

Title: Quantum Error Correction via Hamiltonian Learning

Speakers: Eliska Greplova

Collection: Machine Learning for Quantum Design

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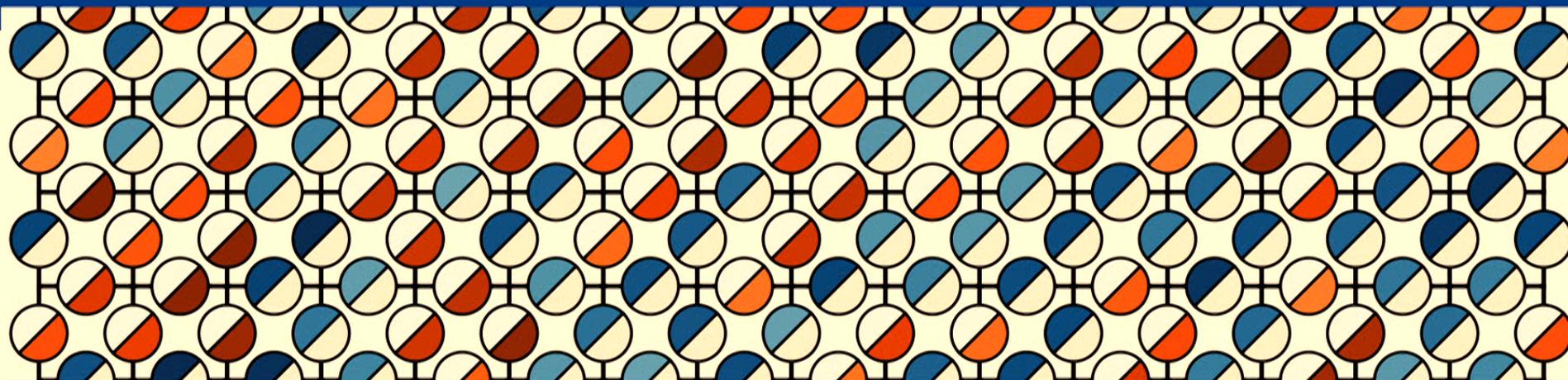
Abstract: Successful implementation of error correction is imperative for fault-tolerant quantum computing. At present, the toric code, surface code and related stabilizer codes are state of the art techniques in error correction.

Standard decoders for these codes usually assume uncorrelated single qubit noise, which can prove problematic in a general setting.

In this work, we use the knowledge of topological phases of modified toric codes to identify the underlying Hamiltonians for certain types of imperfections. The Hamiltonian learning is employed to adiabatically remove the underlying noise and approach the ideal toric code Hamiltonian.

This approach can be used regardless of correlations. Our method relies on a neural network reconstructing the Hamiltonian given as input a linear amount of expectation values. The knowledge of the Hamiltonian offers significant improvement of standard decoding techniques

Eliska Greplova, Agnes Valenti, Evert van Nieuwenburg, Sebastian Huber



# Hamiltonian Learning for Quantum Error Correction

arXiv: 1907.02540

Eliška Greplová, ETH Zürich

**Can Hamiltonian learning provide new ideas for quantum error correction?**

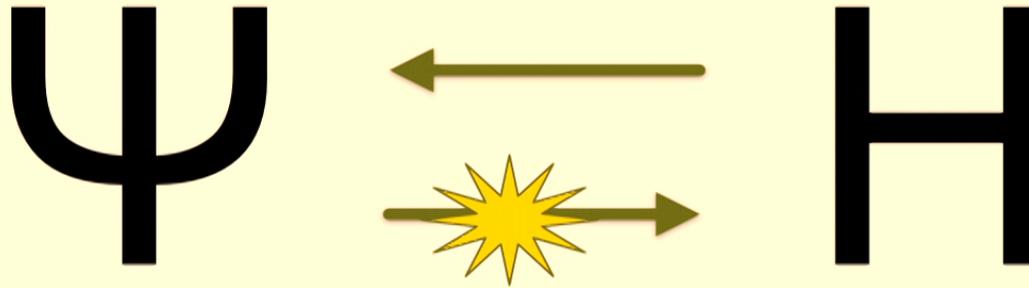
## Goals of this talk

Quantum error correction is important.

Learning Hamiltonians is important.

Bringing them together can be helpful.

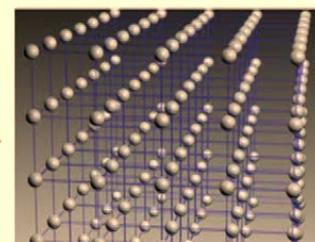
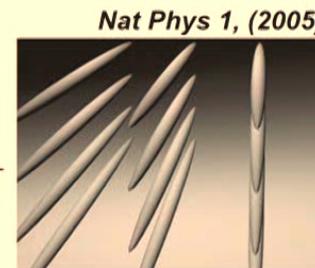
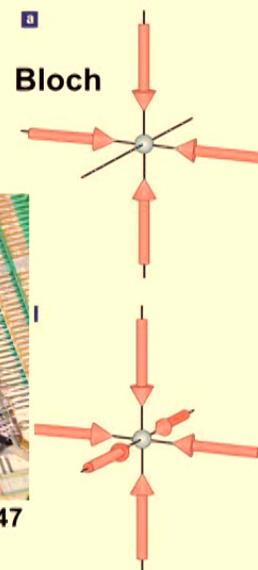
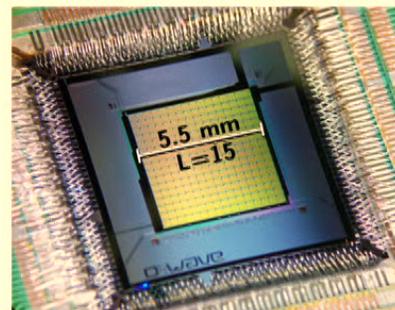
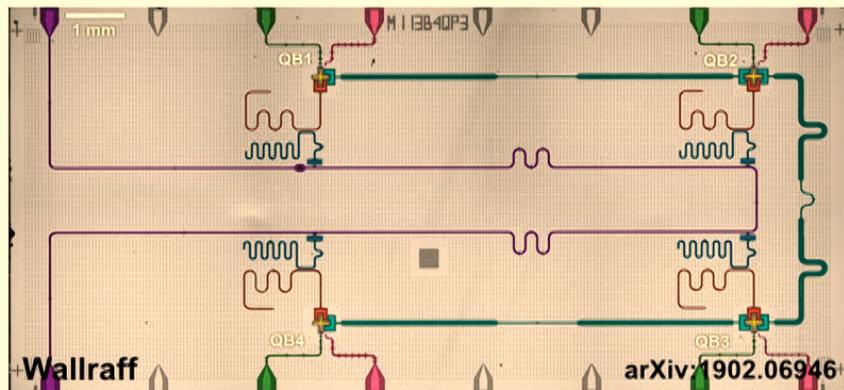
## Wave-function vs. Hamiltonian



