

Title: Lecture 2: Supersymmetric Field Theory and Topological Twists

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Collection: QFT for Mathematicians

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$d=2$ $N=(2,2)$ SUSY σ -model

— top. twist (A/B-model)

— BV formalism

$$V^{N=(2,2)} = \mathbb{R}^{1,1} \times \text{TT} R_+^{\oplus 2} \times \text{TT} R_-^{\oplus 2}$$

$$= \mathbb{R}^{1,1} \times \text{TT} S_+ \times \text{TT} S_-$$

$$Q_{\pm} = Q_{\pm}^1 + i Q_{\pm}^2$$

$$\bar{Q}_{\pm} = Q_{\pm}^1 - i Q_{\pm}^2$$

↑
complex spinor

Susy

$$[Q_{\pm}, \bar{Q}_{\pm}] = -2i \partial_{\pm} = H_{\pm} P$$

$$x^{\pm} = x^0 \pm x^1 \quad \partial_{\pm} = \frac{1}{2} \left(\frac{\partial}{\partial x^0} \pm \frac{\partial}{\partial x^1} \right)$$

Coordinates $V^{N=(1,2)}$: even x^0, x^1 odd $\theta^{\pm}, \bar{\theta}^{\pm}$

Susy diff operators:

$$\begin{cases} Q_{\pm} = \partial_{\theta^{\pm}} - i \bar{\theta}^{\pm} \partial_{\pm} \\ \bar{Q}_{\pm} = \partial_{\bar{\theta}^{\pm}} - i \theta^{\pm} \partial_{\pm} \end{cases} \quad \begin{cases} D_{\pm} = \partial_{\theta^{\pm}} + i \bar{\theta}^{\pm} \partial_{\pm} \\ \bar{D}_{\pm} = \partial_{\bar{\theta}^{\pm}} + i \theta^{\pm} \partial_{\pm} \end{cases}$$

$$[Q, D] = 0$$

Def'n A superfield Φ is called

Chiral

$$\bar{D}_\pm \Phi = 0$$

Observe

$$\bar{D}_\pm (0^\pm) = 0 \quad \bar{D}_\pm (y^\pm) = 0$$

$$(y^\pm = x^\pm + i\bar{\theta}^\pm \theta^\pm)$$

$$\Phi = \Phi(y^\pm, \theta^\pm) = \phi(y^\pm) + \theta^\pm \psi_\pm(y) + \theta^+ \theta^- F(y)$$

Chiral multiplet: $\underline{\Phi} = (\phi, \psi_\pm, F)$

Two types of SUSY inv. action

S_0 is SUSY invariant

① D-term Let $K(z^i, \bar{z}^{\bar{i}})$

be a smooth function on \mathbb{C}^n
(Kähler potential)

$$S_0 = \int d^4x \int d^2\theta d^2\bar{\theta} K(\Phi^i, \bar{\Phi}^{\bar{i}})$$

$$Q_{\pm} S_0 = \int d^4x \int d^2\theta d^2\bar{\theta} (\cancel{\partial_{\theta^{\pm}} - i\bar{\theta}^{\pm} \partial_{\pm}}) K = 0$$

② F-term

Let $W(z_i)$ be a hol. function

$$S_F = \int d^2x \int d^2\theta \left. W(\Phi^i) \right|_{\bar{\theta}=0} + c.c.$$

Gauging: observe \forall hol function $h(z_i)$

Consider $K \mapsto K + h(z_i) + \bar{h}(\bar{z}_i)$

$$\int d^2x \int d^2\theta d^2\bar{\theta} K = \int$$

ion
 + c.c.
 $\bar{\theta} = 0$

hol function $h(z^i)$

$$K + h(z^i) + \bar{h}(\bar{z}^i)$$

$$\int d^2x \int d^2\theta d^2\bar{\theta} K = \int d^2x \int d^2\theta d^2\bar{\theta} (K(\Phi^i, \bar{\Phi}^i) + h(\Phi^i) + \bar{h}(\bar{\Phi}^i))$$

