

Title: Minimalism in modified gravity

Speakers: Shinji Mukohyama

Series: Cosmology & Gravitation

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Abstract: It is generally believed that modification of general relativity inevitably introduce extra physical degree(s) of freedom.

In this talk I argue that this is not the case by constructing modified gravity theories with two local physical degrees of freedom. After classifying such theories into two types, I show explicit examples and discuss their cosmology and phenomenology.

Minimalism in Modified Gravity

Shinji Mukohyama (YITP, Kyoto U)

Based on collaborations with
Katsuki Aoki, Nadia Bolis, Antonio De Felice, Tomohiro Fujita,
Sachiko Kuroyanagi, Francois Larrouturou, Chunshan Lin,
Shuntaro Mizuno, Karim Noui, Michele Oliosi

Dark component in the solar system?

Precession of perihelion
observed in 1800's...

which people tried to
explain with a “dark
planet”, Vulcan,



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But the right answer wasn't “dark planet”, it was
“change gravity” from Newton to GR.

Why modified gravity?

- Can we address **mysteries in the universe?**
Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.

How to unify Quantum Theory with General Relativity?



Probably we need to modify
GR at short distances

Why modified gravity?

- Can we address **mysteries in the universe?**
Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity?**
Superstring, Horava-Lifshitz, etc.
- Do we really **understand GR?**
One of the best ways to understand something may be to break (modify) it and then to reconstruct it.
- ...

of d.o.f. in general relativity

- 10 metric components → 20-dim phase space @ each point

ADM decomposition

- Lapse N , shift N^i , 3d metric h_{ij}

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Einstein-Hilbert action

$$\begin{aligned} I &= \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} {}^{(4)}R \\ &= \frac{M_{\text{Pl}}^2}{2} \int dt d^3\vec{x} N \sqrt{h} \left[K^{ij} K_{ij} - K^2 + {}^{(3)}R \right] \end{aligned}$$

- Extrinsic curvature

$$K_{ij} = \frac{1}{2N} (\partial_t h_{ij} - D_i N_j - D_j N_i)$$

of d.o.f. in general relativity

- 10 metric components → 20-dim phase space @ each point
- Einstein-Hilbert action does not contain time derivatives of N & $N^i \rightarrow \pi_N = 0$ & $\pi_i = 0$
All constraints are independent of N & $N^i \rightarrow \pi_N$ & π_i
“commute with” all constraints → 1st-class

1st-class vs 2nd-class

- **2nd-class constraint S**

$$\{S, C_i\} \neq 0 \text{ for } \exists i$$

Reduces 1 phase space dimension

- **1st-class constraint F**

$$\{F, C_i\} \approx 0 \text{ for } \forall i$$

Reduces 2 phase space dimensions

Generates a symmetry

Equivalent to a pair of 2nd-class constraints

$\{C_i \mid i = 1, 2, \dots\}$: complete set of independent constraints

$A \approx B \iff A = B$ when all constraints are imposed
(weak equality)

of d.o.f. in general relativity

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- 4 generators of 4d-diffeo: 1st-class constraints

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“commute with” all constraints → 1st-class
- 4 generators of 4d-diffeo: 1st-class constraints
- $20 - (4+4) \times 2 = 4 \rightarrow$ 4-dim physical phase space @ each point → 2 local physical d.o.f.

Minimal # of d.o.f. in modified gravity = 2

Can this be saturated?

Is general relativity unique?

- Lovelock theorem says “**yes**” if we assume:
(i) 4-dimensions; (ii) diffeo invariance; (iii) metric only; (iv)
up to 2nd-order eom’s of the form $E_{ab}=0$.
- Effective field theory (derivative expansion) says “**yes**” at
low energy if we assume:
(i) 4-dimensions; (ii) diffeo invariance; (iii) metric only.
- However, cosmological backgrounds break 4d-diffeo while
keeping 3d-diffeo.
- A metric theory with 3d-diffeo but with broken 4d-diffeo
typically has 3 local physical d.o.f. (e.g. scalar-tensor theory,
EFT of inflation/dark energy, Horava-Lifshitz gravity)

Example: simple scalar-tensor theory

- Covariant action

$$I = \frac{1}{2} \int d^4x \sqrt{-g} \left[\Omega^2(\phi) {}^{(4)}R + P(X, \phi) \right] \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

- ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Unitary gauge

$$\phi = t \quad \longrightarrow \quad X = \frac{1}{2} \frac{1}{N^2}$$

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & \frac{N^i}{N^2} \\ \frac{N^j}{N^2} & h^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix}$$

This is a good gauge iff
derivative of ϕ is timelike.