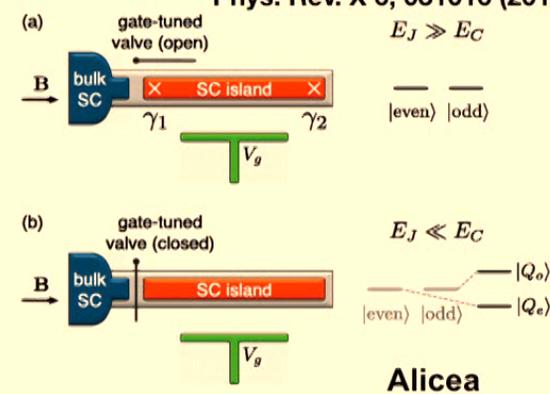
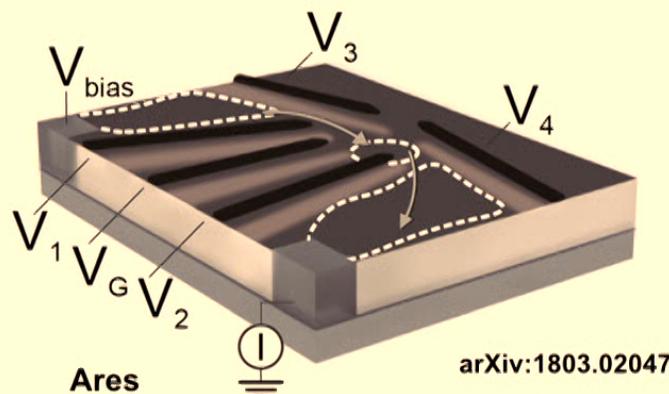
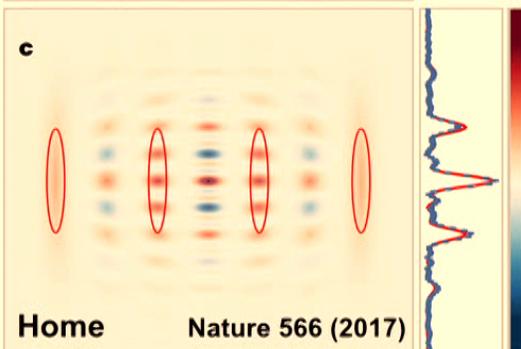
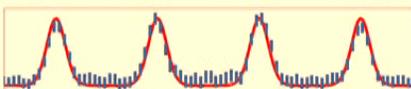
Phys. Rev. Lett. 112, 190501 (2014)

Phys. Rev. Lett. 122, 020504 (2019)

