Title: Lecture 7: Factorization Algebras and the General Structure of QFT

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Collection: QFT for Mathematicians

Date: June 28, 2019 - 11:00 AM

URL: http://pirsa.org/19060024

More on Noether's Theorem

1) Factorization Envelope

$\mathfrak{g}$ Lie algebra

$U\mathfrak{g}$ associative algebra

$\mathfrak{g}$ is { Lie* algebra / Vertex Lie algebra

$U^{ch}\mathfrak{g}$ { chiral envelope / vertex algebra envelope

vertex algebra

Factorization envelope

$\mathcal{L}$ sheaf of dglas on M

(technical assumption:

$\mathcal{L}(U) = C^\infty(U, L)$

L a graded v. bundle

$d, [\ ]$ are differential operators)

$\rightsquigarrow$ $U^{fact}(\mathcal{L})$

a factorization algebra

Encompasses

Ug, $U^{ch}g$ as special cases.

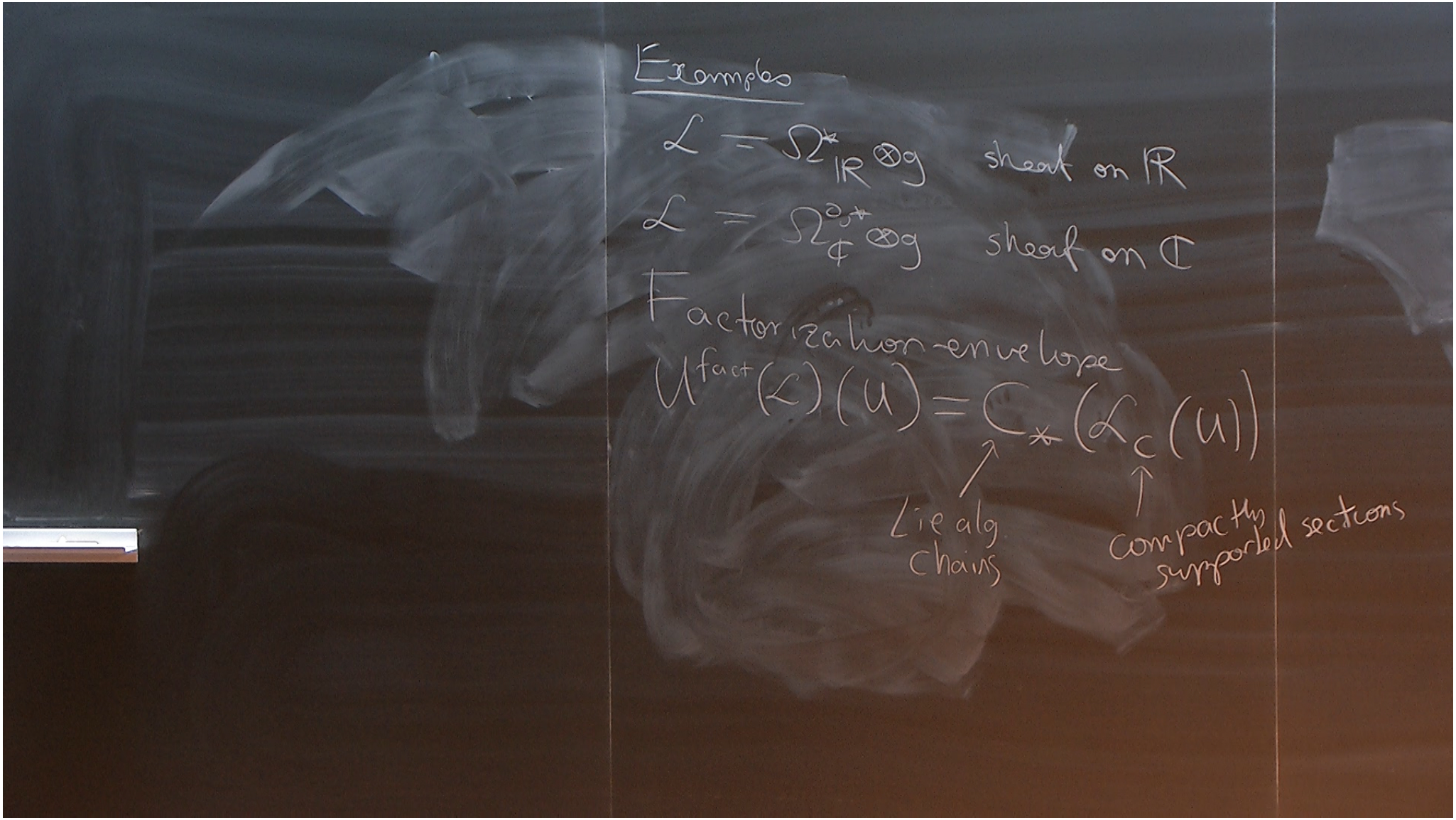
Factorization envelope

$\mathcal{L}$ sheaf of dglas on M $\rightsquigarrow$ $U^{fact}(\mathcal{L})$ a factorization algebra

(technical assumption: $\mathcal{L}(U) = C^\infty(U, L)$, L a graded v. bundle, $d, [\ ]$ are differential operators)

Encompasses U_g, U^{ch}_g as special cases.

Also higher Kac-Moody algebras (Williams, Kapranov et al.)


Examples
$\mathcal{L} = \Omega^*_{\mathbb{R}} \otimes \mathfrak{g}$ sheaf on $\mathbb{R}$
$\mathcal{L} = \Omega^{0,*}_{\mathbb{C}} \otimes \mathfrak{g}$ sheaf on $\mathbb{C}$
Factorization envelope
$U^{fact}(\mathcal{L})(U) = C_*(\mathcal{L}_c(U))$
Lie alg chains
compactly supported sections

⊗g sheaf on $\mathbb{R}$

⊗g sheaf on $\mathbb{C}$

non-enveloping

$U) = C_*(\mathcal{L}_c(U))$

Lie alg. Chains

compactly supported sections

Lemma

If $\mathcal{L} = \Omega^*_{\mathbb{R}} \otimes g$

then $H^*(U^{fact}(\mathcal{L}))$

$= Ug$ as an associative algebra.

Proof: $C_*(\mathcal{L}_c((a,b))$

$= S^*(\Omega^*_c(a,b) \otimes g[1])$

$\rightsquigarrow$ U^{fact}

a fac

Encomp

Ug,

cases.

Also h

algeb

(U

$\Omega^*_{\mathbb{R}\otimes g}$

$^*(U^{fact}(\mathcal{L}))$

g as an associative

lgebra.

