Title: Flux Tube S-matrix Bootstrap

Speakers: Andrea Guerrieri

Collection: Bootstrap 2019

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Flux-Tube S-matrix Bootstrap

with Elias-Miró, Hebbar, Penedones, Vieira

arXiv: 1906.08098

$$S_{ab}^{cd} = \sigma_1 \delta_{ab}\delta^{cd} + \sigma_2 \delta_a^c \delta_b^d + \sigma_3 \delta_a^d \delta_b^c$$

$$\sigma_{sing} = (D-2)\sigma_1 + \sigma_2 + \sigma_3$$

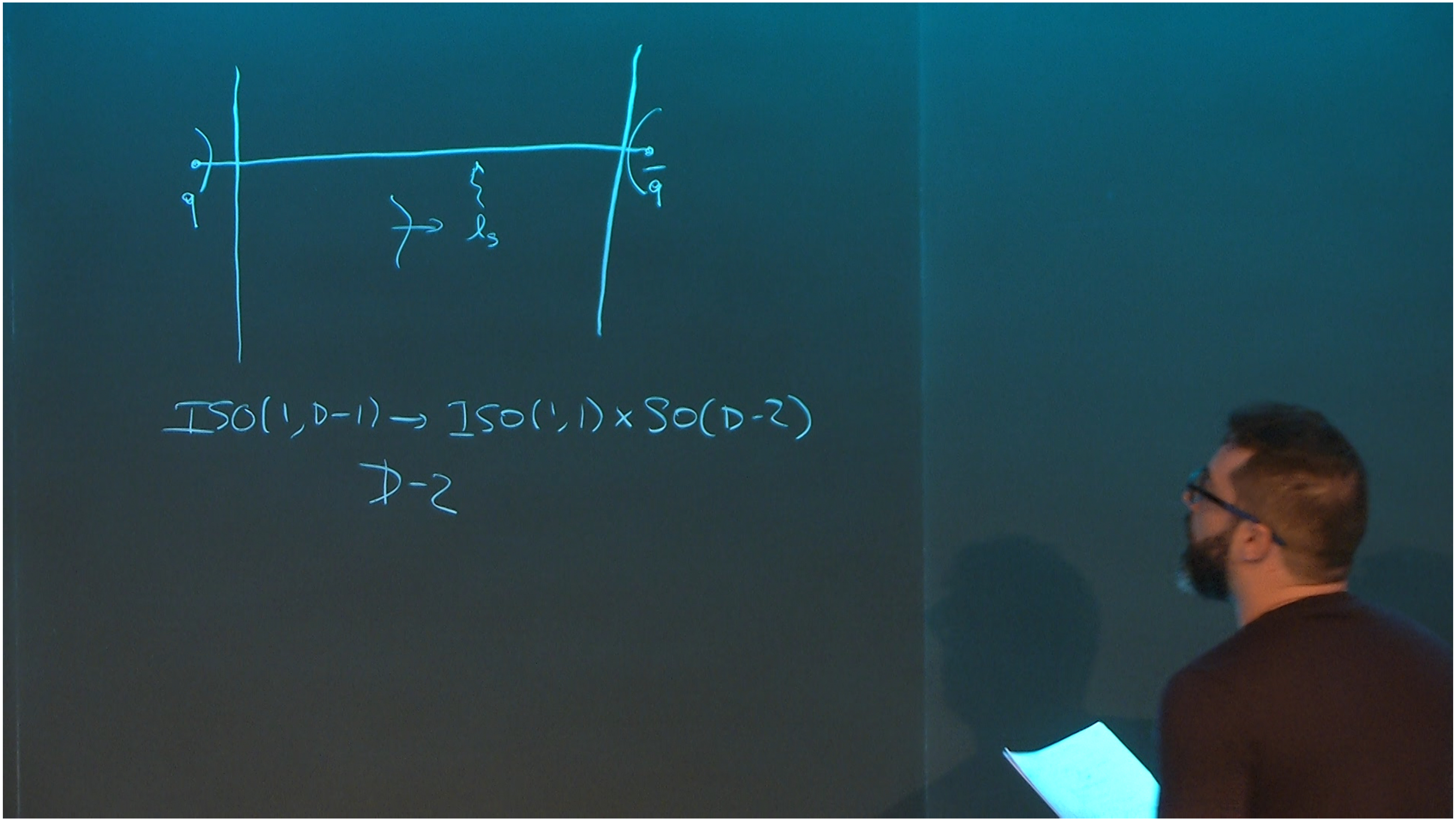
$$\sigma_{anti} = \sigma_2 - \sigma_3$$

$$\sigma_{sym} = \sigma_2 + \sigma_3$$

$$\sigma_{rep} = e^{2i\delta_{rep}(s)}$$

$$\boxed{\begin{aligned}
2\delta_{sym}(s) &= s/4 + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4) \\
2\delta_{anti}(s) &= s/4 - \alpha_2 s^2 + (\alpha_3 + 2\beta_3) s^3 + O(s^4) \\
2\delta_{sing} &= s/4 - (D-3)\alpha_2 s^2 + \\
&\quad (\alpha_3 - (D-2)\beta_3) s^3 + O(s^4)
\end{aligned}}$$

- What we know
- Bootstrap Setup
  - $\times$ $D=3$ analytically
  - $\times$ $D=4$ mostly numerics
- Energy Bounds
- Resonances


q
q̄
l_s
ISO(1,D-1) → ISO(1,1) × SO(D-2)
D-2

$$A = \int d^2\sigma \sqrt{-h}\left(\frac{1}{\ell_s^2} + \cancel{R} + \cancel{K^2} + \ell_s^2 K^4 + \dots\right)$$

universal

$\partial^n X^n$ $\qquad$ $\partial^{n+2} X^n$ $\qquad$ $\partial^{n+4} X^n$

$$h = \det\left(\partial_\alpha X^\mu \partial_\beta X_\mu\right)$$

$$K^\mu_{\alpha\beta} = \nabla_\alpha \partial_\beta X^\mu$$

$$\alpha_3 (K^2)^2 + \beta_3 (K^\mu K^\nu)^2$$

$(D-2)$

$$S_{ab}^{cd} = \sigma_1 \delta_{ab} \delta^{cd} + \dots$$

$$\sigma_{sing} = (D-2)\sigma_1 + \dots$$

$$\sigma_{anti} = \sigma_2 - \sigma_3$$

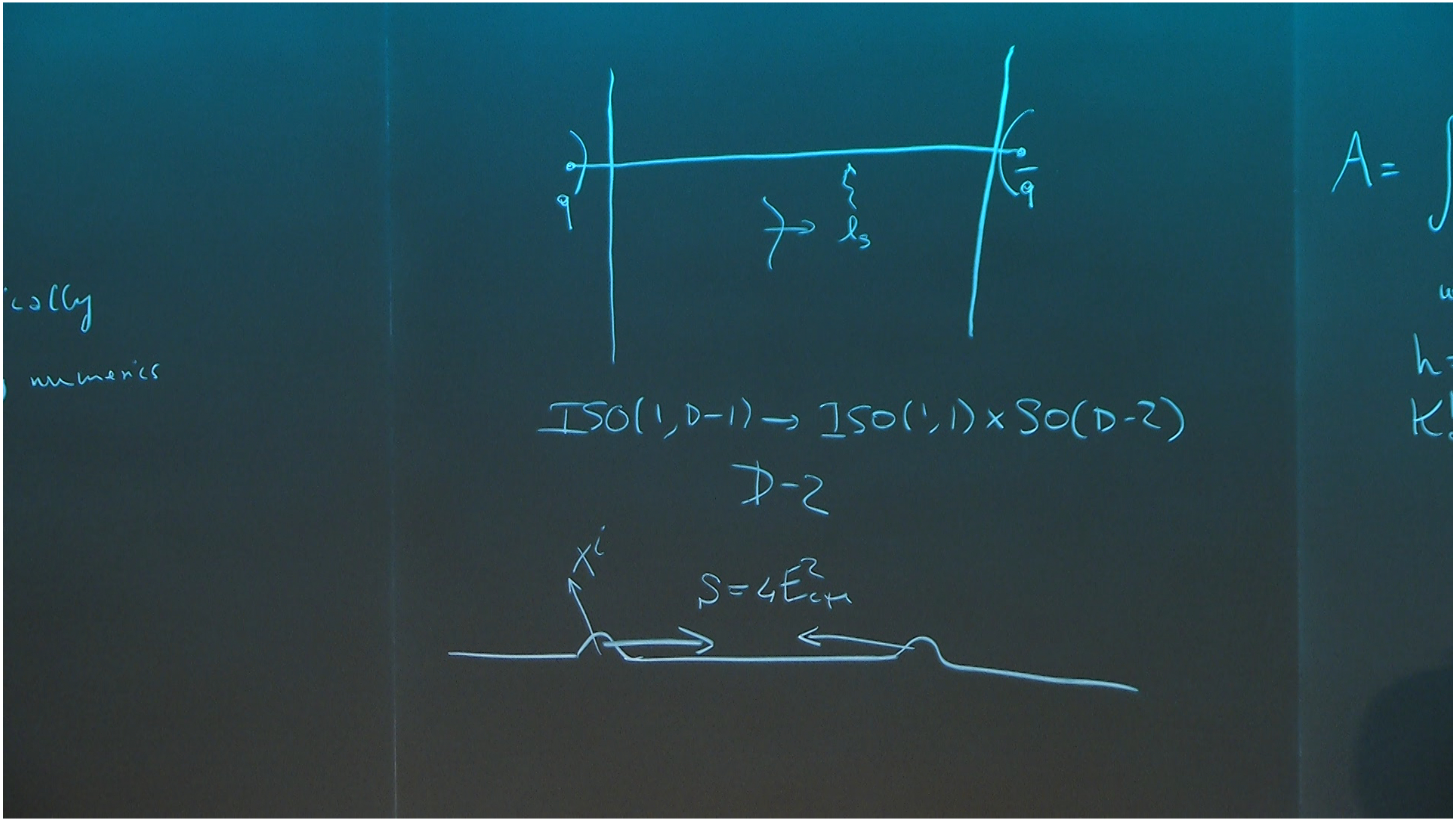
$$\sigma_{sym} = \sigma_2 + \sigma_3$$

$$2\delta_{sym}(s) = \frac{s}{4} + \alpha \dots$$

$$2\delta_{anti}(s) = \frac{s}{4} - \dots$$

$$2\delta_{sing} = \frac{s}{4} - (D-3)\dots$$

$(\alpha \dots$


ically
numerics
q
q̄
ℓ_s
A =
ISO(1,D-1) → ISO(1,1) × SO(D-2)
D-2
X^i
s = 4E^2_cm

$$A = \int d^2\sigma \sqrt{-h}\left(\frac{1}{\ell_s^2} + \cancel{R} + \cancel{K^2} + \ell_s^2 K^4 + \dots\right)$$

universal $\quad \partial^n X^n \qquad \partial^{n+2} X^n \qquad \partial^{n+4} X^n$

$$h = \det\left(\partial_\alpha X^\mu \partial_\beta X_\mu\right)$$

$$K^\mu_{\alpha\beta} = \nabla_\alpha \partial_\beta X^\mu$$

$$\alpha_3 (K^2)^2 + \beta_3 (K^\mu K^\nu)^2$$

$$\alpha_2 = \frac{D-26}{384\pi}$$

$$S_{ab}^{cd} = \sigma_1 \delta_{ab}\delta^{cd} + \sigma_2 \delta_a^c \delta_b^d + \sigma_3 \delta_a^d \delta_b^c$$

