

Title: TA Session: Defects in 3d and 4d Supersymmetric Theories

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Collection: QFT for Mathematicians

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Last time:

$$2d \text{ N=4} \hookrightarrow 3d \text{ N=4} \hookrightarrow 4d \text{ N=4}$$

Time space ZW superheavies can use ten h List. Q: How many translations are in the cohomology

Q_H	1	2	3
Q_A	2	3	4
Q_B	2	3	4

Warmup B-model X Calabi-Yau $\rightarrow G_X \cong U_X \times \mathbb{Z}_2$ needed for orientifolds

$$\text{Map}(\Sigma_{g,h}, T^*\Pi X) = \text{EOM}(\Sigma)$$

Produce a 2d TQFT

$$\text{Ansatz } Z(M) = GQ(\text{EOM}(M))$$

AKSZ formalism tells us

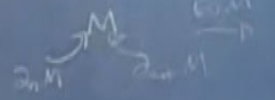
$$\text{Map}(M \text{ or } X \text{ w/ half-structure}, T^*\Pi X)$$

has a symplectic form of degree $1-d$

If I consider $\text{map}(R/\mathbb{Z})$ with ∂

Langmuir correspondence

$$\text{EOM}(M) \rightarrow \text{EOM}(\partial M) \rightarrow \text{EOM}(\text{int } M)$$



Ansatz

$$GQ(T^*\Pi X) = nQGh(X)$$

- $T^*\Pi X$
 - T^*X
 - $T^*\Pi X$
 - $T^*\mathbb{Z}X$
- 1
 - G_X
 - $QGh(X)$
 - $QGh(X)$
 - $=(QGh(X))_{\infty} - \text{mod}$

Warmup B-model $X = \text{Calabi-Yau}$ $(\Omega_X \cong \omega_X \otimes E)$
Usual EOM $\text{Maps}(\Sigma_{dR}, T^* \Pi X) = \text{EOM}(\Sigma)$ needed for oriented TQFT
 Produce a 2d TQFT $(\Pi \Sigma, dR)$ pt 6.2
 $Z: \text{2d Gds} \rightarrow \text{Cat}$
 $\text{Ansatz } Z(M) = \text{GQ}(\text{EOM}(T^*M))$ n-dim symplectic manifold

AKSZ formalism tells us
 $\text{Maps}(M^d_{dR}, T^* \Pi X)$ has a symplectic form of degree $1-d$
 If I consider $\text{Maps}(R/\mathbb{Z})$ with ∂
 $\text{EOM}(M) \rightarrow \text{EOM}(\partial M) \rightarrow \text{EOM}(\text{int } M)$

$Z(\text{pt}) = \text{GQ}(\text{EOM}(\text{pt})) = \text{GQ}(T^* \Pi X) = \text{QCoh}(X)$
 $Z(\mathbb{S}^1) = \text{GQ}(\text{EOM}(\mathbb{S}^1)) = \text{GQ}(\text{Maps}(\mathbb{S}^1, T^* \Pi X)) = \mathbb{P}(\mathbb{S}^1, \mathcal{O}_{\mathbb{S}^1})$
In general there are a $\mathbb{P}(\Sigma, \mathcal{O}_\Sigma)$ worth of circles
 $Z(\mathbb{S}^1) = \mathbb{P}(\mathbb{S}^1, \mathcal{O}_{\mathbb{S}^1}) = \mathbb{P}(X, \mathcal{O}_X)$

Ansatz $\text{GQ}(T^* \Pi X) = \text{ncQCoh}(X)$

$T^* \Pi X$	$T^* X$	1
$(K\mathbb{S}^1)$	$T^* X$	\mathcal{O}_X
	$T^* \Pi X$	$\text{QCoh}(X)$
	$T^* X$	$\text{ncQCoh}(X)$
		$(\text{QCoh}(X))_{\text{nc}}$ - mod



$$Z(\text{pt}) = \mathcal{GQ}(\text{EOM}(\text{pt})) = \mathcal{GQ}(T^*X) = \mathcal{Qch}(X)$$

$$Z(\mathbb{O}) = \mathcal{GQ}(\text{COM}(\mathbb{S})) = \mathcal{GQ}(\text{Maps}(\mathbb{S}^1, T^*X)) = \mathcal{P}(\mathbb{S}^1, \mathbb{O}_X)$$

