Title: Glassy and Correlated Phases of Optimal Quantum Control

Speakers: Marin Bukov

Collection: Machine Learning for Quantum Design

Date: July 12, 2019 - 9:30 AM

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Abstract: Modern Machine Learning (ML) relies on cost function optimization to train model parameters. The non-convexity of cost function landscapes results in the emergence of local minima in which state-of-the-art gradient descent optimizers get stuck. Similarly, in modern Quantum Control (QC), a key to understanding the difficulty of multiqubit state preparation holds the control landscape -- the mapping assigning to every control protocol its cost function value. Reinforcement Learning (RL) and QC strive to find a better local minimum of the control landscape; the global minimum corresponds to the optimal protocol. Analyzing a decrease in the learning capability of our RL agent as we vary the protocol duration, we found rapid changes in the search for optimal protocols, reminiscent of phase transitions. These "control phase transitions" can be interpreted within Statistical Mechanics by viewing the cost function as "energy" and control protocols â€“ as "spin configurations". I will show that optimal qubit control exhibits continuous and discontinuous phase transitions familiar from macroscopic systems: correlated/glassy phases and spontaneous symmetry breaking. I will then present numerical evidence for a universal spin-glass-like transition controlled by the protocol time duration. The glassy critical point is marked by a proliferation of protocols with close-to-optimal fidelity and with a true optimum that appears exponentially difficult to locate. Using a ML inspired framework based on the manifold learning algorithm t-SNE, we visualize the geometry of the high-dimensional control landscape in an effective low-dimensional representation. Across the transition, the control landscape features an exponential number of clusters separated by extensive barriers, which bears a strong resemblance with random satisfiability problems.

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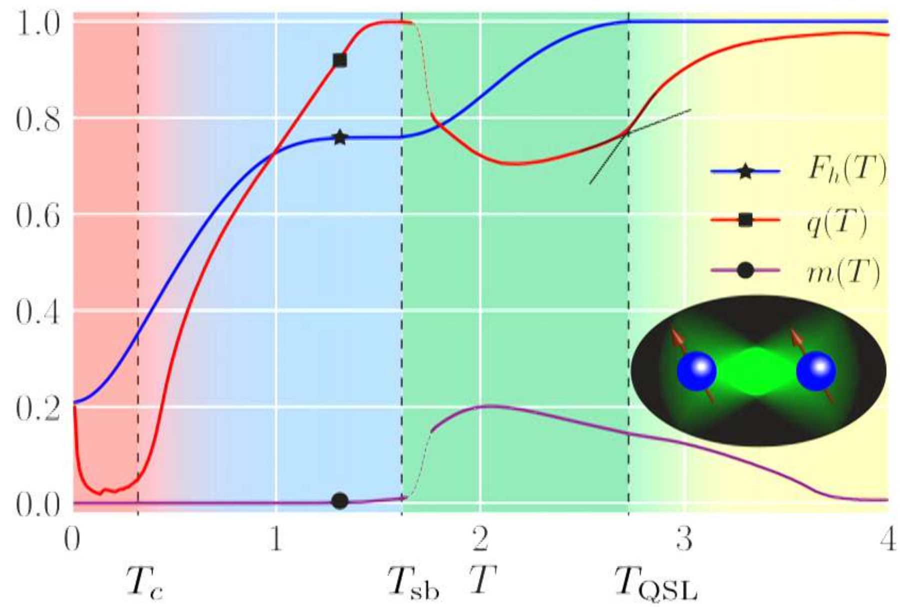
Glassy and Correlated Phases in Optimal Quantum Control



together with A. GR Day, D. Sels, P. Weinberg, A. Polkovnikov and P. Mehta

MB et al, PRX 8 031086 (2018)
MB et al, PRA 97, 052114 (2018)
A. Day, M.B. et al, PRL 122, 020601 (2019)

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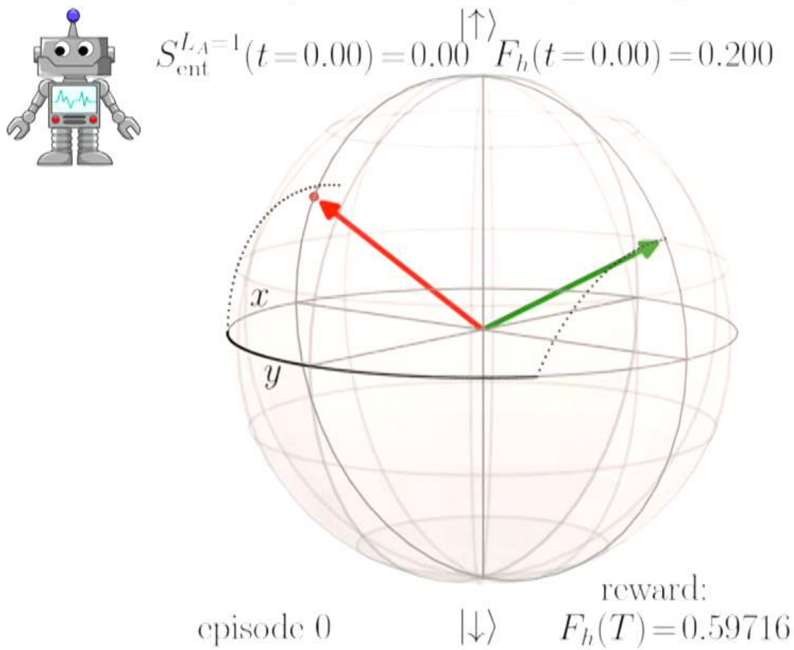
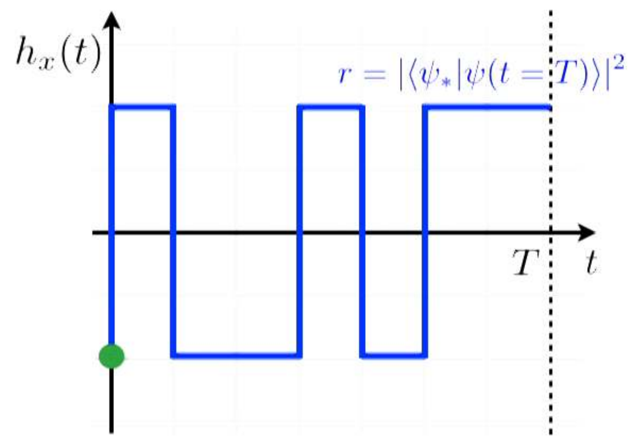
Reinforcement Learning (RL)
Quantum State Preparation

$$H(t) = -S^z - h_x(t)S^x$$

best protocol found by RL agent



$h \in \{\pm 4\}$ bang-bang protocols



episode completed

MB et al, PRX 8 031086 (2018)
Foesel et al, PRX 8 031084 (2018)
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Xiao-Ming et al, arXiv:1902.02157
Jun-Jie et al, arXiv:1901.08748

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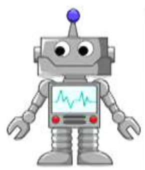
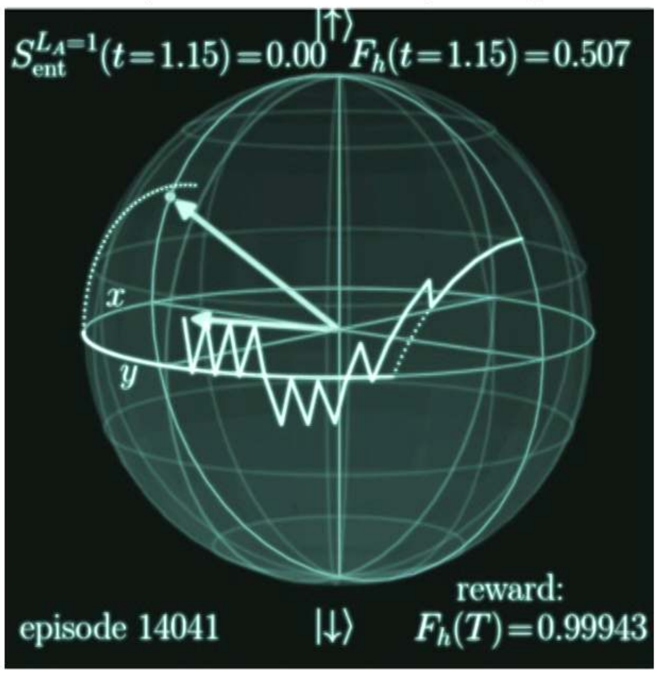
Reinforcement Learning (RL)
Quantum State Preparation

$$H(t) = -S^z - h_x(t)S^x$$

state preparation = fidelity optimization

best protocol found by RL agent





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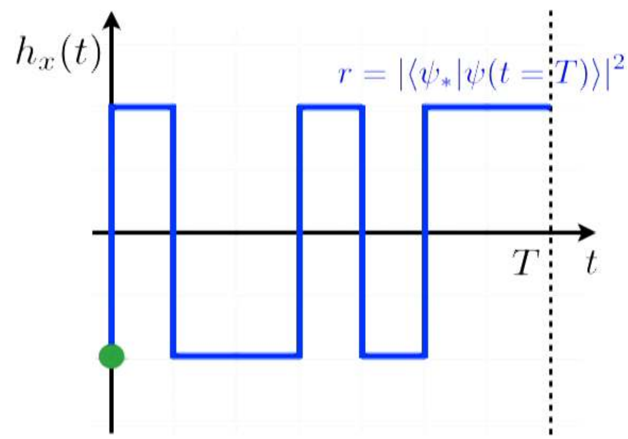

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Reinforcement Learning to Prepare Floquet-Engineered States

$$H_{\rm rot}(t) = H_0 + H_{\rm drive}(t) + H_{\rm control}(t)$$

Kapitza pendulum

fidelity optimization

quantum Kapitza oscillator

$t/T = 0.00,\ \theta(t) =$

periodic drive: ON

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MB PRB 98, 224305 (2018)

Reinforcement Learning to Prepare Floquet-Engineered States

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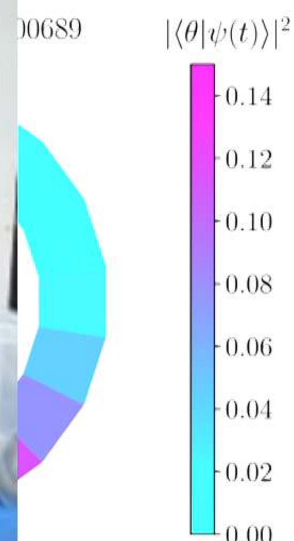

periodic drive: ON

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Reinforcement Learning to Prepare Floquet-Engineered States