$C_*(\mathcal{L}_c((a,b))$

$S^*(\Omega^*_c(a,b)\otimes g[1])$

If $X \in g$

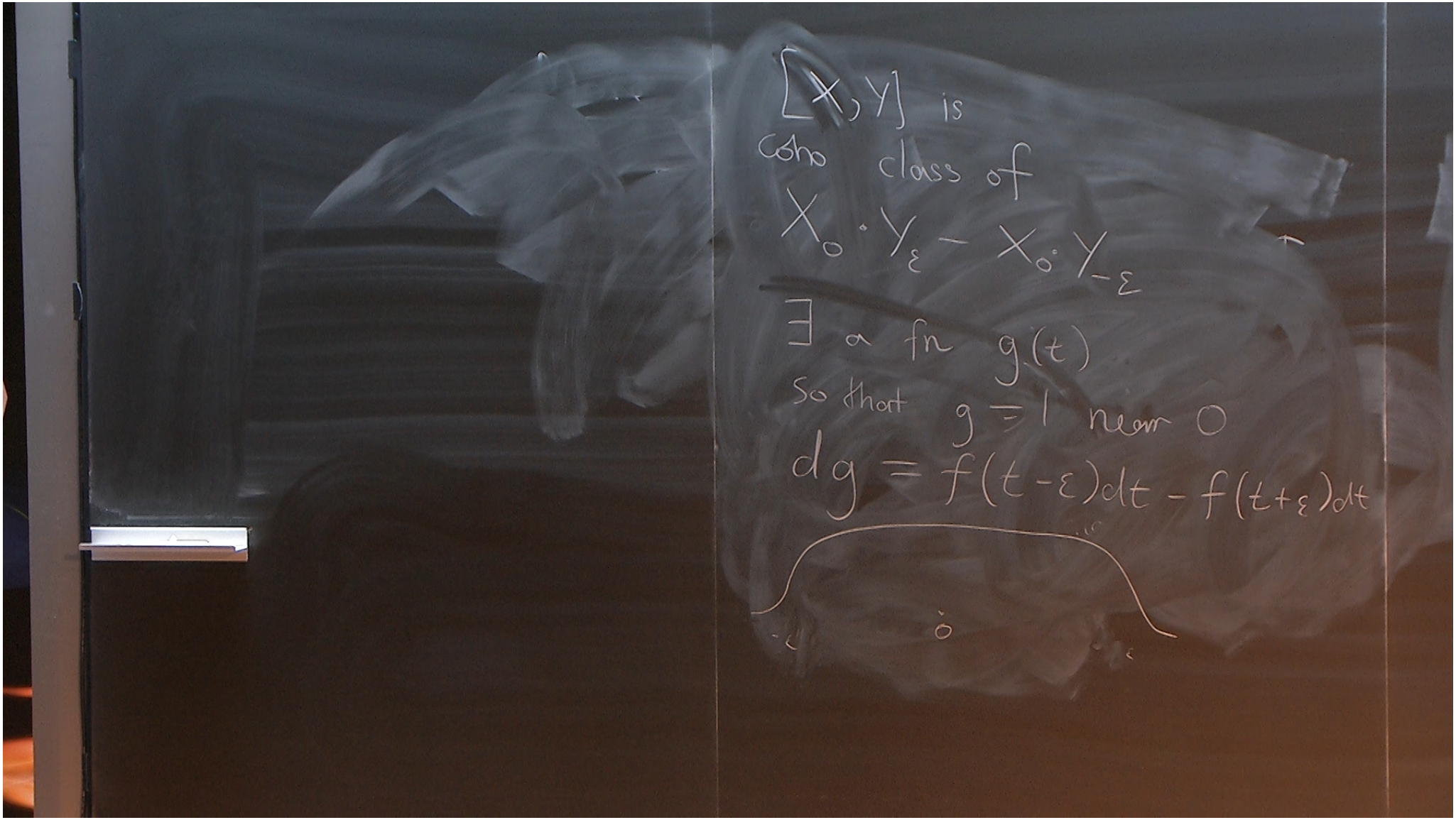
let $X_0 = X \otimes f(t)dt$

where f is a bump function

0

$X_\varepsilon = X \otimes f(t-\varepsilon)dt$

ε


[X, Y] is
Lie class of
$X_0 \cdot Y_\varepsilon - X_0 \cdot Y_{-\varepsilon}$
$\exists$ a fn $g(t)$
so that $g = 1$ near 0
$dg = f(t-\varepsilon)dt - f(t+\varepsilon)dt$

$[X, Y]$ is
Coho class of
$X_0 \cdot Y_\varepsilon - X_0 \cdot Y_{-\varepsilon}$

$\exists$ a fn $g(t)$
so that $g = 1$ near 0
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$d(X_0 \cdot (Y \otimes g))$
$= X_0 \cdot (Y \otimes dg)$
$+ [X_0, Y \otimes g]$

comes from Lie alg. differential
chains

$$d\big(X_0\cdot(Y\otimes g)\big)$$
$$= X_0\cdot(Y\otimes dg)$$
$$+ [X_0, Y\otimes g]$$

comes from Lie alg. chains differential

$[X,Y]$ = comm. class of $X_\varepsilon Y_\varepsilon - X_\varepsilon Y_{-\varepsilon}$

$= [X,Y] f(t) g(t_0)$

$= [X,Y]_0$ ✓

So there's a homomorphism.

2d example

Lemma

$\mathcal{L} = \Omega^{0,\mathrm{ir}}(\mathbb{C}) \otimes \mathfrak{g}$

$H^*(U^{\mathrm{fact}}(\mathcal{L}))$ is a VOA which is level 0 Kac-Moody algebra.

Proof Note $H^*_{\bar\partial, c}(D) \otimes \mathfrak{g}$

$\sim z^{-1}\mathfrak{g}[z^{-1}][-1]$

spanned by δ_0, + derivatives

$H^*(U^{\mathrm{fact}}(\mathcal{L})(D))$

$= H^*(\mathrm{Sym}^*(\Omega^{0,\mathrm{ir}}_c(D) \otimes \mathfrak{g}[1]))$

$= \mathrm{Sym}^*(z^{-1}\mathfrak{g}[z^{-1}])$

= Vacuum module for Kac-Moody.

2d example

Lemma

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spanned by δ_0, + derivatives

$H^*(U^{fact}(\mathcal{L})(D))$

$= H^*(Sym^*(\Omega^{0,*}_c(D) \otimes \mathfrak{g}[t]))$

$= Sym^*(z^{-1}\mathfrak{g}[z^{-1}])$

$=$ Vacuum module for Kac-Moody.

If $X \in \mathfrak{g}$, $X_0 = X \otimes \delta_0$.

$H^*(U^{fact}(\mathcal{L})(D))$

$= H^*(\mathrm{Sym}^*(\Omega^{0,*}_c(D) \otimes g[[t]]))$

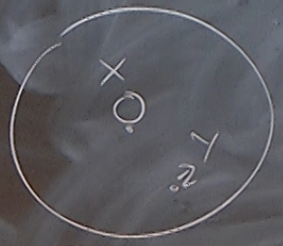
$= \mathrm{Sym}^*(z^{-1}g[z^{-1}])$

= Vacuum module for Kac-Moody.

If $X \in g$, $X_0 = X \otimes \delta_0$

We want to compute cohomology class of

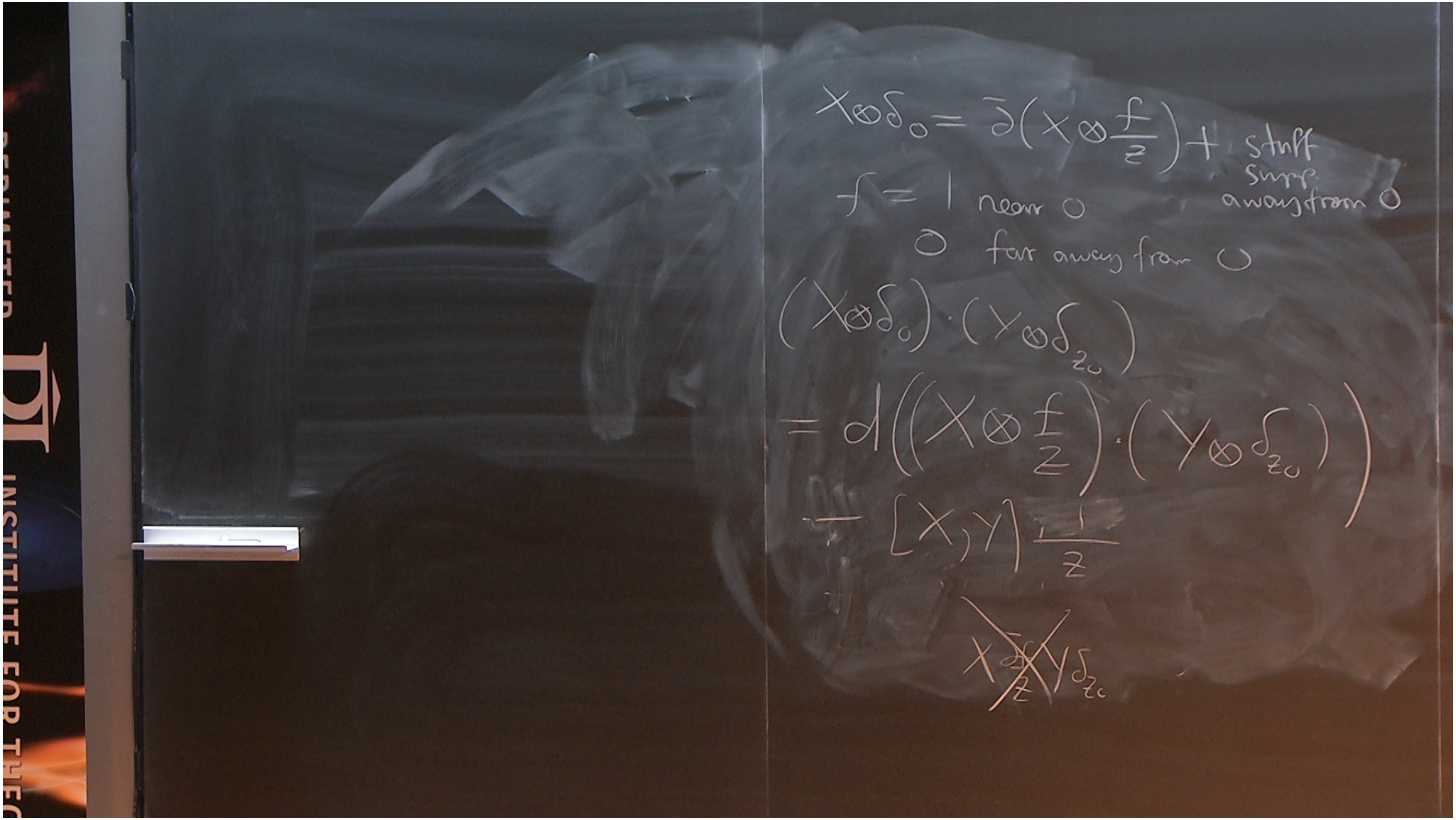
$X_0 \cdot Y_{z_0}$



To compute this, note

$\delta_0 = \bar{\partial}\left(\frac{1}{z}\right)$

$S_0\, X \otimes \delta_0 = \bar{\partial}\left(X \otimes \frac{1}{z}\right)$


$X \otimes \delta_0 = \bar{\partial}(X \otimes \frac{f}{z}) +$ stuff supp away from 0
$f = 1$ near 0
0 far away from 0
$(X \otimes \delta_0) \cdot (Y \otimes \delta_{z_0})$
$= d\left(\left(X \otimes \frac{f}{z}\right) \cdot (Y \otimes \delta_{z_0})\right)$
$- [X, Y] \frac{1}{z}$
$X \otimes \frac{f}{z} Y \delta_{z_0}$