- Action in unitary gauge

$$I = \int dt d^3\vec{x} N \sqrt{h} \left\{ f_1(t) \left[K^{ij} K_{ij} - K^2 + {}^{(3)}R \right] + \frac{2}{N} \dot{f}_1(t) K + f_2(N, t) \right\}$$

$$\Omega^2(\phi) = f_1(t) \quad P(X, \phi) = f_2(N, t)$$

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typically has 3 local physical d.o.f. (e.g. scalar-tensor theory,
EFT of inflation/dark energy, Horava-Lifshitz gravity)
- Is GR unique when we assume: (i) 4-dimensions; (ii) 3d-
diffeo invariance; (iii) metric only; (iv) 2 local physical d.o.f.
(= 2 polarizations of TT gravitational waves)?

A class of minimally modified gravity

Chushan Lin and SM, JCAP1710 (2017), 033

- 4d theories invariant under 3d-diffeo: $x^i \rightarrow x^i + \xi^i(t, x)$

- ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i dt) (dx^j + N^j dt)$$

- Ansatz: actions linear in the lapse function N

$$S = \int dt d^3x \sqrt{h} NF(K_{ij}, R_{ij}, \nabla_i, h^{ij}, t)$$

$$K_{ij} = (\partial_t h_{ij} - \nabla_i N_j - \nabla_j N_i)/(2N)$$

- For simplicity, exclude mixed-derivative terms, i.e. those that contain spatial derivatives acted on K_{ij}

- Relation between K_{ij} and π^{ij} (momenta conjugate to h_{ij}) assumed to be invertible $\det\left(\frac{\partial^2 F}{\partial K_{ij} \partial K_{kl}}\right) \neq 0$

- Seek theories with 2 local physical d.o.f.!

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What we expect/need

- 10 metric components → 20-dim phase space @ each point
- $\pi_N = 0$ & $\dot{\pi}_i = 0$: 1st-class constraints

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An example of MMG: square-root gravity

- Action

$$S = \int d^4x \sqrt{h} N \left[\xi M(t)^4 \sqrt{\left(1 + \frac{c_1(t)}{M(t)^2} \mathcal{K}\right) \left(1 + \frac{c_2(t)}{M(t)^2} R\right)} - \Lambda(t) \right]$$

$$\mathcal{K} = K_{ij} K^{ij} - K^2, \quad K = K^i_i, \quad \xi = \pm 1$$

- In the weak gravity limit,

$$S \simeq \int d^4x \sqrt{h} N \left[\xi M^4 - \Lambda + \frac{\xi}{2} M^2 (c_1 \mathcal{K} + c_2 R) + \dots \right]$$

GR with $M_p^2 = \xi c_1 M^2$, $c_g^2 = \frac{c_2}{c_1}$, $\Lambda_{\text{eff}} = \frac{\Lambda - \xi M^4}{\xi c_1 M^2}$ is recovered.

- Flat FLRW with a canonical scalar $\xi = 1$

$$S = \int dx^3 \int dt a^3 \left[M^4 \sqrt{N^2 - \frac{6c_1}{M^2} \frac{\dot{a}^2}{a^2}} - N\Lambda + \frac{1}{2N} \dot{\phi}^2 - NV(\phi) \right]$$

$$1 - 6c_1 \frac{H^2}{M^2} = \frac{M^8}{(\Lambda + \rho_m)^2}$$

$$H^2 \rightarrow \frac{1}{6c_1^2} M_p^2 \quad \text{as} \quad \rho_m \rightarrow \infty$$

H remains finite

What we found

- The necessary and sufficient condition under which a theory in this class has 2 or less local physical degrees of freedom.
- Simple examples with 2 local physical degrees of freedom
- Hamiltonian-based ansatz [Mukohyama&Noui 2019]
→ more examples, “unification” with cuscuton, ...

Hamiltonian-based ansatz [Mukohyama&Noui 2019]

$$H = \int d^3x \sqrt{h} \left[\mathcal{V}(h_{ij}, \pi^{ij}, \nabla_i) + N\mathcal{H}_0(h_{ij}, \pi^{ij}, \nabla_i) - 2N^i \nabla^j \left(\frac{\pi_{ij}}{\sqrt{h}} \right) \right]$$

with $\{\mathcal{H}_0(x), \mathcal{H}_0(y)\} \approx 0$

- $\pi_N \approx 0$ & $\pi_l \approx 0$ & $\nabla^j \left(\frac{\pi_{ij}}{\sqrt{h}} \right) \approx 0$ are **1st-class**
- If $\{\mathcal{H}_0(x), \mathcal{V}(y)\} \approx 0$ holds trivially then
 $\mathcal{H}_0 \approx 0$ is 1st-class and there is no tertiary constraint.
 $1^{\text{st}}\text{-class} \times 8 \rightarrow [10 \times 2 - 8 \times 2]/2 = 2$ d.o.f.
- If $\{\mathcal{H}_0(x), \mathcal{V}(y)\} \approx 0$ does not hold trivially then
 \exists **tertiary constraint $\mathcal{T}(x) \equiv \{\mathcal{H}_0(x), H\} \approx 0$.**
 \rightarrow **2 or less d.o.f.**

