Title: Astrometric weak lensing from Dark Matter substructures

Speakers: Cristina Mondino

Series: Particle Physics

Date: July 30, 2019 - 1:00 PM

URL: http://pirsa.org/19070000

Abstract: It has been recently pointed out that variable weak gravitational lensing effects on the motion of background stars can be used to probe nonluminous structures inside the Milky Way halo. I will describe one possible detection strategy targeting collapsed dark matter structures in the mass range from million to billion solar masses. The data analysis technique will be discussed in detail with an application to the second data release of Gaia.

Dark Matter

What do we know about DM?

- Gravitational interaction
- Other interactions (if any) are small
- Cold, pressureless fluid

The SM does not provide a DM candidate → Variety of New Physics solutions

Gravitational probes

Why?

- DM interacts gravitationally
- Apply to a wide variety of models
- New information about DM properties (self-interaction, cusp/core)

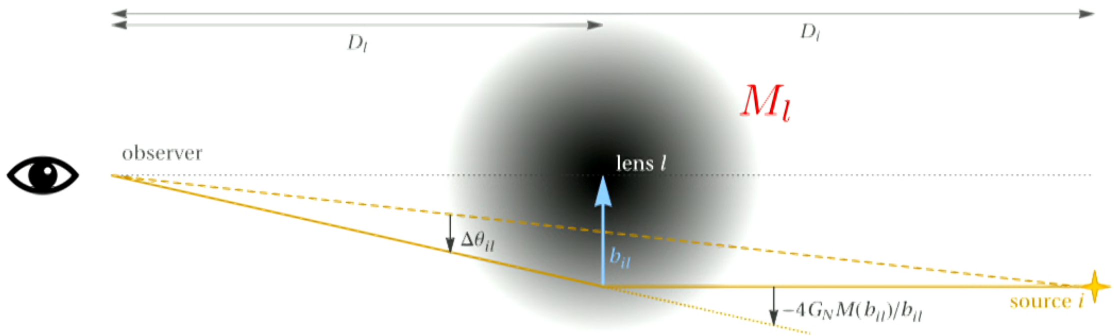
Abell 2218

Bullet Cluster

Outline

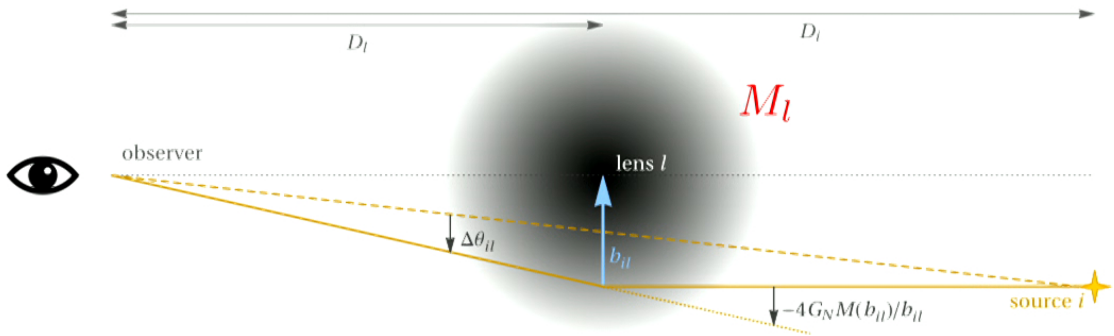
1. Astrometric weak lensing: lens targets
2. Precision astrometry
3. Template method and results

Astrometric weak lensing



- One resolved image
- “Large” impact parameter

Astrometric weak lensing

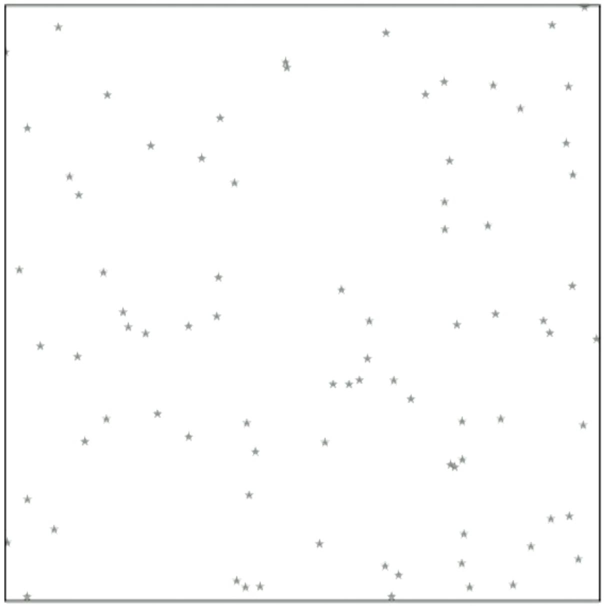


$$\Delta\theta_{il} = -\left(1 - \frac{D_l}{D_i}\right)\frac{4G_N M_l(b_{il})}{b_{il}}\hat{\mathbf{b}}_{il}$$

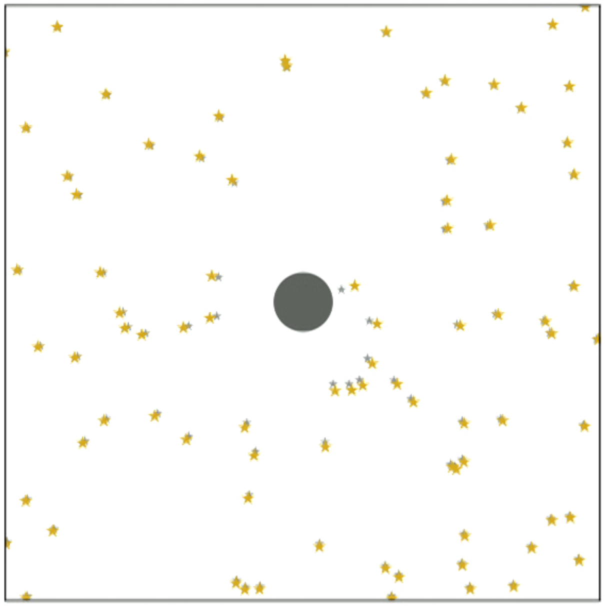
$$\Delta\theta_{il} \sim 40 \text{ mas}\left[\frac{M_l}{10^6\ M_\odot}\right]\left[\frac{1\text{ pc}}{b_{il}}\right]$$

Maybe large enough to detect it?

Astrometric weak lensing



Astrometric weak lensing

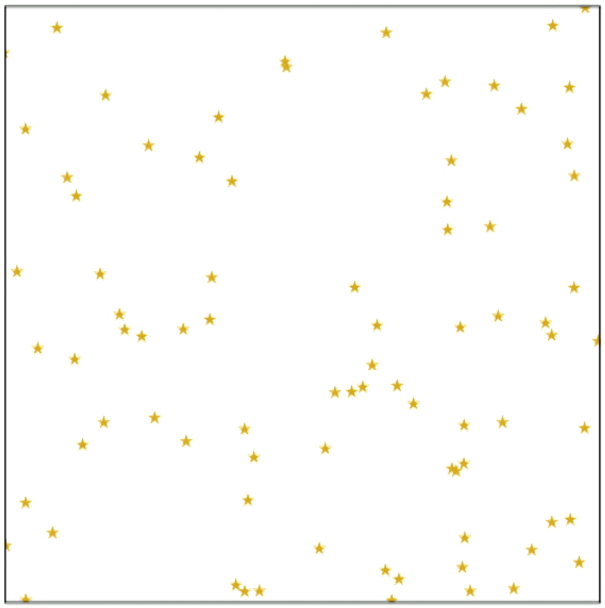


Astrometric weak lensing

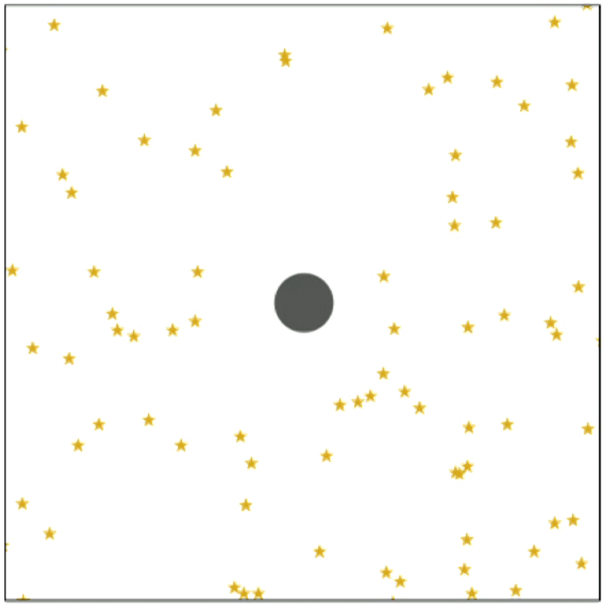
Lensing density fluctuation ~ 10^{-7}

⩓

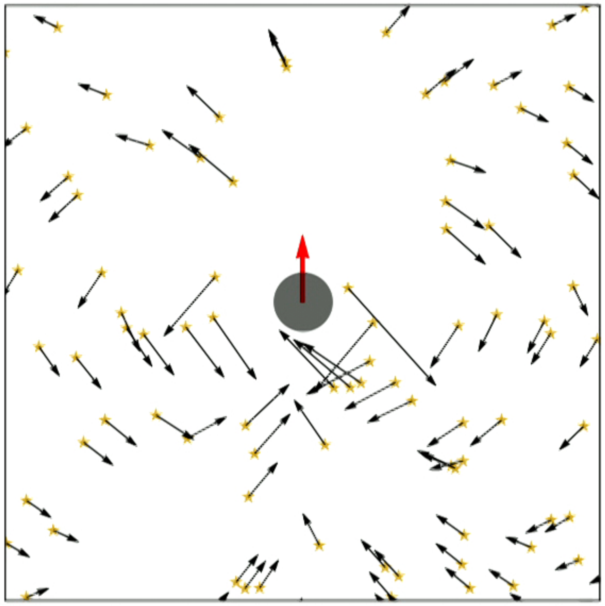
Natural systematic density fluctuations



Time-domain astrometric lensing

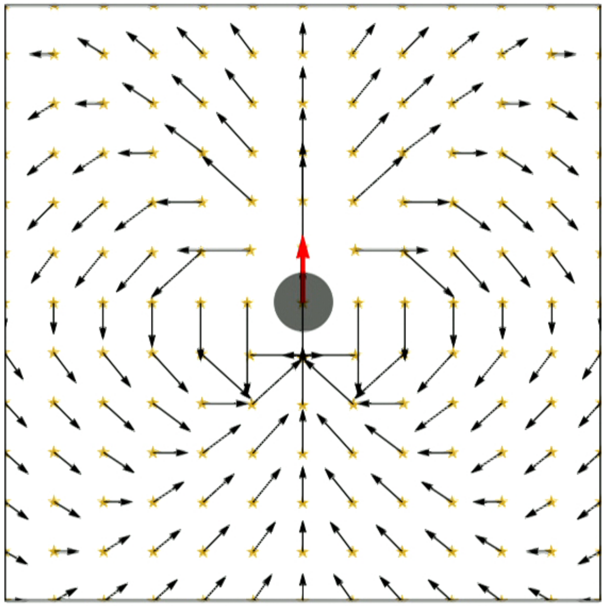


Time-domain astrometric lensing



Time-domain astrometric lensing

Observable:
Local velocity
template



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Halometry from Astrometry

Time-domain astrometric lensing

Motion of the lens during the observation of the source → Impact parameter changes with time

$$\frac{\Delta b_{il}}{b_{il}} \ll 1$$

$$\Delta\dot{\theta}_{il} \sim \frac{4G_N M_l v_{il}}{b_{il}^2}$$

$$\Delta\dot{\theta}_{il} \sim 10\ \frac{\mu\text{as}}{\text{y}} \left[\frac{M_l}{10^6 M_\odot}\right] \left[\frac{(1\ \text{pc})^2}{b_{il}^2}\right] \left[\frac{v_{il}}{10^{-3}c}\right]$$

