Title: Lecture 6: Boundary Conditions and Extended Defects

Speakers: Davide Gaiotto

Collection: QFT for Mathematicians

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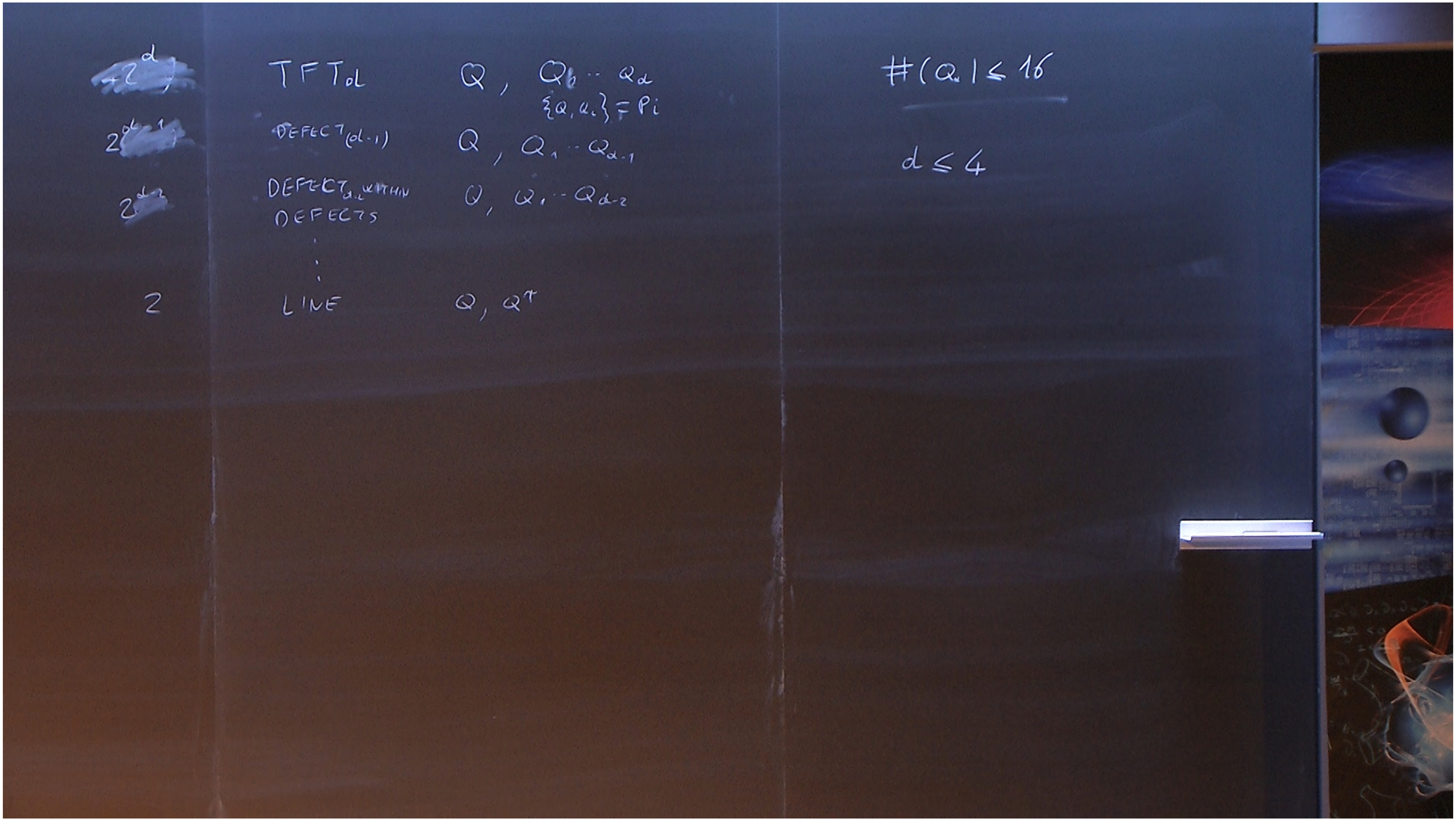


2^d
TFT$_d$
$Q\ ,\ Q_1 \cdots Q_d$
$\{Q, Q_i\} = P_i$
2^{d-1}
DEFECT$_{(d-1)}$
$Q\ ,\ Q_1 \cdots Q_{d-1}$
2^{d-2}
DEFECT$_{d-2}$ WITHIN DEFECTS
$Q,\ Q_1 \cdots Q_{d-2}$
2
LINE
$Q,\ Q^{\dagger}$
$\#(Q_a) \leq 16$
$d \leq 4$

4d	$N=4$	2^d	TFT_d	$Q\,,\ Q_b \cdots Q_d$ $\{Q, Q_i\} = P_i$
	$\cap$			
3d	$N=4$	2^{d-1}	DEFECT$_{(d-1)}$	$Q\,,\ Q_1 \cdots Q_{d-1}$
	$\cap$			
2d	$N=(2,2)$	2^{d-2}	DEFECT$_{d-2}$ WITHIN DEFECTS	$Q\,,\ Q_1 \cdots Q_{d-2}$
	$\cap$		$\vdots$	
1d	$N=2$	2	LINE	$Q\,,\ Q^\dagger$

4d $N=4$ SYM WITH GAUGE ~~LIE~~ ALGEBRA $\mathfrak{g}$

\+ TOPOLOGICAL DATA

$$A_\mu \quad \Omega^1 \otimes \mathfrak{g}$$

$$\lambda_\alpha^A \quad S^+ \otimes \mathfrak{g} \otimes V_4$$

$$\lambda_{\dot\alpha A} \quad S^- \otimes \mathfrak{g} \otimes V_4^*$$

$$\Phi_{AB} \quad \Omega^0 \otimes \mathfrak{g} \otimes \Lambda^2 V_4 \cong \Lambda^2 V_4^*$$

$$P_\mu,\ Q_{\alpha A},\ \bar{Q}_{\dot\alpha}^A,\ M^A{}_B \quad M^A_A = 0$$

$$M^A{}_B \in SU(4)_R$$

4d

3d N

2d $N=$

1d

UGE ~~LIE~~ ALGEBRA of

TOPOLOGICAL DATA

$P_\mu \ , Q_{\alpha A} \ , \bar{Q}^A_{\dot\alpha} \ , \ M^A{}_B \qquad M^A_A = 0$

$\cap$

$SU(4)_R$

$\{Q_{\alpha A}, Q_{\beta B}\} = 0$

$\{Q_{\alpha A}, \bar{Q}^B_{\dot\beta}\} = \delta^B_A P_{\alpha\dot\beta}$

$\{\bar{Q}, \bar{Q}\} = 0$

TFT_d	$Q, Q_b \cdots$
	$\{Q, Q_i\} =$
DEFECT $(d-1)$	$Q, Q_1 \cdots Q_{d-1}$
DEFECT$_{d-2}$ WITHIN DEFECTS	$Q, Q_1 \cdots Q_{d-2}$
⋮	
LINE	Q, Q^+

4d $N=4$ SYM WITH GAUGE ~~LIE~~ ALGEBRA $\mathfrak{g}$

+ TOPOLOGICAL DATA

$$A_\mu \qquad \Omega^1 \otimes \mathfrak{g}$$

$$\lambda^A_\alpha \qquad S^+ \otimes \mathfrak{g} \otimes V_4$$

$$\lambda_{\dot\alpha A} \qquad S^- \otimes \mathfrak{g} \otimes V_4^*$$

$$\Phi_{AB} \qquad \Omega^0 \otimes \mathfrak{g} \otimes \Lambda^2 V_4 \cong \Lambda^2 V_4^*$$

$$P_\mu \,,\; Q_{\alpha A} \,,\; \bar{Q}^A_{\dot\alpha} \,,\; M^A{}_B \qquad M^A_A = 0$$

