Title: Talk 13 via live stream

Speakers: Carlo Meneghelli

Collection: Bootstrap 2019

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Free field realizations from the Higgs Branch

with C. Beem, L. Rastelli 1903.07624 + in Progress

" " + W. Peelaers 1907.08629

F. Bonetti, " 1810.03612

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χ : $\{ \mathcal{N}=2 \ 4d \ SCFTs \} \longrightarrow \{ VOAs \}$

↑ BLLPRvR '13

1. CANONICAL , $\square \in SU(2,2|4)$: $\mathcal{N}=2$ $d=4$ SCA

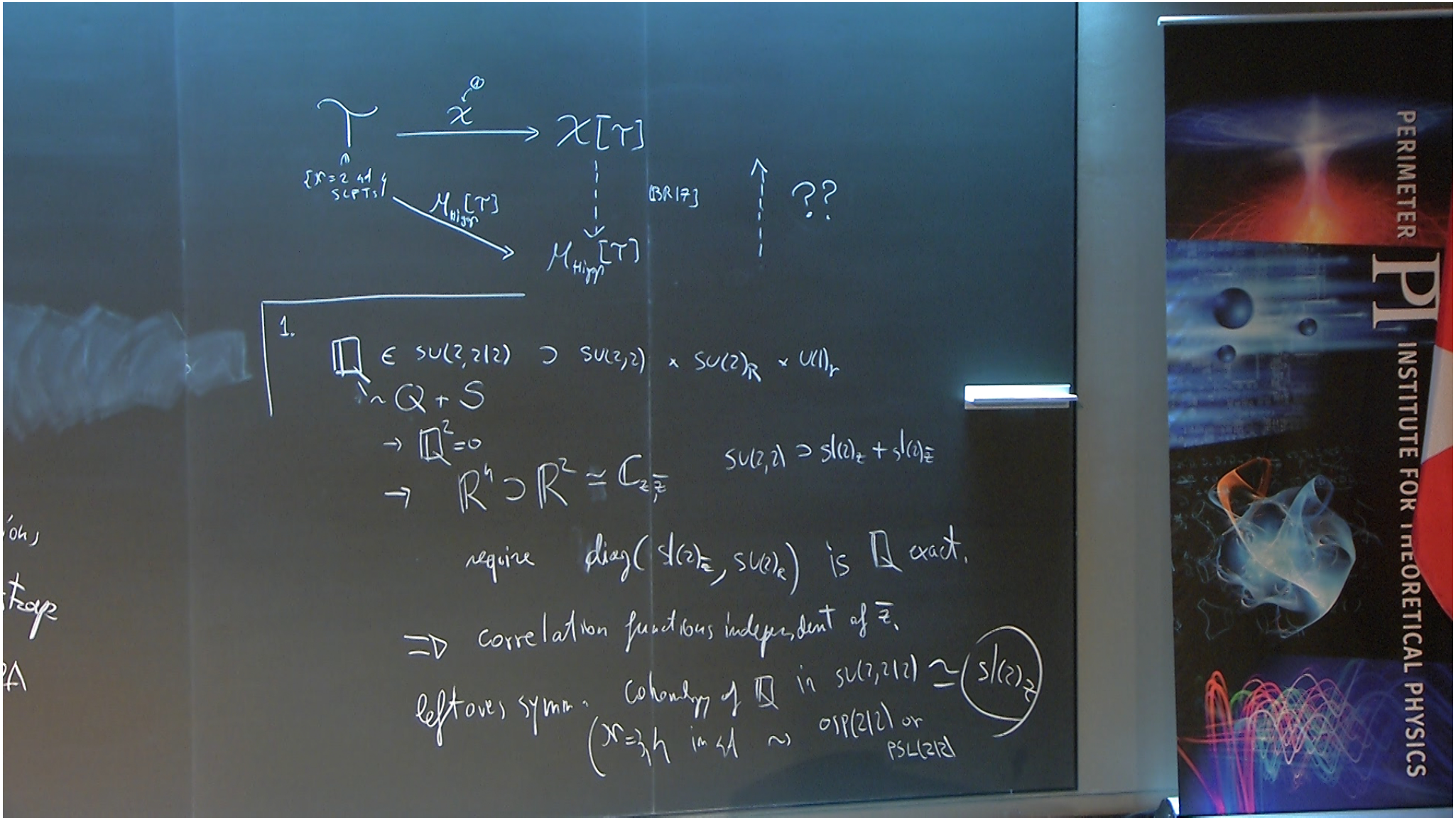
2. "rigid" : $\square$ cohomology $\equiv \chi$-map

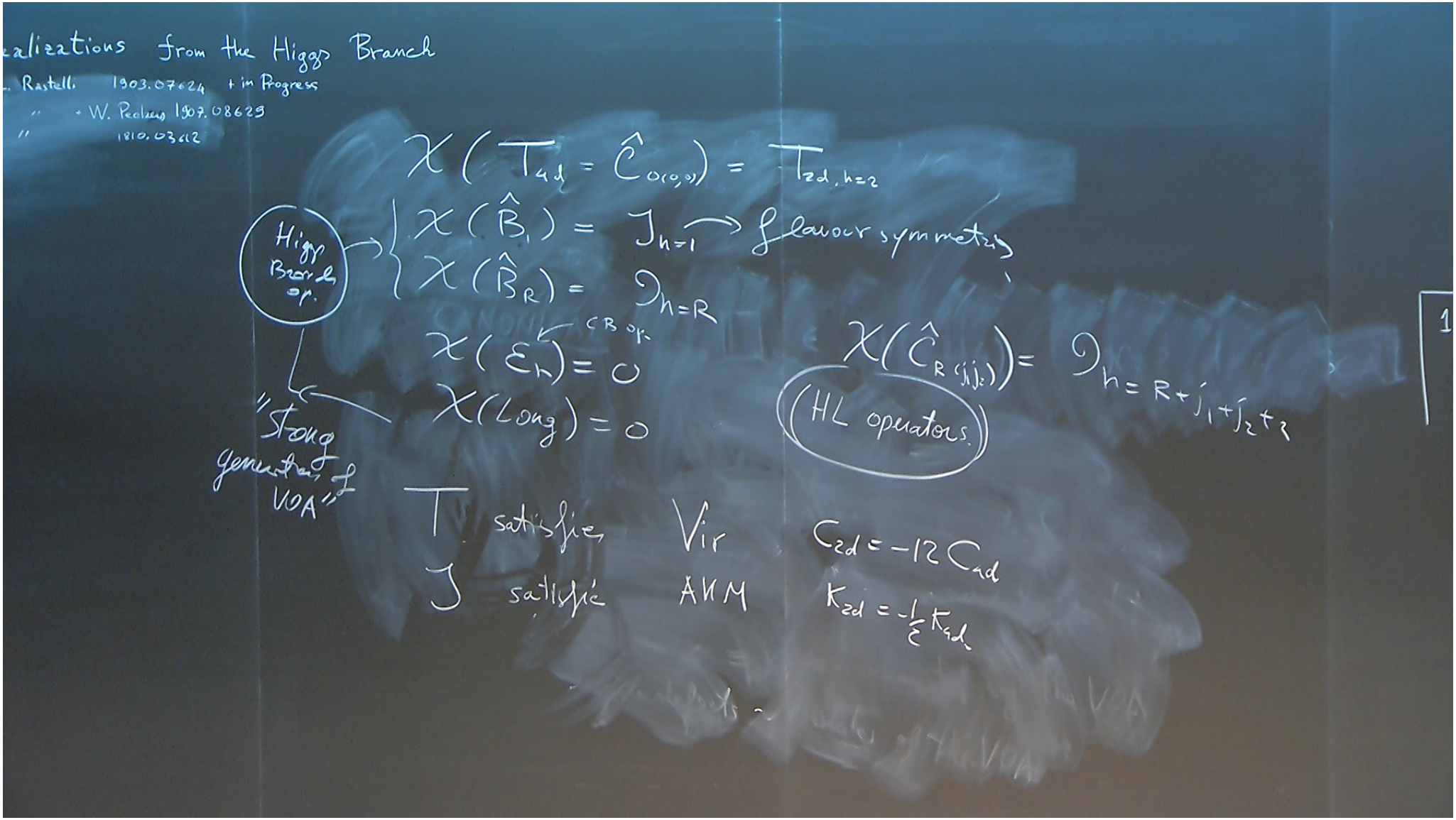
3. Input for the "bulk" : invariant w.r.t. marginal deformations,