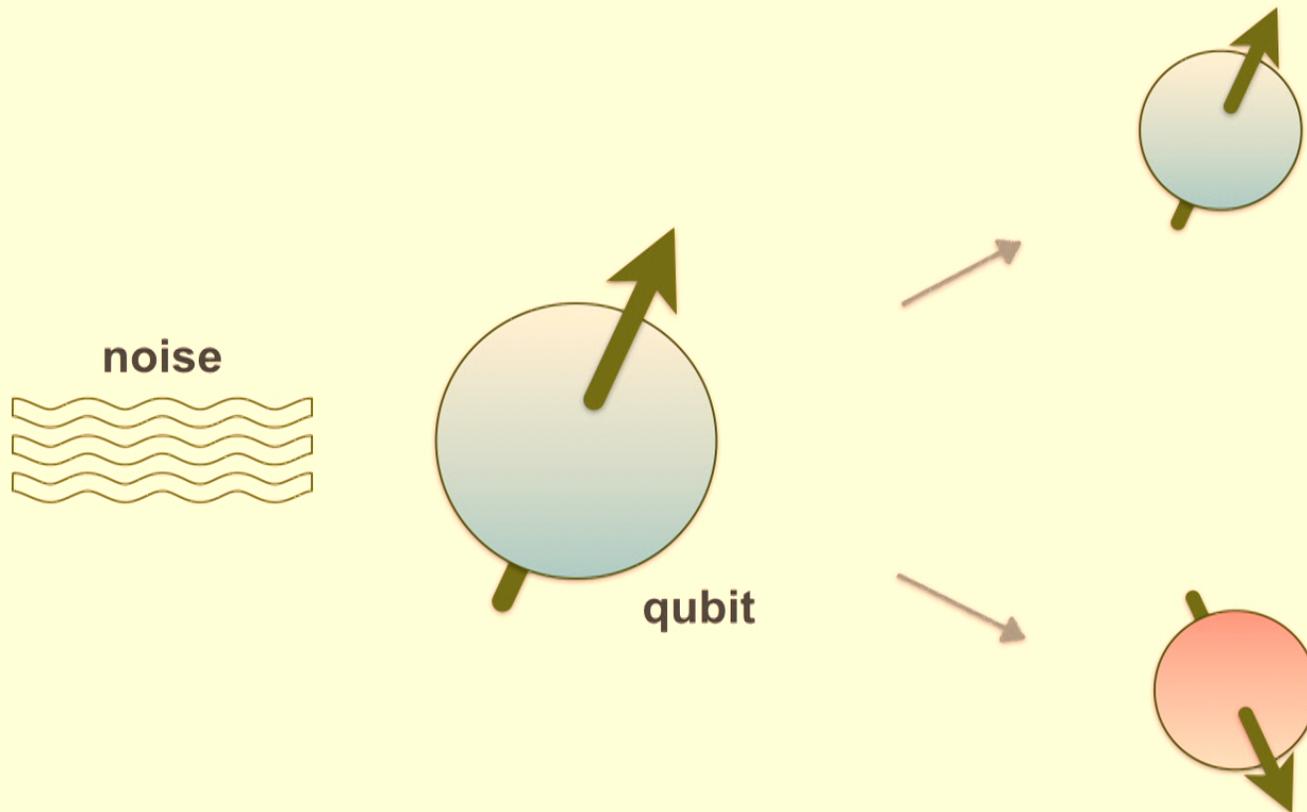
# QIP devices - Hamiltonian Engineering



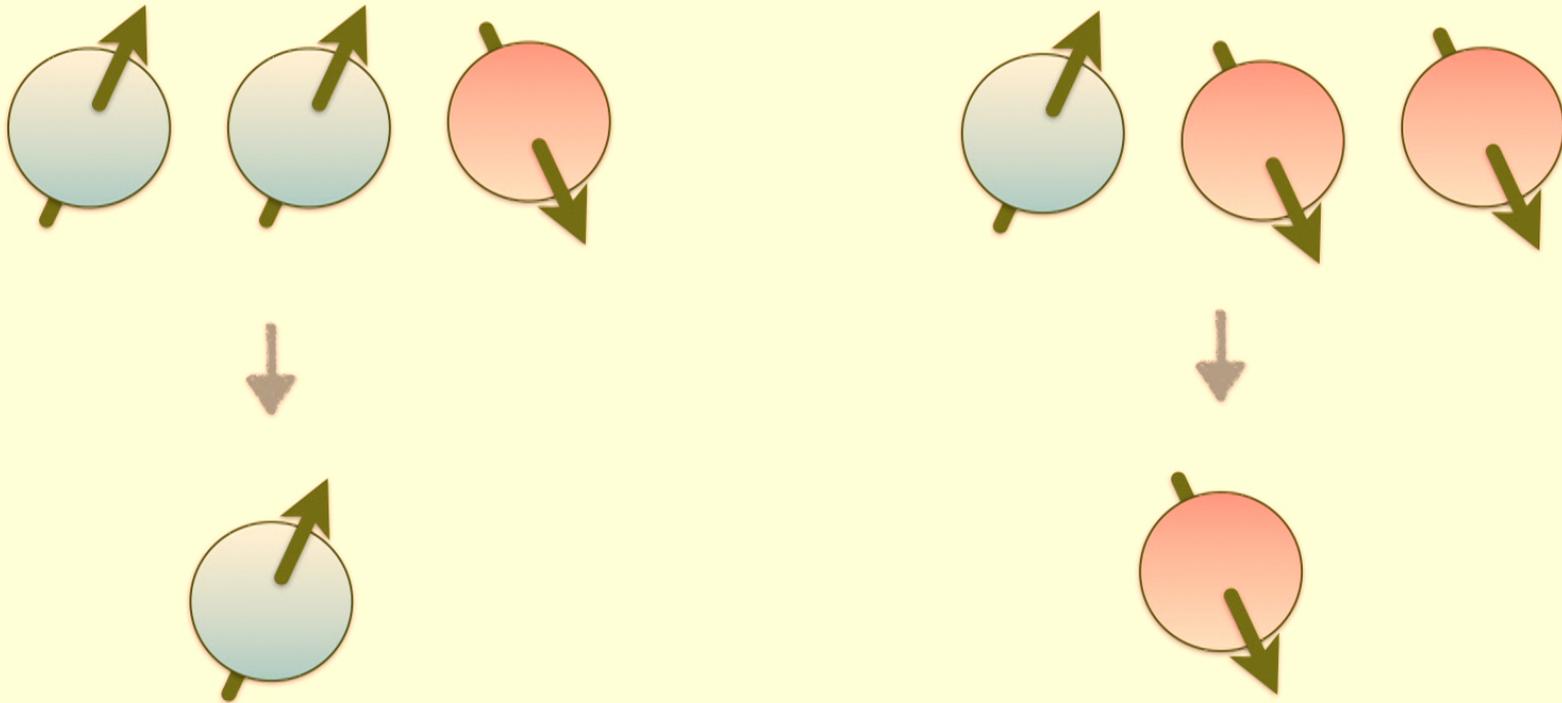
Phys. Rev. X 6, 031016 (2016)



# Quantum Error Correction



# Quantum Error Correction



## Relevant Problem: Quantum Error Correction: Toric Code

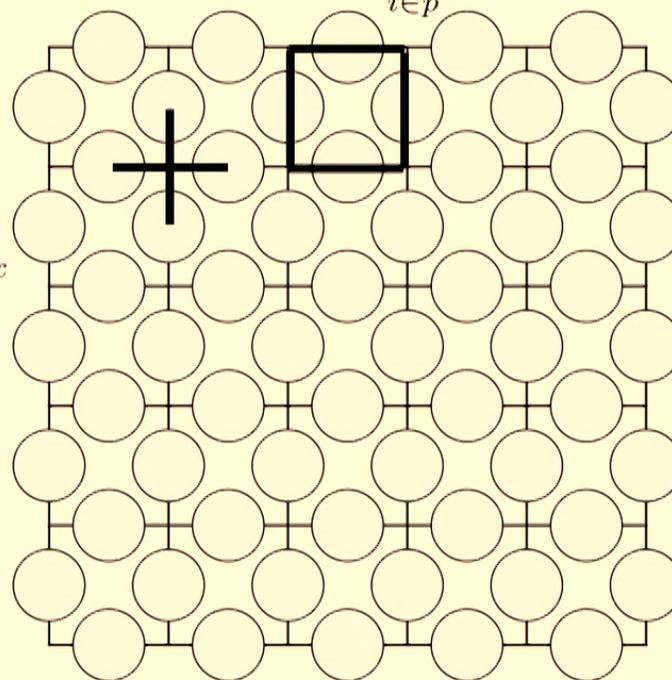
$$H = - \sum_s A_s - \sum_p B_p$$

$$B_p = \prod_{i \in p} \sigma_i^z$$

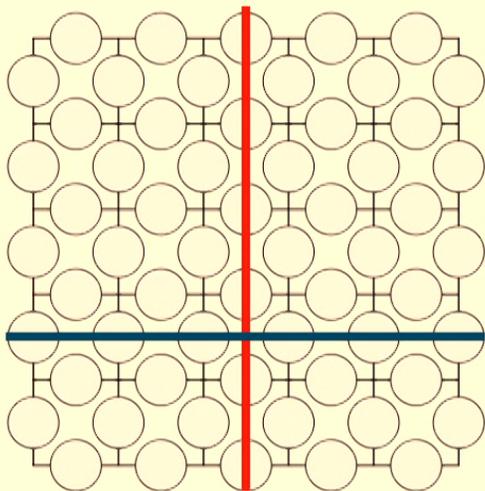
$$A_s = \prod_{i \in s} \sigma_i^x$$

ground state encodes 2 qubits

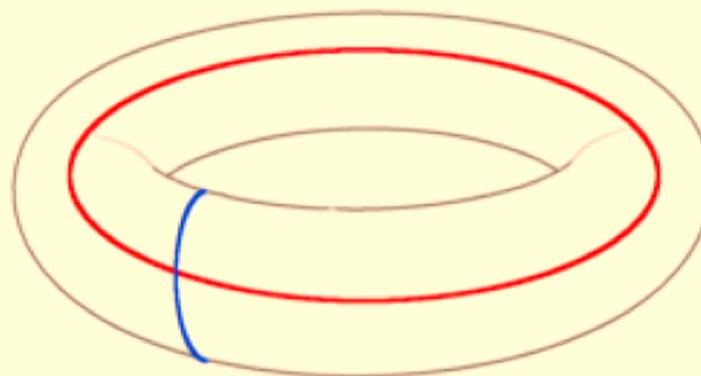
$$|\text{GS}\rangle_{\text{TC}} = \frac{1}{2} \prod_s (1 + A_s) |0\rangle$$