$X \otimes \frac{f}{z}) +$ stuff supp away from 0

away from 0

$\delta_{z_0})$

$\frac{f}{z}) \cdot (Y \otimes \delta_{z_0}))$

$\frac{1}{z}$

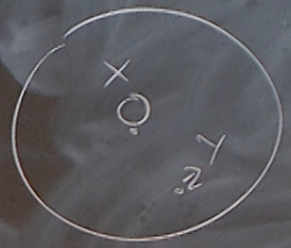
$Y \delta_{z_0}$

$(X \delta_0)(Y \cdot \delta_{z_0})$

$\sim \frac{1}{z}[X, Y]$

We want to compute cohomology class of

$X_0 \cdot Y_{z_0}$



To compute this,

$\delta_0 = \bar{\partial}(\frac{1}{z})$

$\delta_0 \; X \otimes \delta_0 = \bar{\partial}(X$

$X \otimes \delta_0 = \bar{\partial}\left(X \otimes \frac{f}{z}\right) +$ stuff supp away from 0

$f = 1$ near 0

0 far away from 0

$(X \otimes \delta_0) \cdot (Y \otimes \delta_{z_0})$

$= d\left(\left(X \otimes \frac{f}{z}\right) \cdot (Y \otimes \delta_{z_0})\right)$

$= [X, Y] \frac{1}{z_0}$

~~$X \frac{\partial}{\partial z} Y \delta_{z_0}$~~

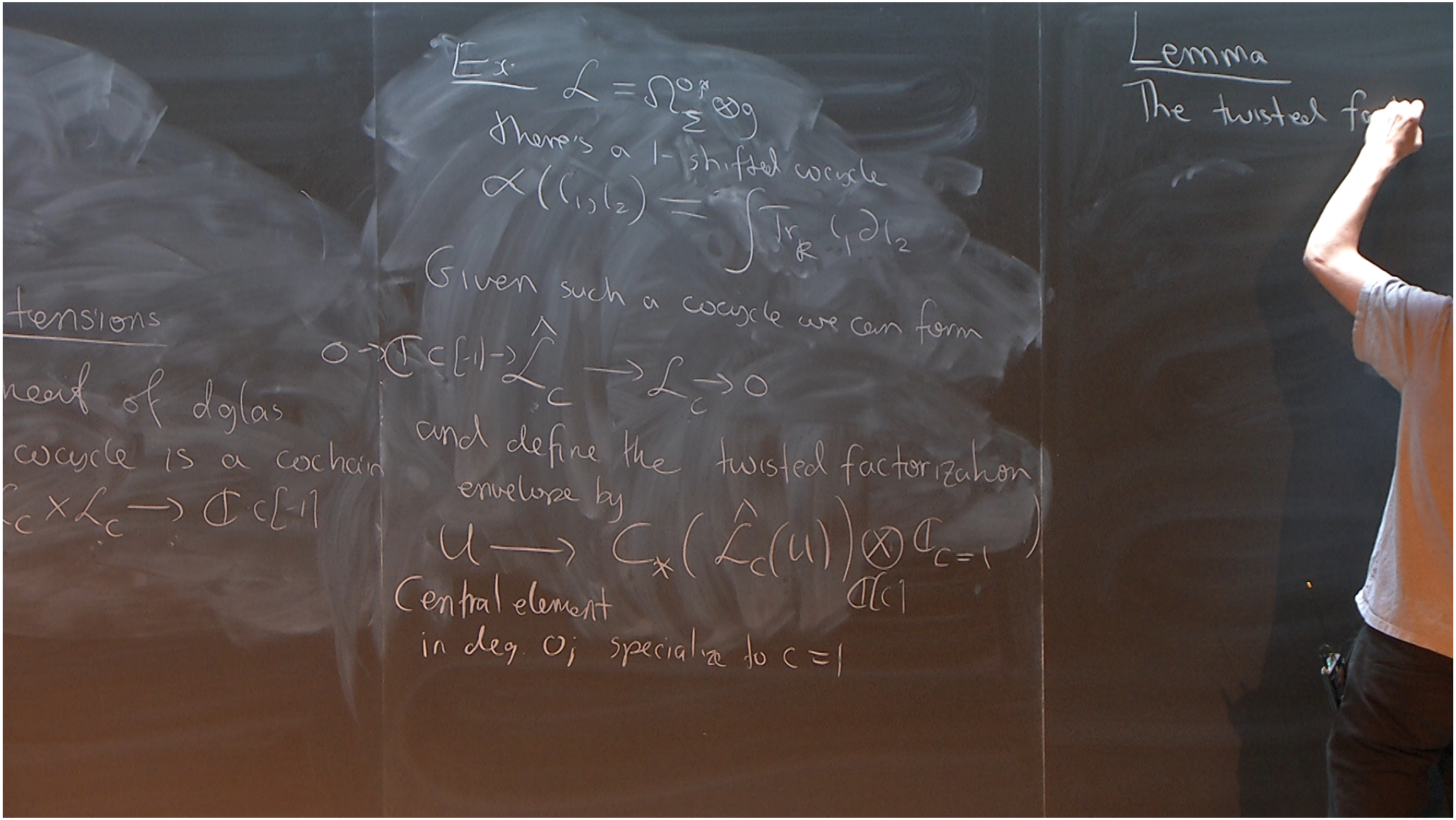
$(X \cdot \delta_0)(Y \cdot \delta_{z_0})$

$\sim \frac{1}{z} [X, Y]$

Central Extensions

$\mathcal{L}$ is a sheaf of dglas

A 1-shifted cocycle is a cochain map $\alpha : \mathcal{L}_c \times \mathcal{L}_c \to \mathbb{C}_c[-1]$


tensions
neard of dglas
cocycle is a cochain
$\mathcal{L}_C \times \mathcal{L}_C \longrightarrow \mathbb{C}[-1]$
Ex. $\mathcal{L} = \Omega^{0,*}_\Sigma \otimes \mathfrak{g}$
there's a 1-shifted cocycle
$\alpha(\ell_1, \ell_2) = \int \mathrm{Tr}_R\, \ell_1 \partial \ell_2$
Given such a cocycle we can form
$0 \to \mathbb{C}[-1] \to \hat{\mathcal{L}}_C \to \mathcal{L}_C \to 0$
and define the twisted factorization envelope by
$\mathcal{U} \longrightarrow C_*\left(\hat{\mathcal{L}}_C(\mathcal{U}) \otimes_{\mathbb{C}[\epsilon]} \mathbb{C}_{\epsilon=1}\right)$
Central element
in deg. 0; specialize to $\epsilon = 1$
Lemma
The twisted fa

Ex. $\mathcal{L} = \Omega^{0,*}_{\Sigma} \otimes \mathfrak{g}$

there's a 1-shifted cocycle

$$\alpha(\ell_1, \ell_2) = \int_{\Sigma} \mathrm{Tr}_{\mathfrak{g}}(\ell_1 \partial \ell_2)$$

Given such a cocycle we can form

$$0 \to \mathbb{C}c[-1] \to \hat{\mathcal{L}}_c \to \mathcal{L}_c \to 0$$

and define the twisted factorization envelope by

$$U \longrightarrow C_*\left(\hat{\mathcal{L}}_c(U) \otimes_{\mathbb{C}[c]} \mathbb{C}_{c=1}\right)$$

Central element in deg. 0; specialize to $c=1$

Lemma

The twisted factorization envelope of $\Omega^{0,*}_{\Sigma} \otimes \mathfrak{g}$ is the Kac-Moody at level given by central extension.