Hamiltonian-based ansatz [Mukohyama&Noui 2019]

$$H = \int d^3x \sqrt{h} \left[\mathcal{V}(h_{ij}, \pi^{ij}, \nabla_i) + N\mathcal{H}_0(h_{ij}, \pi^{ij}, \nabla_i) - 2N^i \nabla^j \left(\frac{\pi_{ij}}{\sqrt{h}} \right) \right]$$

with $\{\mathcal{H}_0(x), \mathcal{H}_0(y)\} \approx 0$

- **Example 1: cusciton** [Afshordi-Chung-Geshnizjani2007]

$$\mathcal{H}_0 = \frac{1}{|h|} \left(\pi_{ij} \pi^{ij} - \frac{1}{2} \pi^2 \right) - R \quad \mathcal{V} = \lambda \pi - \mu \sqrt{|h|}$$

$$S = \int d^4x N \sqrt{h} \left[K_{ij} K^{ij} - K^2 + R + \lambda \left(\frac{K}{N} - \frac{3\lambda}{4N^2} \right) + \frac{\mu}{N} \right]$$

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{|g|} \left[\mathcal{R} - \frac{\lambda}{2} \ln(X^2) \square \phi + \frac{3\lambda^2}{2} X + 2\mu \sqrt{-X} \right]$$

- **Example 2: extended cusciton** [Iyonaga-Takahashi-Kobayashi2018]

$$\mathcal{H}_0 = \frac{1}{|h|} \left(\pi_{ij} \pi^{ij} - \frac{1}{2} \pi^2 \right) - R \quad \mathcal{V} = \frac{1}{|h|} \left(\lambda_1 \pi^{ij} \pi_{ij} - \frac{\lambda_2}{2} \pi^2 \right)$$

- **Example 3: $f(\mathcal{H})$ theories**

$$\mathcal{H}_0 = f(\mathcal{H}_{gr}) \quad \mathcal{H}_{gr} \equiv \frac{1}{|h|} \left(\pi_{ij} \pi^{ij} - \frac{1}{2} \pi^2 \right) - R$$

What we found

- The necessary and sufficient condition under which a theory in this class has 2 or less local physical degrees of freedom.
- Simple examples with 2 local physical degrees of freedom
- Hamiltonian-based ansatz [Mukohyama&Noui 2019]
→ more examples, “unification” with cuscuton, ...
- However, it was not clear how to couple matter to gravity in a consistent way...



Matter coupling in scalar tensor theory

- Jordan (or matter) frame

$$I = \frac{1}{2} \int d^4x \sqrt{-g^J} [\Omega^2(\phi) R[g^J] + \dots] + I_{\text{matter}}[g_{\mu\nu}^J; \text{matter}]$$

- Einstein-frame $g_{\mu\nu}^E = \Omega^2(\phi) g_{\mu\nu}^J$ K.Maeda (1989)

$$I = \frac{1}{2} \int d^4x \sqrt{-g^E} [R[g^E] + \dots] + I_{\text{matter}}[\Omega^{-2}(\phi) g_{\mu\nu}^E; \text{matter}]$$

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- Do we call this GR? No. This is a modified gravity because of **non-trivial matter coupling** → type-I
- There are more general scalar tensor theories where there is **no Einstein frame** → type-II

Type-I & type-II modified gravity

- **Type-I:**

There exists an Einstein frame

Can be recast as GR + extra d.o.f. + matter, which couple(s) non-trivially, by change of variables

- **Type-II:**

No Einstein frame

Cannot be recast as GR + extra d.o.f. + matter by change of variables

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Type-I minimally modified gravity (MMG)

Katsuki Aoki, Chunshan Lin and SM, PRD98 (2018) 044022

- # of local physical d.o.f. = 2
- There exists an Einstein frame
- Can be recast as GR + matter, which couple(s) non-trivially, by change of variables
- The most general change of variables = canonical tr.
- Matter coupling just after canonical tr. → breaks diffeo → 1st-class constraint downgraded to 2nd-class → leads to extra d.o.f. in phase space → inconsistent
- Gauge-fixing after canonical tr. → splits 1st-class constraint into pair of 2nd-class constraints
- Matter coupling after canonical tr. + gauge-fixing → a pair of 2nd-class constraints remain → consistent

Simple example of type-I MMG

Katsuki Aoki, Chunshan Lin and SM, PRD98 (2018) 044022

- Start with the Hamiltonian of GR
phase space: (N, N^i, Γ_{ij}) & (π_N, π_i, Π^{ij})

- Simple canonical tr.** $(\Gamma_{ij}, \Pi^{ij}) \rightarrow (\gamma_{ij}, \pi^{ij})$

$$\Gamma_{ij} = -\frac{\delta F}{\delta \Pi^{ij}} \quad \pi^{ij} = -\frac{\delta F}{\delta \gamma_{ij}} \quad F = -\int d^3x \sqrt{\gamma} f(\tilde{\Pi}) \quad \tilde{\Pi} := \Pi^{ij} \gamma_{ij} / \sqrt{\gamma}$$