only depends on $g_{i\bar{j}} = \partial_i \bar{\partial}_{\bar{j}} K$
 (Kähler metric)

glue to $\bar{\Phi}: V_{ch}^{N=(2,2)} \mapsto X$ Kähler

Two types of

Two types of SUSY in a worldsheet

+ glue on the worldsheet

$$\underline{\Phi}^i = (\phi^i, \psi_{\pm}^i, F^i)$$

$$\phi^i : \Sigma \longrightarrow X$$

$$\begin{cases} \psi_{\pm}^i \in \Gamma(\Sigma, S_{\pm} \otimes \phi^* T_X^{1,0}) \\ \bar{\psi}_{\pm}^i \in \Gamma(\Sigma, S_{\pm} \otimes \phi^* T_X^{0,1}) \end{cases}$$

$N=2,2$ SUSY invariant

$$\bar{\Phi} : V_{ch}(\Sigma) \longrightarrow X$$

Kähler

$\langle \bar{\Phi}^i, \bar{\Phi}^j \rangle$
 $\bar{h}(\bar{\Phi}^i)$
 K
 or metric)
 X Kähler

Two types of SUSY

- glue on the worldsheet

$$\underline{\Phi}^i = (\phi^i, \psi_{\pm}^i, F^i)$$

$$\phi^i : \Sigma \longrightarrow X$$

$$\left\{ \begin{array}{l} \psi_{\pm}^i \in \Gamma(\Sigma, S_{\pm} \otimes \phi^* T_X^{1,0}) \\ \bar{\psi}_{\pm}^i \in \Gamma(\Sigma, S_{\pm} \otimes \phi^* T_X^{0,1}) \end{array} \right.$$

$$D_{\mu} \psi_{\pm}^i = \partial_{\mu} \psi_{\pm}^i + (\partial_{\mu} \phi^j) \Gamma_{jk}^i \psi_{\pm}^k$$

N=1,2

$$\underline{\Phi} : V_{ch}(\Sigma) \longrightarrow X$$

Kähler

$$S_0(\underline{\Phi}) = \int d^2x \left[-\frac{1}{2} g_{i\bar{j}} \partial^{\mu} \phi^i \partial_{\mu} \bar{\phi}^{\bar{j}} + i g_{i\bar{j}} \bar{\psi}_{-}^{\bar{j}} D_{+} \psi_{-}^i + i g_{i\bar{j}} \bar{\psi}_{+}^{\bar{j}} D_{+} \psi_{+}^i + R_{i\bar{j}k\bar{l}} \bar{\psi}_{+}^{\bar{j}} \psi_{-}^k \bar{\psi}_{-}^{\bar{l}} \psi_{+}^i + g_{i\bar{j}} (F^i - \Gamma_{jk}^i \psi_{+}^j \psi_{-}^k) (\bar{F}^{\bar{j}} - \bar{\Gamma}_{\bar{k}\bar{l}}^{\bar{j}} \bar{\psi}_{-}^{\bar{k}} \bar{\psi}_{+}^{\bar{l}}) \right]$$

Topological Twist

$$\bar{\Phi}: V_{\text{ch}}^{N=(2,2)}(\Sigma) \rightarrow X$$

under suS^1 $\delta = \epsilon^\pm Q_\pm + \bar{\epsilon}^\pm \bar{Q}_\pm$

$\epsilon^\pm, \bar{\epsilon}^\pm$ sections of spin bundles.

$$\delta S = \int_\Sigma (\nabla_\mu \epsilon^\pm) G_\pm^\mu + (\nabla_\mu \bar{\epsilon}^\pm) \bar{G}_\pm^\mu$$

Global $\text{suS}^1 \rightsquigarrow \nabla \epsilon^\pm = 0$ covariant constant spinor

$$\int d^2x \int d^2\theta d^2\bar{\theta} K = \int d^2x \int d^2\theta$$

only depends on g .

glue to $\bar{\Phi}: V_{\text{ch}}^{N=(2,2)}$

Top twist

① Consider a theory w/ symmetry

$$\text{Spin}(V) \times G_R$$

(G_R : R-symmetry)