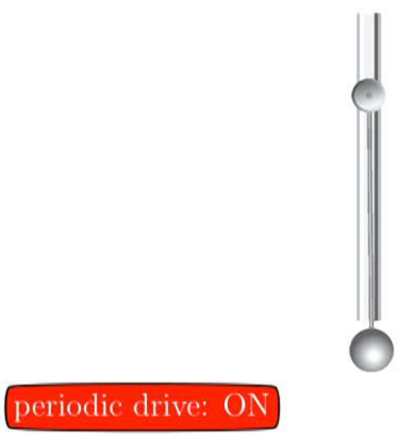
$$H_{\rm rot}(t) = H_0 + H_{\rm drive}(t) + H_{\rm control}(t)$$

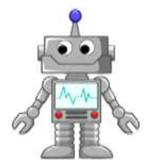
Kapitza pendulum

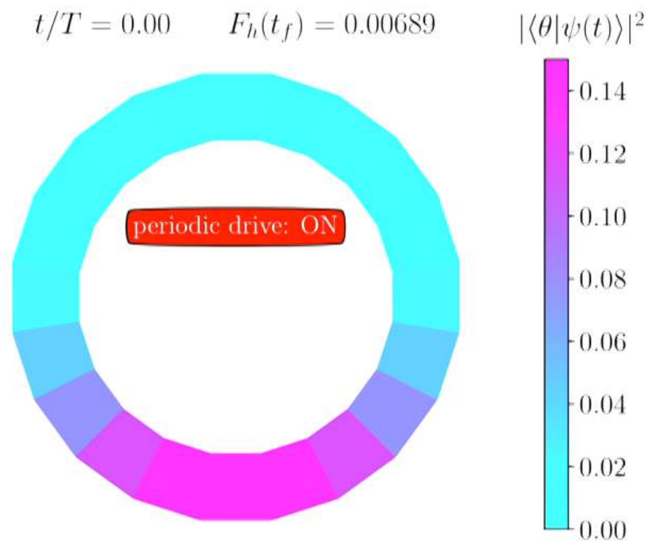
fidelity optimization

quantum Kapitza oscillator

$t/T = 0.00,\ \theta(t) = 0.00\pi,\ p_\theta(t) = 0.00,\ r(t) = 0.00$







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Reinforcement Learning to Prepare Floquet-Engineered States

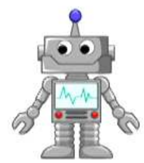
$$H_{\rm rot}(t) = H_0 + H_{\rm drive}(t) + H_{\rm control}(t)$$

Kapitza pendulum

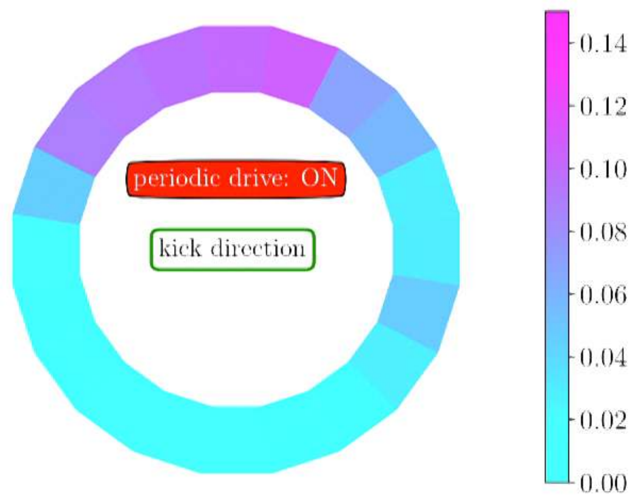
fidelity optimization

quantum Kapitza oscillator

$t/T = 35.83,\ \theta(t) = 0.99\pi,\ p_\theta(t) = 0.03,\ r(t) = 0.97$



$t/T = 12.00 \quad F_h(t_f) = 0.85095 \quad |\langle\theta|\psi(t)\rangle|^2$



periodic drive: OFF

ability to learn depends strongly on *physical* parameters

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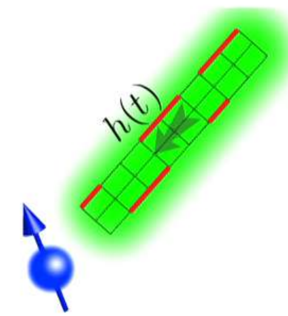
¿QUESTION?

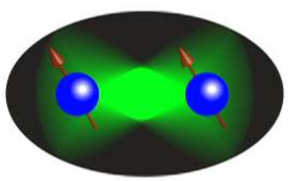
why are there “easy” and “hard” learning regimes and is there a stat. mech. interpretation

$$H(t) = -\sum_j J S^z_{j+1} S^z_j + h_z S^z_j + h_x(t) S^x_j$$

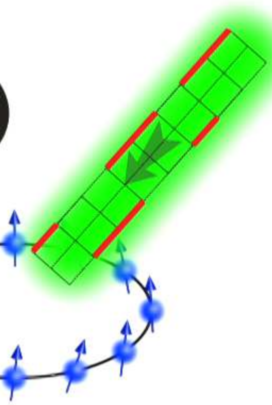
1. Single-qubit System $J = 0$
   - → problem setup
   - → control phase transitions
     overconstrained phase,
     correlated phase,
     controllable phase



2. Two-qubit System $L = 2$
   - → spontaneous symmetry breaking in optimization landscape



3. Many-Body System
   - → strongly correlated (glassy) control phase



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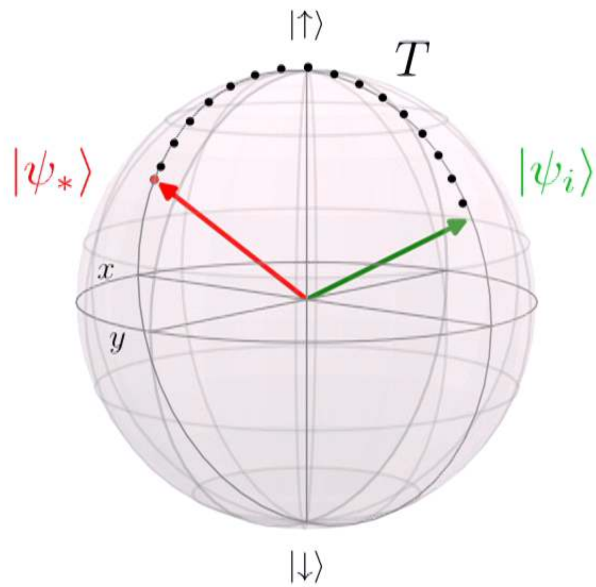
Simple Optimization Problem: Qubit State Preparation

→ Hamiltonian: $H(t) = -S^z - h_x(t)S^x$

initial state: $|\psi_i\rangle$: $\text{GS of } H_i = -S^z - 2S^x$

target state: $|\psi_*\rangle$: $\text{GS of } H_* = -S^z + 2S^x$



TASK: find protocol $h(t) \in [-4, 4]$

such that $|\psi(t=0)\rangle = |\psi_i\rangle,\ |\psi(t=T)\rangle = |\psi_*\rangle$

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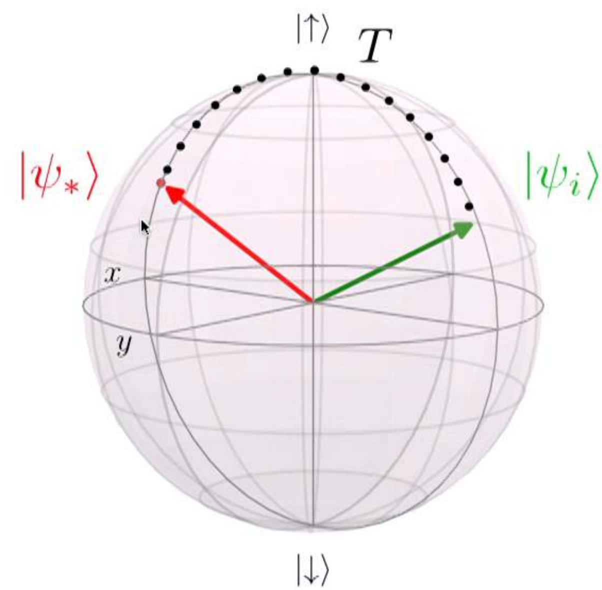
Minimum Ramp Time for Unit Fidelity

$$H(t) = -S^z - h_x(t)S^x$$



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Hegerfeldt, PRL 111, *260501* (2013)