$$\sigma_{sing} = (D-2)\sigma_1 + \sigma_2 + \sigma_3$$

$$\sigma_{anti} = \sigma_2 - \sigma_3$$

$$\sigma_{sym} = \sigma_2 + \sigma_3$$

$$\sigma_{rep} = e^{2i\delta_{rep}(s)}$$

$$\boxed{\begin{aligned} 2\delta_{sym}(s) &= \tfrac{s}{4} + \alpha_2 s^2 + \alpha_3 s^3 + O(s^4) \\ 2\delta_{anti}(s) &= \tfrac{s}{4} - \alpha_2 s^2 + (\alpha_3 + 2\beta_3) s^3 + O(s^4) \\ 2\delta_{sing} &= \tfrac{s}{4} - (D-3)\alpha_2 s^2 + \\ &\quad (\alpha_3 - (D-2)\beta_3) s^3 + O(s^4) \end{aligned}}$$

$$A = \int d^2\sigma \sqrt{-h}\left(\frac{1}{\ell_s^2} + \cancel{R} + \cancel{K^2} + \ell_s^2 K^4 + \dots\right)$$

universal $\quad \partial^n X^n \quad \partial^{n+2} X^n \quad \partial^{n+4} X^n$

$$h = \det\left(\partial_\alpha X^\mu \partial_\beta X_\mu\right)$$

$$K^\mu_{\alpha\beta} = \nabla_\alpha \partial_\beta X^\mu$$

$$\alpha_3 (K^2)^2 + \beta_3 (K^\mu K^\nu)^2$$

$$\alpha_2 = \frac{D-26}{384\pi}$$

$$\mathrm{Im}\,\delta = O(s^6)$$

$$S_{ab}^{cd} = \sigma_1 \delta_{ab}\delta^{cd} + \sigma_2 \delta_a^c \delta_b^d + \sigma_3 \delta_a^d \delta_b^c$$

$$\sigma_{sing} = (D-2)\sigma_1 + \sigma_2 + \sigma_3$$

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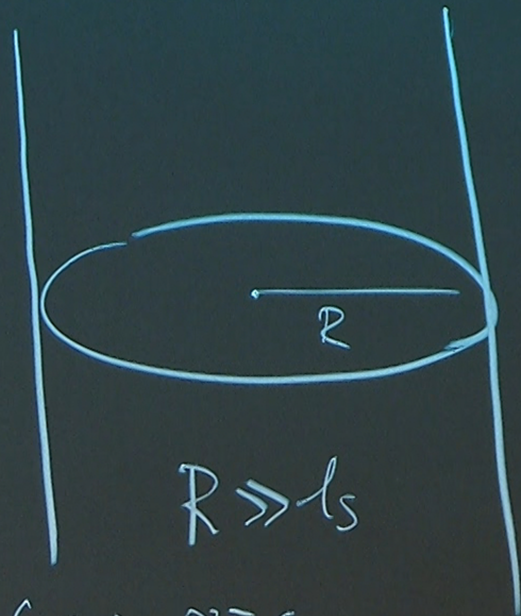
$-2)$

$$K^{\mu}_{\alpha\beta} = \nabla_\alpha \partial_\beta X^\mu$$

$$\alpha_2 = \frac{D-26}{384\pi}$$



$$\mathrm{Im}\,\delta = O(s^9)$$

$$E_0(R) = \sqrt{R^2 - \frac{\pi}{3}(D-2)} + \frac{\delta(D)}{R^7} + O\left(\frac{1}{R^9}\right)$$

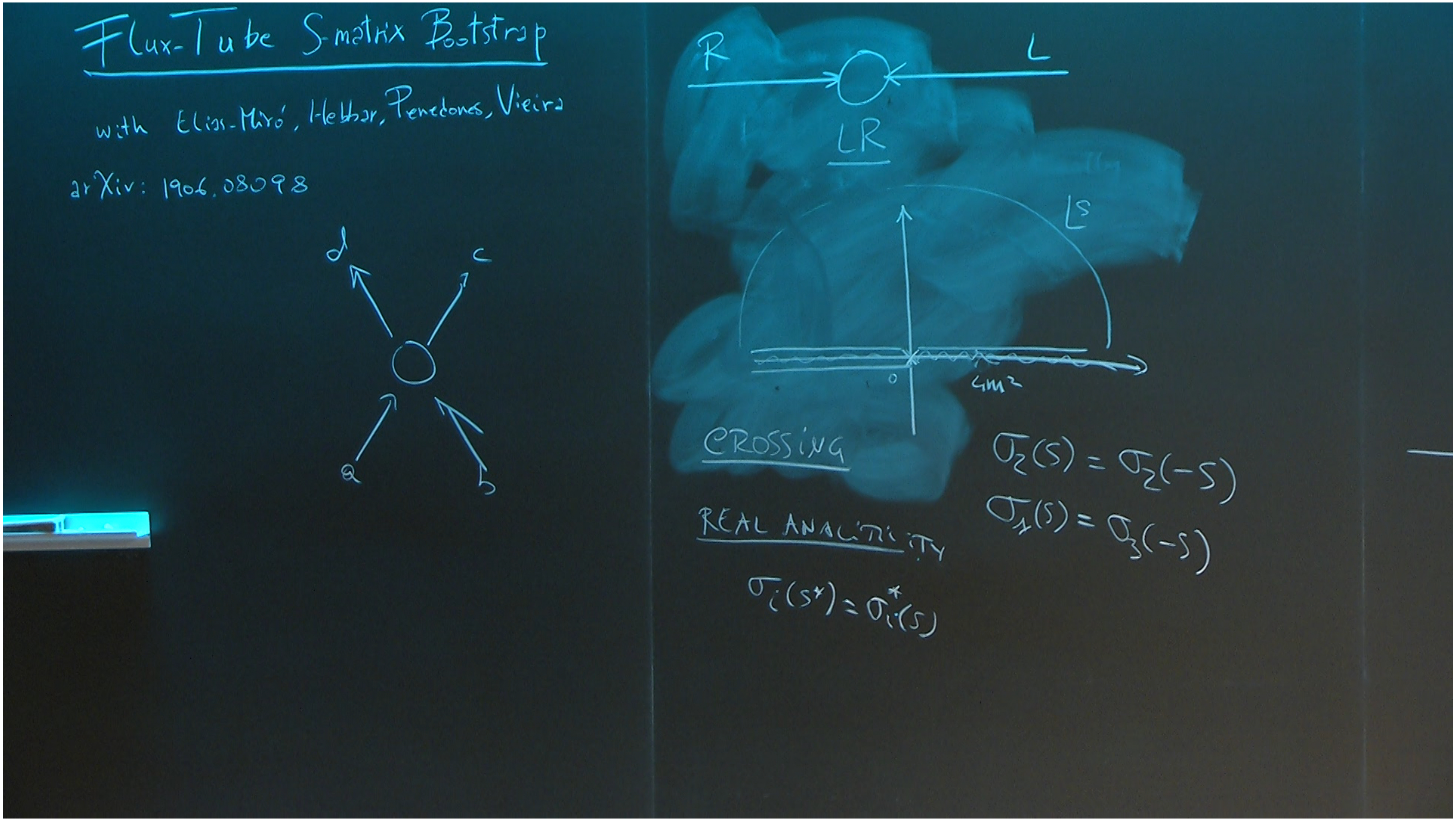
$$\delta(D) = -\frac{32\pi^6}{225}(12-D) \times$$

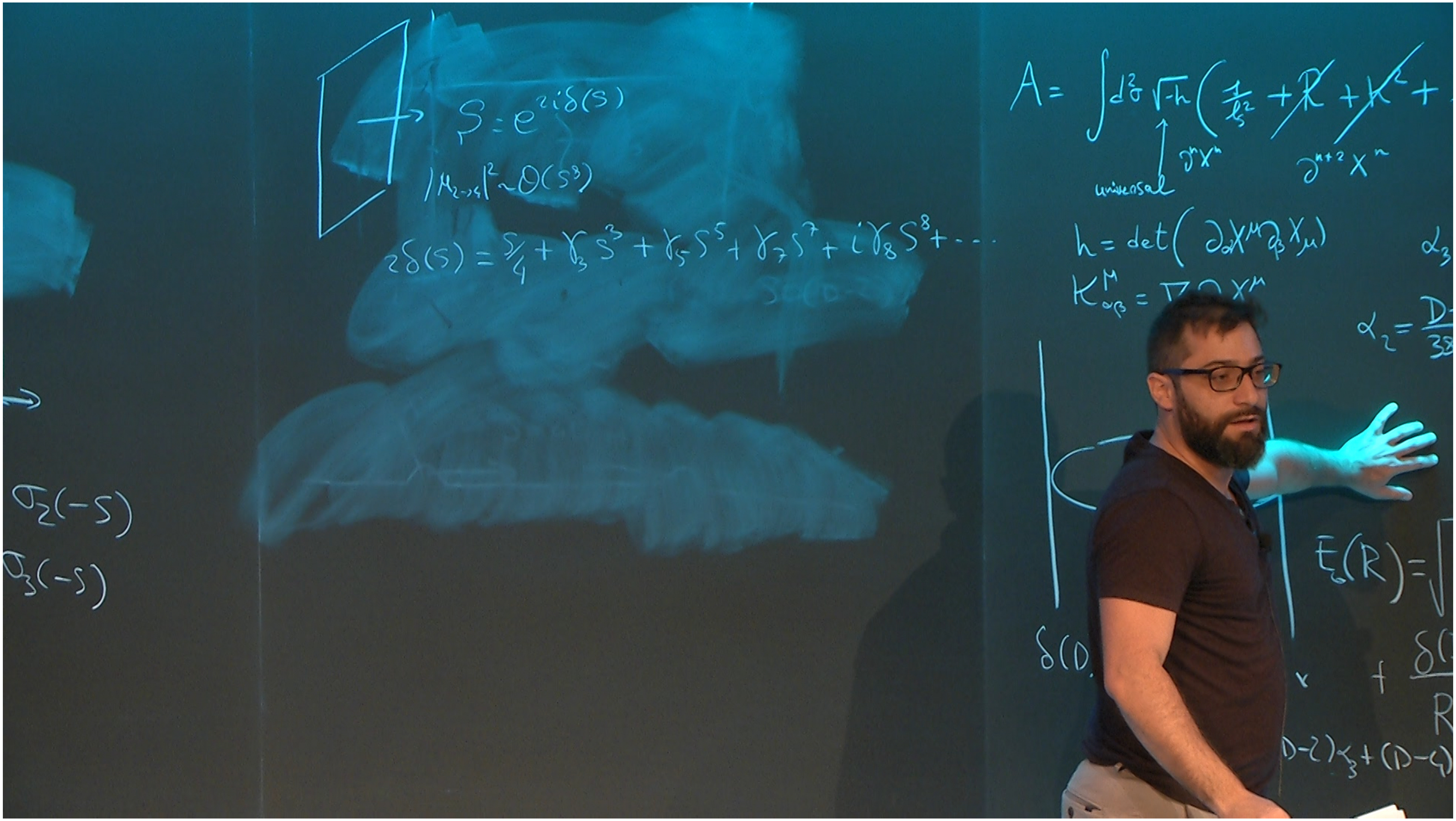
$$\times\left((D-2)\alpha_3 + (D-4)\beta_3\right)$$

