

Title: Lecture 7

Speakers: Crystal Senko

Collection: Many-Body States and Dynamics Workshop II

Date: June 13, 2019 - 11:30 AM

URL: <http://pirsa.org/19060033>



Feasibility of qudit-based quantum computation

(plus: open access quantum computing coming to a lab near you!)

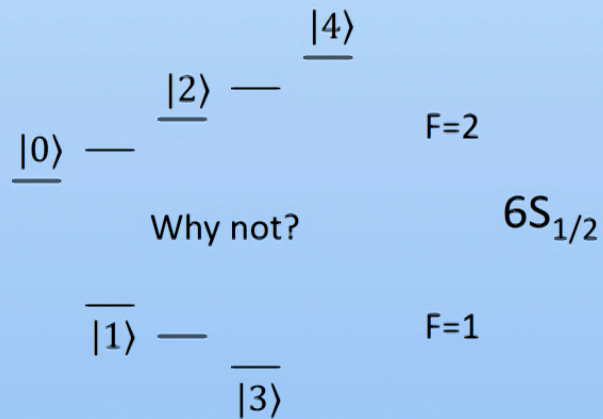
Crystal Senko
IQC-PI workshop
June 13, 2019





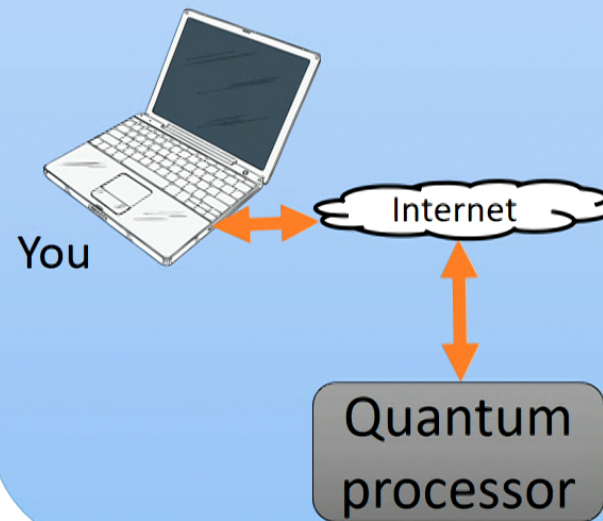
Themes of our research

Qudit based quantum computing



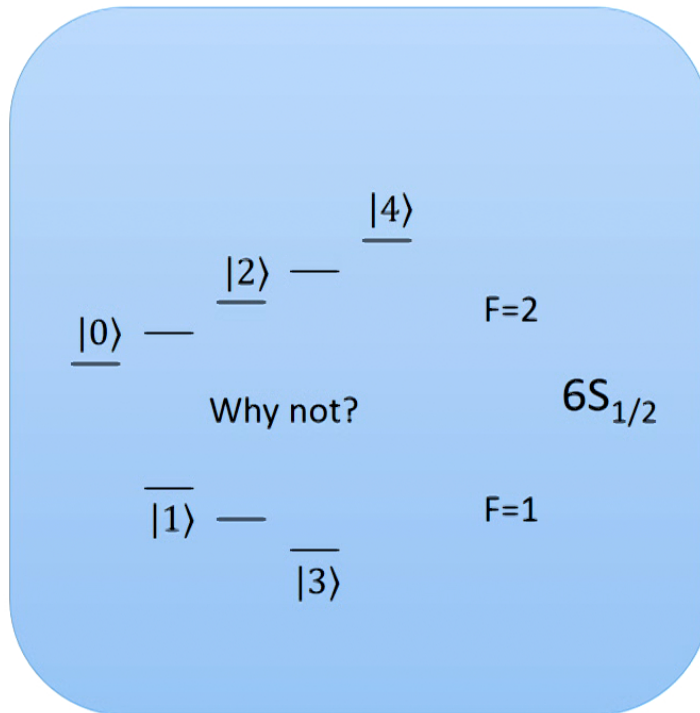
Open access quantum computing

Collaboration with Rajibul Islam



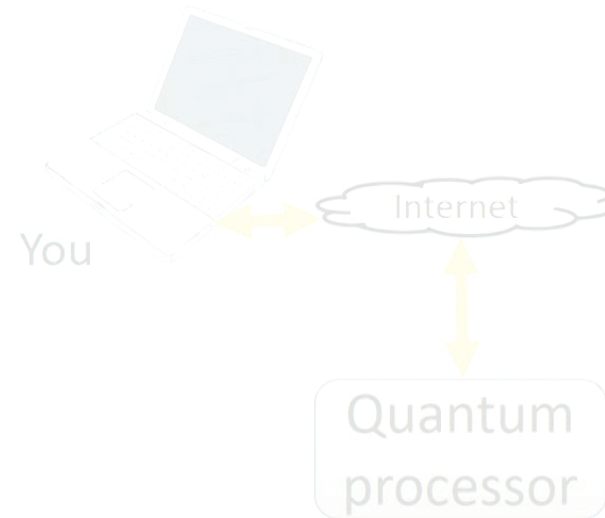
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To start: why qudits?

- Increase Hilbert space: 2^N (qubits) vs d^N (qudits)

$$2^{10} = 1024$$

$$3^{10} = 59049$$

$$5^{10} = 9765625$$

²Fedorov, A. et. al., 2011. *Nature*, 481(7380), 170-172. doi:10.1038/nature10713