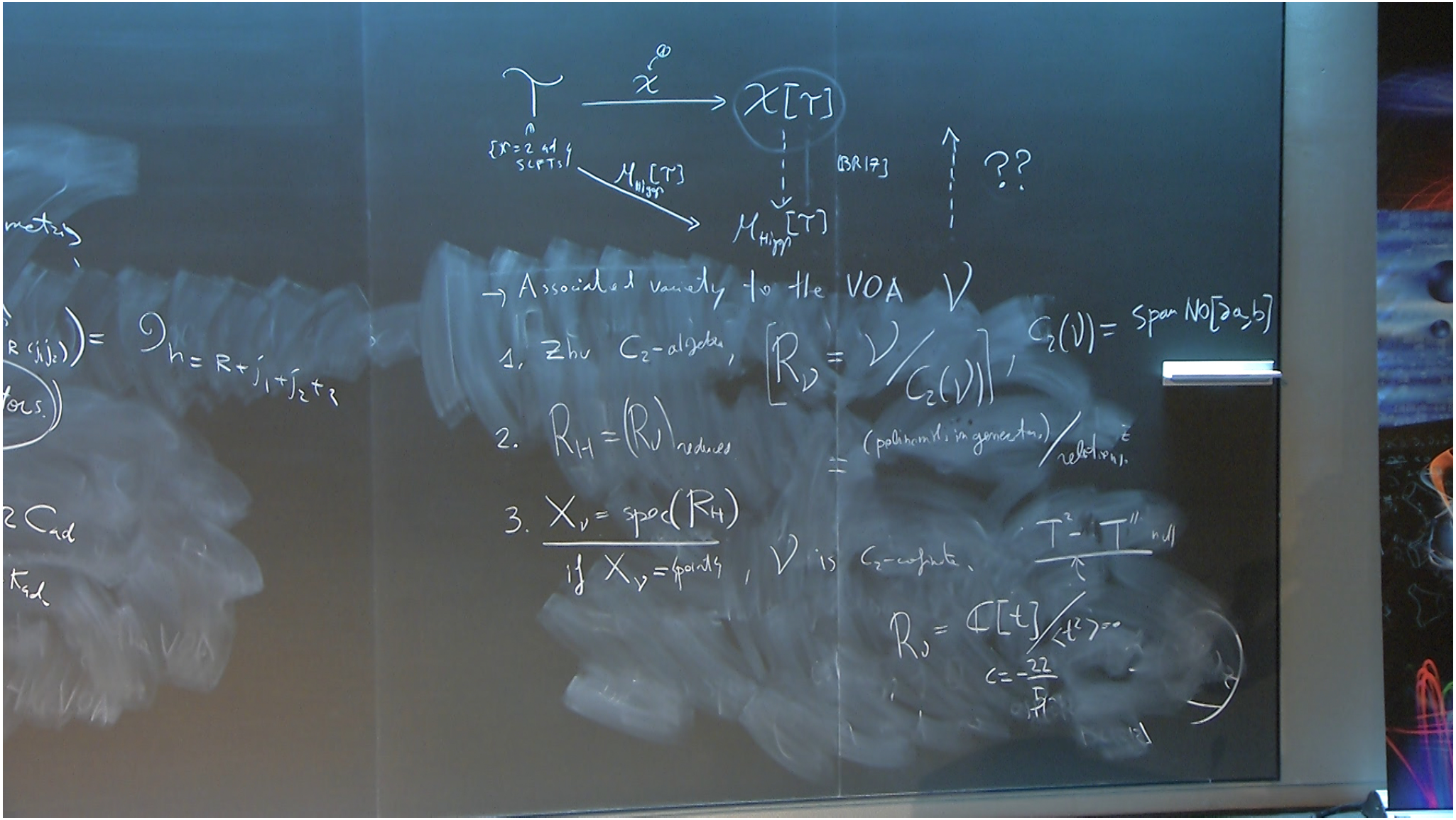
4. $\mathcal{N}=2$ numerical bootstrap

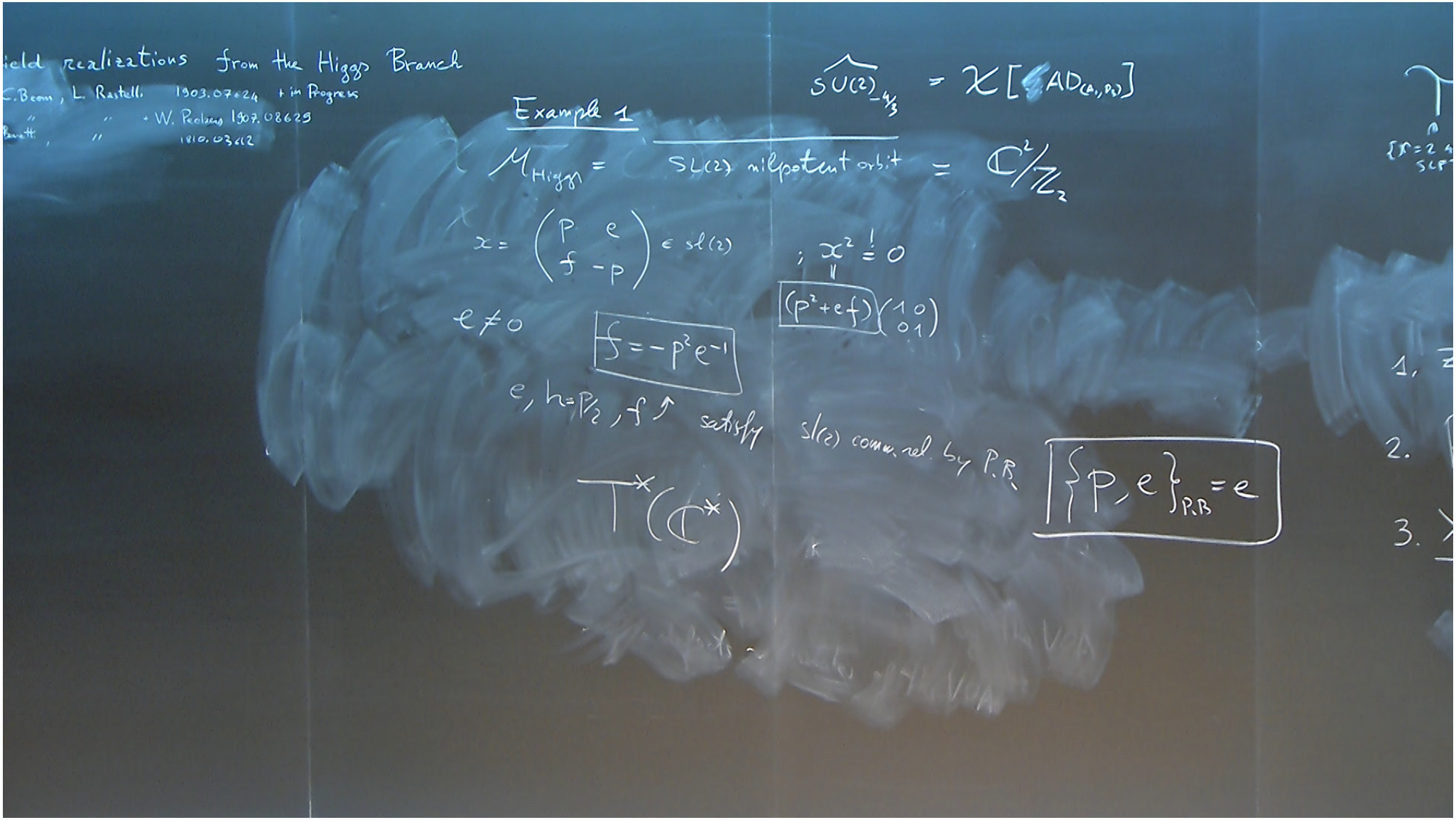
Schur index $\equiv$ vacuum character of the VOA