$$2\delta_{sym}(s) =$$

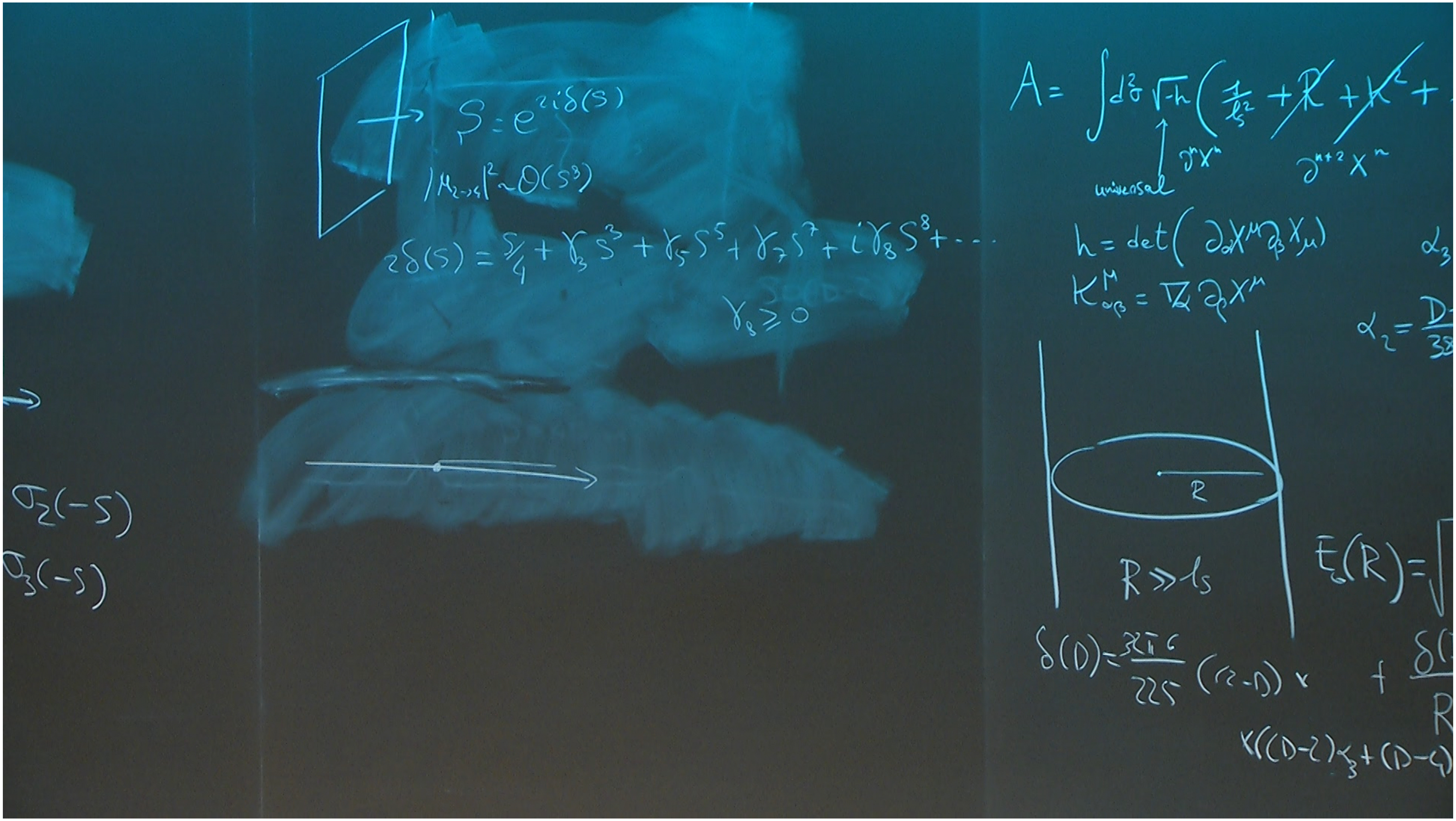
$$2\delta_{anni}(s)$$

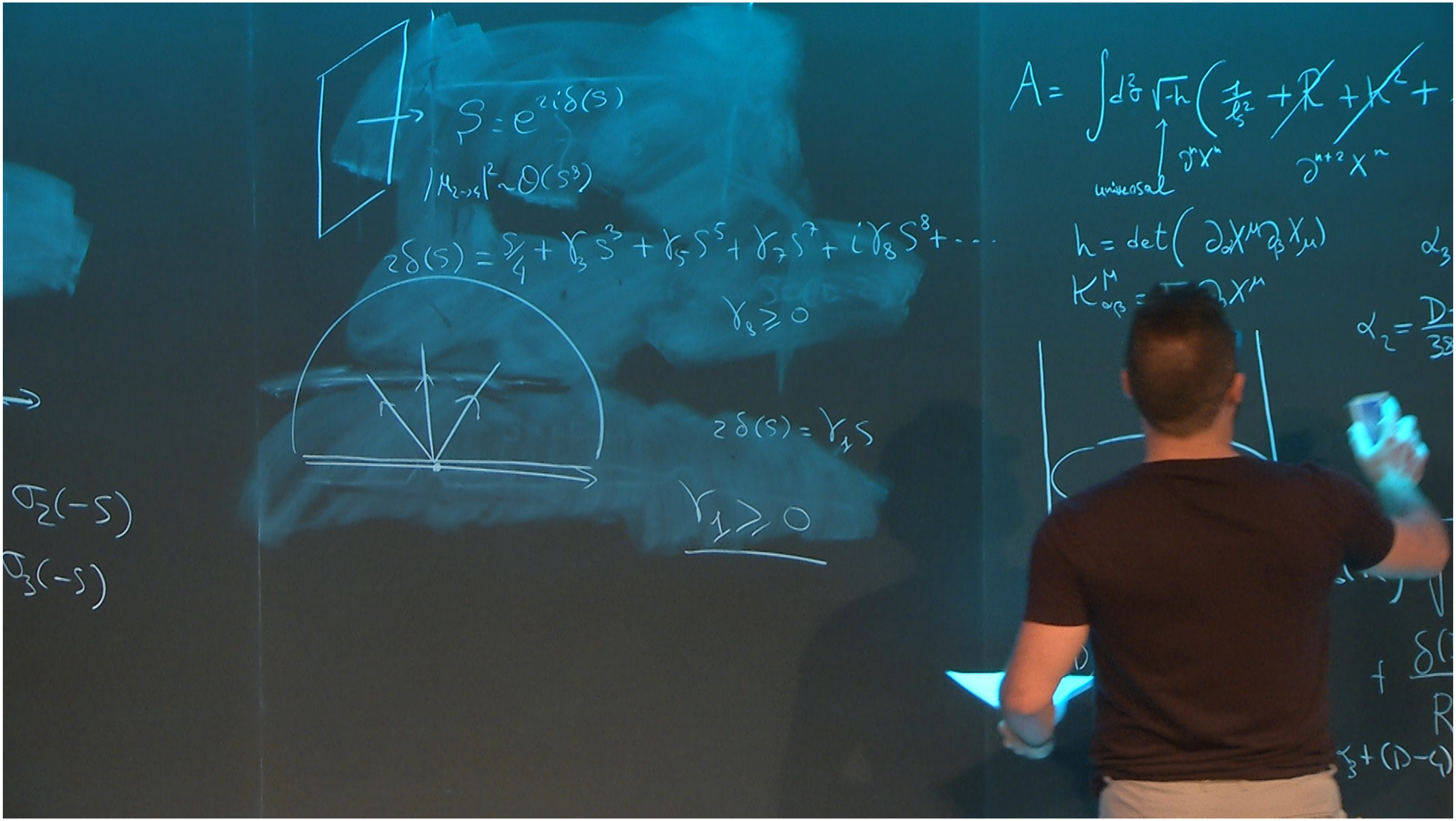
$$2\delta_{sing} = s/4$$

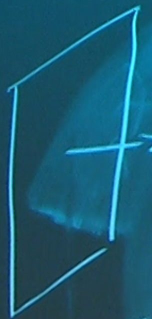

Flux-Tube S-matrix Bootstrap
with Elias-Miró, Hebbar, Penedones, Vieira
arXiv: 1906.08098
d
c
a
b
R
L
LR
s
0
$4m^2$
CROSSING
$\sigma_2(s) = \sigma_2(-s)$
$\sigma_1(s) = \sigma_3(-s)$
REAL ANALYTICITY
$\sigma_i(s^*) = \sigma_i^*(s)$



universal


$S = e^{2i\delta(s)}$
$|M_{2\to 4}|^2 \sim O(s^8)$
$2\delta(s) = s/4 + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 + i\gamma_8 s^8 + \dots$
$\gamma_8 \geq 0$
$\sigma_2(-s)$
$\sigma_3(-s)$
$A = \int d^2\sigma \sqrt{-h}\left(\frac{1}{\ell_s^2} + \cancel{R} + \cancel{K^2} + \right.$
universal
$\partial^n X^n$
$\partial^{n+2} X^n$
$h = \det(\partial_\alpha X^\mu \partial_\beta X_\mu)$
$K^\mu_{\alpha\beta} = \nabla_\alpha \partial_\beta X^\mu$
R
$R \gg \ell_s$
$E_0(R) =$
$\delta(D) = \frac{3\pi^6}{225}((2-D)\times$

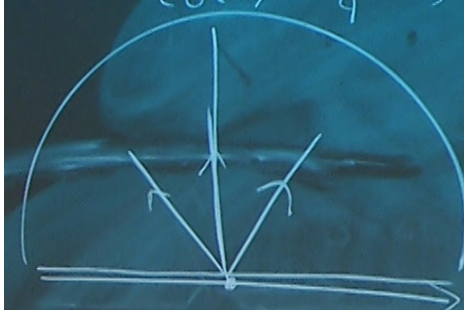

$S = e^{2i\delta(s)}$
$|M_{2\to 4}|^2 \sim O(s^8)$
$2\delta(s) = \frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 + i\gamma_8 s^8 + \dots$
$\gamma_8 \geq 0$
$2\delta(s) = \gamma_1 s$
$\gamma_1 \geq 0$
$\sigma_2(-s)$
$\sigma_3(-s)$
$A = \int d^2\sigma \sqrt{-h}\left(\frac{1}{\ell_s^2} + R + K^2 + \right.$
universal
$\partial^n X^n$
$\partial^{n+2} X^n$
$h = \det(\partial_\alpha X^\mu \partial_\beta X_\mu)$
$K^\mu_{\alpha\beta}$
α_3
$\alpha_2 =$



$S = e^{2i\delta(s)}$

$|M_{2\to 4}|^2 \sim O(s^3)$

$2\delta(s) = \frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 + i\gamma_8 s^8 + \dots$



$\gamma_8 \geq 0$

$2\delta(s) = \gamma_1 s$

$\underline{\gamma_1 \geq 0}$

$S^{(1)}(z|w) = \frac{S(z) - S(w)}{1 - S(z)\bar{S}(w)} \Big/ \frac{z-w}{z-\bar{w}}$