³Parasa et. al., 2011, IEEE 41 <https://doi.org/10.1109/ISMLV.2011.47>

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⁶Lanyon et. al., 2008, NaturePhysics <https://doi.org/10.1038/nphys1150>

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To start: why qudits?

- Increase Hilbert space: 2^N (qubits) vs d^N (qudits)
- More efficient algorithm
 - Toffoli gate: 16 gates (2-level) vs 6 gates (3-level)²
 - More accurate quantum phase estimation³
- More forgiving error threshold for error correcting codes^{4,5,6}

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Number of Ions vs d for QPE up to 5 Decimal Points

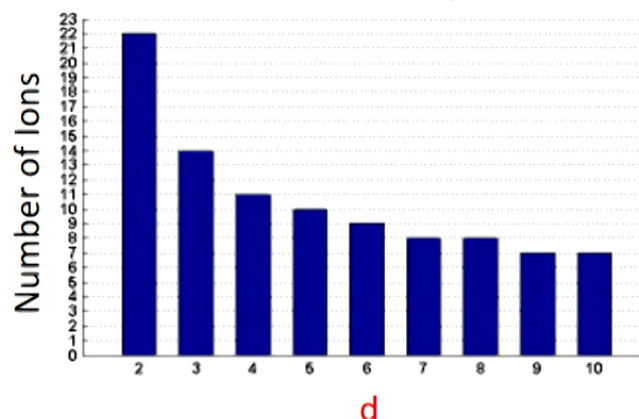


Figure adapted from [3]

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Can we implement *useful* qudit operations in the lab?

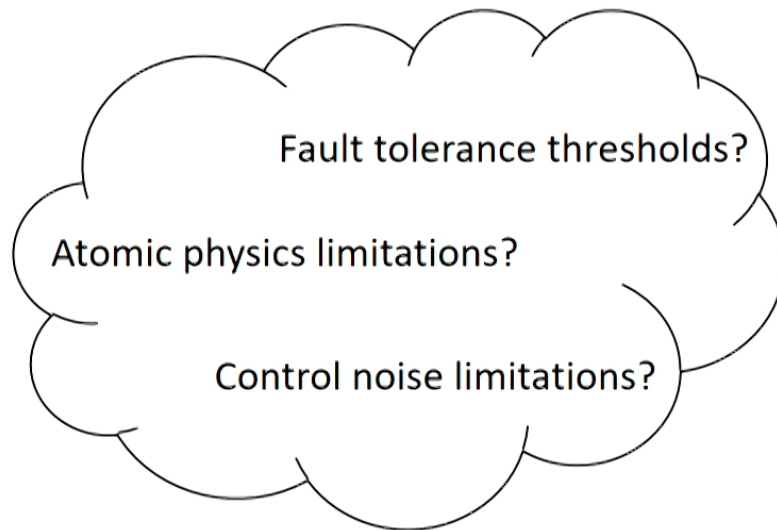
- How do you perform:
 - State preparation and measurement
 - Single-qudit and two-qudit gates