- Gauge-fixing** $\mathcal{G} \approx 0$

$$\{\mathcal{G}, \mathcal{H}_0\} \approx 0 \quad \{\mathcal{G}, \pi_N\} \approx 0 \quad \{\mathcal{G}, \pi_i\} \approx 0 \quad \{\mathcal{G}, \mathcal{H}_i\} \approx 0$$

- Lagrangian for $g^J_{\mu\nu} = (N, N^i, \gamma_{ij})$

$$\sqrt{-g^J} \mathcal{L} = \dot{\gamma}_{ij} \pi^{ij} - \mathcal{H}_{\text{tot}}^{\text{GF}}$$

$\mathcal{H}_{\text{tot}}^{\text{GF}}$: gauge-fixed total Hamiltonian density

- Adding matter

$$I_{\text{matter}}[g^J_{\mu\nu}; \text{matter}]$$

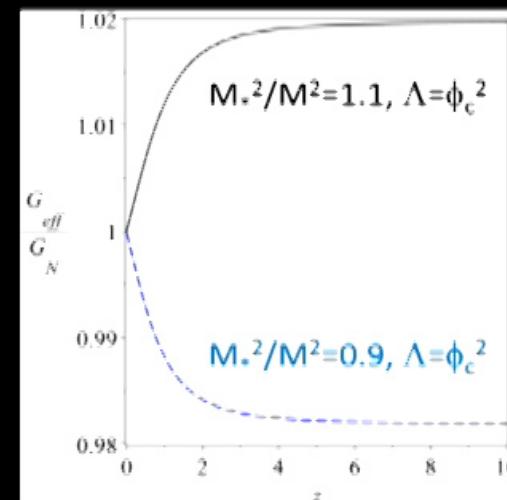
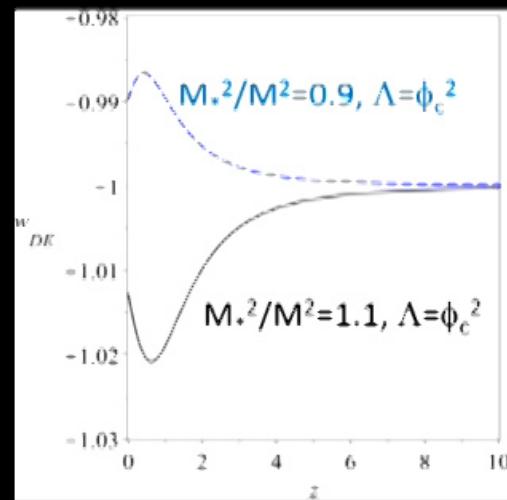
c.f. Carballo-Rubio, Di Filippo & Liberati (2018) argued that the square-root gravity should be of type-I but did not find a consistent matter coupling.

Example with $w_{DE} \neq -1$ & $G_{\text{eff}}/G \neq 1$

- $\Lambda \neq 0$ before canonical tr.

- $c_T^2 = f_1^2/f_0' \rightarrow f_0' = f_1^2$

- A choice of f_0
$$f_0' = \frac{(M_*/M_{\text{pl}})^2 + (\phi/\phi_c)^2}{1 + (\phi/\phi_c)^2}$$



More general example of type-I MMG & phenomenology

Katsuki Aoki, Antonio De Felice, Chunshan Lin, SM and Michele Oliosi, JCAP1901 (2019) 017

- Original phase space: (M, N^i, Γ_{ij}) & (Π_M, π_i, Π^{ij})

- Canonical tr. $(\mathcal{N}, \Gamma_{ij}, \Pi_N, \Pi^{ij}) \rightarrow (N, \gamma_{ij}, \pi_N, \pi^{ij})$

$$\mathcal{N} = -\frac{\delta F}{\delta \Pi_N} \quad \Gamma_{ij} = -\frac{\delta F}{\delta \Pi^{ij}} \quad \pi_N = -\frac{\delta F}{\delta N} \quad \pi^{ij} = -\frac{\delta F}{\delta \gamma_{ij}}$$

$$F = - \int d^3x (M^2 \sqrt{\gamma} f(\hat{\Pi}, \hat{\mathcal{H}}) + N^i \Pi_i) \quad \hat{\Pi} = \frac{1}{M^2 \sqrt{\gamma}} \Pi^{ij} \gamma_{ij}$$

$$f(\phi, \psi) = f_0(\phi) + f_1(\phi)\psi + \mathcal{O}(\psi^2) \quad \hat{\mathcal{H}} = \frac{1}{M^2 \sqrt{\gamma}} \Pi_N N$$

- Same sign for \mathcal{N} & N , Γ_{ij} & $\gamma_{ij} \rightarrow f_0 > 0, f_1 > 0$