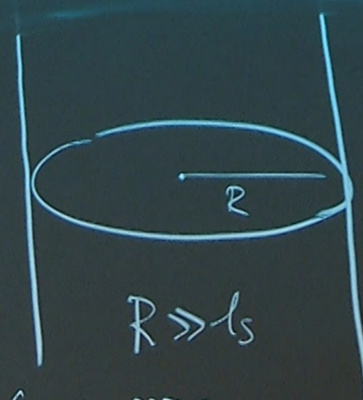
$|w| < 1$

a) holomorphic in $\mathbb{H}$

b) $|S^{(1)}(z|w)| \leq 1$

$S^{(1)}(ix|iy) = -1 + \left(\frac{1}{96} + 8\gamma_3\right) xy + \dots \geq -1$

$\boxed{\gamma_3 \geq -\frac{1}{768}}$

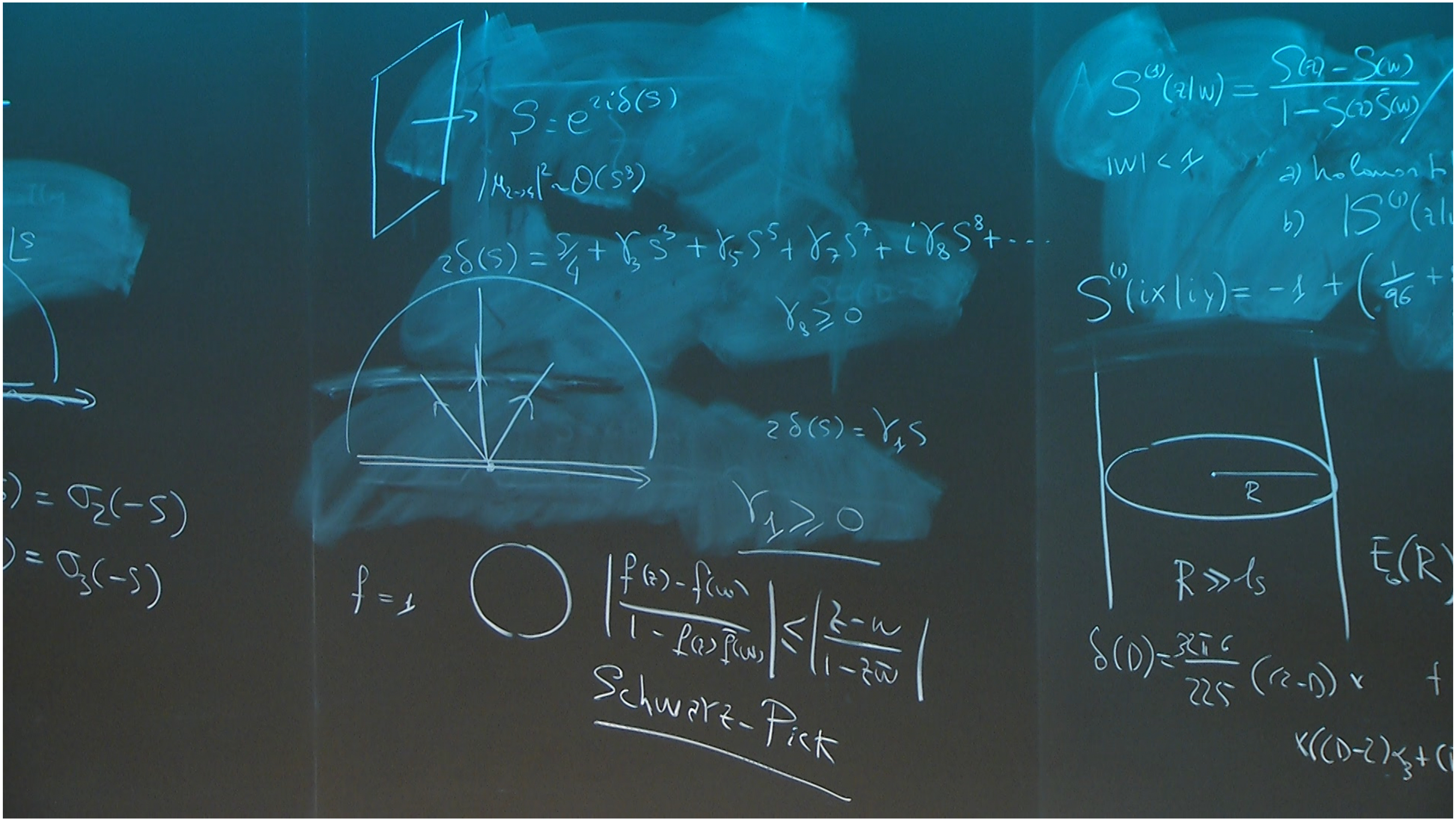


$R \gg l_s$

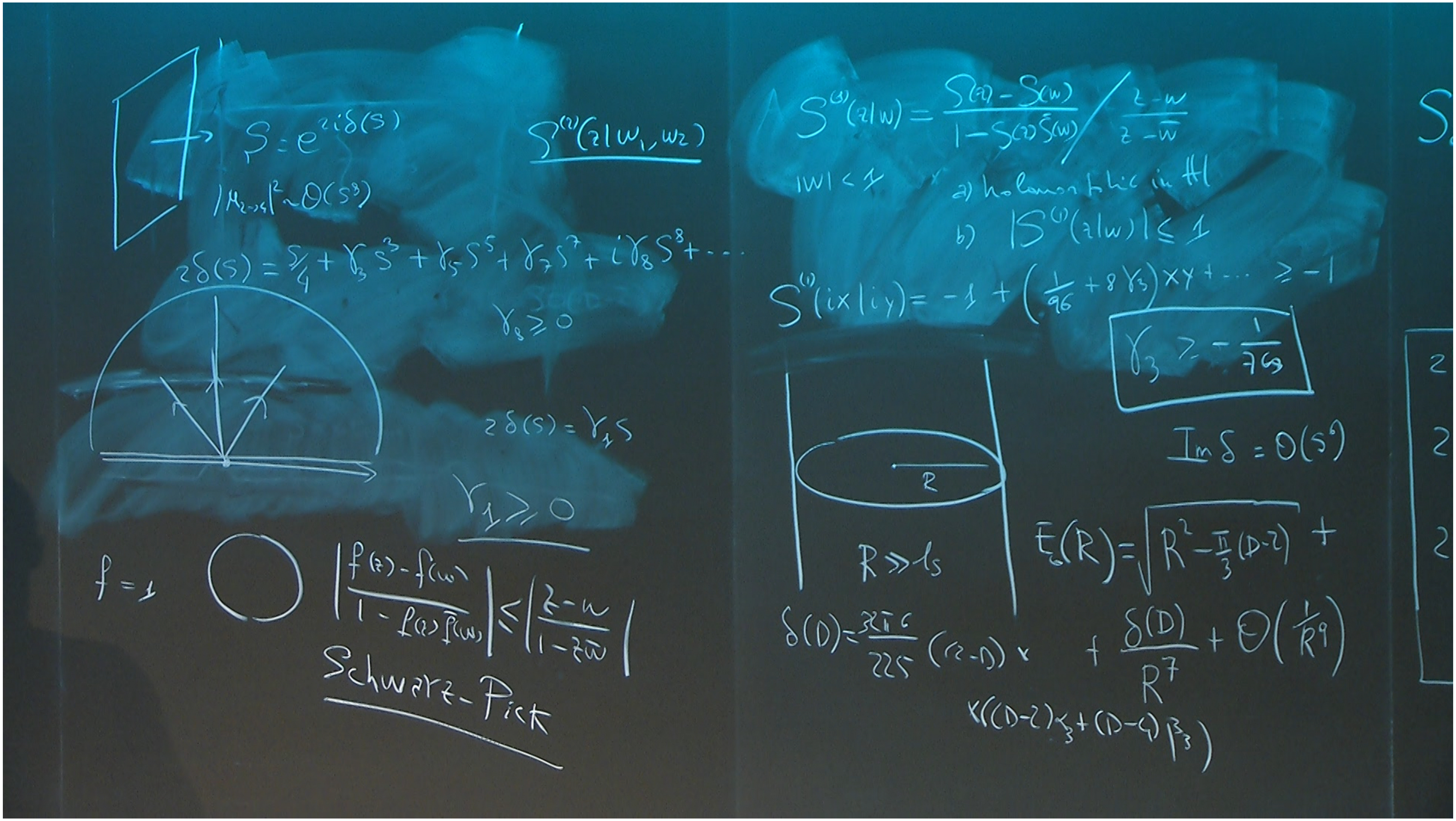
$\mathrm{Im}\,\delta = O(s^9)$

$E_0(R) = \sqrt{R^2 - \frac{\pi}{3}(D-2)} + \frac{\delta(D)}{R^7} + O\left(\frac{1}{R^9}\right)$