## Topological ground state

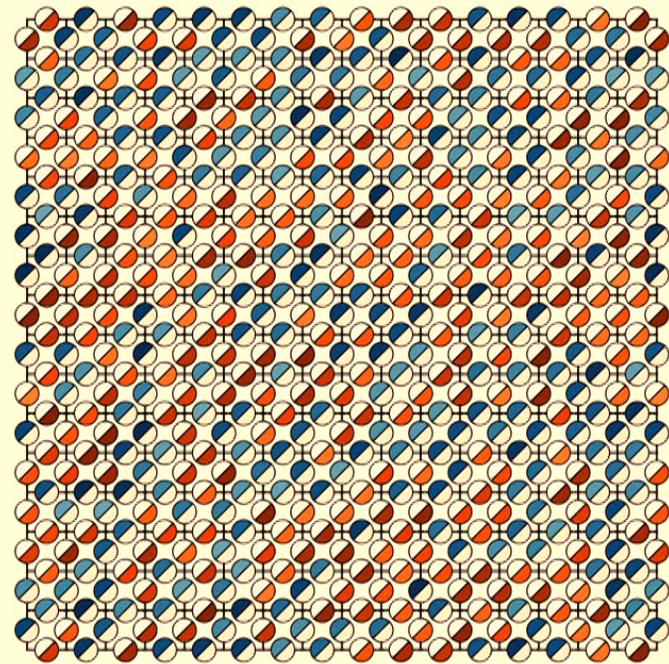
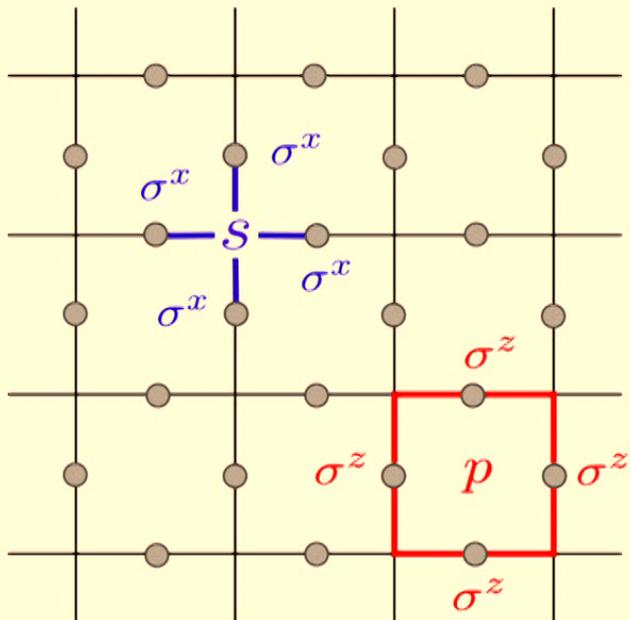


$$|\text{GS}\rangle_{\text{TC}} = \frac{1}{2} \prod_s (1 + A_s) |0\rangle$$

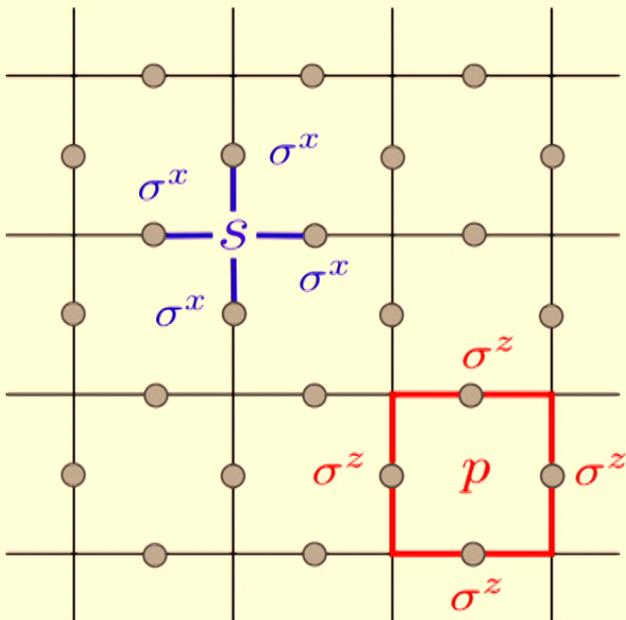


One has to flip the whole row of spins to change the state.  
On the torus this can be done in two directions - two qubits.

# Projective measurements vs. Hamiltonian engineering



## Projective measurements vs. Hamiltonian engineering



Begin from an arbitrary state.

Measure stabilisers with high frequency.

Use decoder to translate measurement results into bit 'fixes'.

Arrive to ground state.

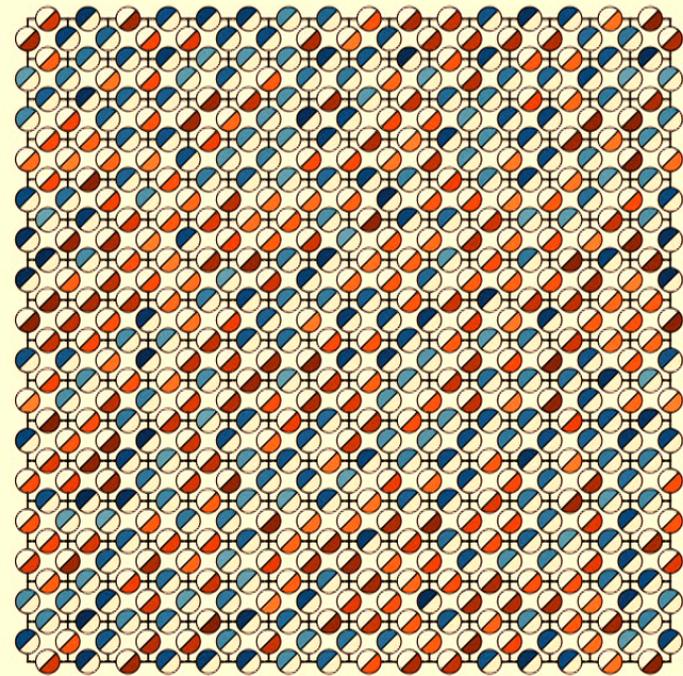
## Projective measurements vs. Hamiltonian engineering

Physically engineer the four-body interaction.

Be in the ground state of engineered interaction.

Do measurements on the state -> learn the Hamiltonian.

Fix errors/imprecisions in the engineering -> correct GS.

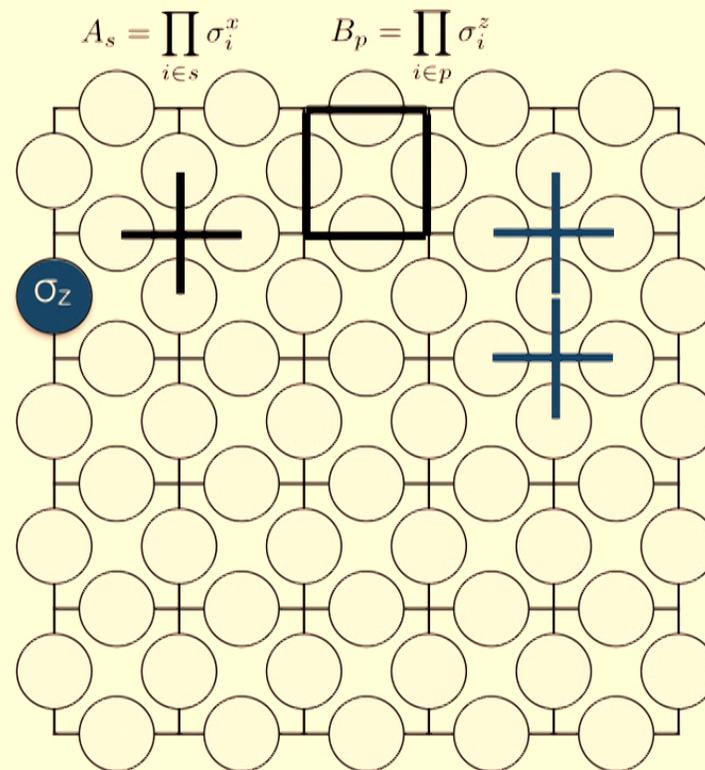


## Relevant Problem: Quantum Error Correction: Toric Code

$$H = \sum_s \left( -A_s + e^{-\sum_{i \in s} \beta_{z,i} \sigma_i^z} \right) - \sum_p B_p$$