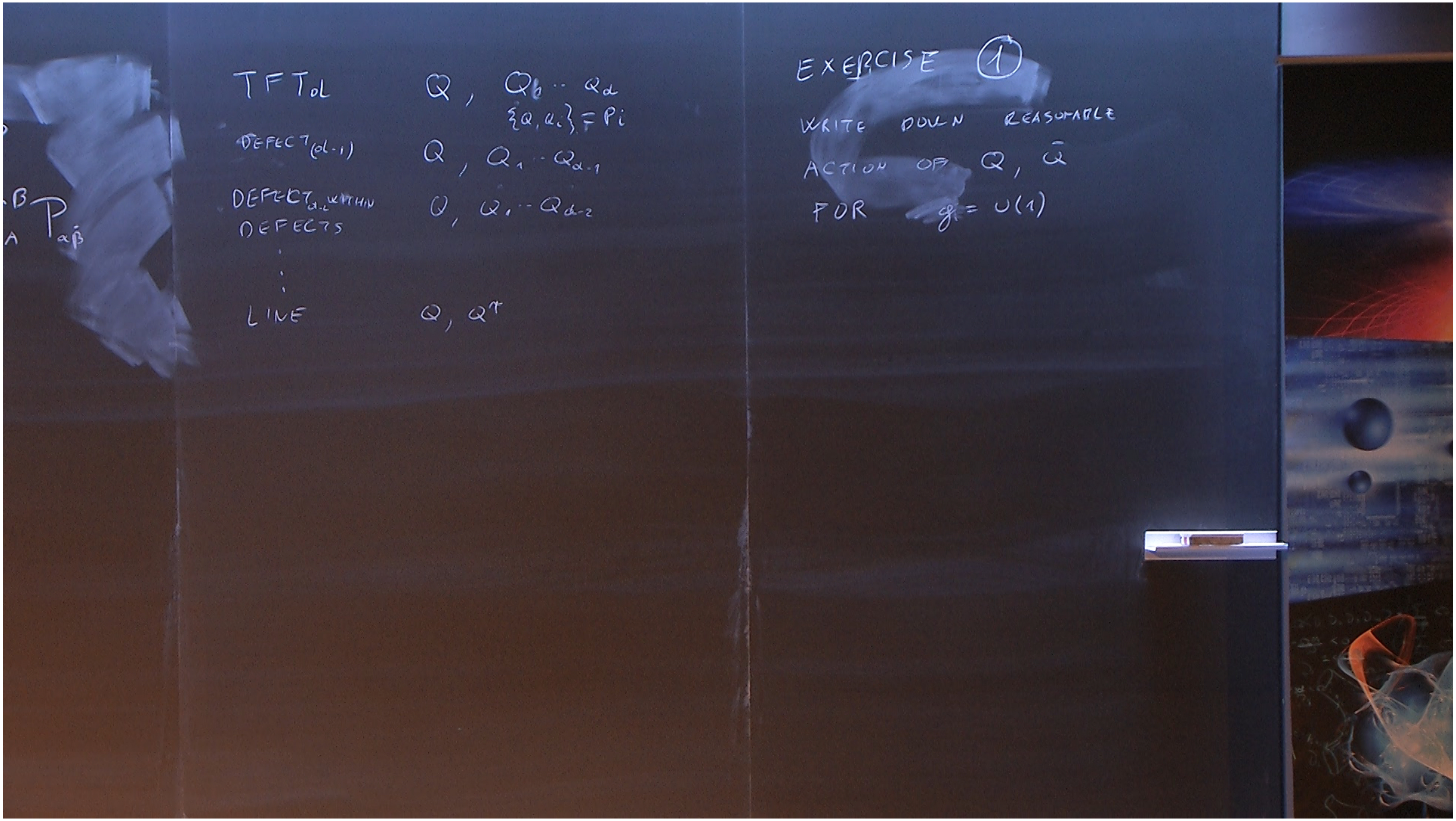
$$M^A{}_B \in SU(4)_R$$

$$\begin{matrix} A \\ \Phi \end{matrix} \xrightarrow[\bar{Q}]{Q} \lambda \xrightarrow[\bar{Q}]{Q} \begin{matrix} F \\ \partial\Phi \end{matrix}$$

$$\{ Q_{\alpha A} , Q_{\beta B} \} = 0$$

$$\{ Q_{\alpha A} , \bar{Q}^B_{\dot\beta} \} = \delta_A^B \ldots$$

$$\{ \bar{Q} , \bar{Q} \} = 0$$


TFT_d Q, Q_b ·· Q_d
{Q, Q_i} = P_i
DEFECT (d-1) Q, Q_1 ·· Q_{d-1}
DEFECT_{d-2} WITHIN DEFECTS Q, Q_1 ·· Q_{d-2}
LINE Q, Q^†
EXERCISE 1
WRITE DOWN REASONABLE
ACTION OF Q, Q̄
FOR g = U(1)

$, Q_{\dot\beta B}\} = 0$

$\bar{Q}^{B}_{\dot\beta}\} = \delta^{B}_{A} P_{\alpha\dot\beta}$

$\bar{Q}, \bar{Q}\} = 0$

$$\frac{1}{g^2_{YM}} \int \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \lambda \not{D} \lambda + (\partial_\mu \phi)^2 + \dots$$

$$+ \theta \int F \wedge F$$

ABELIAN $U(1)$ GAUGE IN 4d

EM DUALITY $\quad F \leftrightarrow * F \quad g^2 \leftrightarrow \frac{1}{g^2} \quad \theta = 0$

$$\frac{1}{g^2} \int |dA|^2 \rightarrow \int |F|^2 + dF \wedge B \rightarrow g^2 \int |dB|^2$$

$$\tau = \frac{\theta}{2\pi} + i \frac{1}{g^2}$$

E.M. $\tau \leftrightarrow -\frac{1}{\tau} \quad g \rightarrow \frac{1}{g}$

$\tau \rightarrow \tau + 1$

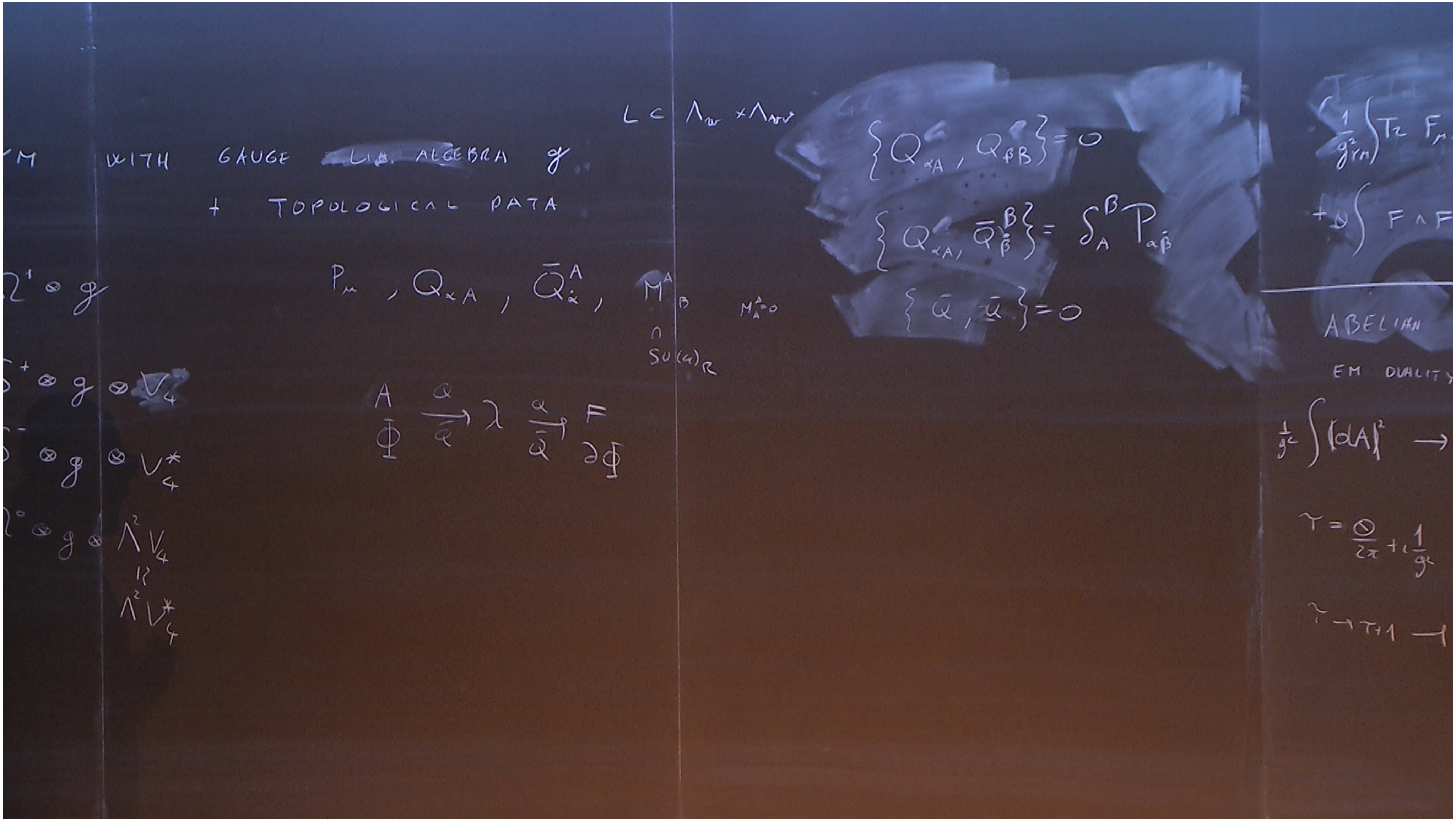
$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}$$

EXERCISE 1

WRITE DOWN REASONABLE ACTION OF Q, $\bar{Q}$ FOR $g = U(1)$

EXERCISE 2

CHECK $\tau \rightarrow -\frac{1}{\tau}$


WITH GAUGE LIE ALGEBRA g
+ TOPOLOGICAL DATA
L ⊂ Λw × Λmw
Pμ , QαA , Q̄Aα̇ , MAB
SU(4)R
{QαA , QβB} = 0
{QαA , Q̄Bβ̇} = δAB Pαβ̇
{Q̄ , Q̄} = 0
ABELIAN
EM DUALITY
τ = θ/2π + i 1/g²
τ → τ+1