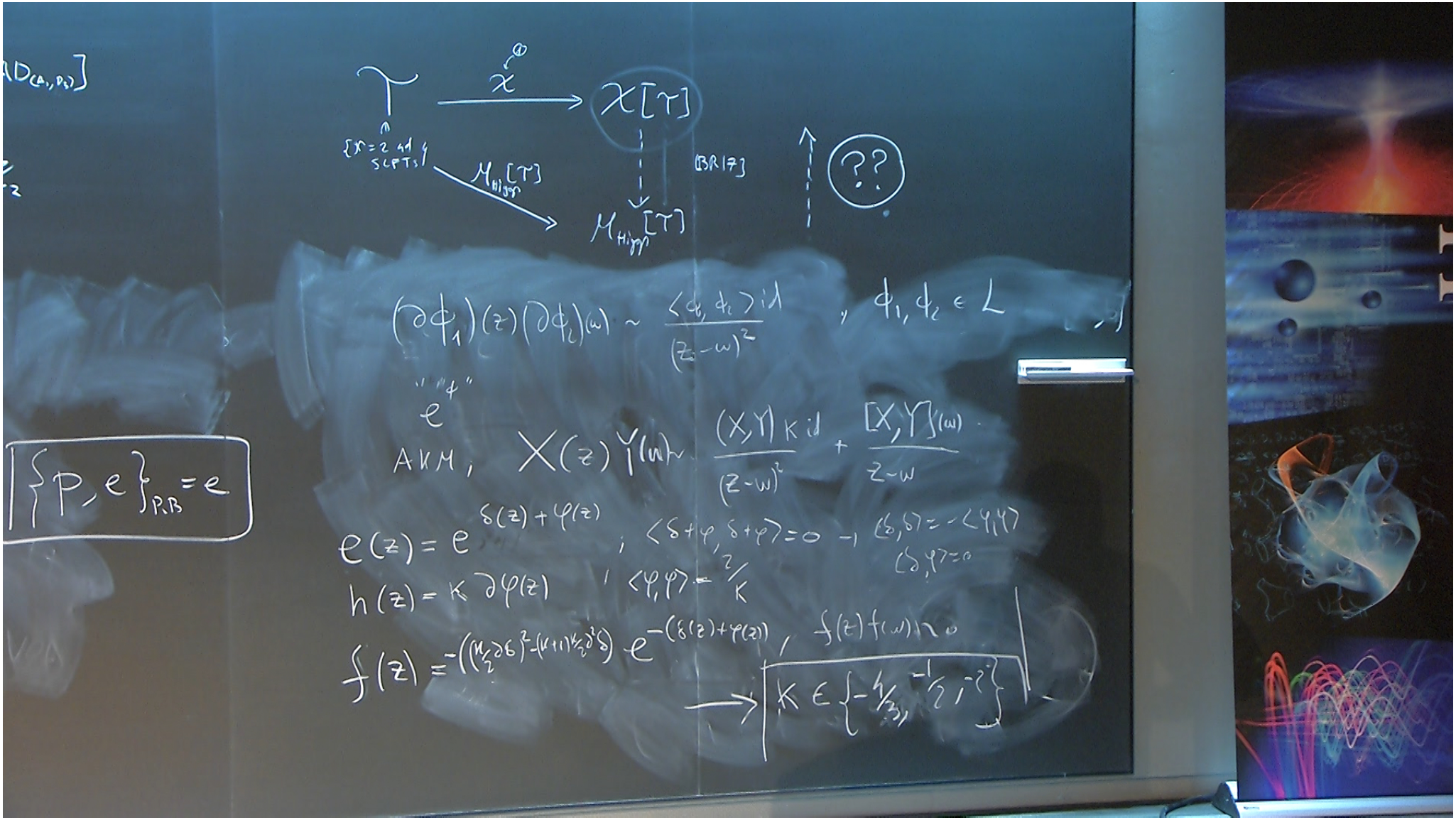
surface defects $\sim$ modules of the VOA


$\mathcal{T}$
$\mathcal{X}^{\mathbb{Q}}$
$\mathcal{X}[\mathcal{T}]$
$\mathcal{M}_{\text{Higgs}}[\mathcal{T}]$
[BR17]
??
$\mathcal{M}_{\text{Higgs}}[\mathcal{T}]$
1.
$\mathbb{Q} \in SU(2,2|2) \supset SU(2,2) \times SU(2)_R \times U(1)_r$
$\sim Q + S$
$\to \mathbb{Q}^2 = 0$
$\to \mathbb{R}^4 \supset \mathbb{R}^2 \simeq \mathbb{C}_{z,\bar{z}}$
$SU(2,2) \supset sl(2)_z + sl(2)_{\bar{z}}$
require $\text{diag}(sl(2)_z, SU(2)_R)$ is $\mathbb{Q}$ exact.
$\Rightarrow$ correlation functions independent of $\bar{z}$.
leftover symm.: Cohomology of $\mathbb{Q}$ in $SU(2,2|2) \simeq sl(2)_z$
($\mathcal{N}=3,4$ in 4d $\sim$ $osp(2|2)$ or $PSL(2|2)$)


alizations from the Higgs Branch
. Rastelli 1903.07624 + in Progress
" + W. Peelaers 1907.08629
" 1810.03612
Higgs Branch op.
"Strong generators of VOA"
CB op.
HL operators
satisfies
Vir
AHM