Choose a homomorphism

$$\rho: \text{Spin}(V) \rightarrow G_R$$

Two types of SUSY

- chiral

n bundles

Constant
norm

Top twist

① Consider a theory w/ symmetry

$$\text{Spin}(V) \times G_R$$

(G_R : R-symmetry)

Choose a homomorphism

$$P: \text{Spin}(V) \rightarrow G_R$$

② Consider the new Poincare symmetry

$$\text{Spin}(V) \xrightarrow{1 \times P} \text{Spin}(V) \times G_R \rightarrow S$$

③ Find $Q \in S$ such that

$Q \in$ trivial rep of $\text{Spin}(V)$
inside S

$$\text{and } [Q, Q] = 0$$

$Q \Rightarrow$ global SUSY

$$D_\mu \psi_\pm =$$

$$\rightarrow \Phi$$

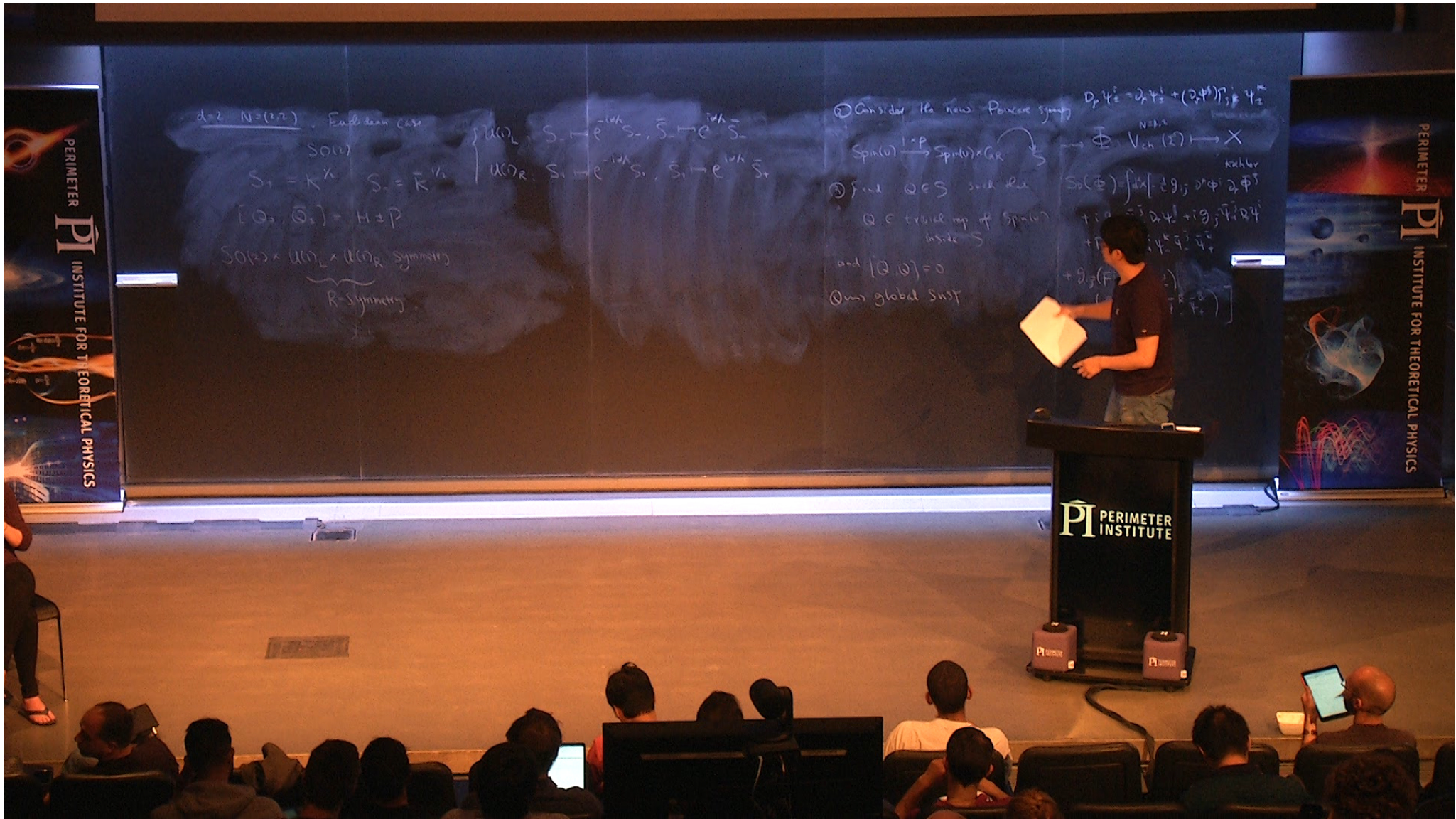
$$S_0(\Phi) =$$

$$+ i g_{ij} \bar{\psi}^i \psi^j$$

$$+ R_{ij} \bar{\psi}^i \psi^j$$

$$+ g_{ij} (\bar{F}^i - F^i) \psi^j$$

$$(F^i - \bar{F}^i)$$



$d=2$ $N=(2,2)$ Euclidean case

$$SO(2) \times U(1)_L \times U(1)_R \text{ symmetry}$$

$$S_+ = K^+ \quad S_- = \bar{K}^-$$

$$[Q_+, Q_-] = H \pm P$$

R-symmetry

Consider the new Poincare group $D_p \psi_\pm^2 = \partial_t \psi_\pm^2 + (2\psi_\pm^2) \partial_r \psi_\pm^2$

$$Spin(1,1) \xrightarrow{1 \rightarrow P} Spin(1,1) \times Gr \xrightarrow{S} \Phi \xrightarrow{N=2} V_{ch}(Z) \rightarrow X$$

Find $Q \in S$ such that $Q \in$ trivial rep of $Spin(1,1)$ inside S

and $[Q, H] = 0$

$Q \rightarrow$ global SUSY

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$d=2$ $N=(2,2)$ Euclidean case