4 SYM WITH GAUGE ~~LIE~~ ALGEBRA $\mathfrak{g}$

+ TOPOLOGICAL DATA

$L \subset \Lambda_{w} \times \Lambda_{wt}$

$\{Q_{\alpha A}, Q_{\beta B}\} = 0$

$\{Q_{\alpha A}, \bar{Q}^{B}_{\dot\beta}\} = \delta_A^B P_{\alpha\dot\beta}$

$\{\bar{Q}, \bar{Q}\} = 0$

$\Omega^1 \otimes \mathfrak{g}$

$S^+ \otimes \mathfrak{g} \otimes V_4$

$S^- \otimes \mathfrak{g} \otimes V_4^*$

$\Omega^0 \otimes \mathfrak{g} \otimes \Lambda^2 V_4 \cong \Lambda^2 V_4^*$

P_μ, $Q_{\alpha A}$, $\bar{Q}^A_{\dot\alpha}$, $M^A{}_B$ $\quad M^A_A = 0$

$\cap$

$SU(4)_R$

$\begin{matrix} A \\ \Phi \end{matrix} \xrightarrow[\bar{Q}]{Q} \lambda \xrightarrow[\bar{Q}]{Q} \begin{matrix} F \\ \partial\Phi \end{matrix}$

$F_{\alpha\beta} \in Sym^2 S_+ \quad = \text{"}F + i{*}F\text{"}$

$F_{\dot\alpha\dot\beta} \in Sym^2 S_- \quad = \text{"}F - i{*}F\text{"}$

$L \subset \Lambda_w \times \Lambda_{w^\vee}$

$$\{Q_{\alpha A}, Q_{\beta B}\} = 0$$

$$\{Q_{\alpha A}, \bar{Q}^{B}_{\dot\beta}\} = \delta_A^B \left(\epsilon_{\alpha\dot\beta} P^{(4)} + P^{(3)}_{(\alpha\dot\beta)}\right)$$

$$\{\bar{Q}, \bar{Q}\} = 0$$

$M^A{}_B$ $\quad M^A_A = 0$

$\cap$

$SU(4)_R$

F

$\partial\Phi$

$3d \subset 4d$ SUSY

$R^4 \to R^3 \oplus R$

$S_+ \to S$

$S_- \to S$

PICK $V_4 \simeq V_4^*$

$SU(4)_R \to SO(4)_R$

$$Q^{3d[\omega]}_{\alpha A} = e^{\frac{i\omega}{2}} Q_{\alpha A} + e^{-\frac{i\omega}{2}} \bar{Q}_{\dot\alpha}{}^{A}$$

$\frac{1}{2}$ BPS b.c.

PRESERVES $Q^{3d}_{\alpha A}[\omega]$

EXERCIS

WRITE D

ACTION

FOR

EXERCIS

CHECK

$, Q_{\beta B} \} = 0$

$\bar{Q}^{B}_{\dot\beta} \} = \delta^{B}_{A} \left(\epsilon_{\alpha\beta} P^{(4)} + P^{(3)}_{(\alpha\dot\beta)} \right)$

$, \bar{u} \} = 0$

$3d \subset 4d$ SUSY

$R^4 \to R^3 \oplus R$

$S_+ \to S$, $S_- \to S$

PICK $V_4 \simeq V_4^*$

$SU(4)_R \to SO(4)_R$

$Q^{3d[\omega]}_{\alpha A} = e^{\frac{i\omega}{2}} Q_{\alpha A} + e^{-\frac{i\omega}{2}} \bar{Q}_{\dot\alpha}^{A}$

$\frac{1}{2}$ BPS b.c.

PRESERVES $Q^{3d}_{\alpha A}[\omega]$

ω IS FIXED AS FUNCTION OF τ AND b.c.

DIRICHLET : $\omega = 0, \pi$

EXERCISE 1

WRITE DOWN REASONABLE ACTION OF Q, $\bar{Q}$ FOR $\mathfrak{g} = U(1)$

EXERCISE 2

CHECK $\tau \to -\frac{1}{\tau}$

EXERCISE 3

$\frac{1}{2}$ BPS DIRICHLET :

$A_i = 0 \quad i = 1,2,3$


τ→τ+1 T
τ→ -1/τ S
U(1) Q T S
DIR → NEU →T^k NEU^(1,k) →S NEU^(-k,1)
NEU^(p,q)
P,q CO-PRIME INTEGERS

$$\{Q_{\alpha A}, Q_{\beta B}\} = 0$$

$$\{Q_{\alpha A}, \bar{Q}^{B}_{\dot\beta}\} = \delta^{B}_{A}\left(\epsilon_{\alpha\dot\beta} P^{(4)} + P^{(3)}_{(\alpha\dot\beta)}\right)$$

$$\{\bar{Q}, \bar{Q}\} = 0$$

3d ⊂ 4d SUSY

$R^4 \to R^3 \oplus R$

$S_+, S_- \to S$

PICK $V_4 \simeq V_4^*$

$SU(4)_R \to SO(4)_R$

$$Q^{3d[\omega]}_{\alpha A} = e^{\frac{i\omega}{2}} Q_{\alpha A} + e^{-\frac{i\omega}{2}} \bar{Q}_{\alpha}^{A}$$

$\frac{1}{2}$ BPS b.c. PRESERVES $Q^{3d}_{\alpha A}[\omega]$

ω IS FIXED AS FUNCTION OF τ AND b.c.

