Title: Lecture 7: Boundary Conditions and Extended Defects

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Collection: QFT for Mathematicians

Date: June 28, 2019 - 9:00 AM

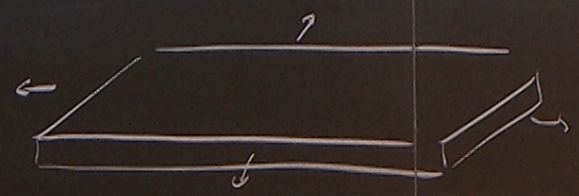
URL: http://pirsa.org/19060023

"Modul. space of vacua"

Classical FT on $\mathbb{R}^{d-1,1}$

a vacuum v

is $v \in \text{Fields}(\mathbb{R}^{d-1} \times [0, \varepsilon))$



s.t. $dS(v) = 0$

AND the energy is minimized on v

$$E = \int_{\mathbb{R}^{d-1}} h$$

h = Hamiltonian density

= Noether current for time translation

E.g. φ^4 th-y, $d = 4$

$$S = \int d^d x \left[\partial_t \varphi^2 - |\partial_x \varphi|^2 - V(\varphi) \right]$$

$$V(\varphi) = m\varphi^2 + \lambda \varphi^4$$

$$h = (\partial_t \varphi)^2 + |\partial_x \varphi|^2 + V(\varphi)$$

$$\text{Vacua} = \{\text{constant } \varphi,\ V(\varphi) = 0\}$$

$$= \text{pt.} \quad \{\varphi \equiv 0\}$$

In Lorentzian QFT

vacua v are states

$$v \in \mathrm{Hilb}(\mathbb{R}^{d-1})$$

s.t. $E = H$ is minimized.

Hamiltonian

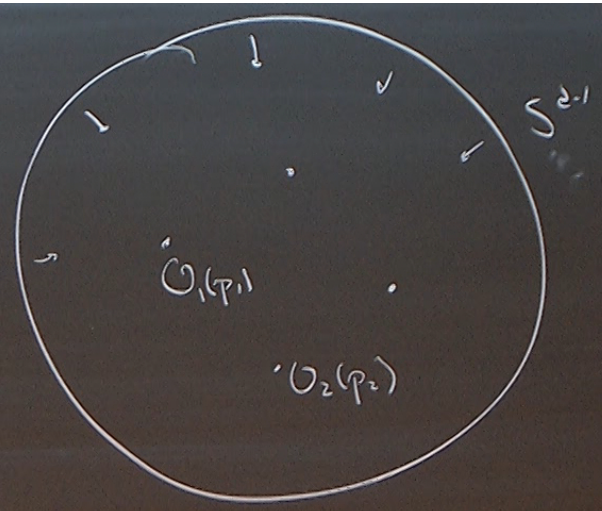
In Euclidean QFT on $\mathbb{R}^d$

look instead at asymptotic bdy S^{d-1}_∞

A (generalized) vacuum is a state

$$v \in \mathrm{Hilb}(S^{d-1}_\infty)$$

s.t. correlation functions of local ops in this state are finite.



In terms of path integral,

v is an asymptotic b.c. on the fields

s.t. $\langle \mathcal{O}_1(p_1) \cdots \mathcal{O}_n(p_n) \rangle_v = \int_{\substack{\text{fields} \\ \text{approaching } v}} e^{-S} (\cdots)$

is finite.

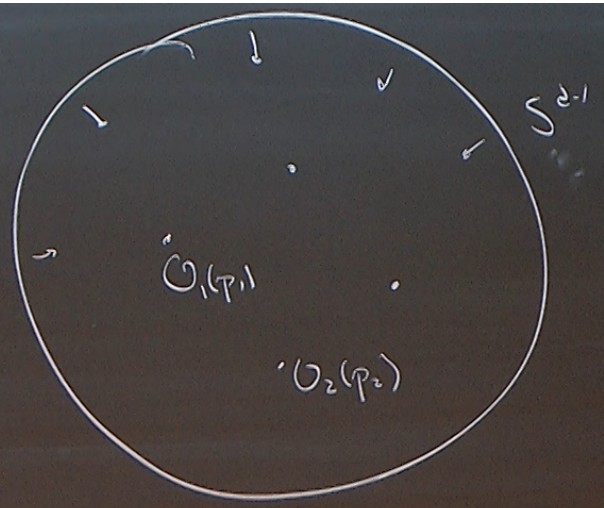
ized.

on $\mathbb{R}^d$

y S^{d-1}_∞

e

local ops



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is finite.