$SO(2)$

$S_+ = K^X \quad S_- = \bar{K}^Y$

$[Q_+, \bar{Q}_+] = H \pm P$

$SO(2) \times U(1)_L \times U(1)_R$ symmetry

R-symmetry

	S_+	S_-	\bar{S}_+	\bar{S}_-
$SO(2) \times U(1)_L \times U(1)_R$	$(\frac{1}{2}, 0, -\frac{1}{2})$	$(\frac{1}{2}, -\frac{1}{2}, 0)$	$(\frac{1}{2}, 0, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2}, 0)$

$U(1)_L: S_+ \mapsto e^{-i\alpha} S_+, \bar{S}_- \mapsto e^{i\alpha} \bar{S}_-$
 $U(1)_R: S_- \mapsto e^{i\alpha} S_-, \bar{S}_+ \mapsto e^{-i\alpha} \bar{S}_+$
 (I) A-twist
 (A) $SO(2) \mapsto U(1)_L \times U(1)_R$
 $e^{\alpha} \mapsto e^{i\alpha} \cdot e^{i\alpha}$

Consider the non-compact group $D_p \mathbb{P}^2 = \mathbb{R}^2 \times \mathbb{P}^1 + (0, \psi^i) \mathbb{P}^1 \times \mathbb{P}^1$
 $Spin(4) \xrightarrow{1 \rightarrow P} Spin(3) \times \mathbb{R} \xrightarrow{N=2} \Phi: V_{ch}(2) \rightarrow X$
 Find QES such that $Q \in$ trivial rep of $Spin(4)$ inside S
 and $[Q, \bar{Q}] = 0$
 $Q \mapsto$ global SWT
 $S_+(\Phi) = \int d^2x [\frac{1}{2} g_{ij} \partial^i \phi^j \partial^k \phi^l \partial^m \phi^n]$
 $\partial_j \bar{\psi}^i \partial_k \psi^j + i \partial_j \bar{\psi}^i \partial_k \psi^j$
 $\partial_j \psi^i \partial_k \bar{\psi}^j + i \partial_j \psi^i \partial_k \bar{\psi}^j$
 $\partial_j \psi^i \partial_k \bar{\psi}^j + i \partial_j \psi^i \partial_k \bar{\psi}^j$