$\delta(D) = -\frac{32\pi^6}{225}(26-D) \times ((D-2)\gamma_3 + (D-4)\beta_3)$



$S = e^{2i\delta(s)}$
$|M_{2\to4}|^2 \sim O(s^3)$
$2\delta(s) = \frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 + i\gamma_8 s^8 + \dots$
$\gamma_8 \geq 0$
$2\delta(s) = \gamma_1 s$
$\gamma_1 \geq 0$
$f = 1$
$\left|\frac{f(z) - f(w)}{1 - f(z)\bar{f}(w)}\right| \leq \left|\frac{z-w}{1-z\bar{w}}\right|$
Schwarz-Pick
$S^{(1)}(z|w) = \frac{S(z) - S(w)}{1 - S(z)\bar{S}(w)}$
$|w| < 1$
$S^{(1)}(ix|iy) = -1 + \left(\frac{1}{96} + \right.$
R
$R \gg l_s$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$|M_{2\to 4}|^2 \sim O(s^8)$
$S^{(2)}(z|w_1, w_2)$
$2\delta(s) = \frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 + i\gamma_8 s^8 + \ldots$
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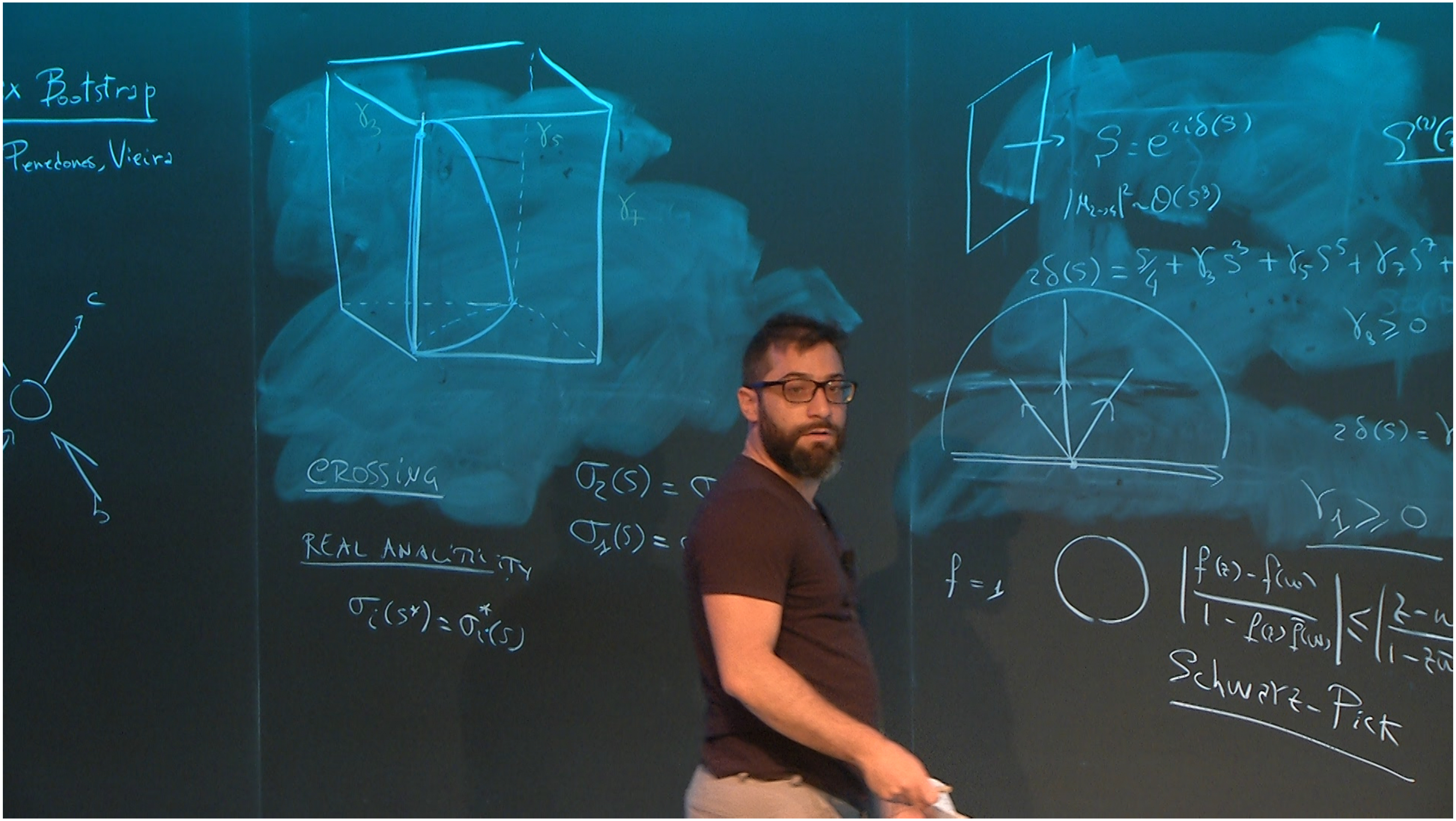
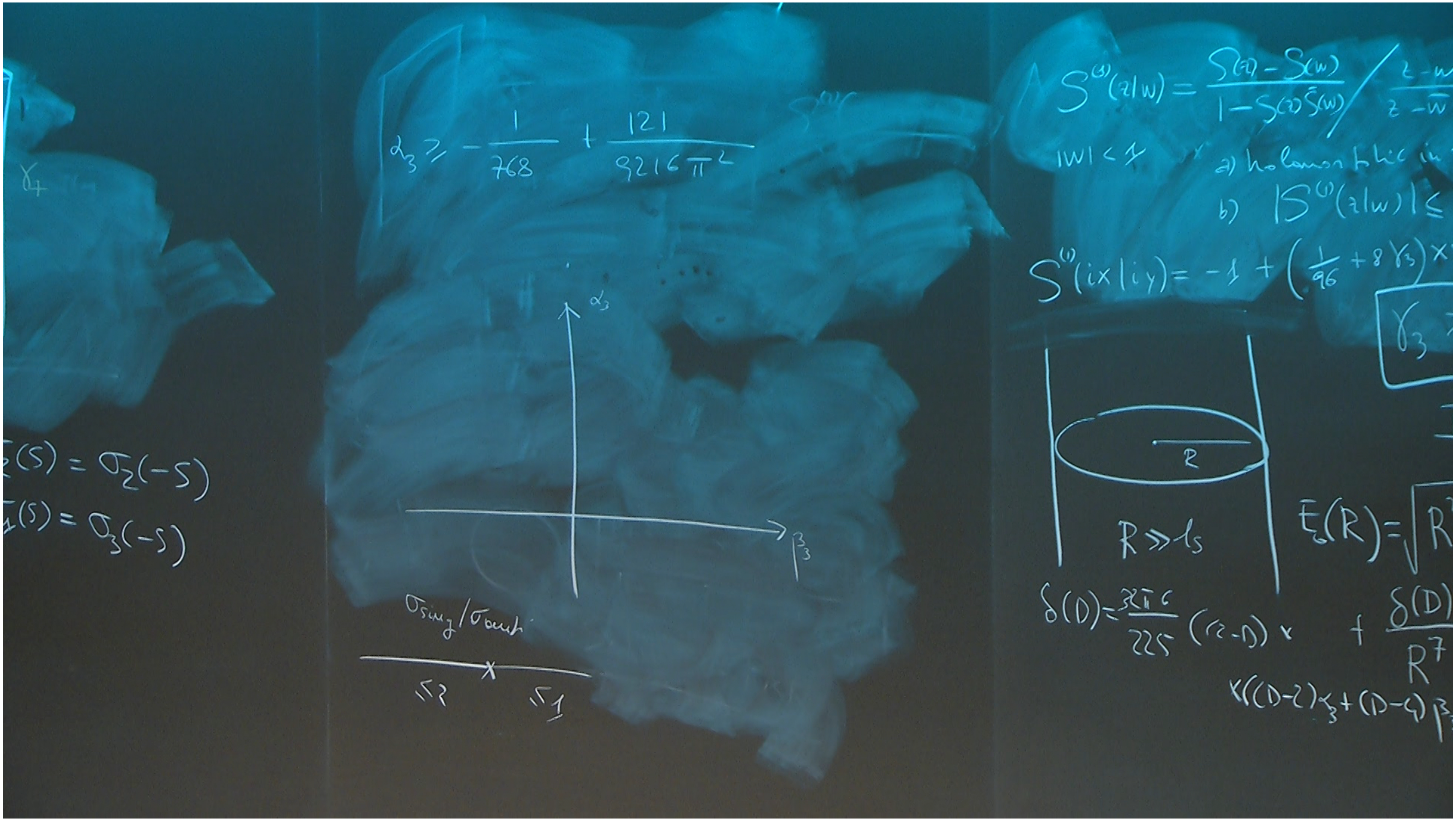
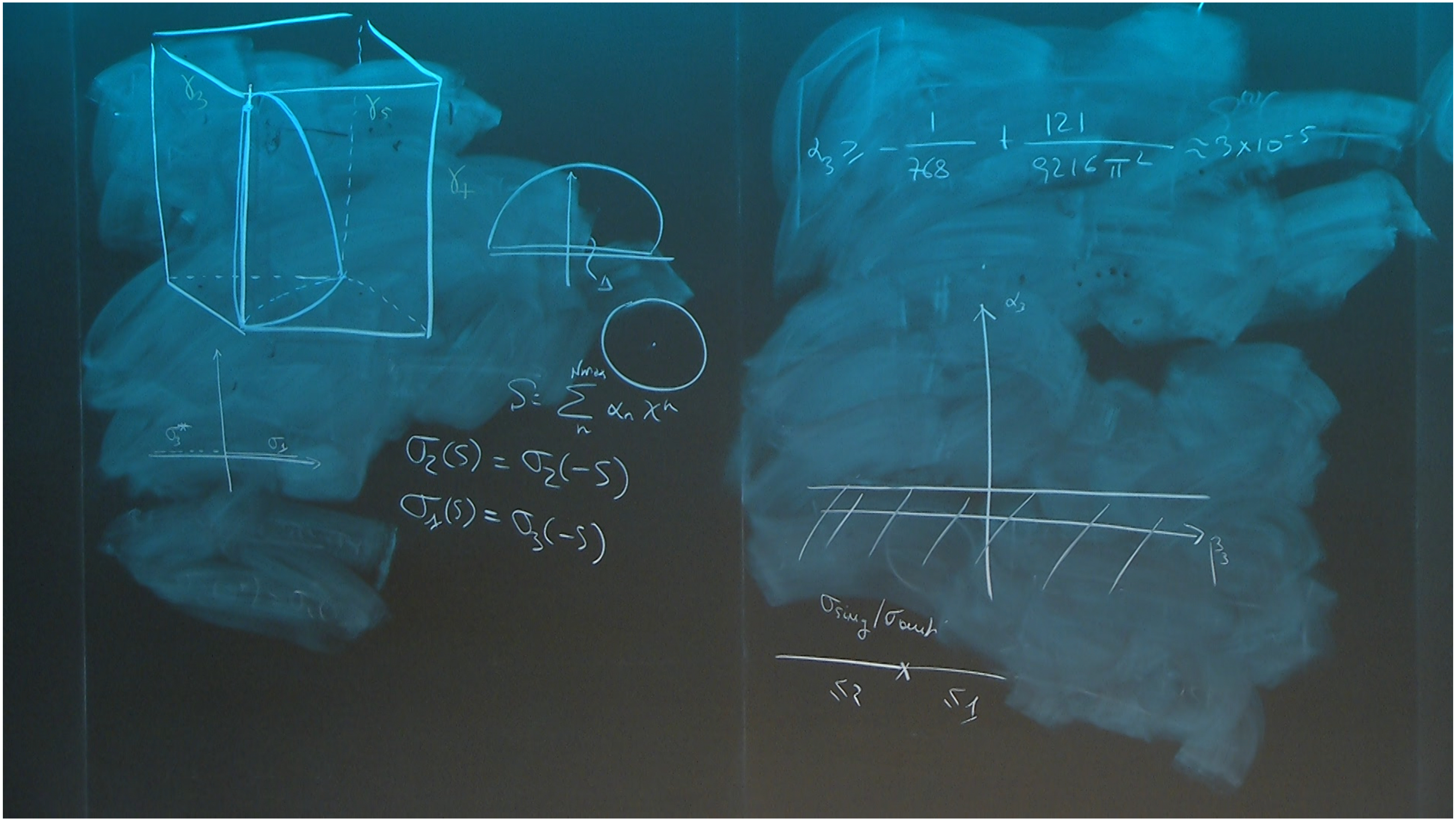
$S^{(1)}(ix|iy) = -1 + \left(\frac{1}{96} + 8\gamma_3\right) xy + \ldots \geq -1$
$\boxed{\gamma_3 \geq -\frac{1}{768}}$
$\mathrm{Im}\, \delta = O(s^8)$
R
$R \gg l_s$
$E_0(R) = \sqrt{R^2 - \frac{\pi}{3}(D-2)} +$
$\delta(D) = -\frac{32\pi^6}{225}(26-D) \times$
$+ \frac{\delta(D)}{R^7} + O\left(\frac{1}{R^9}\right)$
$\times((D-2)\gamma_3 + (D-4)\beta_3)$