Get a pairing

$$\mathrm{Ops}_L \times \mathcal{M}_{vac} \to \mathbb{C}$$

Ops_L = "local" ops in B^d_L (ball of radius L)

$= \mathrm{Fact}(B^d_L)$

$\mathcal{O} \cdots \mathcal{O}$ $\quad v, \quad \langle \mathcal{O} \cdots \mathcal{O} \rangle_v$

Expect (or define) pairing to be non-degen.

Then $\mathcal{M}_{vac} = \lim_{L \to \infty} \mathrm{Spec}\, \mathrm{Fact}(B^d_L)$

?

Spose thy is a cohomological TQFT

e.g. a twist of SUSY thy

$$\mathrm{Ops} = \mathrm{Fact}(\mathbb{R}^d)$$

$$A = H^\bullet_Q(\mathrm{Ops})$$

commutative $d \geq 2$

"topological vacua"

$$\mathrm{Aff}(\mathcal{M}_{vac}) = \mathrm{Spec}(A)$$

line ops form a 1-category

Lines $\ni \mathcal{L}$

Canonical object $\mathbb{1}$

trivial line

$\mathrm{Hom}(\mathbb{1}, \mathcal{L})$

is a module for local ops

$$A = \mathrm{End}(\mathbb{1})$$

By sending any $\mathcal{L}$ to vec space of local ops at its endpoint,

$$\mathrm{Lines} \longrightarrow A\text{-mod} = \mathrm{End}(\mathbb{1})\text{-mod}$$

(some lines may not end!)

$\ni \mathcal{L}$ Canonical object $\mathbb{1}$

trivial line

$\mathrm{Hom}(\mathbb{1}, \mathcal{L})$

is a module for local ops

$$A = \mathrm{End}(\mathbb{1})$$

ing any $\mathcal{L}$ to vec space

local ops at its endpoint,

$\longrightarrow$ A-mod $= \mathrm{End}(\mathbb{1})$-mod

(lines may not end!)

<u>Expect</u> Lines $= \mathrm{Sh}(\mathcal{M}_{vac})$

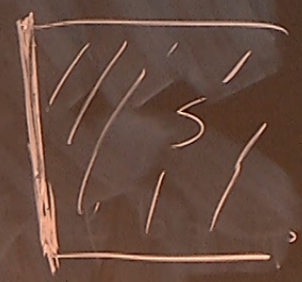
Keep going: Surf 2-cat of surf ops

contains a canonical $S_{\mathbb{1}}$ trivial surface

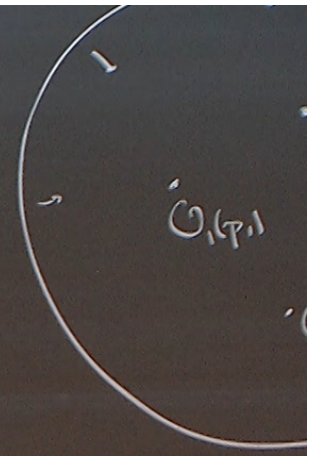
$$\mathrm{End}(S_{\mathbb{1}}) = \text{Lines}, \otimes$$
$$= \mathrm{Sh}(\mathcal{M}_{vac}), \otimes$$