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s

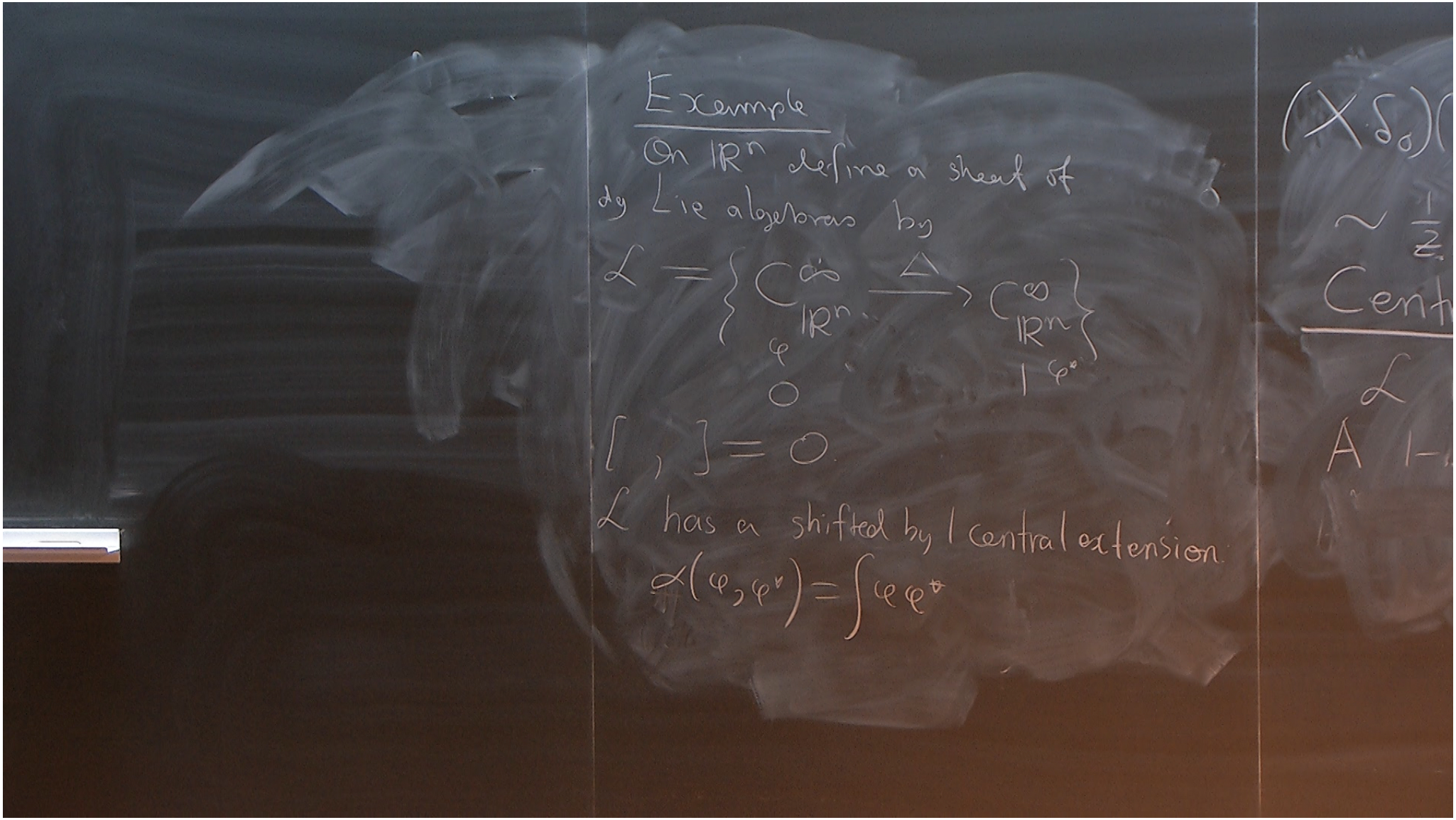
cochain

$[-1]$

Lemma

The twisted factorization envelope of $\Omega^{0,*}_{\Sigma} \otimes \mathfrak{g}$ is the Kac-Moody at level given by central extension

$$\Omega^{*}_{\Sigma}[1] \to \hat{\mathcal{L}} \to \mathcal{L}$$


Example
On $\mathbb{R}^n$ define a sheaf of dg Lie algebras by
$\mathcal{L} = \{ C^\infty_{\mathbb{R}^n} \xrightarrow{\Delta} C^\infty_{\mathbb{R}^n} \}$
φ
0
φ^*
$[\ ,\ ] = 0$
$\mathcal{L}$ has a shifted by 1 central extension.
$\alpha(\varphi, \varphi^*) = \int \varphi \varphi^*$
$(X \delta_0)($
$\sim \frac{1}{2}$
Cent
$\mathcal{L}$
A

Example

On $\mathbb{R}^n$ define a sheaf of dg Lie algebras by

$$\mathcal{L} = \left\{ \begin{array}{ccc} C^\infty_{\mathbb{R}^n} & \xrightarrow{\Delta} & C^\infty_{\mathbb{R}^n} \\ \varphi & & \varphi^* \\ 0 & & 1 \end{array} \right\}$$

$[\,,\,] = 0$

$\mathcal{L}$ has a shifted by 1 central extension.

$\alpha(\varphi, \varphi^*) = \int \varphi \varphi^*$

Lemma

The twisted factorization envelope of this = quantum observables of free scalar.

Proof

$$\mathcal{U}^\alpha(\mathcal{L}) = \mathrm{Sym}^*\left(\begin{array}{ccc} C^\infty_c(U) & \xrightarrow{\Delta} & C^\infty_c(U) \\ \mathcal{O}_g^* & & \mathcal{O}_f \end{array} \right)$$

Lemma

The twisted factorization envelope of this = quantum observables of free scalar

Proof

$$\mathcal{U}^{\star}(\mathcal{L}) = \mathrm{Sym}^{\star}\left(C^{\infty}_{\mathcal{E}}(U) \xrightarrow{\Delta} C^{\infty}_{c}(U) \right)$$

$$\mathcal{O}_g^{\star} \qquad \mathcal{O}_f$$

Central extension gives $d(\mathcal{O}_g, \mathcal{O}_f) = \int fg$

$\Omega^{\star}_{\Sigma}(\,$

Lemma

The twisted factorization envelope of this = quantum observables of free scalar

Proof

$$\mathcal{U}^{\hbar}(\mathcal{L}) = \mathrm{Sym}^{\hbar}\left(C^{\infty}_{c}(U) \xrightarrow{\Delta} C^{\infty}_{c}(U)\right)$$

$\mathcal{O}_g^{\hbar}$ $\quad$ $\mathcal{O}_f$

Central extension gives $d(\mathcal{O}_g, \mathcal{O}_f) = \int fg$

Noether's Theorem

If we have a sheaf $\mathcal{L}$ of dglas which acts on a QFT,

$\Omega^{*}_{\Sigma}(\mathcal{L})$

Lemma

The twisted factorization envelope of this = quantum observables of free scalar

Proof

$$U^{\alpha}(\mathcal{L}) = \mathrm{Sym}^{\bullet}\left(C^{\infty}_{c}(U) \xrightarrow{\Delta} C^{\infty}_{c}(U) \right)$$

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Noether's Theorem

If we have a sheaf $\mathcal{L}$ of dglas, which acts on a QFT, then $\exists$ a cocycle α, (L_∞, t dependent) and a map of f. algebras

$$U^{\alpha}(\mathcal{L}) \longrightarrow \mathrm{Obs}^{Q}$$

Central extension gives
$d(O_g, O_f) = \int fg$

Noether's Theorem

If we have a sheaf $\mathcal{L}$ of dglas, which acts on a QFT, then $\exists$ a cocycle α, ($\mathcal{L}_\alpha$, t dependent) and a map of f. algebras

$U^\alpha(\mathcal{L}) \longrightarrow Obs^Q$

Examples

$\mathcal{L} = \{C^\infty(\mathbb{R}^n) \xrightarrow[0]{\gamma} \xrightarrow[1]{\gamma'} C^\infty(\mathbb{R}^n)\}$

Then $\mathcal{L}$ acts on a free scalar field theory by translation

φ , $\varphi^\vee$ are fields of scalar field theory

$\varphi \longrightarrow \varphi + \gamma$

$\varphi^\vee \longrightarrow \varphi^\vee + \gamma^\vee$ is cpt action

As, e.g.

$H^0(K(n)) = \{\gamma, \Delta\gamma = 0\}$

If φ satisfies EOM, $\Delta\varphi = 0$

then so does $\varphi + \gamma$

Therefore,

$\exists$ a cocycle α s.t.

$\mathcal{U}^{\alpha}_{fact}(\mathcal{L}) \longrightarrow Obs^Q$

The cocycle is of course $\int \partial\gamma^*$

This explains why free field theories have such a simple factorization algebras, they have lots of symmetries

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Example

Consider top' quantum mechanics, fields p_i, q^i

Lagrangian $\sum \int p_i dq^i$

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Consider top' quantum mechanics, fields p_i, q^i

Lagrangian $\sum \int p_i dq^i$

gl_n acts.

Better

$\mathcal{L} = \Omega^*_{\mathbb{R}} \oplus gl_n$

acts.

To see this, use BV

$p_i \rightsquigarrow P_i \in \Omega^{\bullet}(\mathbb{R}) \otimes \mathbb{R}^{\vee}$

$q^i \rightsquigarrow Q^i \in \Omega^{\bullet}(\mathbb{R}) \otimes (\mathbb{R}^n)^{\vee}$

gl_n acts.

Better

$\mathcal{L} = \Omega^*_{\mathbb{R}} \otimes gl_n$

acts.

To see this, use BV

$p_i \rightsquigarrow P_i \in \Omega^+(\mathbb{R}) \otimes \mathbb{R}^\wedge$

$q^i \rightsquigarrow Q^i \in \Omega^\vee(\mathbb{R}) \otimes (\mathbb{R}^\wedge)^\vee$

$\mathcal{L}$ acts on

$\Omega^\vee(\mathbb{R}) \otimes \mathbb{R}^\wedge$

$\Omega(\mathbb{R}) \otimes (\mathbb{R}^\wedge)^\vee$

by fun/anti-fun. representations

Phil Action is

a functional that depends

on $\in \mathcal{L}[1]$ and P, Q.

