

Title: TA Session: Yang-Mills Theory and Asymptotic Freedom

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4d Yang-Mills & Asymptotic Freedom

- 1) Classical Yang-Mills in BV
- 2) The β -function & asymptotic freedom
- 3) Quantization of Yang-Mills

Yang-Mills = \mathbb{R}^4

is single copy Lie group

V rep of G (or)

Fields = A gauge field in $\Omega^1(\mathbb{R}^4, \mathfrak{g})$ $S_4 \cong S_4$ Dim gauge field
 = ψ spinor section in $\Omega^{\frac{1}{2}}(\mathbb{R}^4, S \otimes V)$ - fermions

Quantize with gauge group $\Omega^0(\mathbb{R}^4, \mathfrak{g})$ by BRST
 $A \mapsto A + d_A(c)$

Action G -invariant pairing $\mu: V \otimes V \rightarrow \mathbb{R}$
 $V \rightarrow V$ positive definite inner product

$$S(A, \psi) = \int_{\mathbb{R}^4} \frac{1}{2} \|F_A\|^2 + \mu(\psi, (d_A^* - m)\psi)$$

$$\Omega^0 \xrightarrow{d} \Omega^1 \xrightarrow{d} \Omega^2 \xrightarrow{d} \Omega^3 \xrightarrow{d} \Omega^4 = (\text{E-pairing}) \psi_A \psi = \mu(d_A^* \psi)$$

classical multiplication / Dirac operator
 $\mu: \Omega^2 \otimes S \rightarrow S$

no chiral for Q^{GF}

Deal with this, results in first order formalism

introduce $B \in \Omega^2(\mathbb{R}^4, \mathfrak{g})$ $B = *B$

$$S_{FO}(A, B, \psi) = \int_{\mathbb{R}^4} \langle F_A, B \rangle - \frac{1}{2} \|B\|^2 + \mu(\psi, (d_A^* - m)\psi)$$

Steps for BV quantization

- 1) a) Choose a gauge for \mathcal{L}^{GF} , $[\mathcal{L}, \mathcal{L}^{GF}]$ gauge algebra
 b) Calculate the kernel K_2 using kernel $[\mathcal{L}, \mathcal{L}^{GF}] \in \mathcal{E} \otimes \mathcal{E}$
 c) Calculate the prepotential $\int dt \mathcal{L}^{GF}(e^i) K_2 = P(\mathcal{L}, \mathcal{L}^{GF})$
 d) Calculate $\mathcal{I}(\mathcal{L})$, first step is to try $\int dt W(P(\mathcal{L}, \mathcal{L}^{GF}), \mathcal{I})$, this will be changed, choose $\mathcal{I}^T(\mathcal{L})$ so that $\int dt W(P(\mathcal{L}, \mathcal{L}^{GF}), \mathcal{I} - \mathcal{I}^T(\mathcal{L}))$ exists, many ways of doing this
 e) Try to solve the QME, by adding more \mathcal{J} to $\mathcal{I}(\mathcal{L})$, $\int dt W(P(\mathcal{L}, \mathcal{L}^{GF}), \mathcal{I} - \mathcal{I}^T(\mathcal{L}) + \mathcal{J})$ exists, many ways of doing this

$$\Omega^1(\mathbb{R}^3; \mathfrak{g}) \xrightarrow{\sim} \Omega^1(\mathbb{R}^3; \mathfrak{g}) \xrightarrow{\sim} \Omega^1(\mathbb{R}^3; \mathfrak{g})$$

$$\Omega^2(\mathbb{R}^3; \mathfrak{g}) \xrightarrow{\sim} \Omega^2(\mathbb{R}^3; \mathfrak{g}) \xrightarrow{\sim} \Omega^2(\mathbb{R}^3; \mathfrak{g})$$

$$\Omega^3(\mathbb{R}^3; \mathfrak{g}) \xrightarrow{\sim} \Omega^3(\mathbb{R}^3; \mathfrak{g})$$

$d_A = d + g[A, \cdot]$

$$\mathcal{I} = \langle \mathcal{L}, \mathcal{L}^{GF} \rangle + \langle \mathcal{L}, \mathcal{L}^{GF} \rangle + \langle \mathcal{L}, \mathcal{L}^{GF} \rangle + \langle \mathcal{L}, \mathcal{L}^{GF} \rangle + \langle \mathcal{L}, \mathcal{L}^{GF} \rangle$$

This is likely open to 2^{nd} order terms, should be equal to \mathcal{B} .