Given any other S

$S_{\mathbb{1}}$



Lines $\circlearrowleft$ $\mathrm{Hom}(S_{\mathbb{1}}, S)$



In terms of p

v is an asy

s.t. $\langle \mathcal{O}_1(p,$

$\text{nes} = Sh(\mathcal{M}_{vac})$

Surf 2-cat of surf ops

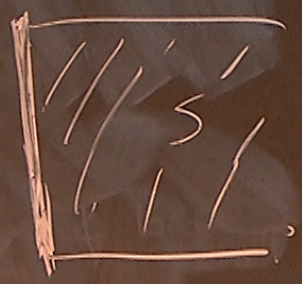
nonical $S_{\mathbb{1}}$ trivial surface

$End(S_{\mathbb{1}}) = \text{Lines}, \otimes$

$= Sh(\mathcal{M}_{vac}), \otimes$

S

$S_{\mathbb{1}}$



$\circlearrowleft \; Hom(S_{\mathbb{1}}, S)$

B-model, target $\mathcal{X}$

$\text{Lines} = Coh(\mathcal{X} \times \mathcal{X})$

$\mathcal{M}_{vac} = T[1]\mathcal{X}$

$\mathcal{A}$ = polyvec fields

nbhd of Δ in $\mathcal{X} \times \mathcal{X}$

$\downarrow$

$Coh(T[1]\mathcal{X}) \overset{HD}{\simeq} Coh(T\mathcal{X})$

$\uparrow$

shift

Get a pairing

$Ops_L \times \mathcal{M}$

"local" ops in B_L^d

(ball of radius L)

$\|$

$Fact(B_L^d)$

$\mathcal{O} \cdots \mathcal{O}$ V,

Expect (or define)

pairing to be non-degen

Then $\mathcal{M}_{vac} = \lim_{L \to \infty} \text{Spec Fa}$

?

3d $N=4$ σ-model to $T^*[2]\mathcal{X}$

$\mathcal{X}$ algebraic

B-twist (RW twist)

(cf. Kapustin-Rozansky-Saulina)

$$A = H^\bullet_Q(\mathcal{O}_{PS})$$
$$= H^{0,\bullet}_{\bar\partial}(T^*\mathcal{X})$$
$$= H^\bullet(T^*\mathcal{X}, \mathcal{O}_{T^*\mathcal{X}})$$

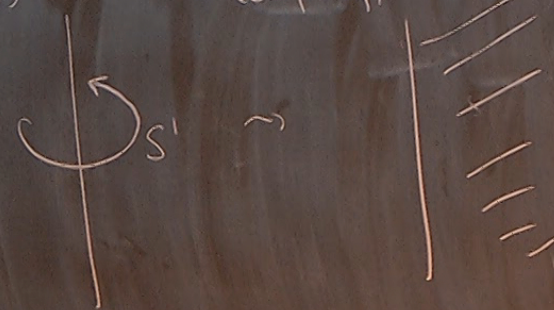
$$\operatorname{Spec} A = \operatorname{Aff}(T^*\mathcal{X})$$

$$\text{Lines} = QCoh(T^*[2]\mathcal{X}), \tilde{\otimes}$$

$$\text{Surf} = Sh\,Cat(T^*[2]\mathcal{X})$$

$\longrightarrow$ Lines-mod.

To find Lines compactify on S^1

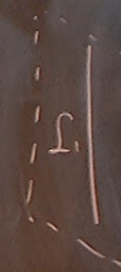


$$\Rightarrow \text{Lines} = QCoh(T^*[2]\mathcal{X})$$

Expect L

Keep going.

contains a



Given any oth

L

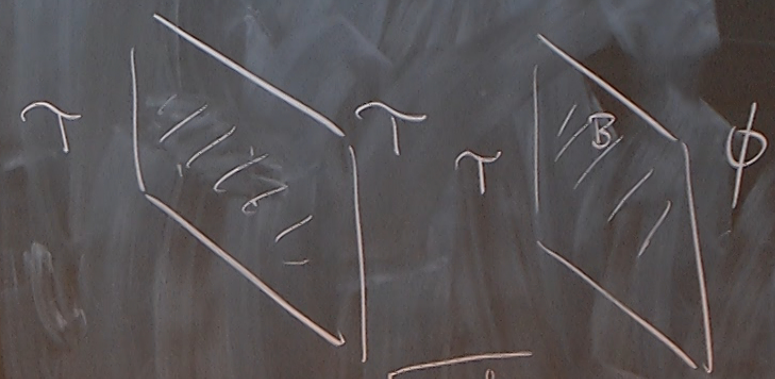
Lines in 3d is an E_2-monoidal 1-category