In general there are a \mathbb{Z} worth of circles

$$\mathbb{Z}(\mathbb{O}) = \mathcal{P}(X, T^*X) \cong \mathcal{P}(X, X)$$

what is the trivial line?

$$\mathbb{I} \rightarrow \mathbb{S}^0 \Rightarrow \text{EOM}(\mathbb{I}) \rightarrow \text{EOM}(\mathbb{S}^0)$$


$$T^*X \xrightarrow{\Delta} T^*X \times X$$

thought of as an integral kernel

$$\mathcal{GQ}(\mathbb{I}) = \Delta_+(X)$$

Ansatz $\mathcal{GQ}(\mathbb{N}^n \times X) \cong \mathcal{P}_+(n, \mathbb{O}_Y) \oplus \mathcal{P}_+(n, \mathbb{O}_Y) \oplus \mathcal{Qch}(Y)$

lets talk about line operators in 2d



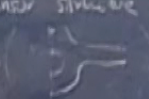
line is $\mathbb{S}^0 \rightarrow \mathbb{R} \times \mathbb{S}^0 \times \mathbb{R} \subset \mathbb{R}^2$

$$\text{Maps}(\mathbb{R} \times \mathbb{S}^0, \mathbb{R}^2) \cong \text{Maps}(\mathbb{S}^1, \mathbb{R}^2) \cong T^*X$$