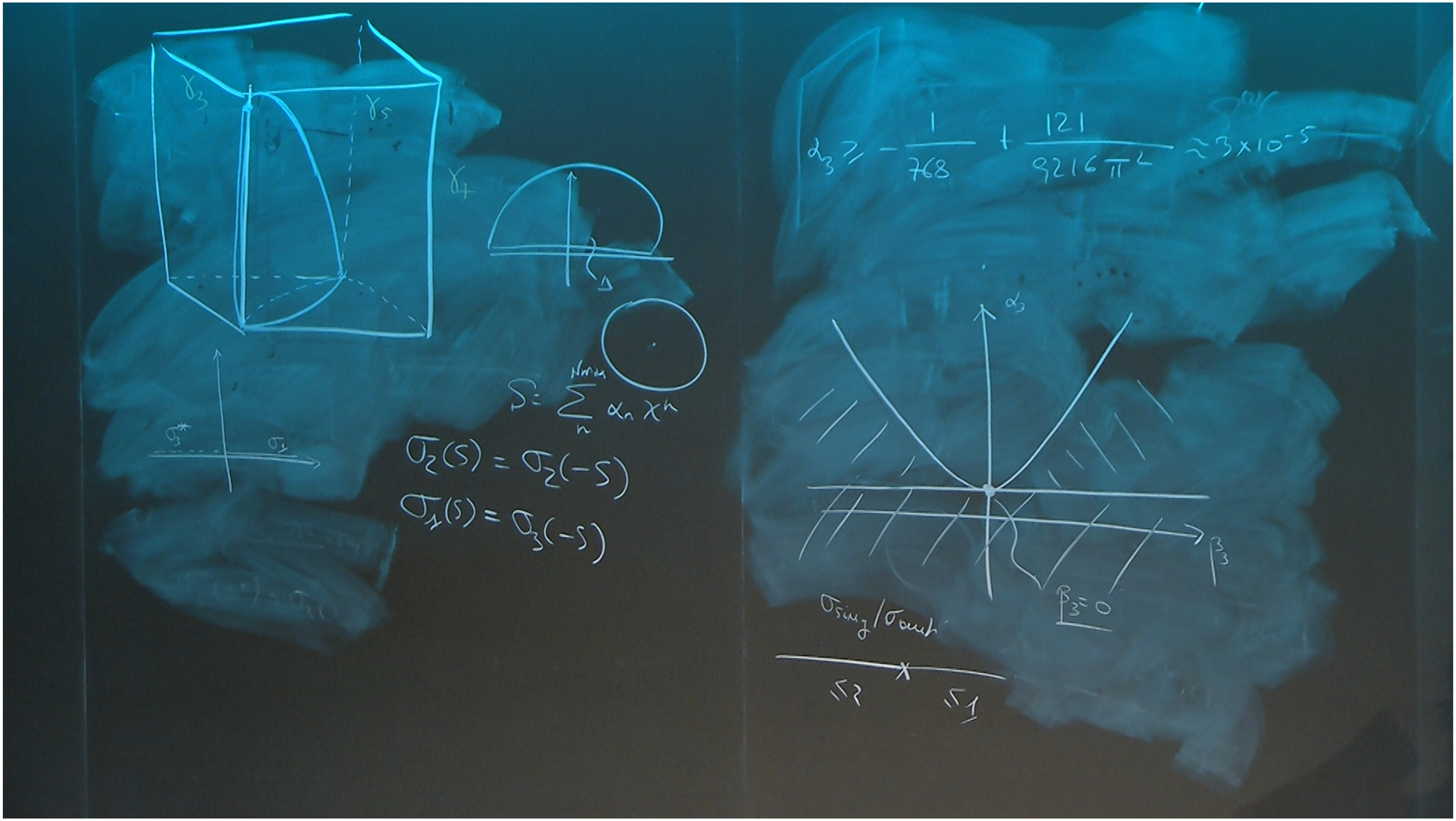


x Bootstrap
Penedones, Vieira
CROSSING
REAL ANALITICITY
$\sigma_i(s^*) = \sigma_i^*(s)$
$\sigma_2(s) =$
$\sigma_1(s) =$
$S = e^{2i\delta(s)}$
$|M_{2\to 4}|^2 \sim O(s^3)$
$2\delta(s) = \frac{s}{4} + \gamma_3 s^3 + \gamma_5 s^5 + \gamma_7 s^7 +$
$\gamma_3 \geq 0$
$2\delta(s) =$
$\gamma_1 \geq 0$
$f = 1$
$\left|\frac{f(z) - f(w)}{1 - f(z)\bar{f}(w)}\right| \leq \left|\frac{z - w}{1 - z\bar{w}}\right|$
Schwarz-Pick


α3 ≥ − 1/768 + 121/9216π²
S(1)(z|w) = (S(z) − S(w))/(1 − S(z)S̄(w))
|w| < 1
a) holomorphic in
R ≫ ls
δ(D) = 3π⁶/225 (12−D)



γ_3
γ_5
γ_4
Δ
$S = \sum_n^{N_{max}} \alpha_n x^n$
σ_3^*
σ_1
$\sigma_2(s) = \sigma_2(-s)$
$\sigma_1(s) = \sigma_3(-s)$
$\alpha_3 \geq -\frac{1}{768} + \frac{121}{9216\pi^2} \approx 3\times10^{-5}$
α_3
β_3
$\sigma_{sing}/\sigma_{out}$
s_2
s_1


γ_3
γ_5
γ_4
$S = \sum_n^{N_{max}} \alpha_n x^n$
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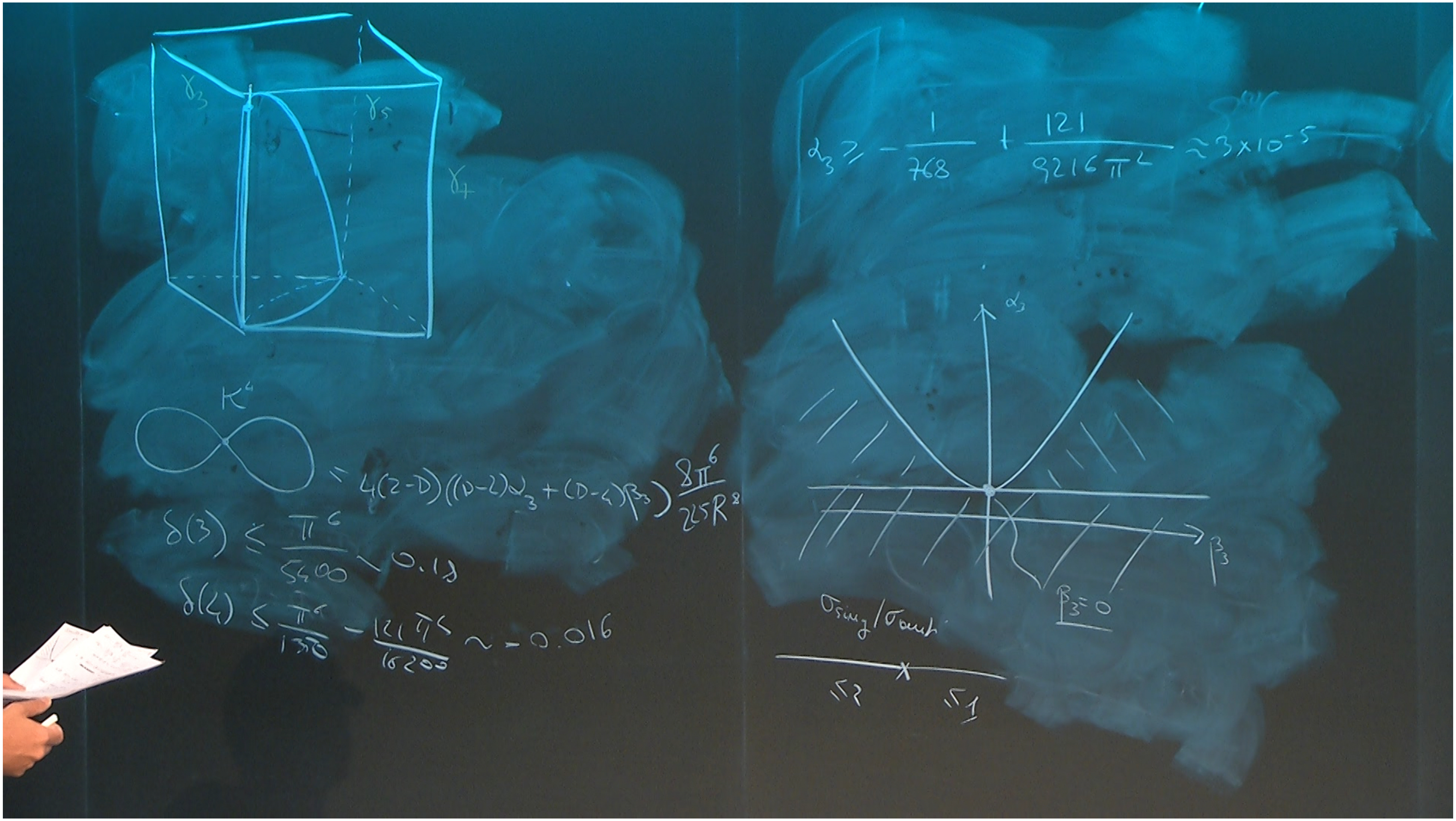
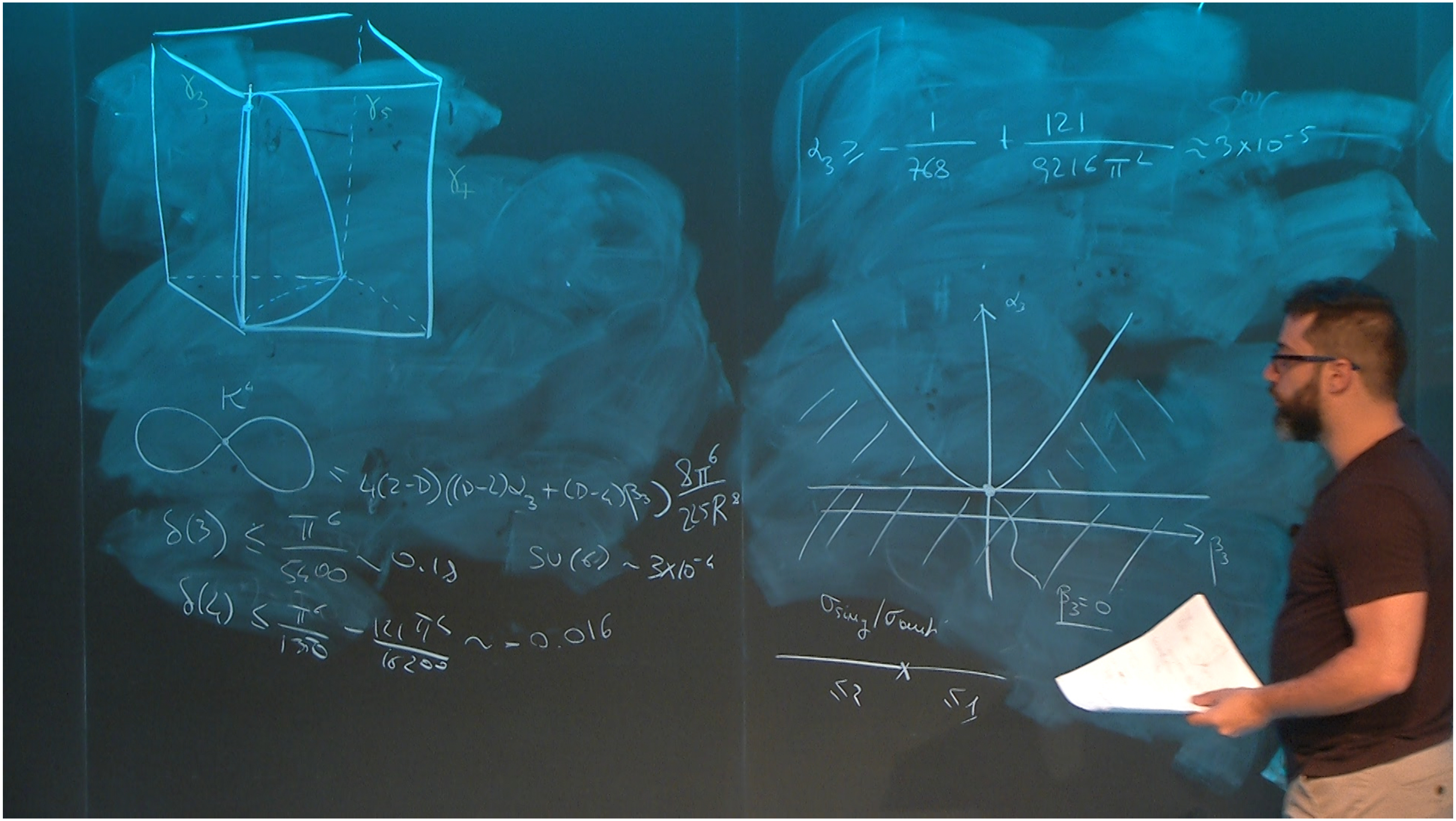
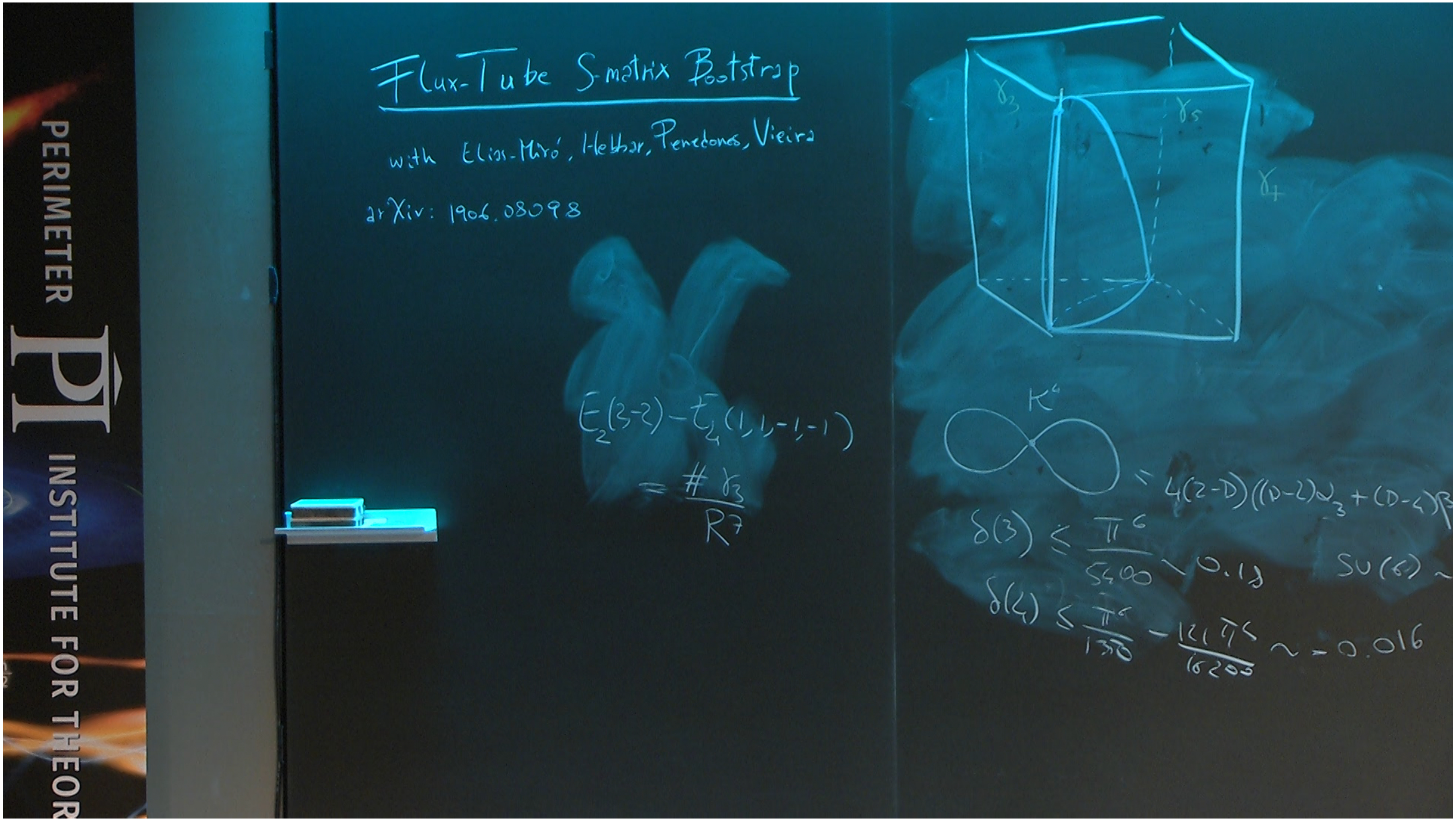
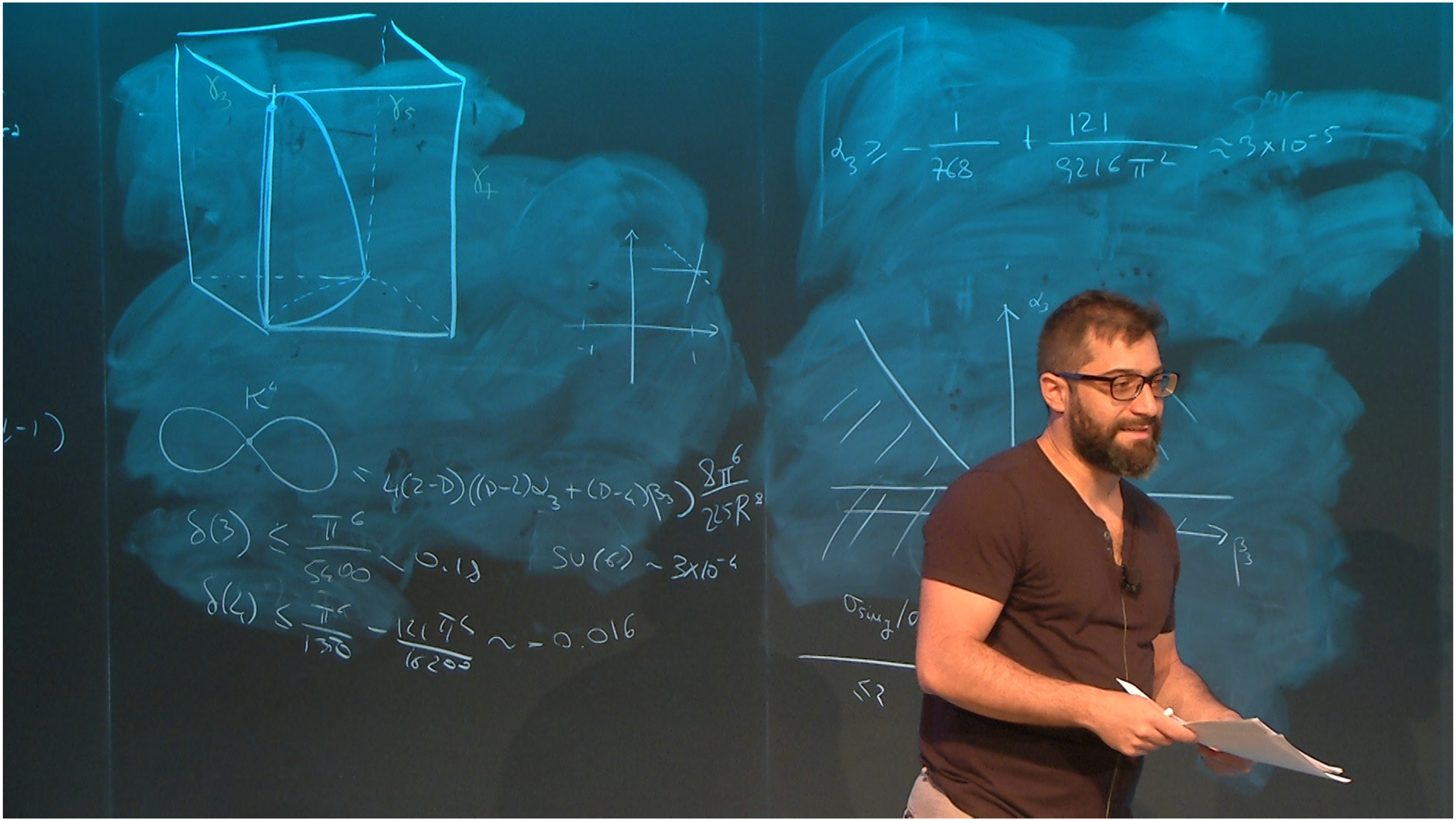
$\alpha_3 \geq -\frac{1}{768} + \frac{121}{9216\pi^2} \approx 3\times10^{-5}$
α_3
β_3
$\beta_3 = 0$
$\sigma_{sing}/\sigma_{cont}$