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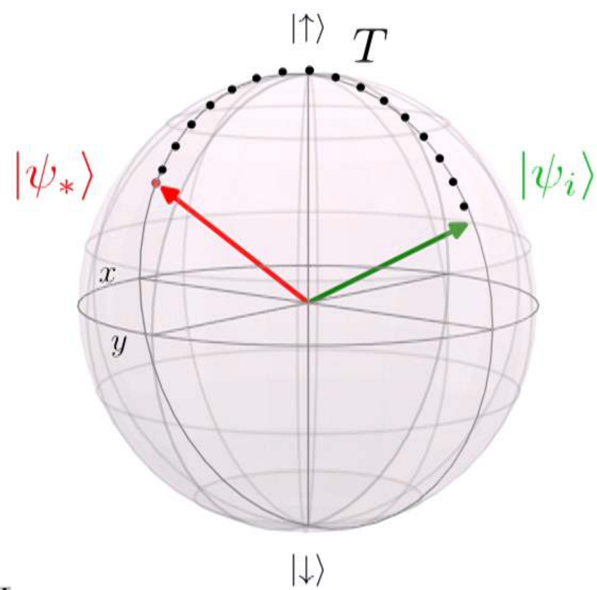
Minimum Ramp Time for Unit Fidelity

$$H(t) = -S^z - h_x(t)S^x + g(t)S^y$$

$h(t) \in [-4, 4]$

→ energy-time uncertainty

$$T_{\mathrm{QSL}} \geq \pi/2$$



→ how about ramp times $T \leq T_{\mathrm{QSL}}$

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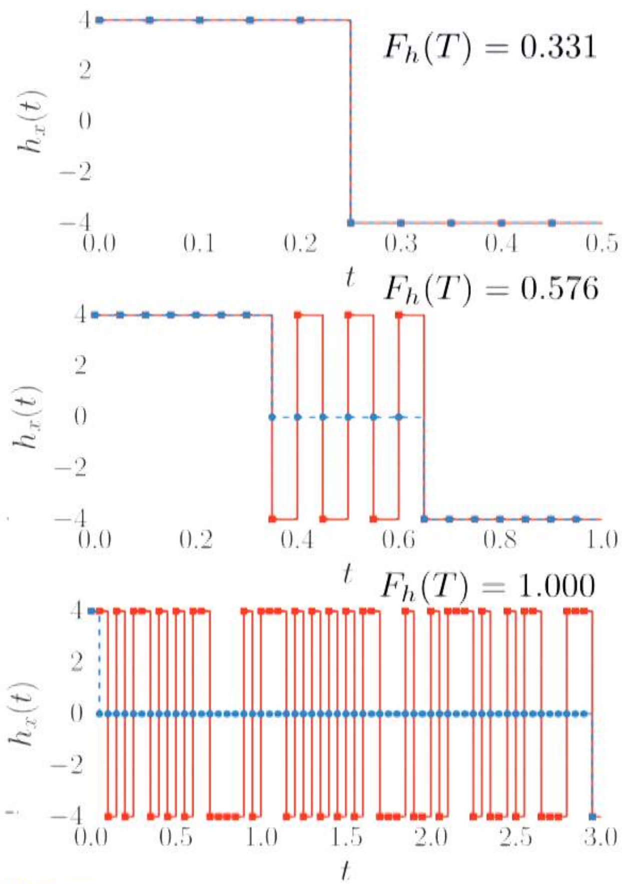
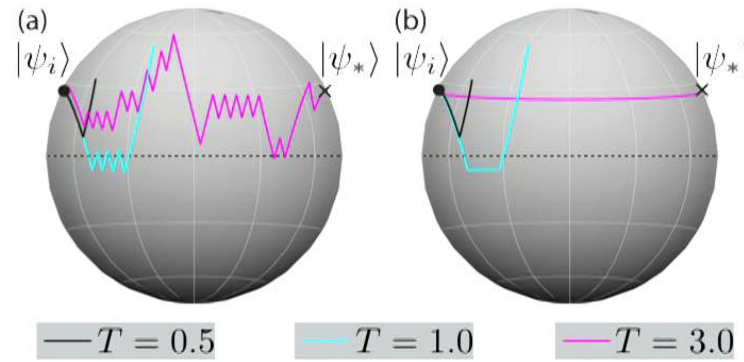
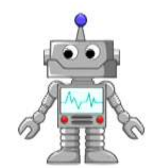
How do Optimal Protocols Look Like?

$$H(t) = -S^z - h_x(t)S^x$$







ability to learn depends strongly on protocol duration!

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MB et al, PRX 8 031086 (2018)

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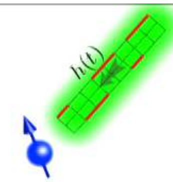
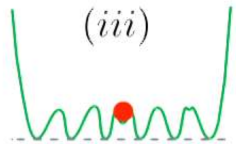
Phase Transitions in the Optimization Landscape



infidelity landscape (schematic)



$$H(t) = -S^z - h_x(t)S^x$$

bang-bang protocols

$$h \mapsto 1 - F_h(T)$$

infidelity landscape
minima: $\{h^\alpha\}$

$$\bar{h}(t) = \frac{1}{\#_{\text{real}}}\sum_\alpha h^\alpha(t)$$

Edwards-Anderson-like order parameter:

$$q(T) \sim \sum_{j=1}^{N_T} \overline{\{h(j\delta t) - \bar{h}(j\delta t)\}^2}$$

MB et al, PRX 8 031086 (2018)
Larocca et al, JPA 51 385305 (2018)

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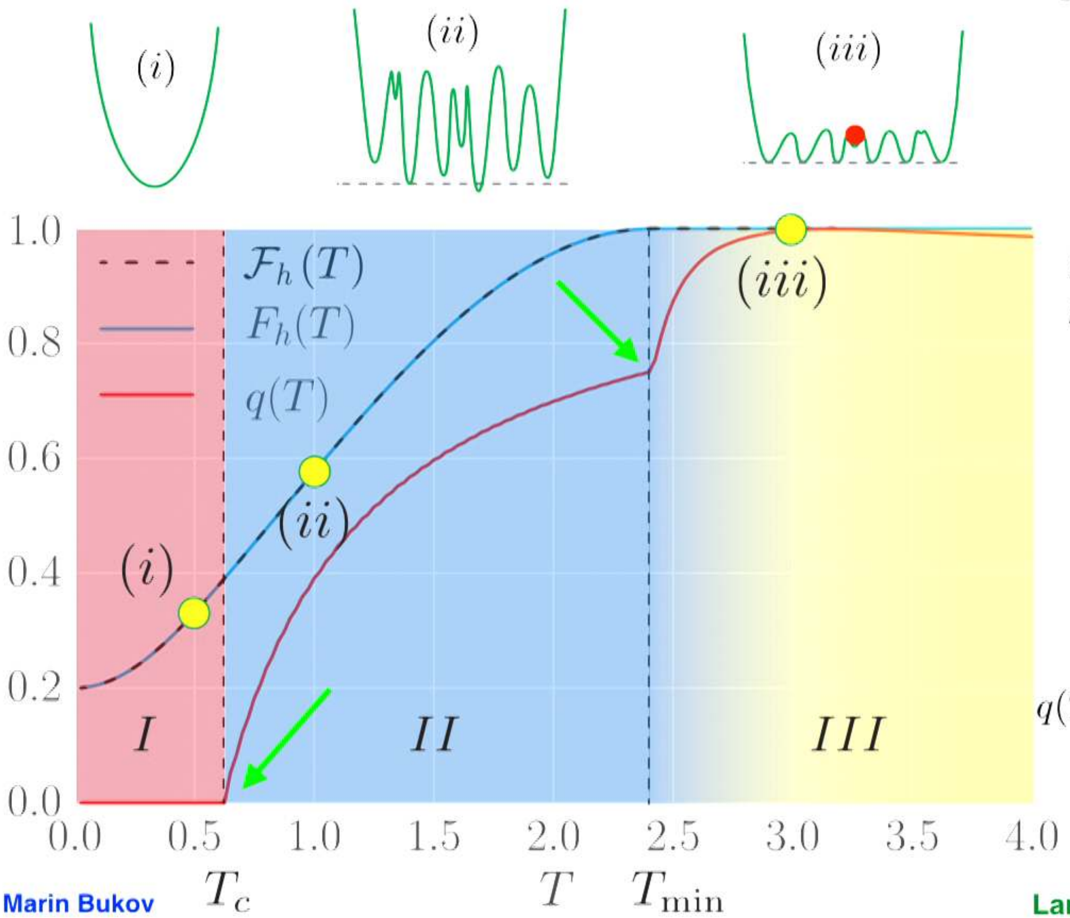

$$H(t) = -S^z - h_x(t)S^x$$

bang-bang protocols

$$h \mapsto 1 - F_h(T)$$

infidelity landscape minima: $\{h^\alpha\}$

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Larocca et al, JPA 51 385305 (2018)

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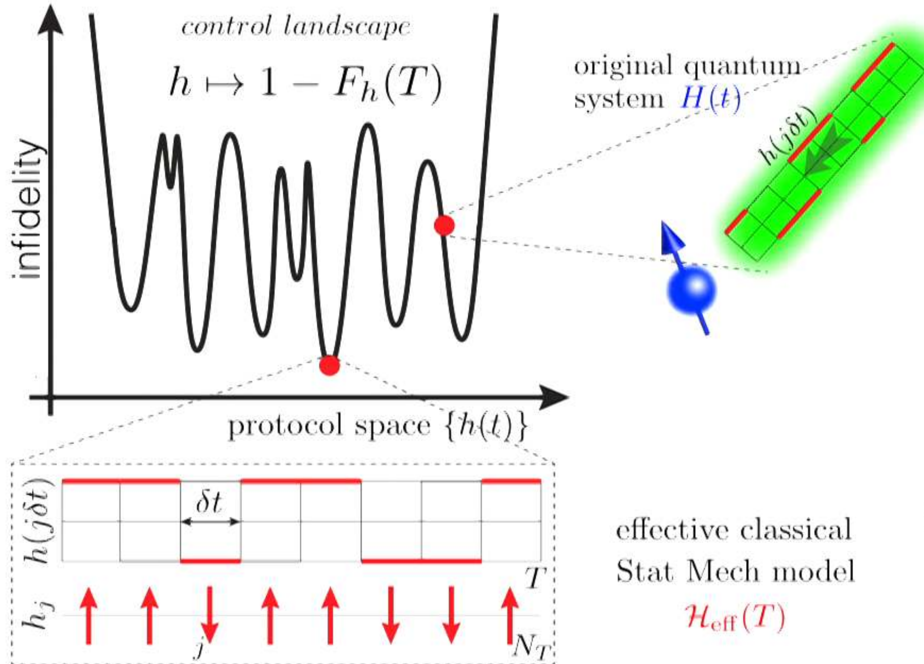
Effective Classical Energy Model

→ one-to-one correspondence:



$$H(t) = -S^z - h_x(t)S^x$$

$$\text{TD limit}: \; N_T \to \infty, \delta t \to 0$$
$$N_T \delta t = const.$$

→ effective *classical* spin energy describes control landscape

$$\mathcal{H}_{\text{eff}}(T) = I(T) + \sum_j G_j(T) h_j + \sum_{ij} J_{ij}(T) h_i h_j + \sum_{ijk} K_{ijk}(T) h_i h_j h_k + \dots$$

j : sites on time lattice

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1. Single-spin System

2. Two-spin System $L = 2$

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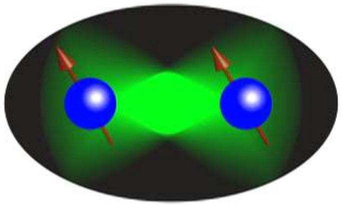