The function is

$S(p_i, \varphi^i, \varphi^i)$

$= \int p_i d\varphi^i + \int p_i \dot{\varphi}^i$

This explains why free field theories have such a simple factorization algebras they have lots of symmetries

Example

Consider top' quantum mechanics, fields p_i, q^i

Lagrangian $\sum \int p_i dq^i$

$S(P_i, C^i, \varphi^i)$

$= \int P_i \, d\varphi^i + \int P_i C^i \varphi^i$

This is

$\omega(P, C \cdot \varphi)$

ω: odd pairing

$\cdot$: action

Noether's Theorem

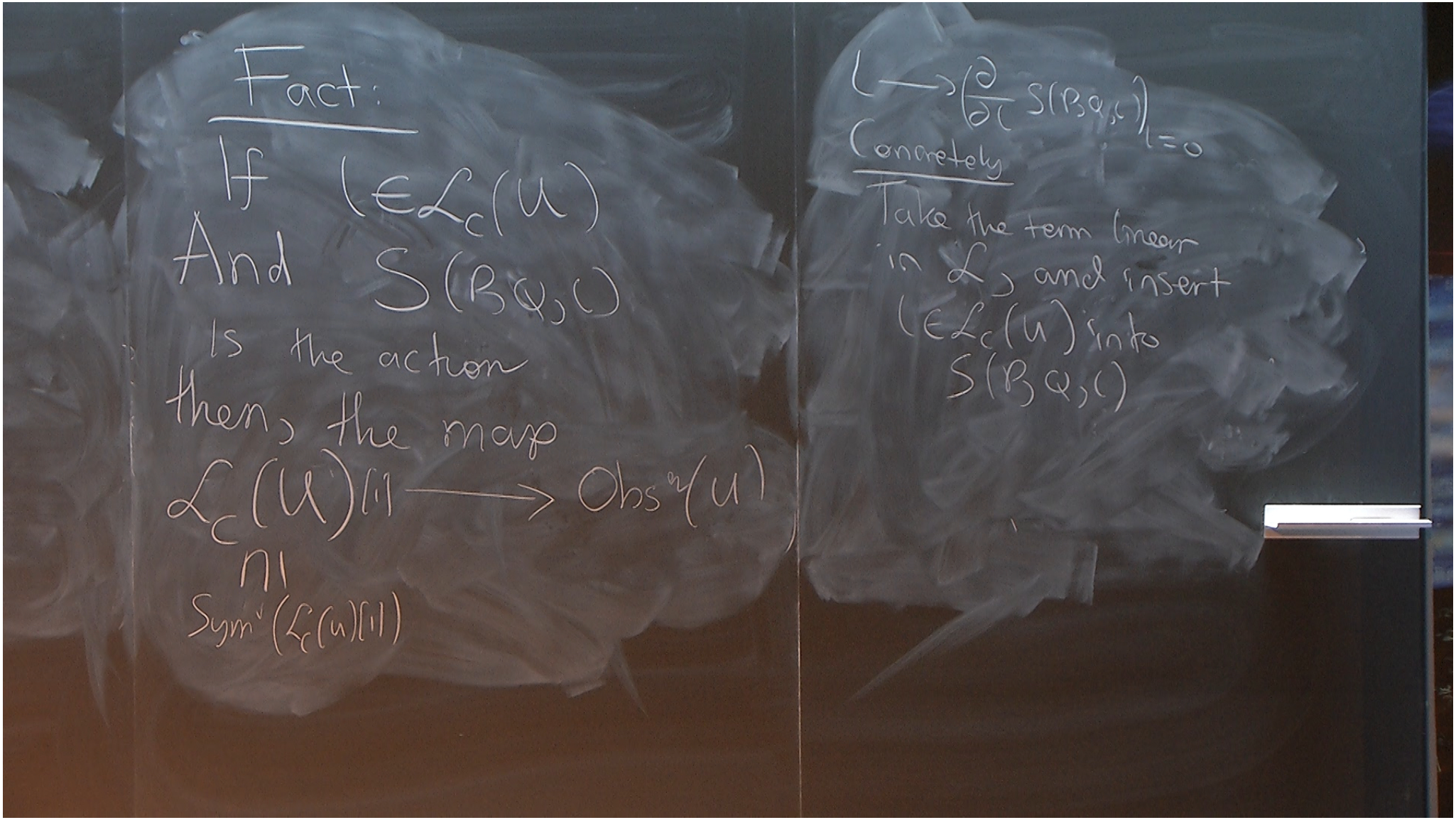
We will have a map

$\mathcal{U}^{fact}(\mathcal{L}) \longrightarrow Obs^2$

At level of cohomology,

$\mathcal{U}^{fact}(\mathcal{L}) = \mathcal{U}\mathfrak{gl}_n$

$Obs^2 = Diff(\mathbb{R}^n)$


Fact:
If $\ell \in \mathcal{L}_c(U)$
And $S(\beta, Q, \ell)$
is the action
then, the map
$\mathcal{L}_c(U)[1] \longrightarrow Obs^{\geq 1}(U)$
$\cap 1$
$Sym^1(\mathcal{L}_c(U)[1])$
$\ell \longrightarrow \left(\frac{\partial}{\partial \ell} S(\beta, Q, \ell)\right)_{\ell=0}$
Concretely
Take the term linear in ℓ, and insert $\ell \in \mathcal{L}_c(U)$ into $S(\beta, Q, \ell)$

If $\ell \in E^i_j \otimes f(t)dt$

then, the element of $f \in C^\infty_c((a,b))$

$Obs_2((a,b))$ is

$$\int P_i(t) \varphi^j(t) f(t) dt$$

At level of cohomology, in $Diff(\mathbb{R}^n)$ this $p_i q^j$ up to some annoying stuff

these give a homomorphism

$$U(gl_n) \longrightarrow Diff(\mathbb{R}^n)$$

Ex

β-γ system

$B \in \Omega^{1,*}(\mathbb{C}, R^*)$

$\Gamma \in \Omega^{0,*}(\mathbb{C}, R)$

R rep. of g

$\mathcal{L} \in \Omega^{0,*}(\mathbb{C}) \otimes g$

Evidently, $\mathcal{L}$ acts

S coupling B, Γ, ζ

$$S(B,\Gamma,\zeta) = \int B \bar{\partial}_\zeta \Gamma$$

$$= \int B \bar{\partial} \Gamma + \int B \zeta \Gamma$$

The map

$$U^{\alpha}_{fant}(\zeta) \longrightarrow Obs^q ?$$

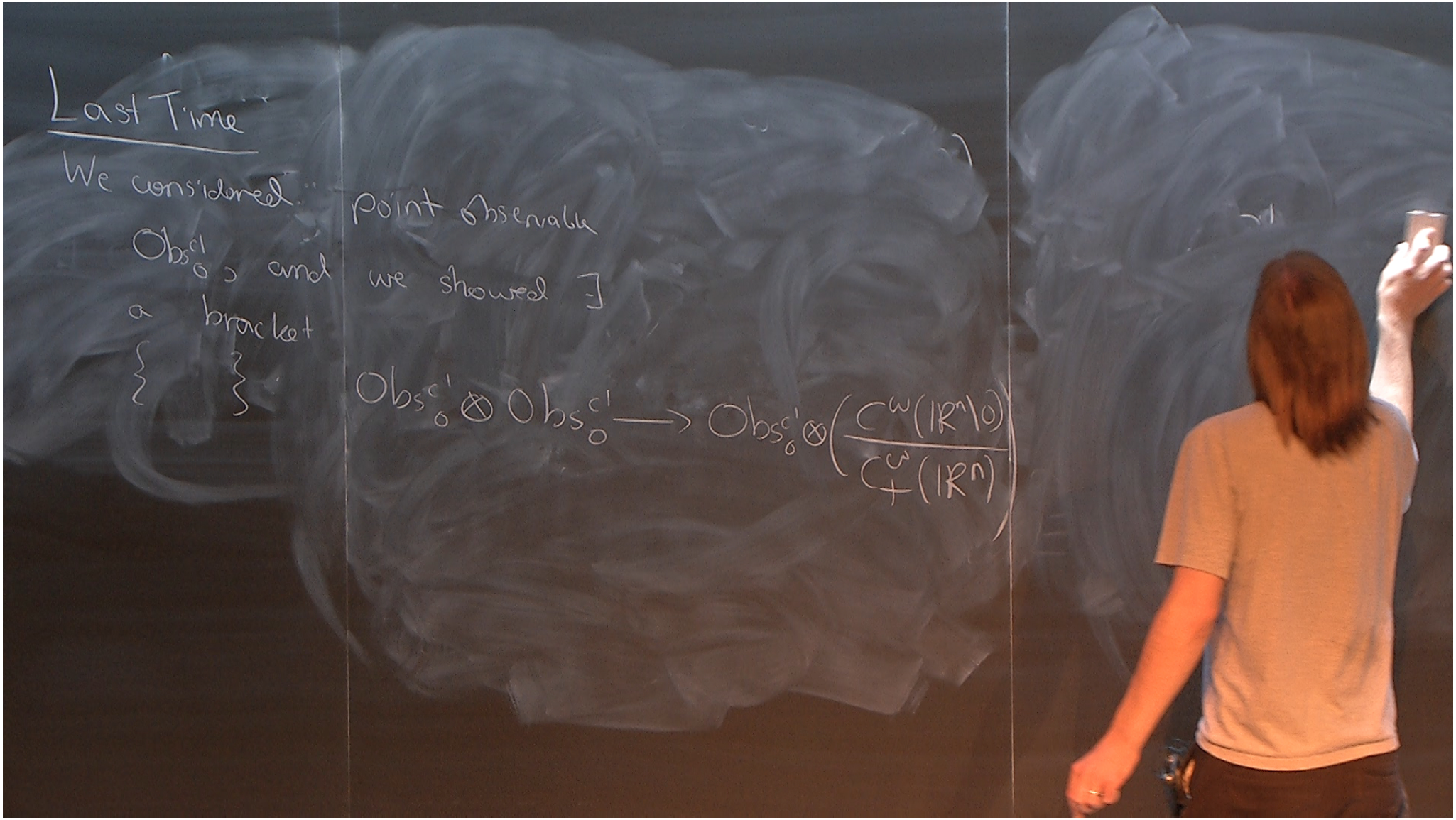
$$X \in g, \quad X \otimes \delta_{z=0} \longrightarrow \langle B, X \cdot \Gamma \rangle (0)$$