DIRICHLET : $\omega = 0, \pi$

NEU^{p,q} $\omega = \omega^{\pm}(\tau, p, q)$

$\theta = 0$ NEU, DIR $\omega = \pm\frac{\pi}{2}$

EXERCISE ①

WRITE DOWN REASONABL[E] ACTION OF $Q, \bar{Q}$ FOR $g = U(1)$

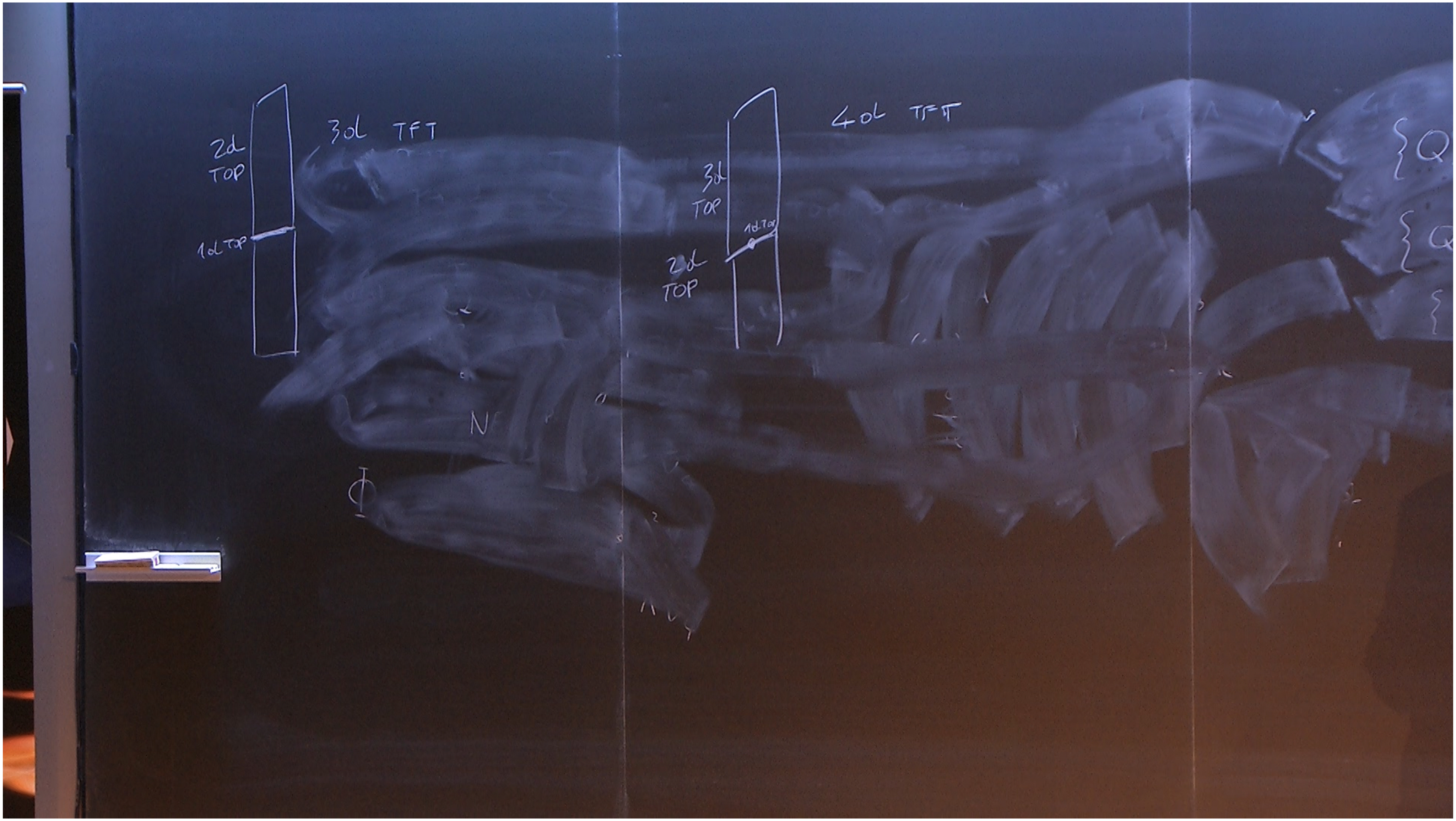
EXERCISE ②

CHECK $\tau \to -\frac{1}{\tau}$

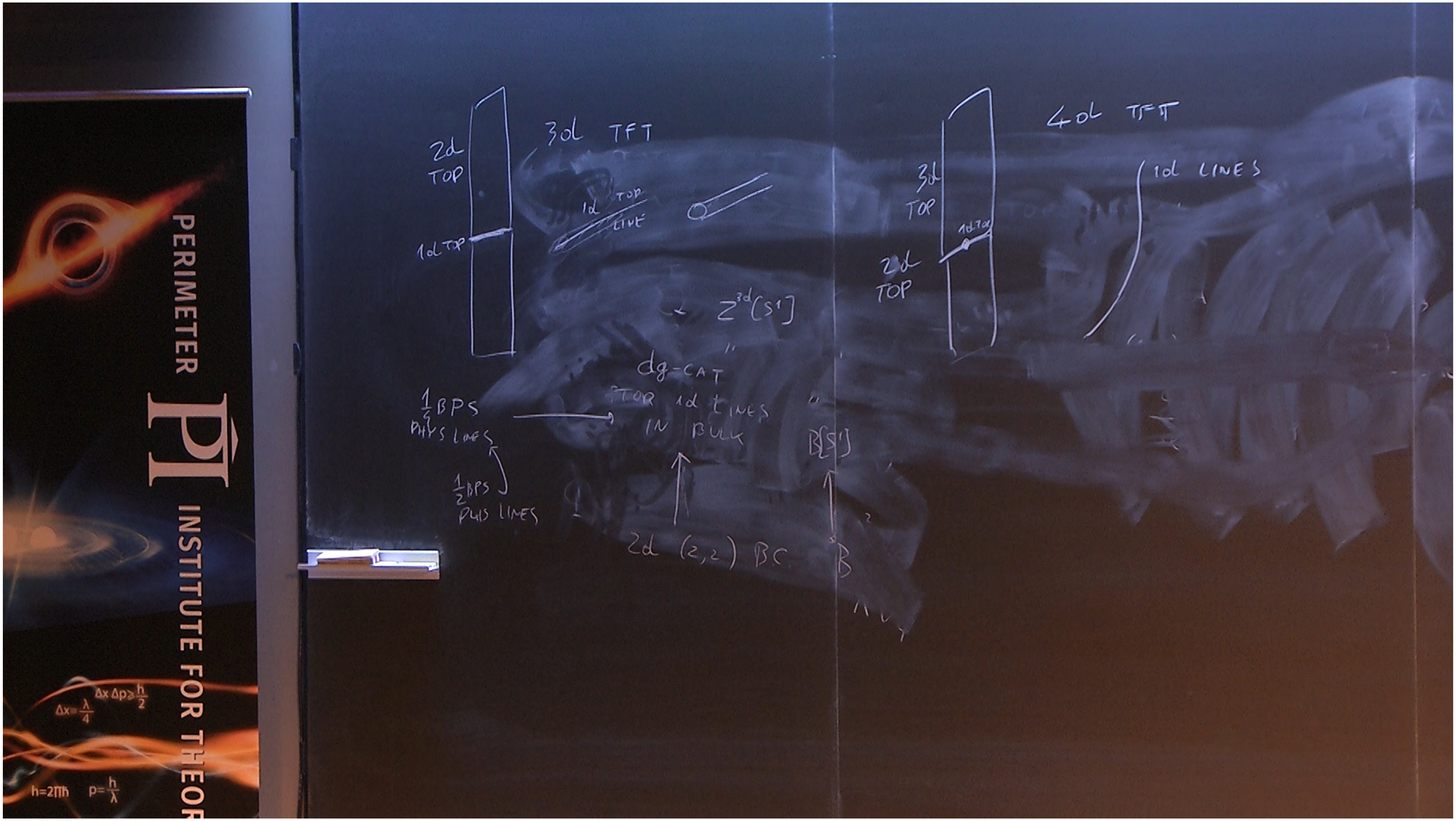
EXERCISE ③

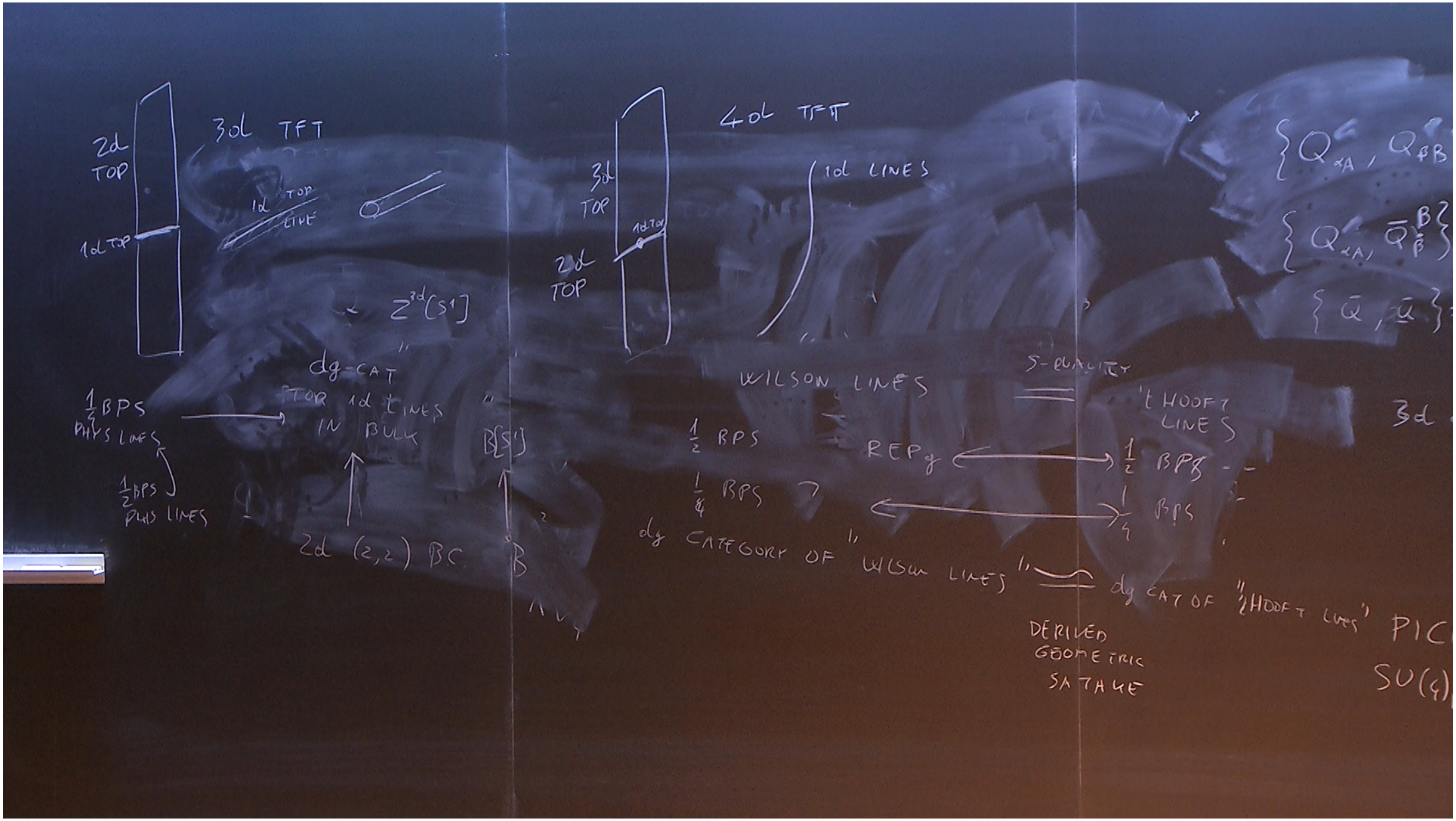
$\frac{1}{2}$ BPS DIRICHLET :