**Remains solvable!**  
**Degenerate ground state!**

$$|\Psi\rangle = \frac{1}{\sqrt{Z}} e^{\frac{\beta}{2} \sum_i \lambda_i \sigma_i^z} |\text{GS}\rangle_{\text{TC}}$$

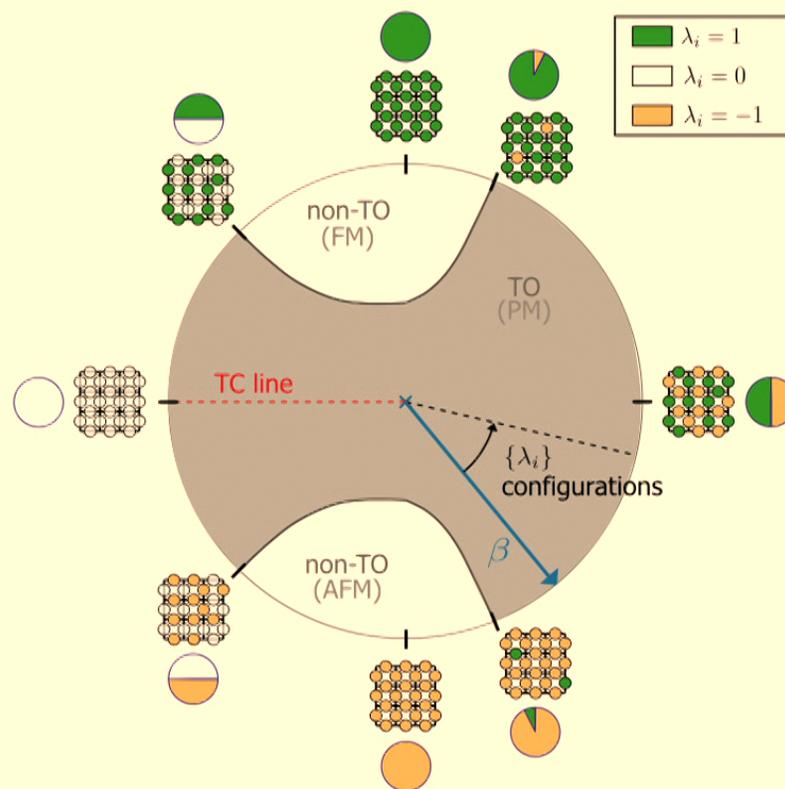


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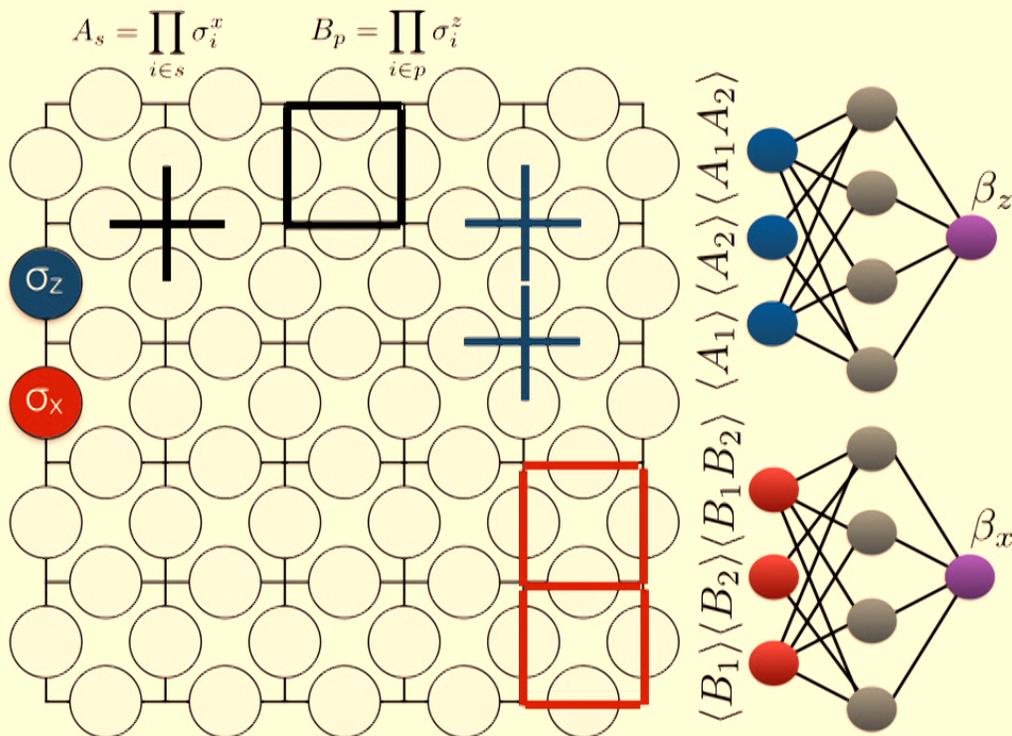
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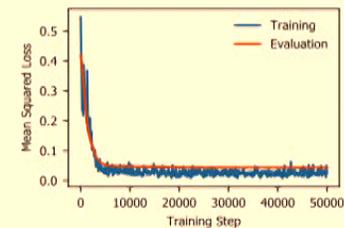
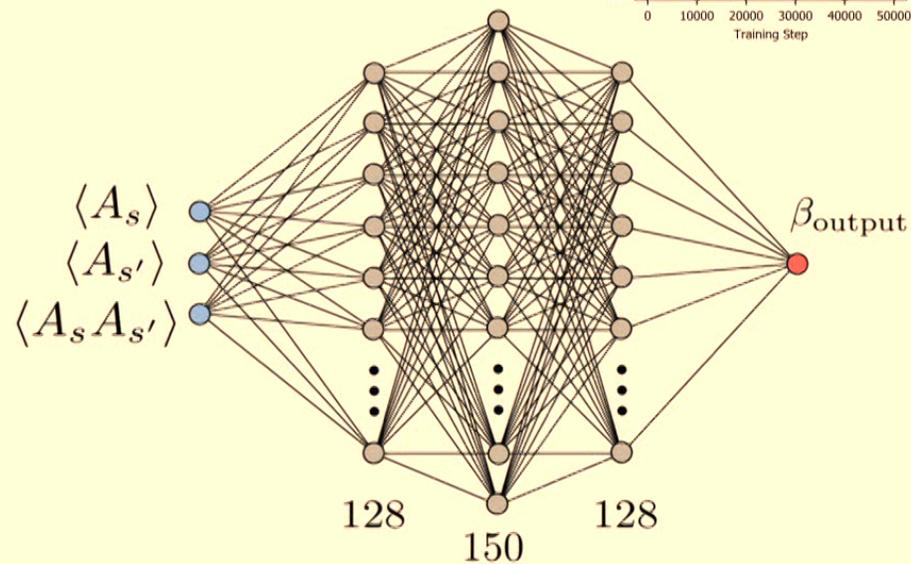
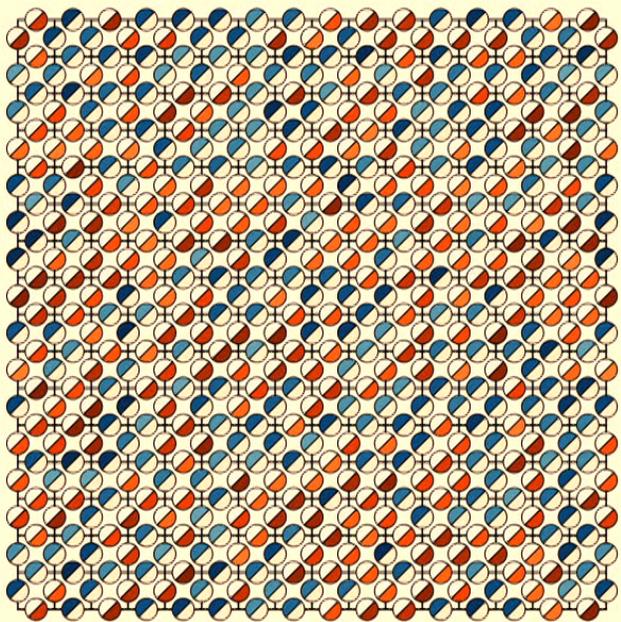
# Relevant Problem: Quantum Error Correction: Toric Code