think about as maps into map from \mathbb{S}^0

category of line operators = $\mathcal{Qch}(X \times X) = Z(\mathbb{S}^0)$

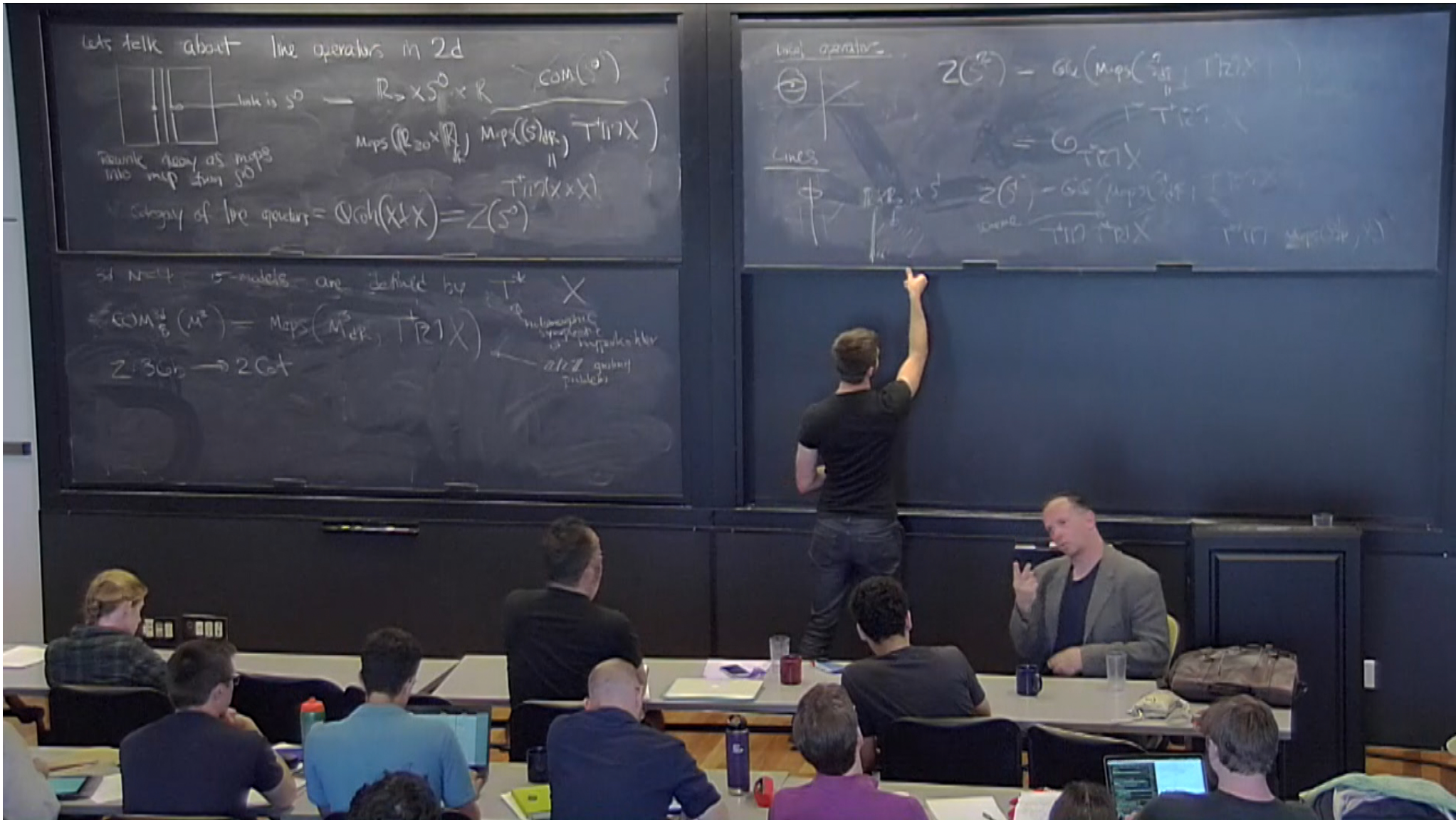
Tensor structure on lines



what is it mean to compatibly be on a \mathbb{S}^0

AKSZ tells us $\text{Maps}(\mathbb{S}^1, T^*X) \rightarrow \text{Maps}(\mathbb{S}^1, T^*X)$

is logarithmic $\mathcal{GQ}(\mathbb{S}^1) = \mathcal{P}(\mathbb{S}^1, X)$



Let's talk about line operators in 2d



line is $S^0 = \mathbb{R} \times S^0 \times \mathbb{R}$ $\text{Com}(S^0)$
 $\text{Maps}(\mathbb{R} \times S^0 \times \mathbb{R}, \mathbb{R}) \cong \text{Maps}(S^0, \mathbb{R}) \cong \mathbb{R}$

Think about as maps into map from S^0

Category of line operators = $\text{Qch}(X \times X) = Z(S^0)$

$Z(S^0) = \text{Qch}(T\mathbb{R} \times X) \cong \text{Qch}(T\mathbb{R} \times X)$

Moral: $\mathbb{R} \times S^0 \xrightarrow{\text{FT}} L(\mathbb{R} \times S^0) \cong \mathbb{R}$

In categorical lang, FT \Rightarrow Kac-Moody

Local operators



$Z(S^1) = \text{GL}(\text{Maps}(S^1, T\mathbb{R} \times X))$
 $\cong \text{GL}(\mathbb{R})$

Lines



$Z(S^1) = \text{GL}(\text{Maps}(S^1, T\mathbb{R} \times X))$
 $\cong \text{GL}(\mathbb{R})$

$D \hookrightarrow S^1 \quad T\mathbb{R} \times X \rightarrow \text{Maps}(S^0, T\mathbb{R} \times X) \cong \mathbb{R} \times \text{Maps}(S^0, X)$

Let A -model $\text{Maps}(C \times M_{\mathbb{R}}, X) \cong \mathbb{R} \times \text{Maps}(C, X)$
 $S^2 = D \cup D \xrightarrow{\text{FT}} \mathbb{H}(\text{Maps}(B, X))$