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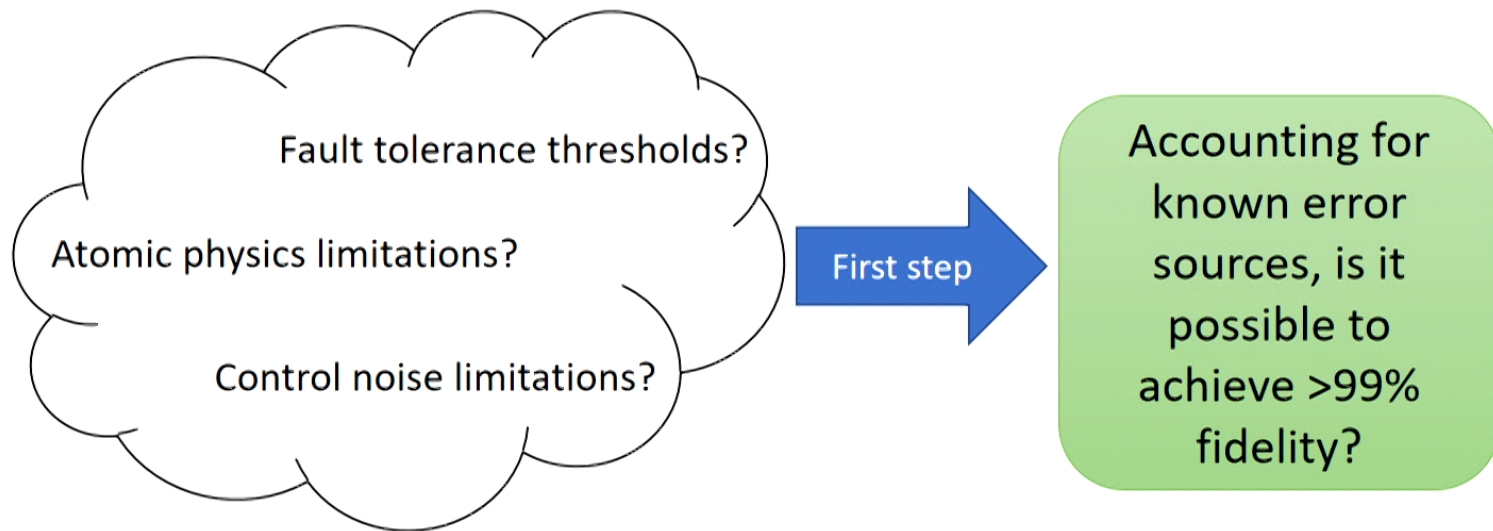
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Caveats to our “fidelity” estimates

What we did here:
fidelity F

- Specific experimental protocol
- *Upper* bound on atomic structure limitations
- Upper bound on noise sources we don't know how to improve

5

Caveats to our “fidelity” estimates

What we'll get in the lab: fidelity $< F$

- All the limitations we estimated, plus:
- Control noise that can be engineered out (voltage noise, etc)

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“Fundamental” limitations: fidelity probably $> F$

- Lots of parameters to optimize
- More clever protocols may be available

State Measurement

- Need more than one bit of information!

1. Shelf states in $S_{1/2}$ to corresponding metastable states in $D_{5/2}$ except for $|0\rangle$.
2. Collect fluorescence from all remaining states in $S_{1/2}$.
3. Bring back state $|1\rangle$ to $S_{1/2}$ level.
4. Repeat steps 2 and 3 for different states.