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$$H = -\sum_s A_s + \sum_p \left( -B_p + e^{-\sum_{i \in p} \beta_{x,i} \sigma_i^x} \right)$$

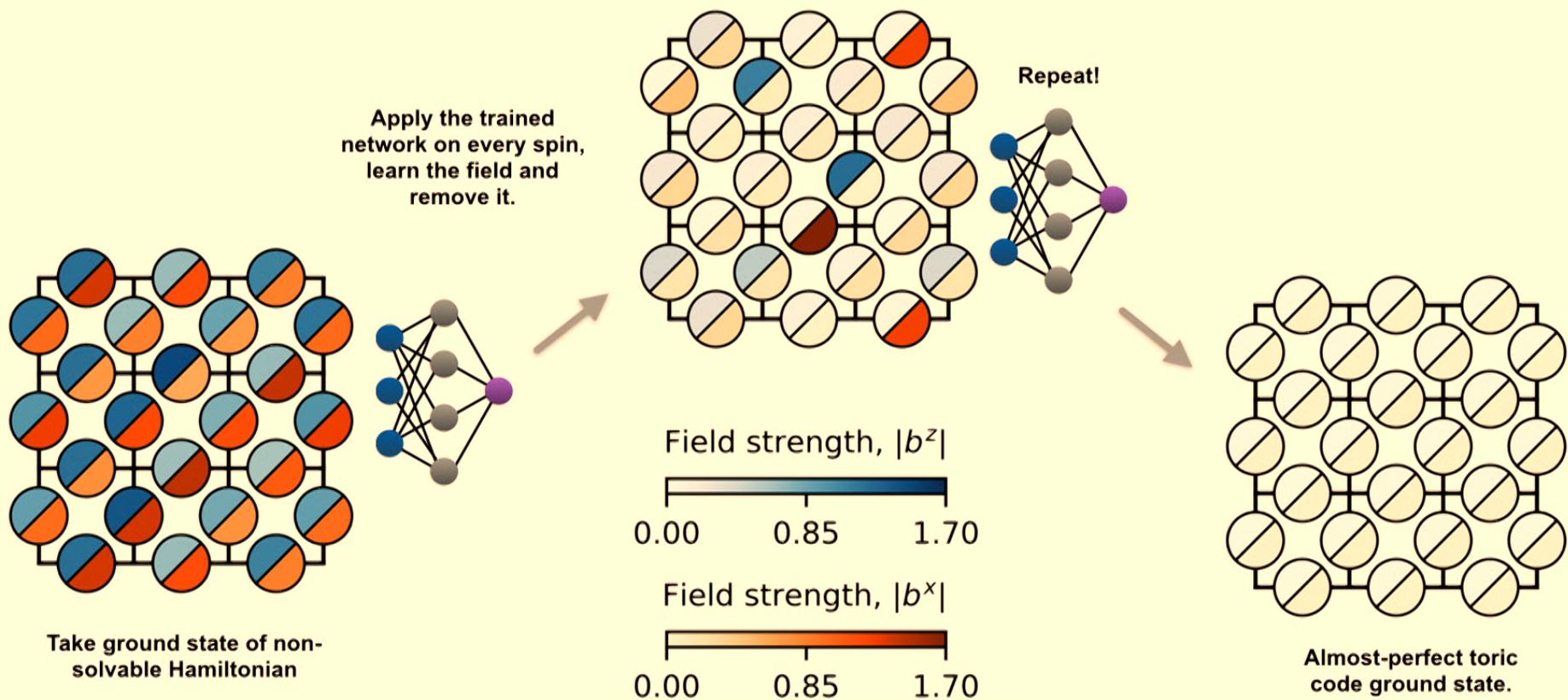


## We train on scalable analytical states



$$H = \sum_s \left( -A_s + e^{-\sum_{i \in s} \beta_i^z \sigma_i^z} \right) + \sum_p \left( -B_p + e^{-\sum_{i \in p} \beta_i^x \sigma_i^x} \right)$$

## Iterative application of the trained model



## Measurements of success

(1) STATE:

"What is the probability that single qubit will flip spin or phase? Can we guarantee established thresholds?"