Local RG flow

known explained $R_\lambda \mathcal{I}(\mathcal{L}) = \mathcal{I}(\mathcal{L}) + \log \lambda$, higher order $\in \mathcal{O}_{\text{loc}}$ $[\lambda^2, \log \lambda]$

Definition

Let \mathcal{M} functional of model is characteristic

$$\mathcal{O}_\beta(\mathcal{L}) = \frac{1}{\beta \log \lambda} R_\lambda \mathcal{I}(\mathcal{L}) \Big|_{\lambda=1} \in \mathcal{O}_{\text{loc}}(\mathbb{R}^3) \otimes \mathfrak{g}$$

$$\mathcal{O}_\beta(\mathcal{L}) = \sum_{i=1}^{\infty} \mathcal{O}_\beta^{(i)}(\mathcal{L}) \beta^i$$

Each: for scale invariant theory, $\lim_{\beta \rightarrow \infty} \mathcal{O}_\beta^{(i)}(\mathcal{L})$ exists
 β is BV closed

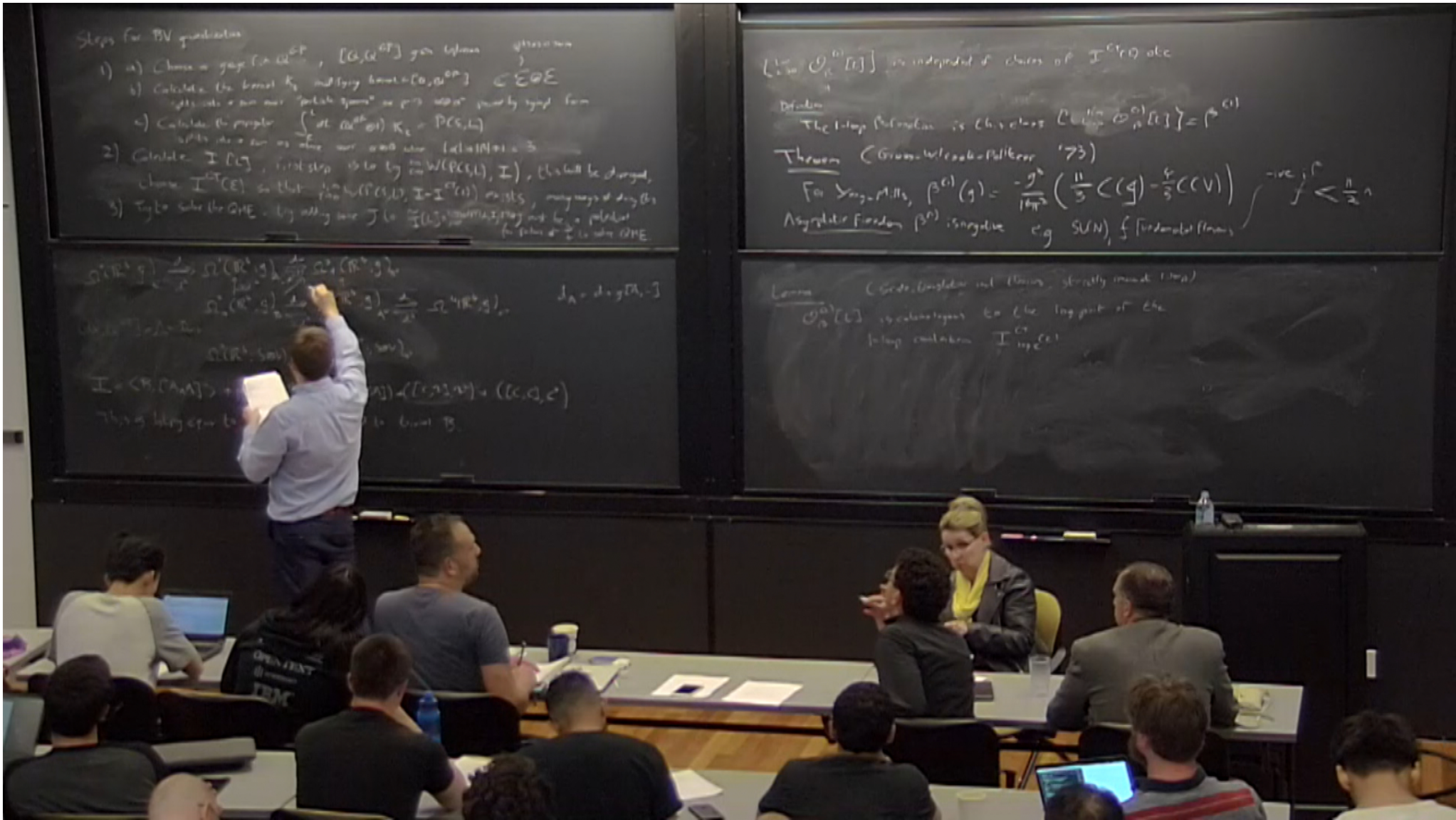
$\{\lim_{\beta \rightarrow \infty} \mathcal{O}_\beta^{(i)}(\mathcal{L})\}$ is independent of choice of $\mathcal{I}^T(\mathcal{L})$ etc