Below current experimental precision

Time-domain astrometric lensing

Motion of the lens during the observation of the source → Impact parameter changes with time

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Below current experimental precision

but

- can be detected in **aggregate**
- precision will get better with time

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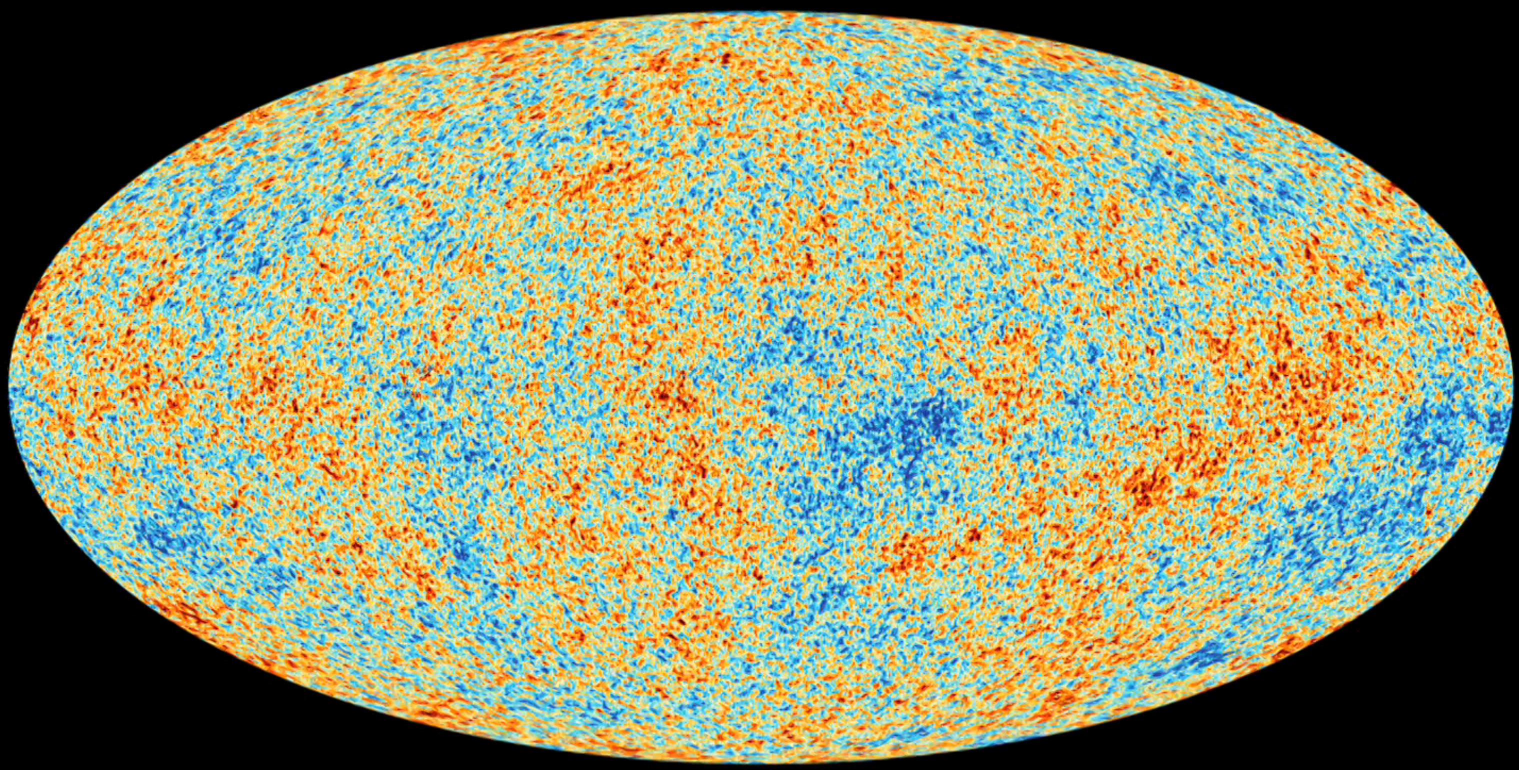
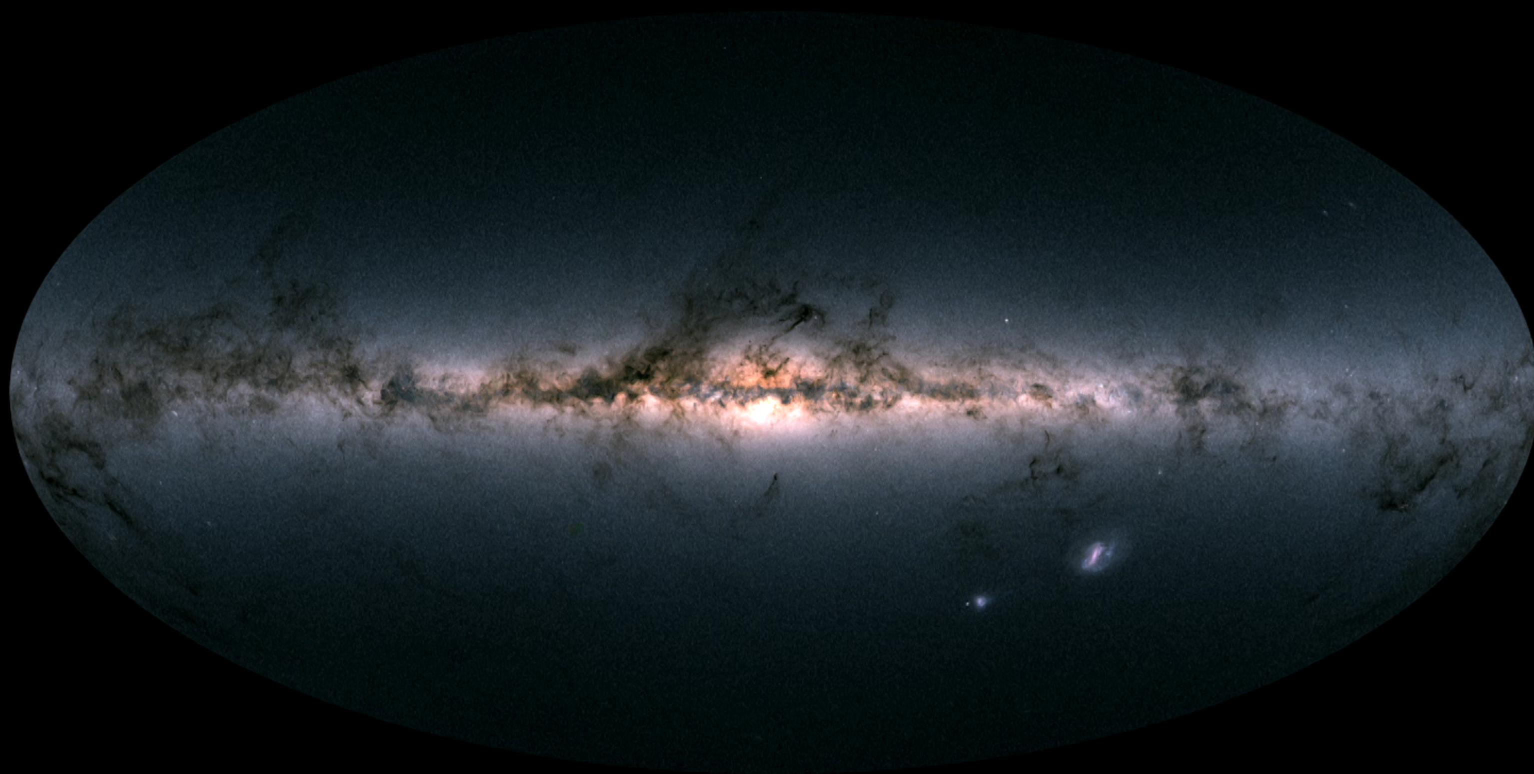
Halometry from Astrometry