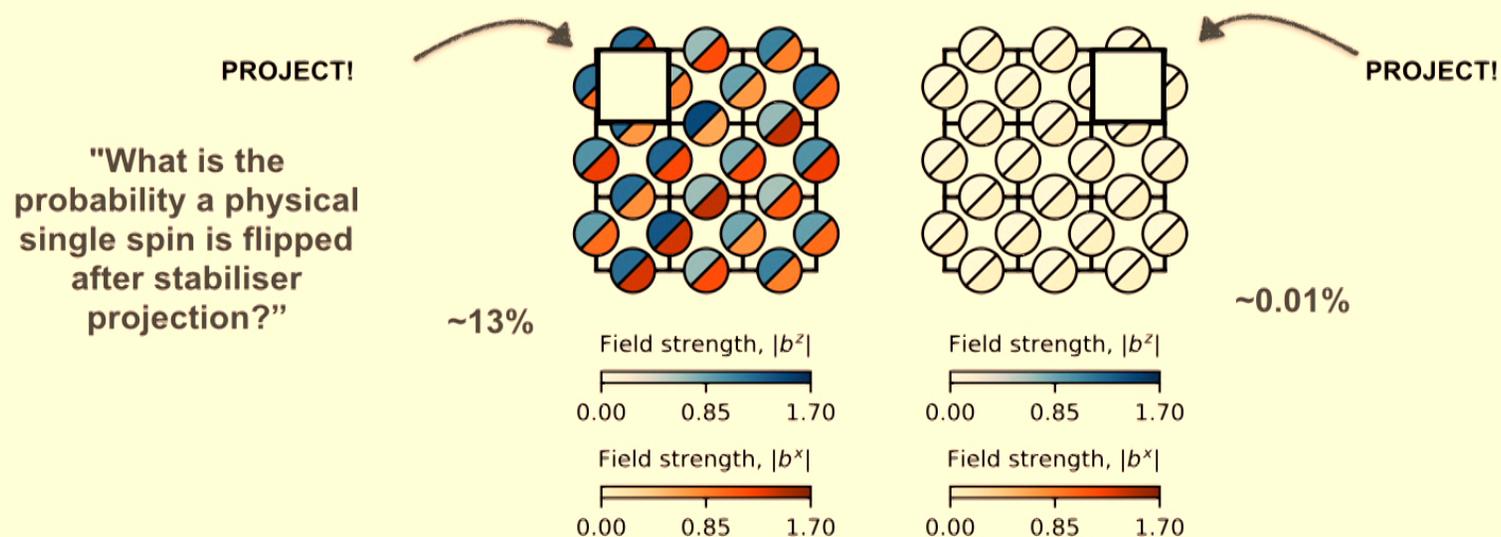
(2) HAMILTONIAN:

"How 'far' are we from the ideal toric code Hamiltonian?"

## Measurements of success

(1) STATE:

"What is the probability that single qubit will flip spin or phase? Can we guarantee established thresholds?"



## Measurements of success

(2) HAMILTONIAN:

“How ‘far’ are we from the ideal toric code Hamiltonian?”

“Find proper basis and calculate L2-distance of the coefficients.”

$$H = \sum_m c_m S_m$$

$$\Delta_H = \|\hat{c}_{\text{true}} - \hat{c}_{\text{recovered}}\|_2$$

## Measurements of success

### (2) HAMILTONIAN:

“How ‘far’ are we from the ideal toric code Hamiltonian?”

