, Vieira

1805.11429 Paulos, Zheng

w/ Gómez Córdova, He, Vieira

In preparation

S-matrix bootstrap for the
 2d $O(N)$ model



He, Irrgang

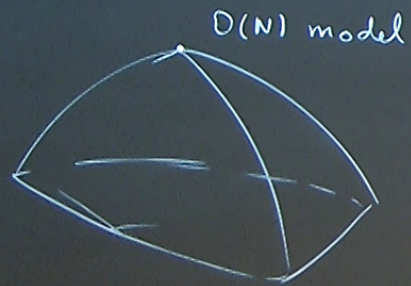
Sómez Cordova, Vieira

Paulos, Zhang

, He, Vieira

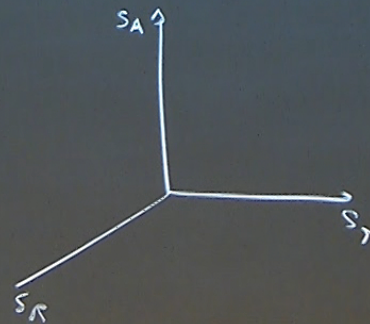
reparation

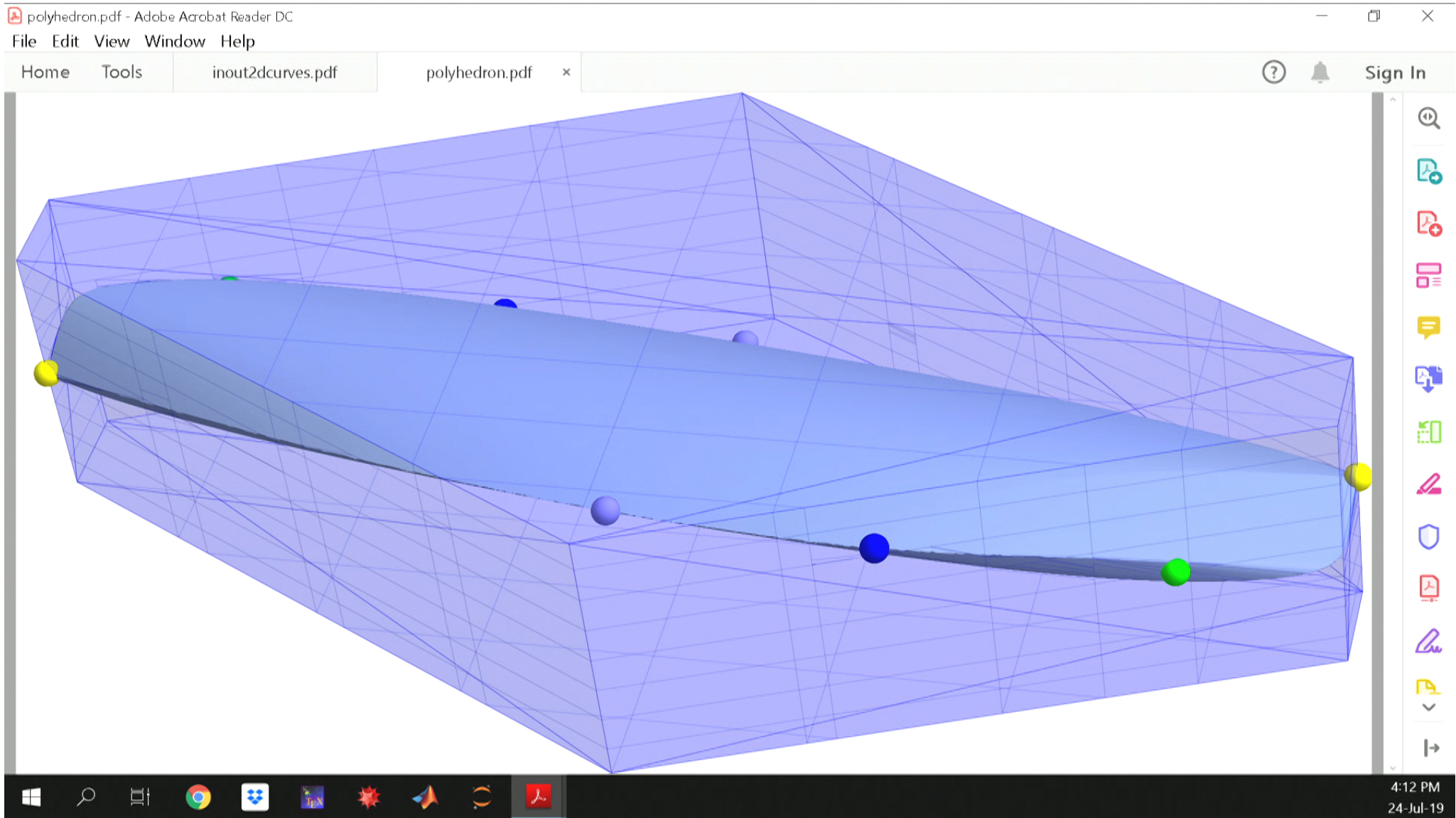
S-matrix bootstrap for the
2d $O(N)$ model