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Two-Qubit Control Phase Diagram

$$H(t) = -2S_1^z S_2^z - (S_1^z + S_2^z) - h_x(t)(S_1^x + S_2^x)$$

infidelity landscape minima: $\{h^\alpha\}$

$$\bar{h}(t) = \frac{1}{\#_{\rm real}} \sum_\alpha h^\alpha(t)$$

Edwards-Anderson-like order parameter:

$$q(T) \sim \sum_{j=1}^{N_T} \overline{\{h(j\delta t) - \overline{h}(j\delta t)\}^2}$$

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MB et al, PRA 97, 052114 (2018)

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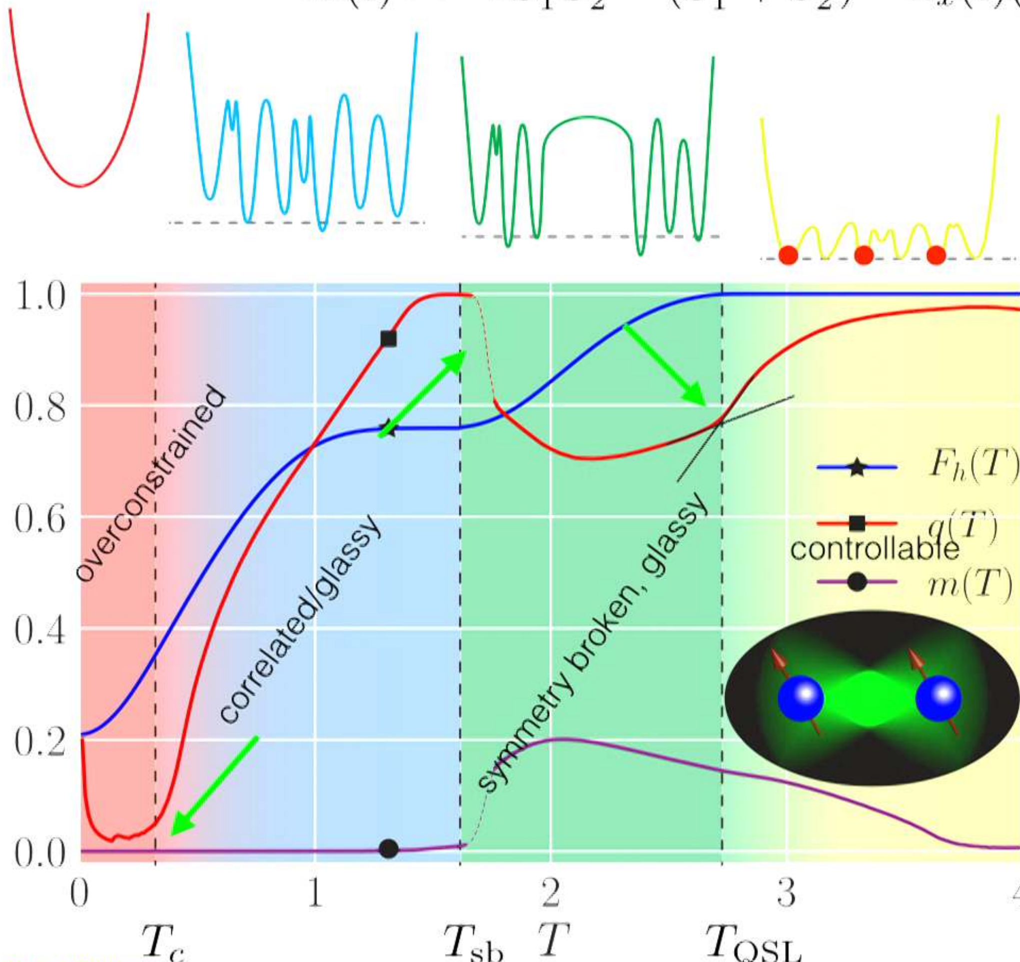

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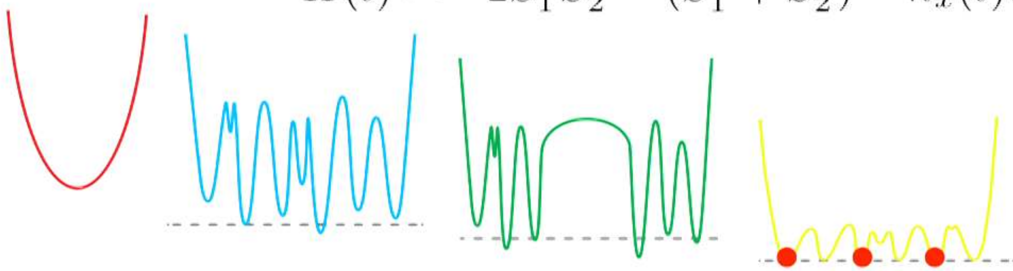

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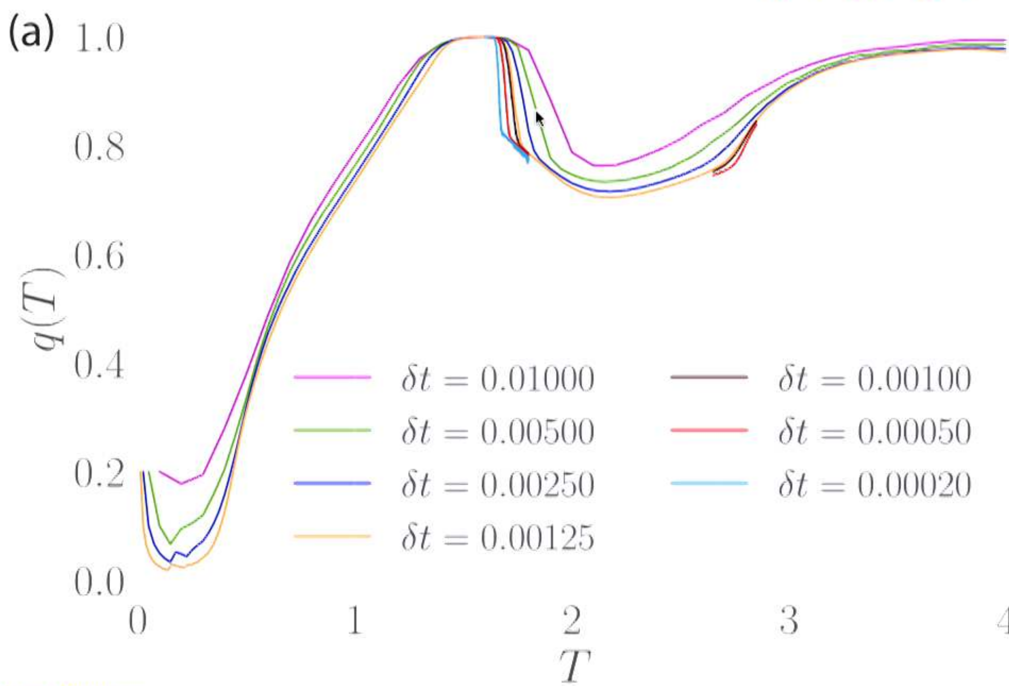

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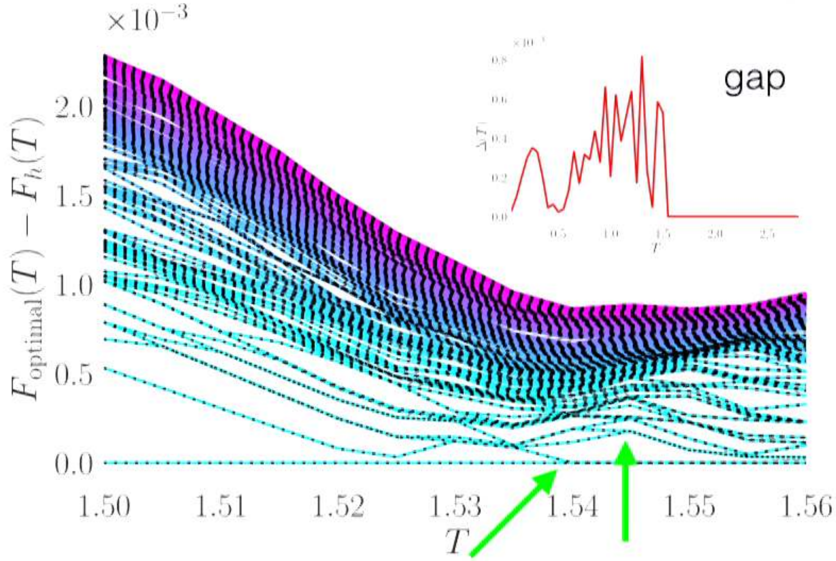
Spontaneous Symmetry Breaking

$$H(t) = -2S_1^z S_2^z - (S_1^z + S_2^z) - h_x(t)(S_1^x + S_2^x)$$

'spectrum' of effective classical model $\mathcal{H}_{\rm eff}(T)$



$\mathbb{Z}_2$ *symmetry of protocols:*

$$|\psi_*\rangle = e^{-i\pi(S_1^z+S_2^z)}|\psi_i\rangle$$

$$F_{h(t)}(T) = F_{-h(T-t)}(T)$$

optimal protocol:

→ *either:* symmetric & unique

→ *or:* symmetry broken & doubly degenerate

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Spontaneous Symmetry Breaking