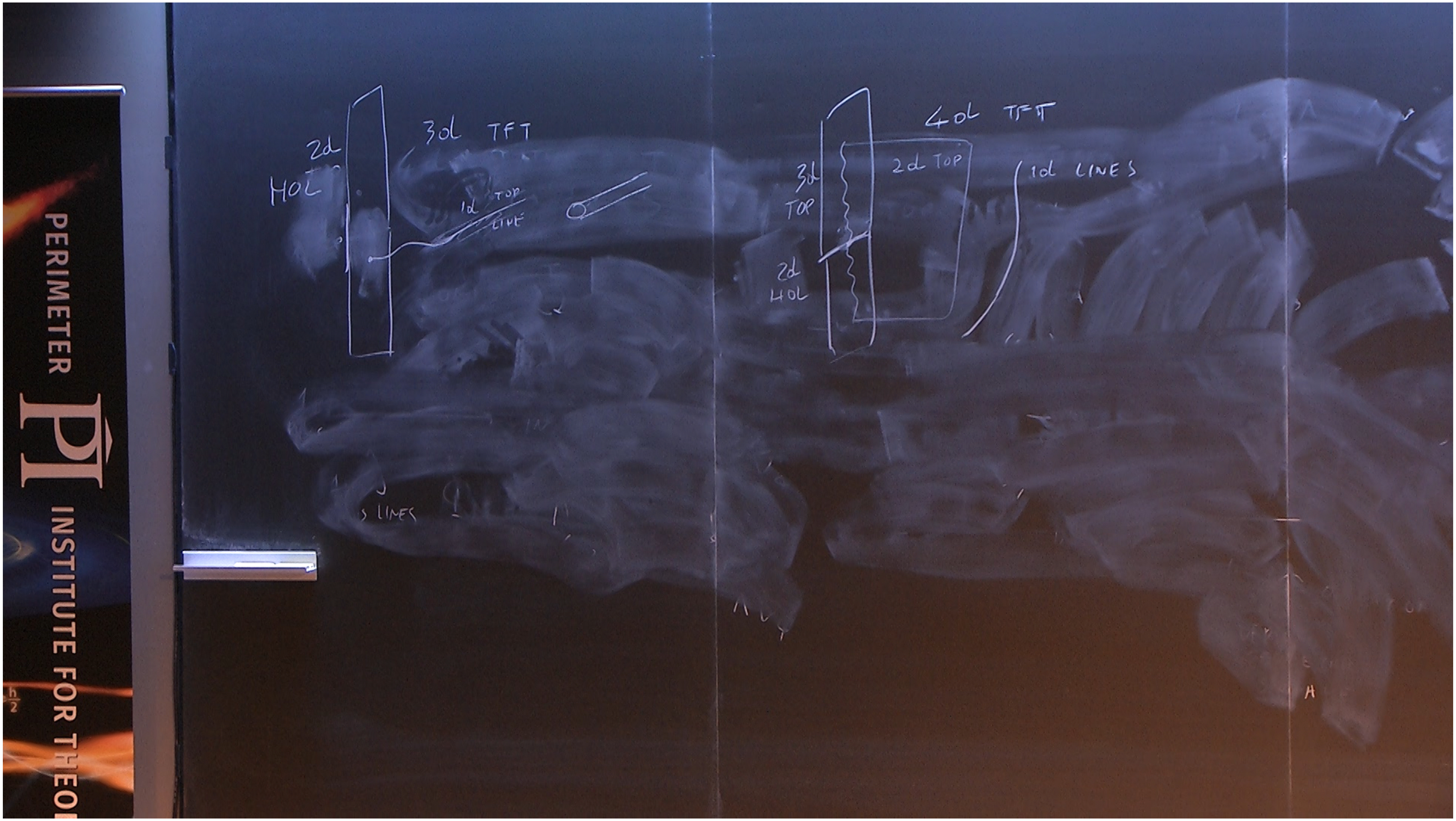
$A_i = 0 \quad i = 1, 2, 3$

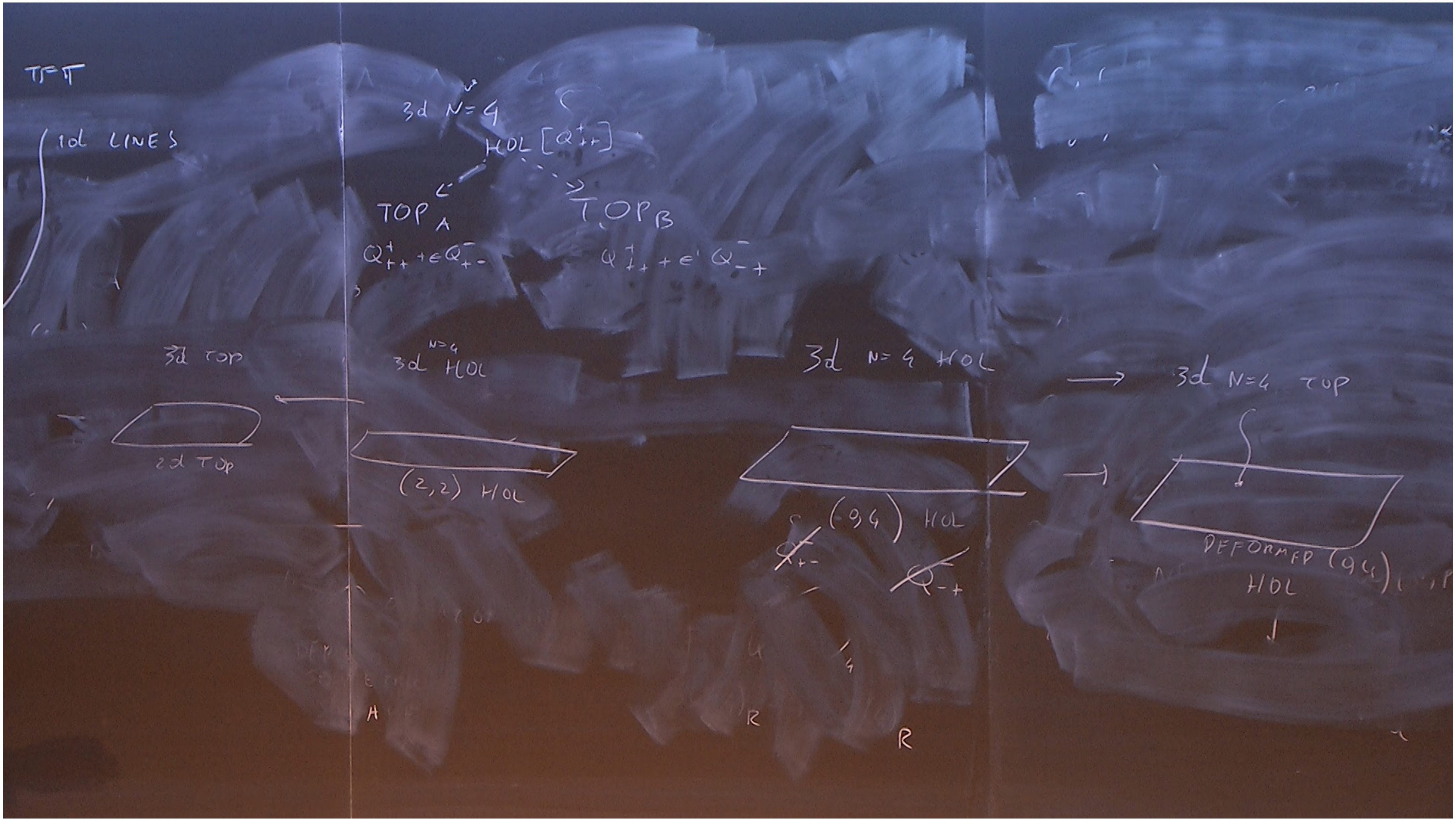


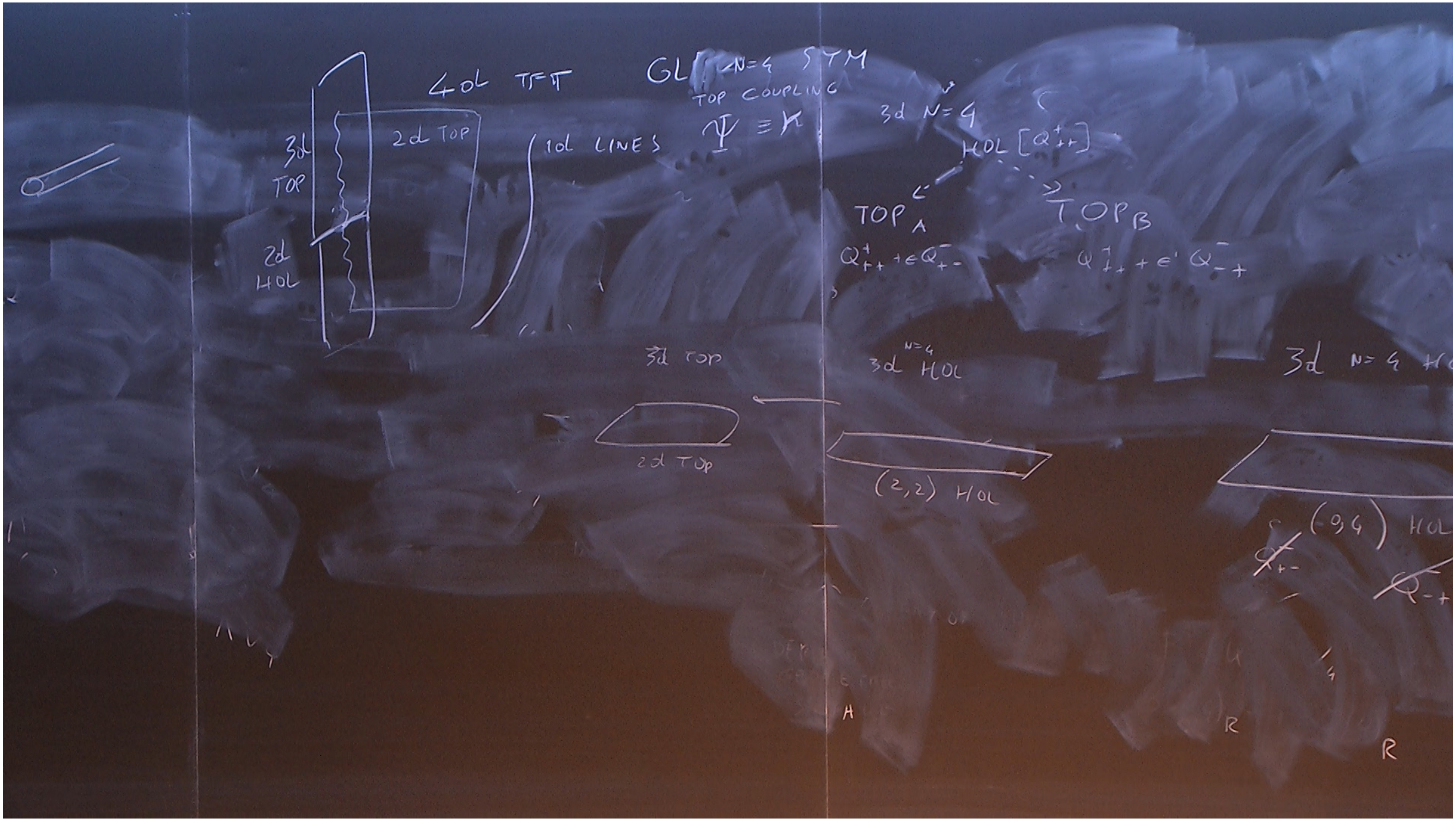
2d TOP
1d TOP
3d TFT
3d TOP
1d TOP
2d TOP
4d TFT