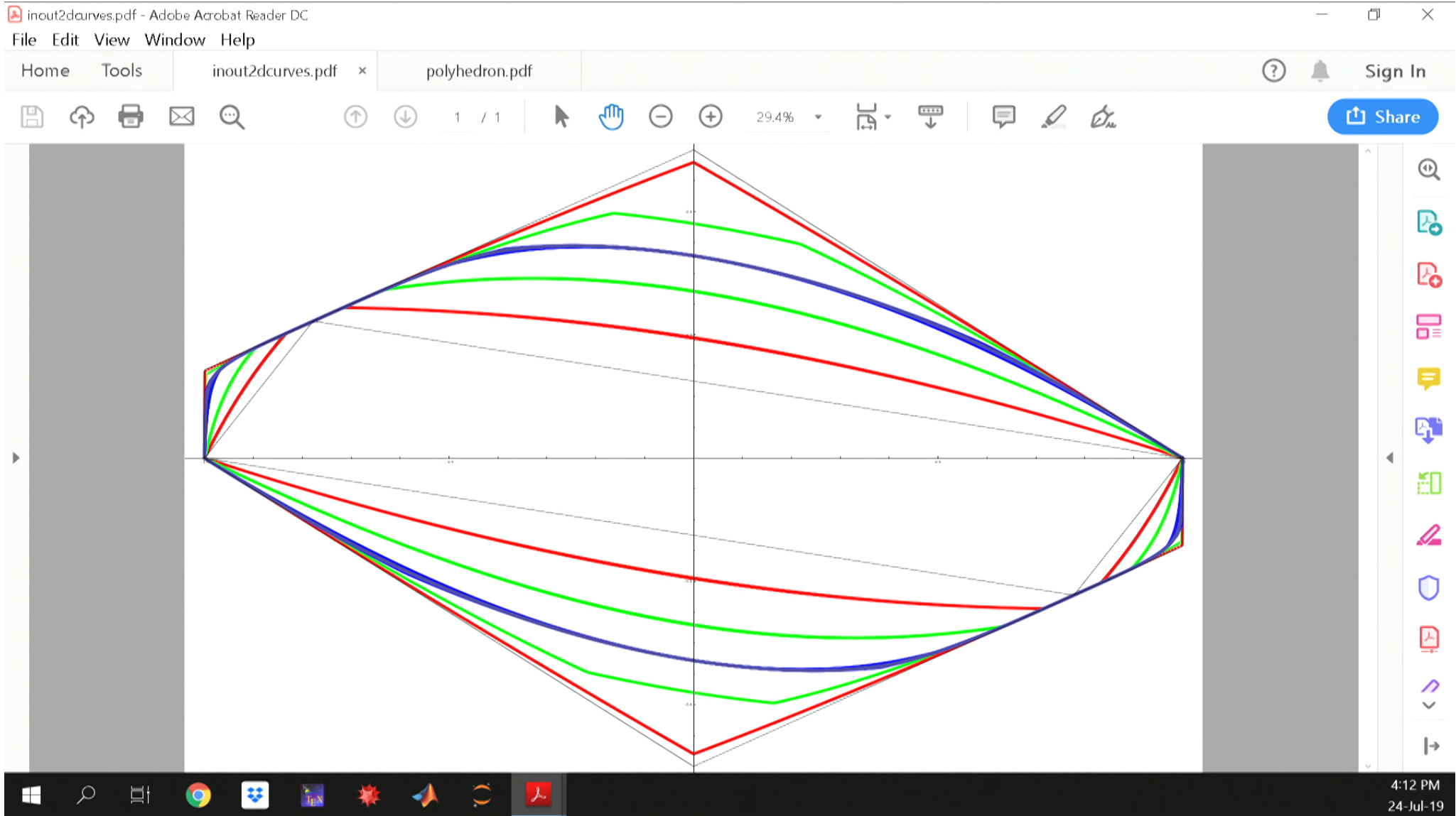


$S_R^{(s)}, S_T^{(s)}, S_A^{(s)}$

$S=3$

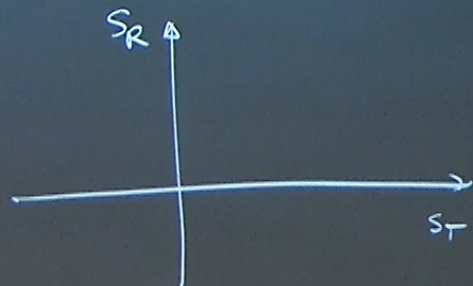




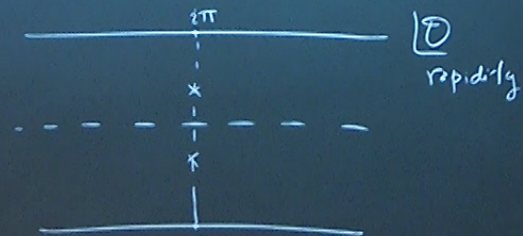


$$S=2$$

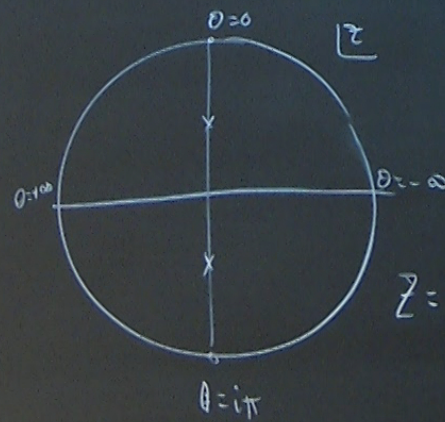
$$t = 4 - s$$



Crossing
 $\theta \rightarrow i\pi - \theta$
 $z \rightarrow -z$



$$S = 4m_1^2 \cosh^2 \theta / 2$$



$$z = \frac{1 - e^\theta}{1 + e^\theta}$$

$$S = \begin{pmatrix} S_D & S_1 \\ S_2 & S_3 \end{pmatrix} \begin{pmatrix} S_D^+ & S_2^+ \\ S_1^+ & S_3^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S_D S_D^+ + S_1 S_1^+ = 1$$

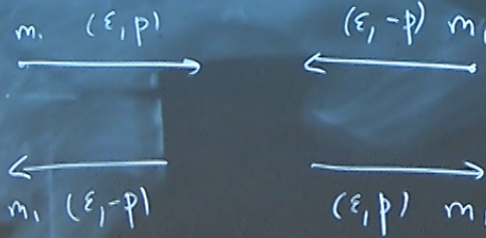
$$S_D S_D^+ - 1 \leq 0$$

$$\begin{pmatrix} 1 & S_D \\ S_D^+ & 1 \end{pmatrix} \geq 0$$

$$\begin{pmatrix} S_2^+ \\ S_3^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