- $c_T^2 = f_1^2/f_0' \rightarrow f_0' = f_1^2$

- $w_{DE} \neq -1$ in general (without dynamical DE)

- $G_{eff}/G = 1/f_0' \neq 1$ in general while $\Psi/\Phi = 1$

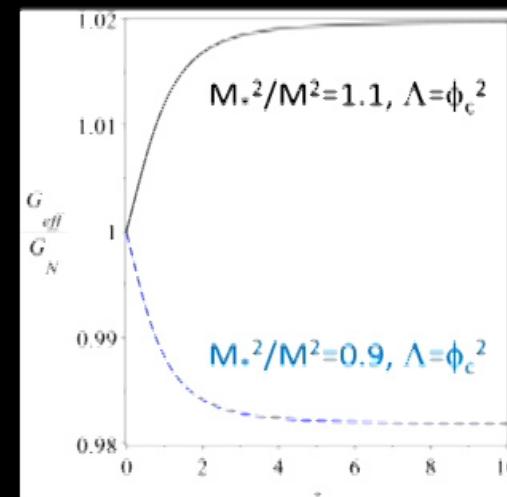
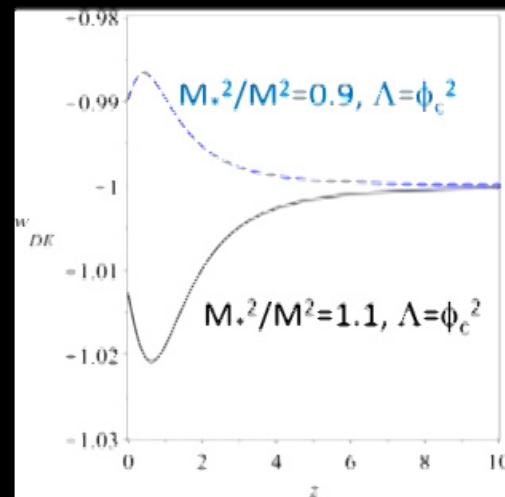
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$$\mathcal{N} = -\frac{\delta F}{\delta \Pi_N} \quad \Gamma_{ij} = -\frac{\delta F}{\delta \Pi^{ij}} \quad \pi_N = -\frac{\delta F}{\delta N} \quad \pi^{ij} = -\frac{\delta F}{\delta \gamma_{ij}}$$

$$F = - \int d^3x (M^2 \sqrt{\gamma} f(\hat{\Pi}, \hat{\mathcal{H}}) + N^i \Pi_i) \quad \hat{\Pi} = \frac{1}{M^2 \sqrt{\gamma}} \Pi^{ij} \gamma_{ij}$$

$$f(\phi, \psi) = f_0(\phi) + f_1(\phi)\psi + \mathcal{O}(\psi^2) \quad \hat{\mathcal{H}} = \frac{1}{M^2 \sqrt{\gamma}} \Pi_N N$$

- Same sign for \mathcal{N} & N , Γ_{ij} & $\gamma_{ij} \rightarrow f_0 > 0, f_1 > 0$

- $c_T^2 = f_1^2/f_0' \rightarrow f_0' = f_1^2$

- $w_{DE} \neq -1$ in general (without dynamical DE)

- $G_{eff}/G = 1/f_0' \neq 1$ in general while $\Psi/\Phi = 1$

Type-II minimally modified gravity (MMG)

- # of local physical d.o.f. = 2
- No Einstein frame
- Cannot be recast as GR + matter by change of variables
- Is there such a theory? Yes!
- Example: Minimal theory of massive gravity
[Antonio De Felice and SM, PLB752 (2016) 302; JCAP1604 (2016) 028; PRL118 (2017) 091104]
- Another example? : Ghost-free nonlocal gravity (if extended to nonlinear level?)

Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Fierz-Pauli theory (1939)
Unique linear theory
without instabilities
(ghosts)



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van Dam-Veltman-
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Massless limit \neq
General Relativity

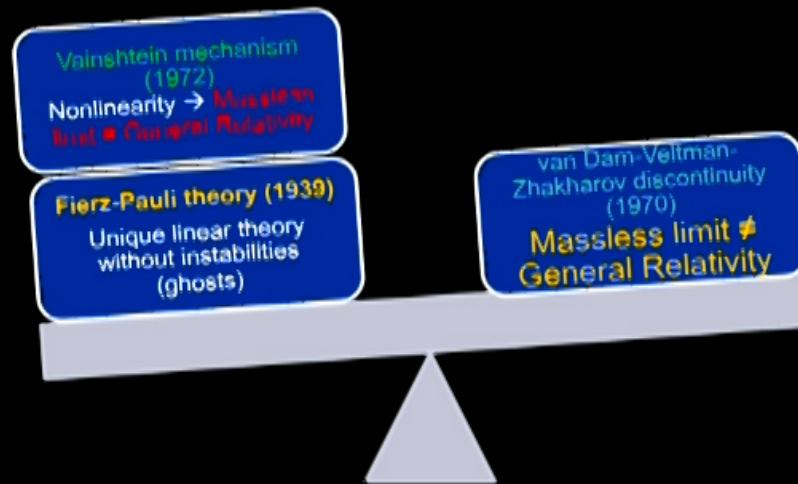
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Vainshtein mechanism
(1972)
Nonlinearity \rightarrow Massless limit \neq General Relativity