$$\begin{aligned} \rightarrow e^{-i\alpha/2} S_-, \bar{S}_- &\mapsto e^{i\alpha/2} S_- \\ \rightarrow e^{-i\alpha/2} S_+, \bar{S}_+ &\mapsto e^{i\alpha/2} \bar{S}_+ \end{aligned}$$

A-twist

$$\begin{aligned} SO(2) &\mapsto U(1)_L \times U(1)_R \\ e^{i\alpha} &\mapsto e^{i\alpha} \times e^{i\alpha} \end{aligned}$$

-twist	S_+	S_-	\bar{S}_+	\bar{S}_-
$U(1)_L \times U(1)_R$	$(0, 0, -\frac{1}{2})$	$(-1, -\frac{1}{2}, 0)$	$(1, 0, \frac{1}{2})$	$(0, \frac{1}{2}, 0)$

the SUSY operators Q_+, \bar{Q}_-
survives globally

$$Q_A = Q_+ + \bar{Q}_-$$

After the twist

$$\begin{cases} \psi_+^i \in \Gamma(\Sigma, \phi^* T_x^{1,0}) \\ \bar{\psi}_+^{\bar{i}} \in \Gamma(\Sigma, K \otimes \phi^* T_x^{0,1}) \\ \psi_-^i \in \Gamma(\Sigma, \bar{K} \otimes \phi^* T_x^{1,0}) \\ \bar{\psi}_-^{\bar{i}} \in \Gamma(\Sigma, \phi^* T_x^{0,1}) \end{cases}$$

$D_\mu \psi_\pm =$
 $\rightarrow \Phi:$
 $S_0(\Phi) =$
 $+ i g_{i\bar{j}} \bar{\psi}^{\bar{i}} \psi^j$
 $+ R_{i\bar{j}k\bar{l}} \bar{\psi}^{\bar{i}} \psi^j \bar{\psi}^{\bar{k}} \psi^l$
 $+ g_{i\bar{j}} (F^i - F^{\bar{j}})$
 $(\bar{F}^{\bar{i}} - F^i)$

$$S_A = Q_A \int d^2z V + \int_{\Sigma} \phi^* \omega$$

$$V = g_{i\bar{j}} (\psi_+^{\bar{j}} \partial_z \phi^i + \psi_-^i \partial_{\bar{z}} \bar{\phi}^{\bar{j}})$$

$SO(2) \times U(1)_L \times U(1)_R$ symmetry

R-symmetry

	S_+	S_-	\bar{S}_+	\bar{S}_-
$SO(2) \times U(1)_L \times U(1)_R$	$(\frac{1}{2}, 0, -\frac{1}{2})$	$(-\frac{1}{2}, -\frac{1}{2}, 0)$	$(\frac{1}{2}, 0, \frac{1}{2})$	$(-\frac{1}{2}, \frac{1}{2}, 0)$

$$\left. \begin{array}{l} U(1)_L: S_- \mapsto e^{-i\alpha/2} S_-, \bar{S}_- \mapsto e^{i\alpha/2} \bar{S}_- \\ U(1)_R: S_+ \mapsto e^{-i\alpha/2} S_+, \bar{S}_+ \mapsto e^{i\alpha/2} \bar{S}_+ \end{array} \right\}$$