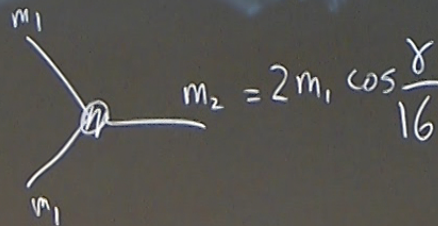
$$\begin{pmatrix} 1 & S_D \\ S_D^+ & 1 \end{pmatrix} \geq 0$$

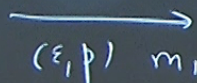
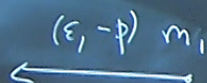
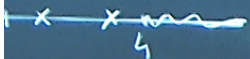
$S^{(1,1)}$



$$S = 4\epsilon^2$$

$$S(s) = S(4m_1^2 - \epsilon)$$





$$S(4m_1^2 - \epsilon)$$

$$m_2 = 2m_1 \cos \frac{\gamma}{16}$$

$$S(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} + \hat{S}(z)$$

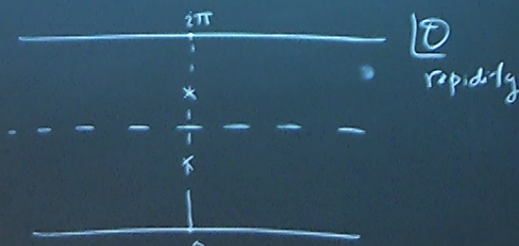
$$|S(e^{i\varphi})| \leq 1$$

$$f(z) = \frac{z^2 + a^2}{1 + a^2 z^2}$$

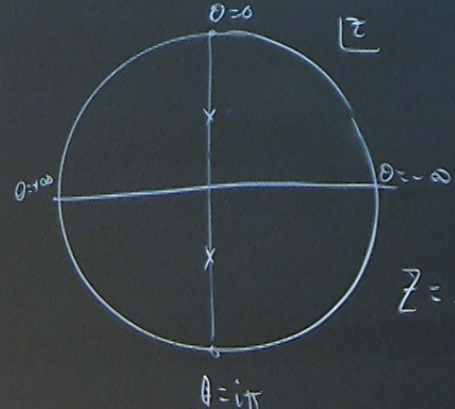
$$1 = S_1(z) = f(z) S(z)$$

$$\underline{S_1(ia)}$$

Crossing
 $\theta \rightarrow i\pi - \theta$
 $z \rightarrow -\bar{z}$



$$S = 4m_1^2 \text{Ch}^2 \theta/2$$



$$z = \frac{1 - e^\theta}{1 + e^\theta}$$

$$S_1(z) = 1 + \xi(z)$$

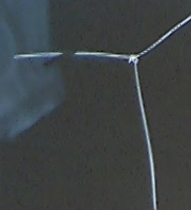
$$\xi(z) = \xi(-z)$$

$$|S_1| \leq 1$$

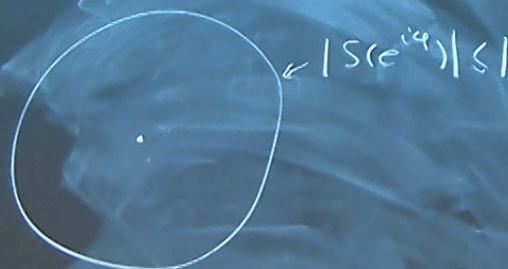
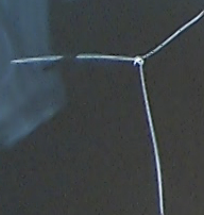
$$(1+\xi)(1+\xi^*) =$$

$$= 1 + 2\operatorname{Re}\xi + \cancel{\xi\xi^*} \leq 1$$

$$\operatorname{Re}\xi \leq 0$$



$$\begin{aligned}
 +\xi)(1+\xi^*) &= \\
 &= 1 + 2\operatorname{Re}\xi + \cancel{\xi\xi^*} \leq 1
 \end{aligned}$$