$6P_{1/2}$ —————

35 s lifetime

————— $5D_{5/2}$

$6S_{1/2}$ $\overline{|0\rangle}$ $\overline{|1\rangle}$ $\overline{|2\rangle}$

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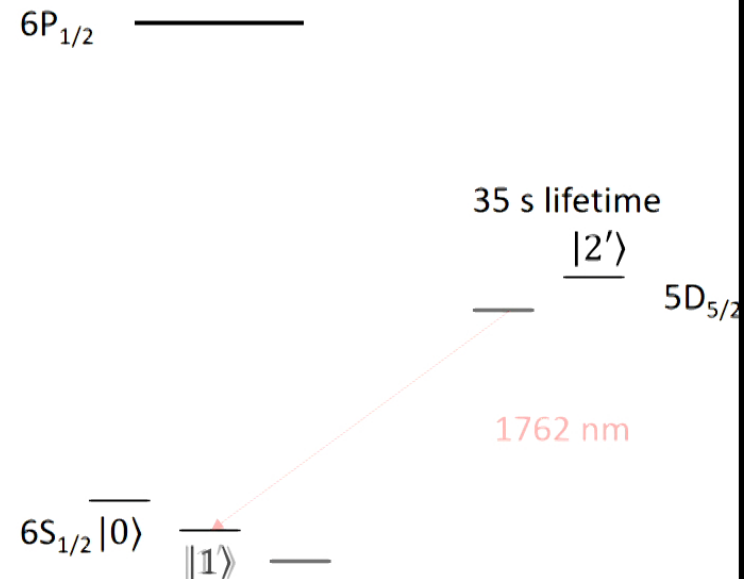
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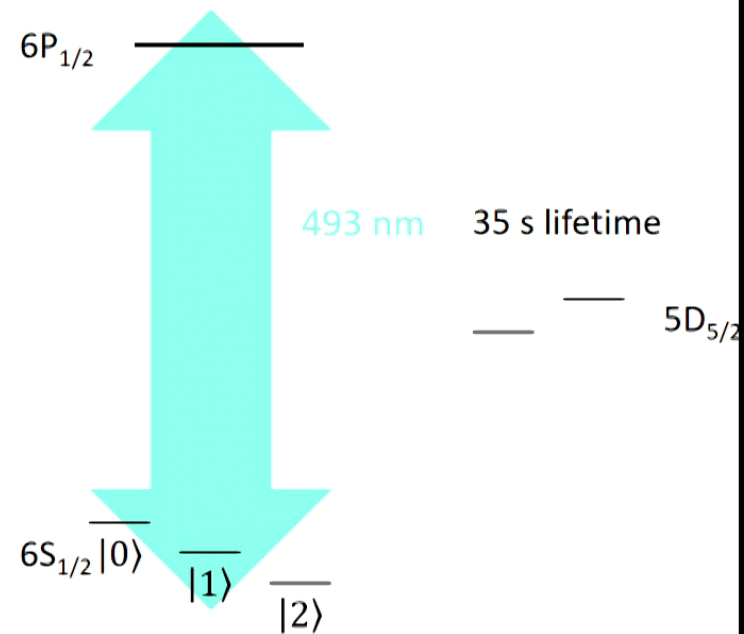
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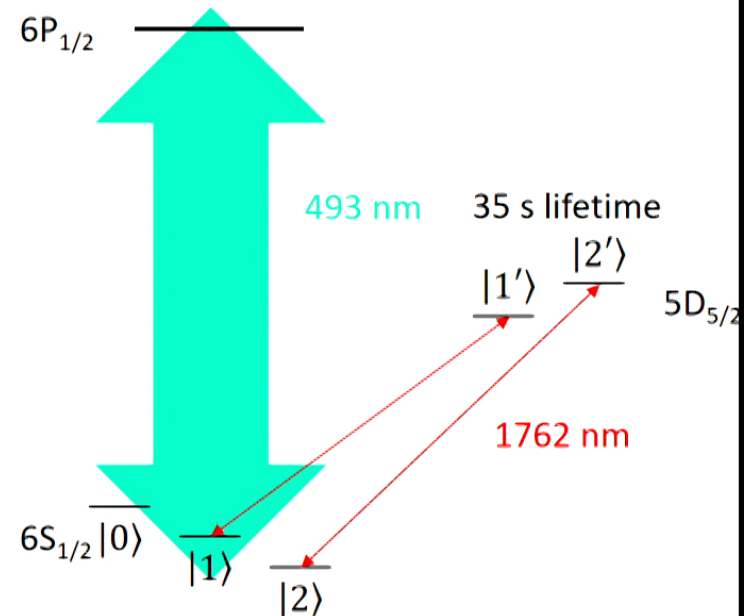