(II) B-twist

$$\rho_A: SO(2) \mapsto U(1)$$

$$e^{i\alpha} \mapsto e^{i\alpha}$$

A-twist	(S_+)
$SO(2) \times U(1)_L \times U(1)_R$	$(0, 0, -\frac{1}{2})$ $(-1, -\frac{1}{2})$

$$\begin{aligned} \bar{S}_- &\mapsto e^{i\alpha/2} \bar{S}_- \\ \bar{S}_+ &\mapsto e^{i\alpha/2} \bar{S}_+ \end{aligned}$$

$$\begin{aligned} &\times U(1)_R \\ &\times e^{-i\alpha} \end{aligned}$$

S_-	\bar{S}_+	\bar{S}_-
$(1, -\frac{1}{2}, 0)$	$(0, 0, \frac{1}{2})$	$(0, \frac{1}{2}, 0)$

The SUSY operators \bar{Q}_+, \bar{Q}_-

$$Q_B = \bar{Q}_+ + \bar{Q}_-$$

$$D_\mu \psi_\pm^i = \partial_\mu \psi_\pm^i + (\partial_\nu \phi^j) \Gamma_{jk}^i \psi_\pm^k$$

$N=1, 2$

$$\Phi: V_{ch}(\Sigma) \mapsto X$$

$$S_0(\Phi) = \int d^4x \left[-\frac{1}{2} g_{ij} \partial^\mu \phi^i \partial_\mu \phi^j \right.$$

$$+ i g_{ij} \bar{\psi}_-^j D_\mu \psi_-^i + i g_{ij} \bar{\psi}_+^j D_\mu \psi_+^i$$

$$+ R_{ijkl} \bar{\psi}_+^i \psi_-^k \bar{\psi}_-^j \psi_+^l$$

$$+ g_{ij} (F^i - \Gamma_{jk}^i \psi_+^j \psi_-^k)$$

$$\left(\bar{F}^j - \bar{\Gamma}_{kl}^j \bar{\psi}_-^k \bar{\psi}_+^l \right)$$

Let $(X, dx), (Y, dy)$ be two dg space, $(D_x, dx), (D_Y, dy)$ dga

X is equipped w/ $\int D_x \mapsto \mathbb{C}$ of $\text{deg} = -k$
 Y is equipped w/ a symplectic form of $\text{deg} = k-1$

$\Rightarrow \text{map}(X, Y)$ is (-1) -shifted symplectic

$dx, dy \rightsquigarrow \{S_{X, -}\}, \{S_{Y, -}\}$

$S = S_X + S_Y$ satisfies $\{S, s\} = 0$
 (CME)

$$\left. \begin{array}{l} U(1)_L : S_- \mapsto e^{-i\alpha/2} S_-, \bar{S}_- \mapsto e^{i\alpha/2} \bar{S}_- \\ U(1)_R : S_+ \mapsto e^{-i\alpha/2} S_+, \bar{S}_+ \mapsto e^{i\alpha/2} \bar{S}_+ \end{array} \right\}$$

(II) B-twist

$$\rho_A : SO(k) \mapsto U(1)_L \times U(1)_R$$

$$e^{i\alpha} \mapsto e^{i\alpha} \times e^{i\alpha}$$

B-twist	S_+	S_-
$SO(2) \times U(1)_L \times U(1)_R$	$(1, 0, -\frac{1}{2})$	$(-1, -\frac{1}{2}, 1)$

be two
 $(Q_X, d_X), (Q_Y, d_Y)$ dga

$\int \mathcal{O}_X \rightarrow \mathbb{C}$ of $\text{deg} = -k$

a symplectic form of $\text{deg} = k-1$

$P(X, Y)$ is (-1) -shifted symplectic

$\rightsquigarrow \{S_X, \cdot\}, \{S_Y, \cdot\}$

$S_X + S_Y$ satisfies $\{S, s\} = 0$
 (CME)

Example

Poisson G-model

① $T[1]\Sigma = (\Sigma, \Omega(\Sigma)) \quad (= \Sigma_{\downarrow R})$

$\mathcal{O}(T[1]\Sigma) = \Omega(\Sigma) \xrightarrow{\int} \mathbb{C} \quad \text{deg} = -2$

(Σ : surface)

② $T^*[1]X = (X, PV(X)) \quad PV(X) = \Lambda^* T^*X$

symplectic of $\text{deg} = 1$

locally (X^I, θ_I)