$\mathcal{T}$
$\mathcal{X}$
$\mathcal{X}[T]$
$\mathcal{M}_{Higgs}[T]$
$\mathcal{M}_{Higgs}[T]$
[BR17]
??
→ Associated variety to the VOA V
1. Zhu C_2-algebra, $R_V = V/C_2(V)$, $C_2(V) = \text{Span}\{:\partial a\, b:\}$
2. $R_H = (R_V)_{\text{reduced}}$ = (polynomials in generators)/relations
3. $X_V = \text{spec}(R_H)$
if X_V = points, V is C_2-cofinite.
$R_V = \mathbb{C}[t]/\langle t^2 \rangle$
$c = -\frac{22}{5}$


ield realizations from the Higgs Branch
C. Beem, L. Rastelli 1903.07624 + in Progress
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Bonetti, " 1810.03612
Example 1
$\mathcal{M}_{Higgs}$ = SL(2) nilpotent orbit = $\mathbb{C}^2/\mathbb{Z}_2$
$\widehat{SU(2)}_{-4/3} = \mathcal{X}[SAD_{(a_1,p_2)}]$
$x = \begin{pmatrix} p & e \\ f & -p \end{pmatrix} \in sl(2)$; $x^2 \overset{!}{=} 0$
$(p^2+ef)\begin{pmatrix}1&0\\0&1\end{pmatrix}$
$e \neq 0$ $f = -p^2 e^{-1}$
e, h=P/2, f ↑ satisfy sl(2) comm. rel. by P.B. $\{P, e\}_{P.B} = e$
$T^*(\mathbb{C}^*)$
1.
2.
3.


$\mathcal{T}$
$\mathcal{X}$
$\mathcal{X}[T]$
$\mathcal{M}_{Higgs}[T]$
$\mathcal{M}_{Higgs}[T]$
[BR17]
??
$(\partial\phi_1)(z)(\partial\phi_2)(w) \sim \frac{\langle\phi_1,\phi_2\rangle id}{(z-w)^2}$, $\phi_1,\phi_2 \in L$
$e^{i\phi}$
AKM, $X(z)Y(w) \sim \frac{(X,Y)K\,id}{(z-w)^2} + \frac{[X,Y](w)}{z-w}$
$\{P,e\}_{P.B} = e$
$e(z) = e^{\delta(z)+\varphi(z)}$; $\langle\delta+\varphi,\delta+\varphi\rangle = 0$, $\langle\delta,\delta\rangle = -\langle\varphi,\varphi\rangle$ $\langle\delta,\varphi\rangle = 0$
$h(z) = K\partial\varphi(z)$, $\langle\varphi,\varphi\rangle = 2/K$
$f(z) = -\left(\left(\frac{K}{2}\partial\delta\right)^2 - (K+1)\frac{K}{2}\partial^2\delta\right)e^{-(\delta(z)+\varphi(z))}$, $f(z)f(w) \sim \cdots$
$K \in \{-\frac{1}{3}, -\frac{1}{2}, -1, -2\}$

from the Higgs Branch

7024 + in Progress

1907.08625

23c12

Example 1

$$\mathcal{M}_{\text{Higgs}} = \overline{\mathcal{O}_{min}[g]} = \overline{(G\text{-orbit of } e_\theta)}$$

θ highest root, $\langle e_\theta, f_\theta, h_\theta \rangle \in sl(2)_\theta \in \mathfrak{g}$

$$\mathfrak{g} = \mathfrak{g}^{\mathfrak{h}}_{\text{commutant}} \oplus sl(2)_\theta \oplus (R, 2) \cong \mathbb{C} f_\theta \oplus R^- \oplus (\mathfrak{g}^{\mathfrak{h}} + \mathbb{C} h_\theta) \oplus R^+ \otimes \mathbb{C} e_\theta$$

h_θ-weights

$$\dim_{\mathbb{C}} \mathcal{O}_{min}[g] = 1 + \dim R + 1 = 2(h^\vee - 1)$$

e_A, $A = 1, \dots, \dim R^+$ "e^+"

$$e_\theta = e^{\delta + \varphi}$$

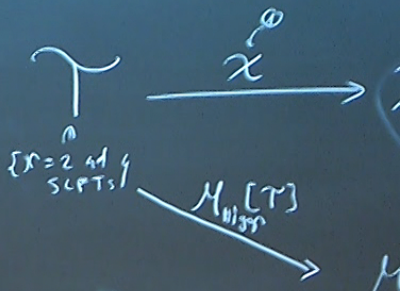
$$e_A = \xi_A(z) e^{\frac{\delta + \varphi}{2}}, \quad \xi_A(z)\xi_B(w) \sim \frac{\Omega_{AB}}{z - w}$$