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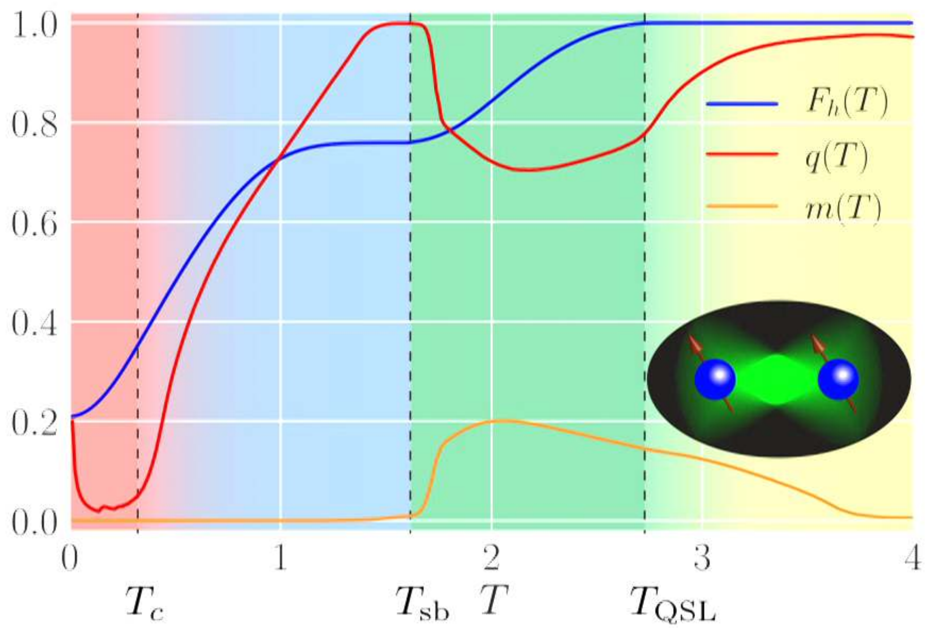

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optimal protocol:

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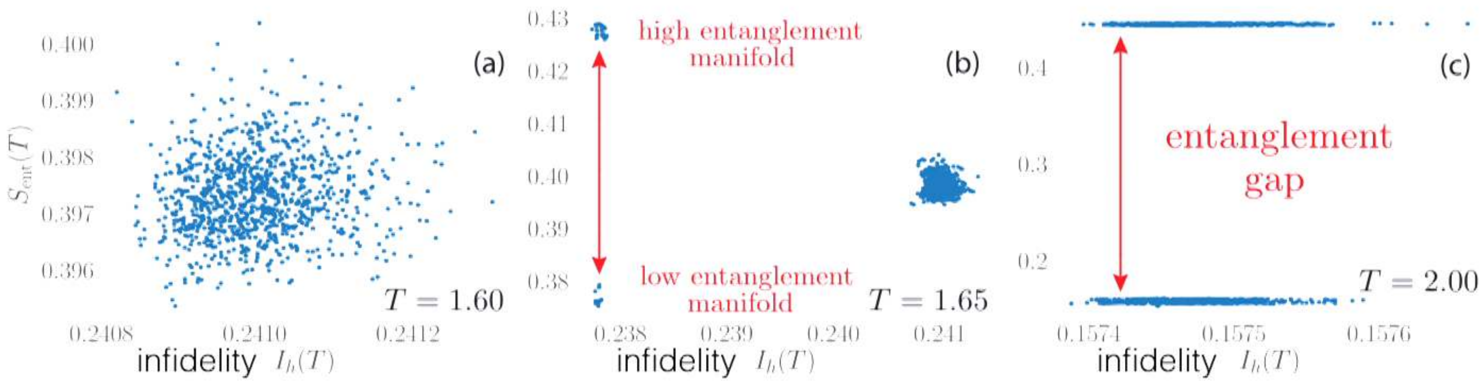
→ “magnetization” of a single protocol: $m_h(T) = N_T^{-1} \sum_{n=1}^{N_T} h_x(n\delta t)$

Marin Bukov   Kappen, PRL 95, 200201 (2005)   MB et al, PRA 97, 052114 (2018)

Implications for Physics

$$H(t) = -2S_1^zS_2^z - (S_1^z + S_2^z) - h_x(t)(S_1^x + S_2^x)$$

→ symmetry breaking ***on top of correlated*** high-fidelity manifold



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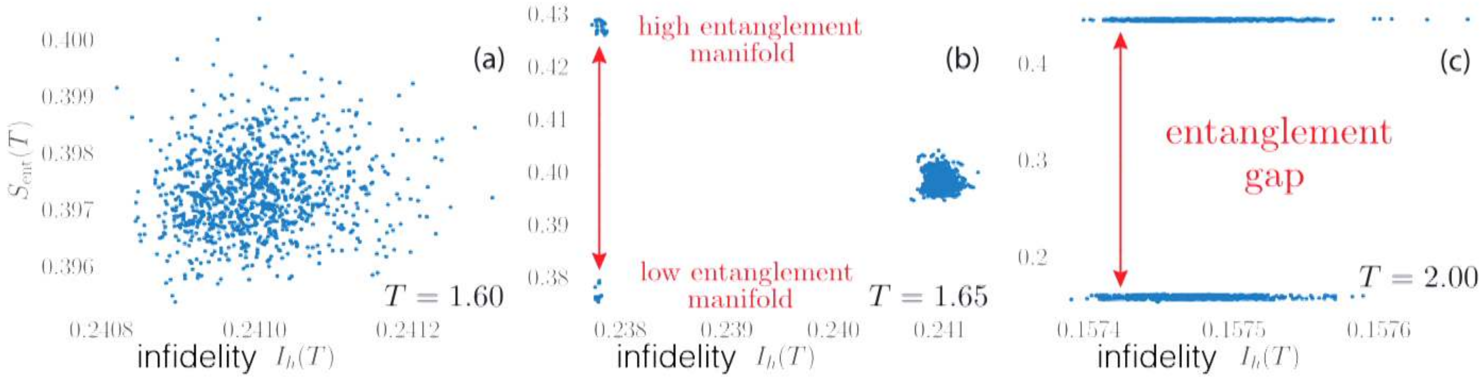
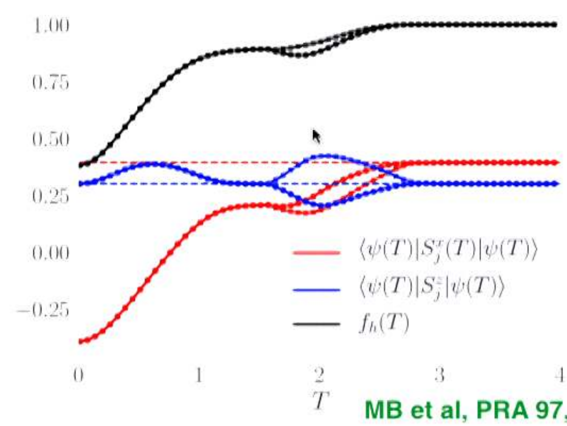


→ can be detected in expectations of local observables



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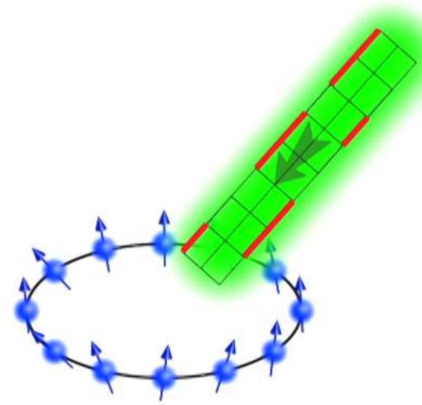
1. Single-spin System

2. Two-spin System

3. Many-Body System

→ strongly correlated (glassy) control phase



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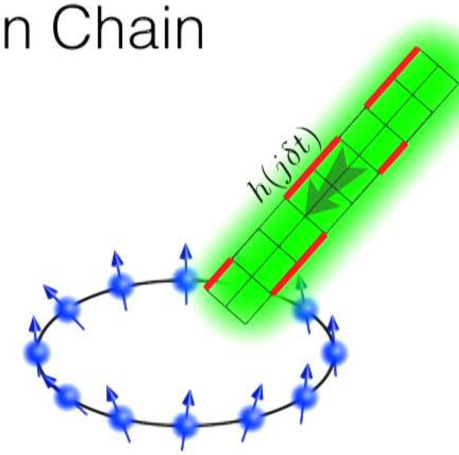
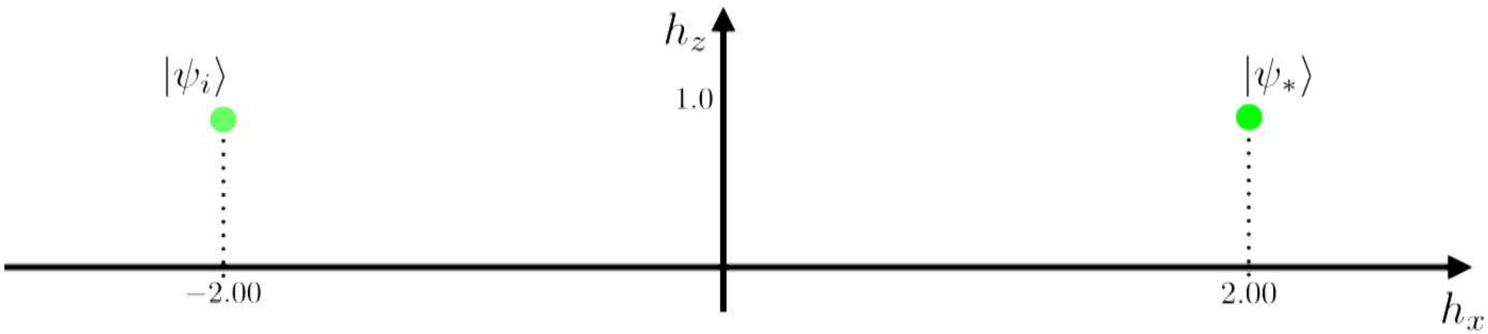
Berkeley UNIVERSITY OF CALIFORNIA Nonintegrable Many-Body Spin Chain

$$H(t) = -\sum_{j=1}^{L} S^z_{j+1} S^z_j + \underbrace{h_z}_{=1} S^z_j + h_x(t) S^x_j$$



→ initial $|\psi_i\rangle$ and target $|\psi_*\rangle$ states are (paramagnetic) GS at:



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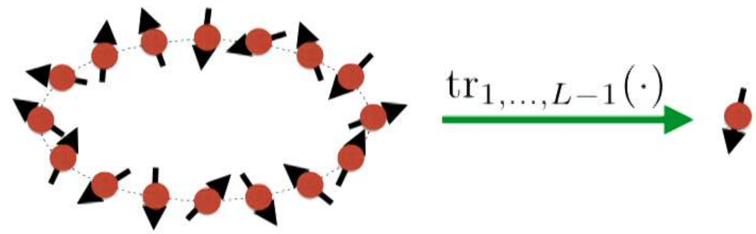
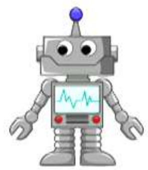
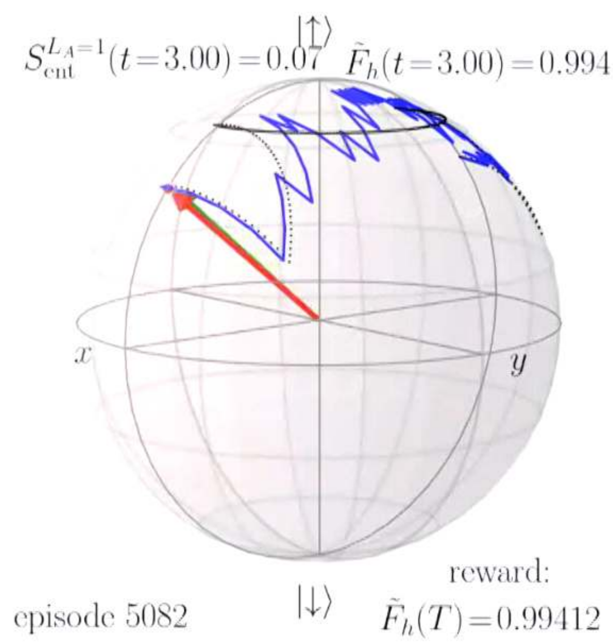
Best RL Many-Body Protocol

$$H(t) = -\sum_{j=1}^{L} S^z_{j+1}S^z_j + S^z_j + h_x(t)S^x_j$$



$h_x \in \{\pm 4\}$ bang-bang protocols





Bloch sphere

$$\tilde{F}_h = \frac{1}{L}\log F_h$$

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MB et al, PRX 8 031086 (2018)

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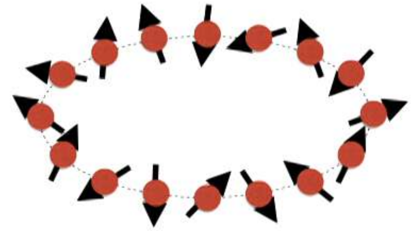
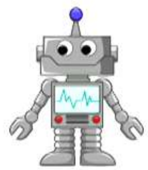
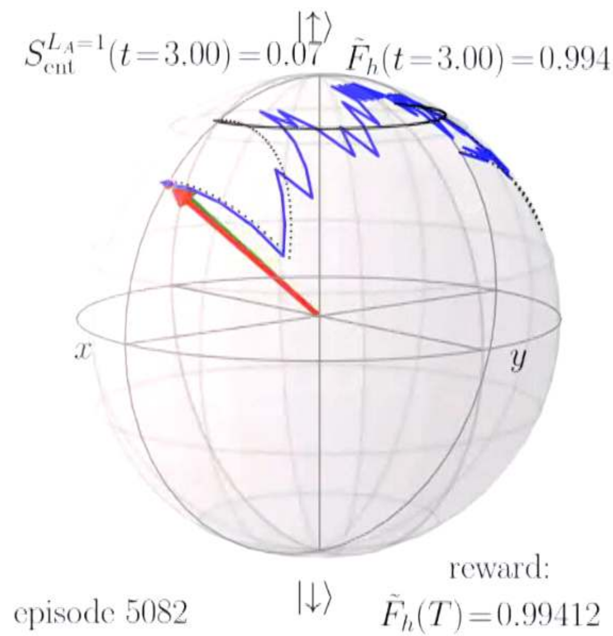


$\mathrm{tr}_{1,\ldots,L-1}(\cdot)$

$h_x \in \{\pm 4\}$ bang-bang protocols





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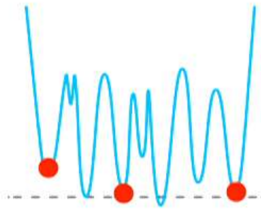
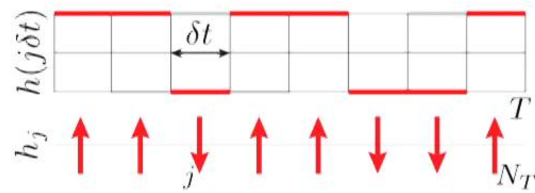
Many-Body Control Phase Diagram

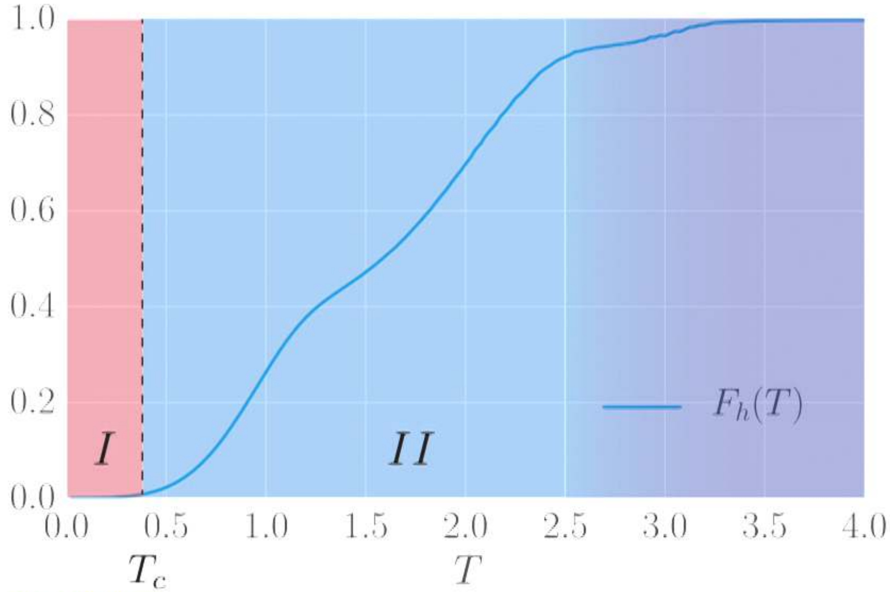
$$H = \sum_j -S^z_{j+1}S^z_j - h_z S^z_j - h_x(t) S^x_j$$

→ ensemble of local minima using Stochastic Descent (zero-temp. MC)



log-fidelity landscape minima: $\{h^\alpha\}$





$$\bar{h}(t) = \frac{1}{\#_{\mathrm{real}}} \sum_\alpha h^\alpha(t)$$

Edwards-Anderson-like order parameter:

$$q(T) \sim \sum_{j=1}^{N_T} \overline{\{h(j\delta t) - \bar{h}(j\delta t)\}^2}$$

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A. Day, M.B. et al, *PRL 122, 020601 (2019)*

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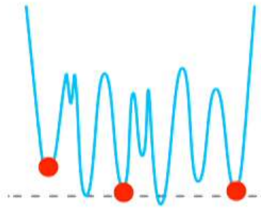

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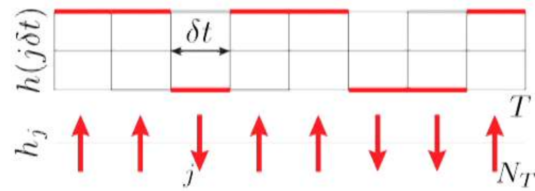

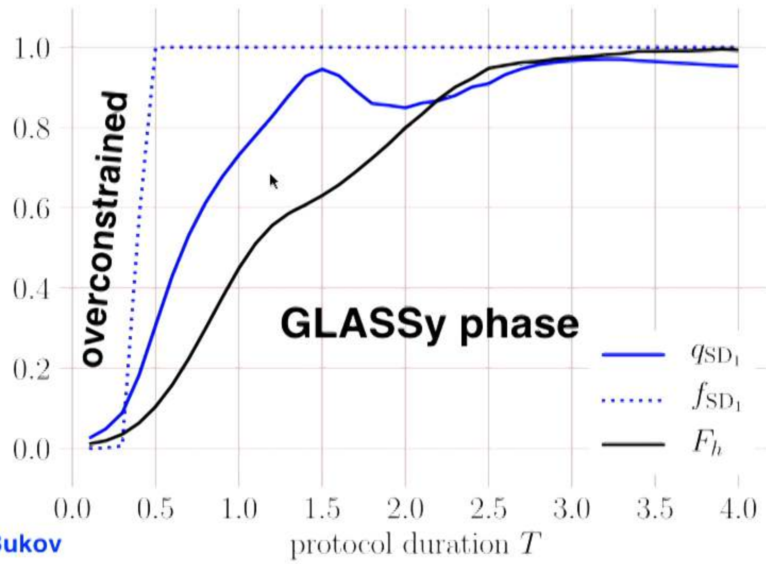

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fraction of unique local log-fidelity minima: $f_h(T)$

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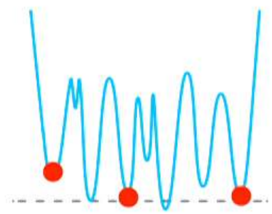

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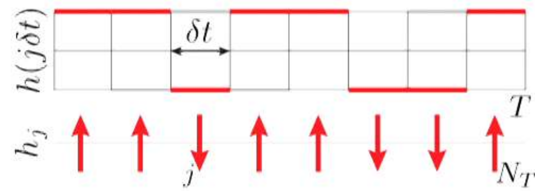

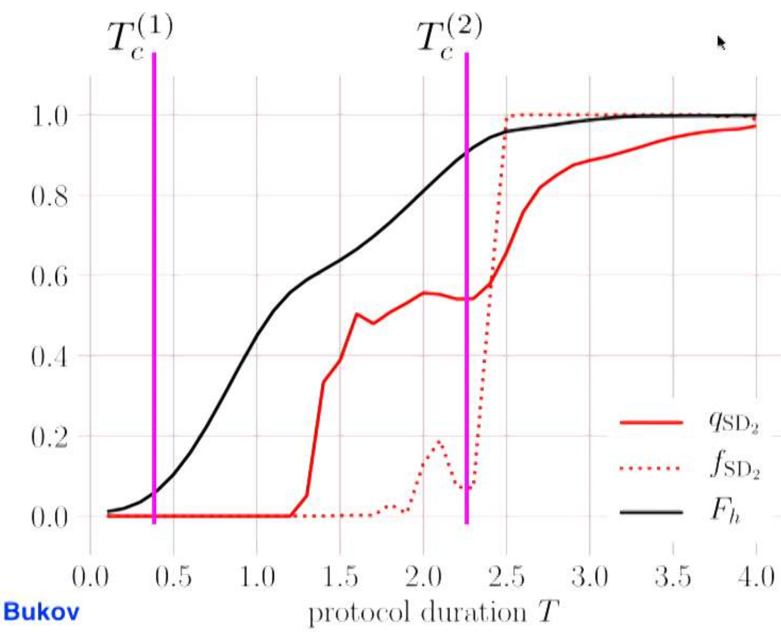

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Visualizing the Glassy Transition with t-SNE