$\omega = dx^I \wedge d\theta_I$

σ -model

$$(\Sigma, \Omega(\Sigma)) \quad (= \Sigma_{\downarrow \mathbb{R}})$$

$$\Omega(\Sigma) \xrightarrow{\int} \mathbb{C} \quad \text{deg} = -2$$

$$x, PV(x)$$

$$PV(x) = \wedge^1 T_x$$

of deg = 1

$$(\theta_I)$$

$$dx^I \wedge d\theta_I$$

$$\text{Let } P = p^{IJ}(x) \partial_{x^I} \wedge \partial_{x^J}$$

Poisson tensor:

$$= p^{IJ}(x) \theta_I \theta_J$$

$$\{P, P\} = 0$$

$$\{P, -\} \curvearrowright \mathcal{O}(T^*(\Sigma, X))$$

Consider

$$\boxed{\text{map}(T(\Sigma), T^*(\Sigma, X))}$$

locally

Let $P = p^{IJ}(x) \partial_{x^I} \wedge \partial_{x^J}$
 Poisson tensor

$$= p^{IJ}(x) \theta_I \theta_J$$

$$\{P, P\} = 0$$

$$\{P, -\} \curvearrowright O(T^*(\Sigma, X))$$

Consider

$$\text{map}(T(\Sigma), T^*(\Sigma, X))$$

locally φ^I, η_I forms on Σ

(-1)-symplectic pairing

$$\langle \varphi^I, \eta_I \rangle = \int_{\Sigma} \varphi^I \wedge \eta_I$$

$$S = \int_{\Sigma} d\varphi^I \wedge \eta_I + p^{IJ}(\varphi) \eta_I \wedge \eta_J$$

$$\{S, S\} = 0$$

In BV formalism

\mathcal{E} fields (-1) -symplectic

S action solving $\Delta(e^{S/\hbar}) = 0$

Choose $\mathcal{L} \subset \mathcal{E}$ super Lagrangian submanifold (UME)

$\int_{\mathcal{L}} e^{-S/\hbar}$ is indep. of continuous def

of \mathcal{L}
(gauge fixing)

A-model

(X, ω) Ka

map $(T[\Sigma], T^*[\Sigma] \times X)$

A-model

(X, ω) Kähler

φ^i, η_i + c.c.
forms on Σ

$$\text{map}(T[\cdot]\Sigma, T^*[\cdot]X)$$

$\omega \rightsquigarrow$ Poisson tensor $\omega^{i\bar{j}}$

$$S_A = \int \omega^{i\bar{j}}(\varphi) \eta_i \eta_{\bar{j}} = \int \omega^{i\bar{j}}$$

Complex structure \rightsquigarrow Lagrangian

model (X, ω) Kähler

$$\text{map}(T[\mathbb{C}]\Sigma, T^*[\mathbb{C}]X)$$

$\omega \rightsquigarrow$ Poisson tensor $\omega^{i\bar{j}}$

$$\int \omega^{i\bar{j}}(\varphi) \eta_i \bar{\eta}_{\bar{j}} = \int \omega^{i\bar{j}}(\varphi) \eta_i \bar{\eta}_{\bar{j}}$$

Complex structure \rightsquigarrow Lagrangian submfld.

$\varphi^i, \eta_i + c.c.$
forms on Σ

$$\varphi^i = (\varphi_0^i, \varphi_z^i, \varphi_{\bar{z}}^i, \varphi_{z\bar{z}}^i)$$

$\varphi_0^i, \varphi_{\bar{z}}^i, \varphi_z^i, \varphi_{z\bar{z}}^i$

$\eta_{i, z\bar{z}}, \eta_{i, z}, \eta_{i, \bar{z}}$

$$\mathcal{M} = \left(\begin{array}{c} \mathbb{C} \\ +c.c. \end{array} \text{ fields} \right)$$