γ3
γ5
γ4
K4
= 4(2-D)((D-2)α3 + (D-4)β3) 8π^6 / 225R^8
δ(3) ≤ π^6 / 5400 ~ 0.18
δ(4) ≤ π^6 / 1350 - 121 π^6 / 16200 ~ -0.016
α3 ≳ - 1/768 + 121 / 9216π^2 ≈ 3×10^-5
α3
β3
β3 = 0
σsing/σauth
≤3
≤1



γ_3
γ_5
γ_4
K^4
$= 4(2-D)((D-2)\alpha_3 + (D-4)\beta_3)\frac{8\pi^6}{225R^8}$
$\delta(3) \leq \frac{\pi^6}{5400} \sim 0.18$
$SU(6) \sim 3\times10^{-4}$
$\delta(4) \leq \frac{\pi^6}{1350} - \frac{121\pi^6}{16200} \sim -0.016$
$\alpha_3 \geq -\frac{1}{768} + \frac{121}{9216\pi^2} \approx 3\times10^{-5}$
α_3
β_3
$\beta_3 = 0$
$\sigma_{sing}/\sigma_{...}$
$\lesssim 2$
$\lesssim 1$


Flux-Tube S-matrix Bootstrap
with Elias-Miró, Hebbar, Penedones, Vieira
arXiv: 1906.08098
PERIMETER
PI
INSTITUTE FOR THEOR
γ_3
γ_5
γ_7
≈ 0.18
SU(6)
~ -0.016