CMB Temperature Fluctuations



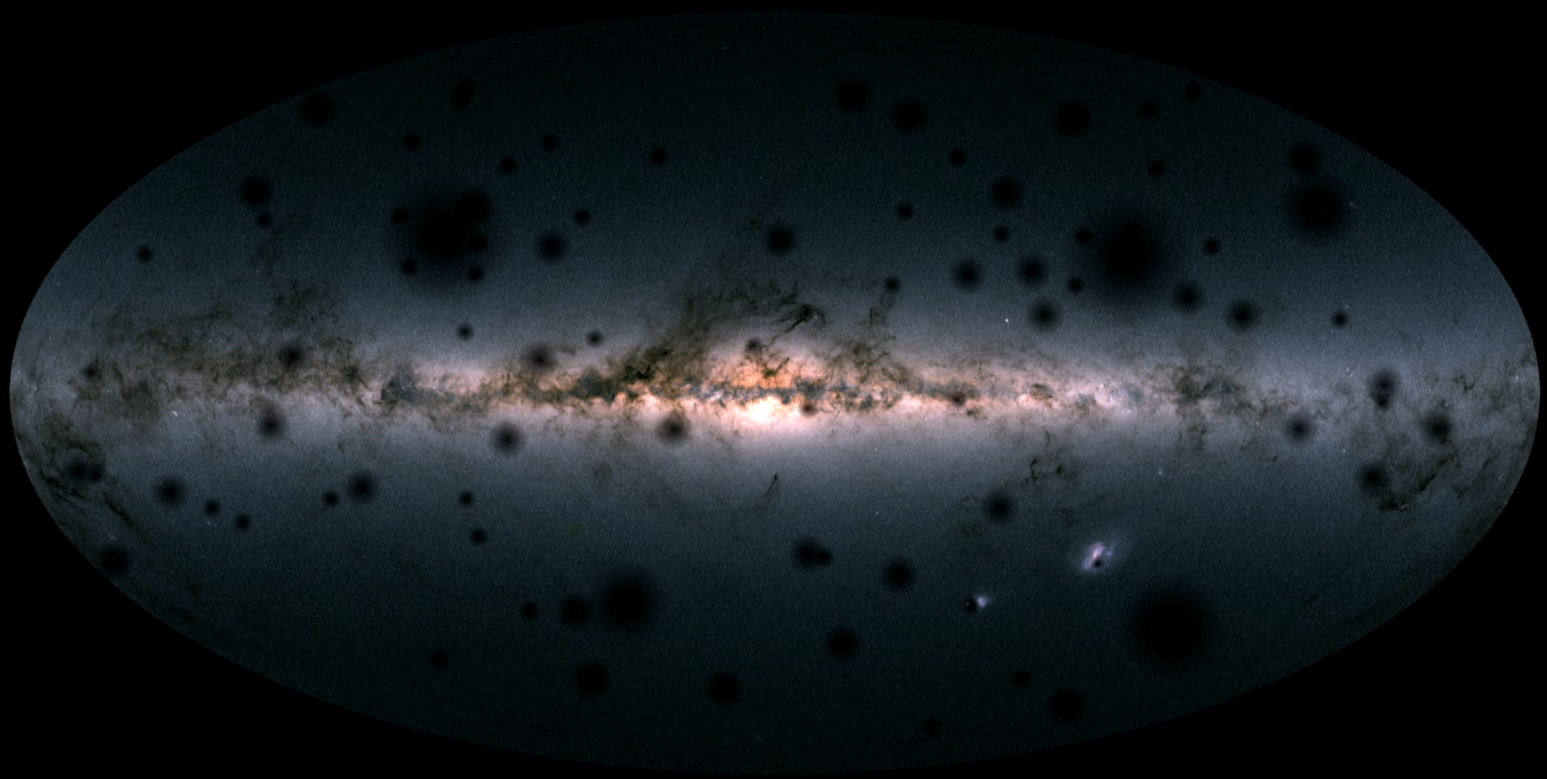
Planck

DM Density Fluctuations



ESA Gaia

DM Density Fluctuations



ESA Gaia

Lensing from DM substructure

Challenge:

For extended subhalos, the angular deflection is suppressed by the lens size

Solution:

Use high statistics and correlated effects!

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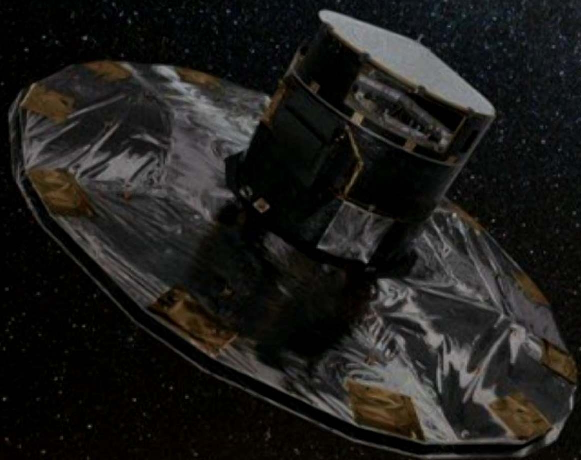
Halometry from Astrometry

Precision astrometry

ESA Gaia

- 1.7 billion stars
- velocities for 1.3 billion stars
- $\mathcal{O}(100\ \mu\mathrm{as})$ precision

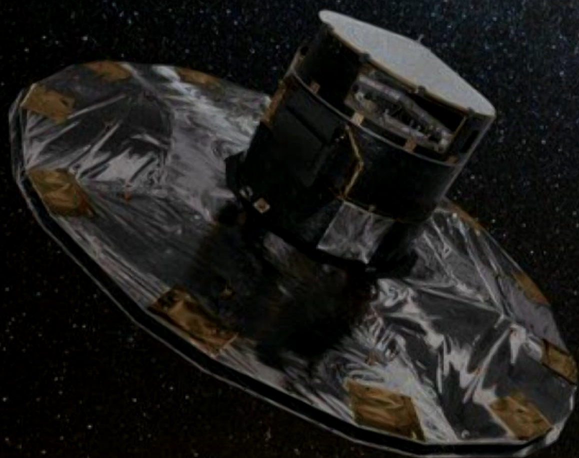


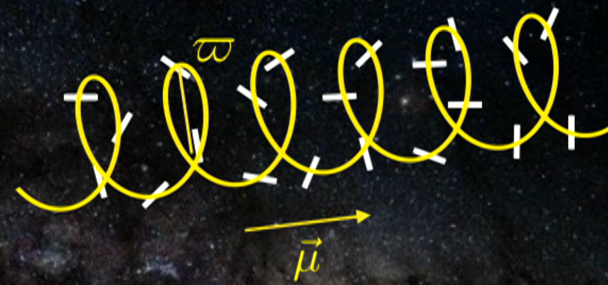
Repeated single star observations

Precision astrometry

ESA Gaia

- DR2 ~ 1 year ago
- 1.7 billion stars
- velocities for 1.3 billion stars
- $\mathcal{O}(100\ \mu\mathrm{as})$ precision





Repeated single star observations

5 parameters astrometric solution:
- 2d position
- 2d velocity $\vec{\mu}$
- parallax ϖ

Stellar targets

- Small intrinsic proper motion dispersion
- A lot of DM along the line of sight
- High number density

} Objects which are far away:

Dwarf Galaxies

Quasars

Stellar targets

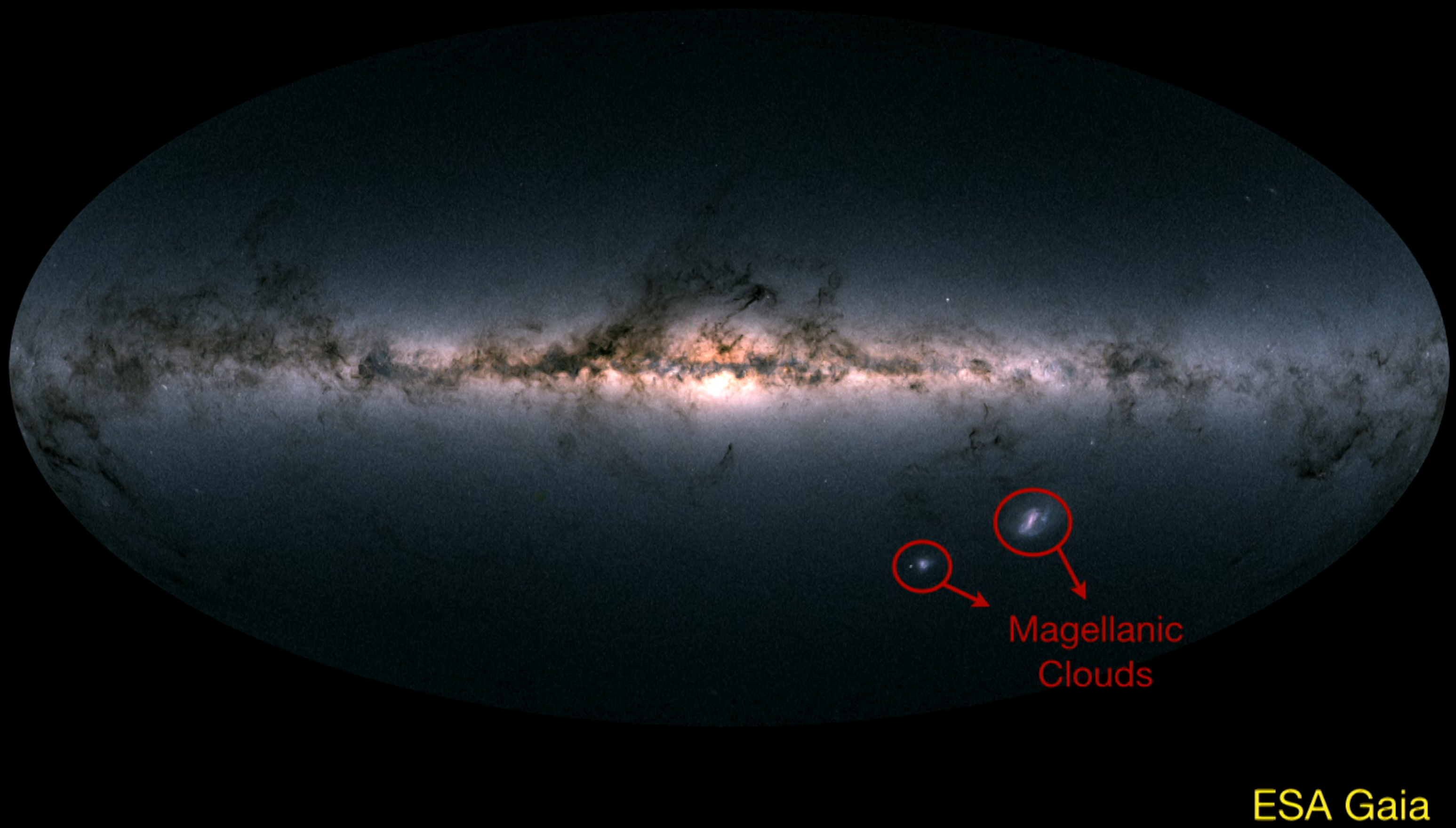
- Small intrinsic proper motion dispersion
- A lot of DM along the line of sight
- High number density

Objects which are far away:

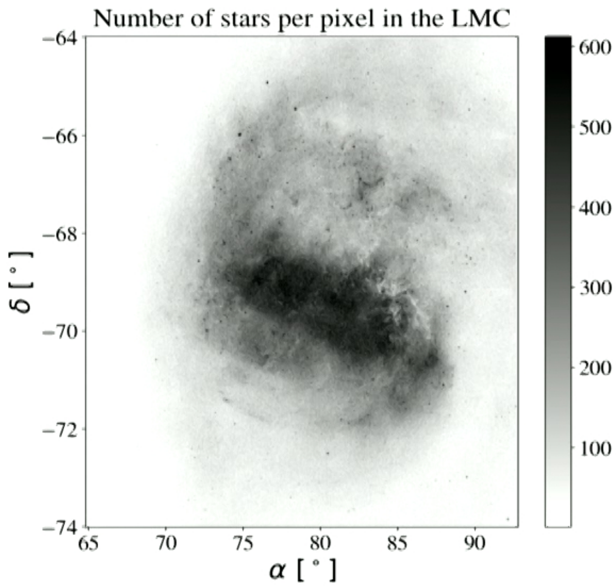
Dwarf Galaxies → With current GAIA data

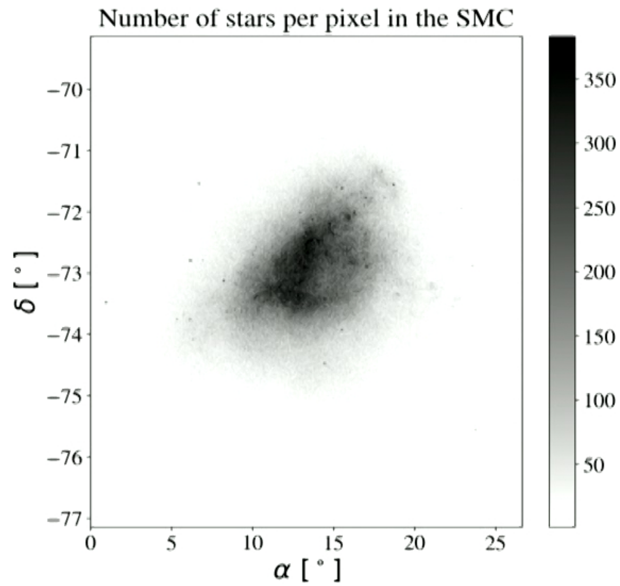
Quasars → Future

Stellar targets



Magellanic Clouds

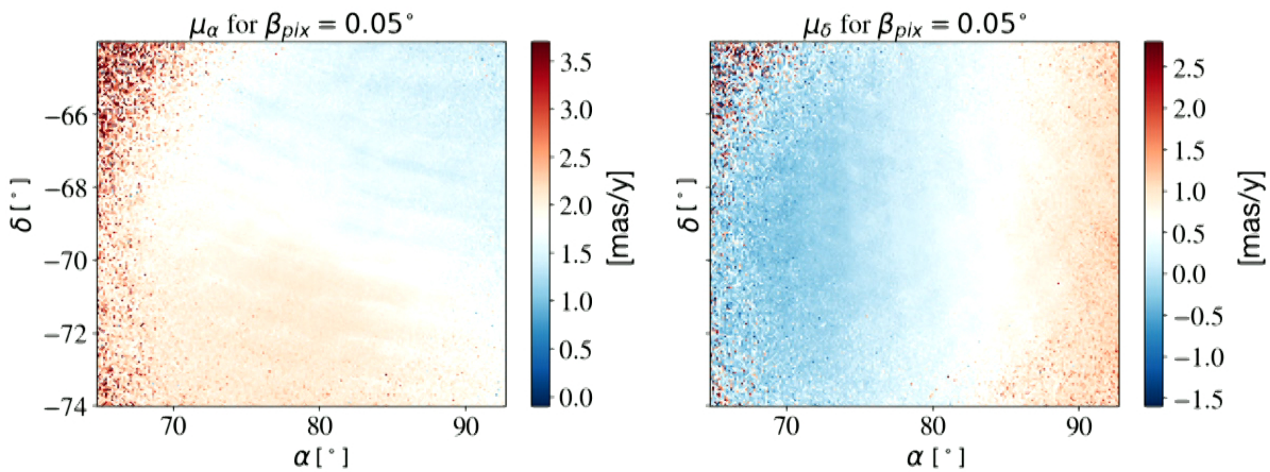




- 50 kpc away
- ~ 10 million stars
- 10^{o}x 10^{o} on the sky

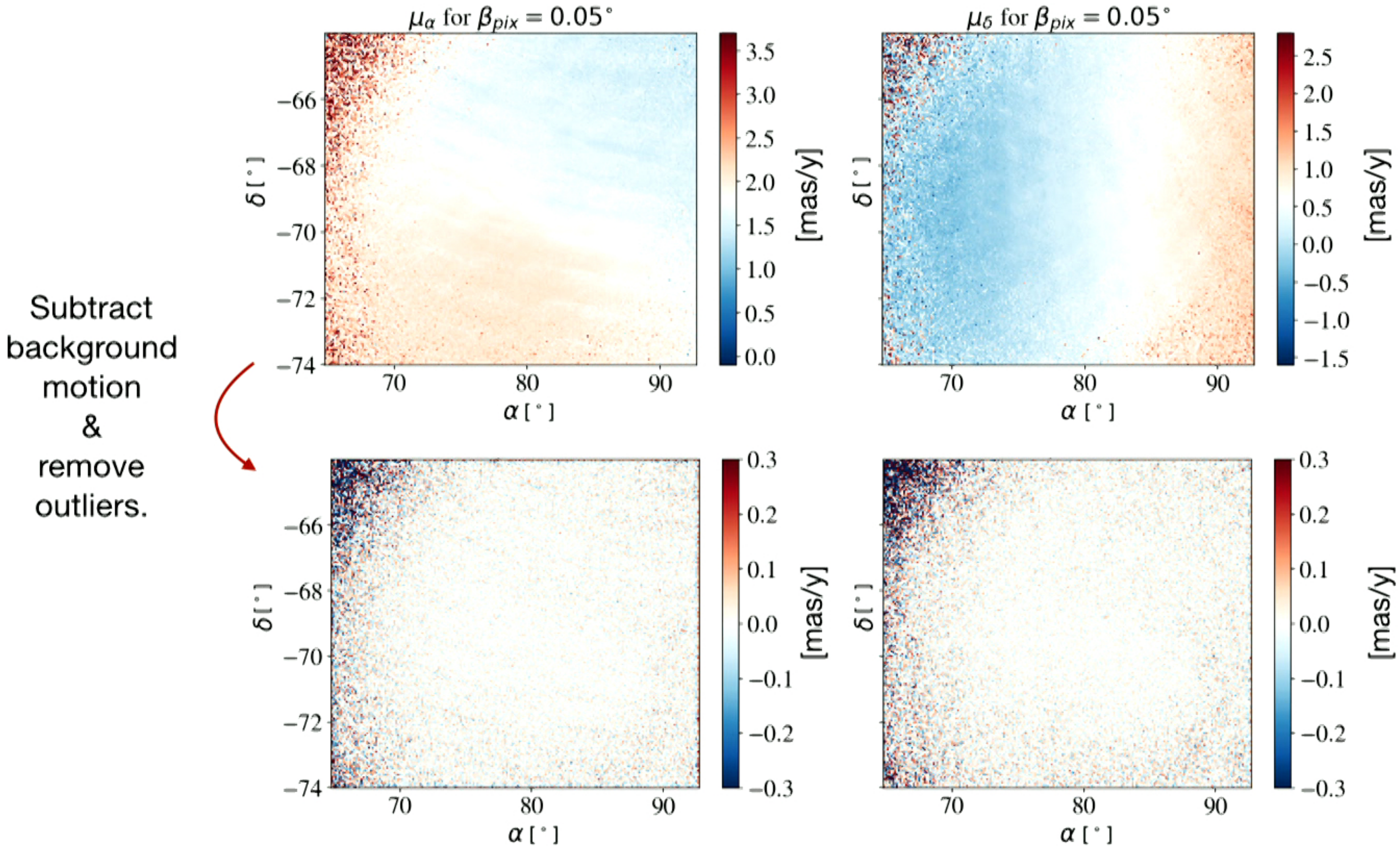
- 60 kpc away
- ~ 1.5 million stars
- 6^{o}x 6^{o} on the sky

Background motion



1. The large scale motion is subtracted at scale 0.1^0 >> lensing effects
2. 3σ velocity outliers are removed to reduce foreground contamination

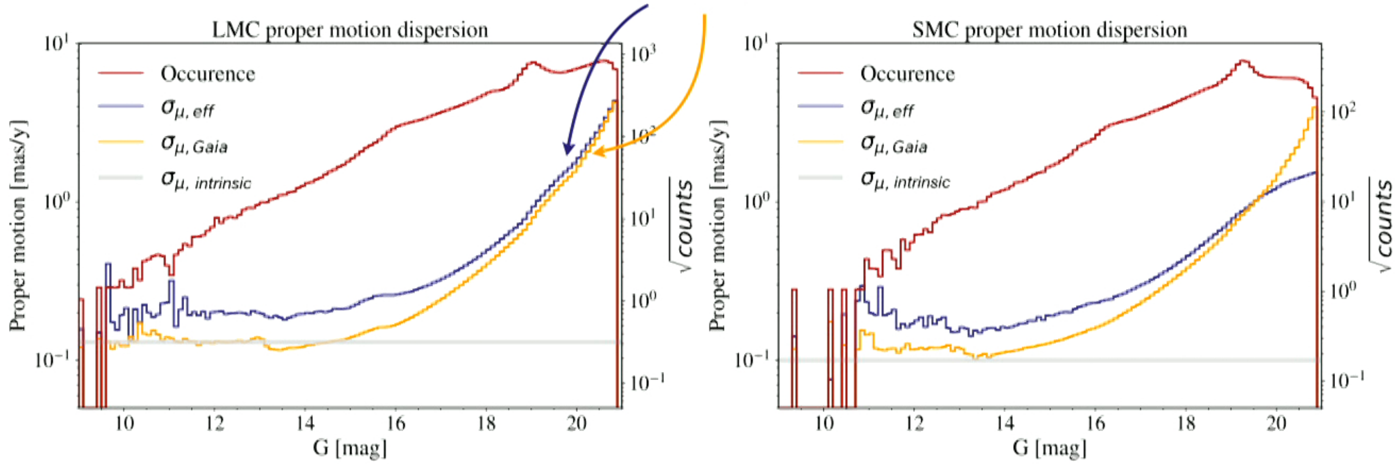
Background motion



Proper motion dispersion

The quality of the astrometric measurement depends on the stellar brightness

Max between
the two



Outline

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3. Template method and results

Local test statistic

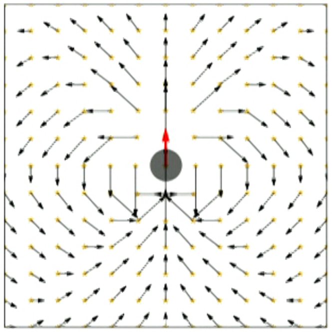
Hypothesis:

- lens with mass M_l and size r_l

Measurement:

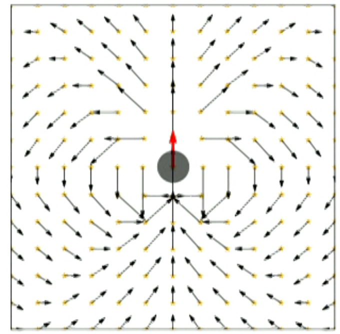
- stars 2d velocities $\vec{\mu}_i$ with uncertainties $\sigma_{\mu,i}$

What is the **best test statistic**?



Local test statistic

What is the **best test statistic**?

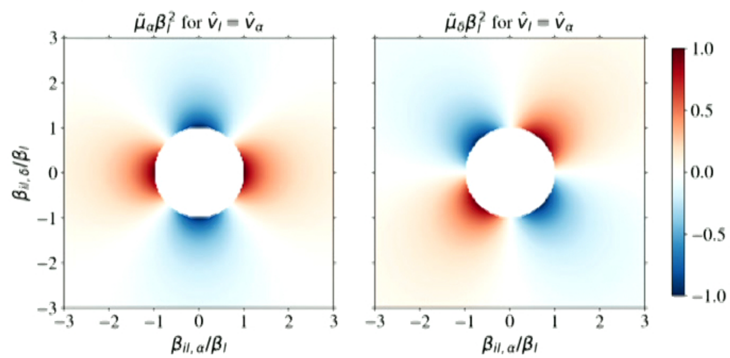


Stellar velocity

$$\mathcal{T} \equiv \sum_i \frac{\vec{\mu}_i \cdot \vec{\tilde{\mu}}_i}{\sigma_{\mu,i}^2}$$

Velocity uncertainty

Matched filter:

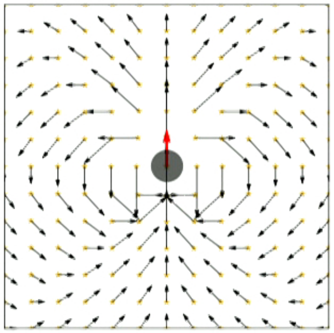


Signal-to-noise ratio

$$\mathcal{T} \equiv \sum_i \frac{\vec{\mu}_i \cdot \vec{\tilde{\mu}}_i}{\sigma^2_{\mu,i}} \qquad \mathcal{N}^2 \equiv \sum_i \frac{|\vec{\tilde{\mu}}_i|^2}{\sigma^2_{\mu,i}}$$

Local test statistic

What is the **best test statistic**?