This is usual Noether charge,

J^a , a an index for a basis of g.

There's a central extension here

$$\Longrightarrow \widehat{K}M_g \longrightarrow Obs^q$$


Last Time
We considered
Point Observable
$Obs^{cl}_{0,0}$ and
we showed
a bracket
$Obs^{cl}_0 \otimes Obs^{cl}_0 \longrightarrow Obs^{cl}_0 \otimes \left(\frac{C^\omega(\mathbb{R}^n, 0)}{C^\omega(\mathbb{R}^n)} \right)$

Last Time

We considered Obs^{cl}_0, and showed $\exists$ a bracket $\{\ ,\ \}$.

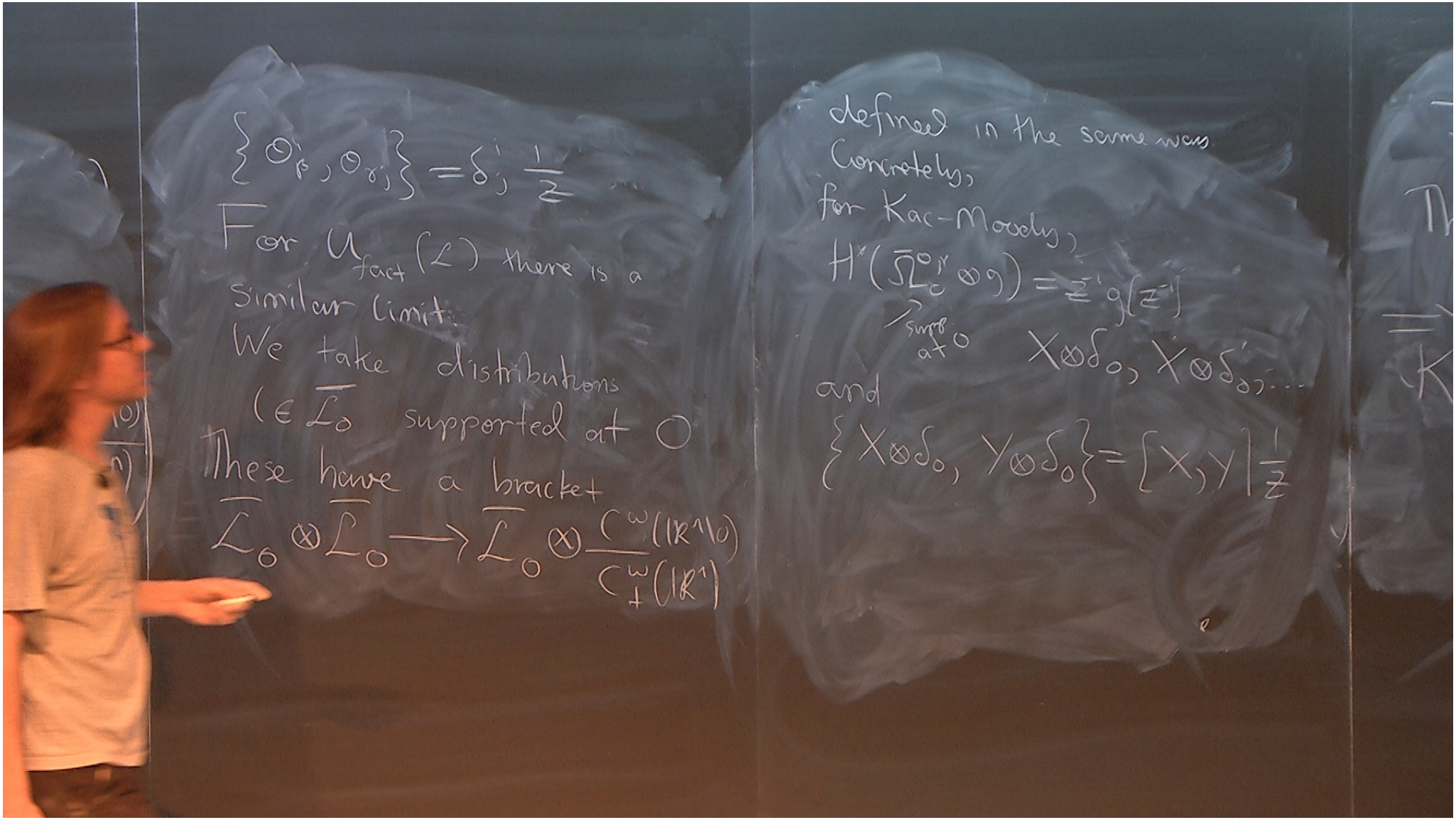
In β-γ system, $\mathcal{O}^i_\beta : \beta \mapsto \beta(0)$

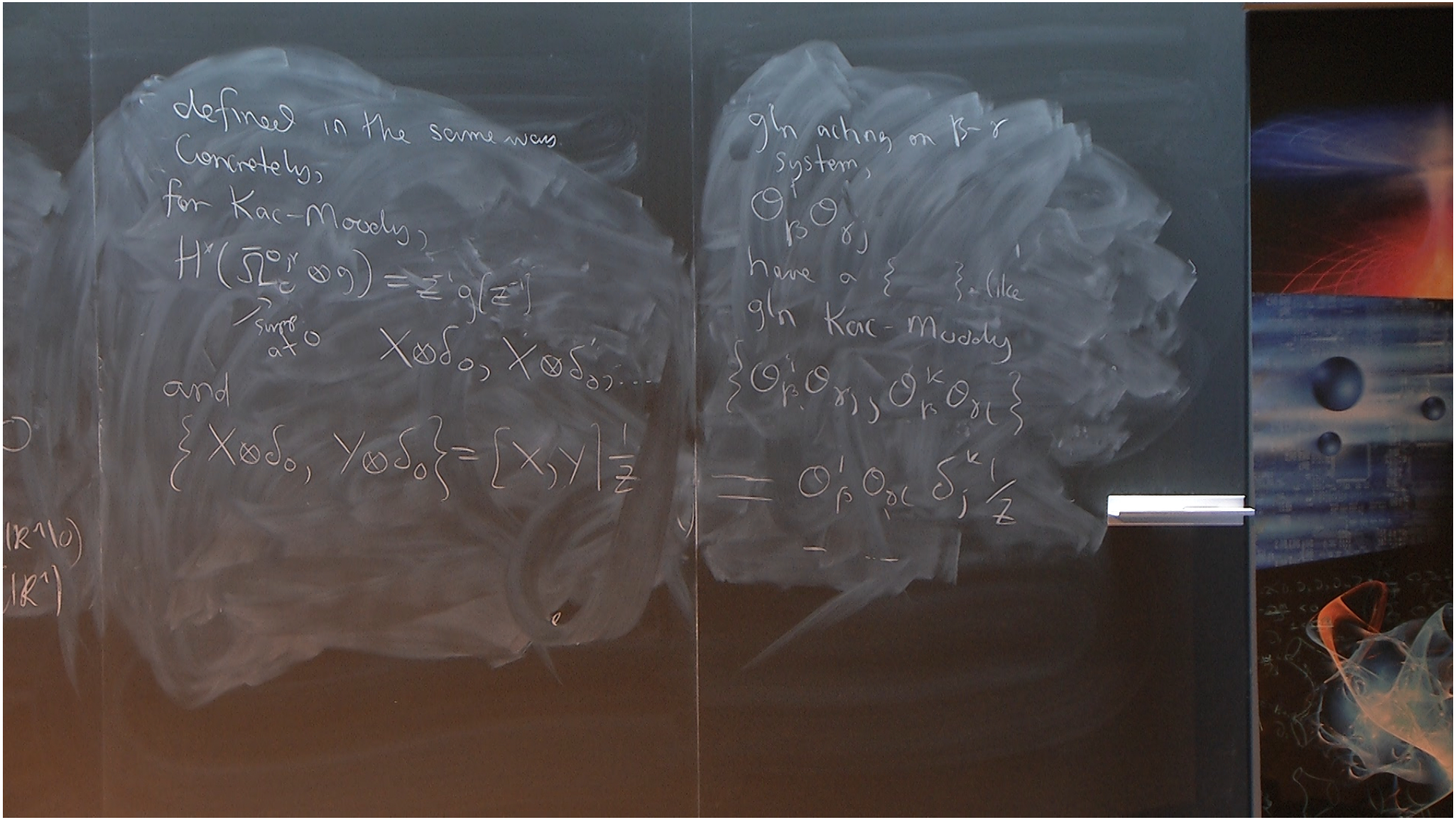
$\mathcal{O}_{\gamma_i} : \gamma \mapsto \gamma(0)$