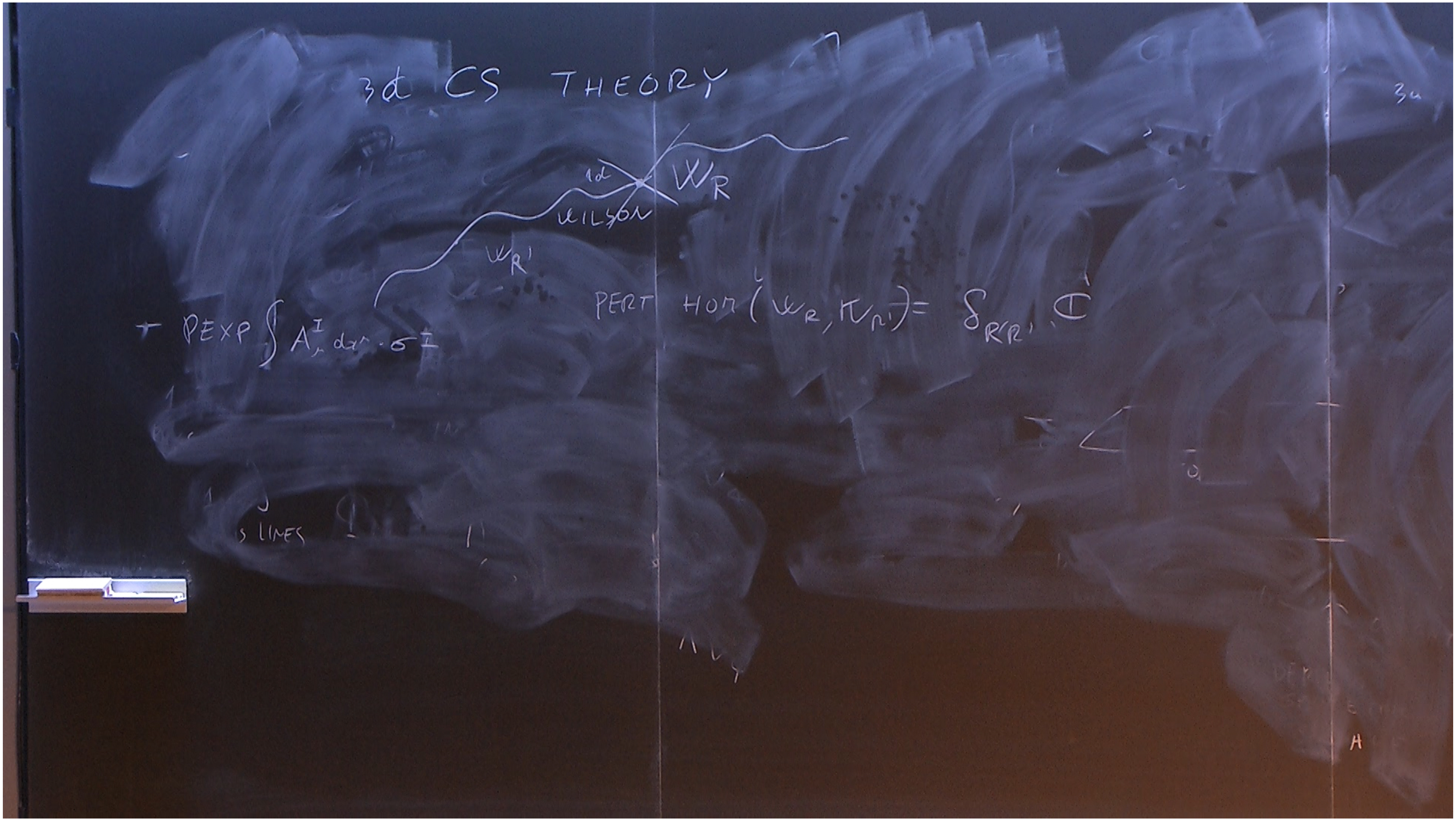

2d TOP
3d TFT
1d TOP LINE
1d TOP
$Z^{3d}[S^1]$
dg-CAT
TOP 1d LINES IN BULK
$\frac{1}{2}$ BPS PHYS LINES
$\frac{1}{2}$ BPS PHYS LINES
2d (2,2) B.C.
$\mathcal{B}[S^1]$
$\mathcal{B}$
3d TOP
2d TOP
4d TFT
1d LINES
PERIMETER PI INSTITUTE FOR THEOR

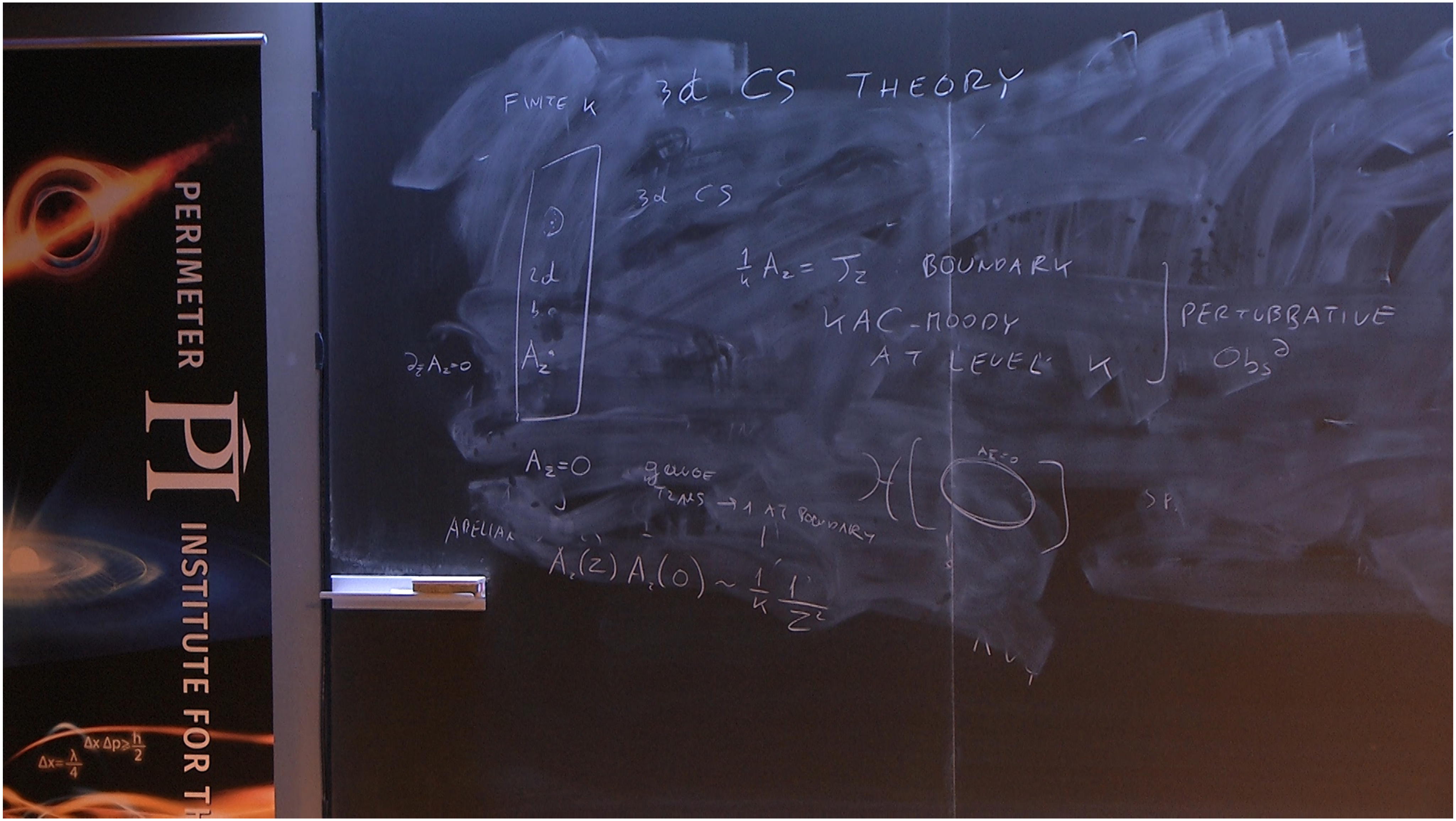

2d TOP
1d TOP
3d TFT
1d TOP LINE
$Z^{3d}[S^1]$
dg-CAT
TOP 1d LINES IN BULK
1/2 BPS PHYS LINES
1/2 BPS PHYS LINES
2d (2,2) BC
B[S1]
4d TFT
3d TOP
2d TOP
1d TOP
1d LINES
WILSON LINES
S-DUALITY
't HOOFT LINES
1/2 BPS
REP
1/2 BPS
1/4 BPS
1/4 BPS
dg CATEGORY OF "WILSON LINES"
dg CAT OF "t HOOFT LINES"
DERIVED GEOMETRIC SATAKE
3d
PIC
SU(4)