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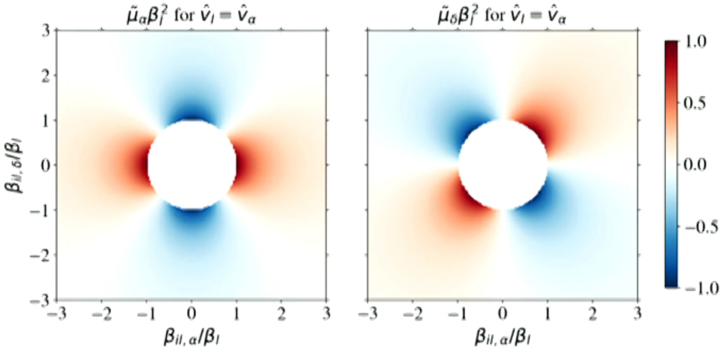

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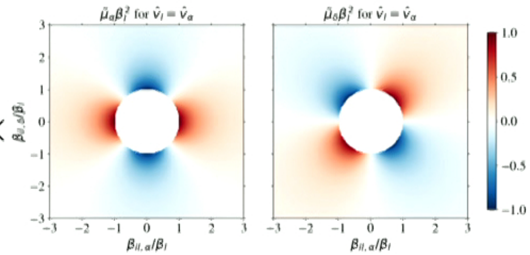
For signal: $\langle \vec{\mu}_i \rangle = \underbrace{\frac{4G_N M_l \beta_l^2 v_l}{r_l^2}}_{C_l} \times$



$$\langle \mathcal{T} \rangle_{\text{signal}} = C_l \mathcal{N}^2$$

Signal-to-noise ratio

$$\mathcal{T} \equiv \sum_i \frac{\vec{\mu}_i \cdot \vec{\tilde{\mu}}_i}{\sigma^2_{\mu,i}} \qquad \mathcal{N}^2 \equiv \sum_i \frac{|\vec{\tilde{\mu}}_i|^2}{\sigma^2_{\mu,i}}$$

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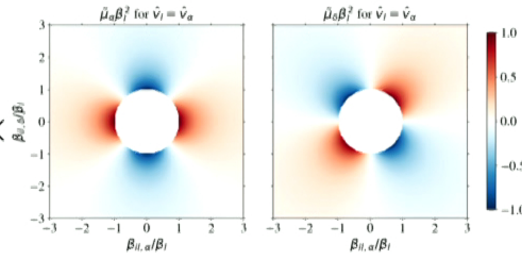


$$\langle \mathcal{T} \rangle_{\text{signal}} = C_l \mathcal{N}^2$$

For noise: $\langle \vec{\mu}_i \rangle = 0 \qquad \langle \vec{\mu}_i{}^2 \rangle = \sigma^2_{\mu,i}$

$$\langle \mathcal{T}^2 \rangle_{\text{noise}} = \mathcal{N}^2$$

Signal-to-noise ratio

For signal: $\langle \mathcal{T} \rangle_{\text{signal}} = C_l \mathcal{N}^2$

For noise: $\langle \mathcal{T}^2 \rangle_{\text{noise}} = \mathcal{N}^2$

$$\text{SNR} = \frac{\langle \mathcal{T} \rangle_{\text{signal}}}{\sqrt{\langle \mathcal{T}^2 \rangle_{\text{noise}}}} = C_l \mathcal{N} \propto \frac{M_l}{r_l D_l} \frac{\sqrt{\Sigma_s}}{\sigma_{\mu,\text{eff}}}$$

Signal-to-noise ratio

For signal: $\langle \mathcal{T} \rangle_{\rm signal} = C_l \mathcal{N}^2$

For noise: $\langle \mathcal{T}^2 \rangle_{\rm noise} = \mathcal{N}^2$

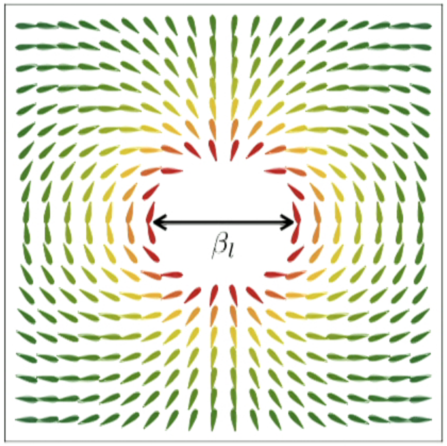
$$\mathrm{SNR} = \frac{\langle \mathcal{T} \rangle_{\rm signal}}{\sqrt{\langle \mathcal{T}^2 \rangle_{\rm noise}}} = C_l \mathcal{N} \propto \frac{M_l}{r_l D_l} \frac{\sqrt{\Sigma_s}}{\sigma_{\mu,\rm eff}}$$

Maximized by the closest lens: $D_{l,\rm min} \propto \left(\frac{M_l}{\Omega_l} \frac{1}{\Delta\Omega} \right)^{1/3}$

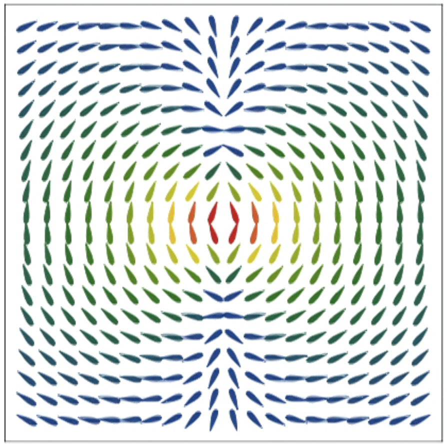
$$\mathrm{SNR} \propto \frac{{M_l}^{2/3} {\Omega_l}^{1/3}}{r_l} \frac{\Delta\Omega^{1/3} \sqrt{\Sigma_s}}{\sigma_{\mu,\rm eff}}$$

Truncated profile vs NFW

Truncated density profile



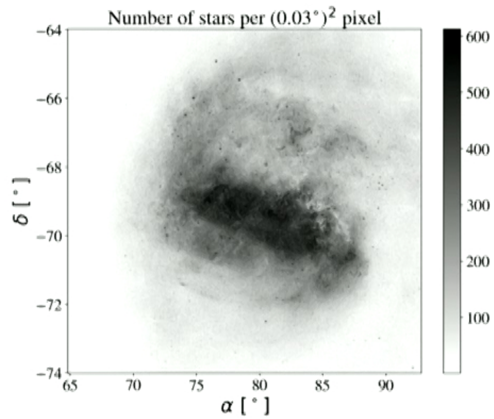
NFW density profile



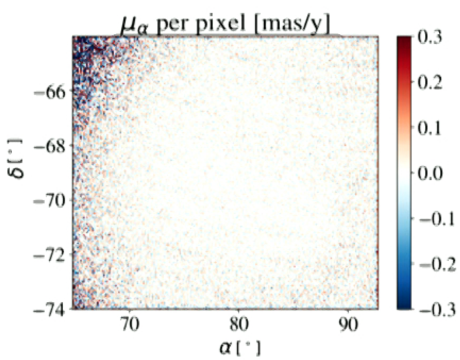
~ 50% of SNR

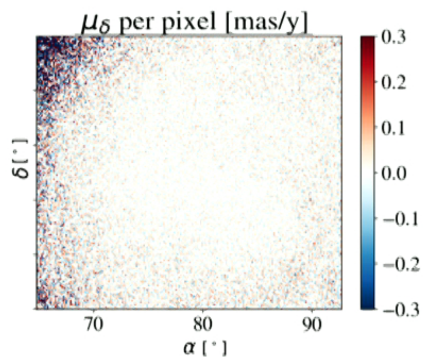
What do we actually do?

$$\mathcal{T} \equiv \sum_i \frac{\vec{\mu}_i \cdot \vec{\tilde{\mu}}_i}{\sigma^2_{\mu,i}}$$