Point Observable

$$Obs^{cl}_0 \otimes Obs^{cl}_0 \longrightarrow Obs^{cl}_0 \otimes \left(\frac{C^\omega(\mathbb{R}^n 0)}{C^\omega_+(\mathbb{R}^n)} \right)$$

$$\{ \mathcal{O}^i_\beta, \mathcal{O}_{\gamma_j} \} = \delta^i_j$$


$\{\mathcal{O}_{\beta,i}, \mathcal{O}_{\gamma,j}\} = \delta_{ij}\frac{1}{2}$
For $\mathcal{U}_{fact}(\mathcal{L})$ there is a similar limit.
We take distributions $\in \bar{\mathcal{L}}_0$ supported at 0
These have a bracket
$\bar{\mathcal{L}}_0 \otimes \bar{\mathcal{L}}_0 \longrightarrow \bar{\mathcal{L}}_0 \otimes \frac{C^\omega(\mathbb{R}^1\setminus 0)}{C^\omega_+(\mathbb{R}^1)}$
defined in the same way.
Concretely,
for Kac-Moody,
$H^*(\bar{\Omega}^{0,*}_C \otimes \mathfrak{g}) = z^{-1}\mathfrak{g}[z^{-1}]$
supp at 0
$X\otimes\delta_0, X\otimes\delta_0', \ldots$
and
$\{X\otimes\delta_0, Y\otimes\delta_0\} = [X,Y]\frac{1}{2}$


defined in the same ways
Concretely,
for Kac-Moody,
$H^*(\bar{\Omega}^{0,r}_c \otimes \mathfrak{g}) = \mathfrak{g}[z^{-1}]$
supp at 0
$X \otimes \delta_0, X \otimes \delta_0', \ldots$
and
$\{X \otimes \delta_0, Y \otimes \delta_0\} = [X,Y] \frac{1}{z}$
$\mathfrak{gl}_n$ acting on β-γ system,
$\theta^i_\beta \theta_{\gamma j}$
have a { } 1 like
$\mathfrak{gl}_n$ Kac-Moody
$\{\theta^i_\beta \theta_{\gamma j}, \theta^k_\beta \theta_{\gamma l}\}$
$= \theta^i_\beta \theta_{\gamma l} \delta^k_j \frac{1}{z}$
$- \ldots$

$= \delta_j^i \frac{1}{z}$

) there is a

distributions
ported at 0

bracket

$\overline{\mathcal{L}}_0 \otimes \frac{C^\omega(\mathbb{R}^1 \setminus 0)}{C^\omega_+(\mathbb{R}^1)}$