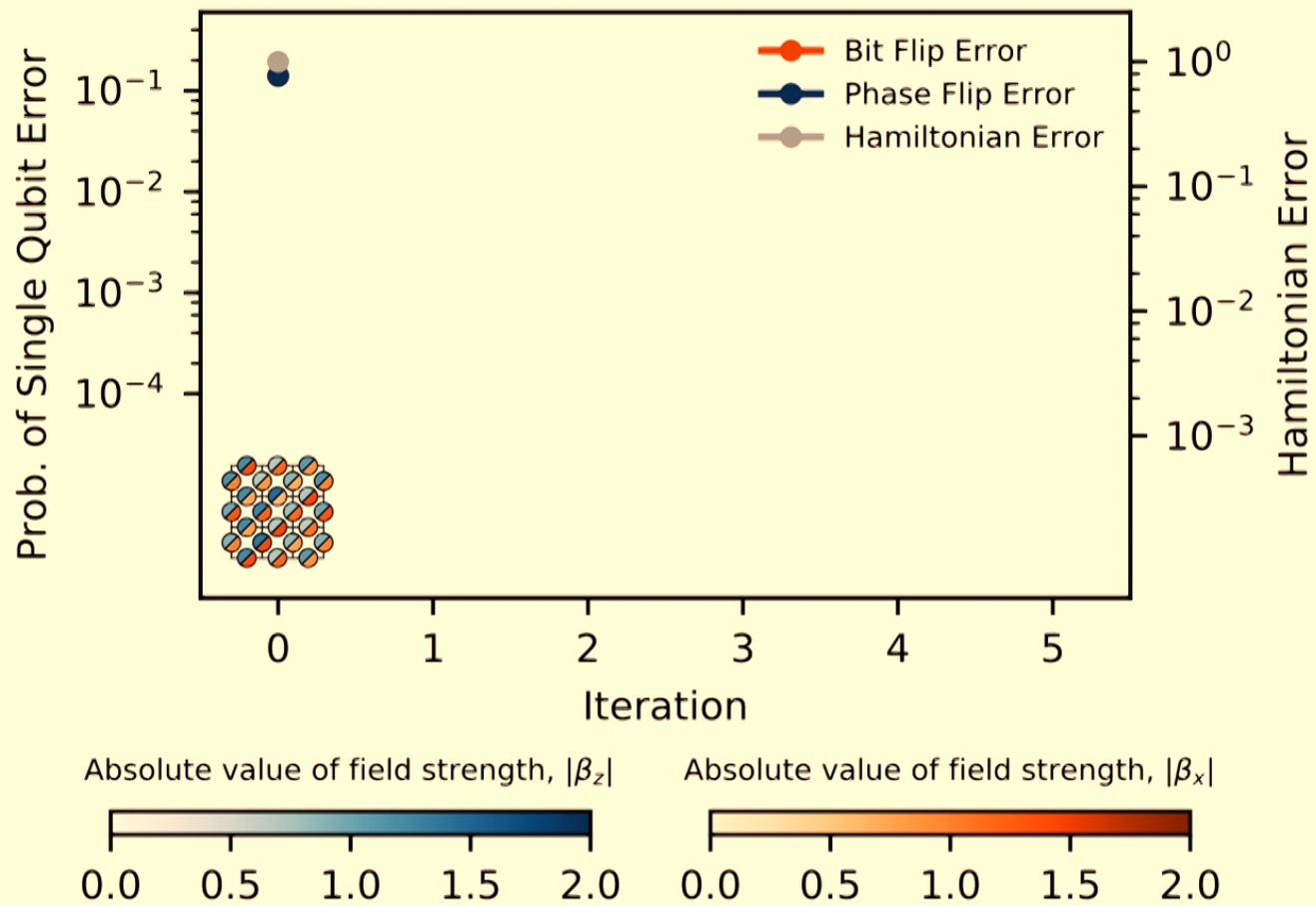
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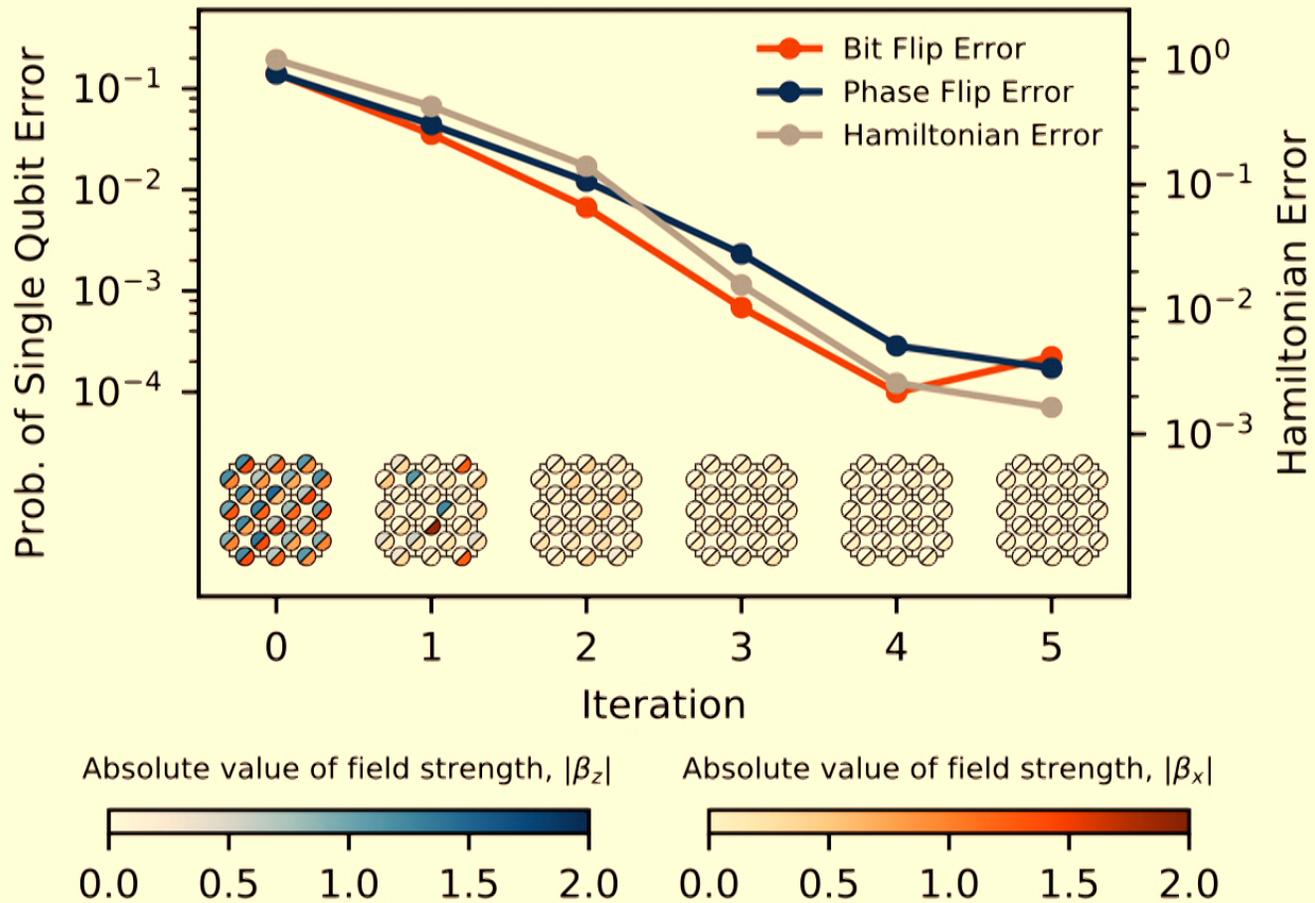
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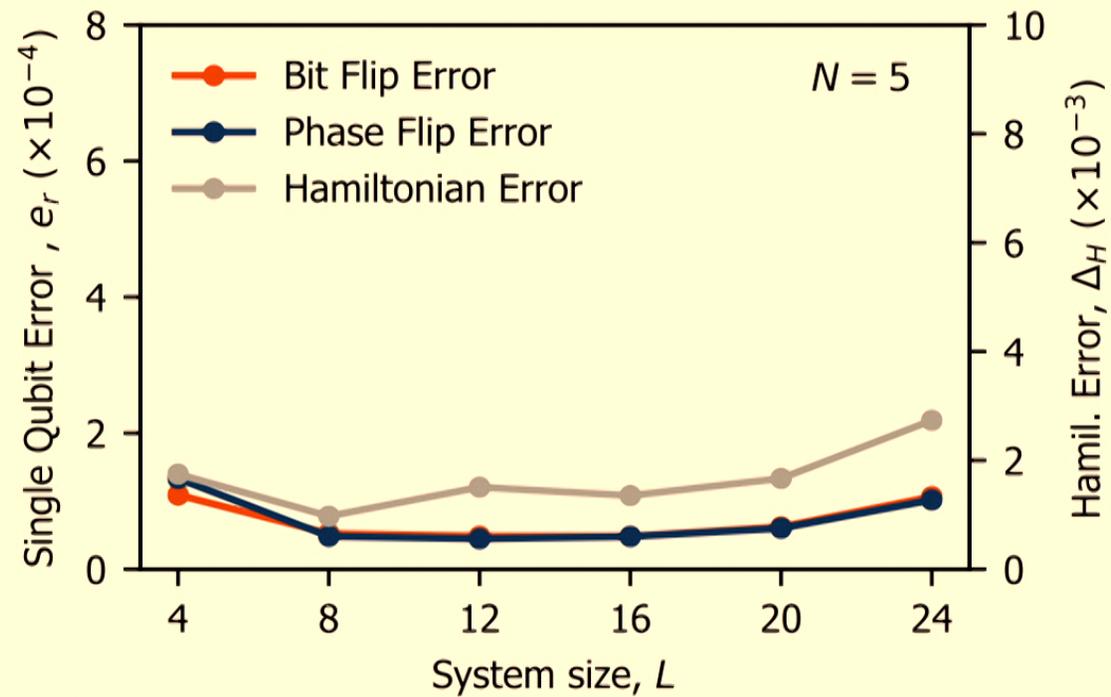
$$\begin{aligned}
 H &= \sum_s (-1 \cdot S_{s,0} + \prod_{i \in s} \cosh(b_i^z)) S_{s,1} \\
 &- \sum_{i \in s} \sinh(b_i^z) \prod_{j \in s, j \neq i} \cosh(b_j^z) S_{s,1+i} \\
 &+ \sum_{i < j \in s} \sinh(b_i^z) \sinh(b_j^z) \prod_{\substack{l < m \in s \\ l \neq i \neq m \neq j}} \cosh(b_l^z) \cosh(b_m^z) S_{s,5+4i+j} \\
 &- \sum_{\substack{j < k < l \in s \\ i \neq j \neq k \neq l \in s}} \cosh(b_i^z) \sinh(b_j^z) \sinh(b_k^z) \sinh(b_l^z) S_{s,11+i} \\
 &+ \prod_{i \in s} \sinh(b_i^z) S_{s,15+i} \\
 &+ (s \leftrightarrow p, x \leftrightarrow z, b_i^z \leftrightarrow b_i^x) \tag{D8} \\
 &= \sum_{s,i} c_{s,i} S_{s,i} + \sum_{p,i} c_{p,i} S_{p,i} = \sum_m c_m S_m. \tag{D9}
 \end{aligned}$$

“technical, but straightforward”





## Error Analysis: Scaling with System Size



**“We learn systematic errors in the  
Hamiltonian engineering.”**



Agnes Valenti  
ETHZ



Evert van Nieuwenburg  
Caltech



Sebastian Huber  
ETHZ

**arXiv: 1907.02540**  
**GIT: cmt-qo/cm-toricCode**



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