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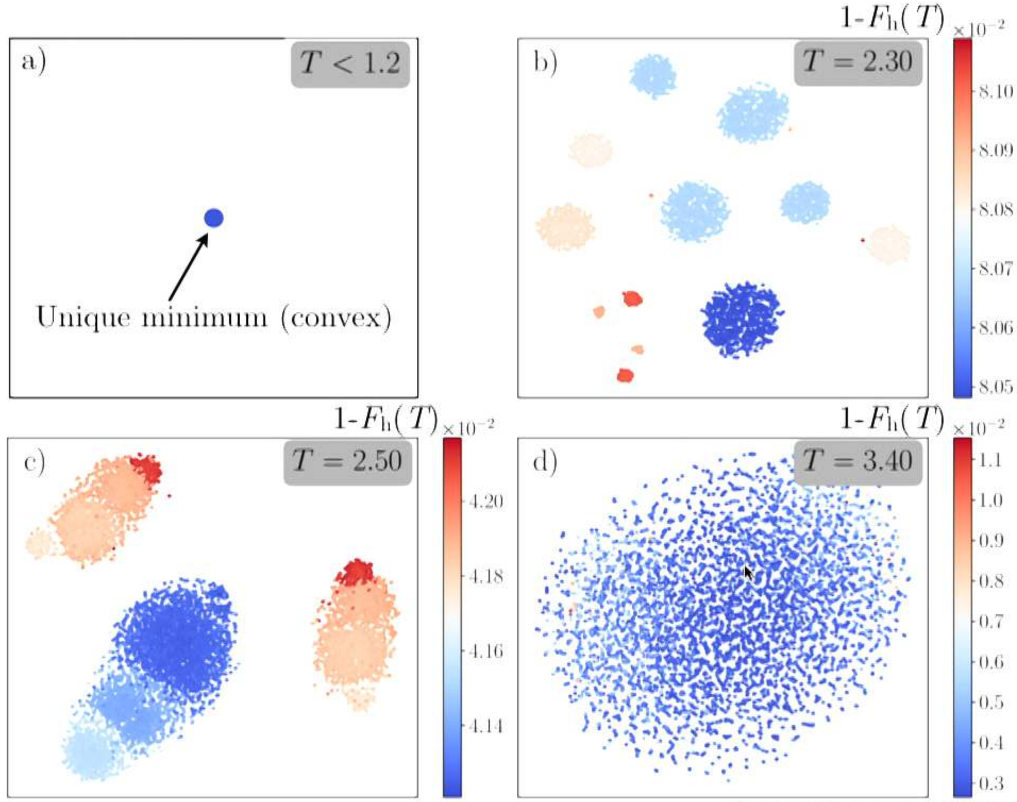

A. Day, M.B. et al, *PRL 122, 020601 (2019)*

Maaten and Hinton, *J of ML Research, 9 2579–2605 (2008)*

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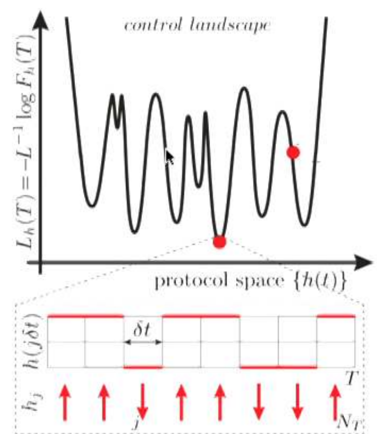
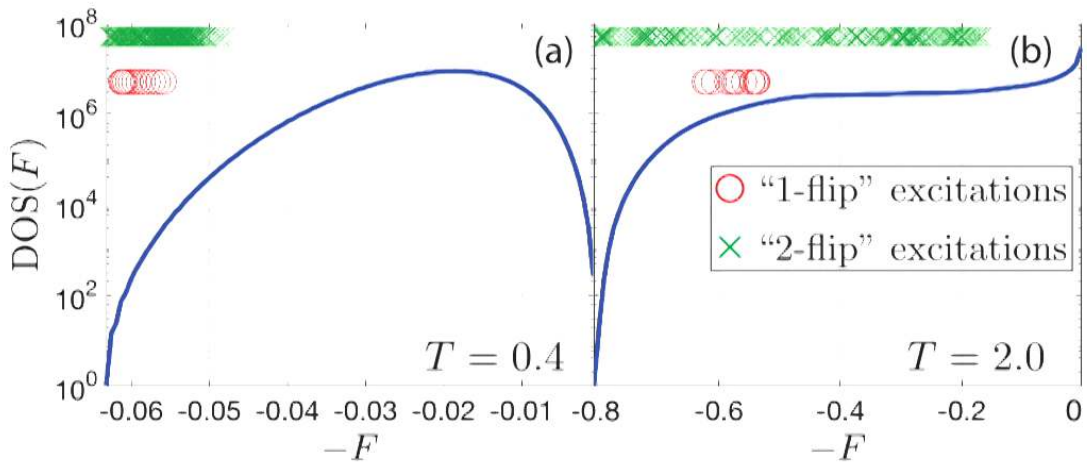
Properties of Effective Model $\mathcal{H}_{\text{eff}}(T)$



→ glassy behavior signatures in the DOS

→ take 28 spins (time steps),
find all energies (fidelities)



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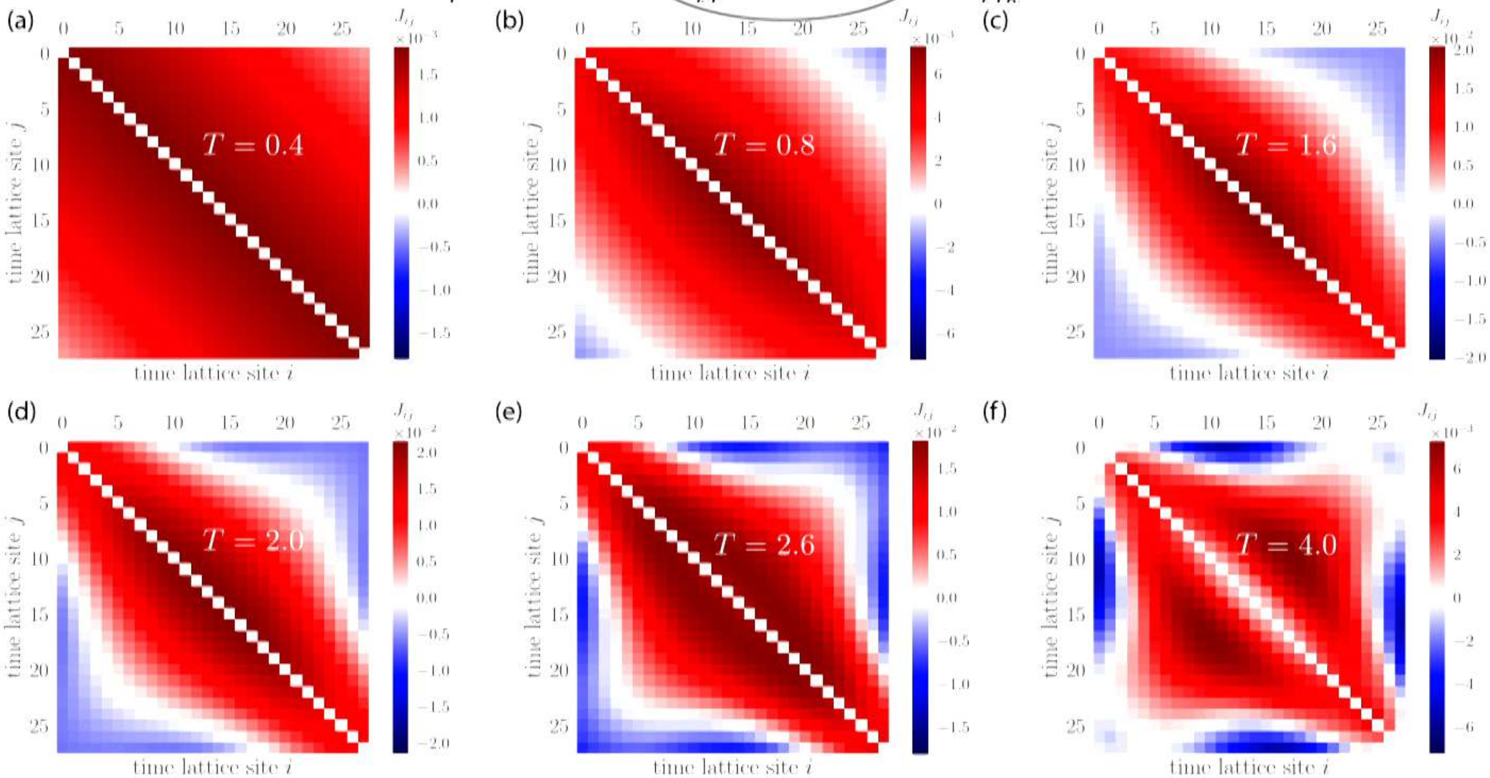
Properties of Effective Model $\mathcal{H}_{\rm eff}(T)$

→ properties of effective coupling strengths: GS is frustrated!

$$\mathcal{H}_{\rm eff}(T) = I(T) + \sum_j G_j(T)h_j + \sum_{ij} J_{ij}(T)h_ih_j + \sum_{ijk} K_{ijk}(T)h_ih_jh_k + \dots$$



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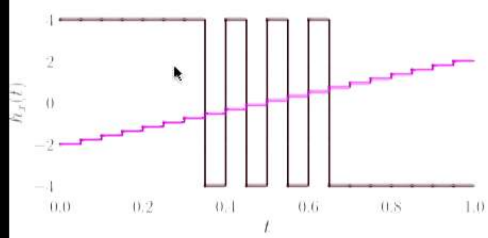
Control Phases: Variational Theory