State Measurement

- Error sources:

- Finite initial frequency detuning
- Landau-Zener probability of diabatic transition
- Dephasing due to laser linewidth
- Off-resonant coupling

d (dimension)	Fidelity F	Error (1-F)
3	99.78%	2E-3
5	99.15%	8E-3
7	98.51%	1.5E-2

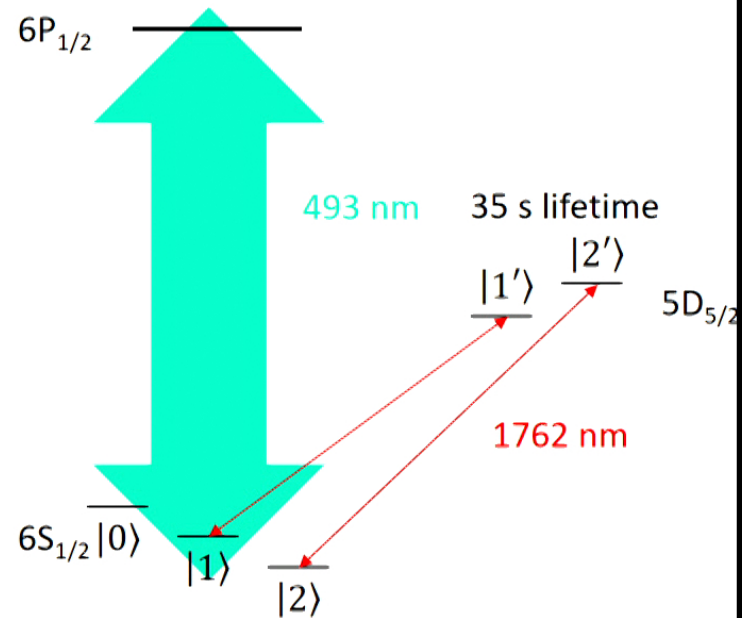


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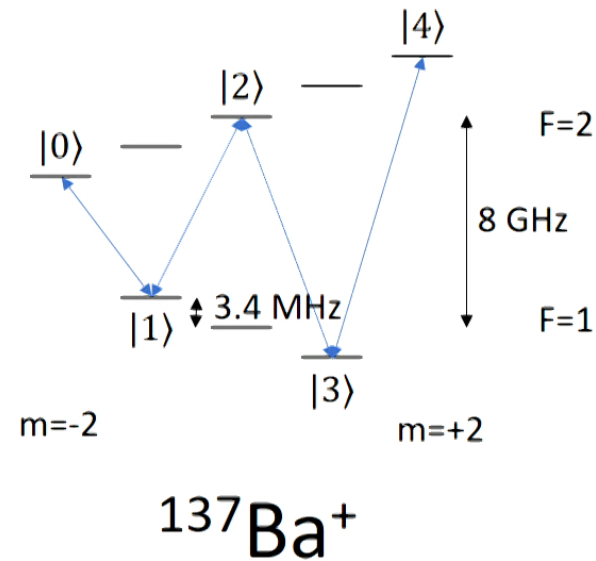
Single Qudit Gate

- With a **fully connected graph**, you can do any single-qudit unitary.
- Need $\frac{d(d-1)}{2} + 2(d-1)$ 2-level transitions

$$\hat{U} = \hat{V}_{d(d-1)/2} \dots \hat{V}_2 \hat{V}_1 \hat{D}$$

$$\hat{V} = \exp\left(i\theta(e^{i\varphi}|j\rangle\langle k| + e^{-i\varphi}|k\rangle\langle j|)\right)$$