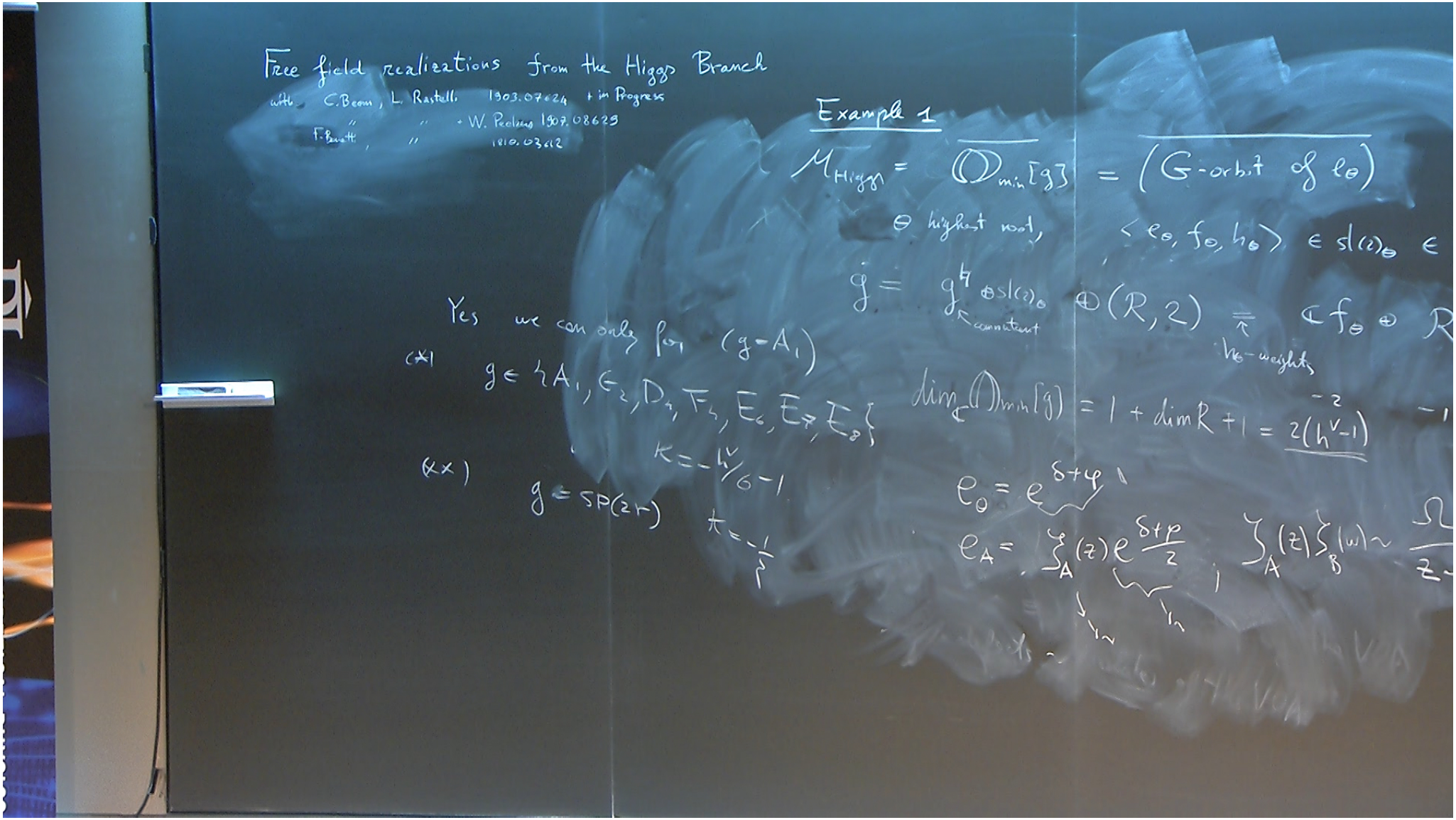
AKM, $X(z)$

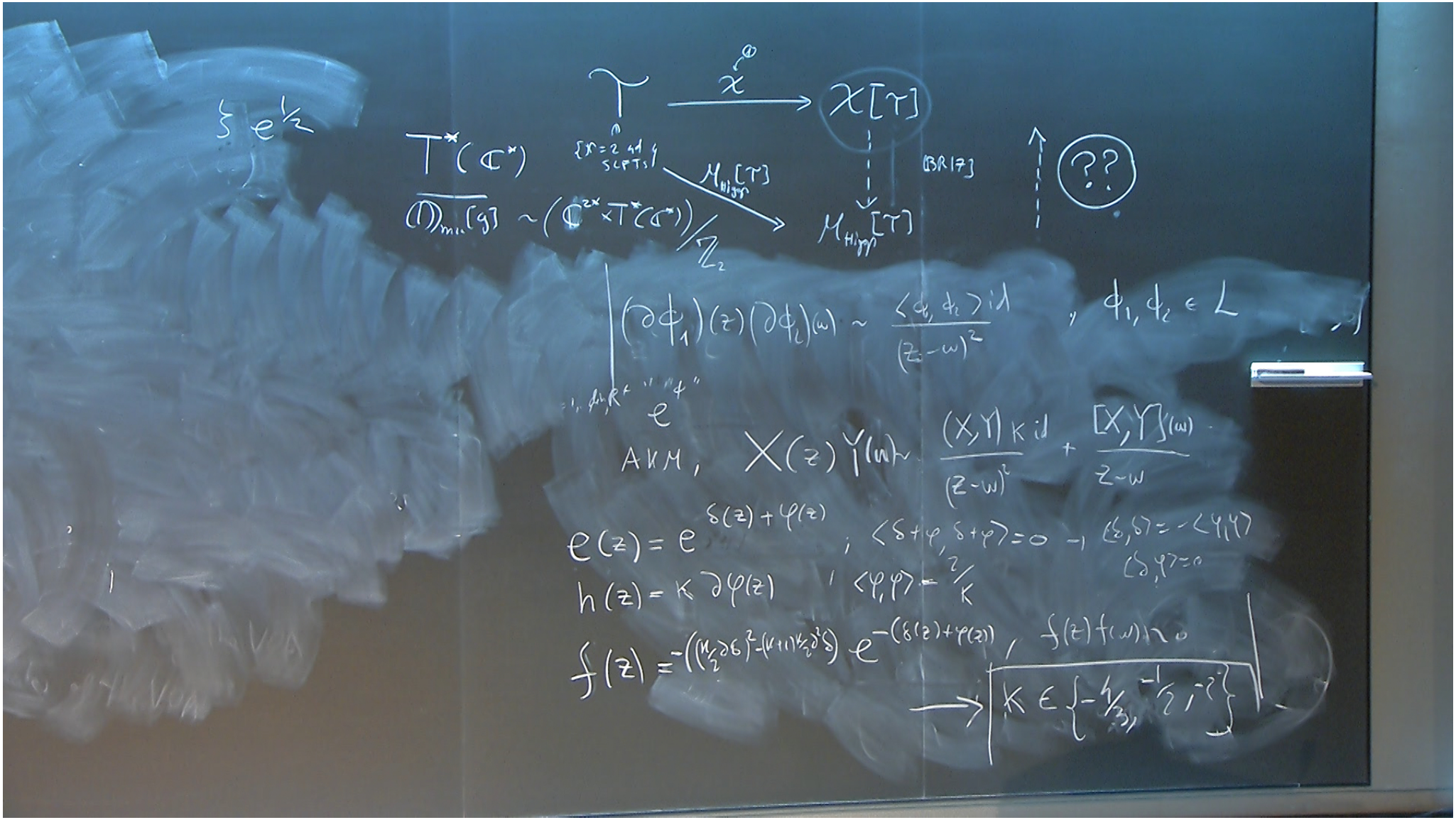
$$e(z) = e^{\delta(z) + \varphi(z)}$$

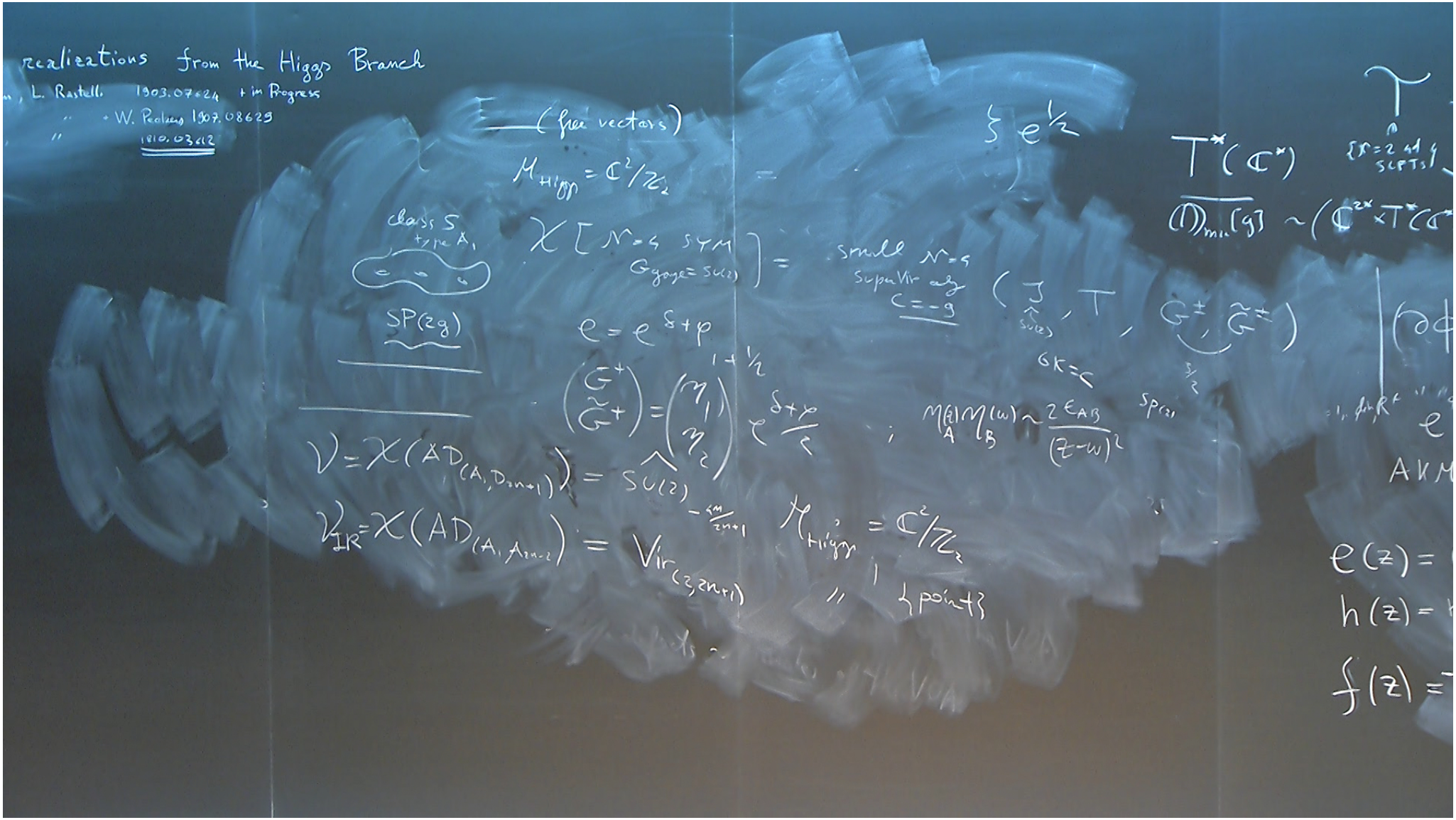