$\alpha_3 \geq -\frac{1}{768} + \frac{121}{9216\pi^2} \approx 3\times10^{-5}$
$= 4(2-D)((D-2)\alpha_3 + (D-4)\beta_3)\frac{8\pi^6}{225R^8}$
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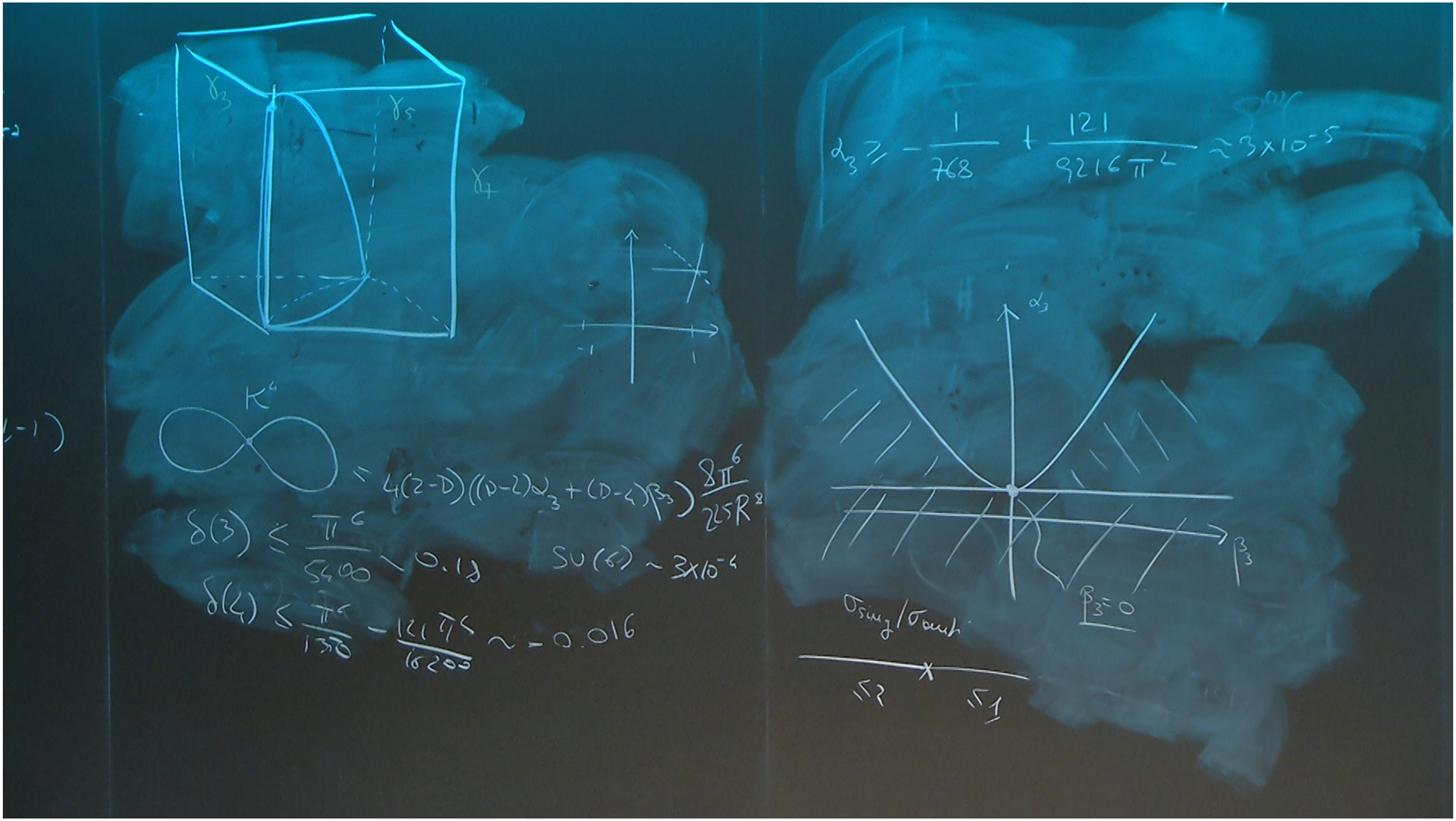
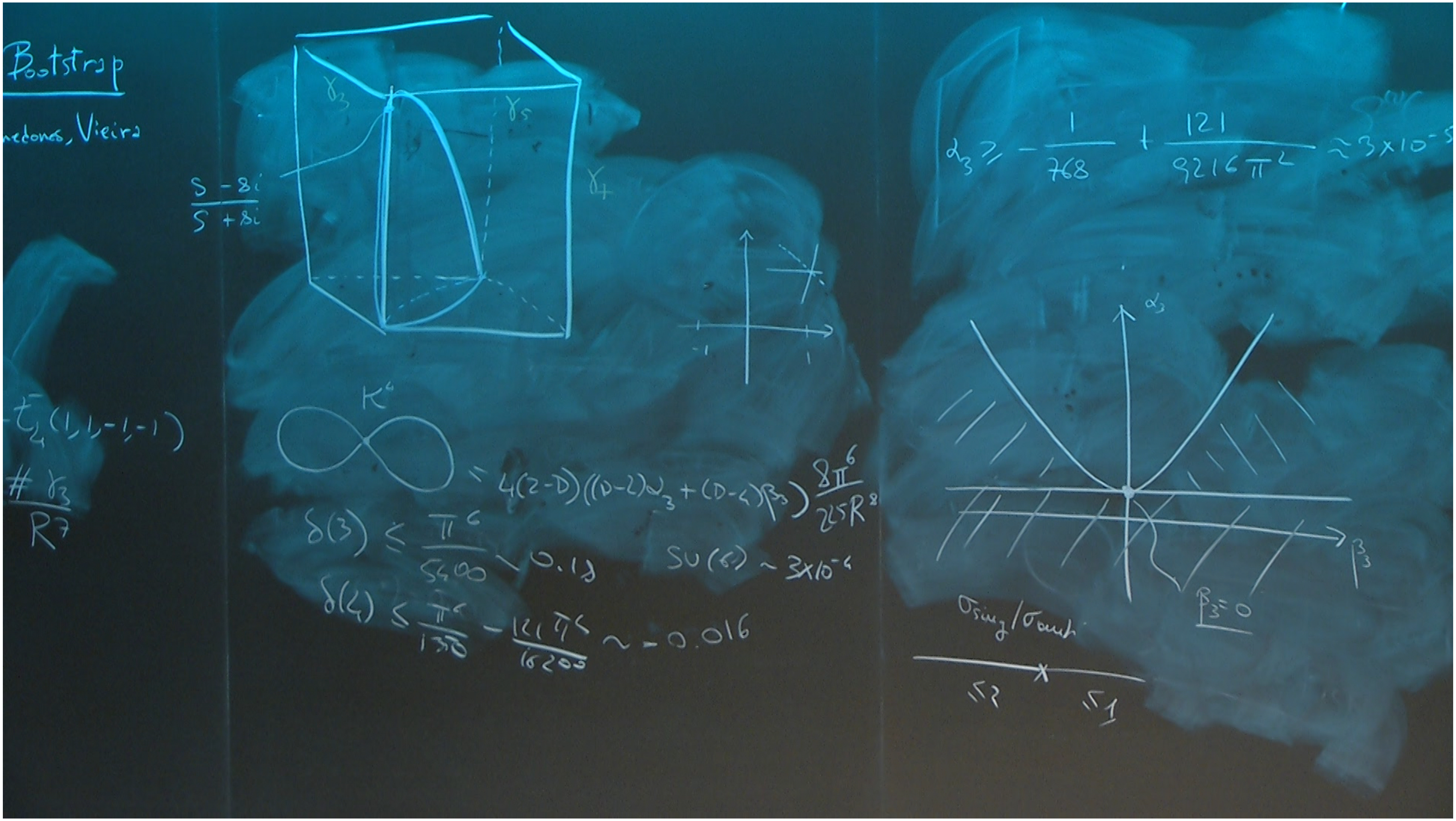
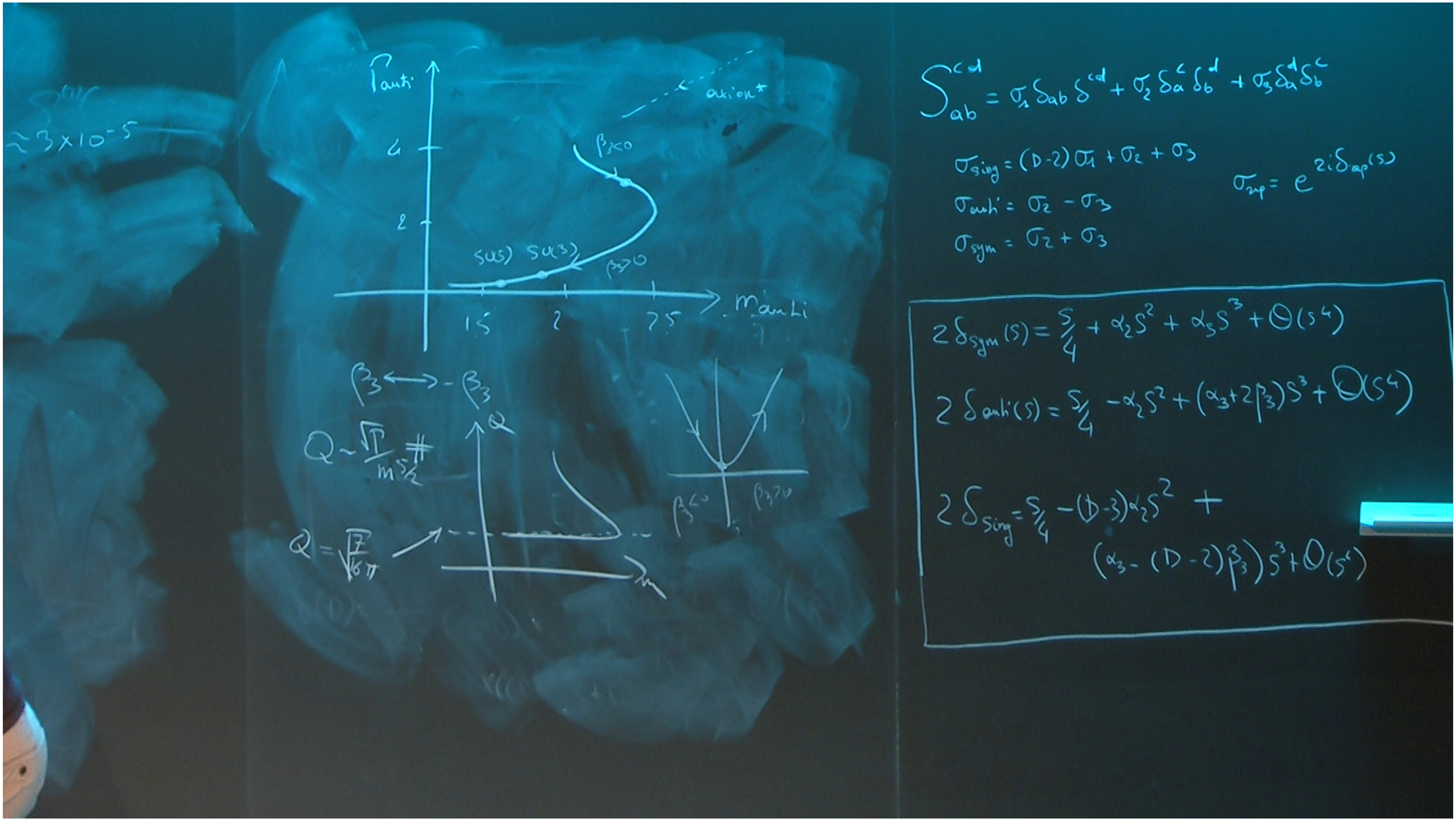


γ_3
γ_5
γ_4
K^4
$= 4(2-D)((D-2)\alpha_3 + (D-4)\beta_3)\frac{8\pi^6}{225R^8}$
$\delta(3) \le \frac{\pi^6}{5400} \sim 0.18$
$SU(6) \sim 3\times10^{-6}$
$\delta(4) \le \frac{\pi^6}{1350} - \frac{121\pi^6}{16200} \sim -0.016$
$\alpha_3 \ge -\frac{1}{768} + \frac{121}{9216\pi^2} \approx 3\times10^{-5}$
α_3
β_3
$\beta_3 = 0$
$\sigma_{sing}/\sigma_{conti}$
$\lesssim 2$
$\lesssim 1$


Bootstrap
Vieira
$\frac{S-8i}{S+8i}$
γ_3
γ_5
γ_4
$\alpha_3 \geq -\frac{1}{768} + \frac{121}{9216\pi^2} \approx 3\times10^{-5}$
R^4
$= 4(2-D)((D-2)\alpha_3 + (D-4)\beta_3)\frac{8\pi^6}{225R^8}$
$\frac{\#\gamma_3}{R^7}$
$\delta(3) \leq \frac{\pi^6}{5400} \sim 0.18$
$SU(6) \sim 3\times10^{-4}$
$\delta(4) \leq \frac{\pi^6}{1350} - \frac{121\pi^4}{16200} \sim -0.016$
α_3
β_3
$\beta_3 = 0$
$\sigma_{sing}/\sigma_{tot}$
$\lesssim 1$


S_ab^cd = σ1 δab δ^cd + σ2 δa^c δb^d + σ3 δa^d δb^c
σsing = (D-2) σ1 + σ2 + σ3
σanti = σ2 - σ3
σsym = σ2 + σ3
σrep = e^{2iδrep(s)}
2δsym(s) = s/4 + α2 s^2 + α3 s^3 + O(s^4)
2δanti(s) = s/4 - α2 s^2 + (α3 + 2β3) s^3 + O(s^4)
2δsing = s/4 - (D-3) α2 s^2 + (α3 - (D-2) β3) s^3 + O(s^4)
axion*
β3 ↔ -β3
Q = √7/√(6π)
≈ 3×10^-5