$\tilde{\otimes}$ is induced from ↗
(braided structure)

actually computed w/ Atiyah bracket

ed
$L(T[2]\mathcal{X})$
th W
$dW=0\}$
"
stant loops
$T^*[2]\mathcal{X})$

In 3d (e.g.) bdy conditions are not in Surf



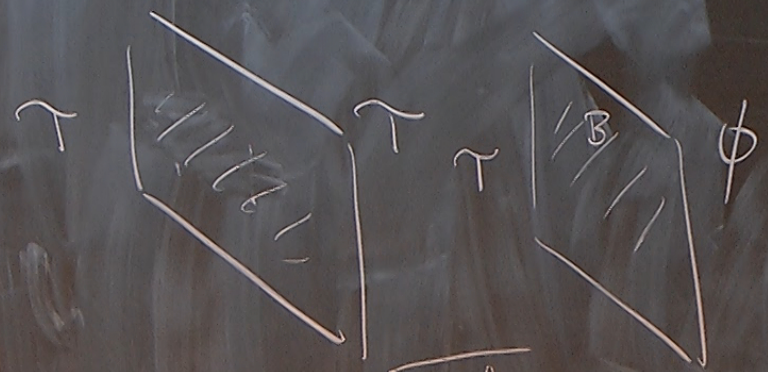
Roughly Bdy $= \sqrt{\text{Surf}}$

B

roidal

bracket

In 3d (e.g.) bdy conditions are not in Surf



Roughly Bdy $= \sqrt{\text{Surf}}$

Basic

Bdy conds are labelled by Lagrangians in $\mathcal{M}_{vac}$.

E.g. 2d B-model $\mathcal{M}_{vac} \simeq T^*[1]\mathcal{X}$

e.g. $N^*[1]Y$, $Y \subseteq \mathcal{X}$

$\updownarrow$

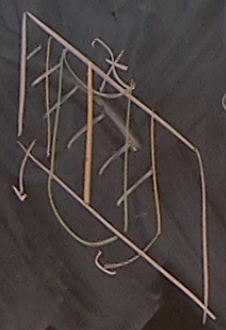
$\mathcal{O}_Y$ in $\mathrm{Coh}(\mathcal{X})$

In 3d B-model to $T^*[2]\mathcal{X}$

<u>One</u> b.c. is $B_{\mathcal{X}}$ — supported on $\mathcal{X}$ (Lag in $T^*[2]\mathcal{X}$)

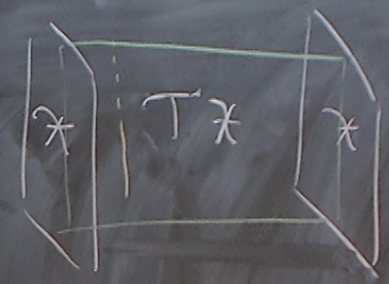
$\mathrm{End}(B_{\mathcal{X}}) = \mathrm{Coh}(\mathcal{X})$