$$\begin{aligned}
 \operatorname{Re} S(0) &= \operatorname{Re} \frac{1}{2\pi i} \oint \frac{S(z)}{z} dz \\
 &= \frac{1}{2\pi} \int_0^{2\pi} \operatorname{Re} S d\varphi \leq \frac{1}{2\pi} \int_0^{2\pi} |S| d\varphi \leq 1 \\
 |S| &= 1 \quad \operatorname{Im} S = 0 \quad \operatorname{Re} S \geq 0
 \end{aligned}$$

$$S(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} +$$

$$|S(e^{i\varphi})| \leq 1$$

$$f(z) = \frac{z^2 + a^2}{1 + a^2 z^2}$$

$$1 = S_1(z) = f(z) S(z)$$

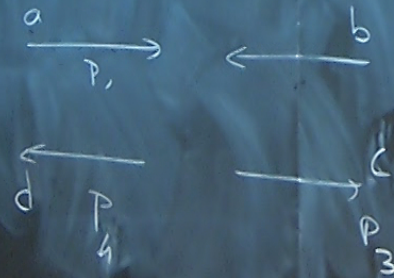
$$S = \frac{1}{2g^2} \int d^2x (\partial_\mu \vec{n})^2$$

$$\vec{n}^2 = 1$$

$$S_T(s) = S_T(4m^2 - s)$$

$$S_R(s) = S_A(4m^2 - s)$$

N bosons m



$$S_{ab \rightarrow cd} = (\delta_{ac} \delta_{bd} S_T(s) +$$

$$+ \delta_{ad} \delta_{bc} S_R(s) +$$

$$+ \delta_{ab} \delta_{cd} S_A(s)) \times$$

$$\times \delta(p_1 - p_3) \delta(p_2 - p_4) + (p_3 \leftrightarrow p_4)(c \leftrightarrow d)$$

$$\underline{n^2 = 1}$$

$$S_T(s) = S_T(4m^2 - s)$$

$$S_R(s) = S_A(4m^2 - s)$$

$$S_{ab \rightarrow cd} = (\delta_{ac} \delta_{bd} S_T(s) +$$

$$+ \delta_{ad} \delta_{bc} S_R(s) +$$

$$+ \delta_{ab} \delta_{cd} S_A(s)) \times$$

$$\times \delta(p_1 - p_3) \delta(p_2 - p_1) + (p_3 \leftrightarrow p_1)(c \leftrightarrow d)$$

$$S_{\pm} = NS_A + S_T + S_R$$

$$S_{\pm} = S_T \pm S_R$$

$$|S_{\pm}(s > 4)| \leq 1$$

$$|S_+| \leq 1$$

$$|S_-| \leq 1$$



$$S(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} + S$$

$$|S(e^{i\varphi})| \leq 1$$

$$f(z) = \frac{z^2 + a^2}{1 + a^2 z^2}$$

$$1 = S_1(z) = f(z) S(z)$$

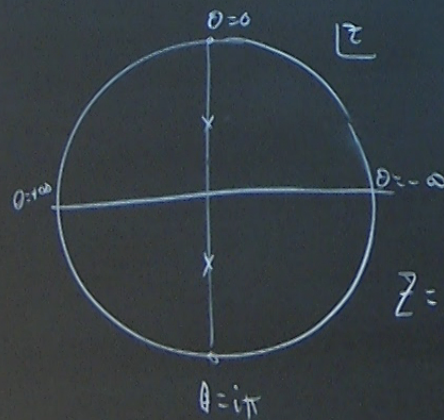
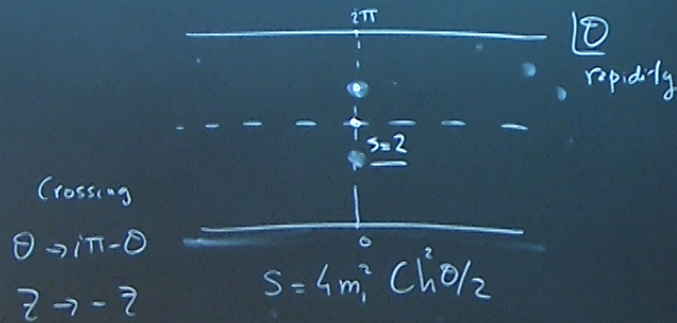
$$t(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} + \hat{S}(z)$$