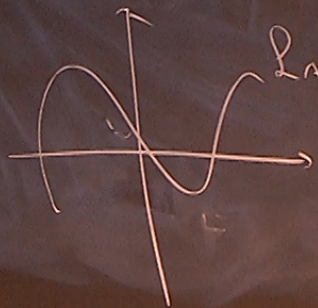
$$\text{map}(T[\mathbb{R}]\Sigma, T^*[\mathbb{R}]X) = T^*[\mathbb{R}]\mathcal{M}$$

$$\text{Consider } \mathcal{I}_h = \int_{\Sigma} d^2z \left(\bar{\psi}_z^i \partial_z \phi_0^i + \psi_z^i \partial_z \bar{\phi}_0^i \right) g_{i\bar{j}}$$

$$\in \mathcal{O}(\mathcal{M})$$

$$\text{Consider } \mathcal{L}_A = \text{Graph}(d\mathcal{I}_h) \subset T^*[\mathbb{R}]\mathcal{M}$$

Check: $S_A|_{\mathcal{L}_A} = A\text{-model action}$



A-model

(X, ω)

$$\text{map}(T[\mathbb{R}]\Sigma, T^*[\mathbb{R}]X)$$

$\omega \rightsquigarrow$ Poisson tensor

$$S_A = \int \omega^{i\bar{j}}(\varphi) \eta_{i\bar{j}}$$

Complex structure \rightsquigarrow \mathcal{L}_A

fields)

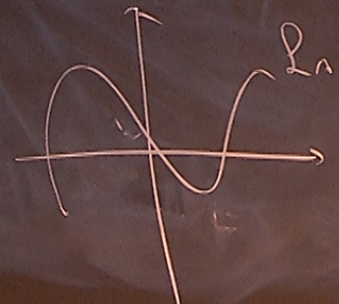
$$\Gamma(X) = T^*[1]M$$

$$\partial_z (\bar{\psi}_z^j \bar{\partial}_z \phi_0^i + \psi_z^i \partial_z \phi_0^j) g_{i\bar{j}}$$

$$\in \mathcal{O}(M)$$

$$\text{graph}(d\Gamma) \subset T^*[1]M$$

model action



B-model

$$\text{map}(T[1]\Sigma, T^*[1]T^{g_1}X)$$

locally

$$\left\{ \begin{array}{l} \phi^i \quad \bar{\phi}^{\bar{i}} \quad \eta^{\bar{i}} \\ \xi_i \quad \bar{\xi}_{\bar{i}} \quad \bar{u}_{\bar{i}} \end{array} \right. \quad \begin{array}{l} \nearrow \\ \searrow \end{array} \text{anti-fields}$$

(X, ω) Kähler

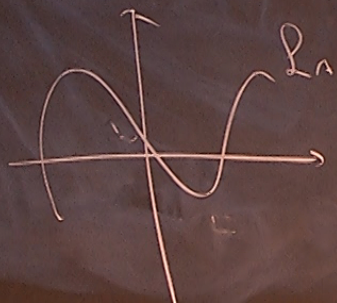
B-model

$T^*[1]M$

$$+ \varphi_z^i \partial_z \phi_0^j g_{i\bar{j}}$$

M)

$C T^*[1]M$



locally

$$S_B = \int_{\Sigma}$$

(X, ω) Kähler

$$\text{map}(T[1]\Sigma, T^*[1]T^*[0]X)$$

$d \uparrow$

$\bar{\partial} \uparrow$

$$\begin{cases} \phi^i & \bar{\phi}^{\bar{i}} & \eta^{\bar{i}} \\ \xi_i & \bar{\xi}_{\bar{i}} & \bar{u}_{\bar{i}} \end{cases} \quad \begin{matrix} \curvearrowright \\ \text{anti-fields} \end{matrix}$$

$$d\phi^i \wedge \xi_i + d\bar{\phi}^{\bar{i}} \wedge \bar{\xi}_{\bar{i}} + d\eta^{\bar{i}} \wedge \bar{u}_{\bar{i}}$$

$$+ \int_{\Sigma} \bar{\xi}_{\bar{i}} \eta^{\bar{i}}$$