$T\mathcal{X}$



$\mathbb{C}$

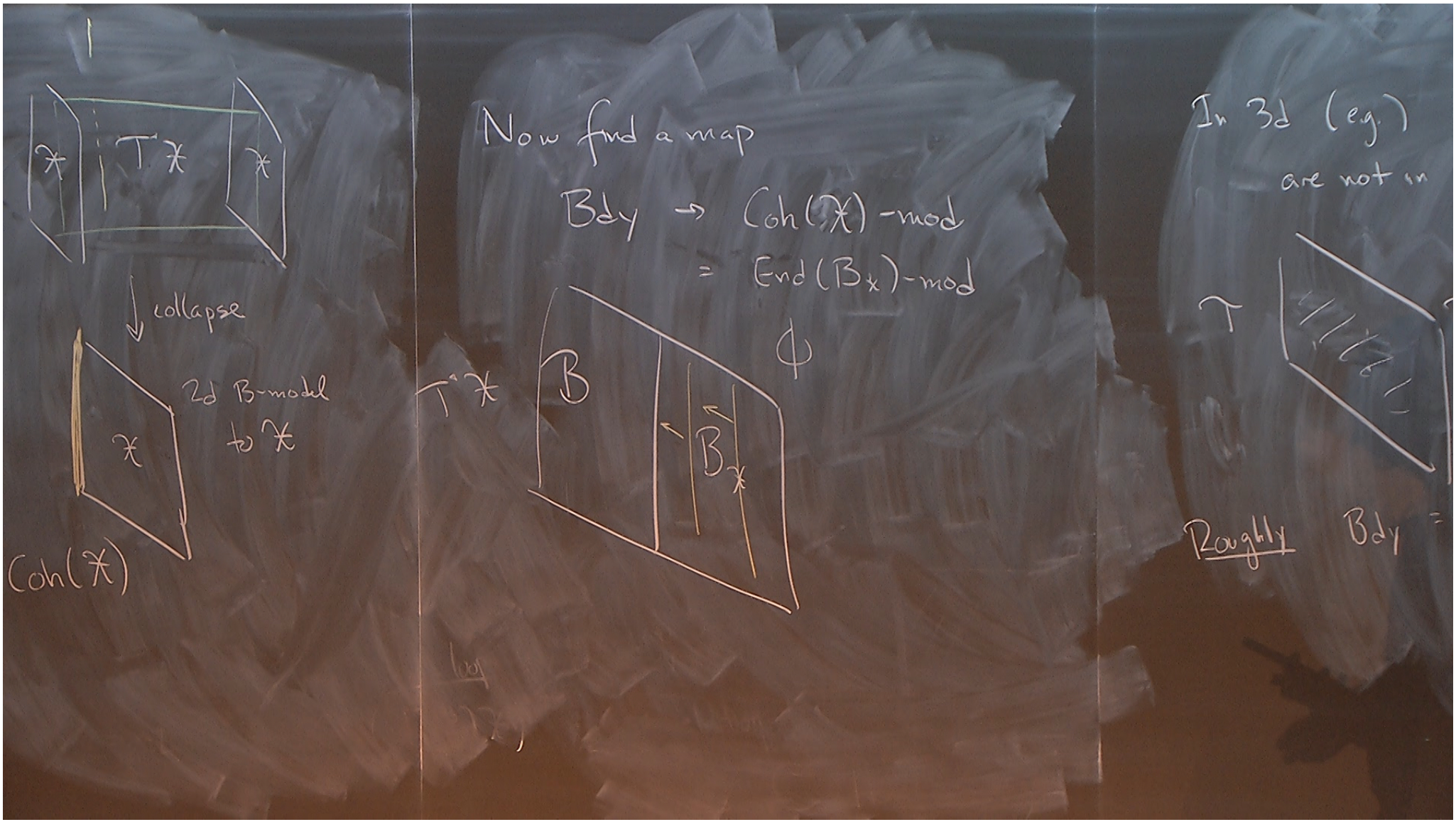
Fold $\rightsquigarrow$ $(\mathbb{R}_t \times \mathbb{R}) \times I$