$$|S(e^{i\varphi})| \leq 1$$

$$f(z) = \frac{z^2 + a^2}{1 + a^2 z^2}$$

$$S_1(z) = f(z) S(z)$$

$$\underline{S_1(ia)}$$



$$z = \frac{1 - e^\theta}{1 + e^\theta}$$



$$n^2 = 1$$

$$S_T(s) = S_T(4m^2 - s)$$

$$S_R(s) = S_A(4m^2 - s)$$

$$S_{ab \rightarrow cd} = (\delta_{ac} \delta_{bd} S_T(s) +$$

$$+ \delta_{ad} \delta_{bc} S_R(s) +$$

$$+ \delta_{ab} \delta_{cd} S_A(s)) \times$$

$$\times \delta(p_1 - p_3) \delta(p_2 - p_4) + (p_3 \leftrightarrow p_4)(c \leftrightarrow d)$$

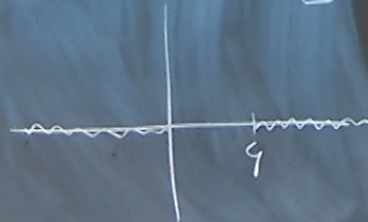
$$S_{\pm} = NS_A + S_T + S_R$$

$$S_{\pm} = S_T \pm S_R$$

$$|S_{\pm}(s > 4)| \leq 1$$

$$|S_{+}| \leq 1$$

$$|S_{-}| \leq 1$$



$$S(z) = \frac{ig}{z+ia} - \frac{ig}{z-ia} + S$$

$$|S(e^{i\varphi})| \leq 1$$

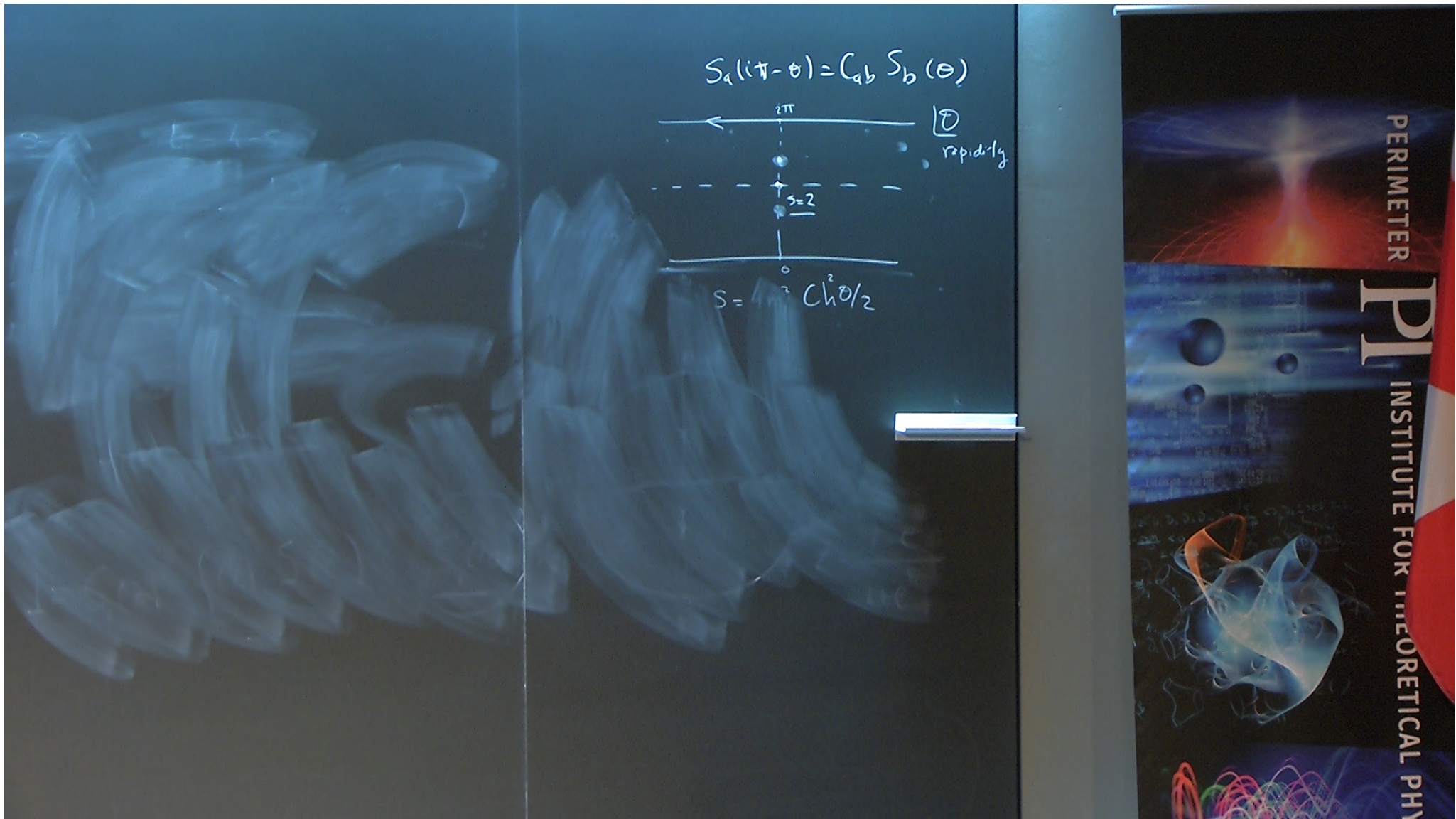
$$f(z) = \frac{z^2 + a^2}{1 + a^2 z^2}$$