$$h(z) = \kappa \partial\varphi(z)$$

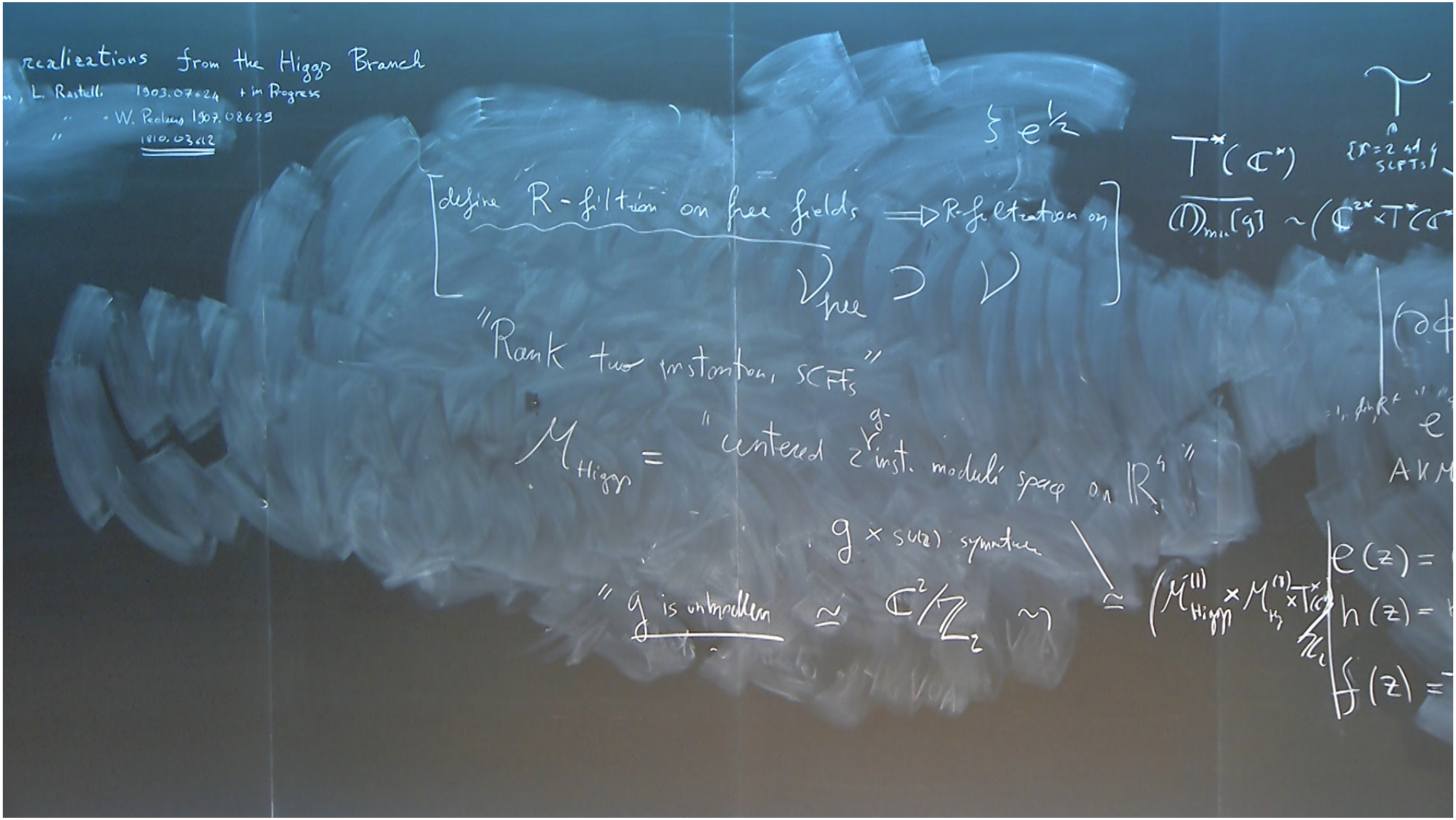
$$f(z) = -\left(\left(\frac{\kappa}{2}\partial\delta\right)^2 - (\kappa+1)\frac{\kappa}{2}\partial^2\delta\right)$$