collapse

2d B-model to $\mathcal{X}$

$\mathrm{Coh}(\mathcal{X})$

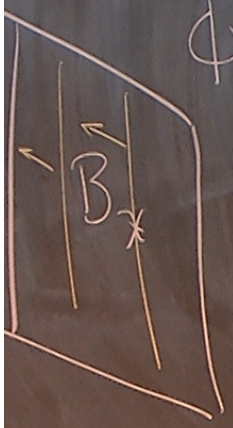

$\mathcal{X}$
$T\mathcal{X}$
$\mathcal{X}$
collapse
2d B-model to $\mathcal{X}$
$\mathcal{X}$
$\mathrm{Coh}(\mathcal{X})$
Now find a map
Bdy $\to$ $\mathrm{Coh}(\mathcal{X})$-mod
$=$ $\mathrm{End}(B_X)$-mod
ϕ
$T^*\mathcal{X}$
B
$B_{\mathcal{X}}$
In 3d (e.g.)
are not in
T
Roughly
Bdy

a map

$\to$ Coh($\mathcal{X}$)-mod

$=$ End(B_*)-mod

Φ

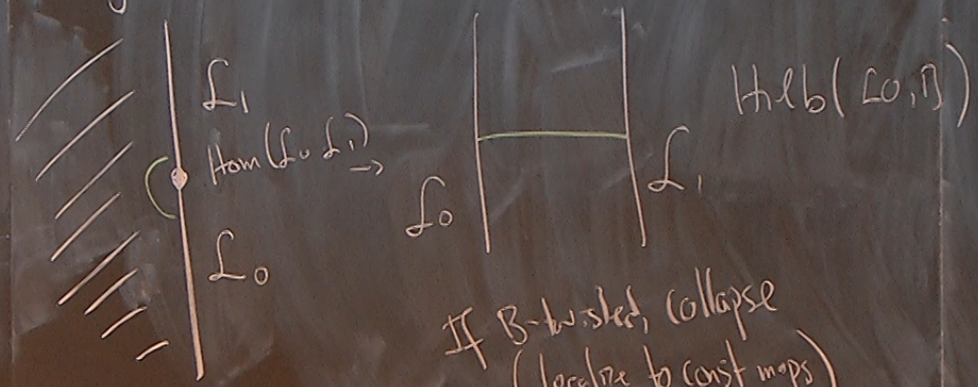


A-twisted analogs: similar,

but in computations, must always keep

loops / paths.

E.g. 2d A-model to $\mathcal{X}$



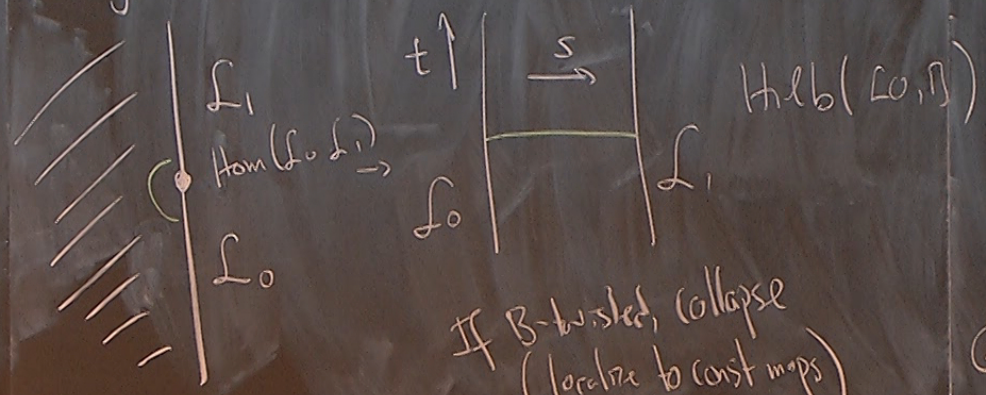
If B-twisted, collapse

(localize to const maps)

A-twisted analogs: similar,

but in computations, must always keep loops / paths.