Fierz-Pauli theory (1939)
Unique linear theory
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(ghosts)

Boulware-Deser ghost
(1972)
Massless limit \neq Nonlinear theory
 \Rightarrow Instability (ghost)

Van Dam-Veltman-
Zakharov discontinuity
(1970)
Massless limit \neq
General Relativity

Massive gravity in a nutshell

Simple question: Can graviton have mass?
May lead to acceleration without dark energy

Yes?

No?

de Rham-Gabadadze-Tolley (2010)

First example of massive massive gravity will go to GR limit when $T \gg T_c$

Vainshtein mechanism (1972)

Nonlinearity \rightarrow Massless limit \leftrightarrow General Relativity

Fierz-Pauli theory (1939)

Unique linear theory without instabilities (ghosts)

Boulware-Deser ghost (1972)

Massless limit \leftrightarrow General Relativity (ghosts)

van Dam-Veltman-Zakharov discontinuity (1970)

Massless limit \neq General Relativity

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

D'Amico, et.al. (2011)
Non-existence of flat
FLRW (homogeneous
isotropic) universe!

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

Open universes with self-acceleration
GLM (2011a)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity

Good?

Bad?

More general fiducial metric $f_{\mu\nu}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

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Cosmological solutions in nonlinear massive gravity

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More general fiducial metric $f_{\mu\nu}$
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GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Minimal theory of massive gravity (MTMG)

De Felice & Mukohyama, PLB752 (2016) 302;
JCAP1604 (2016) 028

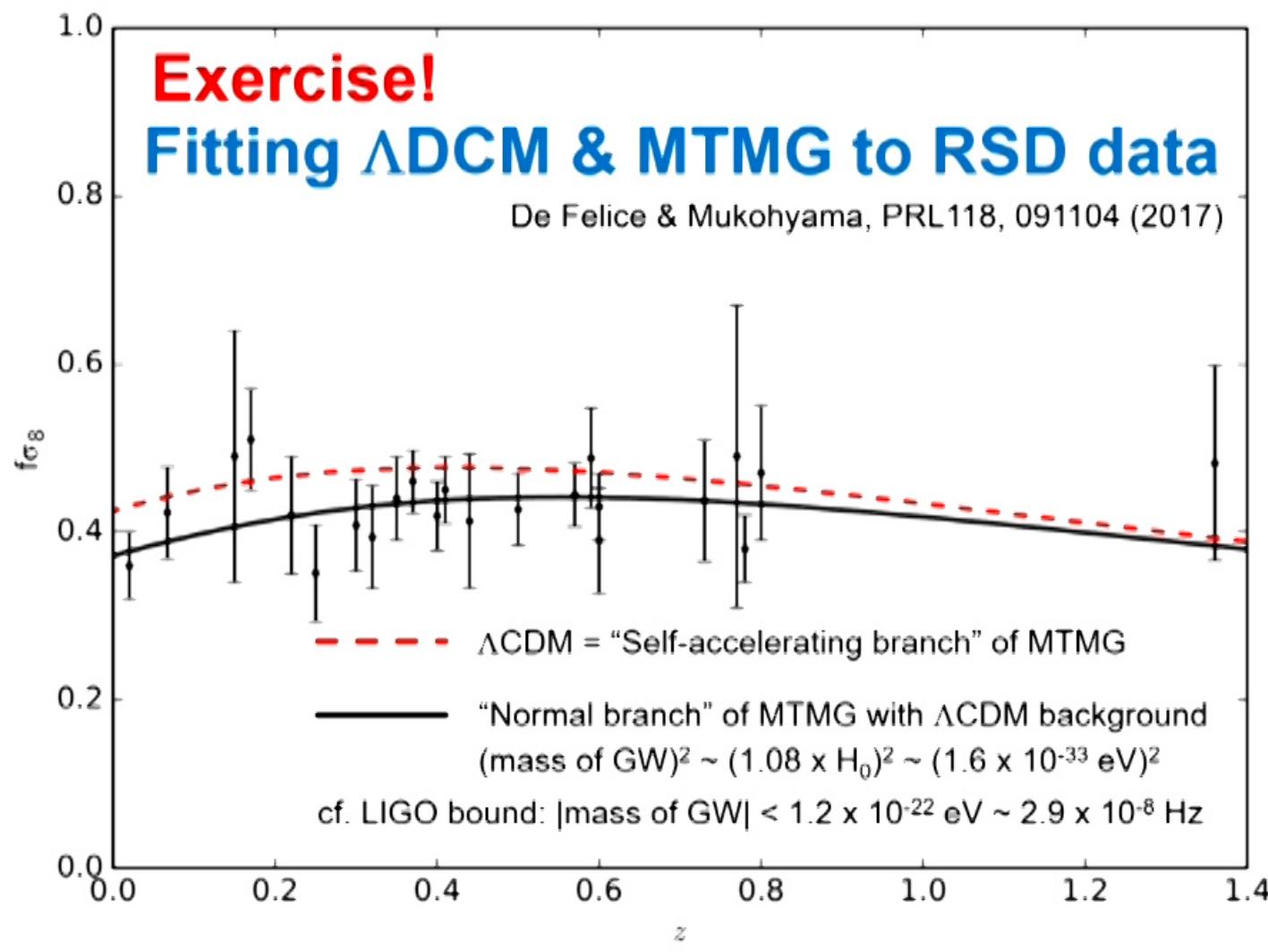
- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost
- positivity bound does not apply