Schirmer et. al., 2001, Journal of Physics A
stacks.iop.org/JPhysA/35/8315



$$n^2 \theta^2$$

stochastic vs. coherent
worst-case accumulation of error

channel C_r $r = 1 - \text{fidelity to } I$

vs coherent only by largest Kraus op

$$(\rho) = \sum_k A_k \rho A_k^\dagger$$

$U D$ diagonal & positive semi-definite

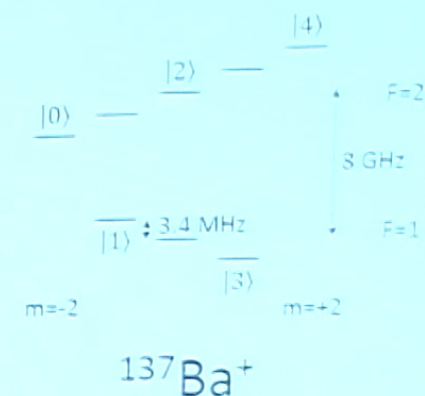
U is a rotation by θ about $\vec{V} = (V_x, V_y, V_z)$

Single Qudit Gate

Error sources:

- Magnetic field noise
- Off-resonant coupling to wrong states

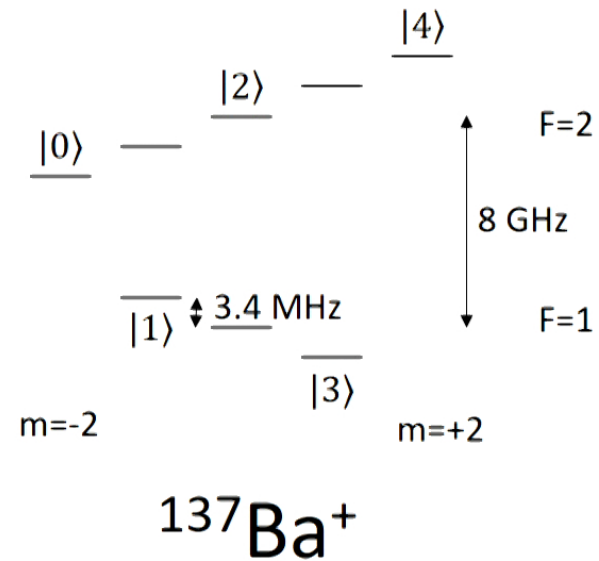
Gate	Error (d=3)	Error (d=5)
X	6.25E-05	6.41E-04
Y	8.93E-05	0.001379
Z	3.51E-05	8.65E-04
T	4.71E-05	8.10E-04
F	1.13E-04	0.001323



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Entangling gate

- Laser-based gate protocol
- Interaction mediated by Coulomb forces
- Form of entangling gate (maps to SUM gate with single qudit operations):

$$U = \exp\left(i\theta \left[S_x^{(1)} + S_x^{(2)}\right]^2\right)$$

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Overall fidelity:

- 99.27% for $d = 3$.
- 32% for $d = 5$ (with experimental configuration we know how to implement)
- 96.6% for $d = 5$ (with a configuration we don't know how to do – could probably be improved)

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Entangling gate fidelity

Error Source	$d = 3$	$d = 5$
Lamb-Dicke approximation	7×10^{-4}	5.8×10^{-3}
Rotating wave approximation	5×10^{-4}	2.2×10^{-3}
Spectator phonon mode	2.9×10^{-3}	1.26×10^{-2}
Photon scattering	8×10^{-4}	1.6×10^{-3}
Imperfect cooling	1×10^{-4}	2.7×10^{-3}
Motional heating	3.3×10^{-3}	4.6×10^{-3}
Magnetic field noise	$< 10^{-4}$	$< 10^{-4}$
Off-resonant frequencies	NA	6.8×10^{-1}