E.g. 2d A-model to $\mathcal{X}$



If B-twisted, collapse (localize to const maps)

ie. $\partial_{\bar z}(x + iy) = 0$

$z = t + is$

Find A-SQM

target $= \mathrm{Maps}([0,1], \mathcal{X})$

$$h = \int_0^1 x \partial_s y \, ds$$

$\mathrm{Hilb} = H_\bullet(\text{MW complex})$

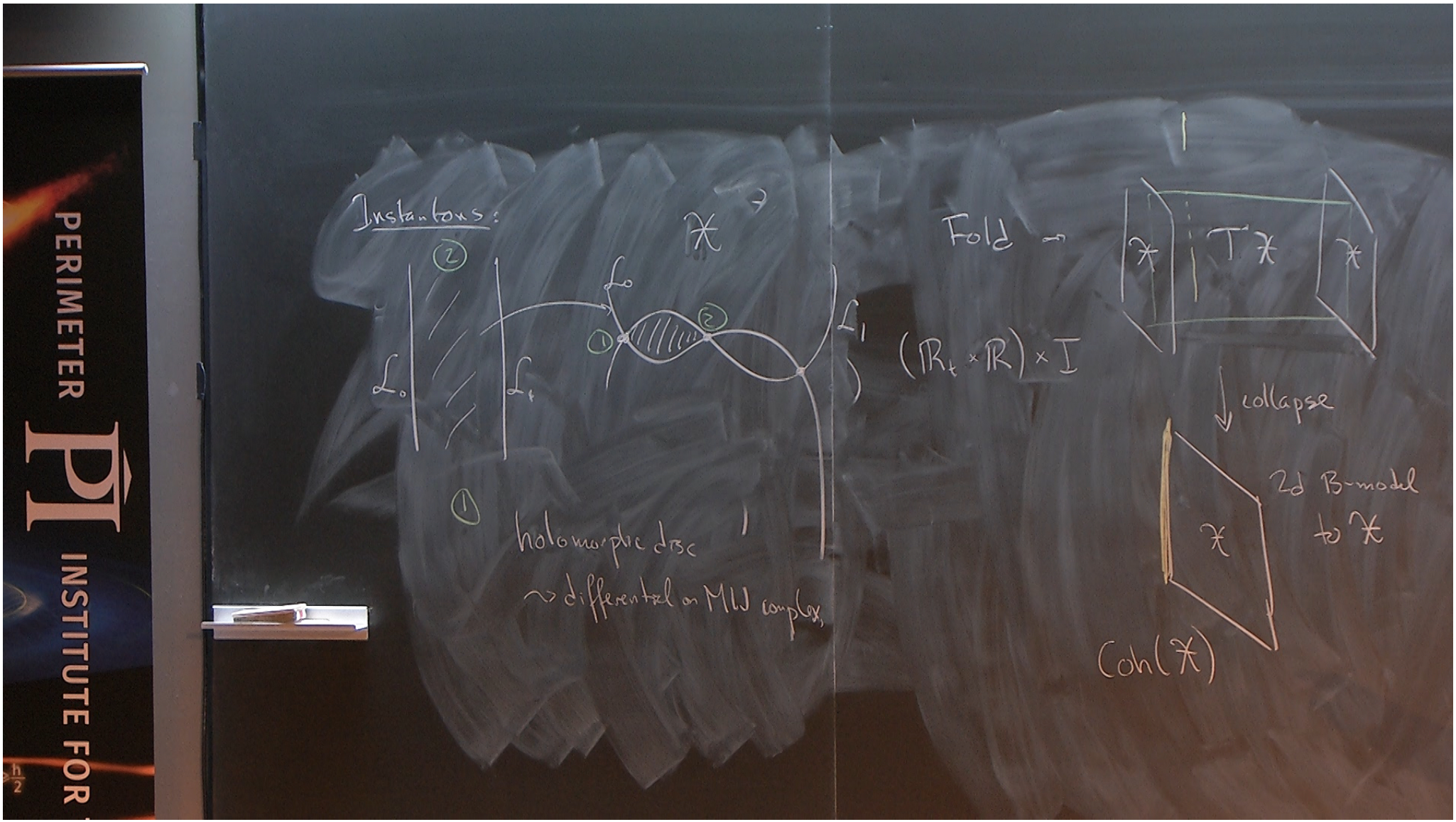
generators $= \{dh = 0\}$

$= \mathcal{L}_0 \cap \mathcal{L}_1$

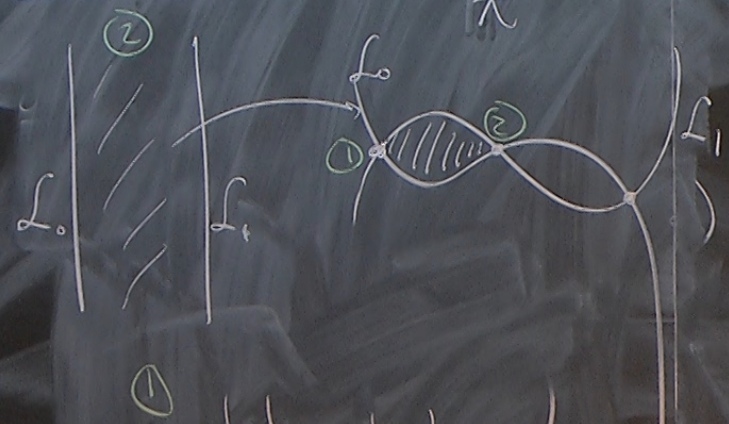
Instantons / Grad flows

$$\partial_t x = \frac{\partial h}{\partial x} = \partial_s y$$

$$\partial_t y = -\partial_s x$$


Instantons:
$\mathcal{X}$
$\mathcal{L}_0$
$\mathcal{L}_1$
f_0
f_1
holomorphic disc
$\rightsquigarrow$ differential on MW complex
Fold $\rightsquigarrow$
$(\mathbb{R}_t \times \mathbb{R}) \times I$
$\mathcal{X}$
$T\mathcal{X}$
$\mathcal{X}$
collapse
$\mathcal{X}$
2d B-model to $\mathcal{X}$
$\mathrm{Coh}(\mathcal{X})$

Instantons:



holomorphic disc

$\rightsquigarrow$ differential on MW complex

Or: local ops

$\mathrm{Hilb}(S^1)$

$\shortparallel$

Hilb of SQM

target: $L\mathcal{X}$