2d
HOL
3d TFT
1d TOP
LINE
4d TFT
2d TOP
3d
TOP
2d
HOL
1d LINES
PERIMETER
PI
INSTITUTE FOR THEO


TFT
1d LINES
3d N=4
HOL [Q+++]
TOP A
TOP B
Q+++ + εQ-+-
Q+++ + ε' Q--+
3d TOP
3d N=4 HOL
2d TOP
(2,2) HOL
3d N=4 HOL
(0,4) HOL
Q+-
Q-+
3d N=4 TOP
DEFORMED (0,4)
HOL
A
R
R


4d TFT
GL N=4 SYM
TOP COUPLING
3d N=4
HOL
TOP A
TOP B
3d TOP
2d TOP
2d HOL
1d LINES
3d TOP
3d HOL
2d TOP
(2,2) HOL
3d N=4
(0,4) HOL


3d CS THEORY
1d
W_R
WILSON
$W_{R'}$
PERT HOM$(W_R, W_{R'}) = \delta_{RR'} \cdot \mathbb{C}$
PEXP $\int A^I_\mu \, dx^\mu \cdot \sigma_I$
LINES


PERIMETER
PI
INSTITUTE FOR TH
FINITE k
3d CS THEORY
3d CS
2d
b.c
A_z
∂_z̄ A_z = 0
1/k A_z = J_z BOUNDARY
KAC-MOODY
AT LEVEL k
PERTURBATIVE
Obs^∂
A_z̄ = 0
gauge TRANS → 1 AT BOUNDARY
ABELIAN
A_z(z) A_z(0) ~ 1/k 1/z²
A_z̄ = 0

EXERCISE 1

WRITE DOWN REASONABLE ACTION OF Q, $\bar{Q}$ FOR $\mathfrak{g} = U(1)$

EXERCISE 2

CHECK $\tau \to -\frac{1}{\tau}$

EXERCISE 3

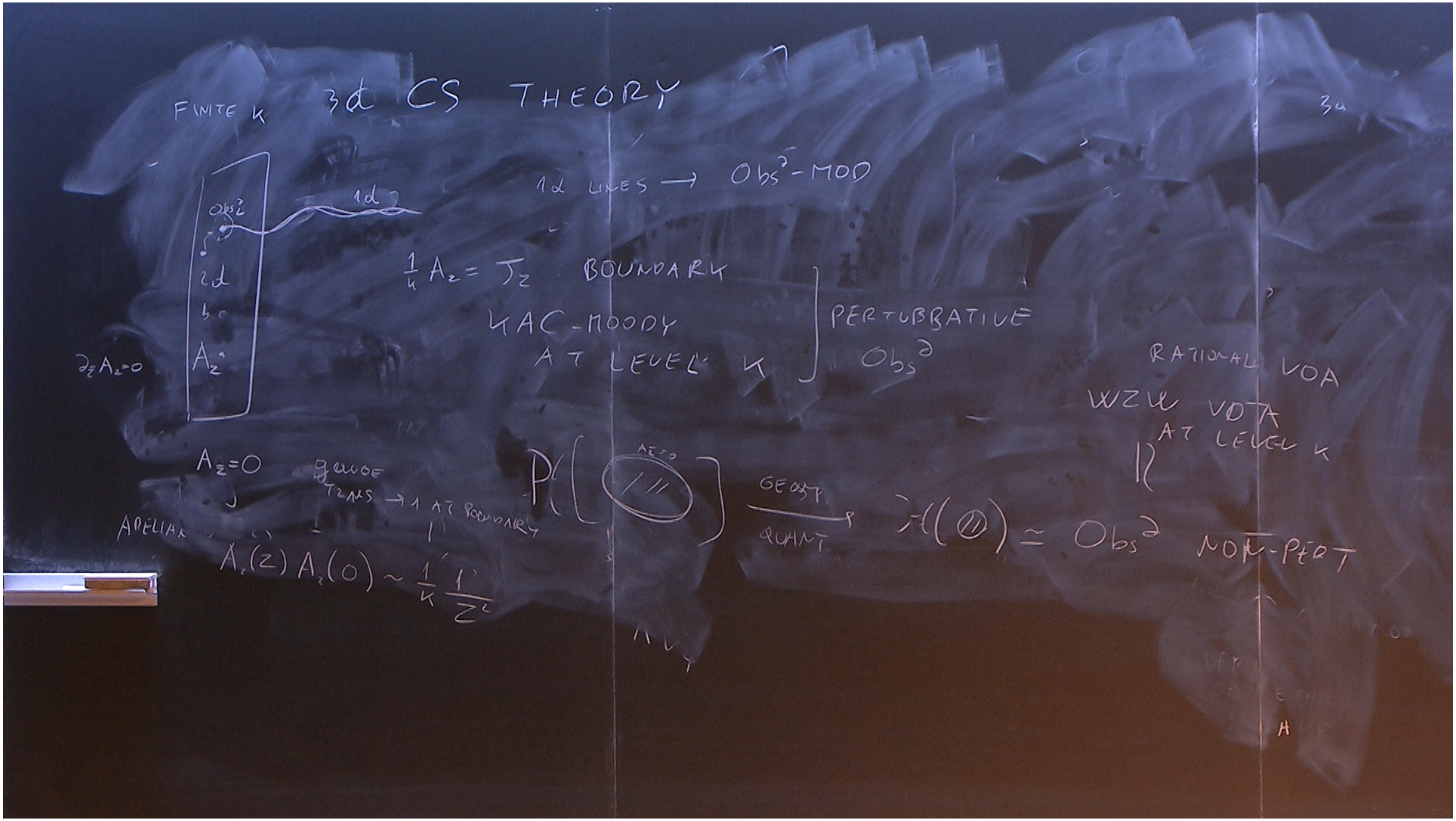
$\frac{1}{2}$ BPS DIRICHLET:

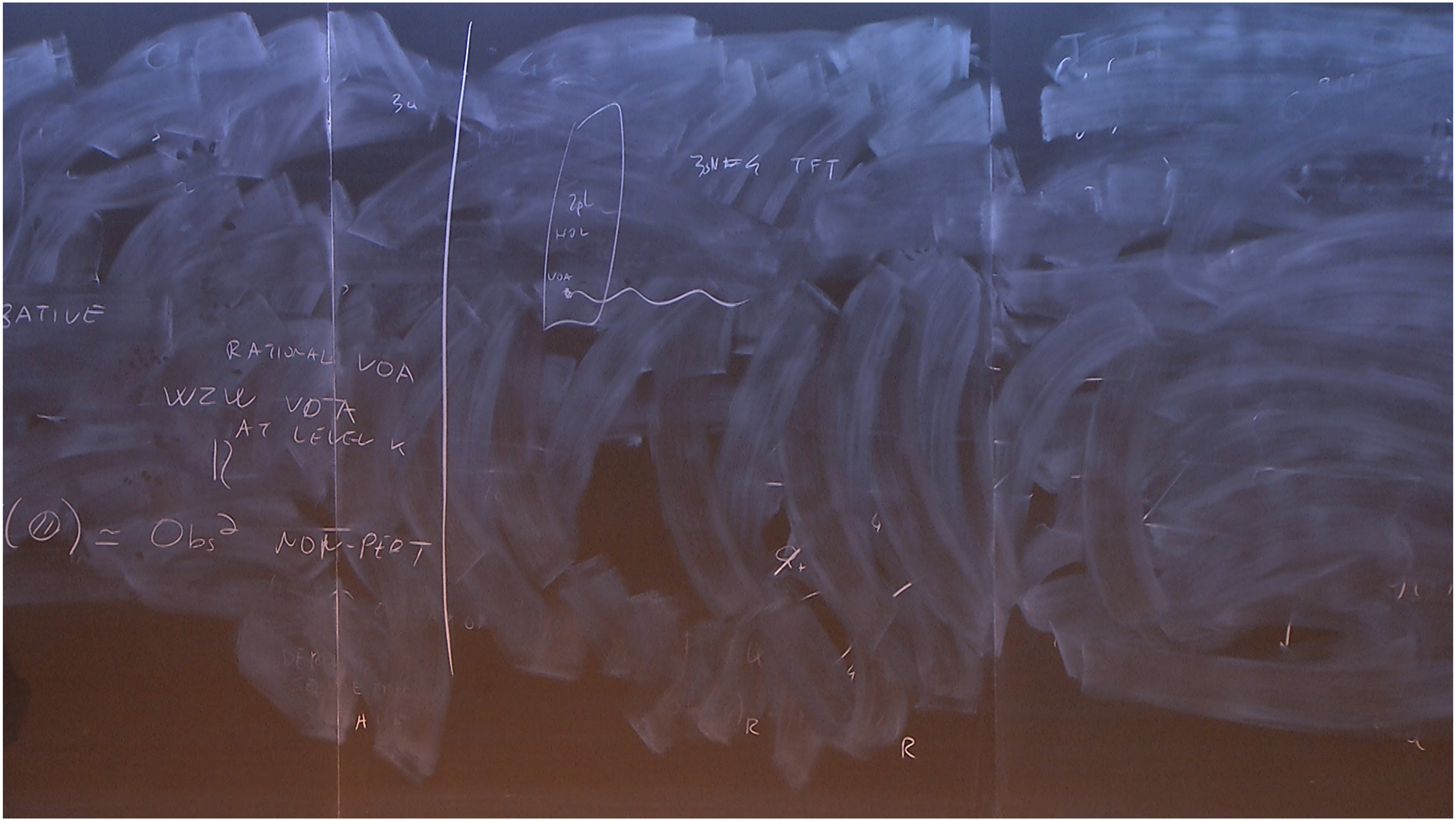
$A_i = 0 \quad i = 1, 2, 3$

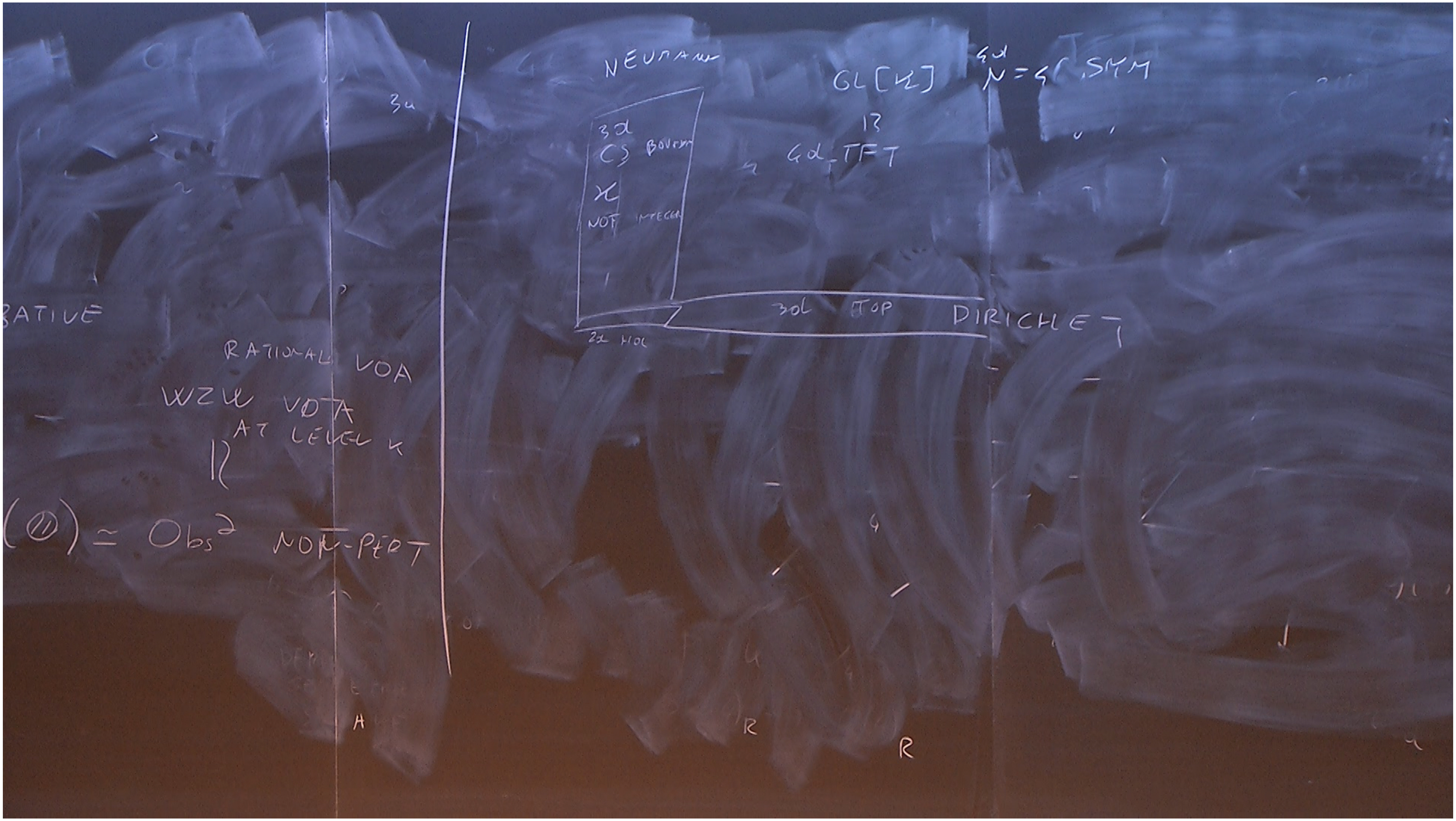
EXERCISE 4

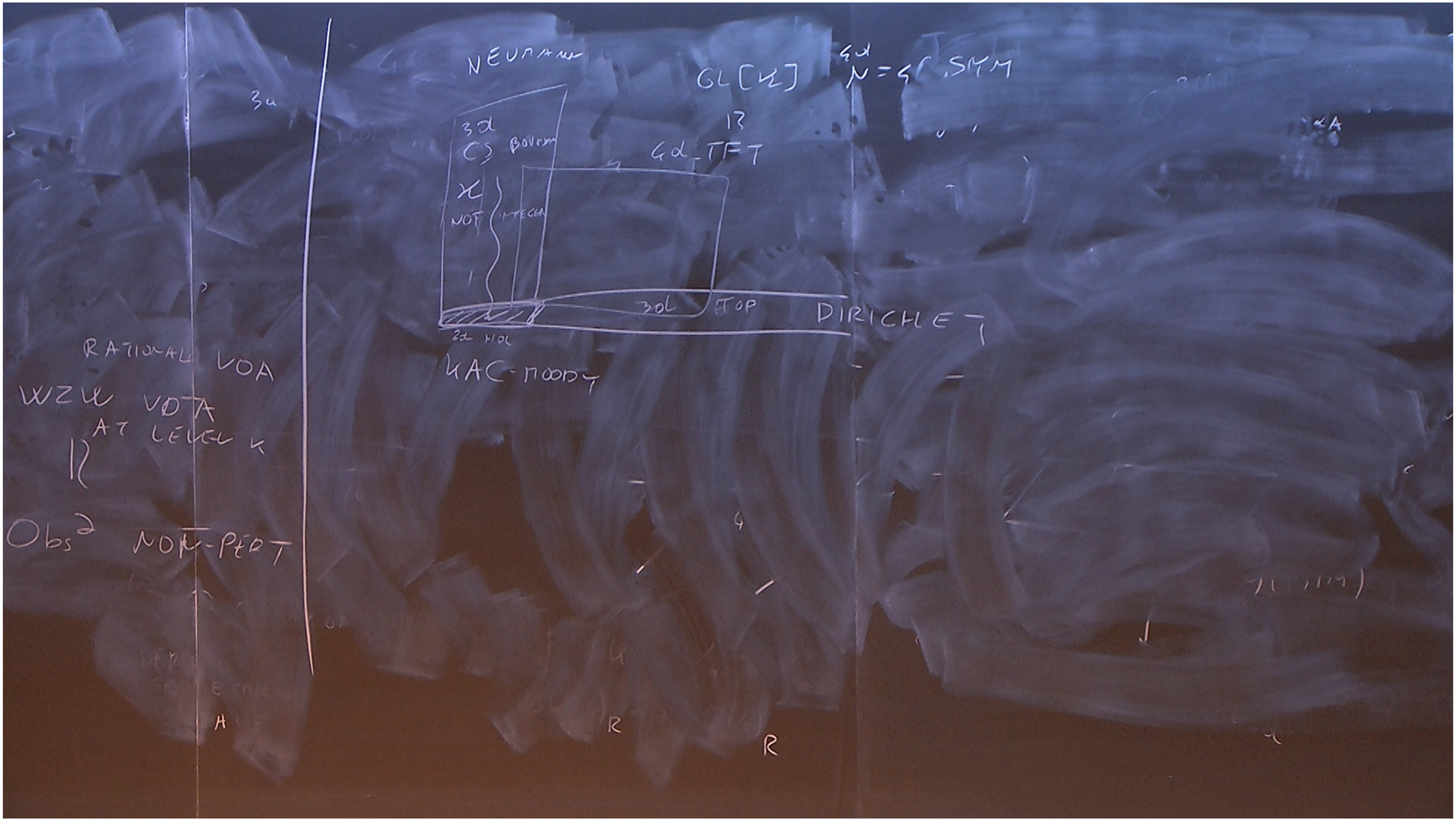
PHASE SPACE OF CS ON

$k = 0$


FINITE k
3d CS THEORY
1d LINES → Obs^∂-MOD
1d
1/k A_z = J_z BOUNDARY
KAC-MOODY
AT LEVEL k
PERTURBATIVE
Obs^∂
RATIONAL VOA
WZW VOA
AT LEVEL k
A_z̄ = 0
gauge TRANS → 1 AT BOUNDARY
ABELIAN
A_z(z) A_z(0) ~ 1/k 1/z^2
GEOM QUANT
≃ Obs^∂
NON-PERT


RATIVE
RATIONAL VOA
WZW VOA
AT LEVEL K
NON-PERT
TFT
VOA
R
R


NEUMANN
3d
CS
NOT INTEGER
4d TFT
3d TOP
DIRICHLET
2d HOL
RATIVE
RATIONAL VOA
WZW VOA
AT LEVEL k
NON-PERT
R
R


NEUMANN
4d N=4 SYM
3d
CS BOUNDARY
NOT INTEGER
4d TFT
3d TOP
DIRICHLET
2d HOL
KAC-MOODY
RATIONAL VOA
WZW VOA
AT LEVEL K
NON-PERT