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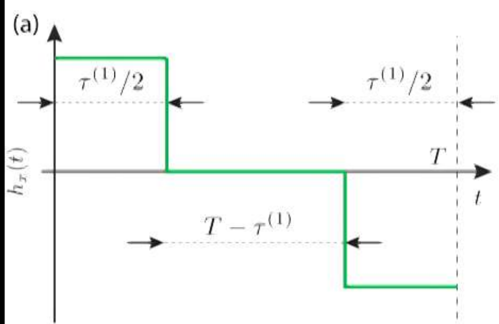

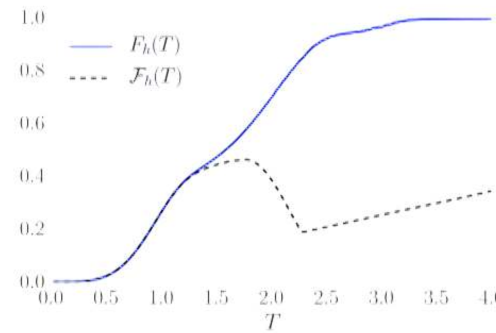

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MB et al, PRX 8 031086 (2018)

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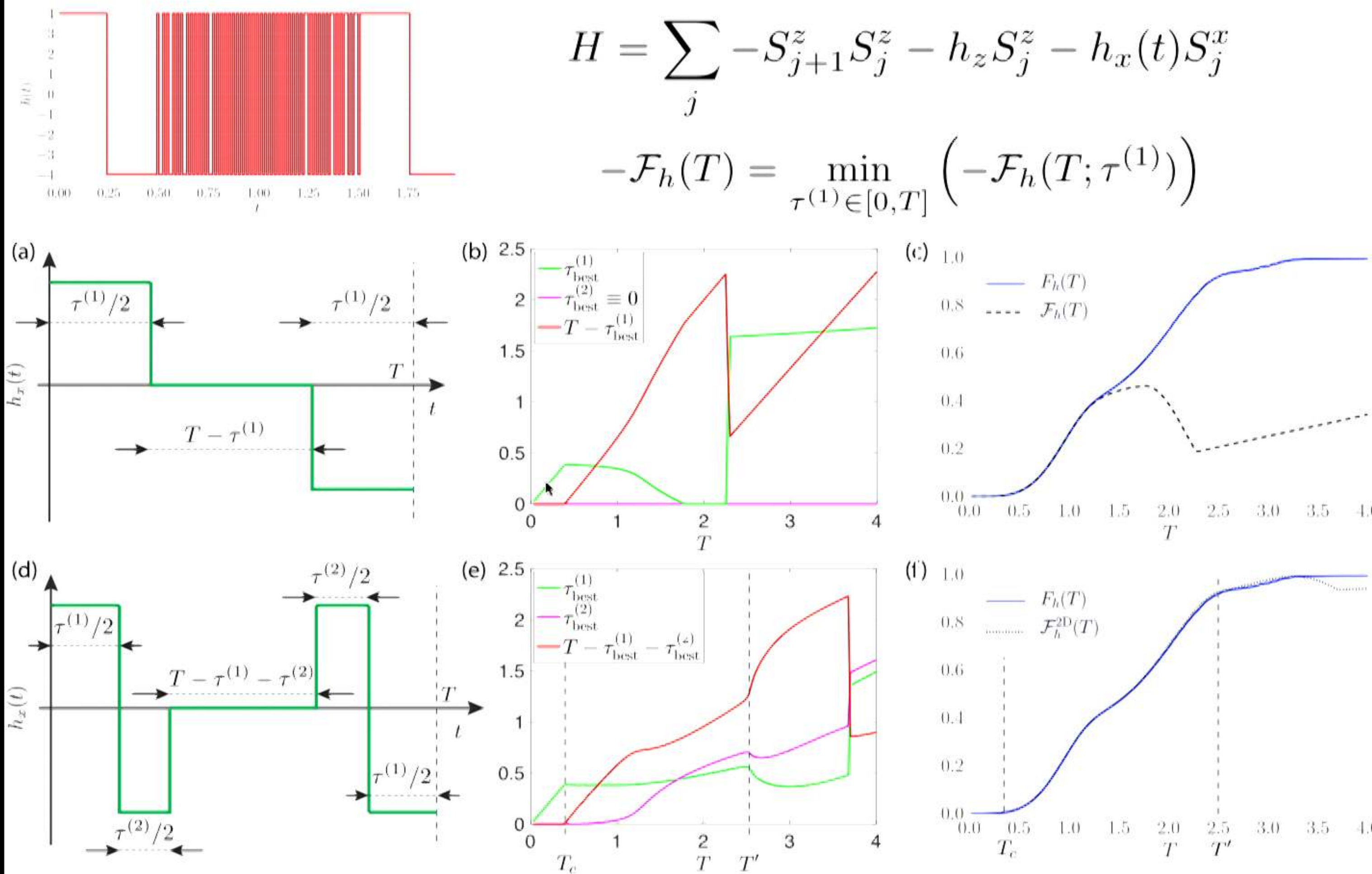

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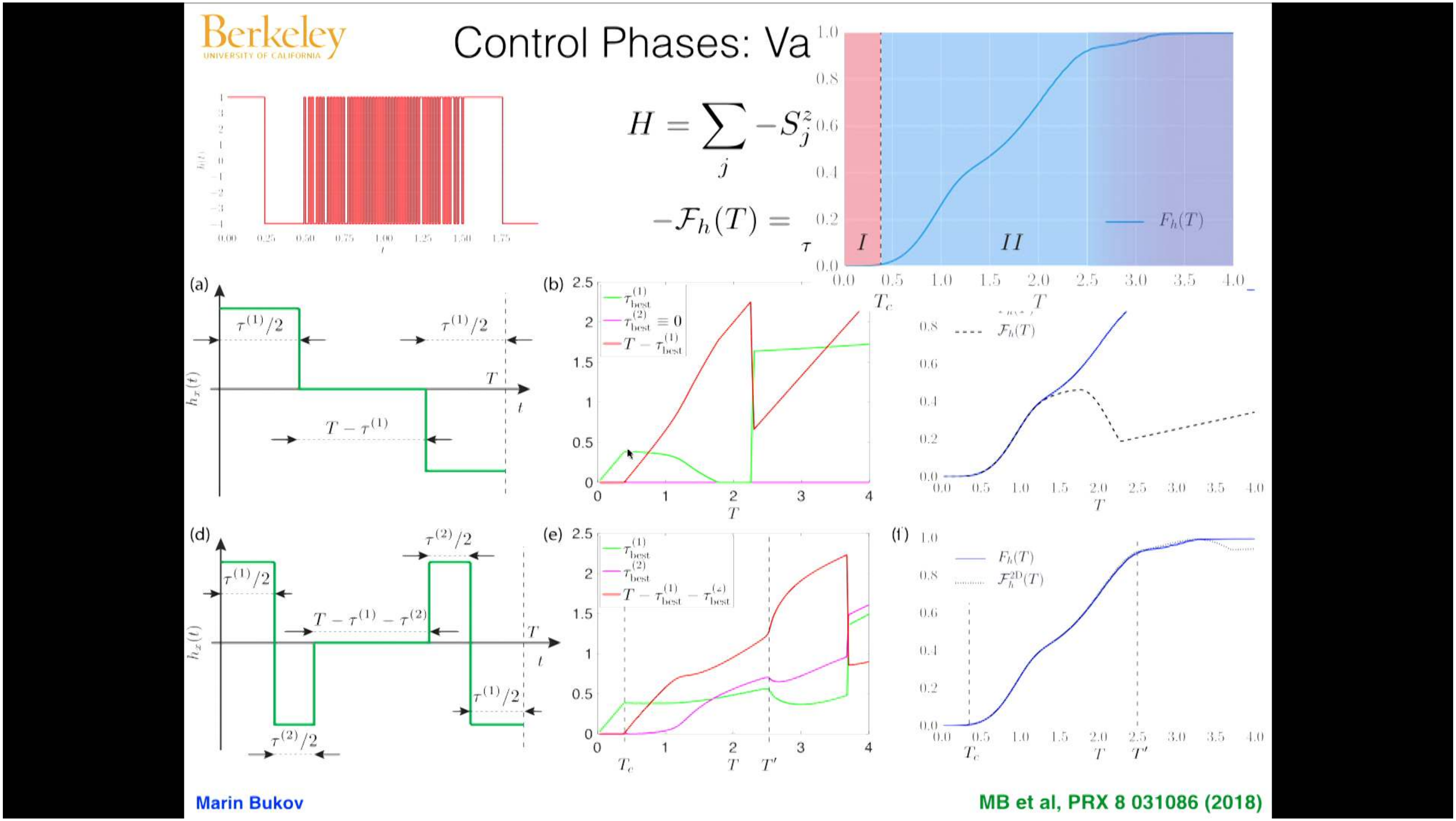

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Control Phases: Va
$H = \sum_j -S^z_j$
$-\mathcal{F}_h(T) =$
τ
I
II
$F_h(T)$
T_c
T
(a)
$\tau^{(1)}/2$
$T - \tau^{(1)}$
$h_x(t)$
(b)
$\tau^{(1)}_{\rm best}$
$\tau^{(2)}_{\rm best} \equiv 0$
$T - \tau^{(1)}_{\rm best}$
$\mathcal{F}_h(T)$
(d)
$\tau^{(2)}/2$
$T - \tau^{(1)} - \tau^{(2)}$
(e)
$\tau^{(2)}_{\rm best}$
T'
(f)
$\mathcal{F}^{\rm 2D}_h(T)$
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Outlook

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- → Control phase transitions found in protocols minimizing work fluctuations
  How generic is this phenomenon? A. Solon et al, *PRL 120, 180605 (2018)*
- → What is the relation of control phase transitions to k-SAT problems?
- → What is the underlying reason for the observed symmetry breaking?
- → How can we understand the success of RL for playing board games?

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spin chain: PRX 8 031086 (2018)
Kapitza oscillator: PRB 98, 224305 (2018)
control phases: PRL 122, 020601 (2019)
PRA 97, 052114 (2018)

QuSpin: *http://weinbe58.github.io/QuSpin*
python package for ED & many-body dynamics (with P. Weinberg, BU)

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web: *mgbukov.github.io*