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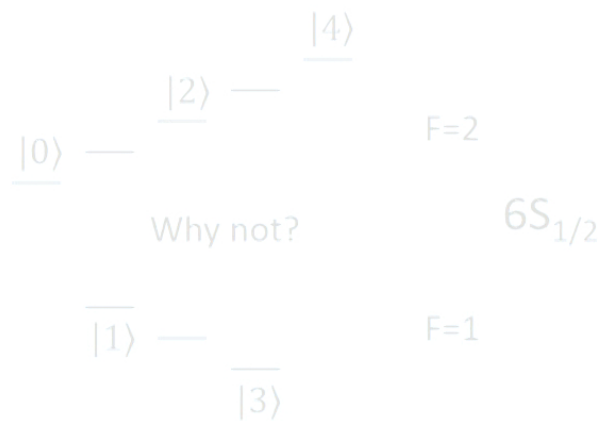
Summary for qudits

- Possible to obtain 99% fidelity for 3-level qudits
- Higher dimensions may also be possible

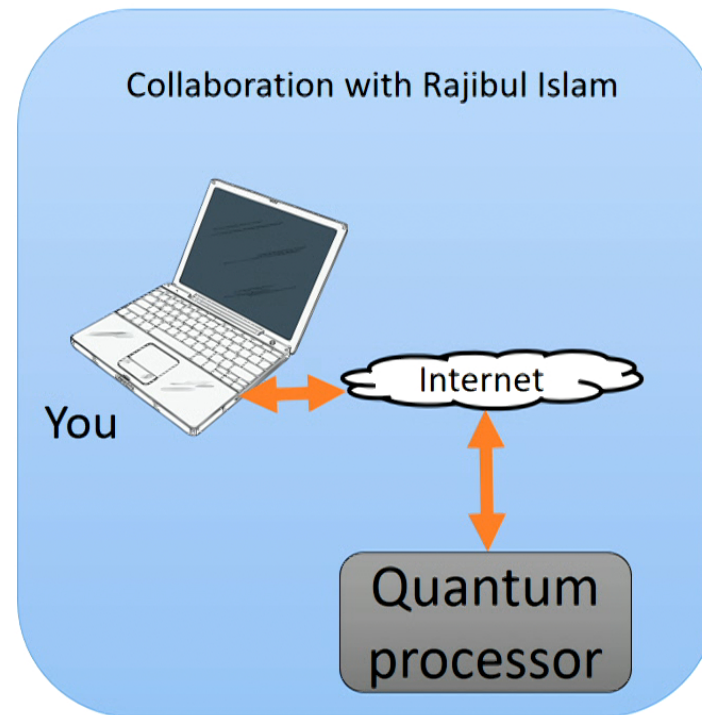
	Fidelity		
d (dimension)	Measurement	Single-qudit gate	2-qudit gate
3	99.78%	99.98%	99.27%
5	99.15%	99.87%	32% (*96.63%)
7	98.51%	Not investigated	Not investigated

Themes of our research

Qudit based
quantum computing



Open access
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Overview of QuantumIon goals

- 10+ qubit ion trap quantum computer
 - Individual qubit control
 - Entangling gates between any pair of qubits
 - Proven architecture (see e.g. UMD, IonQ)
- Remote access platform
- Designed for multiple types of experiment

Quantum error
correction

Quantum
algorithms

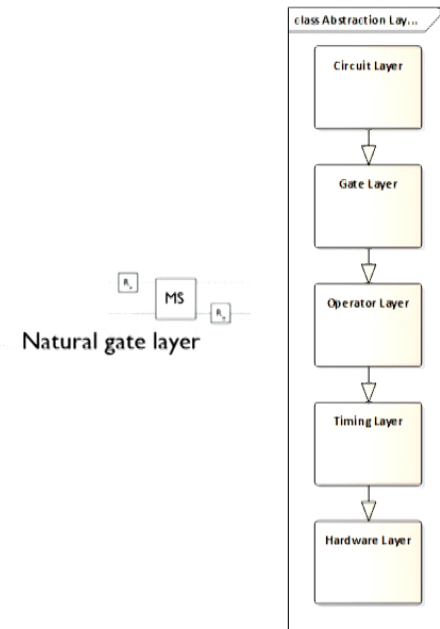
Pulse shaping, gate
optimizations

Quantum optics,
trapped ion physics

Many more

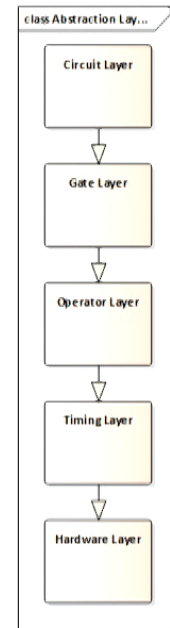
How to describe various controls?