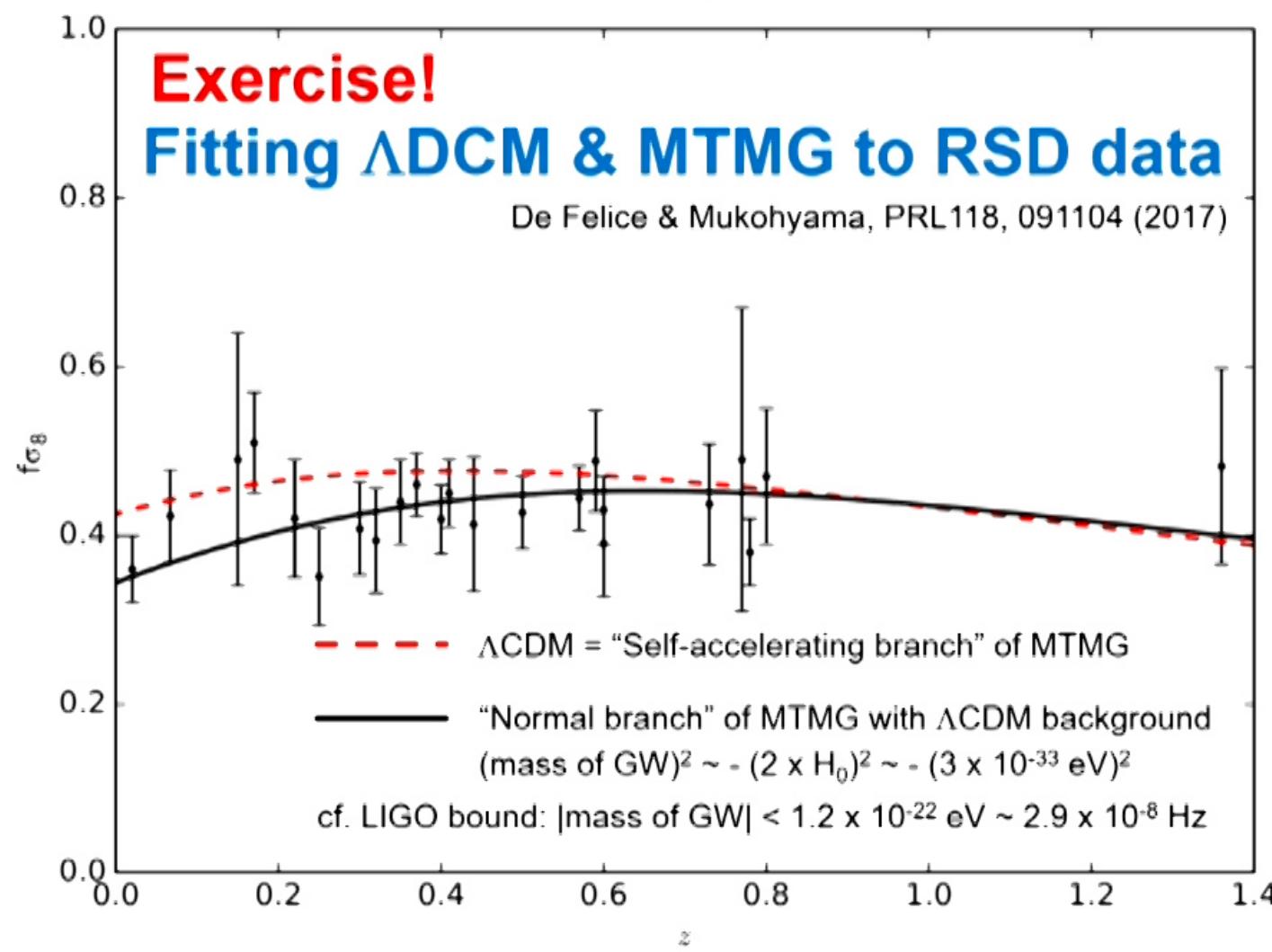
Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. **Add 2 additional constraints**

(It is easy to go back to Lagrangian after 3.)