Definition

The loop functional is characteristic $\mathcal{L}(\lim_{\beta \rightarrow \infty} \mathcal{O}_\beta^{(i)}(\mathcal{L})) = \beta^{(i)}$

Theorem (Gromoll-Lichnerowicz-Pollack, '73)

For Yang-Mills, $\beta^{(i)}(g) = \frac{-g^2}{16\pi^2} \left(\frac{11}{3} C(g) - \frac{2}{3} C(V) \right)$ $\int f < \frac{11}{2} \pi^2$
 Asymptotic freedom $\beta^{(i)}$ is negative e.g. SU(N), $\int F^2$ bounded from above



Steps for BV quantization

- 1) a) Choose a gauge for \mathcal{L}^{GF} , $[\mathcal{L}, \mathcal{L}^{GF}]$ gauge algebra structure
- b) Calculate the kernel K_i using kernel $[\mathcal{L}, \mathcal{L}^{GF}] \in \mathcal{E} \otimes \mathcal{E}$
- c) Calculate the Faddeev-Popov determinant $\int \mathcal{D}\alpha \exp(i\int \mathcal{L}^{FP}(\alpha)) K_i = \text{PFC}(K_i)$
- 2) Calculate $\mathcal{I}(\mathcal{L})$, first step is to fix $\int \mathcal{L}^{FP}(\mathcal{L}, \mathcal{I})$, this will be changed, choose $\mathcal{I}^{FP}(\mathcal{L})$ so that $\int \mathcal{L}^{FP}(\mathcal{L}, \mathcal{I} - \mathcal{I}^{FP}(\mathcal{L}))$ exists, many ways of doing this
- 3) Try to solve the QME by adding more \mathcal{J} to $\mathcal{I}(\mathcal{L})$, many ways of doing this, for \mathcal{L} of the form $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2$ solve QME

$$\Omega^1(\mathbb{R}^n, \mathfrak{g}) \xrightarrow{\text{d}} \Omega^2(\mathbb{R}^n, \mathfrak{g}) \xrightarrow{\text{d}} \Omega^3(\mathbb{R}^n, \mathfrak{g}) \xrightarrow{\text{d}} \Omega^4(\mathbb{R}^n, \mathfrak{g}) \xrightarrow{\text{d}} \dots$$

$$\mathcal{I} = \langle \mathcal{L}, \mathcal{L} \rangle + \langle \mathcal{L}, \mathcal{I} \rangle + \langle \mathcal{I}, \mathcal{L} \rangle + \langle \mathcal{I}, \mathcal{I} \rangle$$

This is being done to...

$\{ \int \mathcal{L}_i^{FP}(\mathcal{L}) \}$ is independent of choice of $\mathcal{I}^{FP}(\mathcal{L})$ etc

Definition
The loop information is the class $\{ \int \mathcal{L}_i^{FP}(\mathcal{L}) \} = \beta^{(1)}$

Theorem (Gromov-Witten-Politzer '73)

For Yang-Mills, $\beta^{(1)}(\mathfrak{g}) = \frac{-2}{16\pi^2} \left(\frac{11}{3} C(\mathfrak{g}) - \frac{4}{3} C(V) \right)$ -ive if $\frac{11}{3} > \frac{4}{3} n$

Asymptotic Freedom $\beta^{(1)}$ negative e.g. SU(N), f. Fermional flows

Lemma (Gromov-Witten-Politzer, strictly proved later)

$\mathcal{L}_i^{FP}(\mathcal{L})$ is cohomologous to the log part of the loop contribution $\int \mathcal{L}_i^{FP}(\mathcal{L})$