$h = \oint_{S^1} ds \, x \, \partial_s y$

Now fro

Bd

T^*

B

s

Hilb(S')

"

Hilb of SQM

target = $L\mathcal{X}$

$h = \oint_{S^1} ds\, x\, \partial_s y$

3d A-model to $T^*[2]\mathcal{X}$

(way more interesting for gauge thys)

Lines?

$\rightsquigarrow$

2d A-model

target = $L(T^*[2]\mathcal{X})$

$W = \oint ds\, X \partial_s Y$

Is?

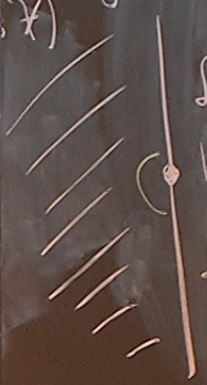
D-mod($L\mathcal{X}$).

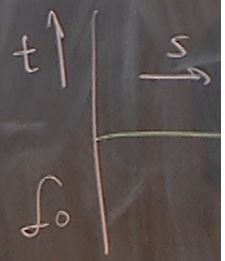
A-twisted analogs: simil

but in computations,

loops / paths.

E.g. 2d A-model to





If B-tw.

(local

ℓ to $T^*[2]\mathcal{X}$

more interesting for gauge thys)

2d A-model

target = $L(T^*[2]\mathcal{X})$

$W = \oint ds\, X \partial_s Y$

is?

D-mod$(L\mathcal{X})$.

QFT 4 Math II

June 2020 (hopefully)

3d $\mathcal{N}=4$ thy's

connections to 4d

and applications!

ie. $\partial_{\bar z}(x+iy)=0$

$z = t+is$

Find A-SQM

target = Maps([0

$h = \int_0^1$

$H_{Hilb} = H_Q^{\cdot}(MW$

generators

$= f_0$

Instantons $\partial_t x =$

Grad flows $=$

$\partial_t y = -$