defined in the same ways
Concretely,
for Kac-Moody,

$H^*(\bar{\Omega}^{0,r}_C \otimes g) = z^i g[z^{-1}]$

supp at 0

$X \otimes \delta_0, X \otimes \delta_0', \ldots$

and

$\{X \otimes \delta_0, Y \otimes \delta_0\} = [X, Y] \frac{1}{z}$

gl_n acting on β-γ system,

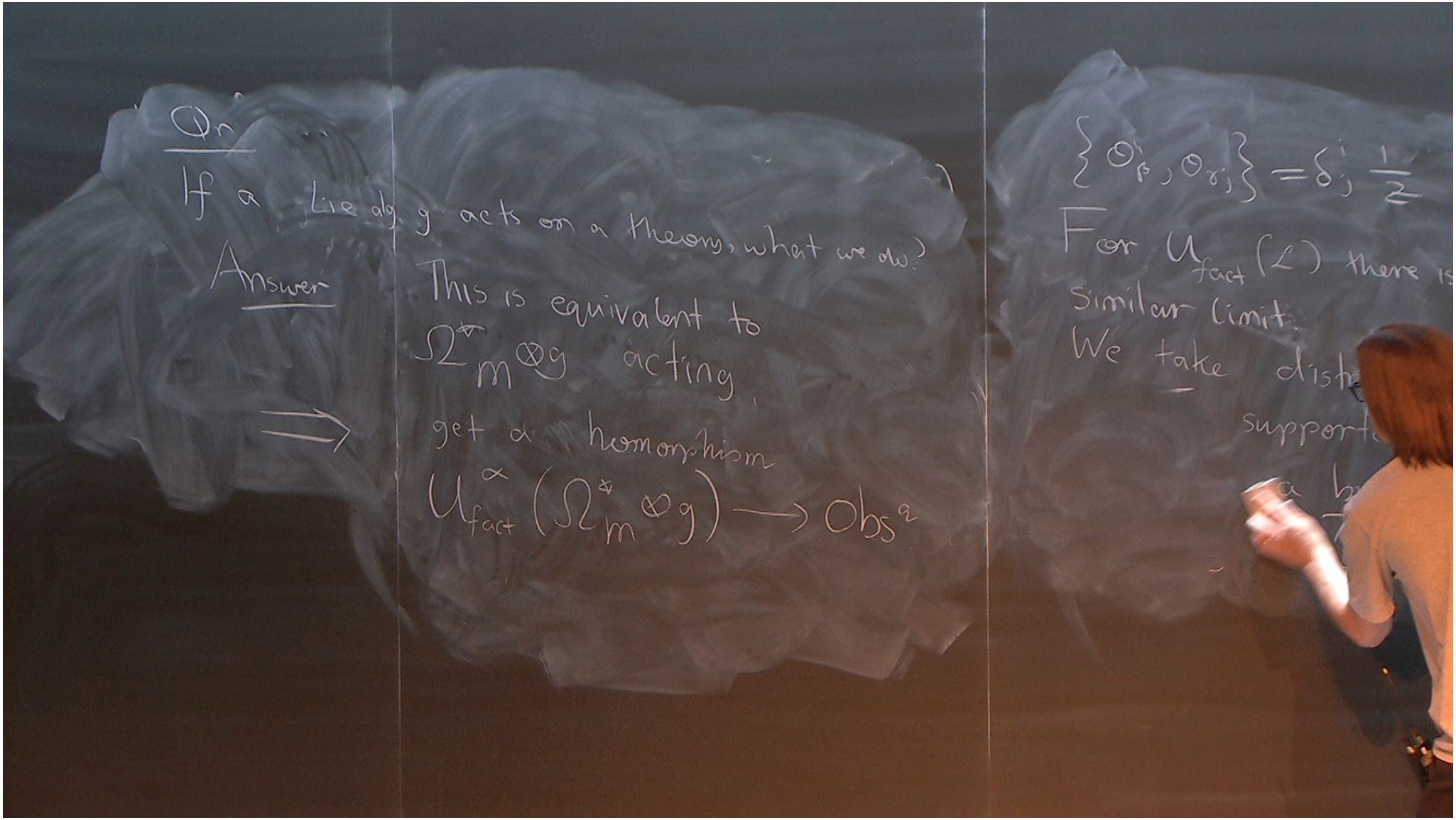
$\theta^i_\beta \theta^j_\gamma$

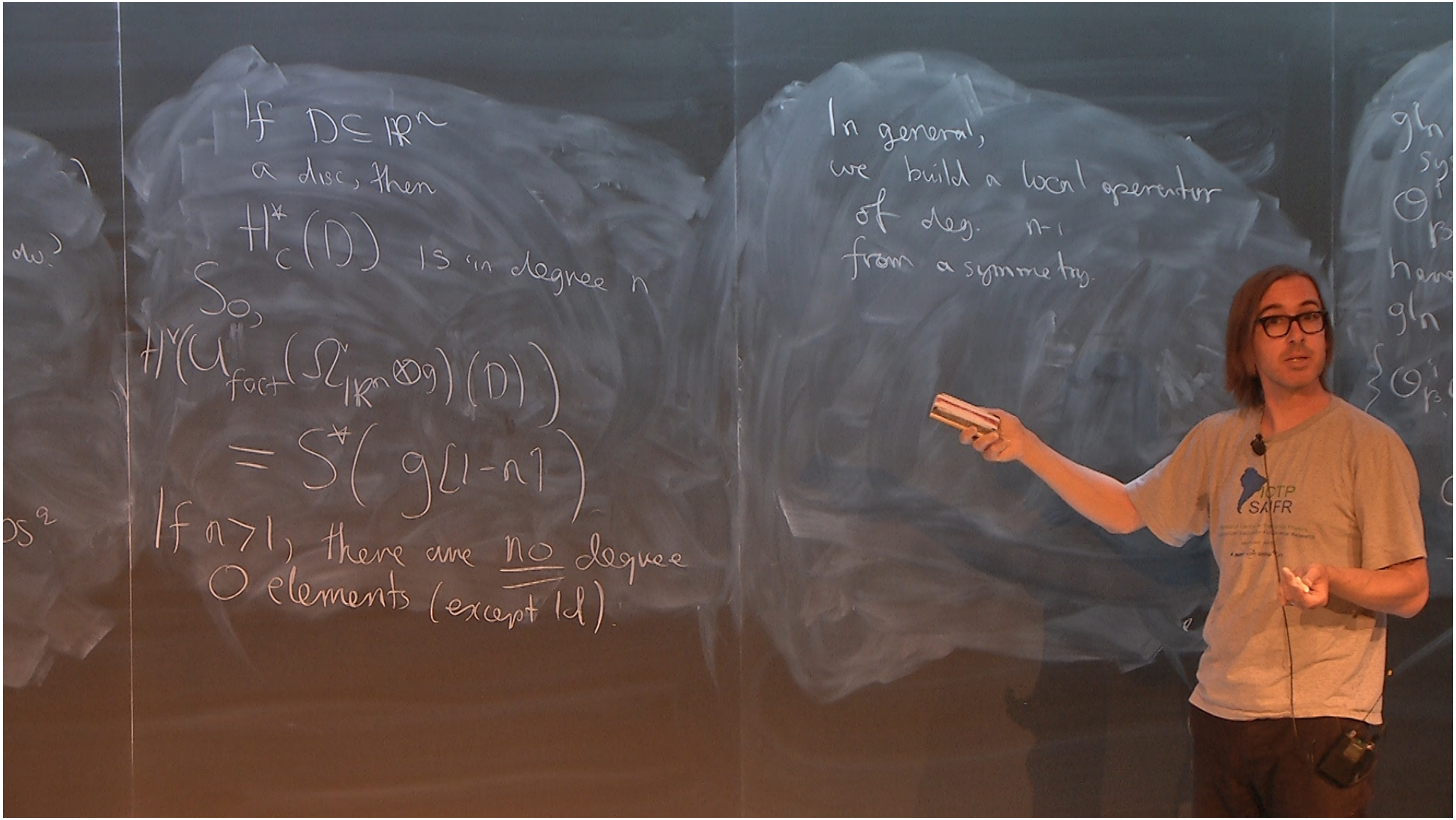
have a $\{\ \}_1$-like

gl_n Kac-Moody

$\{\theta^i_\beta \theta_{\gamma j}, \theta^k_\beta \theta_{\gamma l}\}$

$= \theta^i_\beta \theta_{\gamma l} \delta^k_j \frac{1}{z}$


Q:
If a Lie alg. g acts on a theory, what we do?
Answer
This is equivalent to
$\Omega^*_m \otimes g$ acting.
get a homomorphism
$U^{\propto}_{fact}(\Omega^*_m \otimes g) \longrightarrow Obs^a$
$\{O^i_\beta, O_{\gamma j}\} = \delta^i_j \frac{1}{2}$
For $U_{fact}(\mathcal{L})$ there is
similar limit.
We take


If $D \subseteq \mathbb{R}^n$ a disc, then $H^*_c(D)$ is in degree n
So,
$H^\gamma(\mathcal{U}_{fact}(\Omega^*_{\mathbb{R}^n} \otimes g)(D)) = S^*(g[1-n])$
If $n>1$, there are no degree 0 elements (except Id).
In general, we build a local operator of deg. $n-1$ from a symmetry.
ICTP SAIFR

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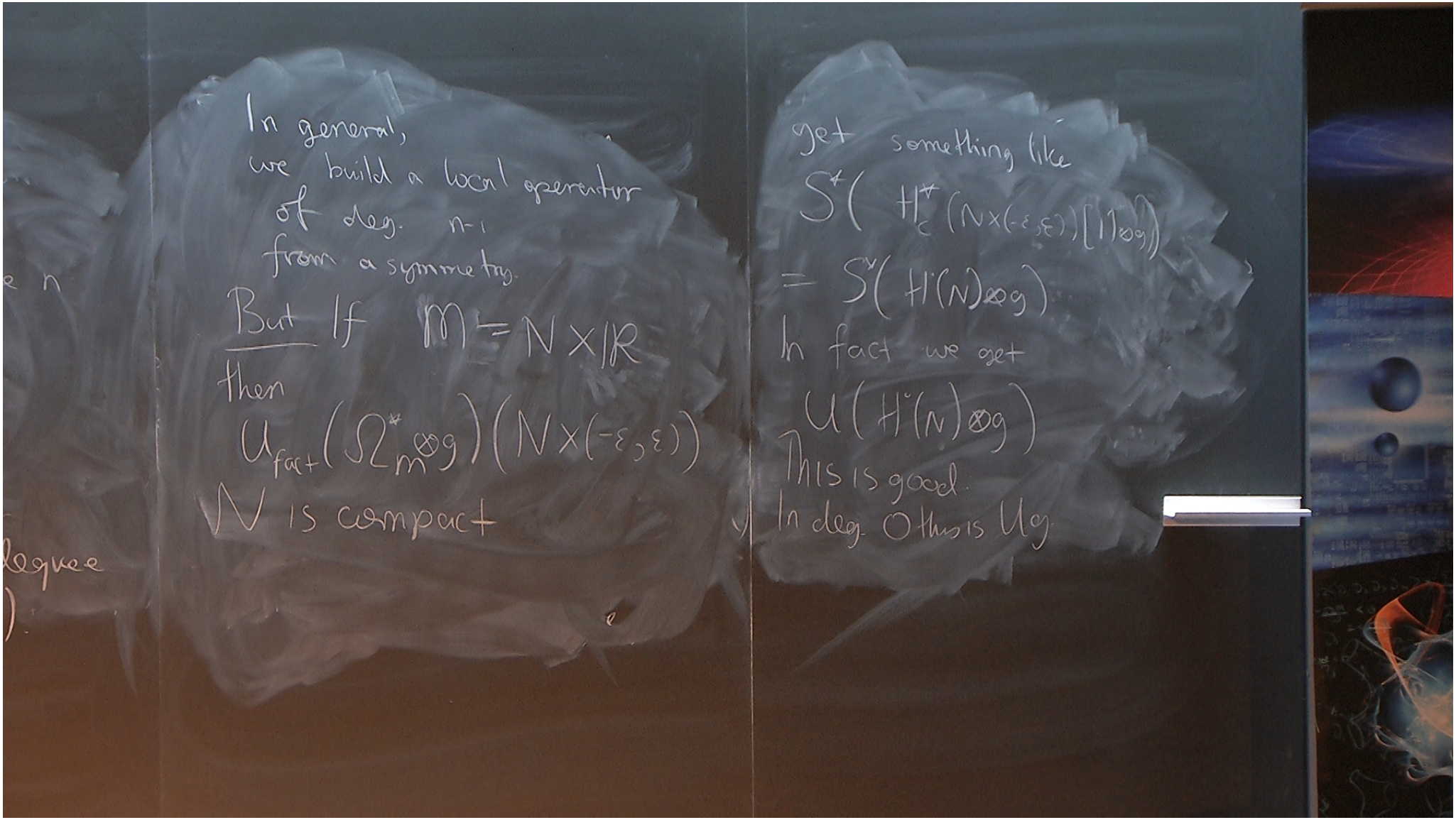
If $n > 1$, there are no degree 0 elements (except Id).

In general, we build a local operator of deg. $n-1$ from a symmetry.

But if $M = N \times \mathbb{R}$ then

$$\mathcal{U}_{fact}(\Omega^*_M \otimes g)(N \times (-\varepsilon, \varepsilon))$$

N is compact


In general,
we build a local operator
of deg. n-1
from a symmetry.
But if $M = N \times \mathbb{R}$
then
$U_{fact}(\Omega^*_m \otimes g)(N \times (-\varepsilon, \varepsilon))$
N is compact
get something like
$S^*(H^*_c(N \times (-\varepsilon, \varepsilon))[1] \otimes g))$
$= S^*(H^*(N) \otimes g)$
In fact we get
$U(H^*(N) \otimes g)$
This is good.
In deg. 0 this is Ug.

Fact

If we have a theory which is not a gauge theory, Observables are in deg. ≤ 0 ...

Then ... an action of $\Omega^{\bullet}_{\mathbb{R}^n \otimes g}$

Factors through $\Omega^{\leq 1}_{\mathbb{R}^n \otimes g}$

$\Rightarrow$ we can get a local Noether charge J.

If $D \subseteq \mathbb{R}^n$ a disc, then

$H^*_c(D)$ is in degree n

So,

$$H^*\left(\mathcal{U}_{fact}\left(\Omega^{\bullet}_{\mathbb{R}^n \otimes g}\right)(D)\right) = S^*\left(g[1-n]\right)$$

If $n > 1$, there are no degree 0 elements (except Id).