

Title: TA Session: Defects in 3d and 4d Supersymmetric Theories

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Collection: QFT for Mathematicians

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Last time:

$$2d \text{ NCS} \hookrightarrow 3d \text{ NCS} \hookrightarrow 4d \text{ NCS}$$

Time space ZW superheals can use the List. ω : the many translations are in the cohomology

Q_H	1	2	3
Q_A	2	3	4
Q_B	2	3	4

Warmup

B-model

X

Calabi-Yau

$G_X \cong U_X \times E_6$

divisor

EOM

$$\text{Map}(\Sigma_{g,h}, T^*M^2) = \text{EOM}(\Sigma)$$

$$(\pi_1 \Sigma, \text{det})$$

Produce a 2d TQFT

$$Z: \text{ZGib} \rightarrow \mathbb{C}^*$$

$$\text{Ansatz } Z(M) = GQ(\text{EOM}(M^2))$$



AKSZ formalism tells us

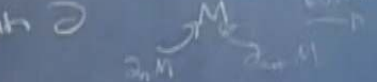
$$\text{Map}(M^d \text{ or } X^d_{\text{dof}}, T^*M^2)$$

has a symplectic form of degree $1-d$

If I consider $\text{map}(R^d)$ with ∂

Lagrangian submanifold

$$\text{EOM}(M)$$



$$\text{EOM}(M)$$

$$\text{EOM}(M)$$

Ansatz

$$GQ(T^*M^2) = nGCh(X)$$

$$T^*M^2$$

$$T^*X$$

$$T^*M^2$$

$$T^*M^2$$

$$1$$

$$G_X$$

$$GCh(X)$$

$$GCh(X)$$

$$= (GCh(X))_{\text{mod}}$$

Warmup B-model $X = \text{Calabi-Yau}$ $(\omega_X \cong \omega_X^{\otimes 2})$
Usual EOM $\text{Maps}(\Sigma_{g,r}, T^*M) = \text{EOM}(\Sigma)$ needed for oriented TQFT
 Produce a 2d TQFT $(\text{EOM}(\Sigma), \text{d}_R)$ part 2
 $Z: \text{Zob} \rightarrow \text{Cat}$
 $\text{Ansatz } Z(M) = \text{GQ}(\text{EOM}(M))$ modified symplectic

AKSZ formalism tells us
 $\text{Maps}(M^d_{\text{or}}, T^*M)$ has a symplectic form of degree $1-d$
 If I consider $\text{Maps}(R^d, \mathcal{D})$ with \mathcal{D}
 $\text{EOM}(M)$
 $\text{EOM}(\text{Cin } M)$ $\text{EOM}(\text{Quot } M)$

$Z(\text{pt}) = \text{GQ}(\text{EOM}(\text{pt})) = \text{GQ}(T^*X) = \text{QCoh}(X)$
 $Z(\text{circle}) = \text{GQ}(\text{EOM}(S^1)) = \text{GQ}(\text{Maps}(S^1, T^*X)) = \mathbb{P}(\text{Coh}(X))$
In general there are a bunch of circles
 $Z(\text{circle}) = \mathbb{P}(\text{Coh}(X))$ $\text{Coh}(X)$ $\text{PV}(X/G)$

Ansatz $\text{GQ}(T^*M) = \text{nc Coh}(X)$

T^*X	T^*X	1
(K3)	T^*X	G_X
	T^*X	$\text{Coh}(X)$
	T^*X	$\text{QCoh}(X)$
		$(\text{QCoh}(X))_{\text{nc}}$ - mod



$$Z(\text{pt}) = \mathcal{G}_k(\text{EOM}(\text{pt})) = \mathcal{G}_k(\text{pt} \times X) = \mathcal{Qch}(X)$$

$$Z(\mathbb{O}) = \mathcal{G}_k(\text{EOM}(\mathbb{S})) = \mathcal{G}_k(\text{Maps}(\mathbb{S}_k, \text{pt} \times X)) = \mathcal{P}(\mathbb{S}_k, \mathbb{O}_k)$$

In general there are a Z worth of $\mathcal{P}(\mathbb{S}_k, \mathbb{O}_k)$

$$\mathcal{Z}(\mathbb{O}) = \mathcal{P}(\mathbb{S}_k, \mathbb{O}_k) \cong \mathcal{P}(X, X)$$

what is the trivial line?

$$\mathbb{I} \rightarrow \mathbb{S}^0 \Rightarrow \text{EOM}(\mathbb{I}) \rightarrow \text{EOM}(\mathbb{S}^0)$$

$$\text{pt} \times X \xrightarrow{\Delta} \text{pt} \times X \times X$$


thought of as an integral kernel

$$\mathcal{G}_k(\mathbb{I}) = \Delta_k(X)$$

Ansatz $\mathcal{G}_k(N \times X)$

$$\cong \mathcal{P}_k(N, \mathbb{O}_k) \quad 1\mathbb{O}_k = \mathbb{O}_k \quad 2\mathbb{O}_k = \mathcal{Qch}(X)$$

lets talk about line operators in 2d




line is $\mathbb{S}^0 \rightarrow \mathbb{R} \times \mathbb{S}^0 \subset \mathbb{R} \times \text{COM}(\mathbb{S}^0)$

$$\text{Maps}(\mathbb{R}_{20} \times \mathbb{R}_k, \text{Maps}(\mathbb{S}_k, \text{pt} \times X)) \cong \text{Maps}(\mathbb{R}, \text{Maps}(\mathbb{S}_k, \text{pt} \times X))$$

think about as maps into map from \mathbb{S}^0

category of line operators = $\mathcal{Qch}(X \times X) = \mathcal{Z}(\mathbb{S}^0)$

Tensor structure on lines



what is it mean to compose, like in a \mathbb{S}^0

AKSZ tells us \rightarrow Legendre transform

$$\mathcal{G}_k(\mathbb{S}^0) = \mathcal{P}_k(\mathbb{O}_k, \mathbb{O}_k)$$

$$\text{Maps}(\mathbb{S}^0, \text{Maps}(\mathbb{S}_k, \text{pt} \times X)) \rightarrow \text{Maps}(\mathbb{R}, \text{Maps}(\mathbb{S}_k, \text{pt} \times X))$$