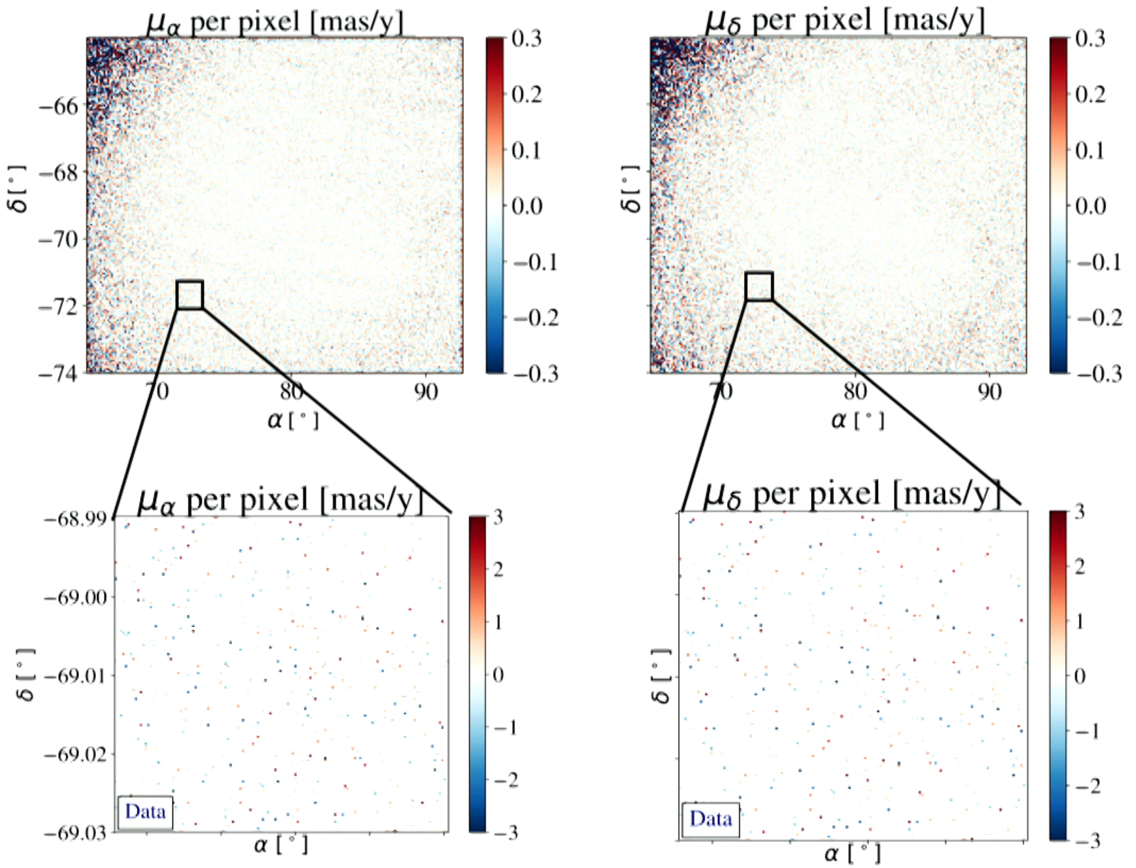


Cleaned stellar motions

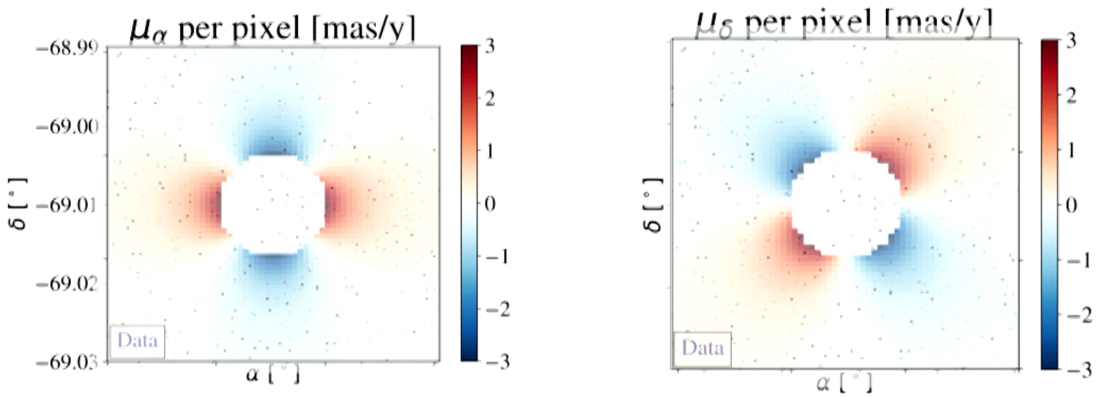




What do we actually do?



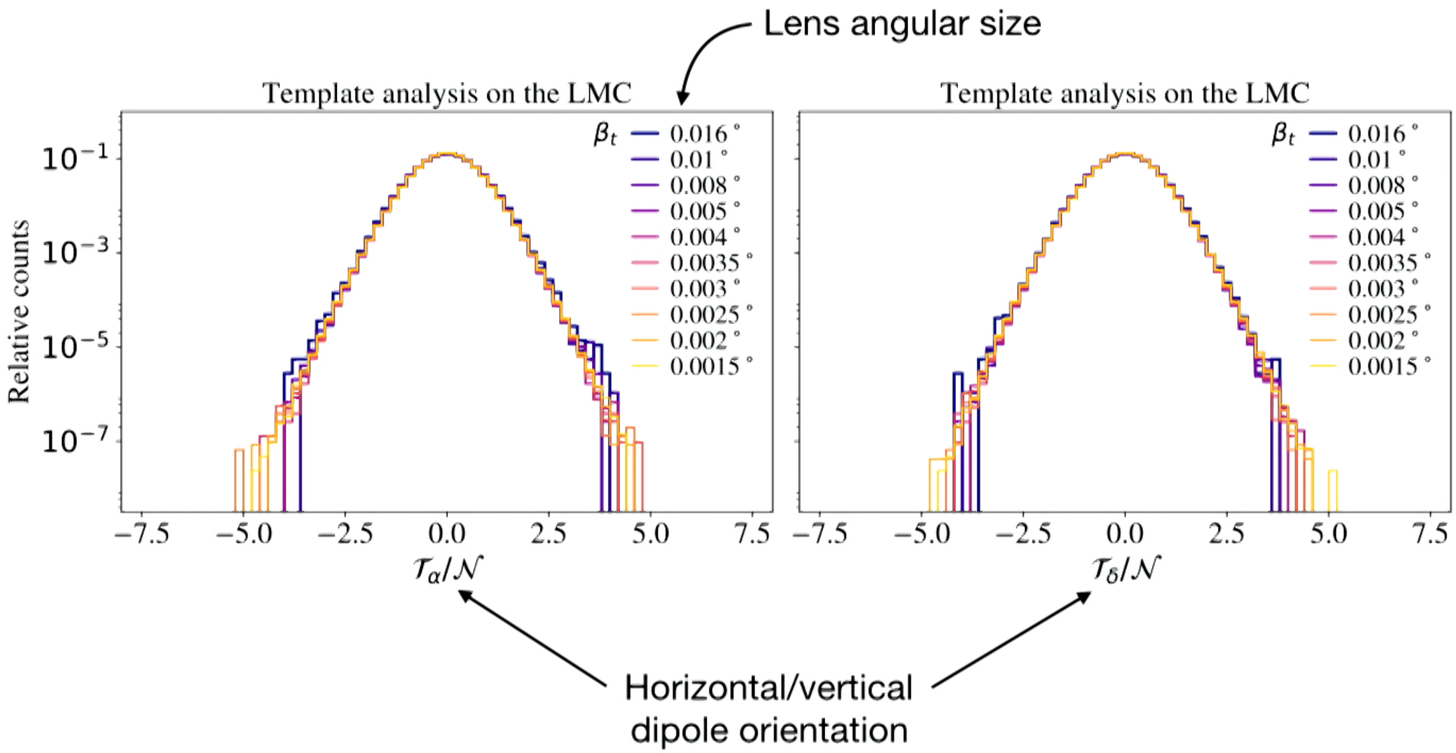
What do we actually do?



$$\Rightarrow \mathcal{T} \equiv \sum_i \frac{\vec{\mu}_i \cdot \vec{\tilde{\mu}}_i}{\sigma^2_{\mu,i}}$$

Repeat for different template locations: scan over the sky

Template sanning



Global test statistic

How do we test for a **population** of lenses?

Global test statistic

Simplifying **assumptions**:

- most of the signal comes from the closest lens (larger angular size $\beta_l = r_l / D_l$)

- the measured errors are gaussian

Likelihood

Simplifying **assumptions**:

- most of the signal comes from the closest lens (larger angular size $\beta_l = r_l / D_l$)

- the measured errors are gaussian

$$\mathcal{L}(\vec{\mu}_i | M_l, r_l) = f_1(\vec{\theta}_l, \beta_l) f_2(\vec{v}_l) \prod_i \frac{1}{2\pi\sigma_{\mu,i}^2} \exp\left[-\frac{(\vec{\mu}_i - \Delta\vec{\mu}_i)^2}{2\sigma_{\mu,i}^2} \right]$$

Probability of a lens:
at position $\vec{\theta}_l$
with size β_l
with velocity $\vec{v}_l$

Probability for the observed star velocities

Likelihood ratio

The best **test statistic** is the maximum **Likelihood Ratio**:

$$\textbf{LR} = \frac{\mathcal{L}\left(\vec{\mu}_i | M_l, r_l\right)}{\mathcal{L}\left(\vec{\mu}_i | \text{no lens}\right)}$$

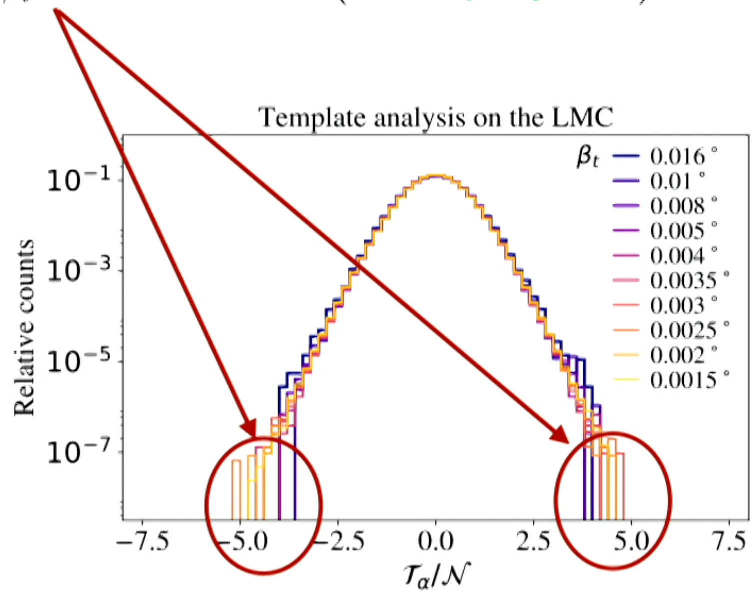
Preferred lens
velocity direction
(observer velocity)

$$\max\{\ln(\mathrm{LR})\} = \max_{\vec{\theta}_l\ \beta_l} \frac{{C_l}^2 {\sigma_v}^2 \mathcal{N}^2 \left(\mathcal{T}^2/\mathcal{N}^2 - {v_0}^2/{\sigma_v}^2\right) + 2 C_l \vec{\mathcal{T}} \cdot \vec{v}_0}{2\left(1 + {C_l}^2 {\sigma_v}^2 \mathcal{N}^2\right)} + A(\rho_l, \beta_l)$$

$$\vec{\mathcal{T}} = \{\mathcal{T}_\alpha, \mathcal{T}_\delta\}$$

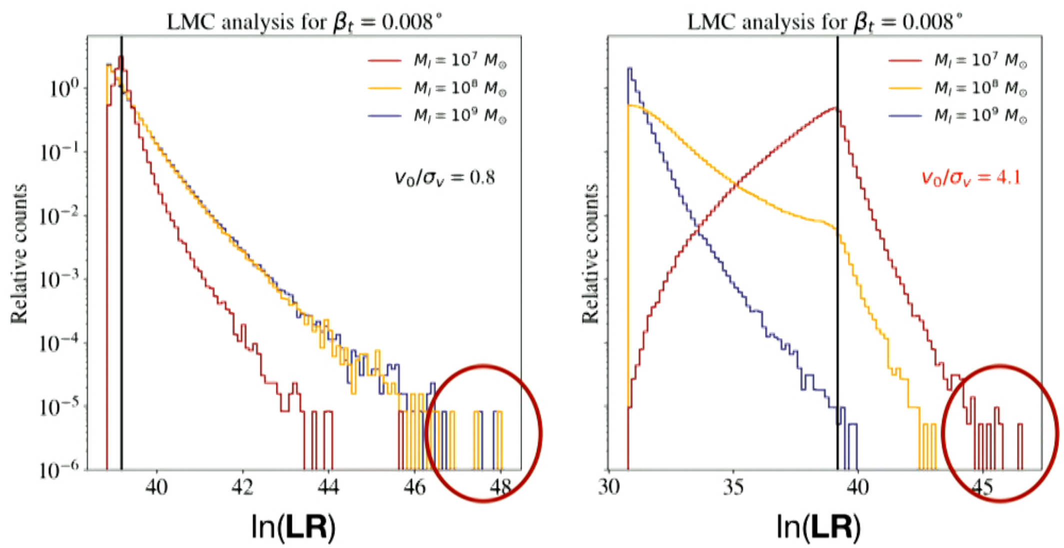
Optimal discriminant

$$\max\{\ln(\mathrm{LR})\} = \max_{\vec{\theta}_l\ \beta_l} \frac{C_l{}^2\sigma_v{}^2\mathcal{N}^2\left(\mathcal{T}^2/\mathcal{N}^2 - v_0{}^2/\sigma_v{}^2\right) + 2C_l\vec{\mathcal{T}}\cdot\vec{v}_0}{2\left(1 + C_l{}^2\sigma_v{}^2\mathcal{N}^2\right)} + A(\rho_l, \beta_l)$$