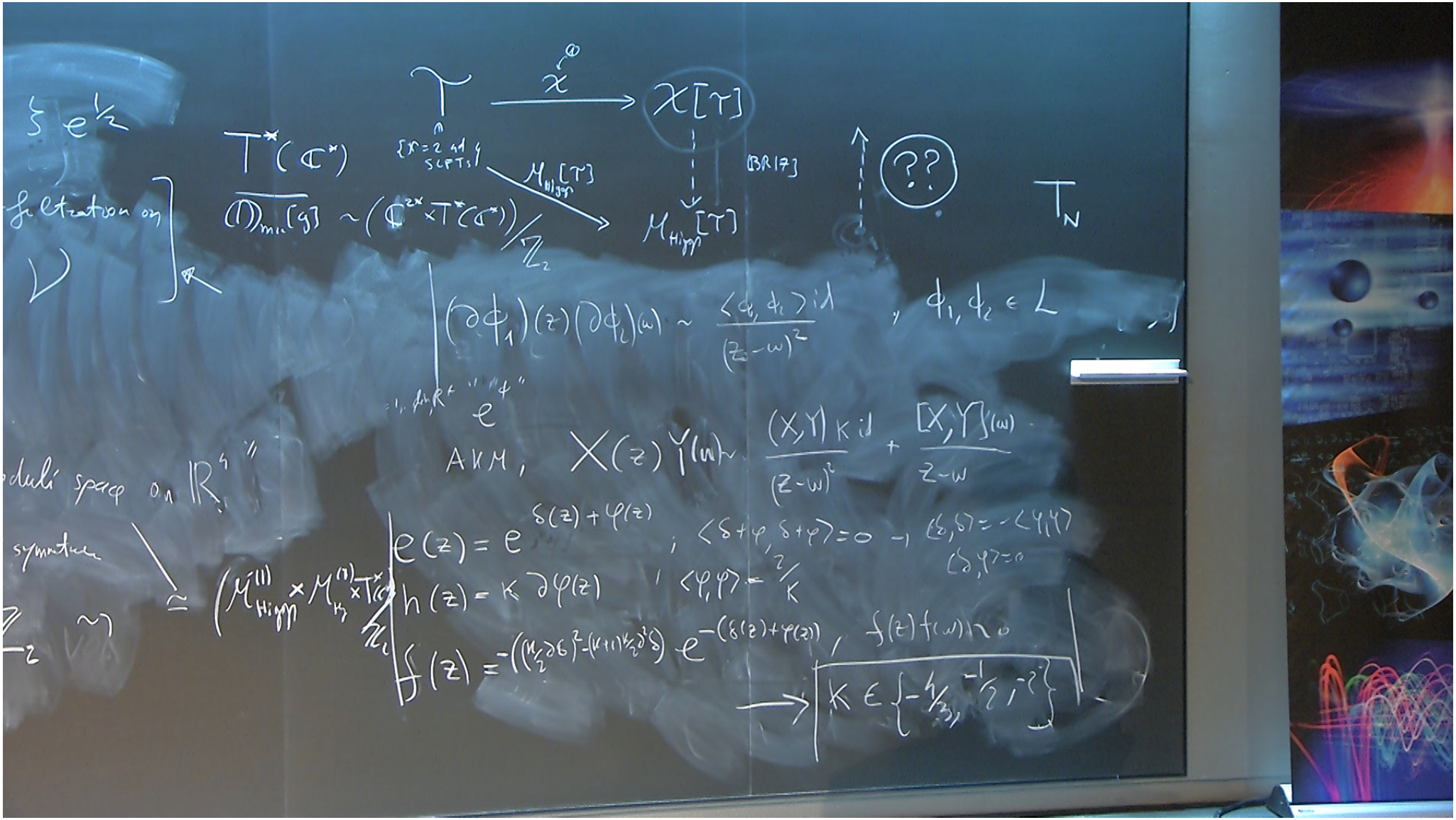


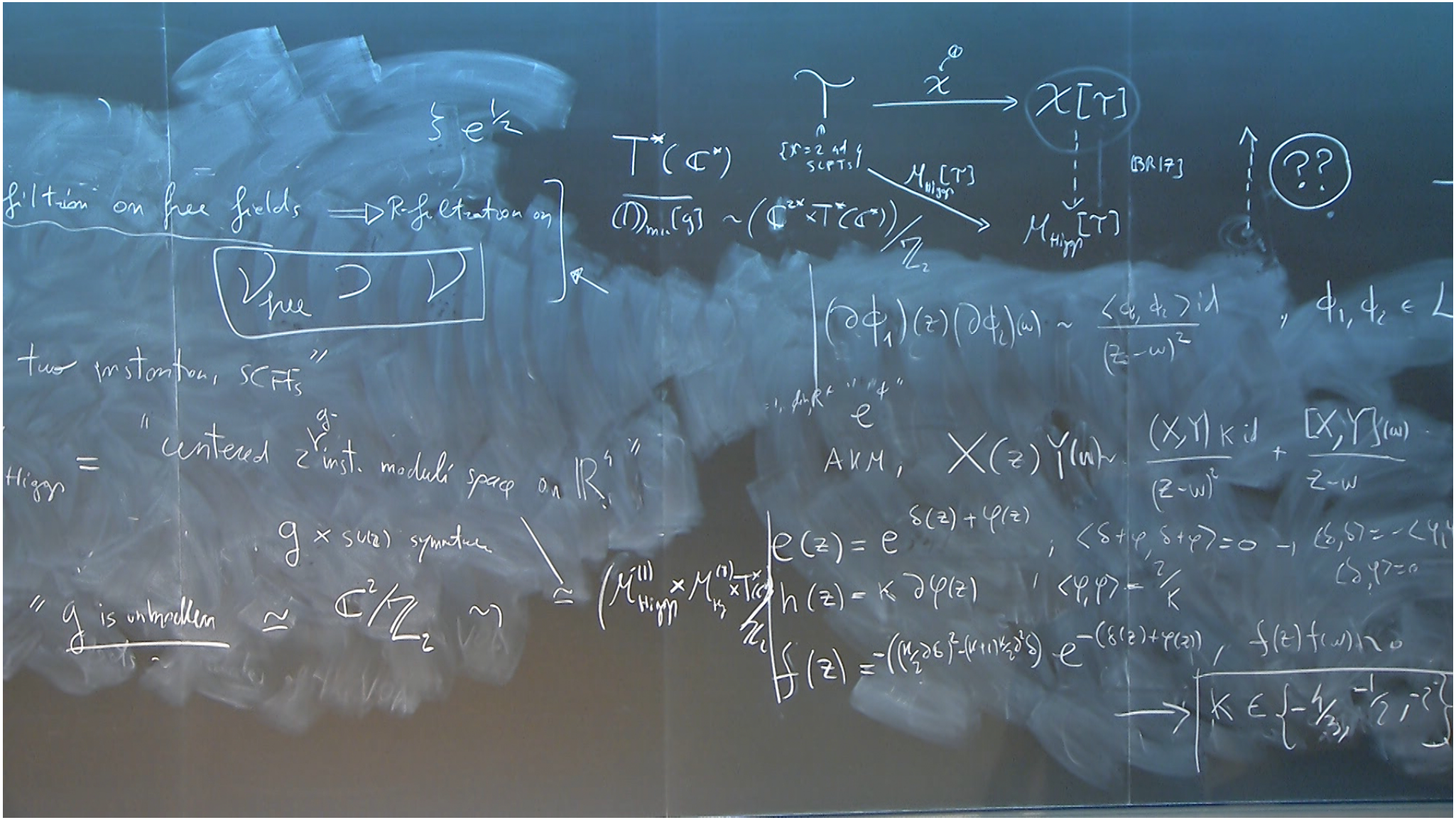

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with C. Beem, L. Rastelli 1303.07624 + in Progress
" " + W. Peelaers 1307.08629
F. Bonetti " " 1810.03612
Example 1
$\mathcal{M}_{Higgs} = \overline{\mathcal{O}_{min}[g]} = \overline{(G\text{-orbit of } e_\theta)}$
θ highest root, $\langle e_\theta, f_\theta, h_\theta \rangle \in sl(2)_\theta \in$
$g = g^h \oplus sl(2)_\theta \oplus (R, 2) \supset f_\theta \oplus R$
h-commutant
h-weights
Yes we can only for $(g - A_1)$
(*) $g \in \{A_1, G_2, D_4, F_4, E_6, E_7, E_8\}$
$\dim_{\mathbb{C}} \mathcal{O}_{min}[g] = 1 + \dim R + 1 = 2(h^\vee - 1)$
$e_0 = e^{\delta + \varphi}$
$e_A = J_A(z) e^{\frac{\delta + \varphi}{2}}$
$J_A(z) J_B(w) \sim \frac{\Omega}{z-}$
(**) $g = SP(2r)$
$K = -h^\vee/6 - 1$
$c = -\frac{1}{2}$