$$1 = S_1(z) = f(z) S(z)$$

$$S_a^{\pm}(\theta)$$

$$\text{Res}\left[K_a(\theta), \frac{i\pi}{2}\right] = n_a$$

$$F = \sum_{a=1}^3 n_a \text{Re} S_a\left(\frac{i\pi}{2}\right) = \text{Re} \oint_{\frac{2\pi i}{\epsilon}} K_a(\theta) S_a(\theta) d\theta$$



$$S_a(\theta) \quad \text{Res}[K_a(\theta), \frac{i\pi}{2}] = n_a$$

$$K_a(i\pi - \theta) = -C_{ba} K_b(\theta)$$

$$F = \sum_a n_a \leq \sum_a |n_a|$$

$$K_a = \frac{i n_a}{c h \theta}$$

$$n_a = C_{ba} n_b$$

$$\sum_a |n_a|$$

$$F = \sum_a n_a \operatorname{Re} S_a\left(\frac{i\pi}{2}\right) = \operatorname{Re} \frac{1}{2\pi i} \oint K_a(\theta) S_a(\theta) d\theta$$

$$= \sum_a \operatorname{Re} \left\{ \frac{1}{i\pi} \int_{-\infty}^{\infty} K_a(\sigma) S_a(\sigma) d\sigma \right\} = \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} \operatorname{Im}(K_a S_a) d\sigma \leq \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} |K_a S_a| d\sigma \leq \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} |K_a| d\sigma$$

$$\left\{ \frac{i\pi}{2} \right\} = n_a$$

$$K_a(i\pi - \theta) = -C_{ba} K_b(\theta)$$

$$n_a = C_{ba} n_b$$

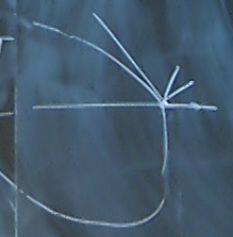
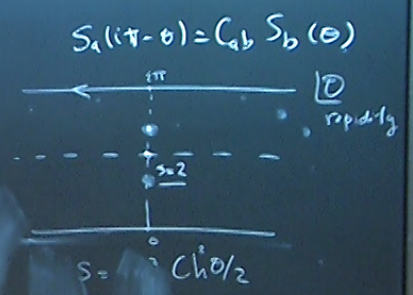
$$F = \sum_a n_a \leq \sum_a |n_a|$$