Optimal discriminant

$$\max\{\ln(\mathrm{LR})\} = \max_{\vec{\theta}_l\ \beta_l} \frac{C_l{}^2\sigma_v{}^2\mathcal{N}^2\left(\mathcal{T}^2/\mathcal{N}^2 - v_0{}^2/\sigma_v{}^2\right) + 2C_l\vec{\mathcal{T}}\cdot\vec{v}_0}{2\left(1 + C_l{}^2\sigma_v{}^2\mathcal{N}^2\right)} + A(\rho_l, \beta_l)$$



Optimal discriminant

$$\max\{\ln(\mathrm{LR})\} = \max_{\vec{\theta}_l\ \beta_l} \frac{{C_l}^2{\sigma_v}^2\mathcal{N}^2\left(\mathcal{T}^2/\mathcal{N}^2 - {v_0}^2/{\sigma_v}^2\right) + 2C_l\vec{\mathcal{T}}\cdot\vec{v}_0}{2\left(1 + {C_l}^2{\sigma_v}^2\mathcal{N}^2\right)} + A(\rho_l, \beta_l)$$

In practice:

- the measured noise is not gaussian
- there is more than one lens

The true **LR** is much more complicated! → Use **signal simulations**

Setting the limit

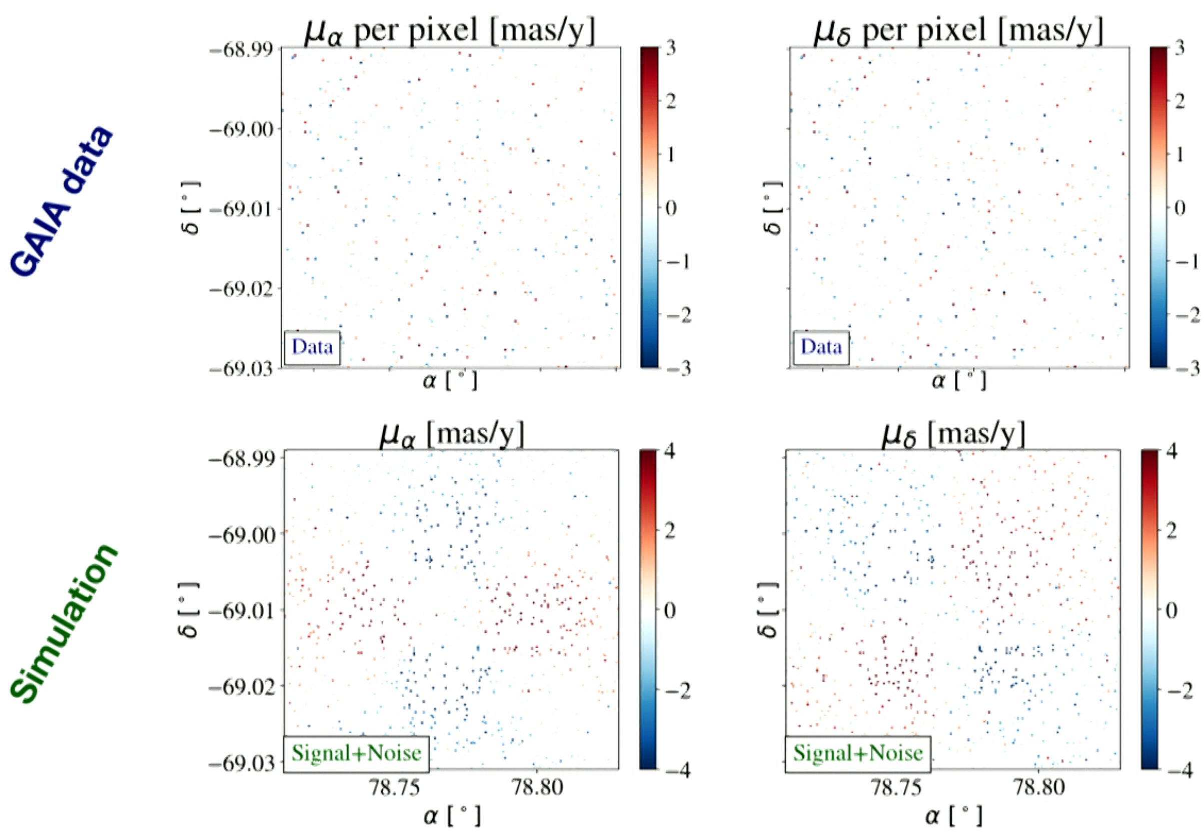
Given the lens parameters: M_l, r_l, Ω_l

1. "Fake" data samples are generated injecting signal + noise (data-driven)
2. The optimal test statistic is computed for ***data*** *and* ***simulations***
3. If in 90% of the ***simulations***

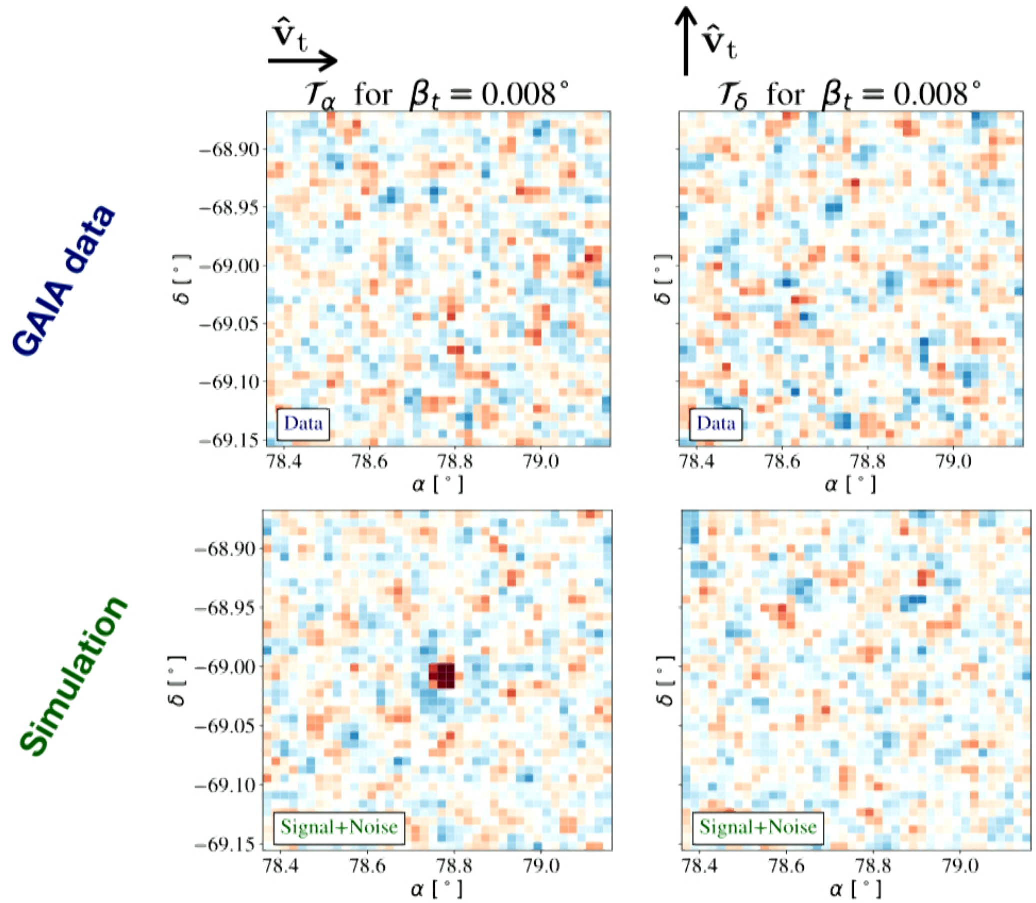
$$\max(\mathbf{LR})_{\text{simulation}} > \max(\mathbf{LR})_{\text{data}}$$

the parameter point is excluded at 90% CL.