- Desired Traits
 - Abstraction layers



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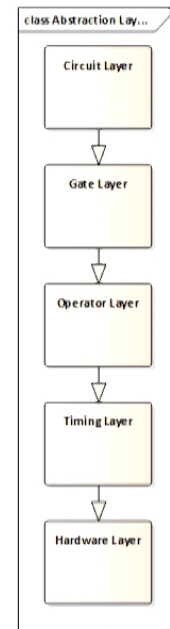
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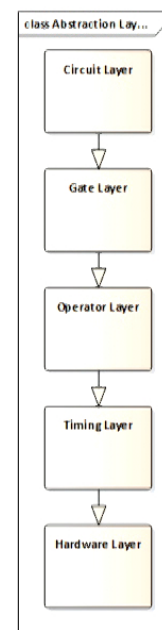
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- Sub-nanosecond timestamps of operation
- Support for multiple languages (e.g. Python, Matlab, GUI???)
- Strong separation between user code & QI machine code
- Internal consistency



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 - Support for multiple languages (e.g. Python, Matlab, GUI???)
 - Strong separation between user code & QI machine code
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- Current languages (IBM, Google, Microsoft) insufficient
- New language of our own
 - Laser pulses are precision timed 'events'
 - Support classical/quantum decisions, loops, etc
 - Allocated resource strategy
 - XML Intermediate language
 - Support timing arithmetic on named constants (e.g. calibration params)
 - Function/macro definitions for design re-use & abstraction



Example User Code

```
// Example program a few pulses and AWG followed by a CCD image capture

// Begin by connecting to the machine this object will have all future transactions
qi = QuantumIon.connect("Username", "Password");

program = qi.CreateQuantumIonExperiment();           // create a new program
img = qi.AllocateQuantumIonImage();                 // the CCD measurement reports an image. this allocates space
MyPulseShape = qi.CreateAWGWaveForm( qi, "mine.mat" ); // same for AWG. It is stored on the QI machine

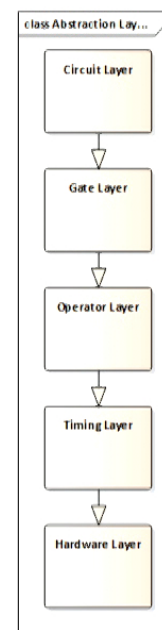
program.AddSteps(                                     // Create program
[
    qi.CreateStandardTrapPrepStep(),                  // ensures the trap is ready
    qi.CreateStatePrepStep( ),                       // state preparation
    qi.CreateSimpleLaserPulseStep( 0.2, 0.3 ),         // pulse @ t=0.2, duration 0.3
    qi.CreateSimpleLaserPulseStep( 0.8, 0.5*qi.cal.rabiperiod), // pulse @ t=0.8, duration calculated
    qi.CreateAWGLaserPulseStep( 1.0, MyPulseShape ),  // AWG pulse @ t=1.0
    qi.CCDRawImageMeasurementStep( 1.1, img )         // CCD raw image and tag for saving later
]);

// perform 10,000 statistical averages
for(int n = 0; n<10000; n++)
{
    qi.QueueProgram( program ); // enqueue on machine
    qi.run();                   // run program

    result = img.download();
    DoStuffWithImage_IDontCareItsOnYourPCNow(); // post process
}
// done
qi.disconnect();
```

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How you run an algorithm:

- Describe quantum program
 - “Get the trap ready”
 - “Prepare the qubits in $|0\rangle$ ”
 - Apply laser pulse from $t=100$ us to $t=200$ us
 - ...
 - Measure qubits at $t=300$ us
- Hand us the quantum program
- Download the measurement results

Acknowledgement



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Post-Doc: Matthew Day

Principal Investigator: Crystal Senko

- QuantumIon: collaboration with Rajibul Islam

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