$$K_a = \frac{i n_a}{\cosh \theta}$$

$$\sum_a |n_a|$$

$$= \operatorname{Re} \frac{1}{2\pi i} \oint K_a(\theta) S_a(\theta) d\theta$$

$$\left\{ \frac{1}{i\pi} \int_{-\infty}^{\infty} K_a(\theta) S_a(\theta) d\theta \right\} = \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} \operatorname{Im}(K_a S_a) d\theta \leq \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} |K_a S_a| d\theta \leq \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} |K_a| d\theta$$



19070040

$$|K_a S_a| = |K_a|$$

$$|K_a|(|S_a| - 1) = 0$$

$$K_a(i\pi - \theta) = -C_{ba}$$

$$F = \sum_{a=1}^3 n_a \operatorname{Re} S_a(i\pi) = \operatorname{Re} \frac{1}{2\pi i} \oint_{\mathcal{C}} K_a(\sigma) S_a(\sigma) d\sigma$$

$$= \sum_a \operatorname{Re} \left\{ \frac{1}{i\pi} \int_{-\infty}^{\infty} K_a(\sigma) S_a(\sigma) d\sigma \right\} = \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} \dots$$

$$= |K_a|$$

$$|S_a| - 1 = 0$$

$$K_a(i\pi - \theta) = -C_{ba} K_b(\theta)$$

$$F = \sum_a n_a \leq \sum_a |n_a|$$

$$K_a = \frac{i n_a}{ch\theta}$$

$$n_a = C_{ba} n_b$$

$$S_a(i\pi/2) = \operatorname{Re} \frac{1}{2\pi i} \oint K_a(\theta) S_a(\theta) d\theta$$

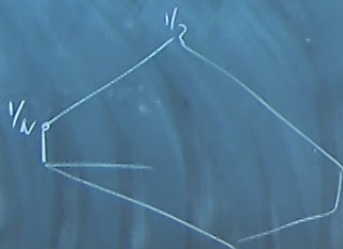
$$\sum_a \operatorname{Re} \left\{ \frac{1}{i\pi} \int_{-\infty}^{\infty} K_a(\sigma) S_a(\sigma) d\sigma \right\} = \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} \operatorname{Im}(K_a S_a) d\sigma \leq \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} |K_a S_a| d\sigma \leq \sum_a \frac{1}{\pi} \int_{-\infty}^{\infty} 1 d\sigma = \sum_a 1$$

$$\operatorname{Re}(K_a S_a) = 0$$

$$\operatorname{Im}(K_a S_a) \geq 0$$

$$S_a = \frac{i \bar{K}_a}{|K_a|}$$

$$K_a = \frac{i\eta_a}{\cosh\theta}$$



$$K_a = \frac{i k_a(\theta)}{\cosh\theta}$$

$$k_a\left(\frac{i\pi}{2}\right) = \eta_a$$

$$k_a(\theta) = k_a(\theta + b)$$

$$S_a(\theta) = S_a(\theta + b)$$

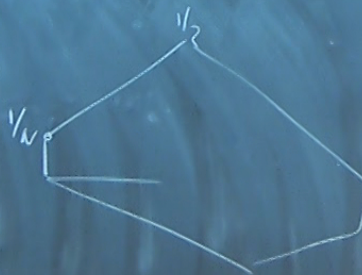
$$\operatorname{Re} S_a = \alpha V_a$$

$$\max \alpha$$

$$n_a V_a = 1$$

$$\min \sum_a |n_a|$$

$$K_a = \frac{i n_a}{\cosh \theta}$$



$$K_a = \frac{i k_a(\theta)}{\cosh \theta}$$

$$k_a\left(\frac{i\pi}{2}\right) = n_a$$

$$k_a(\theta) = k_a(\theta + b)$$