$T^*(\mathbb{C}^*)$
$\{\mathcal{N}=2$ 4d SCFTs$\}$
$\mathcal{M}_{Higgs}[T]$
[BR17]
??
$\mathcal{M}_{Higgs}[T]$
$\mathcal{M}_{mix}[g] \sim (\mathbb{C}^{2\kappa} \times T^*(\mathbb{C}^*))/\mathbb{Z}_2$
$(\partial\phi_1)(z)(\partial\phi_2)(w) \sim \frac{\langle\phi_1,\phi_2\rangle \mathrm{id}}{(z-w)^2}$, $\phi_1, \phi_2 \in L$
AKM, $X(z)Y(w) \sim \frac{(X,Y)K\,\mathrm{id}}{(z-w)^2} + \frac{[X,Y](w)}{z-w}$
$e(z) = e^{\delta(z)+\varphi(z)}$; $\langle\delta+\varphi,\delta+\varphi\rangle = 0$ → $(\delta,\delta) = -\langle\varphi,\varphi\rangle$, $(\delta,\varphi)=0$
$h(z) = K\partial\varphi(z)$, $\langle\varphi,\varphi\rangle = 2/K$
$f(z) = -\left(\left(\frac{K}{2}\partial\delta\right)^2 - (K+1)\frac{K}{2}\partial^2\delta\right) e^{-(\delta(z)+\varphi(z))}$, $f(z)f(w) \sim 0$
$K \in \{-\frac{1}{3}, -\frac{1}{2}, -1, -2\}$


realizations from the Higgs Branch
L. Rastelli 1903.07624 + in Progress
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1810.03612
(free vectors)
$\mathcal{M}_{Higgs} = \mathbb{C}^2/\mathbb{Z}_2$
class $\mathcal{S}$ type A_1
$\chi[\mathcal{N}=4$ SYM $G_{gauge}=SU(2)] =$ small $\mathcal{N}=4$ superVir alg $c=-9$
$SP(2g)$
$e = e^{\delta+\rho}$
$\begin{pmatrix} G^+ \\ \tilde{G}^+ \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} e^{\frac{\delta+\rho}{2}}$
$V = \chi(AD_{(A_1, D_{2n+1})}) = \widehat{SU(2)}_{-\frac{4n}{2n+1}}$
$V_{IR} = \chi(AD_{(A_1, A_{2n-2})}) = Vir_{(3,2n+1)}$
$\mathcal{M}_{Higgs} = \mathbb{C}^2/\mathbb{Z}_2$
// {point}
$T^*(\mathbb{C}^*)$
$\mathcal{M}_{min}[g] \sim (\mathbb{C}^{2g} \times T^*\mathbb{C}^*$
$(J_{\widehat{SU(2)}}, T, G^\pm, \tilde{G}^\pm)$
$GK = c$
$SP(2)$
$\eta_A(w)\eta_B(w) \sim \frac{2\epsilon_{AB}}{(z-w)^2}$
AKM
$e(z) =$
$h(z) =$
$f(z) =$


realizations from the Higgs Branch
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1810.03612
define R-filtration on free fields ⇒ R-filtration on
$V_{free} \supset V$
"Rank two instanton, SCFTs"
$\mathcal{M}_{Higgs}$ = "centered r^{g-} inst. moduli space on $\mathbb{R}^4$"
$g \times SU(2)$ symmetries
"g is unbroken" $\simeq \mathbb{C}^2/\mathbb{Z}_2$
$\simeq (\mathcal{M}^{(1)}_{Higgs} \times \mathcal{M}^{(1)}_{Higgs} \times T(\mathbb{C}^*))/\mathbb{Z}_2$
$T^*(\mathbb{C}^*)$
{N=2 4d SCFTs}
$\mathcal{M}_{min}[g] \sim (\mathbb{C}^{2*} \times T^*\mathbb{C}^*$
AKM
e(z) =
h(z) =
f(z) =


$T^*(\mathbb{C}^*)$
$\mathcal{M}_{Hitchin}[T]$
[BR17]
??
T_N
$(\partial\phi_1)(z)(\partial\phi_2)(w) \sim \frac{\langle\phi_1,\phi_2\rangle \cdot 1}{(z-w)^2}$, $\phi_1, \phi_2 \in L$
AKM, $X(z)Y(w) \sim \frac{\langle X,Y\rangle k\,id}{(z-w)^2} + \frac{[X,Y](w)}{z-w}$
$e(z) = e^{\delta(z)+\varphi(z)}$
$h(z) = k\,\partial\varphi(z)$
$\langle\delta+\varphi,\delta+\varphi\rangle = 0$
$\langle\varphi,\varphi\rangle = \frac{2}{k}$
$f(z) f(w) \sim$
$k \in \{-\frac{1}{3}, -1, 2, 3\}$


filtration on free fields ⇒ R-filtration on
$V_{free} \supset V$
two instanton, SCFTs
$\mathcal{M}_{Higgs}$ = "centered g-inst. moduli space on $\mathbb{R}^4$"
$g \times SU(2)$ symmetries
"g is unbroken" $\simeq \mathbb{C}^2/\mathbb{Z}_2$
$\simeq (\mathcal{M}^{(1)}_{Higgs} \times \mathcal{M}^{(1)}_{Higgs} \times T^*\mathbb{C}^*)/\mathbb{Z}_2$
$T^*(\mathbb{C}^*)$
$\mathcal{M}_{mir}[g] \sim (\mathbb{C}^{2*} \times T^*(\mathbb{C}^*))/\mathbb{Z}_2$
$\mathcal{M}_{Higgs}[T]$
$\mathcal{X}[T]$
[BR17]
??
$(\partial\phi_1)(z)(\partial\phi_2)(w) \sim \frac{\langle \phi_1, \phi_2 \rangle id}{(z-w)^2}$, $\phi_1, \phi_2 \in L$
AKM, $X(z)Y(w) \sim \frac{(X,Y)\kappa id}{(z-w)^2} + \frac{[X,Y](w)}{z-w}$
$e(z) = e^{\delta(z)+\varphi(z)}$
$h(z) = \kappa\,\partial\varphi(z)$
$f(z) = -((\kappa_2\partial\delta)^2 - (\kappa_1)\kappa_2\partial^2\delta)\, e^{-(\delta(z)+\varphi(z))}$
$\langle \delta+\varphi, \delta+\varphi \rangle = 0$, $\langle \varphi, \varphi \rangle = 2/\kappa$
$f(z)f(w)|_{z\to w}$
$\kappa \in \{-1/3, -1/2, -1\}$