Simulated signal



Template scanning



$$M_{\text{lens}} = 10^8 M_\odot$$

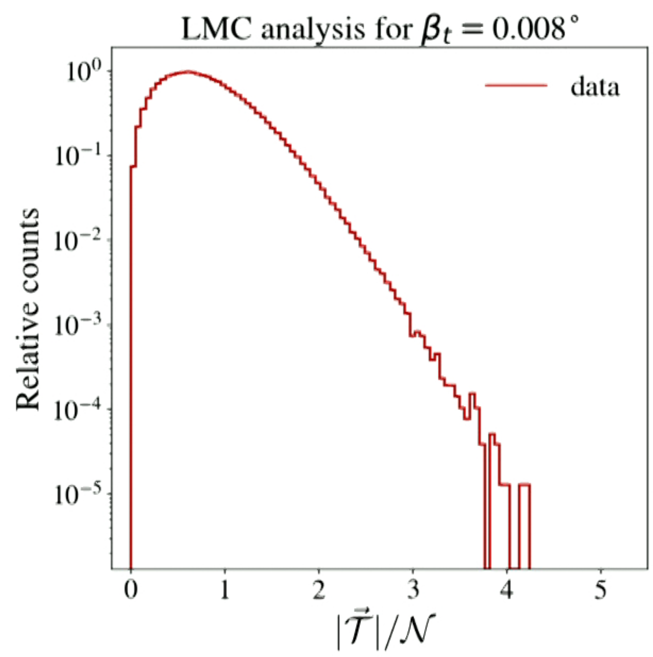
$$r_{\text{lens}} = 1 \text{ pc}$$

$$v_{\text{lens}} = 0.001$$

$$\hat{\mathbf{v}}_{\text{lens}} \longrightarrow$$

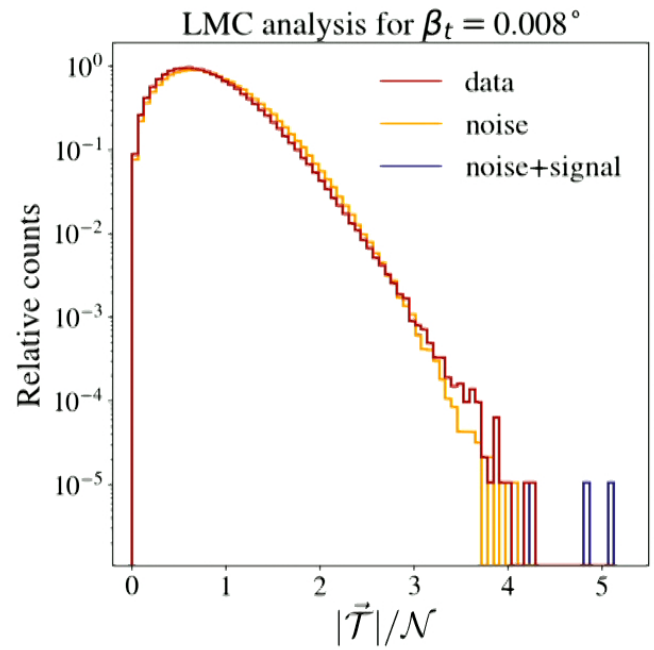
Test statistic

Distribution of the test statistic computed at all possible lens locations



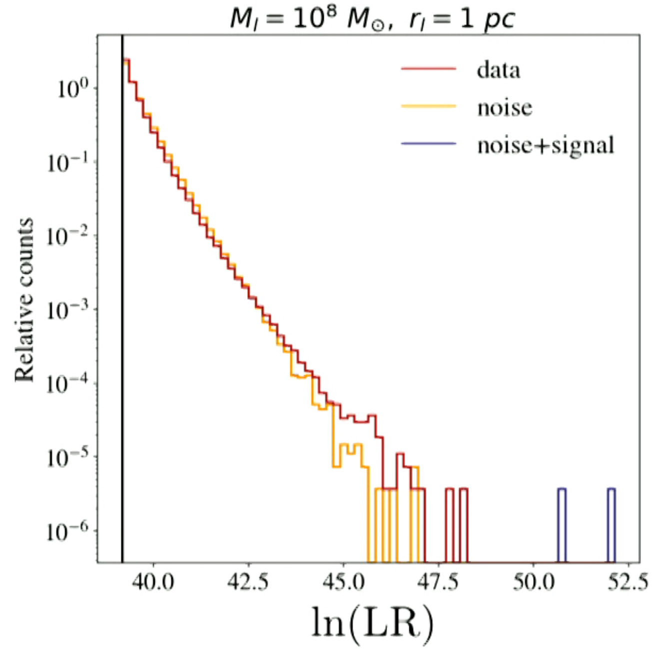
Test statistic

Distribution of the test statistic computed at all possible lens locations

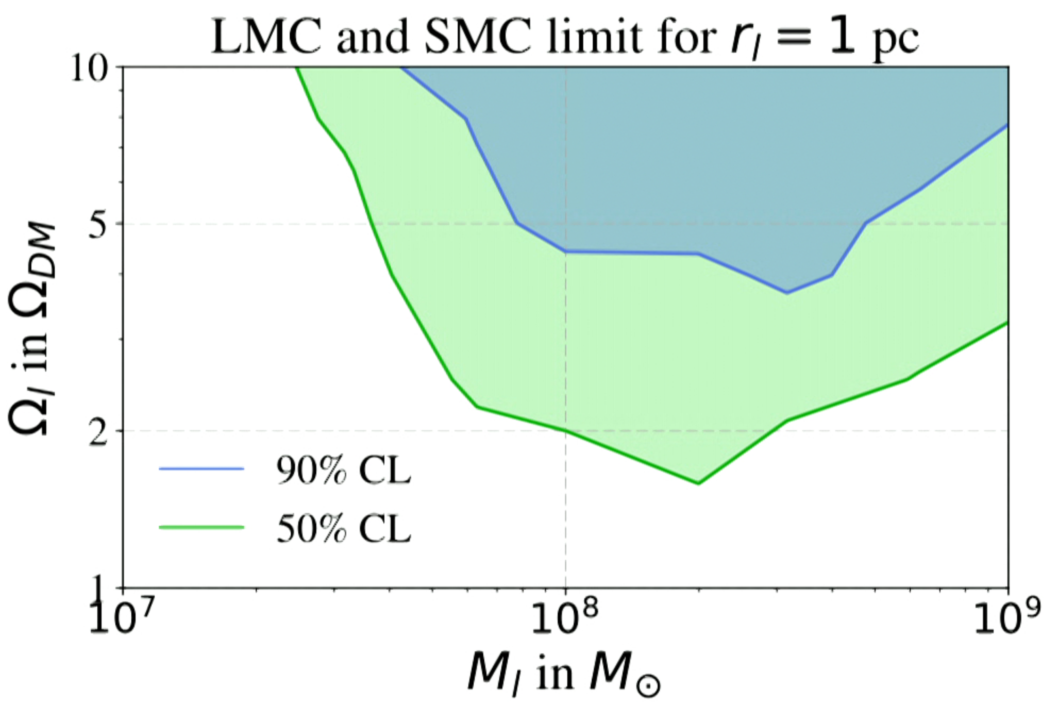


Test statistic

Distribution of the test statistic computed at all possible lens locations



Set limit



LMC and SMC limit for $r_l = 1$ pc
10
5
2
1
90% CL
50% CL
Ω_l in Ω_{DM}
10^7
10^8
10^9
M_l in $M_\odot$

Projections

Lens properties:

- mass M_l
- size r_l
- abundance Ω_l

Sources properties:

- number of stars N_s
- proper motion dispersion $\sigma_{\mu,\mathrm{eff}}$
- solid angle $\Delta\Omega$

$$\mathrm{SNR} \propto \frac{{\Omega_l}^{1/3} {M_l}^{2/3}}{r_l} \frac{N_\mathrm{s}^{1/2}}{\Delta\Omega^{1/6} \sigma_{\mu,\mathrm{eff}}}$$

Projections

$$\mathrm{SNR} \propto \frac{{\Omega_l}^{1/3} {M_l}^{2/3}}{r_l} \frac{N_{\rm s}^{1/2}}{\Delta\Omega^{1/6} \sigma_{\mu,\rm eff}}$$

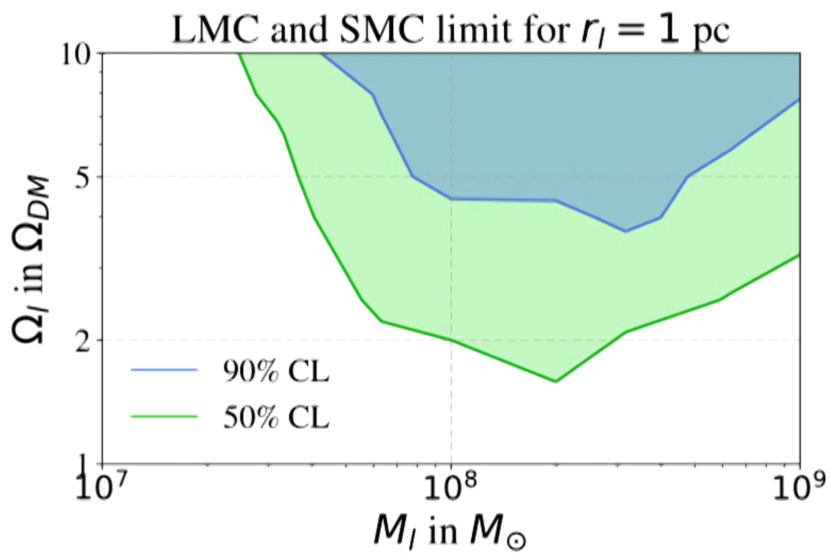
$$\Omega_l \propto \sigma^3_{\mu,\rm eff} \propto t^{-9/2}$$

$$\uparrow$$

$$\sigma_{\mu,\rm eff} \propto t^{-3/2}$$

Projections

$$\Omega_l \propto \sigma^3_{\mu,\text{eff}} \propto t^{-9/2}$$



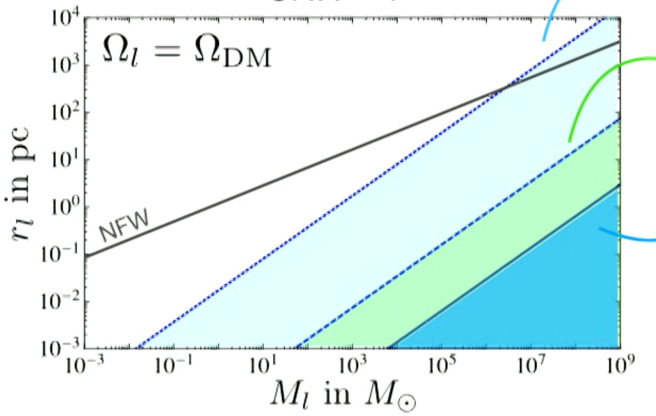
With 10 years

$\mathcal{O}(1000)$

Projections

$$\mathrm{SNR} \propto \frac{{\Omega_l}^{1/3} {M_l}^{2/3}}{r_l} \frac{N_s^{1/2}}{\Delta\Omega^{1/6} \sigma_{\mu,\mathrm{eff}}}$$

SNR = 1



All sky 10^8 quasars $\sigma_\mu \sim 1\ \mu\mathrm{as/y}$ SKA

LMC 10^7 stars $\sigma_\mu \sim 0.2\ \mathrm{mas/y}$ Gaia

LMC 10^7 stars $\sigma_\mu \sim 5\ \mathrm{mas/y}$

Now

High-resolution Space Telescope Proper Motion Collaboration

Hubble Space Telescope repeated exposures:

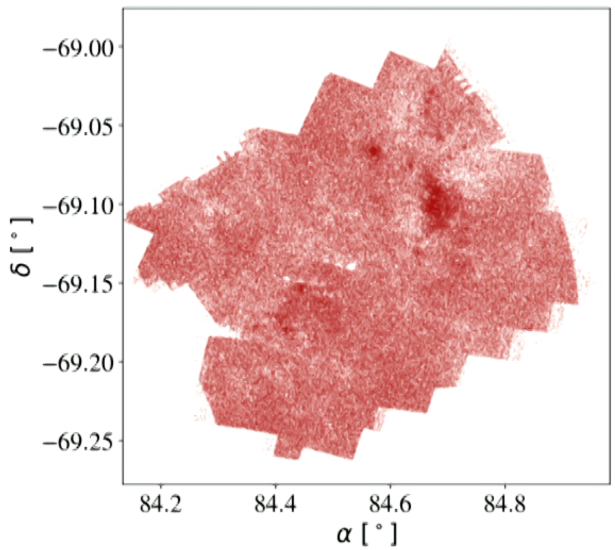
- Smaller field of view
- Larger number density of ~ 10^{10}

(LMC in Gaia 10^{8})

↓

Sensitive to smaller lens masses

30 Doradus (LMC)



Other methods

- Extragalactic strong gravitational lensing (multiply imaged quasars)
  S. Mao and P. Schneider
  Evidence for substructure in lens galaxies?

- Gravitational perturbations in stellar streams
  A. Bonaca, D. Hogg, A. Price-Whelan and C. Convoy
  The Spur and the Gap in GD-1: Dynamical evidence for a dark substructure in the Milky Way halo
  $r_l \lesssim 30$ pc $\quad M_l \sim 10^6 - 10^8\ M_\odot$

- Pulsar timing observations
  E. R. Siegel, M. P. Hertzberg and J. N. Fry
  Probing DM Halo Substructure with Pulsar Timing
  S. Baghram, N. Afshordi and K. Zurek
  Prospects for Detecting DM Halo Substructure with Pulsar Timing

- Milky Way stars phase-space distribution
  M. Buschmann, J. Kopp, B. R. Safdi and C.L. Wu
  Stellar Wakes from Dark Matter Subhalos

Summary

- DM halo substructure: time-domain astrometric lensing effects
- Modern astrometric surveys: precision + statistics
- Velocity template: real data analysis!

no signal ↙ ↘ *positive signal*

Limits on DM halo substructure
(mass, size and abundance)

Further tests:

- effects on the stellar parallaxes
- closer look at the lens location