Lorentz-violation due to graviton loops is suppressed by m^2/M_{Pl}^2 and thus consistent with all constraints for $m = O(H_0)$





BH and Stars in MTMG

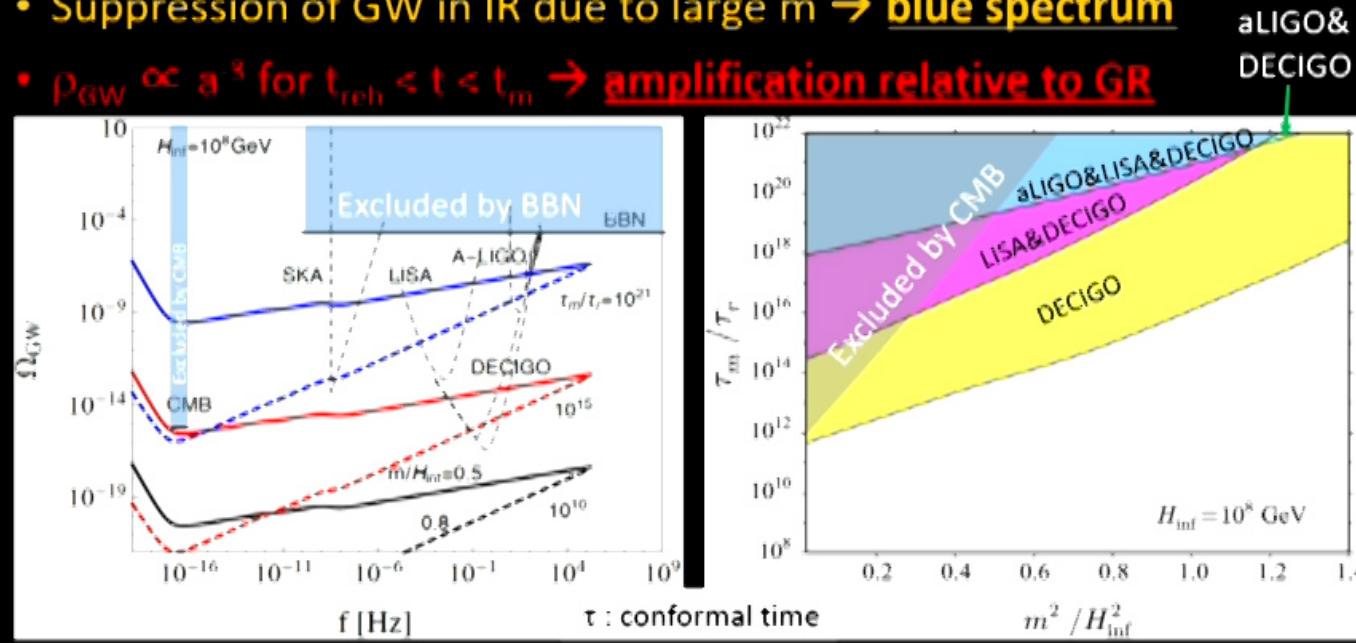
De Felice, Larrouture, Mukohyama, Oliosi,
PRD98, 104031 (2018)

- Any solution of GR that can be rendered spatially flat by a coordinate change is also a solution of the self-accelerating branch of MTMG, with or without matter.
- Schwarzschild sol → **BH, star exterior**
- Spherical GR sol with matter → **gravitational collapse, star interior**
- **No strong coupling**
- **No singularities except for those in GR**

Blue-tilted & amplified primordial GW from MTMG

Fujita, Kuroyanagi, Mizuno, Mukohyama,
PLB789 (2019) 215

- Simple extension: $c_i \rightarrow c_i(\phi)$ with $\phi = \phi(t)$
- m large until t_m ($t_{reh} < t_m < t_{BBN}$) but small after t_m
cf. no Higuchi bound in MTMG
- Suppression of GW in IR due to large $m \rightarrow$ **blue spectrum**
- $\rho_{GW} \propto a^{-3}$ for $t_{reh} < t < t_m \rightarrow$ **amplification relative to GR**



Summary

- Minimal # of d.o.f. in modified gravity = 2 can be saturated → minimally modified gravity (MMG)
- Type-I MMG: \exists Einstein frame
Type-II MMG: no Einstein frame
- Example of type-I MMG
GR + canonical tr. + gauge-fixing + adding matter
Rich phenomenology: w_{DE} , G_{eff} , etc.
- Example of type-II MMG
Minimal theory of massive gravity (MTMG)
Cosmology: self-accelerating branch & normal branch
BHs and stars: no strong coupling, no new singularity
Stochastic GWs: blue-tilted & largely amplified